2022-32-07

Métrique de Schwarzschild

Équation géodésique

$$\rightarrow \dot{u}_i = \frac{1}{2}\partial_i g_{mk} u^m u^k$$
$$u_t = \mathrm{cst} = k$$
$$u_\varphi = \mathrm{cst} = -h$$

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1 Coordonnées de Kottler-Møller

$$\begin{cases} t = \left(x' + \frac{1}{\alpha}\right)\sinh \alpha t \\ x = \left(x' + \frac{1}{\alpha}\right)\cosh \alpha t' - \frac{1}{\alpha} \end{cases}$$

A

$$g'_{ij} = \begin{bmatrix} (1+\alpha x')^2 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\mathrm{d}t - \mathrm{d}x' \sinh \alpha t' + (\alpha x' + 1)\cosh \alpha t' \mathrm{d}t'$$
$$\mathrm{d}x = \mathrm{d}x' \cosh \alpha t' + (\alpha x' + 1)\sinh \alpha t' \mathrm{d}t'$$

$$\mathrm{d}\tau = \mathrm{d}t^2 - \mathrm{d}x^2 = \cdots$$

$$\left[\frac{\partial x^i}{\partial x'^j}\right] = \begin{bmatrix} (\alpha x' + 1)\cosh \alpha t' & \sinh \alpha t' \\ (\alpha x' + 1)\sinh \alpha t' & \cosh \alpha t' \end{bmatrix}$$

B

$$\mathrm{d}\tau = (1 + \alpha x')\,\mathrm{d}t'$$

$$[u^i] = \left(\frac{\mathrm{d}t}{\mathrm{d}\tau}, \frac{\mathrm{d}x}{\mathrm{d}\tau}\right) = \frac{\mathrm{d}t'}{\mathrm{d}\tau}\left(\frac{\mathrm{d}t}{\mathrm{d}t'}, \frac{\mathrm{d}x}{\mathrm{d}t'}\right)\bigg|_{x'=\mathrm{cst}}$$

$$= \frac{1}{1+\alpha x'}\left((1+\alpha x')\cosh \alpha t',\, (1+\alpha x')\sinh \alpha t'\right) = (\cosh \alpha t', \sinh \alpha t')$$

$$[a^i] = \frac{\mathrm{d}t'}{\mathrm{d}\tau}\left(\frac{\mathrm{d}}{\mathrm{d}t'}\cosh\alpha t',\ \frac{\mathrm{d}}{\mathrm{d}t'}\sinh\alpha t'\right) = \frac{\alpha}{1+\alpha x'}\left(\sinh\alpha t', \cosh\alpha t'\right)$$

raccouris je crois

$$a^i = e/f??$$

si

v=0→ $a^i = (0,\, \vec{a}) a_i a^i = -\vec{a}^2 s$