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Bank Marketing Analysis

Designing a Telemarketing Strategy To Reduce Acquisition Costs

A bank sells a product (called term deposit) to prospects mainly through telemarketing. If a prospect customer buys the product, we say that he has 'responded'.

The aim of this analysis is to **reduce the marketing cost by atleast 50%** and acquire a comparable number of customers (say 80-90%).

We'll use *telemarketing data* from a past campaign of the bank. The sales team had recorded customer data like age, salary, whether he has a loan, house, the month of call etc.

The idea is to use machine learning to predict the likelihood of a person 'buying the product 'responding'. We'll identify those who are most likely to respond and telemarket only to them, thereby reducing the total cost of acquisition per customer.

The standard process followed in analytics projects is: 1. Business Understanding 2. Data Understanding

3. Modelling 4. Model Evaluation 5. Model Deployment and Recommendations

Business Understanding

The **overall goal** is to reduce telemarketing costs by about 50% and acquire atleast 80-90% of the customers.

The specific **objective of this analysis** is to build a **'response model'** to predict the likelihood of a prospect buying the product (or responding).

Data Understanding

The datafile is named bank-marketing.csv. You can download it here: https://archive.ics.uci.edu/ml/datasets/Bank+Marketing

```
## 'data.frame': 45211 obs. of 19 variables:
## $ age     : int 58 44 33 47 33 35 28 42 58 43 ...
## $ job            : Factor w/ 12 levels "admin.", "blue-collar", ..: 5 10 3 2 12 5
5 3 6 10 ...
## $ salary     : int 100000 60000 120000 20000 0 100000 120000 55000
60000 ...
```

```
## $ marital : Factor w/ 3 levels "divorced", "married",..: 2 3 2 2 3 2 3 1
2 3 ...
## $ education: Factor w/ 4 levels "primary", "secondary", ..: 3 2 2 4 4 3 3 3
1 2 ...
## $ targeted : Factor w/ 2 levels "no", "yes": 2 2 2 1 1 2 1 1 2 2 ...
## $ default : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 2 1 1 ...
## $ balance : int 2143 29 2 1506 1 231 447 2 121 593 ...
## $ housing : Factor w/ 2 levels "no", "yes": 2 2 2 2 1 2 2 2 2 2 ...
             : Factor w/ 2 levels "no", "yes": 1 1 2 1 1 1 2 1 1 1 ...
## $ loan
## $ contact : Factor w/ 3 levels "cellular", "telephone",..: 3 3 3 3 3 3
3 3 3 ...
## $ day
             : int 555555555...
## $ month
             : Factor w/ 12 levels "apr", "aug", "dec", ...: 9 9 9 9 9 9 9 9 9 9
9 ...
## $ duration : int 261 151 76 92 198 139 217 380 50 55 ...
## $ campaign : int 1 1 1 1 1 1 1 1 1 ...
## $ pdays
           : int -1 -1 -1 -1 -1 -1 -1 -1 -1 ...
## $ previous : int 0000000000...
## $ response : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 1 1 1 ...
```

We have 45211 observations, i.e. we have data of 45211 potential customers. There are 20 variables (or 20 columns). We have two types of variables:

- 1. Two type of attributes (or variables):
- **Customer attributes** like age, job, salary, marital (status), education (primary / secondary / tertiary) etc.
- **Bank related attributes** like targeted (whether he/she was targeted before), loan (yes / no), contact (whether he/she has been contacted before etc.)

The last attribute 'response' tells us whether the person had responded to the bank's marketing campaign. It thus contains only two values - Yes or No. It is called the **target attribute** since that is what we want to predict using the other attributes.

Data Cleaning

Missing Values and Outliers

We always start with cleaning the data i.e. removing the missing values, any erroneous entries etc. Let's see if this data contains **missing values**.

```
sum(is.na(bank_data))
## [1] 0
```

We have simply summed up all the missing values (denoted as 'NA'). There are none of them, so we move ahead.

Next, we should ideally look at **outliers** in the data. Outliers are extreme values for which treatment is done so that the data only represents the general trends and ignores extreme cases. For example, a person having an income of Rs 20 lacs per month will be an outlier in this dataset.

For now, we will not do outlier treatment since this data doesn't have many of them.

Exploratory Analysis

In Exploratory Data Analysis, or EDA, we use plots and summaries of data to understand the patterns in it. Let's see some examples.

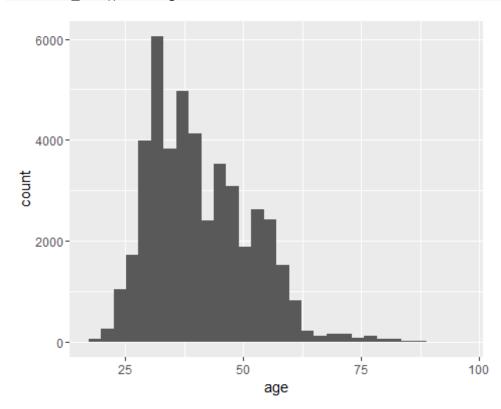
Univariate Analysis

In univariate analysis, we analyse one variable at a time.

The following plot shows the distribution of peoples' ages in the data. The ggplot library is a great data visualisation tool in R.

Since age is a numeric variable, we plot a **histogram**.

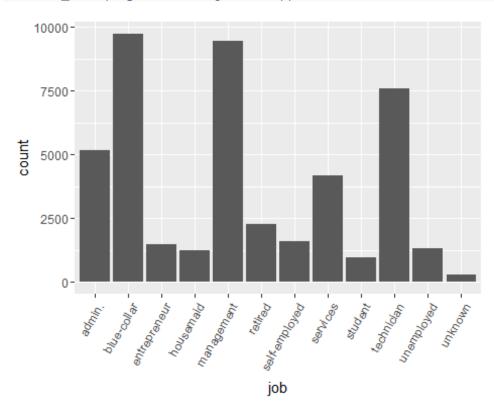
```
library(ggplot2)
ggplot(bank_data, aes(age)) + geom_histogram()
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



You can see that most people are between 25-50 years old. Very few people older than 60 years have been targeted.

Next, let's look at the types of jobs people have. Now *job* is not a numeric variable, it is a **categorical variable** and so we plot a **bar chart** for it.

```
ggplot(bank_data, aes(job)) + geom_bar() + theme(axis.text.x =
element_text(angle = 60, hjust = 1))
```



We have a large number of people with blue-collar jobs and in management, which are about 9000 each. The third highest category is technicians which are about 7500 people. Note that among 45,000 people, about 25,000 or 55% are either blue-collar workers, management employees technicians.

A very important variable for the bank would be **salary**. Let's have a look at the average and the median salary.

```
summary(bank_data$salary)
## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0 20000 60000 57010 70000 120000
```

The average salary is about INR 57000 per month. Note that the maximum is only about INR 1.2 lacs per month.

Let's now look at the summary of the target variable **response**.

```
summary(bank data$response)
```

So out of about 45,000 people, only 5289 had responded. The exact **response rate** can be calculated as:

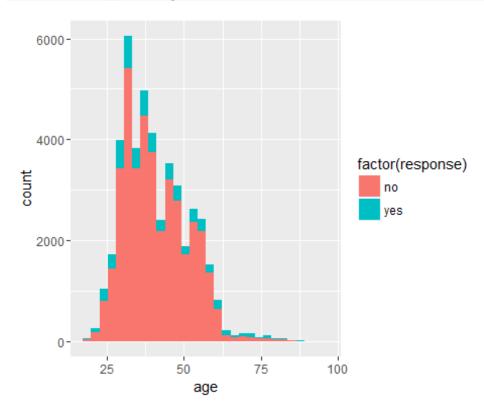
```
## [1] 0.1169848
```

An 11.7% response rate means that if the you make 100 calls to market the product (term deposit), about 11.7% will subscribe for a term-deposit.

Multivariate Analysis

Now let's analyse two variables at a time, one of which should obviously be the target variable 'response'.

```
ggplot(bank_data, aes(age, fill = factor(response))) + geom_histogram()
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



So now the plot shows information of two variables - the age on x-axis and response as a colour. This chart does not show any obvious trend of response rate with age.

The analysis will be easier if we could divide the age into **buckets**, e.g. 0-10 years, 10-20 years etc. This is called bucketing and is often done to divide **numeric variables** into smaller buckets.

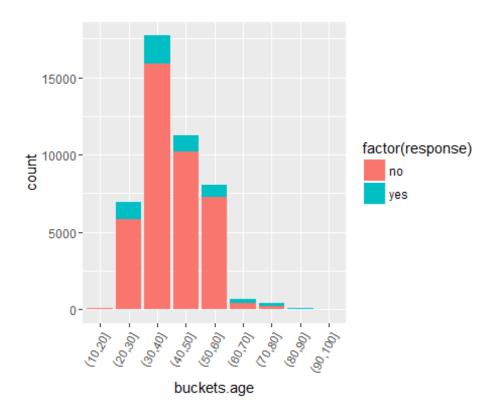
```
bank_data$buckets.age <- cut(bank_data$age, breaks = c(10, 20, 30, 40, 50, 60, 70, 80, 90, 100))
str(bank_data)</pre>
```

```
## 'data.frame': 45211 obs. of 20 variables:
## $ age
                : int 58 44 33 47 33 35 28 42 58 43 ...
## $ job
                : Factor w/ 12 levels "admin.", "blue-collar", ...: 5 10 3 2 12
5 5 3 6 10 ...
## $ salary
                : int 100000 60000 120000 20000 0 100000 100000 120000
55000 60000 ...
               : Factor w/ 3 levels "divorced", "married", ...: 2 3 2 2 3 2 3
## $ marital
1 2 3 ...
## $ education : Factor w/ 4 levels "primary", "secondary", ...: 3 2 2 4 4 3 3
3 1 2 ...
## $ targeted : Factor w/ 2 levels "no", "yes": 2 2 2 1 1 2 1 1 2 2 ...
                : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 2 1 1 ...
## $ default
## $ balance
               : int 2143 29 2 1506 1 231 447 2 121 593 ...
## $ housing
               : Factor w/ 2 levels "no", "yes": 2 2 2 2 1 2 2 2 2 2 ...
                : Factor w/ 2 levels "no", "yes": 1 1 2 1 1 1 2 1 1 1 ...
## $ loan
               : Factor w/ 3 levels "cellular", "telephone", ...: 3 3 3 3 3 3
## $ contact
3 3 3 3 ...
## $ day
                : int 555555555...
                : Factor w/ 12 levels "apr", "aug", "dec", ...: 9 9 9 9 9 9 9 9
## $ month
99 ...
## $ duration : int 261 151 76 92 198 139 217 380 50 55 ...
## $ campaign : int 1 1 1 1 1 1 1 1 1 ...
## $ pdays : int -1 -1 -1 -1 -1 -1 -1 -1 ...
## $ previous
                : int 0000000000...
## $ poutcome : Factor w/ 4 levels "failure", "other", ...: 4 4 4 4 4 4 4 4 4 4
4 ...
## $ response : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 1 1 1 ...
## $ buckets.age: Factor w/ 9 levels "(10,20]","(20,30]",..: 5 4 3 4 3 3 2 4
5 4 ...
sum(is.na(bank_data))
## [1] 0
```

Note that a new variable named *buckets.age* is now added to bank_data. Ages are now bucketed into this variable.

