2022-32-07

$$\mathscr{H} = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{B}^2\right)$$

Comme on étudie le champ électromagnétique seul (sans source), on a

$$\boldsymbol{\nabla} \cdot \mathbf{A} = 0 \qquad \Phi = 0$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \qquad \mathbf{B} = \boldsymbol{\nabla} \wedge \mathbf{A}$$

$$L = \frac{1}{2}\int \mathrm{d}^3\tau \left(\mathbf{E}^2 - \mathbf{B}^2\right) = \frac{1}{2}\int \mathrm{d}^3 r \left\{\dot{\mathbf{A}}^2 - \left(\boldsymbol{\nabla}\mathbf{A}\right)^2\right\}$$

$$\mathbf{A}(\mathbf{r}) = \frac{1}{\mathcal{V}}\sum_q \mathbf{A}_q e^{i\mathbf{qr}}$$

$$\boldsymbol{\nabla} \wedge \mathbf{A}(\mathbf{r}) = \frac{-i}{\sqrt{\mathcal{V}}}\sum_q \mathbf{A}_q \wedge \mathbf{q} e^{i\mathbf{qr}}$$

$$\begin{aligned}
L &= \frac{1}{2\mathcal{V}}\int \mathrm{d}^3 r \sum_{q,q'} \left\{\dot{A}_q \dot{A}'_q + (\mathbf{q} \wedge \mathbf{A}_q) \cdot (\mathbf{q} \wedge \mathbf{A}_q)\right\} e^{-i\mathbf{r}\left(\mathbf{q}-\mathbf{q}'\right)} \\
&= \frac{1}{2}\sum_{q,q'} \dot{A}_q^* \cdot \dot{A}_{q'} - \left(q^2 \mathbf{A}_q^* \cdot \mathbf{A}_q - (\mathbf{q} \cdot \mathbf{A}_q)(\mathbf{q} \cdot \mathbf{A}_q)\right) \\
&= \frac{1}{2}\sum_q \left\{\dot{\mathbf{A}}_q \dot{\mathbf{A}}_{\mathbf{q}-\mathbf{q}^2 A_{\mathbf{q}^* \cdot \mathbf{A}_\mathbf{q}}}\right\} \\
&= \frac{1}{2}\sum_{q,j=1,2} \left\{\dot{A}_{jq}^* \dot{A}_{jq} - \omega_q^2 A_{jq}^* A_{jq}\right\}
\end{aligned}$$

Comme avec la champ scalaire, on va pouvoir définir des opérateurs de création et d'annihilation

flashback du champ scalaire

$$L = \frac{1}{2}\sum_q \left\{\dot{\phi}_q^* \dot{\phi}_q - \omega_q^2 \phi_q^* \phi_q\right\}$$

$$\phi(\mathbf{r}) = \frac{1}{\sqrt{\mathcal{V}}}\sum_p \frac{1}{\sqrt{2\omega_p}}\left(e_\mathbf{p} e^{i\mathbf{pr}} + a_p^\dagger e^{-i\mathbf{pr}}\right)$$

$$[a_{jq}, a_{j'q'}^\dagger] = \delta_{jj'}\delta_{qq'}$$

$$A(\mathbf{r},t) = \frac{1}{\sqrt{\mathcal{V}}} \sum_{p,j} \frac{1}{\sqrt{2\omega_p}} \left(a_{jq}\epsilon_{jq} e^{i\mathbf{qr}+i\omega t} + a^\dagger_{jq}\epsilon^*_{jq} e^{-i\mathbf{qr}+i\omega t} \right)$$

on a ici utilisé la *jauge transverse* ou $\mathbf{q}\cdot\mathbf{A}_q = 0$

$$H = \sum_{q,j} \omega_q \left(a^\dagger_{jq} a_{jq} + 12 \right)$$

$$\omega_q = |q| \implies \text{masse nulle}$$

vecteur de Poynting

$$\mathbf{P} = \int \mathrm{d}^3 r \mathbf{E} \wedge \mathbf{B} = \sum_{jq} q a^\dagger_{jq} a_{jq}$$

à partir de la densité de quantité de mouvement $\mathbf{E}\wedge\mathbf{B}$ on construit la densité que momement cinétique $\mathbf{r}\wedge(\mathbf{E}\wedge\mathbf{B})$

$$\mathbf{S} = \int \mathrm{d}^3 r \mathbf{r} \wedge (\mathbf{E}\wedge\mathbf{B}) = \mathbf{S}_{\text{orb}} + \sum_{pj} a^\dagger_{jq} a_{jq} \left(\epsilon^*_{jq} \wedge \epsilon_{jq} \right)$$

