2.4.7 :

Calculer le transport parallèle d'un vecteur autour du cercle $u = u_0$ sur le cône $p(u,v) = (u\cos v,\, u\sin v,\, cu)$

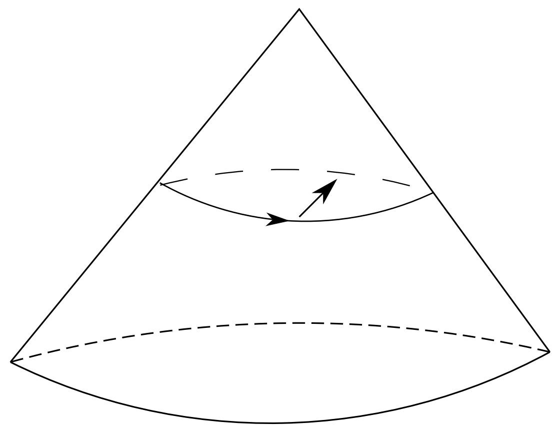

Figure 1 – Cône

Rappel

$f(t)p_u + g(t)p_v$ est $\parallel$ le long de $\alpha(t) = p(u(t), v(t))$ ssi

$$\begin{pmatrix} f' \\ g' \end{pmatrix} = -\begin{pmatrix} u'\Gamma^u_{uu} + v'\Gamma^u_{uv} & u'\Gamma^u_{uv} + v'\Gamma^u_{vv} \\ u'\Gamma^v_{uu} + v'\Gamma^v_{uv} & u'\Gamma^v_{uv} + v'\Gamma^v_{vv} \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}$$

$$\left(\nabla_{\alpha'} e_1\right) e_2$$

si $e_1, e_2 = p_u, p_v$

$$Dp = \begin{pmatrix} \cos v & -u\sin v \\ \sin v & -u\cos v \\ c & 0 \end{pmatrix}$$

$$\alpha = \left(u_0\cos(v), u_0\sin(v), cu_0\right)$$

$$\frac{\partial\alpha}{\partial u} p_u + \frac{\partial\alpha}{\partial v} p_v$$

$$n = \frac{p_u \times p_v}{\|p_u \times p_v\|} = \frac{1}{\sqrt{1+c^2}} \begin{pmatrix} -c\cos v \\ -c\sin v \\ 1 \end{pmatrix}$$

$$p_{uu} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \Gamma_{uu}^u = \Gamma_{uu}^v = 0$$

$$p_{uv} = \begin{pmatrix} -\sin v \\ \cos v \end{pmatrix}$$

$$p_{uv} = u p_v$$

$$\Rightarrow \Gamma_{uv}^u = 0$$

$$\Gamma_{uv}^v = 0$$

$$p_{vv} = \begin{pmatrix} 0 \\ -u\cos v \\ -u\sin v \end{pmatrix}$$

$$P_{vv} = \frac{P_{vv}\cdot P_u}{P_u \cdot P_u} P_u + \frac{P_{vv} \cdot P_v}{P_v \cdot P_v} P_v + P_{vv} \cdot n$$

$$= \frac{-u}{1+c^2} P_u + 0 P_v + \diagdown$$

$$\Gamma^u_{vv} = \frac{-u}{1+c^2}$$

$$\Gamma^v_{vv} = 0$$

$$\Rightarrow \begin{pmatrix} f' \\ g' \end{pmatrix} = \begin{pmatrix} 0 & v'\left(\frac{-u}{1+c^2}\right) \\ v'\left(\frac{1}{u}\right) & u'\frac{1}{u} \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}$$

chemin α:

$u(t) = u_0$

$v(t) = t \quad 0 \leq t \leq 2\pi$

$$\begin{pmatrix} f' \\ g' \end{pmatrix} = \begin{pmatrix} 0 & \frac{u_0}{1+c^2} \\ \frac{-1}{u_0} & 0 \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix}$$

$$\Rightarrow f' = \frac{u_0}{1+c^2} g, \quad g' = \left(\frac{-1}{u_0}\right) f \Rightarrow \ldots$$

$$\Rightarrow r(t) = A\cos\left(\frac{1}{\sqrt{1+c^2}}t\right) + B\sin\left(\frac{1}{\sqrt{1+c^2}}t\right)$$

$$\varphi(t) = \frac{1+c^2}{v_0}\left[\frac{-A}{\sqrt{1+c^2}}\sin(\ldots) + \frac{B}{\sqrt{\ldots}}\cos(\ldots)\right]$$

À la fin du chemin α $t = 2\pi$

$$r(2\pi) = A\cos\left(\frac{2\pi}{\sqrt{\ldots}}\right) + B \ldots$$

Angle de rotation !

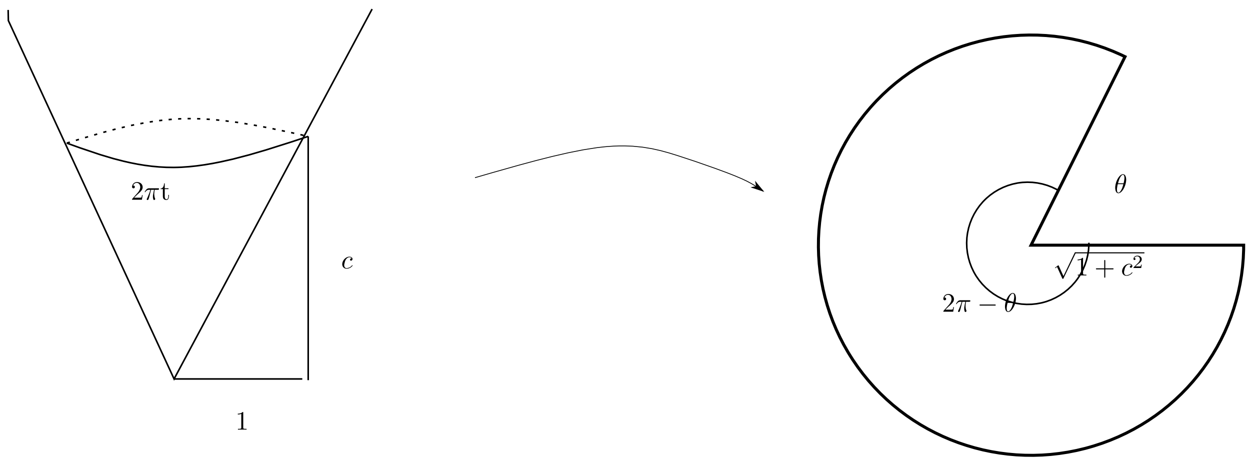

FIGURE 2 – Isomorphisme Euclidien

Exer