Let's use the buckets to see if age affects the response rate.

```
ggplot(bank_data, aes(buckets.age, fill = factor(response))) + geom_bar() +
theme(axis.text.x = element text(angle = 60, hjust = 1))
```



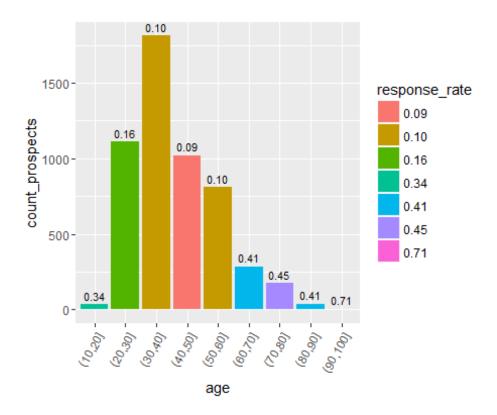
It will be easier if we could see the response rate in numbers as well. We can convert the 'yes-no' values to '1-0' respectively and then calculate the response rate by summing up the 1s.

```
bank_data$response.numeric <- ifelse(bank_data$response == "yes", 1, 0)</pre>
str(bank_data)
## 'data.frame':
                    45211 obs. of 21 variables:
                              58 44 33 47 33 35 28 42 58 43 ...
## $ age
## $ job
                       : Factor w/ 12 levels "admin.", "blue-collar", ...: 5 10 3
2 12 5 5 3 6 10 ...
                            100000 60000 120000 20000 0 100000 100000 120000
## $ salary
                       : int
55000 60000 ...
                       : Factor w/ 3 levels "divorced", "married", ...: 2 3 2 2 3
## $ marital
2 3 1 2 3 ...
                      : Factor w/ 4 levels "primary", "secondary", ...: 3 2 2 4
## $ education
4 3 3 3 1 2 ...
## $ targeted
                      : Factor w/ 2 levels "no", "yes": 2 2 2 1 1 2 1 1 2 2
                       : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 2 1 1
##
   $ default
. . .
                      : int 2143 29 2 1506 1 231 447 2 121 593 ...
##
    $ balance
    $ housing
                       : Factor w/ 2 levels "no", "yes": 2 2 2 2 1 2 2 2 2 2
##
                       : Factor w/ 2 levels "no", "yes": 1 1 2 1 1 1 2 1 1 1
##
   $ loan
## $ contact
                      : Factor w/ 3 levels "cellular", "telephone", ...: 3 3 3 3
```

```
3 3 3 3 3 ...
## $ day
                    : int 555555555...
## $ month
                    : Factor w/ 12 levels "apr", "aug", "dec", ...: 9 9 9 9 9 9
9 9 9 9 ...
## $ duration
                    : int 261 151 76 92 198 139 217 380 50 55 ...
## $ campaign
                   : int 111111111...
## $ pdays
                   : int -1 -1 -1 -1 -1 -1 -1 -1 -1 ...
## $ previous
                   : int 0000000000...
                   : Factor w/ 4 levels "failure", "other", ...: 4 4 4 4 4 4
## $ poutcome
4 4 4 4 ...
## $ response
                  : Factor w/ 2 levels "no", "yes": 1 1 1 1 1 1 1 1 1 1
## $ buckets.age : Factor w/ 9 levels "(10,20]","(20,30]",..: 5 4 3 4 3
3 2 4 5 4 ...
## $ response.numeric: num 0000000000...
```

We can now aggregate the 1s by each age bucket and see the response rates for each bucket.

```
agg_age <- merge(aggregate(response.numeric ~ buckets.age, bank_data, mean),</pre>
                 aggregate(response.numeric~buckets.age, bank data, sum),
                 by = "buckets.age")
colnames(agg_age) <- c("age", "response_rate", "count_prospects")</pre>
agg_age$response_rate <- format(round(agg_age$response_rate, 2))</pre>
agg_age
##
          age response_rate count_prospects
## 1 (10,20]
                       0.34
                                          33
## 2 (20,30]
                       0.16
                                        1112
## 3 (30,40]
                       0.10
                                        1812
## 4 (40,50]
                       0.09
                                        1019
## 5 (50,60]
                       0.10
                                         811
## 6 (60,70]
                       0.41
                                         284
## 7 (70,80]
                       0.45
                                         175
## 8 (80,90]
                       0.41
                                          38
## 9 (90,100]
                       0.71
                                           5
ggplot(agg_age, aes(age, count_prospects, fill = response_rate, label =
response rate)) + geom bar(stat = 'identity') + theme(axis.text.x =
element text(angle = 60, hjust = 1)) + geom text(size = 3, vjust = -0.5)
```



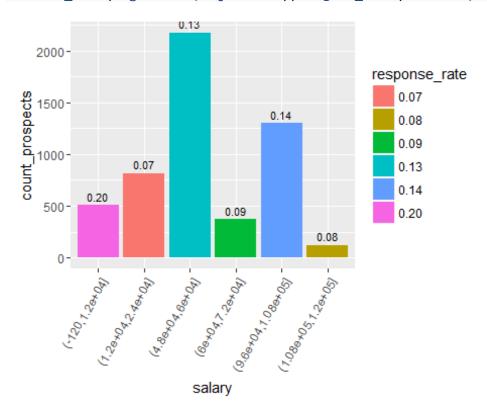
Note that although the bucket 10-20 has about 34% response rate, there are only a few prospects in that bucket (only 33, see code below) and thus we should ignore the bucket.

The buckets 20-30 has 16% response rate while 40-50 and 50-60 have low response rates around 10%.

Similarly, we can measure the response rate with salary and jobs.

```
bank data$buckets.salary <- cut(bank data$salary, breaks = 10)</pre>
agg_salary <- merge(aggregate(response.numeric ~ buckets.salary, data =</pre>
bank data, mean),
                     aggregate(response.numeric ~ buckets.salary, bank_data,
sum),
                                by = "buckets.salary")
colnames(agg_salary) <- c("salary", "response_rate", "count_prospects")</pre>
agg salary$response rate <- format(round(agg salary$response rate, 2))</pre>
agg_salary
##
                  salary response_rate count_prospects
## 1
         (-120,1.2e+04]
                                   0.20
                                                     505
## 2 (1.08e+05,1.2e+05]
                                   0.08
                                                     123
      (1.2e+04,2.4e+04]
                                                     817
## 3
                                   0.07
## 4
        (4.8e+04,6e+04]
                                   0.13
                                                    2174
        (6e+04,7.2e+04]
## 5
                                   0.09
                                                     369
## 6 (9.6e+04,1.08e+05]
                                   0.14
                                                    1301
```

```
ggplot(agg_salary, aes(salary, count_prospects, fill = response_rate, label =
response_rate)) + geom_bar(stat = 'identity') + theme(axis.text.x =
element_text(angle = 60, hjust = 1)) + geom_text(size = 3, vjust = -0.5)
```



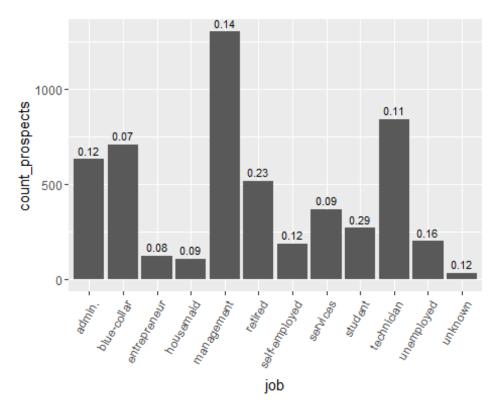
You can see that the response rate is highest for the lowest salary band (19.96%) while it contains 505 prospects. This might tell you something about the banking products you should be selling (which are used by people in this salary band.)

The number of prospects is highest in the fourth salary bucket at 2174 where the response rate is about 13.08% (higher than the average of 11.7%).

Let's also compare response rates across various jobs.

```
agg_job <- merge(aggregate(response.numeric~job, data = bank_data, mean),</pre>
                  aggregate(response.numeric~job, bank_data, sum),
                  by = "job")
colnames(agg_job) <- c("job", "response_rate", "count_prospects")</pre>
agg job$response rate <- format(round(agg job$response rate, 2))</pre>
agg_job
##
                 job response_rate count_prospects
              admin.
## 1
                               0.12
                                                 631
## 2
        blue-collar
                               0.07
                                                 708
## 3
       entrepreneur
                               0.08
                                                 123
## 4
          housemaid
                               0.09
                                                 109
## 5
         management
                               0.14
                                                1301
## 6
            retired
                               0.23
                                                 516
```

```
## 7
     self-employed
                              0.12
                                                187
## 8
           services
                              0.09
                                               369
## 9
            student
                              0.29
                                               269
## 10
         technician
                              0.11
                                               840
## 11
         unemployed
                              0.16
                                               202
## 12
            unknown
                              0.12
                                                34
ggplot(agg job, aes(job, count prospects, label = response rate)) +
geom_bar(stat = 'identity') + theme(axis.text.x = element_text(angle = 60,
hjust = 1) + geom_text(size = 3, vjust = -0.5)
```

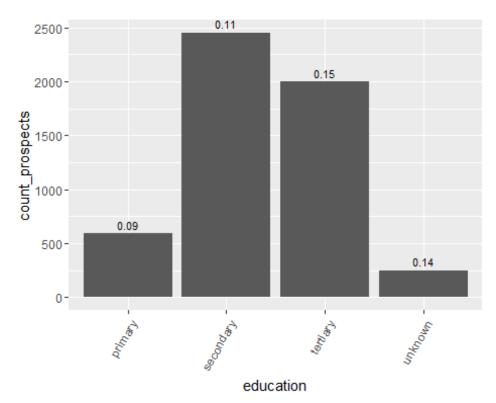


Interestingly, response rate is highest for students and second highest for retired people. It is quite low for blue-collar workers, housemaids and entrepreneurs.

Similarly, you can analyse response rates with other variables like education, marital status, loan etc. The plots below show how response rate varies with education, marital status, contact method and monh of contact.

```
## 2 secondary    0.11     2450
## 3 tertiary    0.15     1996
## 4 unknown    0.14     252

ggplot(agg_ed, aes(education, count_prospects, label = response_rate)) +
geom_bar(stat = 'identity') + theme(axis.text.x = element_text(angle = 60,
hjust = 1)) + geom_text(size = 3, vjust = -0.5)
```



```
plot_response <- function(cat_var, var_name){
    a <- aggregate(response.numeric~cat_var, bank_data, mean)
    b <- aggregate(response.numeric~cat_var, bank_data, sum)

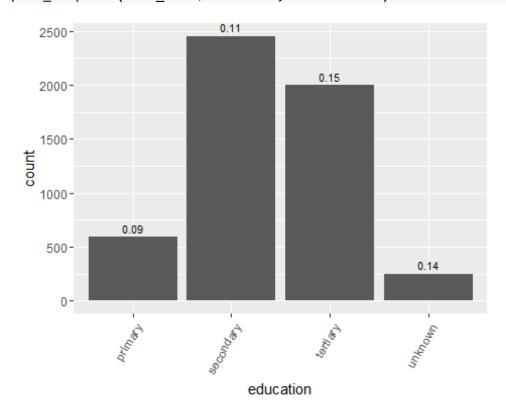
colnames(a)[1] <- var_name
    colnames(b)[1] <- var_name
    agg_response <- merge(a, b, by = var_name)

colnames(agg_response) <- c(var_name, "response_rate","count")
    agg_response[, 2] <- format(round(agg_response[, 2], 2))

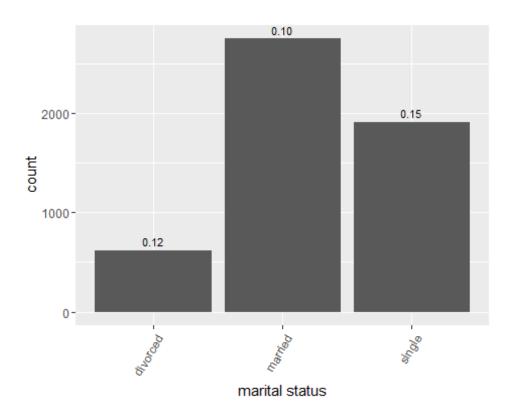
ggplot(agg_response, aes(agg_response[, 1], count, label = response_rate))
+ geom_bar(stat = 'identity') + theme(axis.text.x = element_text(angle = 60, hjust = 1)) + geom_text(size = 3, vjust = -0.5) + xlab(var_name)

# return(agg_response)
}</pre>
```

