# Épisode 2

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<u>Spineurs, bases et rerésentations</u>

$$\text{ECOC} : X, Y, Z, S_z, (S^2) : \mathcal{E}_{\vec{r}} \otimes \mathcal{E}_s = \mathcal{E} \ \ |\vec{r}, s\rangle \tag{1}$$
$$\text{ECOC} : P_x, P_y, P_z, S_z; |\vec{p}, s\rangle \tag{2}$$
$$\text{ECOC} : H_0, \mathbf{L}^2, L_z, S_z; |n, l, m, s\rangle \tag{3}$$

Relation de fermeture dans $\mathcal{E}$ :

$$1 = 1_{\vec{r}} \otimes 1_S = \int \mathrm{d}^3 r \, |\vec{r}\rangle \langle \vec{r}| \otimes \sum_{\epsilon} |\epsilon\rangle \langle \epsilon|$$

$$\implies 1 = \sum_{\epsilon} \int \mathrm{d}^3 r \, |\vec{r}\epsilon\rangle\langle \vec{r}, \epsilon|$$

Preuve très similaire pour les autres bases.

$$|\psi\rangle = 1 \, |\psi\rangle = \sum_{\epsilon} \int \mathrm{d}^3 r \, |\vec{r}, \epsilon\rangle \underbrace{\langle \vec{r}, \epsilon | \psi\rangle}_{\Psi_\epsilon(\vec{r})}$$

Représentation matricielle :

$$|\psi\rangle = \int \mathrm{d}^3 r \begin{pmatrix} \psi_+(\vec{r}) \\ \psi_-(\vec{r}) \end{pmatrix} |\vec{r}\rangle$$

$$\langle \vec{r} | \psi\rangle = \begin{pmatrix} \psi_+(\vec{r}) \\ \psi_-(\vec{r}) \end{pmatrix} = [\psi] \ (\textit{Spineur!})$$

$$\langle \psi | = \int \mathrm{d}^3 r \begin{pmatrix} \psi_+^*(\vec{r}) & \psi_-^*(\vec{r}) \end{pmatrix} \langle \vec{r}|$$

$$|\psi\rangle = 1 \, |\psi\rangle = \sum_{\epsilon} \sum_{n,l,m} |n, l, m, \epsilon\rangle \overbrace{\langle n, l, m, \epsilon | \psi\rangle}^{C_{n,l,m,\epsilon}} \tag{4}$$

si

$$|\vec{r}\rangle\langle n, l, m| = R_n, l(r) Y_l^m(\theta, \phi)$$

$$\langle \vec{r} | \psi\rangle = \sum_{n,l,m} \sum_{\epsilon} \underbrace{\langle \vec{r} | n, l, m\rangle}_{R_n, l(r) Y_l^m(\theta,\phi)} |\epsilon\rangle \, C_{n,l,m,\epsilon} = \sum_{n,l,m} \begin{pmatrix} c_{n,l,m,+} R_n, l(r) Y_l^m(\theta, \phi) \\ c_{n,l,m,-} R_n, l(r) Y_l^m(\theta, \phi) \end{pmatrix}$$