Charge 1

Jean-Baptiste Bertrand

19 janvier 2022

$$H_p \frac{1}{2m} = \left\{ \vec{p} \cdot \underbrace{\left[\vec{p} - \frac{q}{c}\vec{A} \right]}_{\vec{\Pi}} \right\}^2 + qV(\vec{R})$$

$$(\vec{\sigma} \cdot \vec{\pi})^2 = \vec{\pi} \cdot \vec{\pi} + i\vec{\sigma} \cdot (\vec{\pi} \times \pi)$$

<u>Preuve :</u>

$$\boxed{\begin{array}{c} \sigma_i^2 = 1 \\ \sigma_i \sigma_j = -\sigma_j \sigma_i \\ \sigma_1 \sigma_2 = i\sigma_3 \end{array}}$$

$$(\vec{\sigma} \cdot \vec{\pi})^2 = \sum_{ij} \sigma_i \pi_i \sigma_j \pi_j = \sum_{ij} + \sum_{i \neq j}$$

... Pas le temps de retranscrire

$$\vec{\pi} \times \vec{\pi} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \left(\frac{\hbar}{i}\vec{\nabla} - \frac{q}{c}\vec{A} \right) \times \left(\frac{\hbar}{i}\vec{\nabla} - \frac{q}{c}\vec{A} \right) \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix}$$

$$= \underbrace{\left(\frac{\hbar}{i} \right)^2 \boldsymbol{\nabla} \times \boldsymbol{\nabla}(f)}_{0} - \frac{q}{c}\vec{A} \times \frac{\hbar}{i}\boldsymbol{\nabla}(f) - \frac{\hbar}{i}\boldsymbol{\nabla} \times \boldsymbol{\nabla}\frac{q}{c}\vec{A}(F) + \underbrace{\left(\frac{q}{c} \right)^2 \vec{A} \times A(f)}_{0}$$

Expenssion du produit vectorielle : On se rend compte que sur A ou que sur f

$$= -\frac{\hbar}{i}\frac{q}{c} \underbrace{\boldsymbol{\nabla} \times \mathbf{A}}_{\mathbf{B}}(f)$$

$$H_p = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\mathbf{A}\right)^2 + \frac{i\vec{\sigma}}{2m}\cdot\left(-\frac{\hbar}{i}\frac{q}{c}\mathbf{B}\right) + qV(\mathbf{R})$$

Le 2eme terme est genre $S \cdot B$ ou dequoi

$$\boxed{Y_0^0 = \frac{1}{\sqrt{4\pi}}}$$

1 Spineurs et mesures

$$[\psi](\vec{r}) = Ne^{-\alpha r^2/2}\begin{pmatrix}\sin\theta\cos\varphi + \sin\theta\sin\varphi \\ 1+\cos\theta\end{pmatrix} = \begin{pmatrix}\psi_+(\vec{r}) \\ \psi_-(\vec{r})\end{pmatrix}$$

$$\psi_0(\vec{r}) = f_0(\mathbf{r})\sum_{l,m} Y_l^m(\theta,\varphi)a_{lm\sigma}$$

$$\mathcal{N}([\phi]) = \int \mathrm{d}^3r\left(|\psi_+(\mathbf{r})|^2 + |\psi_-(\mathbf{r})|^2\right) = \int \mathrm{d}r r^2\left(\sum_{lm\sigma} f_0(r)^2|a_{lm\sigma}|^2\right)$$

$$P(l,m,\sigma) = \frac{1}{\mathcal{N}[\psi]} \times \int \mathrm{d}r r^2 f_0(r)^2|a_{lm\sigma}|^2$$