

Nord Quantique Internship summary

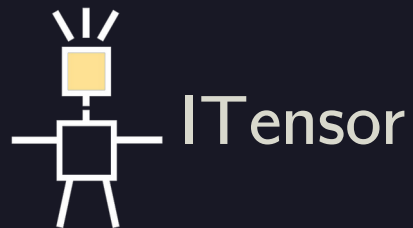
by Jean-Baptiste Bertrand

Similarity to my master's project

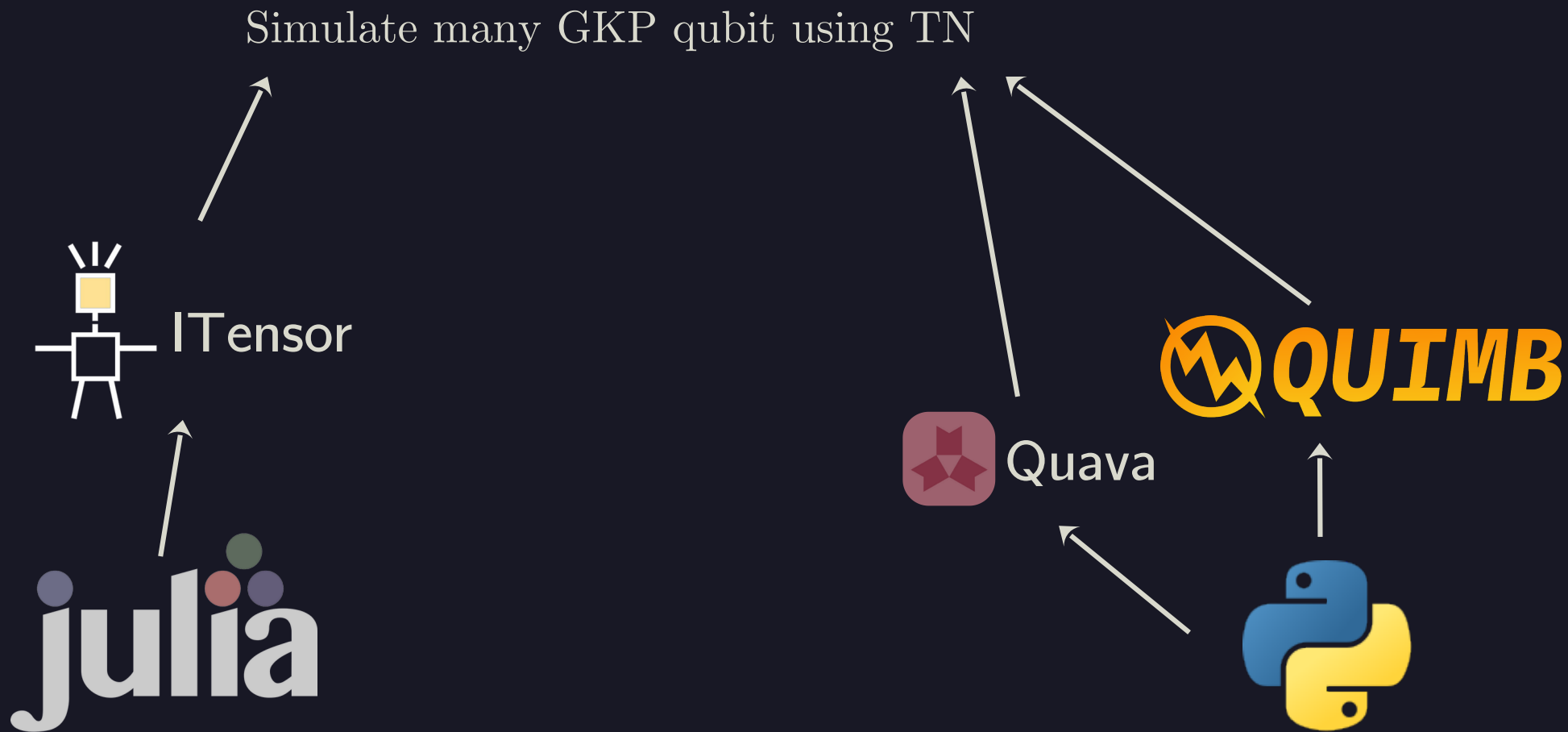
Simulate many GKP qubit using TN

Similarity to my master's project

Simulate many GKP qubit using TN



Similarity to my master's project

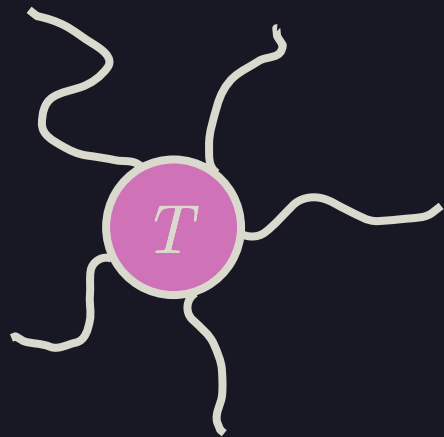


The main ideas of using Tensor Networks

A pictorial representation introduced by Roger Penrose

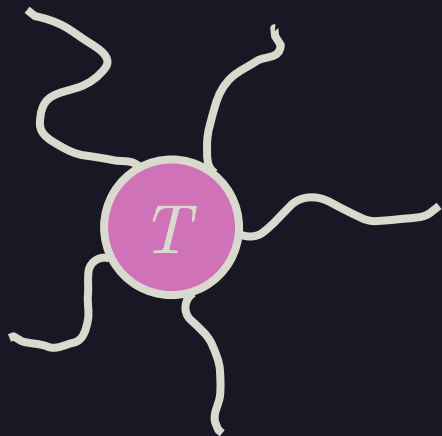
A pictorial representation introduced by Roger Penrose

rank n tensor \rightarrow n legged *blob*



A pictorial representation introduced by Roger Penrose

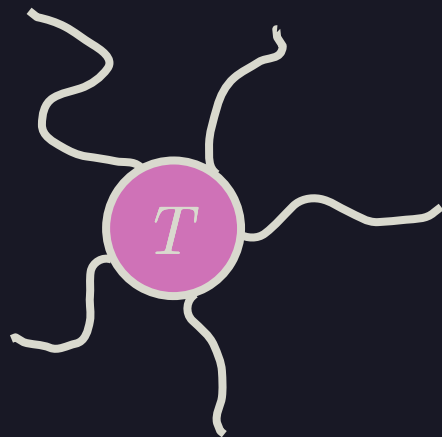
rank n tensor \rightarrow n legged *blob*



$$\psi_i \rightarrow \text{blob}(\psi)_i$$
