2022-09-12

Section différentielle de diffusion

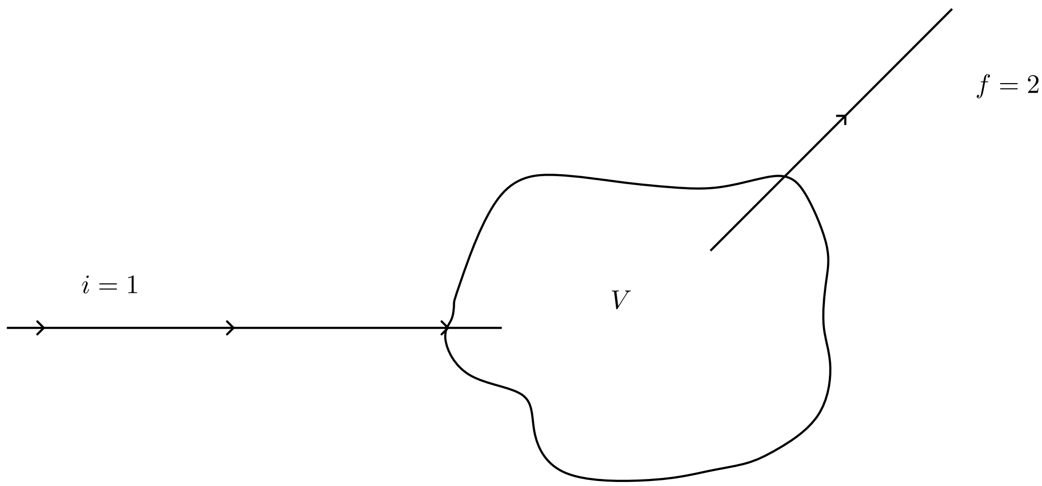

FIGURE 1 – diffusion par un potentiel fixe

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{\Phi}\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega}$$

$$\Gamma = 2\pi\frac{1}{\nu}\int \frac{\mathrm{d}^3 P_?}{(2\pi)^3}|M_{fi}|^2\delta(E_2 - E_1)$$

L'intégrale deviens, en coord sphérique :

$$\frac{1}{\nu}\int \frac{p_2^2 \mathrm{d}p_2 \mathrm{d}\Omega}{(2\pi)^3}$$

Non relativiste : $E_2 = \frac{p_2^2}{2m}$

$$\mathrm{d}E_2 = \frac{p_2}{m}\mathrm{d}p_2$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} = \frac{1}{(2\pi)^3}\nu\int |M_f i|^2 p_2 m \mathrm{d}E_3 \delta(E_{2-E_1}) = \frac{1}{(2\pi)^3}\nu|M_f i|^2|\mathbf{p}|m$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m}{2\pi}\right)^2 \nu^2 |M_{fi}|^2$$

$$\text{Flux : } \rho \underbrace{v}_{\text{vitesse} = \frac{|\mathbf{p}|}{m}}$$

$$M_{fi} = \langle f| V |i\rangle = \langle \mathbf{p}_2| V |\mathbf{p}_1\rangle = d^3r \langle \mathbf{p}_2|\mathbf{r}\rangle V(\mathbf{r}) \langle \mathbf{r}|\mathbf{p}_1\rangle$$

$$= \frac{1}{\nu} \int \mathrm{d}^3 r e^{-i(\mathbf{p}_2 - \mathbf{p}_1)\cdot\mathbf{r}}$$

$$= \frac{1}{\nu} \tilde{V}(\underbrace{\mathbf{p}_2 - \mathbf{p}_1}_{\mathbf{q}=\text{tansfert de } p})$$

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m}{2\pi}\right)^2 \left|\tilde{V}(\mathbf{q})\right|^2}$$