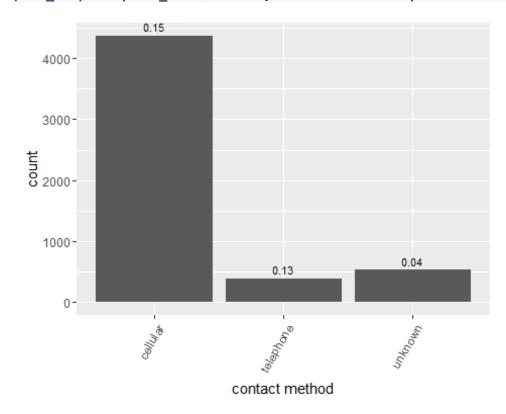
plot_response(bank_data\$education, "education")



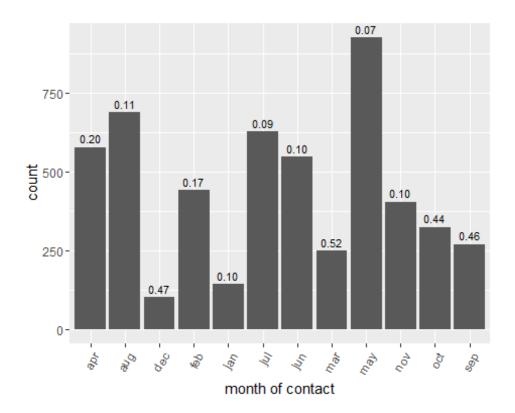
plot_response(bank_data\$marital, "marital status")



plot_response(bank_data\$contact, "contact method")



plot_response(bank_data\$month, "month of contact")



- From the plots shown above, we can draw the following observations:
- **Education**: Response rate is only 9% for primary education, 11% for secondary and 15% for tertiary; implying that education clearly plays a crucial role in predicting response
- __Marital Status_: Response rate is 15% for single prospects and 10% for married
- **Contact method**: Response rate is 15% and 13% for cellular and telephonic contact methods, respectively
- **Month of contact**: Interestingly, response rate varies drastically with month of contact; it is 52% in March, 47% in December and only 7% in May

We now have a decent understanding of the variable which are important in predicting response.

But this way, we can only only analyse the effect of each variable separately. We saw that multiple attributes like month of contact, marital status etc. affect the reponse rate. How do we analyse the *combined effect* of the variables? Also, how can we know which variables affect response rate more than others?

This is why we need machine learning **models**. We'll see that in the next section.

Modelling

Let's now build some machine learning models to predict the type of potential customers who are more likely to respond.

To build machine learning models, we use only a part of the data to train the model. This is called **training data**.

Rest of the data is used to test or evaluate the model, which is called **test data**.

We'll use 70% data to train the model and the rest 30% to test it.

```
Data Preparation
```

```
library(caret)
## Loading required package: lattice
library(caTools)
library(dummies)
## dummies-1.5.6 provided by Decision Patterns
bank_data <- bank_data[, -c(20, 21, 22)]
#creating dummy variables
bank data$response <- as.integer(bank_data$response)</pre>
bank data <- dummy.data.frame(bank data)</pre>
bank data$response <- as.factor(ifelse(bank data$response == 1, "no", "yes"))</pre>
# splitting into train and test data
set.seed(1)
split indices <- sample.split(bank data$response, SplitRatio = 0.70)</pre>
train <- bank data[split indices, ]</pre>
test <- bank data[!split indices, ]</pre>
nrow(train)/nrow(bank_data)
## [1] 0.6999845
nrow(test)/nrow(bank_data)
## [1] 0.3000155
```

