# Épisode 2

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Spineurs, bases et rerésentations

$$\text{ECOC} : X, Y, Z, S_z, (S^2) : \mathcal{E}_{\vec{r}} \otimes \mathcal{E}_s = \mathcal{E}\ \ |\vec{r}, s\rangle \tag{1}$$
$$\text{ECOC} : P_x, P_y, P_z, S_z; |\vec{p}, s\rangle \tag{2}$$
$$\text{ECOC} : H_0, \mathbf{L}^2, L_z, S_z; |n, l, m, s\rangle \tag{3}$$

Relation de fermeture dans $\mathcal{E}$ :

$$1 = 1_{\vec{r}} \otimes 1_S = \int \mathrm{d}^3 r \, |\vec{r}\rangle \langle\vec{r}| \otimes \sum_\epsilon |\epsilon\rangle \langle\epsilon|$$

$$\implies 1 = \sum_\epsilon \int \mathrm{d}^3 r \, |\vec{r}\epsilon\rangle\langle\vec{r}, \epsilon|$$

Preuve très similaire pour les autres bases.

$$|\psi\rangle = 1\,|\psi\rangle = \sum_\epsilon \int \mathrm{d}^3 r \, |\vec{r}, \epsilon\rangle \underbrace{\langle\vec{r}, \epsilon|\psi\rangle}_{\Psi_\epsilon(\vec{r})}$$

Représentation matricielle :

$$|\psi\rangle = \int \mathrm{d}^3 r \begin{pmatrix} \psi_+(\vec{r}) \\ \psi_-(\vec{r}) \end{pmatrix} |\vec{r}\rangle$$

$$\langle\vec{r}|\psi\rangle = \begin{pmatrix} \psi_+(\vec{r}) \\ \psi_-(\vec{r}) \end{pmatrix} = [\psi] \ (\mathit{Spineur!})$$

$$\langle\psi| = \int \mathrm{d}^3 r \begin{pmatrix} \psi_+^*(\vec{r}) & \psi_-^*(\vec{r}) \end{pmatrix} \langle\vec{r}|$$

$$|\psi\rangle = 1\,|\psi\rangle = \sum_\epsilon \sum_{n,l,m} |n, l, m, \epsilon\rangle \overbrace{\langle n, l, m, \epsilon|\psi\rangle}^{C_{n,l,m,\epsilon}} \tag{4}$$

si

$$|\vec{r}\rangle\langle n, l, m| = R_n, l(r) Y_l^m(\theta, \phi)$$

$$\langle\vec{r}|\psi\rangle = \sum_{n,l,m} \sum_\epsilon \underbrace{\langle\vec{r}|n, l, m\rangle}_{R_n, l(r) Y_l^m(\theta,\phi)} |\epsilon\rangle\, C_{n,l,m,\epsilon} = \sum_{n,l,m} \begin{pmatrix} c_{n,l,m,+} R_n, l(r) Y_l^m(\theta, \phi) \\ c_{n,l,m,-} R_n, l(r) Y_l^m(\theta, \phi) \end{pmatrix}$$

Norme

$$\langle\psi|\psi\rangle = \int \mathrm{d}^3r[\psi^*][\psi]$$

<u>Produit interieur</u>

$$\langle\psi|\phi\rangle = \int \mathrm{d}^3r[\psi^*][\phi]$$

<u>Élément de matrice</u>

$$\langle\Psi|\nVdash A \nVdash|\Phi\rangle = \sum_{\epsilon,\epsilon'} \int \mathrm{d}^3r\mathrm{d}^3r' \underbrace{\langle\psi|\vec{r}',\epsilon'\rangle}_{\psi^*_\epsilon(\vec{r}')} \underbrace{\langle\vec{r}',\epsilon'|\,A\,|\vec{r},\epsilon\rangle}_{A_{\epsilon'\epsilon}(\vec{r}',\vec{r})} \underbrace{\langle\vec{r}',\epsilon'|\psi\rangle}_{\psi_\epsilon(\vec{r})} = \int \mathrm{d}^3r\mathrm{d}^3r'[\psi^*][\![A]\!][\phi]$$

$$L_z \rightarrow_{|\vec{r}\rangle} \frac{\hbar}{i}\frac{\partial}{\partial\varphi} \rightarrow_{\mathcal{E}_{\vec{r}}\otimes\mathcal{E}_\epsilon} = \frac{\hbar}{i}\begin{pmatrix} \frac{\partial}{\partial\phi} & 0 \\ 0 & \frac{\partial}{\partial\phi} \end{pmatrix}$$

<u>Mesure</u>

La quatrième postula reste valable :

$$|\psi\rangle : \text{ vecteur d'état}$$

$$\mathcal{P}(\underbrace{a_n}_{\text{val dicrete d'un obs}}) = |\,\langle\varphi_n|\psi\rangle\,|^2$$

$$\mathrm{d}\mathcal{P}(\underbrace{\alpha}_{\text{val continue d'un obs}}) = |\,\langle\omega_\alpha|\psi\rangle\,|^2\mathrm{d}\alpha$$

$$\mathcal{P}(a_n) = \sum_{i=1}^{g_n} |\,\langle\varphi_n^i|\psi\rangle\,|^2$$

Dans notre cas, qui est une combinaisons de discret et continue, on a :

$$\mathrm{d}\mathcal{P}(\vec{r},\pm) = |\,\langle\vec{r},\pm|\psi\rangle\,|^2\mathrm{d}^3r$$

$$\mathcal{P}_\pm = \int \mathrm{d}\mathcal{P} = \int \mathrm{d}^3r|\psi(\vec{r})|^2$$

Si $[\psi] = \begin{pmatrix} \psi_+(r,\theta,\varphi) \\ \psi_-(r,\theta,\varphi) \end{pmatrix}$

$$\mathcal{P}_{\vec{L}^2} = \left|\int \sum_{l',m'} Y_l^{m*} a_{l',m',+}(r) Y_{l'}^{m'}\mathrm{d}\Omega\right|^2 + \left|\int \sum_{l',m'} Y_l^{m*} a_{l',m',-}(r) Y_{l'}^{m'}\mathrm{d}\Omega\right|^2$$