Méthode des variations

$$-\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} - (\lambda - |x|)\psi = 0$$

$$\psi_E(x) = \begin{cases} c(a - |x|) & |x| < a \\ 0 & \text{sinon} \end{cases}$$

$$\int |\psi|^2 \mathrm{d}^d n = 1$$

$$\psi \sim L^{-1/2}$$

$$c \sim L^{-1/2} \implies \lambda \propto \alpha^{-3/2}$$

$$H = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + |x|$$

$$E(\alpha) = C_1\alpha^{-2} + C_2\alpha$$

$$\frac{\mathrm{d}|x|}{\mathrm{d}x} = \Theta(x) - \Theta(-x)$$

$$\frac{\mathrm{d}^2|x|}{\mathrm{d}x^2} = \delta(x) + \delta(-x) = 2\delta(x)$$

Il ne reste qu'a trouver les coefficients C_1, C_2

On fait l'intégrale

$$\frac{1}{|\psi\rangle\,\langle\psi|} t_{-a} - aa^a - \psi(s)[2][x + |x|\psi^2 \d{\mathrm{x}} = E(\alpha)$$

On peut ensuite minimiser le résultat par rapport à α

Particules identiques

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$\left|\varphi_n^{(1)}\epsilon^{(1)}\right\rangle \left|\varphi_m^{(2)}\epsilon^{(2)}\right\rangle$$

État fondamental :

$$\left|\varphi_0^{(1)}, \epsilon^{(1)}, \varphi_0^{(2)} \bar{\epsilon}^{(2)}\right\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[\left|\varphi_0^{(1)}, \epsilon^{(1)}, \varphi_0^{(2)}, \bar{\epsilon}^{(2)}\right\rangle - \left|\varphi_0^{(1)} \bar{\epsilon}^{(1)}, \varphi_0^{(2)} \epsilon^{(2)}\right\rangle\right]$$

$$\langle\psi| H |\psi\rangle = 2\hbar\omega \frac{1}{2}$$

Diffusion de potentiel :

Approximation de born et potentiel central !

$V(\mathbf{r}) = v(r)$

$$\text{Q} : f_k(\theta, \varphi) = ?$$

$$f_k(\theta, \varphi) = \frac{\mu}{2\pi\hbar^2} \int V(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} \mathrm{d}^3 r$$

$$\mathbf{q} = \mathbf{k}_f - \mathbf{k}_i$$

$$\sigma(\theta, \varphi) = |f_k(\theta, \varphi)|^2$$

On a donc,

$$\begin{aligned}
f_k(\theta, \varphi) &= \frac{\mu}{2\pi\hbar^2} \int \sin\tilde{\theta} \mathrm{d}\tilde{\theta} \mathrm{d}\tilde{\varphi} r^2 \mathrm{d}r V(r) e^{-iqr\cos\tilde{\theta}} \\
&= \frac{\mu}{\hbar^2} \int_0^\infty \int_{-1}^1 r 2 V(r) e^{-iqr} \mathrm{d}u \mathrm{d}r = \frac{\mu}{\hbar^2} \int_0^\infty \mathrm{d}r r^2 V(r) \left[\frac{e^{-iqr} - e^{iqr}}{-2iqur}\right] 2 \\
&= \frac{2\mu}{\hbar^2 q} \int_0^\infty \mathrm{d}r r V(r) \sin(qr) = \\
&= \frac{2\mu}{\hbar^2 \sin(\theta/2)} \int_0^\infty \mathrm{d}r r V(r) (kr \sin(\theta/2)
\end{aligned}$$

Intégrale commune

$$I_1 = \int_0^\infty \mathrm{d}r r e^{-\alpha r^2 + \beta r}$$

On peut obtenir le résultat de cette intégrale à partir d'une intégrale connue :

$$I_0 = \int_0^\infty \mathrm{d}r e^{-\alpha r^2 + \beta r} = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}} e^{\frac{\beta^2}{4\alpha}}$$

$$I_1 = \frac{\mathrm{d}I_0}{\mathrm{d}\beta} = \frac{\beta}{4\alpha} = \sqrt{\frac{\pi}{\alpha}} e^{\beta^2/4\alpha}$$

$$f_k(\theta) = \frac{\mu}{\hbar^2 k \sin(\theta/2) 2i} \int_0^\infty \mathrm{d}r r V_0 e^{-r^2/a^2} \left[e^{iqr} - e^{-iqr}\right]$$

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