Model 1: Logistic Regression

Let's build the first model - **logistic regression**.

```
library(MASS)
library(car)
logistic_1 <- glm(response ~ ., family = "binomial", data = train)</pre>
summary(logistic 1)
##
## Call:
## glm(formula = response ~ ., family = "binomial", data = train)
## Deviance Residuals:
##
       Min
                       Median
                                     30
                                             Max
                  10
## -4.7547
           -0.3728
                      -0.2496
                               -0.1458
                                          3.4916
## Coefficients: (11 not defined because of singularities)
##
                         Estimate Std. Error z value Pr(>|z|)
                                   3.990e-01 -11.170
## (Intercept)
                       -4.457e+00
                                                       < 2e-16 ***
                                               -0.928
## age
                       -2.450e-03
                                   2.641e-03
                                                        0.35354
                                                1.323
## jobadmin.
                        3.545e-01
                                   2.679e-01
                                                        0.18579
  `jobblue-collar`
                                                0.078
                        2.078e-02
                                   2.673e-01
                                                        0.93802
## jobentrepreneur
                                               -0.232
                       -6.857e-02
                                   2.951e-01
                                                        0.81627
## jobhousemaid
                       -2.042e-01
                                   2.988e-01
                                               -0.684
                                                        0.49427
## jobmanagement
                        2.123e-01
                                   2.659e-01
                                                0.799
                                                        0.42448
## jobretired
                        5.813e-01
                                   2.727e-01
                                                2.132
                                                        0.03304 *
## `jobself-employed`
                        2.009e-01
                                   2.833e-01
                                                0.709
                                                        0.47819
## jobservices
                        6.568e-02
                                   2.735e-01
                                                0.240
                                                        0.81021
## jobstudent
                        7.528e-01
                                                2.665
                                                        0.00769 **
                                   2.825e-01
## jobtechnician
                        1.799e-01
                                    2.659e-01
                                                0.677
                                                        0.49866
## jobunemployed
                        1.390e-01
                                    2.852e-01
                                                0.487
                                                        0.62599
## jobunknown
                               NA
                                           NA
                                                   NA
                                                             NA
## salary
                               NA
                                           NA
                                                   NA
                                                             NΑ
## maritaldivorced
                       -4.294e-02
                                   8.107e-02
                                               -0.530
                                                        0.59636
## maritalmarried
                       -2.074e-01
                                   6.652e-02
                                               -3.118
                                                        0.00182 **
## maritalsingle
                               NA
                                           NA
                                                   NA
                                                             NA
## educationprimary
                       -2.118e-01
                                    1.511e-01
                                               -1.402
                                                        0.16080
## educationsecondary -9.074e-02
                                               -0.640
                                   1.418e-01
                                                        0.52218
## educationtertiary
                        8.320e-02
                                   1.224e-01
                                                0.680
                                                        0.49654
## educationunknown
                               NA
                                           NA
                                                   NA
                                                             NA
## targetedno
                        8.358e-03
                                    9.161e-02
                                                0.091
                                                        0.92731
## targetedyes
                               NA
                                           NA
                                                   NA
                                                             NA
## defaultno
                       -8.588e-03
                                    1.928e-01
                                               -0.045
                                                        0.96447
## defaultves
                               NA
                                           NA
                                                   NA
                                                             NA
## balance
                        1.529e-05
                                    6.019e-06
                                                2.539
                                                        0.01111 *
                                               14.500
                                                        < 2e-16
## housingno
                        7.693e-01
                                    5.306e-02
## housingyes
                               NA
                                                    NA
                                           NA
                                                             NΑ
## loanno
                                                5.067 4.04e-07 ***
                        3.636e-01
                                   7.175e-02
## loanyes
                               NA
                                           NA
                                                   NA
                                                             NA
## contactcellular
                        1.643e+00
                                   8.798e-02
                                               18.670
                                                        < 2e-16 ***
## contacttelephone
                        1.509e+00
                                    1.210e-01
                                               12.475
                                                        < 2e-16
## contactunknown
                               NA
                                           NA
                                                   NA
                                                             NA
                                                        0.00757 **
## day
                        7.950e-03
                                   2.977e-03
                                                2.671
```

```
1.425e-01 -5.745 9.21e-09 ***
## monthapr
                      -8.189e-01
                                  1.372e-01 -11.149 < 2e-16 ***
## monthaug
                      -1.530e+00
## monthdec
                      -1.471e-01
                                  2.332e-01
                                             -0.631
                                                      0.52825
## monthfeb
                                  1.441e-01
                                             -6.562 5.31e-11 ***
                      -9.456e-01
                                  1.817e-01 -11.686 < 2e-16 ***
## monthjan
                      -2.124e+00
## monthjul
                      -1.659e+00
                                  1.406e-01 -11.798
                                                      < 2e-16
## monthjun
                      -3.797e-01
                                  1.473e-01
                                             -2.577
                                                      0.00996
## monthmar
                       7.240e-01
                                  1.733e-01
                                               4.177 2.95e-05
                                             -8.725
## monthmay
                      -1.191e+00
                                  1.366e-01
                                                      < 2e-16
## monthnov
                      -1.678e+00
                                  1.456e-01 -11.524
                                                      < 2e-16 ***
                                               0.692
## monthoct
                       1.125e-01
                                  1.627e-01
                                                      0.48908
                                                  NA
## monthsep
                              NA
                                         NA
                                                           NA
## duration
                       4.272e-03
                                  7.820e-05
                                              54.631
                                                      < 2e-16 ***
## campaign
                      -9.677e-02
                                  1.230e-02
                                              -7.866 3.65e-15 ***
## pdays
                       5.737e-04
                                  3.556e-04
                                               1.613
                                                      0.10667
## previous
                       1.169e-02
                                  7.290e-03
                                              1.604
                                                      0.10872
## poutcomefailure
                      -6.733e-02
                                  1.113e-01
                                             -0.605
                                                      0.54529
## poutcomeother
                       4.195e-02
                                  1.279e-01
                                               0.328
                                                      0.74290
                                                      < 2e-16 ***
## poutcomesuccess
                       2.269e+00
                                  1.007e-01
                                             22.535
## poutcomeunknown
                              NA
                                         NA
                                                  NA
                                                           NA
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 22840
                             on 31646
                                       degrees of freedom
## Residual deviance: 15016
                                       degrees of freedom
                             on 31603
## AIC: 15104
##
## Number of Fisher Scoring iterations: 6
#stepAIC(logistic 1, direction = "both")
# stepAIC has removed some variables and only the following ones remain
logistic 2 <- glm(formula = response ~ jobadmin. + jobhousemaid +
jobmanagement +
    jobretired + jobstudent + jobtechnician + maritalmarried +
    educationprimary + educationsecondary + balance + housingno +
    loanno + contactcellular + contacttelephone + day + monthapr +
    monthaug + monthfeb + monthjan + monthjul + monthjun + monthmar +
    monthmay + monthnov + duration + campaign + pdays + previous +
    poutcomesuccess, family = "binomial", data = train)
##
            jobadmin.
                            jobhousemaid
                                               jobmanagement
##
             1.279574
                                1.075275
                                                    1.848829
##
           iobretired
                              jobstudent
                                               jobtechnician
##
             1.269010
                                1.186903
                                                    1.356935
##
       maritalmarried
                        educationprimary educationsecondary
##
             1.094159
                                1.481352
                                                    1.639156
##
              balance
                               housingno
                                                      loanno
##
             1.033559
                                1.410136
                                                    1.057855
```

```
##
      contactcellular
                        contacttelephone
                                                          day
##
                                                    1.315402
             2.472082
                                 1.951957
##
             monthapr
                                 monthaug
                                                    monthfeb
##
             2.146210
                                 2.555384
                                                    1.980877
##
             monthjan
                                 monthjul
                                                    monthjun
##
             1.398465
                                 2.495161
                                                    2.511823
##
                                                    monthnov
             monthmar
                                 monthmay
##
             1.337560
                                 3.180751
                                                    1.937072
##
             duration
                                                       pdays
                                 campaign
##
             1.131015
                                                    1.357429
                                 1.102743
##
             previous
                         poutcomesuccess
##
             1.161571
                                 1.133248
##
## Call:
  glm(formula = response ~ jobadmin. + jobhousemaid + jobmanagement +
##
       jobretired + jobstudent + jobtechnician + maritalmarried +
       educationprimary + educationsecondary + balance + housingno +
##
##
       loanno + contactcellular + contacttelephone + day + monthapr +
##
       monthaug + monthfeb + monthjan + monthjul + monthjun + monthmar +
##
       monthmay + monthnov + duration + campaign + pdays + previous +
##
       poutcomesuccess, family = "binomial", data = train)
##
## Deviance Residuals:
                      Median
##
       Min
                 10
                                    3Q
                                            Max
           -0.3727
                     -0.2504
  -4.7557
                              -0.1459
                                         3.4618
##
## Coefficients:
##
                        Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                      -4.406e+00
                                   1.614e-01 -27.295 < 2e-16 ***
## jobadmin.
                       2.964e-01
                                   7.371e-02
                                               4.021 5.79e-05 ***
## jobhousemaid
                       -2.749e-01
                                   1.541e-01
                                              -1.784 0.074388
## jobmanagement
                       1.560e-01
                                   6.858e-02
                                               2.274 0.022951 *
## jobretired
                                   8.931e-02
                                               5.198 2.01e-07 ***
                       4.643e-01
                                               6.101 1.05e-09 ***
## jobstudent
                       7.148e-01
                                   1.171e-01
## jobtechnician
                       1.234e-01
                                   6.775e-02
                                               1.821 0.068580
## maritalmarried
                       -2.172e-01
                                   4.532e-02
                                              -4.792 1.65e-06 ***
## educationprimary
                       -3.120e-01
                                   8.208e-02
                                              -3.800 0.000144 ***
## educationsecondary -1.738e-01
                                   5.534e-02
                                              -3.141 0.001685 **
## balance
                       1.519e-05
                                   5.983e-06
                                               2.539 0.011128 *
## housingno
                                              14.646 < 2e-16 ***
                       7.677e-01
                                   5.242e-02
## loanno
                                               5.126 2.95e-07 ***
                       3.658e-01
                                   7.136e-02
                                              18.894
## contactcellular
                       1.651e+00
                                   8.739e-02
                                                      < 2e-16
## contacttelephone
                                   1.199e-01
                                              12.534
                                                      < 2e-16 ***
                       1.503e+00
                                               2.847 0.004420 **
## day
                       8.415e-03
                                   2.956e-03
                                              -8.232
## monthapr
                       -8.538e-01
                                   1.037e-01
                                                      < 2e-16
## monthaug
                                   9.726e-02 -16.039
                      -1.560e+00
                                                      < 2e-16
## monthfeb
                       -9.689e-01
                                   1.087e-01
                                              -8.910
                                                       < 2e-16
                                                      < 2e-16 ***
## monthjan
                      -2.153e+00
                                   1.516e-01 -14.204
## monthjul
                      -1.692e+00
                                   1.003e-01 -16.878 < 2e-16 ***
```

```
1.128e-01 -3.608 0.000309 ***
## monthjun
                      -4.069e-01
                                              4.854 1.21e-06 ***
## monthmar
                       6.990e-01
                                  1.440e-01
## monthmay
                      -1.222e+00
                                  9.672e-02 -12.638
                                                     < 2e-16
                                  1.077e-01 -15.931
                                                     < 2e-16
## monthnov
                      -1.717e+00
## duration
                       4.272e-03
                                  7.812e-05
                                            54.690 < 2e-16 ***
## campaign
                      -9.724e-02
                                  1.229e-02
                                             -7.915 2.48e-15 ***
                       4.559e-04
                                              2.089 0.036703 *
## pdays
                                  2.182e-04
## previous
                       1.183e-02
                                  7.001e-03
                                              1.690 0.091013
                                  8.089e-02 28.309 < 2e-16 ***
## poutcomesuccess
                       2.290e+00
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
       Null deviance: 22840
                             on 31646
##
                                       degrees of freedom
## Residual deviance: 15025
                             on 31617
                                       degrees of freedom
## AIC: 15085
##
## Number of Fisher Scoring iterations: 6
##
## Call:
## glm(formula = response ~ jobadmin. + jobhousemaid + jobmanagement +
       jobretired + jobstudent + jobtechnician + maritalmarried +
##
       educationprimary + educationsecondary + balance + housingno +
##
       loanno + contactcellular + contacttelephone + day + monthapr +
##
       monthaug + monthfeb + monthjan + monthjul + monthjun + monthmar +
##
       +monthnov + duration + campaign + pdays + previous + poutcomesuccess,
       family = "binomial", data = train)
##
##
## Deviance Residuals:
##
       Min
                 10
                      Median
                                   30
                                           Max
## -4.6947
           -0.3816
                     -0.2524 -0.1457
                                        3.4782
##
## Coefficients:
                        Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
                      -5.564e+00 1.343e-01 -41.441 < 2e-16 ***
                                              4.418 9.98e-06 ***
## jobadmin.
                       3.234e-01
                                  7.321e-02
## jobhousemaid
                      -2.133e-01
                                  1.530e-01
                                            -1.394 0.163357
                                  6.815e-02
## jobmanagement
                                              2.766 0.005680 **
                       1.885e-01
                                              6.473 9.61e-11 ***
## jobretired
                       5.705e-01
                                  8.814e-02
                                              6.523 6.88e-11 ***
## jobstudent
                       7.570e-01
                                  1.161e-01
## jobtechnician
                                  6.744e-02
                                              2.232 0.025612 *
                       1.505e-01
## maritalmarried
                      -1.959e-01
                                  4.503e-02
                                             -4.351 1.35e-05 ***
                                             -4.037 5.41e-05 ***
## educationprimary
                      -3.293e-01
                                  8.158e-02
## educationsecondary -1.842e-01
                                  5.502e-02
                                             -3.347 0.000817 ***
                                             2.894 0.003800 **
## balance
                       1.712e-05
                                  5.917e-06
## housingno
                       9.394e-01
                                  5.098e-02
                                             18.425
                                                    < 2e-16 ***
                                             5.359 8.36e-08 ***
## loanno
                       3.815e-01 7.118e-02
## contactcellular
                       1.834e+00 8.457e-02 21.681 < 2e-16 ***
```

```
## contacttelephone
                                               14.633 < 2e-16 ***
                        1.716e+00
                                   1.173e-01
                                                3.289 0.001004 **
## day
                        9.709e-03
                                   2.951e-03
## monthapr
                       -4.432e-03
                                   8.090e-02
                                               -0.055 0.956316
                                   7.815e-02 -10.225
## monthaug
                       -7.991e-01
                                                        < 2e-16
## monthfeb
                       -1.644e-01
                                   9.005e-02
                                               -1.826 0.067853
## monthjan
                       -1.392e+00
                                   1.404e-01
                                               -9.915
                                                        < 2e-16
                                   7.845e-02 -11.168
## monthjul
                       -8.762e-01
                                                        < 2e-16
## monthjun
                        4.922e-01
                                   8.870e-02
                                                5.549 2.87e-08
## monthmar
                        1.467e+00
                                   1.328e-01
                                               11.045
                                                       < 2e-16
## monthnov
                       -9.104e-01
                                   8.835e-02 -10.305
                                                        < 2e-16
## duration
                        4.246e-03
                                   7.769e-05
                                               54.648
                                                        < 2e-16
## campaign
                       -1.018e-01
                                   1.235e-02
                                               -8.240
                                                       < 2e-16 ***
## pdays
                        4.146e-04
                                   2.194e-04
                                                1.890 0.058793 .
## previous
                        1.414e-02
                                   7.667e-03
                                                1.844 0.065154 .
                        2.354e+00
                                   8.025e-02
                                               29.327
                                                      < 2e-16 ***
## poutcomesuccess
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 22840
                              on 31646
                                         degrees of freedom
## Residual deviance: 15183
                              on 31618
                                         degrees of freedom
## AIC: 15241
##
## Number of Fisher Scoring iterations: 6
##
            jobadmin.
                             jobhousemaid
                                                jobmanagement
##
             1.278345
                                 1.073900
                                                     1.852500
##
           jobretired
                               jobstudent
                                                jobtechnician
##
             1.255725
                                 1.186584
                                                     1.355827
##
       maritalmarried
                         educationprimary educationsecondary
##
             1.093686
                                 1.478612
                                                     1.640305
##
              balance
                                housingno
                                                        loanno
##
                                                     1.057721
             1.034041
                                 1.347969
##
      contactcellular
                         contacttelephone
