1 Théorème de Wigner-Eckart

$\mathbf{v}$ est vectroiel si $[J_i, v_i] = i\hbar\epsilon_{\text{ijk}}V_k$

$\mathbf{J}$ est vectoriel. Si $\mathbf{J} = \mathbf{L} + \mathbf{S}$, $\mathbf{S}$, $\mathbf{L}$ le sont aussi

$$[J_i, L_j + S_j] = [J_i, L_i] + [J_i, S_i] = i\hbar\epsilon_{\text{ijk}}(L_k + S_k)$$

$$[J_x, V_x] = 0$$

$$[J_x, v_y] = i\hbar V_z$$
$$[J_x, \underbrace{V_x \pm iV_y}_{V_\pm}] = \mp\hbar V_z$$

$$[J_z, V_z] = 0$$

$$\mathcal{P}_\mathcal{E} = \sum_{\mathrm{m}} |k, j, m\rangle\langle k, j, m|$$

$$\mathcal{P}_\mathcal{E} V_{\mathrm{z}} \mathcal{P}_\mathcal{E} = \alpha P_\mathcal{E} J_z P_\mathcal{E}$$

$$\langle k, j, m| V_\pm |k', j', m'\rangle = \pm\frac{1}{\hbar} \langle k, j, m| [J_z, V_p m] |k', j', m'\rangle$$

frac12

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2 Charge

2.1 Composition de 2 spins

$$H_1 \otimes H_1 = H_2 \oplus H_2 \oplus H_0$$

$$|j_1 - j_2| = 0 \leq H \leq j_1 + j_2 = 2$$

$J = 2$

$$|2. + 2\rangle\, |1, +1; 1, +1\rangle$$
$$|2, -2\rangle = |1, -1; 1, -1\rangle$$

M/S	2	1	0
$+2$	$\|2,+2\rangle$		
$+1$	$\|2,+1\rangle$	$\|1,+1\rangle$	
0	$\|2,0\rangle$	$\|1,0\rangle$	$\|0,0\rangle$
$+1$	$\|2,-1\rangle$	$\|1,-1\rangle$	
$+1$	$\|2,-2\rangle$		

Table 1 – Tableau de toutes les valeurs possible

$$J_- |2,+2\rangle = \hbar\sqrt{2(2+1)-2(2-1)}\,|2,+1\rangle = (J_{1-}+J_{2-}\,|1,+1,1,+1\rangle) = \hbar\sqrt{1(1+1)-1(1-1)}\,|1,0;1,+1\rangle + \hbar\sqrt{2}\,|1,+1,1,0\rangle$$

$$|2,\pm1\rangle = \frac{1}{\sqrt{2}}(|1,\pm1,1,0\rangle + |1,0,1,\pm1\rangle)$$

$$J_1 |2,+1\rangle = \hbar\sqrt{2(2+1)-1(1-1)}\,|2,0\rangle = \frac{1}{\sqrt{2}}(J_{1-} + J_{2-}[|1,+1,1,0\rangle + |1,0;1,+1\rangle])$$

$$= \frac{\hbar}{\sqrt{2}}\Big[\sqrt{2}\,|1,0,1,0\rangle + \sqrt{2}\,|1,1,1,-1\rangle + \sqrt{2}\,|1,0,1,0\rangle + \sqrt{2}\,|1,1,1,-1\rangle\Big]$$

$$|2,0\rangle = \frac{1}{\sqrt{6}}(|1,-1,1,1\rangle + |1,1,1,-1\rangle + 2\,|1,0,1,0\rangle)$$

On a fini la première colone !

$$|1,+1\rangle = \alpha\,|1,+1,1,0\rangle + \beta\,|1,-;1,+1\rangle$$

$$J_+ |1,+1\rangle = 0 = \hbar\sqrt{2}\alpha\,|1,+1,1,0\rangle + \hbar\sqrt{2}\,|1,0,1,+1\rangle$$

$$\implies \alpha = -\beta$$

$$|1,+1\rangle = \frac{1}{\sqrt{2}}(|1,+1;1,0\rangle - |1,0;1,+1\rangle)$$

$$|1,-1\rangle = \frac{1}{\sqrt{2}}\,|1,-1;1,0\rangle - |1,0;1,-1\rangle$$

$$J_- |1,+1\rangle = \cdots \implies |1,0\rangle\,\frac{1}{\sqrt{2}}(|1,+1;1,-1\rangle - |1.-1;1,+1\rangle)$$

$$|0,0\rangle = \alpha\,|1,0,1,0\rangle + \beta\,|1,+1,1,-1\rangle + \gamma\,|1,-1,1,+1\rangle$$

$$0 = J_- |0,0\rangle = \hbar\alpha\sqrt{2} + \cdots \implies \alpha+\beta+\alpha+\gamma = 0$$

$$\implies |0,0\rangle = \frac{1}{\sqrt{3}}[|1010\rangle - |11;1-1\rangle - |1,-1,1,+1\rangle]$$

(On a utlisé la normalisation comme 3eme équation)