Action je crois

$$S = \int_{t_i}^{t_f} \mathrm{d}t \frac{1}{2} m \mathbf{v}^2 (T) \quad \text{non-relativiste}$$

$$\to -m \int_A^V \mathrm{d}\tau = -m \int_A^B \mathrm{d}t \sqrt{1-\mathbf{v}^2} = -m \int_A^B \mathrm{d}t \{1 - \frac{1}{2}\mathbf{v}^+ \frac{12}{\mathbf{v}^2} - \cdots \} = -m(t_B - T_a) + \frac{1}{2} \int_A^B \mathrm{d}\tau m \mathbf{v}^2 - \cdots$$

$L = -m\sqrt{1-\mathbf{v}^2}$

$$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}} = \frac{m\mathbf{v}}{\sqrt{1-\mathbf{v}^2}}$$

$$H = \mathbf{p}\mathrm{d}\mathbf{v} - L = \frac{m\mathbf{v}^2}{\sqrt{1-v^2}} + m\sqrt{1-v^2} = \frac{m}{\sqrt{1-\mathbf{v}^2}} = \sqrt{\mathbf{p}^2 + m^2}$$

<u>4-impulsion</u>

$p^\mu = (E, \mathbf{p}) = m u^\mu$

$$u^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \left(\frac{\mathrm{d}t}{\mathrm{d}\tau}, \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\tau} \right) = \left(\frac{1}{\sqrt{1-v^2}}, \frac{\mathbf{v}}{\sqrt{1-v^2}} \right)$$

Invarient associé au quadri-vecteur

$$p^\mu p_{mu} = p^2 = E^2 - \mathbf{p}^2 = m^2 \cancelto{1}{u^2} = m^2$$

$$p^2 = m^2$$

$$\mathbf{v} = \frac{\mathbf{p}}{E}$$

masse nulle $\to p^2 = 0 \to T = |\mathbf{p}|$

$$p_\pi = (m_\pi, \mathbf{0})$$

$$p_\pi = p_\mu + p_\nu$$

$p_\nu = p_\pi - p_\mu$

$$p_\nu^2 = p_\pi^2 + p_\mu^2 - 2 p_\pi p_\mu$$

$$0 = m_\pi^2 + m_\mu^2 - 2 m_\pi E_\mu$$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}$$

$$E_{nu} = m_\pi - E_\mu = \frac{m_\pi^2 - m_\mu^2}{m_\pi} = |\mathbf{p}_\nu| = |\mathbf{p}_\nu|$$

$$|\mathbf{v}_\mu| = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_\mu^2}$$

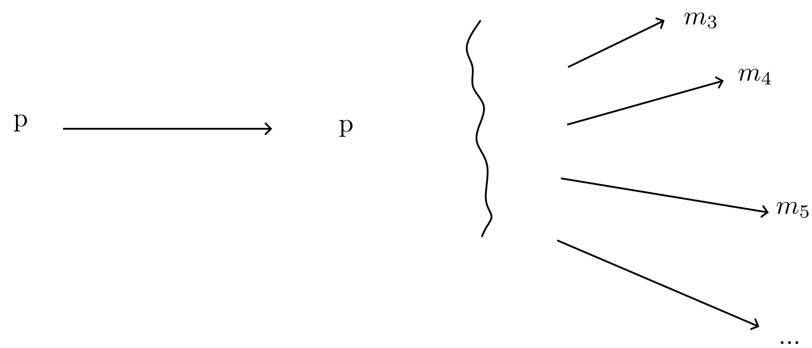

FIGURE 1 – proton incident

Énérgie de seuil ? (Plus d'Énérgie cinétique à la fin)

$$E = \sum_{i=3}^{N} m_i$$

$$p_1^\mu + p_2^\mu = \sum_{i=3}^{N} p_i^\mu$$

$$(p_1^\mu + p_2^\mu)^2 = \left[\sum_{i=3}^{N} p_i^\mu\right]^2 = \left(\sum_{i=3}^{N} m_i\right)^2$$

au <u>seuil</u> $p_i = (m_i, \mathbf{0})$

$$E_p = \frac{M_{\text{tot}}^2 - 2m_p^2}{2m_p}$$

L'énérige requise va comme le carré des masses.