                                                           day
##
             2.331771
                                                     1.314812
                                 1.885092
##
             monthapr
                                                     monthfeb
                                 monthaug
                                 1.649712
##
             1.295233
                                                     1.344307
##
             monthjan
                                 monthjul
                                                     monthjun
##
             1.191206
                                 1.524433
                                                     1.517659
##
             monthmar
                                 monthnov
                                                     duration
##
             1.119970
                                 1.296945
                                                     1.127920
##
             campaign
                                     pdays
                                                     previous
##
             1.100228
                                 1.390133
                                                     1.197572
##
      poutcomesuccess
##
             1.136761
##
## Call:
## glm(formula = response ~ jobadmin. + jobmanagement + jobretired +
```

```
##
       jobstudent + jobtechnician + maritalmarried + educationprimary +
       educationsecondary + balance + housingno + loanno + contactcellular +
##
       contacttelephone + day + monthapr + monthaug + monthfeb +
##
       monthjan + monthjul + monthjun + monthmar + +monthnov + duration +
##
       campaign + pdays + previous + poutcomesuccess, family = "binomial",
##
##
       data = train)
##
## Deviance Residuals:
       Min
                 10
                      Median
                                   3Q
                                           Max
## -4.6895
           -0.3818
                     -0.2529
                             -0.1457
                                        3.4820
##
## Coefficients:
##
                        Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                      -5.569e+00
                                  1.342e-01 -41.485 < 2e-16 ***
## jobadmin.
                       3.346e-01
                                  7.282e-02
                                              4.594 4.34e-06 ***
## jobmanagement
                       2.019e-01
                                  6.753e-02
                                              2.990 0.002790 **
## jobretired
                       5.886e-01
                                  8.724e-02
                                              6.747 1.51e-11 ***
                                              6.656 2.82e-11 ***
## jobstudent
                                  1.157e-01
                       7.701e-01
## jobtechnician
                       1.628e-01
                                  6.691e-02
                                              2.433 0.014984 *
## maritalmarried
                      -1.967e-01
                                  4.503e-02
                                             -4.367 1.26e-05 ***
## educationprimary
                                  8.122e-02
                                             -4.196 2.71e-05 ***
                      -3.408e-01
## educationsecondary -1.823e-01
                                  5.500e-02
                                             -3.314 0.000919 ***
## balance
                       1.714e-05
                                  5.919e-06
                                             2.896 0.003784 **
## housingno
                       9.350e-01
                                  5.088e-02
                                             18.376 < 2e-16
## loanno
                                              5.342 9.17e-08 ***
                       3.804e-01
                                  7.119e-02
## contactcellular
                       1.833e+00
                                  8.459e-02
                                             21.671 < 2e-16 ***
                       1.713e+00
                                                     < 2e-16 ***
## contacttelephone
                                  1.173e-01
                                             14.606
## day
                       9.628e-03
                                  2.951e-03
                                              3.263 0.001104 **
## monthapr
                      -4.825e-03
                                  8.091e-02
                                             -0.060 0.952441
                      -8.037e-01
## monthaug
                                  7.807e-02 -10.296 < 2e-16 ***
## monthfeb
                                             -1.833 0.066842 .
                      -1.650e-01
                                  9.003e-02
## monthjan
                                  1.403e-01
                                             -9.896 < 2e-16 ***
                      -1.388e+00
## monthjul
                      -8.788e-01
                                  7.845e-02 -11.202 < 2e-16 ***
                                              5.501 3.78e-08 ***
## monthjun
                       4.877e-01
                                  8.865e-02
## monthmar
                       1.464e+00
                                  1.328e-01
                                             11.025
                                                     < 2e-16 ***
## monthnov
                                  8.834e-02 -10.314
                                                     < 2e-16 ***
                      -9.111e-01
                       4.243e-03
                                             54.641
                                                     < 2e-16 ***
## duration
                                  7.766e-05
## campaign
                                  1.234e-02
                                             -8.222
                                                     < 2e-16 ***
                      -1.015e-01
## pdays
                       4.166e-04
                                  2.193e-04
                                              1.900 0.057494 .
## previous
                       1.408e-02
                                  7.651e-03
                                              1.840 0.065727 .
## poutcomesuccess
                       2.354e+00
                                  8.023e-02 29.345 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
       Null deviance: 22840
                             on 31646
                                       degrees of freedom
## Residual deviance: 15185
                             on 31619
                                       degrees of freedom
## AIC: 15241
```

```
##
## Number of Fisher Scoring iterations: 6
##
            jobadmin.
                            jobmanagement
                                                   jobretired
##
             1.265142
                                 1.818987
                                                     1.230380
##
                                               maritalmarried
           iobstudent
                            iobtechnician
##
             1.179704
                                 1.335063
                                                     1.093653
##
     educationprimary educationsecondary
                                                      balance
##
             1.464271
                                 1.639060
                                                     1.034000
##
            housingno
                                   loanno
                                              contactcellular
##
             1.342635
                                 1.057621
                                                     2.333056
##
     contacttelephone
                                      day
                                                     monthapr
##
             1.884936
                                 1.314397
                                                     1.295365
##
             monthaug
                                 monthfeb
                                                     monthjan
##
             1.647107
                                 1.344399
                                                     1.190980
##
             monthjul
                                 monthjun
                                                     monthmar
##
             1.523644
                                 1.516960
                                                     1.119701
##
             monthnov
                                 duration
                                                     campaign
##
             1.297082
                                 1.126930
                                                     1.099934
##
                                 previous
                pdays
                                              poutcomesuccess
##
             1.389528
                                 1.196844
                                                     1.136652
##
## Call:
## glm(formula = response ~ jobadmin. + jobmanagement + jobretired +
##
       jobstudent + jobtechnician + maritalmarried + educationprimary +
##
       educationsecondary + balance + housingno + loanno + contactcellular +
##
       contacttelephone + day + monthaug + monthjan + monthjul +
       monthjun + monthmar + +monthnov + duration + campaign +
##
poutcomesuccess,
       family = "binomial", data = train)
##
##
## Deviance Residuals:
##
       Min
                 10
                       Median
                                    30
                                             Max
            -0.3811
                      -0.2531
  -4.6847
                              -0.1468
                                          3.4937
##
## Coefficients:
                         Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
                                   1.326e-01 -42.146 < 2e-16 ***
                       -5.587e+00
                                                4.686 2.78e-06
## jobadmin.
                        3.410e-01
                                   7.276e-02
## jobmanagement
                        2.061e-01
                                   6.746e-02
                                                3.055
                                                       0.00225 **
                                                6.787 1.15e-11 ***
## jobretired
                        5.915e-01
                                   8.716e-02
                                                6.763 1.35e-11 ***
## jobstudent
                        7.819e-01
                                   1.156e-01
## jobtechnician
                                   6.690e-02
                                                2.437
                                                       0.01482 *
                        1.630e-01
## maritalmarried
                                   4.497e-02
                                               -4.288 1.80e-05 ***
                       -1.928e-01
## educationprimary
                                               -4.152 3.29e-05 ***
                       -3.370e-01
                                   8.117e-02
## educationsecondary -1.787e-01
                                   5.494e-02
                                               -3.253
                                                       0.00114 **
## balance
                        1.724e-05
                                   5.909e-06
                                                2.917
                                                       0.00353 **
## housingno
                        8.965e-01
                                   4.937e-02
                                               18.159
                                                       < 2e-16 ***
                                                5.349 8.84e-08 ***
## loanno
                        3.806e-01 7.115e-02
```

```
## contactcellular
                                   8.052e-02
                                              23.183
                                                       < 2e-16 ***
                       1.867e+00
                                                       < 2e-16 ***
## contacttelephone
                       1.745e+00
                                   1.147e-01
                                              15.213
## day
                       1.090e-02
                                   2.785e-03
                                               3.915 9.04e-05
## monthaug
                                   7.053e-02 -11.287
                       -7.961e-01
                                                       < 2e-16
## monthjan
                      -1.382e+00
                                   1.372e-01 -10.078
                                                       < 2e-16
## monthjul
                      -8.852e-01
                                   7.136e-02 -12.405
                                                       < 2e-16
## monthjun
                                               6.019 1.75e-09
                       5.168e-01
                                   8.586e-02
## monthmar
                       1.493e+00
                                   1.292e-01
                                              11.561
                                                       < 2e-16
## monthnov
                                   8.293e-02 -10.900
                      -9.040e-01
                                                       < 2e-16
## duration
                       4.241e-03
                                   7.748e-05
                                              54.728
                                                       < 2e-16
                                                       < 2e-16 ***
## campaign
                      -1.030e-01
                                   1.229e-02
                                              -8.383
## poutcomesuccess
                       2.433e+00
                                   7.682e-02
                                              31.668
                                                      < 2e-16 ***
## ---
## Signif. codes:
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 22840
                              on 31646
                                        degrees of freedom
## Residual deviance: 15200
                              on 31623
                                        degrees of freedom
## AIC: 15248
##
## Number of Fisher Scoring iterations: 6
##
            jobadmin.
                            jobmanagement
                                                  jobretired
##
             1.264792
                                 1.817724
                                                     1.230185
##
                                              maritalmarried
           jobstudent
                            jobtechnician
##
             1.177893
                                                     1.091911
                                 1.334885
##
     educationprimary educationsecondary
                                                      balance
##
                                 1.636925
                                                     1.033673
             1.463544
##
            housingno
                                   loanno
                                             contactcellular
##
                                 1.056964
             1.265137
                                                     2.115809
##
     contacttelephone
                                      day
                                                     monthaug
##
             1.806276
                                 1.178723
                                                     1.346639
##
             monthjan
                                 monthjul
                                                     monthjun
##
             1.137174
                                 1.264032
                                                     1.423663
##
                                                     duration
             monthmar
                                 monthnov
##
             1.062195
                                 1.144795
                                                     1.123197
             campaign
##
                          poutcomesuccess
##
             1.092087
                                 1.041965
##
## Call:
## glm(formula = response ~ jobadmin. + jobmanagement + jobretired +
       jobstudent + maritalmarried + educationprimary + educationsecondary +
##
##
       balance + housingno + loanno + contactcellular + contacttelephone +
##
       day + monthaug + monthjan + monthjul + monthjun + monthmar +
##
       +monthnov + duration + campaign + poutcomesuccess, family =
"binomial",
##
       data = train)
##
```

```
## Deviance Residuals:
##
       Min
                 10
                      Median
                                   30
                                           Max
## -4.6982
           -0.3810
                    -0.2534
                             -0.1468
                                        3.4930
##
## Coefficients:
##
                        Estimate Std. Error z value Pr(>|z|)
                      -5.531e+00
## (Intercept)
                                 1.304e-01 -42.419 < 2e-16 ***
## jobadmin.
                       2.856e-01
                                  6.893e-02
                                              4.144 3.42e-05 ***
## jobmanagement
                       1.448e-01
                                  6.241e-02
                                              2.321 0.020292 *
## jobretired
                       5.463e-01
                                  8.499e-02
                                              6.427 1.30e-10 ***
## jobstudent
                       7.216e-01
                                  1.128e-01
                                             6.396 1.60e-10 ***
## maritalmarried
                      -1.989e-01
                                  4.491e-02
                                            -4.429 9.48e-06 ***
## educationprimary
                                  7.942e-02
                                            -4.741 2.13e-06 ***
                      -3.765e-01
## educationsecondary -1.838e-01
                                            -3.346 0.000818 ***
                                  5.492e-02
## balance
                                  5.895e-06
                                              2.929 0.003395 **
                       1.727e-05
## housingno
                       8.976e-01
                                  4.938e-02
                                            18.178 < 2e-16 ***
## loanno
                       3.794e-01
                                  7.113e-02
                                              5.333 9.64e-08 ***
                                                     < 2e-16 ***
## contactcellular
                                            23.234
                       1.871e+00
                                  8.055e-02
## contacttelephone
                       1.746e+00
                                  1.147e-01
                                             15.226
                                                     < 2e-16
                                              3.990 6.59e-05 ***
## day
                       1.111e-02
                                  2.784e-03
                                  7.008e-02 -11.078
                                                     < 2e-16 ***
## monthaug
                      -7.763e-01
## monthjan
                      -1.384e+00
                                  1.371e-01 -10.095
                                                     < 2e-16
## monthjul
                                  7.136e-02 -12.426 < 2e-16 ***
                      -8.868e-01
## monthjun
                       5.181e-01
                                  8.590e-02
                                              6.031 1.63e-09 ***
                                                     < 2e-16 ***
## monthmar
                      1.499e+00
                                  1.292e-01
                                             11.609
## monthnov
                      -9.047e-01
                                  8.290e-02 -10.912
                                                     < 2e-16
## duration
                                                     < 2e-16 ***
                       4.237e-03
                                  7.744e-05
                                             54.717
                                                     < 2e-16 ***
## campaign
                      -1.033e-01
                                  1.228e-02
                                            -8.405
                       2.436e+00 7.680e-02 31.719
## poutcomesuccess
                                                     < 2e-16 ***
## ---
## Signif. codes:
                  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 22840
                             on 31646 degrees of freedom
## Residual deviance: 15206
                             on 31624
                                       degrees of freedom
## AIC: 15252
##
## Number of Fisher Scoring iterations: 6
##
## Call:
## glm(formula = response ~ jobadmin. + jobretired + jobstudent +
##
       maritalmarried + educationprimary + educationsecondary +
##
       balance + housingno + loanno + contactcellular + contacttelephone +
       day + monthaug + monthjan + monthjul + monthjun + monthmar +
##
       +monthnov + duration + campaign + poutcomesuccess, family =
##
"binomial",
##
       data = train)
##
```

```
## Deviance Residuals:
##
       Min
                 10
                      Median
                                    30
                                            Max
## -4.6989
            -0.3814
                     -0.2541
                              -0.1467
                                         3.4880
##
## Coefficients:
                         Estimate Std. Error z value Pr(>|z|)
##
## (Intercept)
                       -5.461e+00
                                   1.266e-01 -43.123 < 2e-16 ***
                                                3.794 0.000148 ***
## jobadmin.
                       2.570e-01
                                   6.773e-02
                                                6.134 8.57e-10 ***
## jobretired
                       5.138e-01
                                   8.376e-02
## jobstudent
                                   1.110e-01
                                                6.085 1.17e-09 ***
                       6.756e-01
                                              -4.388 1.15e-05 ***
## maritalmarried
                       -1.970e-01
                                   4.489e-02
                                              -5.945 2.76e-09 ***
## educationprimary
                       -4.411e-01
                                   7.419e-02
## educationsecondary -2.457e-01
                                   4.777e-02
                                              -5.143 2.70e-07 ***
## balance
                       1.757e-05
                                   5.890e-06
                                               2.983 0.002854 **
## housingno
                                   4.938e-02
                                              18.196
                                                      < 2e-16
                       8.985e-01
                                                5.351 8.72e-08 ***
## loanno
                        3.806e-01
                                   7.113e-02
## contactcellular
                        1.878e+00
                                   8.052e-02
                                              23.328
                                                       < 2e-16 ***
                                                       < 2e-16 ***
## contacttelephone
                                              15.258
                       1.750e+00
                                   1.147e-01
## day
                       1.113e-02
                                   2.784e-03
                                                3.997 6.42e-05 ***
## monthaug
                                   7.006e-02 -11.040
                                                       < 2e-16
                       -7.735e-01
                                   1.371e-01 -10.122
## monthjan
                       -1.388e+00
                                                       < 2e-16
## monthjul
                       -8.876e-01
                                   7.135e-02 -12.440
                                                       < 2e-16
## monthjun
                       5.188e-01
                                   8.592e-02
                                                6.038 1.56e-09 ***
## monthmar
                       1.506e+00
                                   1.291e-01
                                              11.664
                                                       < 2e-16
## monthnov
                       -9.020e-01
                                   8.292e-02 -10.878
                                                       < 2e-16
## duration
                       4.234e-03
                                   7.739e-05
                                               54.718
                                                       < 2e-16 ***
                                                       < 2e-16 ***
## campaign
                       -1.029e-01
                                   1.227e-02
                                              -8.389
                                   7.676e-02 31.736
## poutcomesuccess
                       2.436e+00
                                                       < 2e-16 ***
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 22840
                              on 31646
                                        degrees of freedom
## Residual deviance: 15211
                              on 31625
                                        degrees of freedom
## AIC: 15255
##
## Number of Fisher Scoring iterations: 6
##
            jobadmin.
                               jobretired
                                                   jobstudent
##
             1.095196
                                 1.136371
                                                     1.085759
##
       maritalmarried
                         educationprimary educationsecondary
##
             1.088615
                                 1.226031
                                                     1.238695
##
              balance
                                housingno
                                                       loanno
##
                                                     1.057005
             1.032898
                                 1.266552
##
      contactcellular
                         contacttelephone
                                                          day
##
             2.116738
                                 1.808209
                                                     1.177530
##
                                                     monthjul
             monthaug
                                 monthjan
##
             1.327510
                                 1.137262
                                                     1.264405
##
                                                     monthnov
             monthjun
                                 monthmar
```

```
## 1.426528 1.061397 1.144361
## duration campaign poutcomesuccess
## 1.121823 1.091999 1.041863
```

