Formule d'échange

2 particules 2 états $\psi_a\psi_b$

$$\langle\psi_a|\psi_b\rangle$$

$$\psi_s(x_1,x_2) = S\psi_a + \psi_b$$

$$\left\langle\Delta x^2\right\rangle_s = \frac{1}{2}\int \mathrm{d}x_1\mathrm{d}x_2(x_1-x_2)^2\left[\psi_a(x_1)^*\psi_b(x_2)^* + s\psi)a(x_2)^*\psi_b(x_1)^*\right]\times\left[\psi_a(x_1)\psi_b(x_2) + s\psi_a(x)2)\psi_b(x_1)\right]$$

$$= \frac{1}{2}\int \mathrm{d}x_1\mathrm{d}x_2(x_1-x_2)^2\left[|\psi_a(x_1)|^2|\psi_b(x_2)|^2 + |\psi_a(x_2)|^2|\psi_b(x_1)|^2 + s\left(\psi_b(x_1)^*\psi_a(x_1)\psi_a(x^2)^*\psi_b(x_2) + \psi_a(x_1)^*\psi_b(x_1)\psi_b(x_2)^*\psi_a(x_2)\right)\right]$$

$$= \left\langle\Delta x^2\right\rangle_d - 2s\left|\langle x\rangle_{ab}\right|^2$$

Premier terme : $= \int \mathrm{d}x_1\mathrm{d}x_2(x_1-x_2)^2|\psi_a(x_1)|^2|\psi_b(x_2)|^2$

Donc :

$$\begin{cases}\text{bosons :} & \left\langle\Delta x^2\right\rangle_{\text{bosons}} = \left\langle\Delta_x^2\right\rangle_d - 2\left|\langle x\rangle_{ab}\right|^2\\ \text{fermions :} & \left\langle\Delta x^2\right\rangle_{\text{fermions}} = \left\langle\Delta_x^2\right\rangle_d + 2\left|\langle x\rangle_{ab}\right|^2\end{cases}$$

Méthode variationnelle

$$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

$$\psi_E(x) = \begin{cases}a^{-\frac{1}{2}}\cos(\pi x/2a) & |x| < a\\ 0 & \text{sinon}\end{cases}$$

$$E(a) = \langle\varphi_E|\,H\,|\varphi_E\rangle$$

$$E(a) = \int \mathrm{d}x - \frac{\hbar^2}{2m}\varphi_E(x)\partial^n\varphi_E(x) + \int \mathrm{d}x\frac{1}{2}kx^2\varphi_E(x)^2$$

$$E(a) = \frac{hbar^2}{2km}\int \mathrm{d}x(\partial_x\varphi_E)^2 + V(a)$$

$$E(a) = \frac{\hbar^2\pi^2}{8ma^2} + ka^2\left(\frac{1}{6} - \frac{1}{pi^2}\right)$$

$$E(a_0) = \frac{1}{2}\hbar\omega \left(\frac{\pi^2 - 6}{3}\right)^2$$

Ce qui est plus grand que la véritable valeur, soit $\frac{1}{2}\hbar\omega$