Unités naturelles

$$\hbar c = 197\text{MeV}\cdot\text{fm}$$

$$\frac{\hbar c}{m_e c^2} = \frac{197\text{MeVfm}}{0,511\text{MeV}} = 400\text{fm} \quad \text{longueur d'onde de Compton}$$

Constante de structure fine

$$\alpha = \frac{e^2}{4\pi\hbar c} \sim \frac{1}{137}$$

Heaviside-Lorentz

$$\epsilon_0 = 1 \qquad \mu_0 = 1$$

$$\frac{\alpha\hbar}{m_e c} = \frac{e^2}{4\pi m_e c^2} = \text{ rayon classique de l'éléctron}$$

$$\frac{\hbar}{m_e \alpha c} = \frac{4\pi\hbar c\hbar}{e^2 m_e c} = \frac{4\pi\hbar^2}{m_e e^2} = \text{ rayon de bohr}$$

onde plane

$$\psi(\mathbf{r}) = \frac{1}{\sqrt{\nu}} e^{i\mathbf{p}\cdot\mathbf{r}}$$

Condition au limite périodiques à l'univers (une boîte bien sûr)

$$e^{ip_x L_x} = 1$$

$$\mathbf{p} = 2\pi\left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z}\right)$$

où $n \in \mathbb{Z}$

$$\Delta p_x = \frac{2\pi}{L_x} \leftrightarrow \Delta n_x = 1$$

$$\Delta p_x \Delta p_y \Delta p_z = \frac{(2\pi)^3}{L_x L_y L_z} = \frac{(2\pi)^3}{\nu}$$

$$\sum_{\mathbf{p}} f_{\mathbf{p}} = \nu \int \frac{\mathrm{d}P}{(2\pi)^3} f_{\mathbf{p}}$$

$$\langle \mathbf{p}' | \mathbf{p} \rangle = \delta_{\mathbf{p}\mathbf{p}'} \& \sum_{\mathbf{p}} |\mathbf{p}\rangle \langle \mathbf{p}| = \mathbb{1}$$

$$\langle \mathbf{r} | \mathbf{p} \rangle = \frac{1}{\sqrt{\nu}} e^{i\mathbf{p}\cdot\mathbf{r}}$$

$$\delta_{\mathbf{p}\mathbf{p}'} \to \frac{(2\pi)^3}{\nu} \delta(\mathbf{p} - \mathbf{p}')$$

$$\text{Normalisation continue} \begin{cases} \langle \mathbf{p}' | \mathbf{p} \rangle = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}'') \\ \int \frac{\mathrm{d}^3 P}{(2\pi)^3} \langle \mathbf{p} | \mathbf{p} \rangle = \mathbb{1} \end{cases}$$

On a le problème que $\mathrm{d}^3 P$ n'est pas invarient de Lorentz
$\mathrm{d}^3 p \mathrm{d} p^0$ en revanche l'est

$$\mathrm{d}^3 \gamma \mathrm{d} t = \mathrm{d}^4 x = \mathrm{d}^4 x'$$

Le Jacobien

$$J = 1$$

$$\int \frac{\mathrm{d}^3 P}{(2\pi)^3} \to \int \frac{\mathrm{d}^4}{(2\pi)^3} \delta(p^2 - m) \theta(p^0)$$

$$\int \frac{\mathrm{d}^4 P}{(2\pi)^3} \delta\left((p^0 - E_{\mathbf{p}})(p^0 + E_{\mathbf{p}})\right) \Theta(p^0)$$

$$E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$$

$$\delta(\beta x) = \frac{1}{|\beta|} \delta(x)$$

$$\int \frac{\mathrm{d}^4 P}{(2\pi^3) 2E_p} \delta(p^0 - E_p) = \int \frac{\mathrm{d}^3 P}{(2\pi)^3 2E_p}$$

4

$$\text{Normalisation relativiste} \begin{cases} \int \frac{\mathrm{d}^3 P}{(2\pi)^3 2E_p} \langle \mathbf{p} | \mathbf{p} \rangle = \mathbb{1} \\ \langle \mathbf{p} | \mathbf{p}' \rangle = 2E_{\mathbf{p}} \delta(\mathbf{p} - \mathbf{p}'(2\pi)^2) \end{cases}$$