upGrad Campus

Course : Understanding Problem & Formulating Hypothesis

Lecture On : Optimization

Session Agendas

In this revision session, we will explore the concept of optimization, its business relevance, and terminologies associated with it.

You will also learn to perform optimization and visualise the results using Excel.

Optimization

Optimization is the process of making the best or most effective use of available resources or a situation.The areas in which they are applied are:

- Minimal cost
- Maximal profit
- Minimal error
- Optimal design
- Optimal management

Some of the popular examples of optimization found in industries are Airlines revenue management, Making a project estimate based on NPV (Net Present Value), Google adwords bidding.

Linear Optimization

As Linear optimization technique is used for 80-90% cases, the scope of our course is limited to that technique only. First, before you get into the linear optimization technique, you need to get familiar with the some terminologies, such as:

- Objective function
- Decision variable
- Constraints
- Sensitivity analysis

Let’s take an example and look at the definitions of these terminologies in that context.

Airline Seat allotment problem

Let's take an example of a fictitious airline FlyIndia. FlyIndia will cater to two types of customers: "early birds," or customers who purchase tickets far in advance and thus qualify for a discount, and "late buyers," or customers who purchase tickets at regular prices. The main task here is to allocate the 150 seats in their aircraft between the two segments in such a way that the company's revenue is maximised.

A regular ticket costs 3000, while a discounted ticket costs 1200. Assume FlyIndia allocates x number of regular tickets and y number of discounted tickets.

Understanding terminologies

Objective function

The objective function essentially describes the entity in the given business situation that we want to optimize (maximise or minimise).

In case of example discussed in previous slide, it would be the revenue for airlines. So if we calculate the total revenue generated by the tickets, it would be given by this formula 3000x + 1200y. This expression is also objective function of this example.

Decision variables

Decision variables are the quantities that decision makers control in order to optimize the objective function.

In case of example discussed in previous slide, decision variables would be x and y. Because by properly adjusting the value of the variables, we can maximise the revenue.

Understanding terminologies

Constraints

The constraints are the limitations on the value of decision variables. Some types of constraints are that constraint where the values should not exceed the available resources, non negativity constraint, demand constraint, "positive integer" constraint.

In case of airlines seat allocation example, Constraints would be:

- The first constraint would be x+y=150, x<=150, y<=150 (so that value does not exceed the no. of available seats)
- Non negativity constraint: In this case, that negative number of seats won't make sense. Hence x and y should not be negative. (x>=0; y>=0)
- Demand constraints give a realistic estimate of the maximum no. of tickets that can be allocated to either of the customer segments.
- The positive integer constraint makes sure that the solution that you obtain is a positive integer since we're dealing with allocating seats here and seats can't be a decimal number.

Understanding terminologies

Sensitivity analysis

Sensitivity Analysis is essentially a comparison of the various optimal solutions obtained for different constraints, as changes in constraints result in different optimal solutions.

In case of airlines seat allocation example, consider the comparison between two aircrafts each having 150 and 200 seats respectively. Now the objective functions would be the same. $3000x+1200y$. But there would be change in constraints ($x+y=150$ for 1st case; $x+y=200$ for 2nd case)

Eventually this would lead to change in optimal solution as well. This exact process of taking into account of different multiple scenarios and doing the comparison is Sensitivity analysis.

By doing this analysis, we would be able to get an efficient solution.

Optimization using Excel

Let’s take an example for carrying out the optimization using excel.

An airline company, XYZ has an Aircraft that can hold 166 seats and the demand shared by the demand forecasting team is approximately:

- 100 for non-discounted
- 150 for discounted

The flight prices for Delhi to Bangalore are:

- Discounted: 1190 INR
- Non-Discounted: 3085 INR

Inputting Objective function in excel

Objective function for this case is, 3085x + 1190y where x and y were the regular and discounted seats respectively.

Let's see how to input objective function in excel. We can make a table as shown in the picture below and objective function can be calculated using the SUMPRODUCT() function. In the picture, Cells C6 and C7 contain the regular and discounted ticket prices, respectively. Cells E6 and E7 will be used to fill in the seats that we decide to allocate in order to maximise revenue. These cells are essentially the decision variables x and y.

