Нечего сказать о H_0 но можем

$\boxed{T10}$

$H_0: \varphi = p_0 = \begin{cases} 1, & x \in (0,1) \\ 0 & x \notin (0,1) \end{cases}$ $\quad H_1: \varphi = p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$

а) $n = 1 \quad \alpha$

$l = \frac{p_1}{p_0} = \frac{e}{e-1} e^{-x} \geq C \Rightarrow e^{-x} \geq B \Rightarrow G: x \leq A$.

$P(x \leq A | H_0) = \alpha$

$\int_0^A dx = \alpha \Rightarrow A = \alpha$

$\alpha_1 = P(X_1 \leq A | H_0) = \alpha$

$W = P(X \leq A | H_1) = \int_0^\alpha \frac{e^{1-x}}{e-1} dx = \frac{e}{e-1}(1 - e^{-\alpha})$

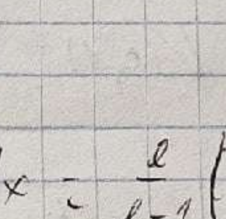
$\alpha_2 = 1 - W = 1 - \frac{e^0}{e-1}(1 - e^{-\alpha})$

б) $n = 2 \quad \alpha$

$l = \frac{L_1}{L_0} = \frac{e^2}{(e-1)^2} e^{-x_1} e^{-x_2} \geq C$

$e^{-x_1} e^{-x_2} \geq B$

$G: x_1 + x_2 \leq A$



$P(x_1+x_2 \le A \mid H_0) = \frac{A^2}{2} = \alpha \Rightarrow A = \sqrt{2\alpha}$

$G: x_1+x_2 \le \sqrt{2\alpha}$

$\alpha_1 = \alpha$

$W = P(x_1+x_2 \le A \mid H_1) = \int_0^{\sqrt{2\alpha}} dx_1 \int_0^{\sqrt{2\alpha}-x_1} \left(\frac{e}{e-1}\right)^2 e^{-x_1-x_2} dx = \frac{e^2}{(e-1)^2}\left(1-e^{-\sqrt{2\alpha}}-\sqrt{2\alpha}e^{-\sqrt{2\alpha}}\right)$

$\alpha_2 = 1-W$

c) асимпт. крит. n, α

$l = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \ge c \Rightarrow \ln l = \sum_{i=1}^{n} \underbrace{\ln \frac{p_1(x_i)}{p_0(x_i)}}_{\eta_i}$

$P(l \ge c \mid H_0) = \alpha$

$\frac{\sum_{i=1}^{n} \eta_i - n M[\eta]}{\sqrt{n D_\eta}} \sim N(0,1)$

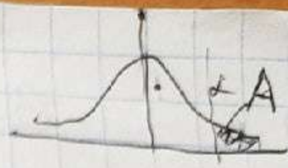
$H_0: M[\eta] = M[\ln \frac{e}{e-1} e^{-x_i}] = M[\ln \frac{e}{e-1} - x_i] = \ln \frac{e}{e-1} - \frac{1}{2}$

$D[\eta] = D[\ln \frac{e}{e-1} e^{-x_i}] = D[\ln \frac{e}{e-1} - x_i] = D[x_i] = \frac{1}{12}$ равн. распр.

$P(\ln l \ge \ln c \mid H_0) = P\left(\frac{\ln l - n(\ln \frac{e}{e-1} - \frac{1}{2})}{\sqrt{\frac{n}{12}}} \ge \frac{\ln c - n(\ln \frac{e}{e-1} - \frac{1}{2})}{\sqrt{\frac{n}{12}}} \right) = \alpha$

$\sim N(0,1)$

$$\ln c = \sqrt{\frac{n}{12}}\, u_{1-\alpha} + n \ln \frac{e}{e-1} - \frac{n}{2}$$



$A = u_{1-\alpha}$

$$G: \ln l \geq \ln c$$

$$\ln l = \sum l_i = \sum \ln\left(\frac{e}{e-1} e^{-x_i}\right) = \sum\left(\ln \frac{e}{e-1} - x_i\right) \Rightarrow$$

$$\Rightarrow n \ln \frac{e}{e-1} - n\bar{x} \geq \sqrt{\frac{n}{12}}\, u_{1-\alpha} + n \cancel{\ln \frac{e}{e-1}} - \frac{n}{2}$$

$$G: \bar{x} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}$$

$$\alpha_1 = \alpha$$

$$W = P\left(\bar{x} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} \,\middle|\, H_1\right)$$

$$\frac{\bar{x} - M\xi}{\sqrt{D\xi}} \sqrt{n} \Rightarrow N(0,1)$$

$$H_1: \; M[\xi] = \int_0^1 \frac{e}{e-1} x e^{-x}\, dx = \frac{e-2}{e-1}$$

$$M[\xi^2] = \int_0^1 \frac{e}{e-1} x^2 e^{-x}\, dx = \frac{2e-5}{e-1}$$

$$D = M\xi^2 - M\xi^2 = \frac{e^2 - 3e + 1}{(e-1)^2}$$

$$W = P\Bigg(\underbrace{\frac{\bar{x} - M\xi}{\sqrt{D\xi}} \sqrt{n}}_{\to N(0,1)} \leq \overbrace{\frac{\frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}} - M\xi}{\sqrt{D\xi}} \sqrt{n}}^{B}\Bigg) = \int_{-\infty}^{B} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}\, dx$$

$$\alpha_2 = 1 - W$$

d) n $\quad G: x_{min} < c$

$$P(x_{min} < c \mid H_0) = \alpha$$

$$F_{min}(y) = 1-(1-F(x))^n = \cancel{1(0,1)} = 1-(1-x)^n \ (0,1)$$

$$P(x_{min} < c) = F_{min}(c) \Rightarrow \underline{1-(1-c)^n = \alpha} \Rightarrow \boxed{c = -\sqrt[n]{1-\alpha}+1}$$

$$\alpha_1 = \alpha$$

$$W = P(x_{min} < c \mid H_1) =$$

$$F_1(x) = \int_0^x \frac{e}{e-1} e^{-t}\,dt = \frac{e}{e-1}(1-e^{-x})$$

$$W = F_{min}(c) = 1-(1-F_1(c))^n = \cancel{1-\frac{e}{e-1}(1-e^{-c})}$$

$$W = 1-\left(1-\frac{e}{e-1}(1-e^{-c})\right)^n = 1-\left[1-\frac{e}{e-1}\left(1-e^{\sqrt[n]{1-\alpha}-1}\right)\right]^n$$

$$\alpha_2 = 1-W$$