2022-34-29

Retour sur les spineurs

On peut montrer que $\psi^\dagger \vec{\sigma} X$ est un vecteur.

Pour se faire on va transformer que la quantité $\psi^\dagger \vec{\sigma} \cdot \vec{A}$, qui devrait être un scalaire, par une rotation.

$$\to \psi^\dagger R^\dagger \vec{\sigma} \vec{\sigma} R X \cdot \mathcal{R} \vec{A}$$

$$\psi_i^\dagger X A_i = \psi^\dagger R^\dagger \sigma_i R X \mathcal{R}_{ij} A_j = \psi^\dagger R^\dagger \sigma_j R X \mathcal{R}_{ji} A_i$$

$$\psi^\dagger \sigma_i X = \psi^\dagger R^\dagger \sigma_j R X \mathcal{R}_{ji}$$

$$R \sigma_i R^\dagger = \mathcal{R}_{ji} \sigma_j$$

$$\boxed{R^\dagger \sigma_i R = \mathcal{R}_{ijj}}$$

On représente un vecteur à trois composante par un matrice

$$\mathbf{r} = \begin{pmatrix} x & y & z \end{pmatrix}$$

$$X = \vec{r} \cdot \vec{\sigma} = \begin{bmatrix} z & x - iy \\ x + iy & -z \end{bmatrix}$$

$$\operatorname{tr} X = 0 \qquad \det X = -\mathbf{r}^2$$

alors

$$X' = R \sigma_i R^\dagger x_i = R X R^\dagger$$

On fait maintenant la même chose pour des 4-vecteurs.

$$X(x) = \begin{bmatrix} x^0 - x^3 & -x^1 + ix^2 \\ -x^1 - ix^2 & x^0 + x^3 \end{bmatrix} = x^0 \mathbb{1} - x^3 \sigma_3 - x^1 \sigma_1 -^2 \sigma^3 = x_\mu \sigma^\mu$$

$$\det X = x_\mu x^\mu$$

$$\det X = \det X' \qquad X = N^\dagger X' N \qquad \det N = 1$$

$$\det X = \det X'$$

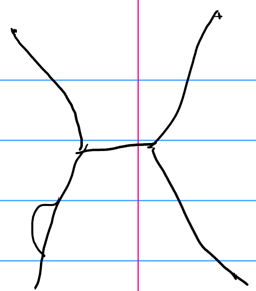
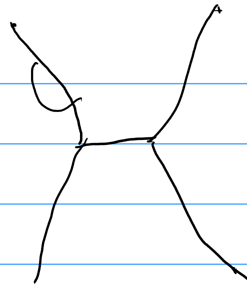
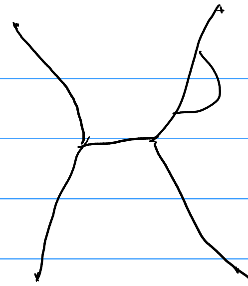
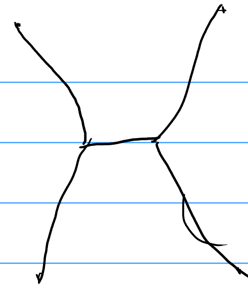
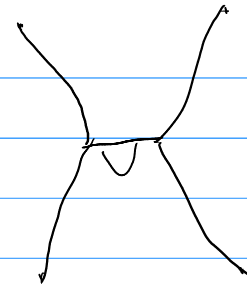
On a deux contraintes sur 8 degrés de liberté (4 degrés complex) et on impose deux contraintes sur les determinant. On a donc 6 degrés de liberté qui correspondent à ceux des trasformation de Lorentz.

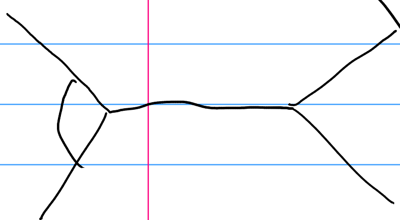
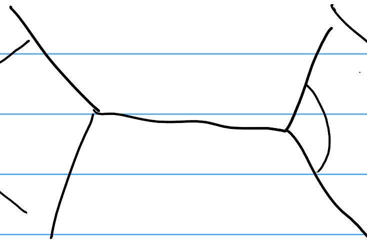
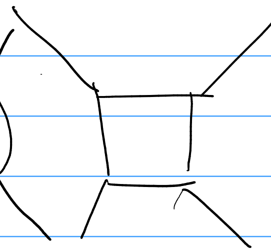
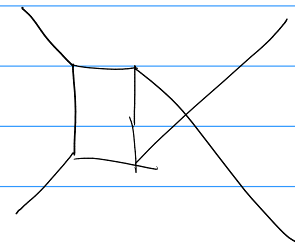
$$\begin{aligned} x'^\nu &= \Lambda^\mu_\nu x^\nu \\ x'_\mu &= \Lambda^\nu_\mu x_\nu = (\Lambda^{-1})^\nu_\mu x_\nu \\ X' = x'_\mu \sigma^\mu &= \Lambda^\nu_\mu x_\nu \sigma_\mu \\ &\dots \end{aligned}$$

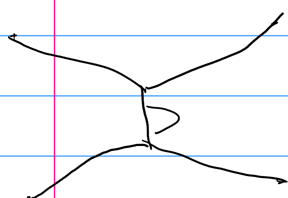
$$\boxed{N^\dagger \sigma^\mu N = \Lambda^\mu_\nu \sigma^\nu}$$

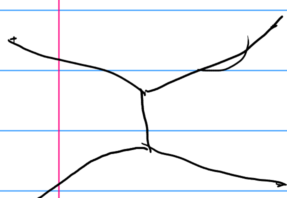
C'est vrai pour un définiton de type 1, pour le type 2 (avec les $+$) on aurait

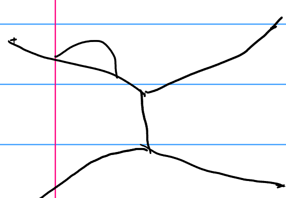
$$M^\dagger \tilde{\sigma}^\mu M = \Lambda^\mu_\nu \sigma^\nu$$

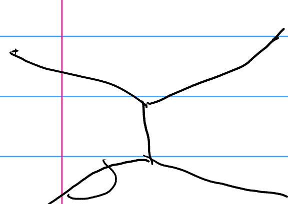






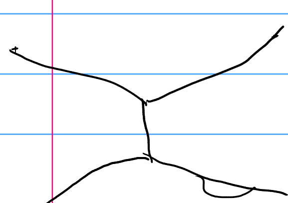















Théorie en ϕ^4

$$\mathscr{L} = \frac{1}{2}_{mu}\phi^{\mu}\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{4!}\phi^4$$

Le premier ordre en énérgie ne s'annule pas, c'est donc plus simple

$$H_1 = \frac{g}{4!}\int \mathrm{d}^3 vr\phi^4 \left(a + a^\dagger\right)\left(a + a^\dagger\right)\left(a + a^\dagger\right)\left(a + a^\dagger\right)$$

$$M_{fi} = \langle f|H|i\rangle = g$$