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Retour

Chaîne de masse

$$L = \frac{1}{2}\mu \sum_{r=1}^{N} \left\{ \dot{u}_r^2 - \Omega^2 u_r^2 - \Gamma^2 \left(u_r - u_{r+1}\right)^2 \right\}$$

$$U_i = Ae^{i(qr-\omega t)}$$

$$\omega_q = \sqrt{\Omega^2 + 2\Gamma^2\left(1-\cos q\right)} = \sqrt{\Omega^2 + 4\Gamma^2 \sin^2 \frac{q}{2}}$$

Limite continue

$$ra \to x,\ u_r \to u(x),\ u_{r+1} - u_r \to a\partial_x u,\ \sum_r \to \int \mathrm{d}x/a,\ \phi(x) = \sqrt{\frac{mu}{\omega}} u(x)$$

On a donc

$$L = \frac{1}{2}\mu \int_0^\ell \mathrm{d}x \frac{1}{a} \left[\dot{u}^2 - \Omega^2 u^2 - \Gamma^2 a^2 \left(\partial_x u\right)^2\right]$$

$$L = \frac{1}{2}\int_0^t \mathrm{d}x \left[\dot{\phi}^2 - \Omega^2\phi^2 - c^2(\partial_x \phi)^2\right]$$

On définir la densité lagrangienne telle que

$$L = \int \mathrm{d}x \mathscr{L}$$

$$\mathscr{L} = \frac{1}{2}\left(\dot{\phi}^2 - \Omega^2\phi^2 - c^2({}_x\phi)^2\right)$$

Équations de Lagrange :

$$\frac{\partial^2 u}{\partial t^2} + \Omega^2 u - a^2\Gamma^2 \frac{\partial^2 u}{\partial x^2} = 0$$

$$\implies \sqrt{m^2c^4 + c^2p^2} \quad p := \frac{q}{a} \quad m := \frac{\Omega}{c^2}$$

Hamiltonien

$$H = \sum_r p_r \dot{u}_r - L$$

Dans le cas quasi-continu on a

$$L = \sum_r a\mathscr{L}(\phi(x_r), \dot{\phi}(x_r))$$

Le moment conjugé est alors

$$\frac{\partial L}{\partial \dot{\phi}} = a\pi(x_r) \quad \text{où} \quad p_r = \frac{\partial L}{\partial \dot{u}_r}$$

$$[\phi(x_r), \pi(x_s)]_p = \frac{1}{a}\delta_{rs}$$

donc, pour un système continue

$$[(x), \pi(x')]_p = \delta(x - x')$$

donc

$$H = \sum_r a\pi(x_r)\dot{\phi}(x_r) - L = \int \mathrm{d}c \left(\pi(x)\dot{\phi}(x) - \mathscr{L}\right)$$

On peut donc le représenter comme

$$H = \int \mathrm{d}x\mathscr{H} \quad \text{où} \quad \mathscr{H} = \pi(x)\dot{\phi}(x) - \mathscr{L}$$

Généralisation à trois dimensions

$$L = \frac{1}{2}\int \mathrm{d}^3r \left\{\dot{\phi}^2 - \Omega^2\phi^2 - c^2(\nabla\phi)^2\right\}$$

L'équation de Lagrange deviens alors

$$\frac{\partial^2\phi}{\partial t^2} + \Omega^2\phi - c^2\nabla^2\phi = 0$$

$$H = \int \mathrm{d}^3x\mathscr{H}$$

$$H = \pi(\mathbf{r})\dot{\phi}(\mathbf{r}) - \mathscr{L}$$

$$\boxed{[\phi(\mathbf{r}), \pi(\mathbf{r}')]_p = \delta(\mathbf{r} - \mathbf{r}')}$$

Action

$$S = \int \mathrm{d}^4 x \left(\partial_\mu \phi \partial^u \phi - m^2 \phi^2 \right)$$

Équation de continuité

$$\boxed{\frac{\partial P}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{J} = 0}$$

shro

$$i \frac{\partial \psi}{\partial t} = -\frac{1}{2}^2 \psi$$

$P = |\psi|^2$

$\mathbf{J} = \frac{1}{2m} \left(\psi * \boldsymbol{\nabla} \psi - \psi \boldsymbol{\nabla} \psi^* \right)$

$\frac{\mathrm{d}P}{\mathrm{d}t} = \psi^* \dot{\psi} - \dot{\psi}^* \psi$

$\boldsymbol{\nabla} \mathbf{J} = \frac{1}{2m} \{ \cdots \}$

$\frac{1}{2} \psi^* (0) + (0) \psi$

L'équation de Klein-Gordon n'as pas cette propriété

Cette dernière conserve bien le quadri-courrant mais $J^2 = p \ngtr 0$