Électrodynamique quantique (QED)

$$S = \int \mathrm{d}^4 x i \bar{\psi} \gamma^\mu \underbrace{\partial_\mu}_{\to \mathcal{D}_\mu = \partial_\mu + ieA_\mu} \psi - m\bar{\psi}\psi = -\frac{1}{4} \int \mathrm{d}^4 x F_{\mu\nu} F^{\mu\nu}$$

$$S = S_0 + S_{\text{int}}$$
$$S_{\text{int}} = -e \int \mathrm{d}^4 x \bar{\psi} \gamma^\mu \psi A_\mu$$

$$L_{\text{int}} - e \int \mathrm{d}^3 r \bar{\psi} \gamma^\mu \psi A_\mu$$

$$H_{\text{int}} = e \int \mathrm{d}^3 r \bar{\psi} \vec{\gamma} \psi \cdot \vec{A}$$

$$H = -\frac{e}{\mathscr{V}} \int \mathrm{d}^3 r \sum_{j,\mathbf{k}} \sum_{s,s',\mathbf{p},\mathbf{p}'} \frac{1}{\sqrt{2\omega_k \mathscr{V}}} \left(a_{j\mathbf{k}} \varepsilon_{j\mathbf{k}} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}} + a^\dagger_{j\mathbf{k}} \varepsilon^*_{j\mathbf{k}} \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{r}} \right)$$
$$\left(c^\dagger_{\mathbf{p},s} \bar{u}_{\mathbf{p},s} \mathrm{e}^{-i\mathbf{p}\cdot\mathbf{r}} + d_{\mathbf{p},s} \bar{v}_{\mathbf{p},s} \mathrm{e}^{i\mathbf{p}\cdot\mathbf{r}} \right) \gamma \left(c_{\mathbf{p}',s'} u_{\mathbf{p}',s'} \mathrm{e}^{i\mathbf{p}'\cdot\mathbf{r}} + d^\dagger_{\mathbf{p}',s'} v_{\mathbf{p}',s'} \mathrm{e}^{-i\mathbf{p}'\cdot\mathbf{r}} \right)$$

$$i\mathcal{M} = u_\alpha(p_1 s_1) u_\gamma(p_2, s_2) \bar{u}_\beta \bar{u}_{delta} \left(-ie\gamma^\mu_{\beta\alpha} \right) \left(-ie\gamma^\nu_{\delta\gamma} \right) \frac{-ig_{\mu\nu}}{(p_1 - p_2)^2}$$