A diagram showing a central pink circle with a white border, containing the symbol ψ in a white serif font. A single straight white line, representing a leg, extends from the top of the circle, ending with the index i in a white serif font.

A pictorial representation introduced by Roger Penrose

rank n tensor \rightarrow n legged *blob*



$$\psi_i \rightarrow \text{diagram}$$

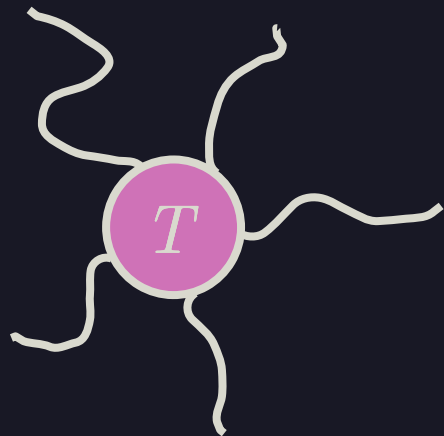
The diagram on the right is a pink circle containing the script letter ψ . A single vertical line extends upwards from the top of the circle, ending with the index i .

$$A^i_j \rightarrow \text{diagram}$$

The diagram on the right is a light blue square containing the letter A . A vertical line extends upwards from the top of the square, ending with the index j . Another vertical line extends downwards from the bottom of the square, ending with the index i .