	A	B	C	D	E
1	**AIRLINE REVENUE MANAGEMENT**				
2	Flight Duration	6 hr			
3	Number of seats	166			
4					**Decisions**
5			Price	Demand	Seats
6	DEL to BLR	Regular	3085	100	
7		Discount	1190	150	
8					
9	**Objective:**	=SUMPRODUCT(C6:C7,E6:E7)			
10					

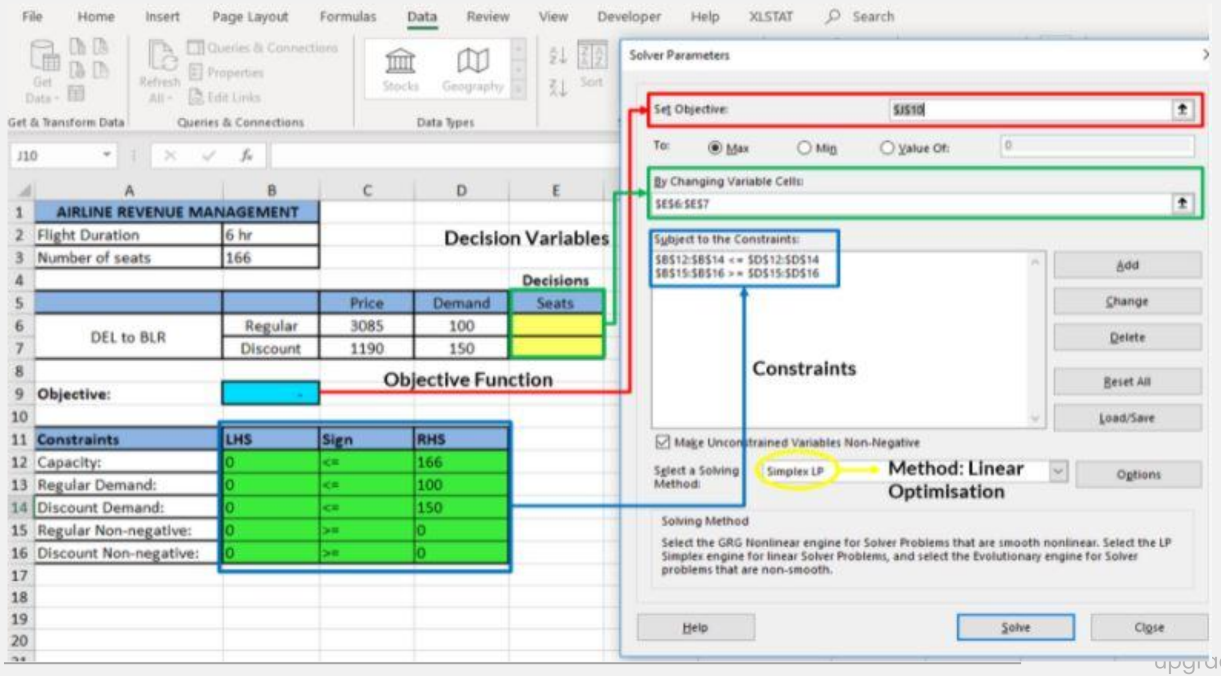
Constraints in Excel

- Flight capacity is 166 seats, so the total number of seats that can be allocated $x+y \le 166$.
- Demand forecasting states that there is demand for 100 Regular & 150 discounted seats. ($x \le 100$ & $y \le 150$)
- The non-negativity constraint that the number of seats (both regular and discounted) cannot be negative. ($x>=0$; $y>=0$)

Constraints	LHS	Sign	RHS
Capacity:	0	<=	166
Regular Demand:	0	<=	100
Discount Demand:	0	<=	150
Regular Non-negative:	0	>=	0
Discount Non-negative:	0	>=	0

Optimization using Solver

Once inputted all the values, optimization can be done using Solver add-in. With Solver add-in installed, go to Data -> Solver. In Solver, input the data as shown in the picture and select Simplex LP to perform the linear optimization to get optimal revenue. Click solve.



Optimization using Solver

The optimal revenue was calculated to be ₹3,87,040. And the decision variables, i.e., the number of seats allocated to regular and discounted, came out to be 100 and 66, respectively, to arrive at this optimal revenue.

Keep in mind that the algorithm to arrive at these values are out of scope of this course. Let's visualize the results for better understanding.

Visualization

Plot the constraints one by one, and you will get the feasible area (Fig. 1) where the possible solution lies. Now if we plot the optimum solution, we can see how our optimum solution (purple line) lies in that feasible area. (Fig 2)