We now have a logistic model named logistic_final. The variables in the final model can be seen above, like job, education, balance, housing, contact method, month of contact etc.

Next, we'll use the model to predict the response in the test data.

```
predictions_logit <- predict(logistic_final, newdata = test[, -55], type =
"response")
summary(predictions_logit)

## Min. 1st Qu. Median Mean 3rd Qu. Max.
## 0.0000564 0.0190000 0.0446700 0.1161000 0.1090000 1.0000000</pre>
```

So now we have predicted the 'probabilities of responding' (for the test data). Note that the average probability (as shown above in summary(predictions_logit) is 11.6%, which is the average response rate.

Next comes the interesting part. We need to convert the probabilities to an actual prediction, i.e. **yes or no**. Can we just say that anything *above 50% probability of response is yes and no otherwise*? Yes, we could, but we can do better.

We can rather experiment with other **probability thresholds** like 30%, 40% etc. We will go with whatever gives us the highest (loosely speaking) **accuracy**. In fact, apart from accuracy, there are other metrics to **evaluate the model** like sensitivity, specificity etc.

Model Evaluation

In model evaluation, we use the test data to evaluate how good the model is (note that it was trained on 'train data' and hasn't seen the test data, so we are not cheating).

Let us first look at how **accurate** the predictions are. For now, let's use a probability cutoff of 50% and then we'll iterate.

```
predicted_response <- factor(ifelse(predictions_logit >= 0.50, "yes", "no"))
conf <- confusionMatrix(predicted response, test$response, positive = "yes")</pre>
conf
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction
                 no
                      yes
##
          no 11684
                     1046
##
                293
                       541
          yes
##
##
                  Accuracy : 0.9013
##
                    95% CI: (0.8961, 0.9063)
##
       No Information Rate: 0.883
##
       P-Value [Acc > NIR] : 6.515e-12
```

```
##
##
                     Kappa : 0.3984
   Mcnemar's Test P-Value : < 2.2e-16
##
##
               Sensitivity: 0.34089
##
               Specificity: 0.97554
##
##
            Pos Pred Value: 0.64868
            Neg Pred Value: 0.91783
##
                Prevalence: 0.11700
##
            Detection Rate: 0.03988
##
      Detection Prevalence: 0.06149
##
##
         Balanced Accuracy: 0.65822
##
##
          'Positive' Class : yes
##
```

Firstly, note that the **accuracy** is approx. 90% which means that the model has made about 90% predictions correct (whether yes or no).

There are two other important metrics - **sensitivity** and **specificity**.

Sensitivity is the fraction of correctly identified responses, i.e. out of those who will actually respond, how many has the model identified.

Specificity is the fraction of **incorrectly identified responses**, i.e. out of those who will actually NOT respond, how many has the model identified.

These two metrics can be calculated using the table of predictions as follows:

```
sensitivity <- conf$byClass[1]
specificity <- conf$byClass[2]
sensitivity

## Sensitivity
## 0.3408948

specificity

## Specificity
## 0.9755364</pre>
```

The values of sensitivity and specificity are about 31.75% and 97.72% respectively. This means that the model predicts 97.72% of those who will NOT buy correctly while only 31.75% of those who'll buy.

Since the number of "yes" responders are few (only 11% respond), it is hard to predict them. So if you market the product to about 10,000 people, you know that about 1100 will respond. The model will identify about 31% or 350 of them correctly.

But these predictions are based on an arbitrary cut-off of 0.50 probability. Now that we know what accuracy, specificity and sensitivity mean, we can find a cutoff which optimises the most important metric. In our case, it is sensitivity.

```
library(ROCR)

## Loading required package: gplots

##

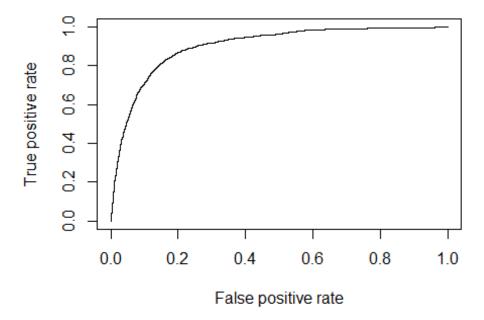
## Attaching package: 'gplots'

## The following object is masked from 'package:stats':

##

## lowess

predictions_object <- prediction(predictions_logit, test$response)
perf_object <- performance(predictions_object, "tpr", "fpr")
plot(perf_object)</pre>
```

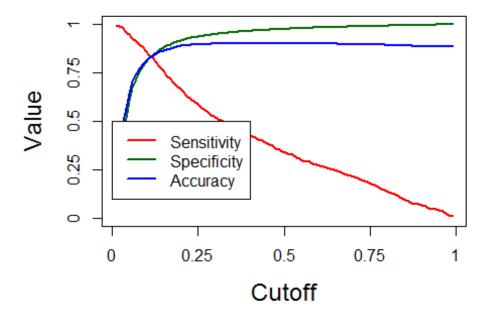


The plot shown above is called the **ROC** curve. It has False Positive Rate (FPR) and True Positive Rate (TPR, or sensitivity) on the x and y axes respectively.

The objective is to **maximise the TPR** and **minimise the FPR** which means that we want the curve to be aligned towards the **top-left**.

Now, let's find oput the optimal probabilty cutoff, i.e. the value above which we'll predict "yes" and "no" otherwise. We can plot the three metrics against cutoff values ranging from 0% to 100% and choose the one which gives high accuracy, sensitivity and specificity.

```
perform_fn <- function(cutoff)</pre>
  {
  predicted_response <- factor(ifelse(predictions_logit >= cutoff, "yes",
"no"))
  conf <- confusionMatrix(predicted_response, test$response, positive =</pre>
"yes")
  acc <- conf$overall[1]</pre>
  sens <- conf$byClass[1]</pre>
  spec <- conf$byClass[2]</pre>
  out <- t(as.matrix(c(sens, spec, acc)))</pre>
  colnames(out) <- c("sensitivity", "specificity", "accuracy")</pre>
  return(out)
}
# creating cutoff values from 0.01 to 0.99 for plotting and initialising a
matrix of size 1000x4
s = seq(.01,.99, length=100)
OUT = matrix(0,100,3)
# calculate the sens, spec and acc for different cutoff values
for(i in 1:100)
  OUT[i,] = perform_fn(s[i])
# plotting cutoffs
plot(s,
OUT[,1],xlab="Cutoff",ylab="Value",cex.lab=1.5,cex.axis=1.5,ylim=c(0,1),type=
"1", lwd=2, axes=FALSE, col=2)
axis(1, seq(0,1, length=5), seq(0,1, length=5), cex.lab=1.5)
axis(2,seq(0,1,length=5),seq(0,1,length=5),cex.lab=1.5)
lines(s,OUT[,2],col="darkgreen",lwd=2)
lines(s,OUT[,3],col=4,lwd=2)
box()
legend(0,.50,col=c(2,"darkgreen",4,"darkred"),lwd=c(2,2,2,2),c("Sensitivity",
"Specificity", "Accuracy"))
```



The plot above shows the sensitivity, specificity and accuracy for cutoff probabilities ranging from 0 to 100. It is clear that a cutoff around 12-13% will optimise the three metrics.

```
cutoffs <- s[which(abs(OUT[, 1] - OUT[, 2]) < 0.01)]</pre>
```

Let's choose a cutoff value of 12% for the final model.

```
## Accuracy
## 0.8409024

## Sensitivity
## 0.8204159

## Specificity
## 0.8436169
```

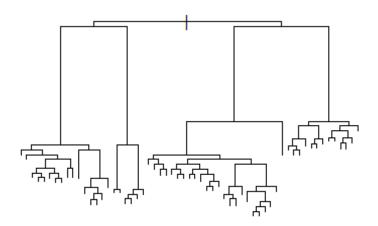
We have accuracy = 82.62%, sensitivity = 75.29% and specificity = 83.59%. This is a remarkable improvement over cutoff = 0.50, where the sensitivity was around 31% only.

Now, if you market the product to 10,000 people (out of which around 1100 usually respond), the model will be able to identify 75% of 1100 or approx. 825 people correctly.