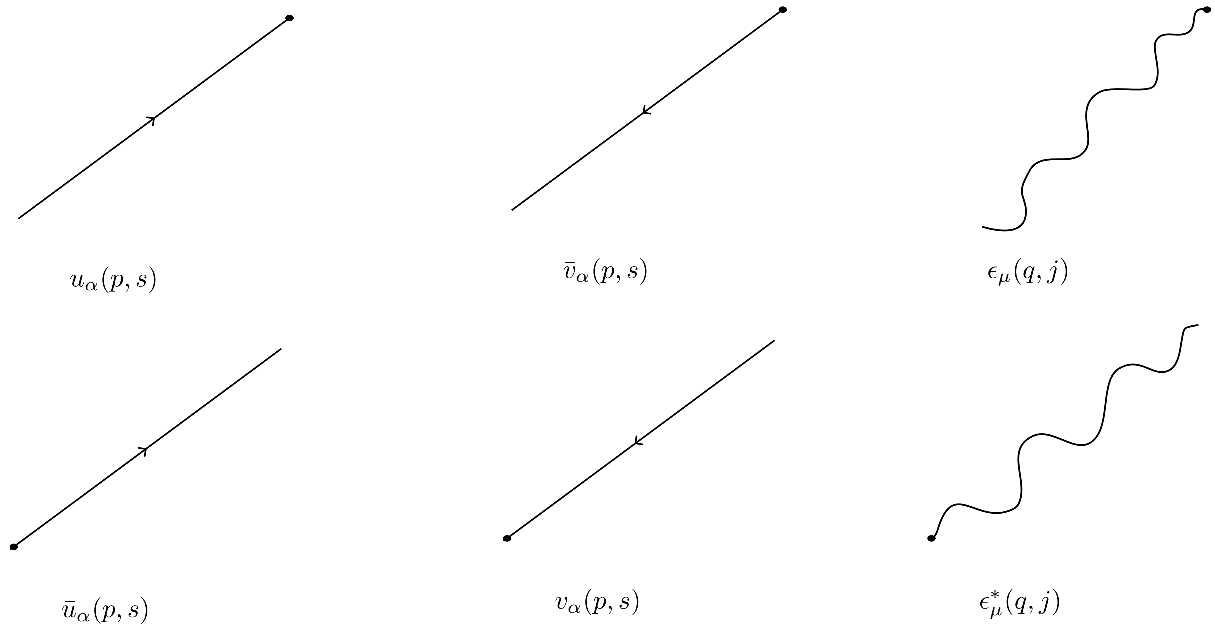

Figure 1 – Diagramme de Feynman : ligne externes

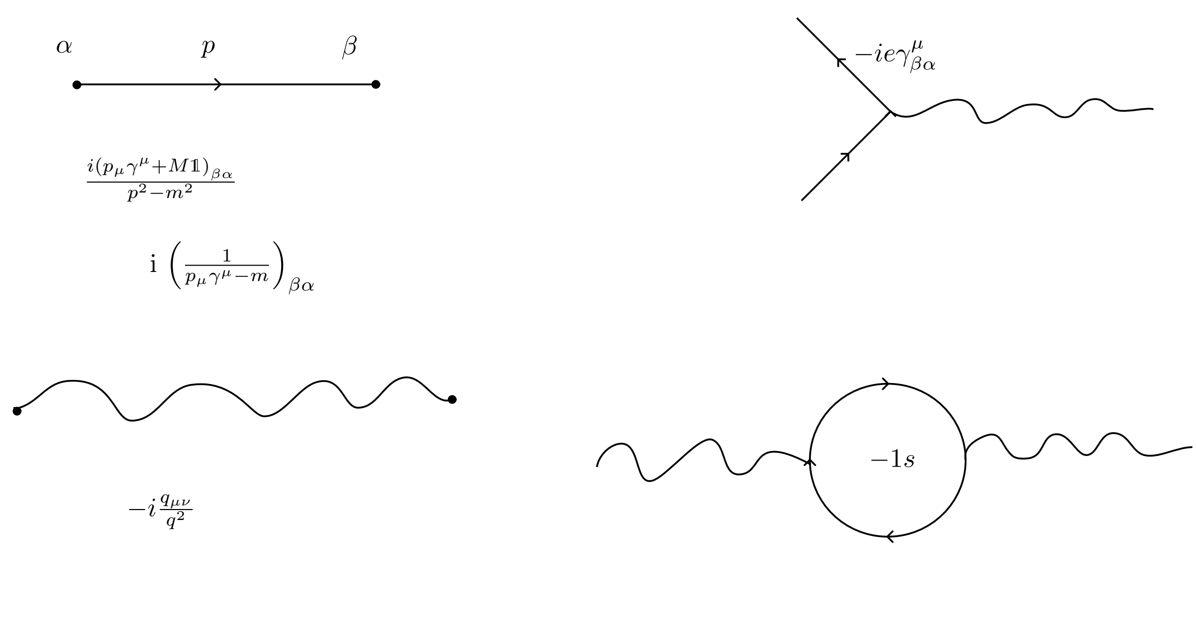

Figure 2 – lignes internes

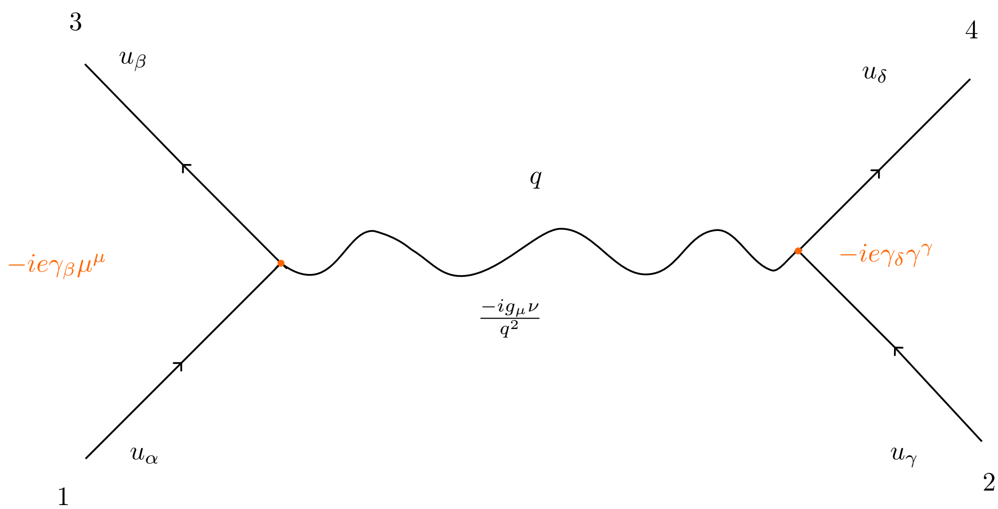

FIGURE 3 – Diffusion électron muon