

14

$$\varphi \sim R[0, 2\theta]$$

$$p(x) = \begin{cases} \frac{1}{\theta}, & x \in [0, 2\theta] \\ 0, & x \notin [0, 2\theta] \end{cases}$$

а) ОММ:

$$L_1(\theta) = M_\varphi = \int_0^{2\theta} \frac{x}{\theta} dx = \frac{x^2}{2\theta} \Big|_0^{2\theta} = 2\theta - \frac{\theta}{2} = \frac{3}{2}\theta$$

$$\tilde{L}_1 = \frac{1}{n} \sum x_i = \bar{x}$$

$$L_1(\theta) = \tilde{L}_1 \Rightarrow \frac{3}{2}\theta = \bar{x} \Rightarrow \boxed{\tilde{\theta}_1 = \frac{2}{3} \bar{x}}$$

ОМП:

$$L(\theta) = \frac{1}{\theta^2} (\theta \leq x_i \leq 2\theta) = \frac{1}{\theta^2} (\max x_i \leq 2\theta) \rightarrow \max$$

$$\tilde{\theta}_2 = \frac{x_{\max}}{2}$$

$$\boxed{\tilde{\theta}_3 = \frac{1}{5} (x_{\min} + 2x_{\max})}$$

б) 1) $\tilde{\theta}_1 = \frac{2}{3} \bar{x}$

• Несмещаемость:

$$M[\tilde{\theta}_1] = \frac{2}{3} M\left[\frac{1}{n} \sum x_i\right] = \frac{2}{3} M_\varphi = \frac{2}{3} \cdot \frac{3}{2} \theta = \theta \quad \underline{\text{несмещ.}}$$

• Состоятельность:

$$D[\tilde{\theta}_1] = \frac{4}{9} D\left[\frac{1}{n} \sum x_i\right] = \frac{4}{9n} D\theta \xrightarrow{n \rightarrow \infty} 0 \rightarrow \text{состояем}$$

$$\textcircled{2} \tilde{\theta}_2 = \frac{x_{\max}}{2}$$

~~Нечетность~~ • Нечетность

$$M[\tilde{\theta}_2] = \frac{1}{2} M[x_{\max}] = \frac{1}{2} \int_0^{2\theta} x h F(x)^{n-1} p(x) dx = \left[\frac{1}{2} \int_0^{2\theta} x h \left(\frac{x}{\theta}\right)^{n-1} \frac{1}{\theta} dx \right] =$$

$$= \left[\frac{h\theta}{2} \int_0^{2\theta} \left(\frac{x}{\theta}\right)^n d\left(\frac{x}{\theta}\right) = \frac{h\theta}{2} \left(\frac{x}{\theta}\right)^{\frac{n+1}{2}} \Big|_0^{2\theta} = \frac{2^n h\theta^{\frac{n+1}{2}}}{n+1} = \frac{h\theta}{2} \right]$$

$$= \frac{h}{2} \int_0^{2\theta} x \left(\frac{x-\theta}{\theta}\right)^{n-1} \frac{1}{\theta} dx = \frac{1}{2} \int_0^{2\theta} x d\left(\frac{x-\theta}{\theta}\right)^n = \frac{1}{2} x \left(\frac{x-\theta}{\theta}\right)^n \Big|_0^{2\theta} -$$

$$- \frac{1}{2} \int_0^{2\theta} \left(\frac{x-\theta}{\theta}\right)^n dx = \theta - \frac{\theta}{2} \int_0^{2\theta} \left(\frac{x-\theta}{\theta}\right)^n d\left(\frac{x-\theta}{\theta}\right) =$$

$$= \theta - \frac{\theta}{2} \left(\frac{x-\theta}{\theta}\right)^{\frac{n+1}{2}} \cdot \frac{1}{\frac{n+1}{2}} \Big|_0^{2\theta} = \theta - \frac{\theta}{2(n+1)} = \theta \frac{2n+1}{2(n+1)} - \text{сметать}$$

$$\tilde{\theta}_2 = \frac{2n+2}{2n+1} \frac{x_{\max}}{2} = \frac{n+1}{2n+1} x_{\max} - \text{нечетность}$$

• Состояем

$$D[\tilde{\theta}_2] = \left(\frac{n+1}{2n+1}\right)^2 D[x_{\max}]$$

$$D[x_{\max}] = M[x_{\max}^2] - M[x_{\max}]^2$$

$$M[x_{\max}^2] = \int_0^{2\theta} x^2 h \left(\frac{x-\theta}{\theta}\right)^{n-1} \frac{1}{\theta} dx = \int_0^{2\theta} x^2 d\left(\frac{x-\theta}{\theta}\right)^n = x^2 \left(\frac{x-\theta}{\theta}\right)^n \Big|_0^{2\theta} -$$

$$- \int_0^{2\theta} \left(\frac{x-\theta}{\theta}\right)^n 2x dx = 4\theta^2 - \frac{2\theta}{n+1} \int_0^{2\theta} x d\left(\frac{x-\theta}{\theta}\right)^{n+1} =$$

