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Théorie des champs quantique

$$\text{Classique} \to \text{Quantique}$$
$$[A,B]_p \to \frac{1}{i\hbar}[\hat{A},\hat{B}]$$

$$L = \int \mathrm{d}\mathscr{L} = \int \mathrm{d}^3 r \left\{ \dot{\phi}^2 - m^2\phi^2 - (\boldsymbol{\nabla}\phi)^2 \right\}$$

$$H = \int \mathrm{d}^3 \tau \left\{ \pi(\mathbf{r}) + m^2\phi^2 + (\boldsymbol{\nabla}\phi)^2 \right\} \quad \text{où} \quad \pi(\mathbf{r}) = \frac{\partial \mathscr{L}}{\partial \phi}\dot{\phi}$$
$$[\phi(\mathbf{r}), \pi(\mathbf{r'})]_p = \delta(\mathbf{r}-\mathbf{r'}) \xrightarrow{M.Q.} [\phi(\mathbf{r}), \pi(\mathbf{r'})] = i\delta(\mathbf{r}-\mathbf{r'}) \qquad \hbar = 1$$

On introduit un transformé de Fourrier pour profiter de la symétrie de translation :

$$\phi(\mathbf{r}) = \frac{1}{\nu}\sum_{\mathbf{p}} \phi_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} \quad , \quad \phi_{\mathbf{p}} = \int \mathrm{d}^3 r \phi(\mathbf{r}) e^{-i\mathbf{p}\cdot\mathbf{r}}$$

L'opérateur, une fois passé dans la TF, n'est plus hermitien. Sa conjugason hermitien préserve quand même une expression simple :

$$\phi^\dagger(\mathbf{r}) = \phi(\mathbf{r}) \qquad \phi_{\mathbf{p}^\dagger = \phi_{-\mathbf{p}}}$$

Pareil pour π

$$\pi^\dagger(\mathbf{r}) = \pi(\mathbf{r}) \qquad \pi_{\mathbf{p}^\dagger = \pi_{-\mathbf{p}}}$$

$$H = \frac{1}{2}\int \mathrm{d}^3 r \frac{1}{\nu^2}\sum_{\mathbf{p},\mathbf{p'}} \left\{ \pi_{\mathbf{p}}\pi_{\mathbf{p'}} e^{i\left(\mathbf{p}\cdot\mathbf{p'}\right)} + \cdots \right\}$$

L'intégrale sur les exponentielles donne des $\delta(p-p')$

$$H = \frac{1}{2}\frac{1}{\nu}\sum_{p} \left\{ \pi_{\mathbf{p}}\pi_{-\mathbf{p}} + \left(m^2 - \mathbf{p}^2\right)\phi_{\mathbf{p}}\phi_{-\mathbf{p}} \right\}$$
$$H = \frac{1}{2}\frac{1}{\nu}\sum_{p} \left\{ \pi_{\mathbf{p}}^\dagger\pi_{\mathbf{p}} + \omega_p^2 \phi_{\mathbf{p}}^\dagger\phi_{\mathbf{p}} \right\}$$

$$[\phi_{\mathbf{p}}, \pi_{\mathbf{p'}}] = \int \mathrm{d}^3 r \mathrm{d}^3 r' e^{-i\left(\mathbf{p}\cdot\mathbf{r} + \mathbf{p'}\cdot\mathbf{r'}\right)} [\phi(r), \pi(r')] = \cdots = i\nu\delta_{\mathbf{p},-\mathbf{p'}}$$

$$[\phi_{\mathbf{p}}, \phi_{\mathbf{p}'}] = 0 \qquad [\pi_{\mathbf{p}}, \pi_{\mathbf{p}'}] = 0$$

On introduit alors

$$a_{\mathbf{p}} = \sqrt{\frac{\omega_{\mathbf{p}}}{2\nu}} \left(\phi_{\mathbf{p}} + i \frac{\pi_{\mathbf{p}}}{\omega_{\mathbf{p}}} \right) \qquad a_{\mathbf{p}}^\dagger = \sqrt{\frac{\omega_{\mathbf{p}}}{2\nu}} \left(\phi_{\mathbf{p}}^\dagger - i \frac{\pi_{\mathbf{p}}^\dagger}{\omega_{\mathbf{p}}} \right)$$

$$[a_{\mathbf{p}}, a_{\mathbf{q}}^\dagger] = \cdots = \delta_{p,q}$$

$$H = \sum_{\mathbf{p}} \omega_{\mathbf{p}} \left(a_{\mathbf{p}}^\dagger a_{\mathbf{p}} + \frac{1}{2} \right)$$