Exemple : Loi de Coulomb

$$V(\mathbf{r}) = \frac{e_1 e_2}{4\pi r}$$

$$\nabla^2 \phi = -\delta(\mathbf{r}) \qquad \phi(\mathbf{r}) = \frac{1}{4\pi r}$$

$$-\mathbf{q}^2 \tilde{\phi}(\mathbf{q}) = -1 \to \tilde{\phi}(\mathbf{q}) = \frac{1}{|q|^2}$$

$$\mathbf{q}^2 = \cdots 4 = \mathbf{p}^2 \sin^2 \frac{\theta}{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}\Omega} = \left(\frac{m e_1 e_2}{8\pi p^2}\right)^2 \mathrm{cosec}^4 \frac{\theta}{2}$$

$$\sigma = \int \mathrm{d}\Omega \frac{\mathrm{d}}{\mathrm{d}\Omega} \to \infty$$

distribution de charge

$$V(\mathbf{r}) = \frac{e_1 e_2}{4\pi r} \to \frac{e_1 e_2}{4\pi r} \int \mathrm{d}^3 r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

c'est une convolution !

$$\tilde{V}(\mathbf{q}) = \frac{1}{|\mathbf{q}|^2} \tilde{\rho}(\mathbf{q})$$

On obtiens donc un simple facteur de correction

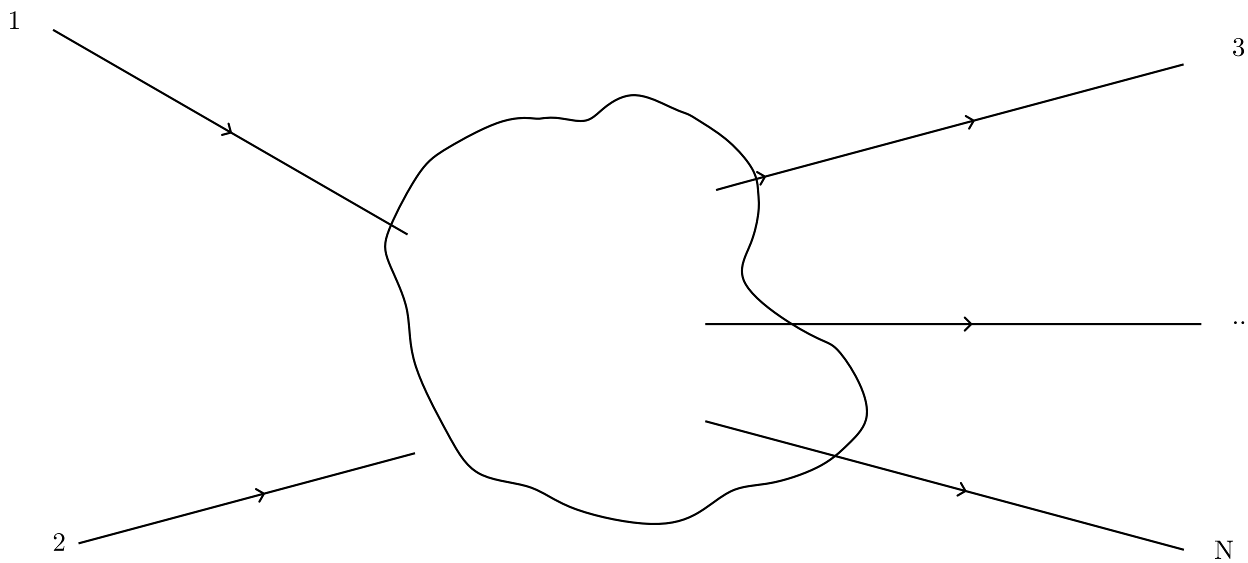

Figure 2 – diffusions à plusieurs particules

Diffusion à plusieurs particules

$$\mathrm{d}\Gamma = |\mathcal{M}_{fi}|^2 \frac{\mathrm{d}^3 P}{(2\pi)^3} (2\pi)^4 \delta^4 (p_{1+p_2} - p_3 - p_4 - \cdots - p_N) \qquad \text{[N.C.]}$$

$$\mathrm{NC} \to \mathrm{NR} \qquad |\mathbf{p}\rangle_{\mathrm{NC}} = \frac{1}{\sqrt{2E}} |\mathbf{p}\rangle_{\mathrm{NR}}$$

$$\mathrm{d}\sigma = |\mathcal{M}_{fi}|^2 \frac{E_1}{|\mathbf{p}|} \frac{\mathrm{d}^3 p_3}{(2\pi)^3} \cdots$$