A pictorial representation introduced by Roger Penrose

rank n tensor \rightarrow n legged *blob*



$$\psi_i \rightarrow$$



$$A^i_j \rightarrow$$



$$T^d_{abc} \rightarrow$$



Examples

$$\sum_i A_j^i \psi_i \rightarrow A_j^i \psi_i \rightarrow \begin{array}{c} \text{j} \\ | \\ \boxed{T} \\ | \\ \text{i} \\ \textcircled{\psi} \end{array} = \begin{array}{c} \text{j} \\ | \\ \textcircled{\psi'} \end{array}$$

Examples

$$\sum_i A_j^i \psi_i \rightarrow A_j^i \psi_i \rightarrow \begin{array}{c} j \\ | \\ \boxed{T} \\ | \\ i \\ \textcircled{\psi} \end{array} = \begin{array}{c} j \\ | \\ \textcircled{\psi'} \end{array}$$

$$\sum_j A_j^i B_k^j \rightarrow A_j^i B_k^j \rightarrow \begin{array}{c} i \\ | \\ \boxed{A} \end{array} \begin{array}{c} j \\ | \\ \boxed{B} \end{array} \begin{array}{c} k \\ | \\ \end{array} = \begin{array}{c} i \\ | \\ \boxed{C} \end{array} \begin{array}{c} k \\ | \\ \end{array}$$

Examples

$$\sum_i A_j^i \psi_i \rightarrow A_j^i \psi_i \rightarrow \begin{array}{c} j \\ | \\ \boxed{T} \\ | \\ i \\ \bigcirc \psi \end{array} = \begin{array}{c} j \\ | \\ \bigcirc \psi' \end{array}$$

$$\sum_j A_j^i B_k^j \rightarrow A_j^i B_k^j \rightarrow \begin{array}{c} i \\ | \\ \boxed{A} \end{array} \begin{array}{c} j \\ | \\ \boxed{B} \end{array} \begin{array}{c} k \\ | \\ \end{array} = \begin{array}{c} i \\ | \\ \boxed{C} \end{array} \begin{array}{c} k \\ | \\ \end{array}$$

$$\vec{v} \cdot \vec{v} \rightarrow \begin{array}{c} \bigcirc v \\ \text{---} \\ \bigcirc v \end{array}$$

Examples

$$\sum_i A_j^i \psi_i \rightarrow A_j^i \psi_i \rightarrow \begin{array}{c} j \\ | \\ \boxed{T} \\ | \\ i \\ \textcircled{\psi} \end{array} = \begin{array}{c} j \\ | \\ \textcircled{\psi'} \end{array}$$

$$\sum_j A_j^i B_k^j \rightarrow A_j^i B_k^j \rightarrow \begin{array}{c} i \\ | \\ \boxed{A} \end{array} \begin{array}{c} j \\ | \\ \boxed{B} \end{array} \begin{array}{c} k \\ | \end{array} = \begin{array}{c} i \\ | \\ \boxed{C} \end{array} \begin{array}{c} k \\ | \end{array}$$

$$\vec{v} \cdot \vec{v} \rightarrow \textcircled{v} - \textcircled{v} \quad \text{tr}(T) = T_i^i \rightarrow \boxed{T} \text{ with a loop}$$

The goal is to use less memory via powerful approximation

Singular Value Decomposition

- Similar to a diagonalisation
- Can be performed on any complex matrix, including rectangular ones

Singular Value Decomposition

- Similar to a diagonalisation
- Can be performed on any complex matrix, including rectangular ones

$$\begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{M} \end{array} \right\} \\ m \end{matrix} = \begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{U} \end{array} \right\} \\ n \end{matrix} \times \begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{S} \end{array} \right\} \\ n \end{matrix} \times \begin{matrix} \boxed{V^\dagger} \\ n \end{matrix}$$

$$\boxed{S} = \begin{pmatrix} s_1 & 0 & 0 & \cdots \\ 0 & s_2 & 0 & \cdots \\ 0 & 0 & s_3 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Singular Value Decomposition

- Similar to a diagonalisation
- Can be performed on any complex matrix, including rectangular ones

$$\begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{M} \\ \underbrace{\hspace{1cm}} \\ m \end{array} \right\} \end{matrix} = \begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{U} \\ \underbrace{\hspace{1cm}} \\ n \end{array} \right\} \end{matrix} \times \begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{S} \\ \underbrace{\hspace{1cm}} \\ n \end{array} \right\} \end{matrix} \times \begin{matrix} \underbrace{\boxed{V^\dagger}}_n \\ \left\{ \hspace{1cm} \right\} n \end{matrix}$$

$$\boxed{S} = \begin{pmatrix} s_1 & 0 & 0 & \cdots \\ 0 & s_2 & 0 & \cdots \\ 0 & 0 & s_3 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Uniquely defined when singular values are ordered in a specific way