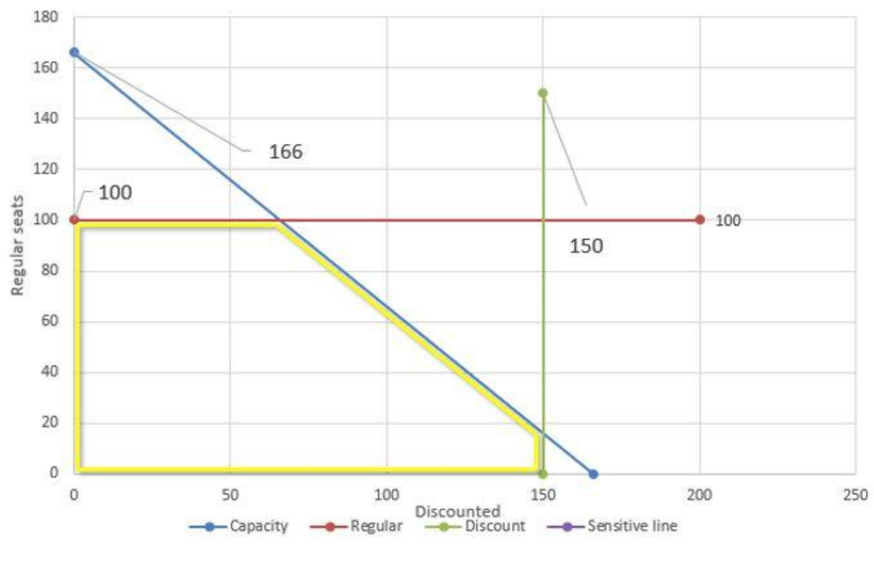


Fig. 1

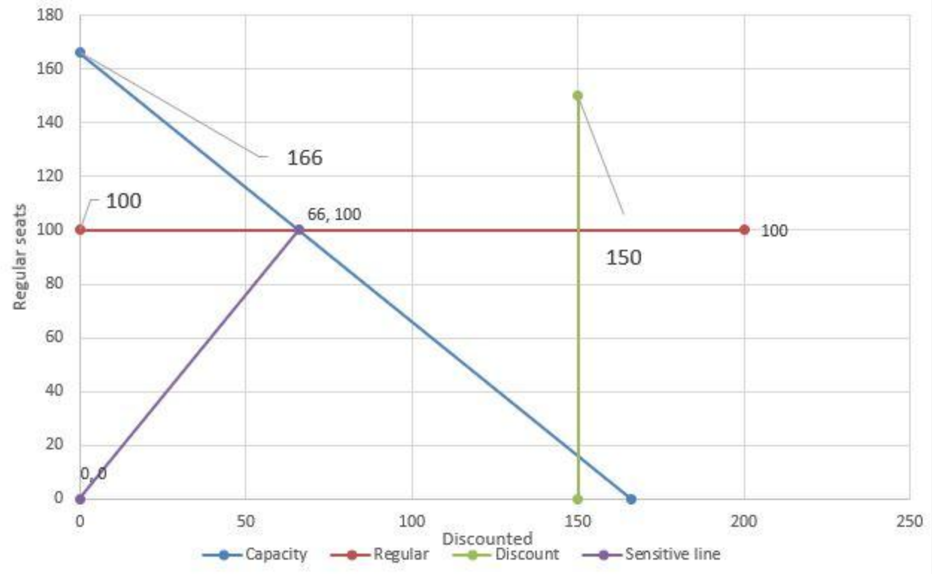


Fig.2

Airline Optimization: Sensitivity Analysis-I (Marketing)

Remember that the demand for regular and discounted tickets was 100 and 150, respectively. Now, suppose the Airlines is planning some marketing in order to increase these demand numbers. Will they generate more revenue after marketing?

Answer is indeed yes. The demand increased for regular tickets. In-fact the demand can even be increased and we can decrease the amount of discounted seats. However, we have to chose to keep some tickets aside for discounted buyers because we don't want to lose out on one market.

This was done because if demand drops at some point, we may not be able to fill all of the seats because discounted buyers may not check our flight given the history of our pricing.

Airline Optimization: Sensitivity Analysis-II (Airplane sizes)

This time, we are presented with this scenario where we have three aircraft with capacities of 166, 176, and 218 seats, and we must select the best option. We calculated optimum and marginal revenue for each of the case.

Air craft rental	Cost/hr	Total cost	Seats	Regular	Discounted	Revenue	Marginal revenue
Basic 166	60335	362010	166	100	66	387040	25030
176 seater	63825	382950	176	100	76	398940	15990
218 seater	72785	436710	218	100	118	448920	12210

Now we can select among these options for our objective:

Objective	Choice
Profitability	166 seater
Market share	218 seater
Balance between Profitability and market share	176 seater

Airline Optimization: Sensitivity Analysis-III (Connecting flights)

This time, we are presented with this scenario where we have aircraft with connecting flight. We have to incorporate constraints and all information as follows:

Constraints	LHS	sign	RHS
Capacity DEL-MUM	166	<=	166
Capacity MUM-BLR	166	<=	166
Demand Constraints	Seats	<=	Demand
Non-negative	Seats	>=	0

Once we inputted all the information, we can arrive at the optimal solution using the exact same steps we did before.

		Price	Demand	Seats
DEL to BLR	Regular	4280	80	80
	Discount	1900	120	0
DEL to MUM	Regular	6420	75	75
	Discount	2240	100	11
MUM to BLR	Regular	5120	60	60
	Discount	1900	110	26

Takeaways from this module

Understanding optimization & terminologies associated with it ✔

Perform optimization using excel & solver for given scenario and visualise the results ✔

Perform sensitivity analysis for different scenarios ✔

Any Questions?

Thank You!