On veut trouver une quanité qui est egale à $\mathbf{p}_1$ dans le référentiel du laboratoire mais est aussi un invariant

$$(\underbrace{\mathbf{p}_1}_{(E_1,\mathbf{p}_1)} \underbrace{\mathbf{p}_2}_{(m_2,\mathbf{0})})^2 - (m_1 m_2)^2$$

$$E_1^2 m_2^2 - m_1^2 m_2^2 = (E_1^2 - m_1^2) m_2^2 = \mathbf{p}_1^2 m_1^2 (m_2, \mathbf{0})$$

$$
\mathrm{d} = |\mathcal{M}_{fi}|^2 \frac{1}{4\sqrt{(p_1 p_2)^2 - (m_1 m_2)^2}} \frac{\mathrm{d}^3 p_2}{2E_3 (2\pi)^3} \cdots (2\pi)^4 \delta(p_1 + p_2 - p_3 - \cdots - p_N)
$$

Résonances & masse invariante

Masse invariente de N particules

$$
M^2 = \underbrace{(p_1 + p_2 + \cdots + p_N)}_{p_{\text{tot}}}^{2} = (E_1 + \cdots + E_n) - (\mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_N)^2
$$

$$
\rho(E) = \frac{1}{2\pi} \frac{\Gamma}{(E - M)^2 + \left(\frac{\Gamma}{2}\right)^2}
$$

