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Révision

<u>Spineur de Dirac</u>

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} R$$

Le lagrangien de Dirac,

$$\mathscr{L}_D = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

mène à l'équation de Dirac

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

$$\gamma^0 = \begin{bmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{bmatrix} \qquad \gamma^k = \begin{bmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{bmatrix}$$

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2q^{\mu\nu}\mathbb{1}$$

$$\gamma^0 \gamma^\mu \gamma^0 = (\gamma^\mu)^\dagger$$

$$\gamma^\mu \to U\gamma^\mu U^\dagger$$

$$\psi \to U\psi\$$$

<u>Représentation de Dirac</u>

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbb{1} & \mathbb{1} \\ -\mathbb{1} & \mathbb{1} \end{bmatrix}$$

$$\psi = \frac{1}{\sqrt{2}} \begin{bmatrix} \psi_L + \psi_R \\ -\psi_L + \psi_R \end{bmatrix}$$

$$\gamma^0 = \begin{bmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{bmatrix} \qquad \gamma^k = \begin{bmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{bmatrix}$$

<u>Ondes plannes</u>

$$\psi = \psi_{\mathbf{p}} e^{-ip_\mu x^\mu}$$

$$(p_\mu \gamma^\mu - m)\, \psi_{\mathbf{p}} = 0$$

$$\left(E\gamma^0 - \sum_i \gamma^i p^i - m\right) \psi_{\mathbf{p}} = 0$$

En multipliant par γ^0 de la gauche, on obtiens ($(\gamma^0)^2 = \mathbb{1}$)

$$\left(E - \sum_i \gamma^0\gamma^i p^i - m\right) \psi_{\mathbf{p}} = 0$$

$$\underbrace{\begin{bmatrix} m & \mathbf{p}\cdot\sigma \\ \mathbf{p}\cdot\sigma & -m \end{bmatrix}}_{"H"} \psi_p = E\psi_p \quad \text{ou} \quad \underbrace{\begin{bmatrix} m & -i\sigma\cdot\boldsymbol{\nabla} \\ -i\sigma\cdot\boldsymbol{\nabla} & -m \end{bmatrix}}_{"\mathscr{H}"} \psi_p = i\frac{\partial\psi}{\partial t}\psi_p$$

Au repos ($\mathbf{p} = 0$), on a

$$\begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix} \psi_0 = E\psi_0$$

$$E = m \qquad u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$E = -m \qquad u_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$(o_\mu\gamma^\mu + m)\,(p_\mu\gamma^\nu - m) = p_\mu p_\nu \gamma^\mu\gamma^\nu - m^2 = p_\mu p_\mu \frac{1}{2}\left(\gamma^\mu\gamma^\nu + \gamma^\mu\gamma^\nu\right) - m^2 = g^{\mu\nu}p_\mu p_\nu - m^2 = p^2 - m^2$$

$$(p_\nu\gamma^\nu + m) \boxed{(p_\mu\gamma^\mu - m)\, \psi_p} = 0$$

$$\left(p^2 - m^2\right) \psi_{\mathbf{p}} = 0$$

$$p^2 = m^2 \qquad E^2 - \mathbf{p}^2 = m^2$$

m se comporte donc bel et bien comme une masse

$$(p_\nu\gamma^\nu + m) = \begin{bmatrix} p^0 + k & 0 & -p_z & -p_x + ip_y \\ 0 & p^0 + m & -p_x - ip_y & p_z \\ p_z & p_x - ip_y & -p^0 + m & 0 \\ p_x + ip_y & -p_z & 0 & -p^0 + m \end{bmatrix}$$

Les quatres colonnes sont les vecteurs $u_{\mathbf{p}_i}$ si on les normalise par $\frac{1}{\sqrt{2E_{\mathbf{p}}(E_p+m)}}$