Model 2: Decision Tree

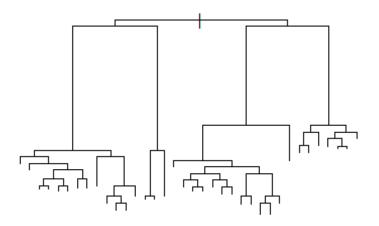
Let's build a decision tree and compare that with the logistic regression model.

```
library(rpart)
library(rattle)
## Rattle: A free graphical interface for data mining with R.
## Version 4.1.0 Copyright (c) 2006-2015 Togaware Pty Ltd.
## Type 'rattle()' to shake, rattle, and roll your data.
bank <- read.csv("bank-marketing.csv")</pre>
split_indices <- sample.split(bank$response, SplitRatio = 0.70)</pre>
train dt <- bank[split indices, ]</pre>
test_dt <- bank[!split_indices, ]</pre>
nrow(train_dt)/nrow(bank)
## [1] 0.6999845
nrow(test_dt)/nrow(bank)
## [1] 0.3000155
# building a tree with arbitrary minsplit and cp
banktree_1 <- rpart(response ~ ., data=train_dt, method= "class",</pre>
                      control=rpart.control(minsplit=65, cp=0.001))
plot(banktree_1)
```



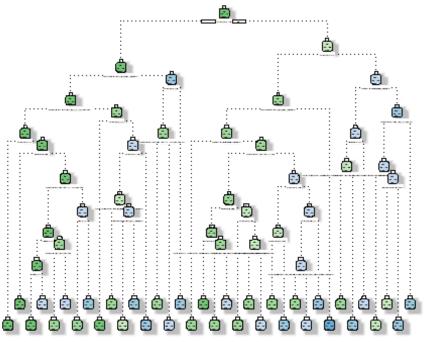
```
# This is clearly an overfitted tree
# Increasing the minsplit two fold to 130
banktree_2 <- rpart(response ~ ., data=train_dt, method= "class",</pre>
```

```
control=rpart.control(minsplit=130, cp=0.001))
plot(banktree_2)
```



This one is better, but still looks a little too complex
fancyRpartPlot(banktree_2)

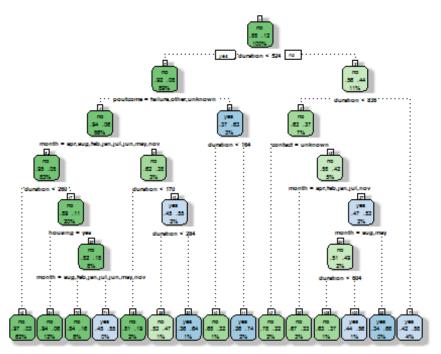
Warning: labs do not fit even at cex 0.15, there may be some overplotting



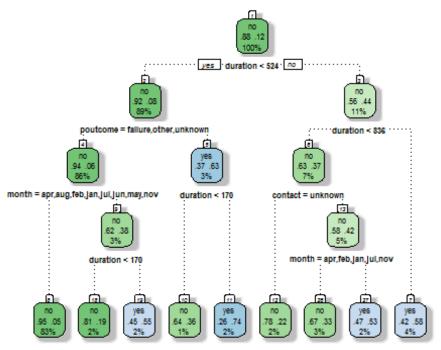
Rattle 2016-Dec-21 18:28:25 Pratika

```
# Listing the variables by importance: Duration, poutcome, month are the top
banktree_2$variable.importance
##
       duration
                     poutcome
                                     month
                                                 housing
                                                                   age
## 1100.0806054
                               355.8465846
                                                            47.2998892
                 629.1219282
                                              58.5726337
##
            job
                      contact
                                      pdays
                                                previous
                                                                   day
##
     41.7918574
                   39.2661168
                                31.0693077
                                              24.2778212
                                                            22.1378873
##
                   education
                                   balance
                                                targeted
                                                               marital
         salary
     11.4936286
                   10.8707432
                                 9.5013333
                                               4.8280236
                                                             2.1555044
##
##
       campaign
                         loan
                                   default
##
      1.9067268
                    1.6462427
                                 0.2932873
# We can further simplify the tree by increasing minsplit
banktree 3 <- rpart(response ~ ., data=train dt, method= "class",
                      control=rpart.control(minsplit=400, cp=0.001))
banktree_3$variable.importance
##
       duration
                     poutcome
                                     month
                                                 housing
                                                               contact
## 1086.3435731
                 563.7976032
                               336.4775041
                                              48.0529388
                                                            41.8905974
##
            day
                                        job
                                                   pdays
                                                               balance
                          age
                                 6.5981194
     10.2809741
                    7.9403807
##
                                               5.1803734
                                                             3.5911400
##
                     previous
                                       loan
                                                             education
         salary
                                                campaign
##
      3.3759324
                    2.7766546
                                 1.6462427
                                                             0.3502029
                                               1.1720273
```

fancyRpartPlot(banktree_3)



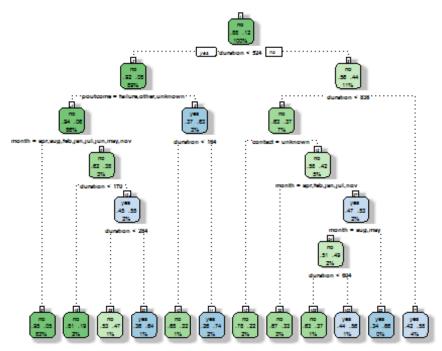
Rattle 2016-Dec-21 18:28:33 Pratika



Rattle 2016-Dec-21 18:28:37 Pratika

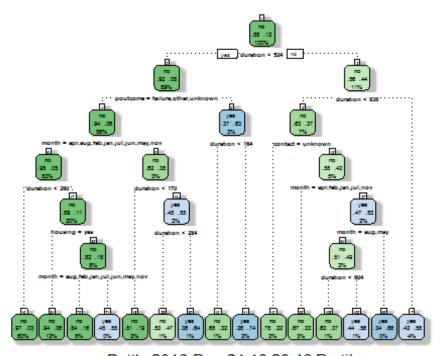
```
banktree_4$variable.importance
##
      duration
                  poutcome
                                 month
                                            contact
                                                            day
                                                                      pdays
## 994.0661890 563.7976032 267.4717306
                                        40.2783307
                                                      8.8805044
                                                                  4.4344924
##
       balance
                       age
                               housing
                                           previous
                                                           loan
                                                                   campaign
##
     3.4058204
                 3.2271714
                             2.7437378
                                          2.1176411
                                                      1.6462427
                                                                  0.9971059
# We'll evaluate banktree _3 and banktree_4 using cross validation and choose
the optimal value of cp parameter
# The xerror is the average error measured during cross validation. The cp
value for which the xerror is minimum is our ideal choice for the cp
parameter.
# cp = 0.0024 minimises the xerror
printcp(banktree 3)
##
## Classification tree:
## rpart(formula = response ~ ., data = train_dt, method = "class",
       control = rpart.control(minsplit = 400, cp = 0.001))
##
## Variables actually used in tree construction:
## [1] contact duration housing month
                                           poutcome
##
## Root node error: 3702/31647 = 0.11698
##
## n= 31647
##
```

```
CP nsplit rel error xerror
                        1.00000 1.00000 0.015444
## 1 0.0374572
## 2 0.0232307
                        0.88763 0.89762 0.014731
                    3
## 3 0.0074284
                    4
                        0.86440 0.87061 0.014533
## 4 0.0051324
                        0.84954 0.85521 0.014419
                    6
## 5 0.0015307
                   11
                        0.81929 0.83657 0.014278
## 6 0.0010000
                   14
                        0.81469 0.83522 0.014268
banktree_3 <- rpart(response ~ ., data=train_dt, method= "class",</pre>
                     control=rpart.control(minsplit=400, cp=0.0024))
fancyRpartPlot(banktree_3)
```



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```
##
##
            CP nsplit rel error xerror
                                             xstd
                    0
                        1.00000 1.00000 0.015444
## 1 0.0374572
## 2 0.0197191
                        0.88763 0.89870 0.014739
## 3 0.0074284
                        0.86791 0.89141 0.014686
## 4 0.0051324
                    6
                        0.85305 0.89492 0.014712
## 5 0.0010000
                        0.84279 0.88682 0.014653
banktree_4 <- rpart(response ~ ., data=train_dt, method= "class",</pre>
                     control=rpart.control(minsplit=400, cp=0.001))
fancyRpartPlot(banktree 4)
```



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```
banktree_4$variable.importance
##
       duration
                                     month
                     poutcome
                                                 housing
                                                               contact
                                              48.0529388
## 1086.3435731
                 563.7976032
                               336.4775041
                                                           41.8905974
##
            day
                                                   pdays
                                                               balance
                          age
                                        job
##
     10.2809741
                    7.9403807
                                 6.5981194
                                               5.1803734
                                                             3.5911400
##
         salary
                    previous
                                      loan
                                                campaign
                                                            education
##
      3.3759324
                    2.7766546
                                 1.6462427
                                               1.1720273
                                                             0.3502029
## Model Evaluation for banktree_3 and banktree_4
# using test data from now on
# banktree 3
banktree_3_pred <- predict(banktree_3, test_dt[, -19], type = "class")</pre>
confusionMatrix(banktree_3_pred, test_dt[, 19], positive = "yes")
```

```
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction
                 no
                      yes
##
          no 11577
                      898
##
          yes
                       689
                400
##
##
                  Accuracy : 0.9043
##
                    95% CI: (0.8992, 0.9092)
##
       No Information Rate: 0.883
##
       P-Value [Acc > NIR] : 1.152e-15
##
##
                     Kappa: 0.4639
##
    Mcnemar's Test P-Value : < 2.2e-16
##
##
               Sensitivity: 0.43415
##
               Specificity: 0.96660
##
            Pos Pred Value: 0.63269
            Neg Pred Value: 0.92802
##
##
                Prevalence: 0.11700
##
            Detection Rate: 0.05080
##
      Detection Prevalence: 0.08029
##
         Balanced Accuracy: 0.70038
##
##
          'Positive' Class : yes
##
# Accuracy is 90.14%, sensitivity is only 42.91%
# banktree 4
banktree 4 pred <- predict(banktree 4, test dt[, -19], type = "class")</pre>
confusionMatrix(banktree_4_pred, test_dt[, 19], positive = "yes")
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction
                 no
                      yes
##
          no 11547
                      867
##
                430
                      720
          yes
##
##
                  Accuracy : 0.9044
##
                    95% CI: (0.8993, 0.9093)
##
       No Information Rate: 0.883
##
       P-Value [Acc > NIR] : 9.166e-16
##
##
                     Kappa: 0.4745
    Mcnemar's Test P-Value : < 2.2e-16
##
##
##
               Sensitivity: 0.45369
##
               Specificity: 0.96410
```

```
##
            Pos Pred Value: 0.62609
##
            Neg Pred Value: 0.93016
                Prevalence: 0.11700
##
##
            Detection Rate: 0.05308
      Detection Prevalence: 0.08478
##
##
         Balanced Accuracy: 0.70889
##
##
          'Positive' Class : yes
##
# Sensitivity is only about 42%; we can improve the model quite a bit since
logistic model has sensitivtiy around 75%
# Model 3: Random forest
library(randomForest)
## randomForest 4.6-12
## Type rfNews() to see new features/changes/bug fixes.
##
## Attaching package: 'randomForest'
## The following object is masked from 'package:ggplot2':
##
##
       margin
split indices <- sample.split(bank$response, SplitRatio = 0.70)</pre>
train rf <- bank[split indices, ]</pre>
test_rf <- bank[!split_indices, ]</pre>
nrow(train_rf)/nrow(bank)
## [1] 0.6999845
nrow(test_rf)/nrow(bank)
## [1] 0.3000155
bank_rf <- randomForest(response ~., data = train_rf, proximity = F, do.trace</pre>
= T, mtry = 5
## ntree
              00B
                       1
##
       1: 12.27% 7.04% 53.87%
##
       2: 12.53% 7.17% 53.72%
##
       3: 12.46% 7.12% 52.98%
##
       4: 12.30% 7.13% 51.77%
##
       5: 11.97% 6.71% 51.98%
##
       6: 12.00% 6.69% 52.32%
       7: 11.67% 6.30% 52.49%
##
##
       8: 11.41% 6.03% 52.11%
       9: 11.21% 5.87% 51.64%
##
```