$$= 4Q^2 - \frac{2Q}{\lambda+1} \left(2Q - \int_0^{2Q} \left(\frac{x-Q}{Q} \right)^{\lambda+1} dx \right) = 4Q^2 \frac{\lambda}{\lambda+1} + \frac{2Q^3}{(\lambda+1)(\lambda+2)} =$$

$$= \frac{2Q^2(2\lambda^2 + 4\lambda + 1)}{(\lambda+1)(\lambda+2)}$$

~~$$D[X_{\max}] = \frac{2Q^2(2\lambda^2 + 4\lambda + 1)}{(\lambda+1)(\lambda+2)} - \frac{Q^2(2\lambda+1)^2}{(\lambda+1)^2} = \frac{\lambda Q^2}{(\lambda+1)^2(\lambda+2)}$$~~

$$D[X_{\max}] \xrightarrow{n \rightarrow \infty} 0 \Rightarrow \tilde{Q}_c' - \text{состояем}$$

$$D[\tilde{Q}_c'] = \frac{\lambda Q^2}{(\lambda+2)(2\lambda+1)} \xrightarrow{n \rightarrow \infty} 0$$

$$\textcircled{3} \tilde{Q}_3 = \frac{1}{5}(X_{\min} + 2X_{\max})$$

~~Решаем~~ • Решаем момент 6.

$$M[\tilde{Q}_3] = \frac{1}{5}M[X_{\min}] + \frac{2}{5}M[X_{\max}]$$

$$M[X_{\min}] = \int_0^{2Q} x \cdot n \left(1 - \frac{x-Q}{Q} \right)^{\lambda+1} \frac{1}{Q} dx = \int_0^{2Q} n \left(\frac{x}{Q} \right)^{\lambda} dx = \frac{Q(\lambda+2)}{(\lambda+1)}$$
~~$$= \frac{n}{\lambda+1} \int_0^{2Q} \left(\frac{x}{Q} \right)^{\lambda} dx = \frac{n}{\lambda+1} \left(\frac{x}{Q} \right)^{\lambda+1} \frac{1}{\lambda+1} \Big|_0^{2Q} = \frac{nQ}{\lambda+1} \left(\frac{2^{\lambda+1}}{\lambda+1} - 0 \right) = \frac{nQ 2^{\lambda+1}}{(\lambda+1)^2}$$~~

$$M[\tilde{Q}_3] = \frac{1}{5} \frac{Q(\lambda+2)}{(\lambda+1)} + \frac{2}{5} \frac{Q(2\lambda+1)}{(\lambda+1)} = \frac{Q}{5} \frac{\lambda^2 + 3\lambda + 2 + 4\lambda^2 + 4\lambda + 6}{(\lambda+1)(\lambda+1)}$$

$$= \frac{Q(5\lambda+4)}{5(\lambda+1)} = Q \frac{5\lambda+4}{5\lambda+5} - \text{вылез}$$

$$\tilde{Q}_3' = \frac{5\lambda+5}{5\lambda+4} \tilde{Q}_3 \Rightarrow \tilde{Q}_3' = \frac{\lambda+1}{5\lambda+4} (X_{\min} + 2X_{\max}) - \text{нечем}$$

• Комогам:

$$D[\tilde{\theta}_3'] = \frac{(\lambda+1)^2}{(5\lambda+4)^2} (D[X_{\min}] + 4D[X_{\max}] + 4\text{COV}(X_{\min}, X_{\max}))$$

$$M[X_{\min}^2] = \int_0^{2\theta} x^2 \lambda \left(1 - \frac{x-\theta}{\theta}\right)^{\lambda-1} \frac{1}{\theta} dx = \frac{(\lambda(\lambda+5)+8)\theta^2}{(\lambda+1)(\lambda+2)}$$

$$D[X_{\min}] = \frac{\lambda^2 + 5\lambda + 8}{(\lambda+1)(\lambda+2)} \theta^2 - \frac{\theta^2(\lambda+2)^2}{(\lambda+1)^2} = \frac{\lambda}{(\lambda+1)^2(\lambda+2)} \theta^2$$

$$\text{COV}(X_{\min}, X_{\max}) = M[X_{\min} \cdot X_{\max}] - M[X_{\min}] M[X_{\max}]$$

$$K(y, z) = \begin{cases} F(z) - (F(z) - F(y))^{\lambda}, & z \geq y \\ F(z), & z < y \end{cases} \quad \text{— сдвиг за к. макс.}$$

$$\partial^2 K(y, z) = \frac{\partial^2 K}{\partial y \partial z} = \lambda(\lambda-1)(F(z) - F(y))^{\lambda-2} F'(y) F'(z) \quad z \geq y =$$