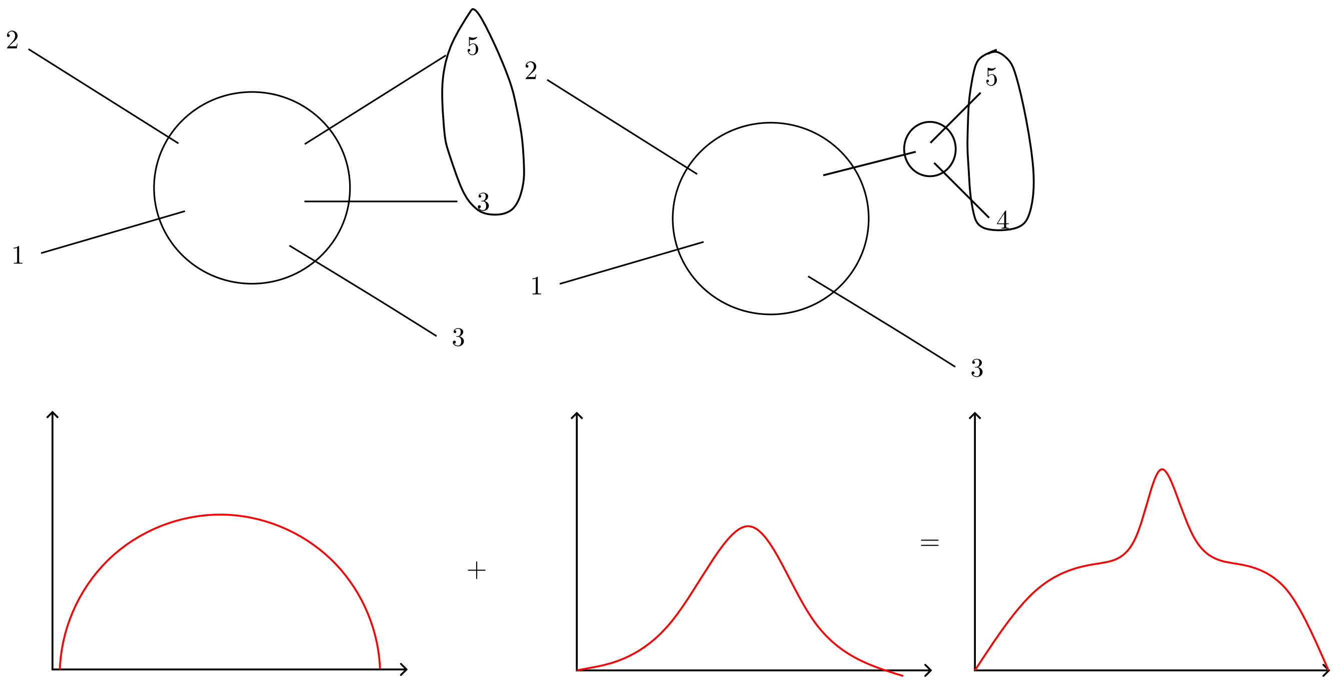

Figure 3 – Désintégration 2

Chaîne de masse μ

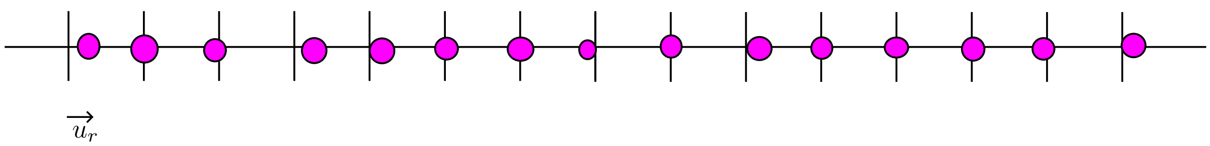

Figure 4 – Chaîne de masse

$$\mathcal{L} = \frac{1}{2}\mu \sum_{r=1}^{N} \left\{ \dot{u}_r^2 - \Omega^2 u_r^2 - \Gamma^2 \left(u_r - u_{r+1}\right)^2 \right\}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{u}_r} - \frac{\partial \mathcal{L}}{\partial u_r} = 0$$

On tourne la manivelle :

$$\omega_q = \sqrt{\Omega^2 + 2\Gamma^2(1 - \cos q)}$$

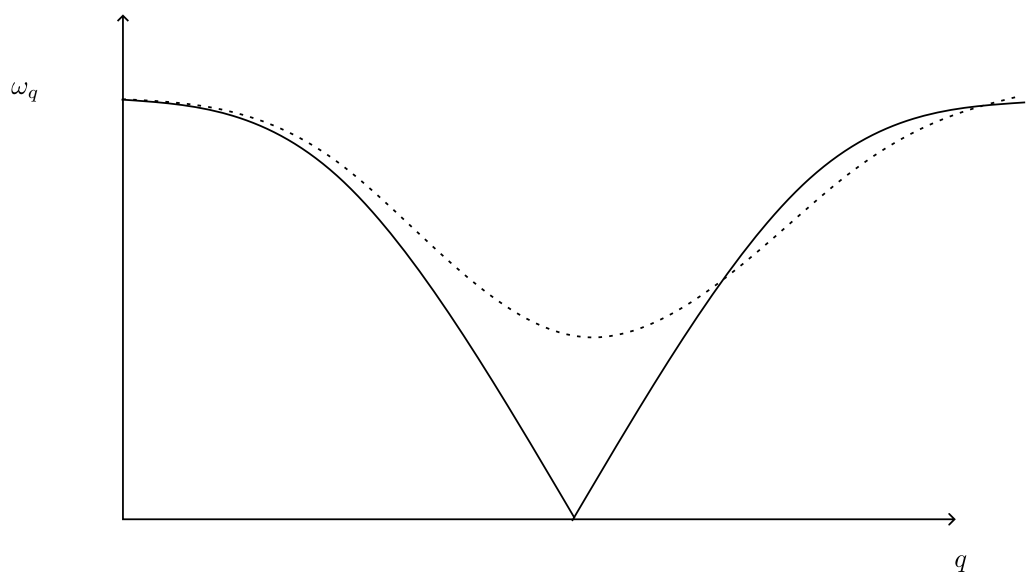

Figure 5 – relation de dispersion