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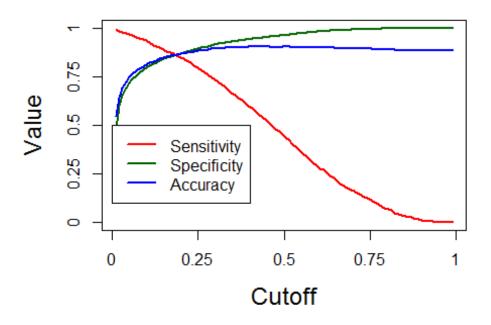
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rf pred <- predict(bank rf, test rf[, -19], type = "prob")
perform fn rf <- function(cutoff)</pre>
  predicted_response <- factor(ifelse(rf_pred[, 2] >= cutoff, "yes", "no"))
  conf <- confusionMatrix(predicted_response, test_rf$response, positive =</pre>
"yes")
```

```
acc <- conf$overall[1]</pre>
  sens <- conf$byClass[1]</pre>
  spec <- conf$byClass[2]</pre>
  out <- t(as.matrix(c(sens, spec, acc)))</pre>
  colnames(out) <- c("sensitivity", "specificity", "accuracy")</pre>
  return(out)
}
# creating cutoff values from 0.01 to 0.99 for plotting and initialising a
matrix of size 1000x4
s = seq(.01,.99, length=100)
OUT = matrix(0,100,3)
# calculate the sens, spec and acc for different cutoff values
for(i in 1:100)
  OUT[i,] = perform fn rf(s[i])
}
## Warning in confusionMatrix.default(predicted_response, test_rf$response, :
## Levels are not in the same order for reference and data. Refactoring data
## to match.
## Warning in confusionMatrix.default(predicted response, test rf$response, :
## Levels are not in the same order for reference and data. Refactoring data
## to match.
## Warning in confusionMatrix.default(predicted response, test rf$response, :
## Levels are not in the same order for reference and data. Refactoring data
## to match.
## Warning in confusionMatrix.default(predicted response, test rf$response, :
## Levels are not in the same order for reference and data. Refactoring data
## to match.
## Warning in confusionMatrix.default(predicted response, test rf$response, :
## Levels are not in the same order for reference and data. Refactoring data
## to match.
# plotting cutoffs
plot(s.
OUT[,1],xlab="Cutoff",ylab="Value",cex.lab=1.5,cex.axis=1.5,ylim=c(0,1),type=
"1", lwd=2, axes=FALSE, col=2)
axis(1, seq(0,1, length=5), seq(0,1, length=5), cex.lab=1.5)
axis(2,seq(0,1,length=5),seq(0,1,length=5),cex.lab=1.5)
lines(s,OUT[,2],col="darkgreen",lwd=2)
lines(s,OUT[,3],col=4,lwd=2)
box()
legend(0,.50,col=c(2,"darkgreen",4,"darkred"),lwd=c(2,2,2,2),c("Sensitivity",
"Specificity", "Accuracy"))
```



```
# the plot shows that cutoff value of around 22% optimises sensitivity and
accuracy
predicted_response_22 <- factor(ifelse(rf_pred[, 2] >= 0.22, "yes", "no"))
confusionMatrix(predicted_response_22, test_rf[, 19], positive = "yes")
## Confusion Matrix and Statistics
##
##
             Reference
## Prediction
                 no
                      yes
##
          no 10548
                      271
##
          yes 1429
                     1316
##
##
                  Accuracy : 0.8747
                    95% CI: (0.869, 0.8802)
##
##
       No Information Rate: 0.883
       P-Value [Acc > NIR] : 0.9987
##
##
##
                     Kappa: 0.5393
##
    Mcnemar's Test P-Value : <2e-16
##
##
               Sensitivity: 0.82924
##
               Specificity: 0.88069
            Pos Pred Value: 0.47942
##
            Neg Pred Value: 0.97495
##
##
                Prevalence: 0.11700
##
            Detection Rate: 0.09702
##
      Detection Prevalence: 0.20237
```

```
## Balanced Accuracy : 0.85496
##
## 'Positive' Class : yes
##
# Final RF important variables
importance <- bank_rf$importance</pre>
```

Variable importance:

	MeanDecreaseGini *
duration	1790.85243
month	743.77911
balance	629.50472
age	578.96925
day	528.41167
poutcome	449.43560
job	354.25112
pdays	274.70875
campaign	233.54896
salary	164.17749
previous	135.32161
education	131.74432
contact	127.12283
housing	120.88369
marital	114.72936
loan	44.22241
targeted	43.70625
default	10.90698

We'll choose the **Random Forest** as the final model. The top 5 important variables are duration, month, balance, age and day.

Model Deployment and Recommendations

Now that we have a model which predicts the probability of response, we can arrive at some interesting recommendations.

Our objective is to reduce the marketing cost and get almost the same number of customers as before.

The usual response rate is 11%, which means that if we telemarket to 10,000 people, 1100 will buy the product.

We can rather telemarket to only thoso whose **probability of purchase is high**. Let's look at the probabilities of purchase. Note that we will use only test data for this analysis.

```
test rf$predicted probs <- rf pred[, 2]
test rf$predicted response <- predicted response 22
test_predictions_rf <- test_rf[, c("response", "predicted_probs",</pre>
"predicted response")]
head(test_predictions_rf)
##
      response predicted_probs predicted_response
## 2
            no
                          0.000
                                                 no
## 3
                          0.004
            nο
                                                 nο
## 5
                          0.018
            no
                                                 no
## 6
                          0.002
            no
                                                 no
## 8
            no
                          0.040
                                                 no
## 12
                          0.000
            no
                                                 no
write.csv(test_predictions_rf, file = "response_predictions_rf.csv")
```

We have 13,564 observations in test data. Since we now have the probabilities of response, we can sort them and market only to those with high probabilities.

Reducing Customer Acquision Cost

Let's assume that telemarketing to each person costs INR 1. In the test data, we have 13,564 observations, so the total cost is INR 13564.

Among these, about 11.7% respond, so we get 1587 customers for INR 13564, or Rs 8.54 per customer.

```
summary(test_predictions_rf$response)
## no yes
## 11977 1587
```

Let's sort the observations in decreasing order of probability.

```
test predictions rf <-
test_predictions_rf[order(test_predictions_rf$predicted_probs, decreasing =
T), ]
head(test_predictions_rf)
##
         response predicted_probs predicted_response
## 44159
                             0.944
              yes
                                                   yes
## 44864
              yes
                             0.942
                                                   yes
## 44745
                             0.934
              yes
                                                   yes
                             0.932
## 45007
              yes
                                                   yes
```

```
## 41473 yes 0.930 yes
## 40149 no 0.924 yes
```

Now if we market to, say, only 50% population (approx. 6800 people), then about 1576 will respond (see below). The response rate is improved to 23.1%, almost double of what you'll get by randomly marketing. The acquision cost comes down to Rs 4.31 per customer.

```
summary(test_predictions_rf$response[1:6800])

## no yes
## 5232 1568

1576/6800

## [1] 0.2317647

6800/1576

## [1] 4.314721
```

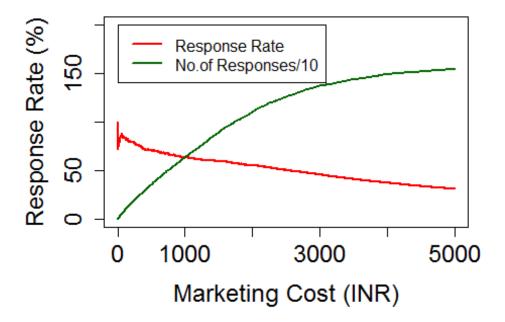
We can also visualise how the response rate varies with the marketing cost.

```
seq_prospects \leftarrow seq(1, 5000, by = 1)
cost_matrix <- matrix(0, length(seq_prospects), 3)</pre>
for (i in seq_prospects)
  cost_matrix[i, 1] = i
  response <- length(which(test predictions rf$response[1:i] == "yes"))</pre>
  cost matrix[i, 2] = response/i
  cost_matrix[i, 3] = response
}
colnames(cost_matrix) <- c("number of prospects targeted (marketing cost)",</pre>
"response rate", "number of responses")
head(cost_matrix)
##
        number of prospects targeted (marketing cost) response rate
## [1,]
                                                              1.0000000
## [2,]
                                                        2
                                                              1.0000000
## [3,]
                                                        3
                                                              1.0000000
## [4,]
                                                        4
                                                              1.0000000
                                                        5
## [5,]
                                                              1.0000000
## [6,]
                                                              0.8333333
##
        number of responses
## [1,]
                            1
## [2,]
                            2
                            3
## [3,]
## [4,]
                            4
## [5,]
                            5
## [6,]
                            5
```

The cost_matrix stores the number of prospects targeted, the response rates and the number of responses. The marketing cost is same as number of people targeted since we've assumed Re 1 per call.

```
plot(cost_matrix[, 1], cost_matrix[,2]*100,xlab="Marketing Cost
(INR)",ylab="Response Rate (%)",cex.lab=1.5,cex.axis=1.5,
ylim=c(0,200),type="l",lwd=2,axes=TRUE,col=2)

lines(seq_prospects, cost_matrix[, 3]/10, col="darkgreen",lwd=2)
box()
legend(0, 200,col=c(2,"darkgreen"),lwd=c(2,2),c("Response Rate","No.of
Responses/10"))
```



The plot shows how the number of responses and the response rate varies with marketing cost (no. of prospects targeted).

You can see that for INR 3000, almost 1379 prospects are expected to respond. Earlier, about 1587 would respond at a cost of Rs 13500.

```
cost matrix[3000:3010, ]
##
         number of prospects targeted (marketing cost) response rate
    [1,]
##
                                                    3000
                                                              0.4566667
##
    [2,]
                                                    3001
                                                              0.4565145
    [3,]
                                                              0.4563624
##
                                                    3002
                                                    3003
                                                              0.4562105
##
    [4,]
##
                                                    3004
                                                              0.4560586
    [5,]
```

```
##
    [6,]
                                                   3005
                                                            0.4559068
   [7,]
##
                                                   3006
                                                            0.4557552
##
   [8,]
                                                   3007
                                                            0.4556036
##
                                                   3008
                                                            0.4554521
   [9,]
## [10,]
                                                   3009
                                                            0.4553008
## [11,]
                                                   3010
                                                            0.4551495
         number of responses
##
##
   [1,]
                        1370
##
                        1370
    [2,]
## [3,]
                        1370
##
  [4,]
                        1370
## [5,]
                        1370
## [6,]
                        1370
## [7,]
                        1370
## [8,]
                        1370
## [9,]
                        1370
## [10,]
                        1370
## [11,]
                        1370
1379/1587
## [1] 0.8689351
3000/13500
## [1] 0.2222222
```

Thus, we can acquire **about 86% of the customers for only about 22% of the marketing cost.**