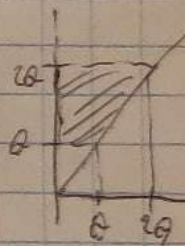
$$= \lambda(\lambda-1) \left(\frac{z-y}{\theta}\right)^{\lambda-2} \frac{1}{\theta^2}$$

$$M[X_{\min} \cdot X_{\max}] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy \partial^2 K(y, z) dy dz = \int_{\theta}^{2\theta} dy \int_{\theta}^y dz \lambda(\lambda-1) \left(\frac{z-y}{\theta}\right)^{\lambda-2} \frac{1}{\theta^2} =$$

$$= \frac{\theta^2(2\lambda+3)}{\lambda+2}$$

$$\text{COV}(X_{\min}, X_{\max}) = \frac{\theta^2(2\lambda+3)}{\lambda+2} - \frac{\theta(\lambda+2)}{\lambda+1} \cdot \frac{\theta(2\lambda+1)}{(\lambda+1)} = \frac{\theta^2}{(\lambda+1)^2(\lambda+2)}$$

$$D[\tilde{\theta}_3'] = \frac{(\lambda+1)^2}{(5\lambda+4)^2} \left(\frac{5\lambda\theta^2}{(\lambda+1)^2(\lambda+2)} + \frac{4\theta^2}{(\lambda+1)^2(\lambda+2)} \right) = \frac{\lambda\theta^2}{(5\lambda+4)(\lambda+2)} \xrightarrow{\lambda \rightarrow \infty} 0 \quad \text{Комогам}$$



$$c) p(y) = \begin{cases} e^{1-y}, & y \geq 1 \\ 0, & y < 1 \end{cases}$$

$$p(\theta | \bar{x}_n) = \frac{p(\theta) L(\theta)}{p(\bar{x}_n)} = \begin{cases} \frac{e^{1-\theta} \prod_{i=1}^n p(x_i, \theta)}{p(\bar{x}_n)} B, & \theta \geq 1 \\ 0, & \theta < 1 \end{cases}$$

$$p(\theta | \bar{x}_n) = \begin{cases} e^{1-\theta} \prod_{i=1}^n p(x_i, \theta) B, & \theta \geq 1 \\ 0, & \theta < 1 \end{cases}$$

Нормировка:

$$\int_1^{\infty} e^{1-\theta} \prod_{i=1}^n p(x_i, \theta) B d\theta = 1 \Rightarrow B =$$

$$\int_1^{\infty} e^{1-\theta} \prod_{i=1}^n \frac{\theta-1}{x_i^{\theta}} B d\theta = \int_1^{\infty} e^{1-\theta} (\theta-1)^n \left(\prod_{i=1}^n \frac{1}{x_i} \right)^{\theta} B d\theta = 1$$

$$\ln p(\theta | \bar{x}_n) = \ln B + 1 - \theta + n \ln(\theta - 1) - \theta \sum \ln x_i$$

$$\frac{d}{d\theta} = 0 \Rightarrow -1 + \frac{n}{\theta-1} - \sum \ln x_i = 0 \Rightarrow \hat{\theta} = 1 + \frac{n}{\sum \ln x_i + 1}$$

$$\int_{f_1}^{f_2} p(\theta | \bar{x}_n) d\theta = 0.95$$

$$\int_{f_2}^{f_1} p(\theta | \bar{x}_n) d\theta = 0.95$$

$$\Rightarrow I = (f_1; f_2)$$

$$\int_{f_1}^{f_2} \lambda(y-1)^{\lambda-1} dy = \frac{1-0,95}{2} = \frac{1-0,95}{2}$$

$$(f_1 - 1)^{\lambda} = 0,025 \Rightarrow f_1 = \sqrt[2]{0,025} + 1$$

$$\int_{f_2}^2 \lambda(y-1)^{\lambda-1} dy = \frac{1-0,95}{2}$$

$$1 - (f_2 - 1)^{\lambda} = 0,975 \Rightarrow f_2 = \sqrt[2]{0,975} + 1$$

$$1 - (f_2 - 1)^{\lambda} = 0,975 \Rightarrow f_2 = \sqrt[2]{0,975} + 1$$

Доверит. интервал. (модуль):

$$\left(\sqrt[2]{0,025} + 1, \sqrt[2]{0,975} + 1 \right) \Rightarrow \left[\frac{\max x_i}{1 + \sqrt[2]{0,975}} < \theta < \frac{\max x_i}{1 + \sqrt[2]{0,025}} \right]$$

е) ОММ

$$\tilde{\theta} = \frac{2}{3} \bar{x} = \frac{2}{3} \bar{L}_1$$

$$f(L) = \frac{2}{3} L_1 \Rightarrow \nabla f(L) = \frac{2}{3}$$

$$k = (K_{11}) = L_2 - L_1 = \tilde{L}_2 - \tilde{L}_1$$

$$\Rightarrow \sigma = \sqrt{\frac{4}{9} (L_2 - L_1)^2}$$

$$\frac{\tilde{\theta} - \theta}{\sqrt{\frac{4}{9} (L_2 - L_1)^2}} \sim N(0,1) \sim p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\int_{f_1}^{f_2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 0,025 \quad -1,96 < \frac{\tilde{\theta} - \theta}{\frac{2}{3} \sqrt{L_2 - L_1}} \sqrt{n} < 1,96$$

$$\left[-1,96 \cdot \frac{2}{3} \sqrt{L_2 - L_1} + \tilde{\theta} < \theta < 1,96 \cdot \frac{2}{3} \sqrt{L_2 - L_1} + \tilde{\theta} \right] - \text{гов. интервал асимпт.}$$