Singular Value Decomposition

- Similar to a diagonalisation
- Can be performed on any complex matrix, including rectangular ones

$$\begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{M} \end{array} \right\} \\ m \end{matrix} = \begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{U} \end{array} \right\} \\ n \end{matrix} \times \begin{matrix} m \\ \left\{ \begin{array}{c} \boxed{S} \end{array} \right\} \\ n \end{matrix} \times \begin{matrix} \begin{array}{c} \boxed{V^\dagger} \\ n \end{array} \\ n \end{matrix}$$

$$\boxed{S} = \begin{pmatrix} s_1 & 0 & 0 & \cdots \\ 0 & s_2 & 0 & \cdots \\ 0 & 0 & s_3 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- Uniquely defined when singular values are ordered in a specific way
- Singular values are always real and positive

Splitting Hilbert space



$$\sum_i c_i |i\rangle \longrightarrow \sum_{\mu, \nu} c_{\mu, \nu} |\mu, \nu\rangle$$

Splitting Hilbert space

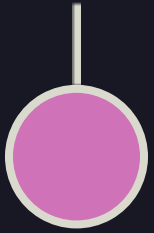


E.g. :

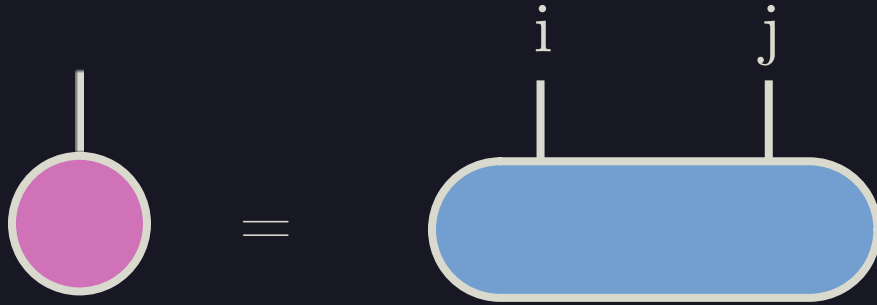
$$|5\rangle \rightarrow |1\rangle|0\rangle|1\rangle$$

$$\sum_i c_i |i\rangle \longrightarrow \sum_{\mu, \nu} c_{\mu, \nu} |\mu, \nu\rangle$$

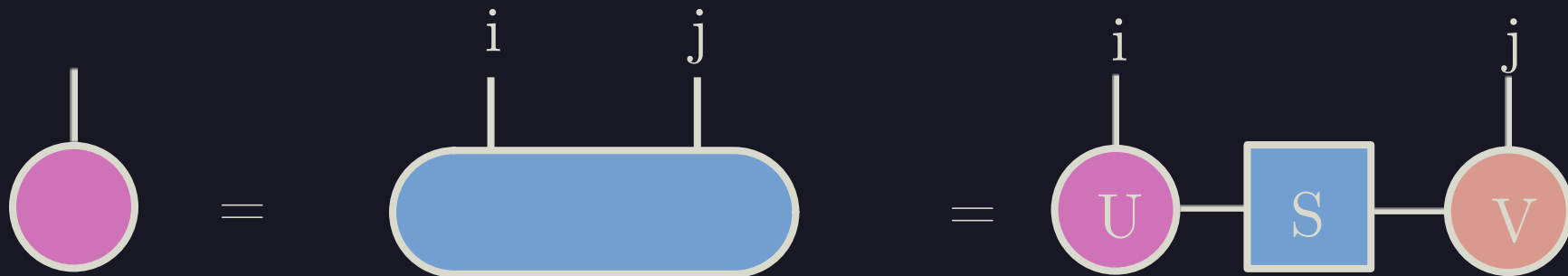
Hilbert space is a small(er) place!



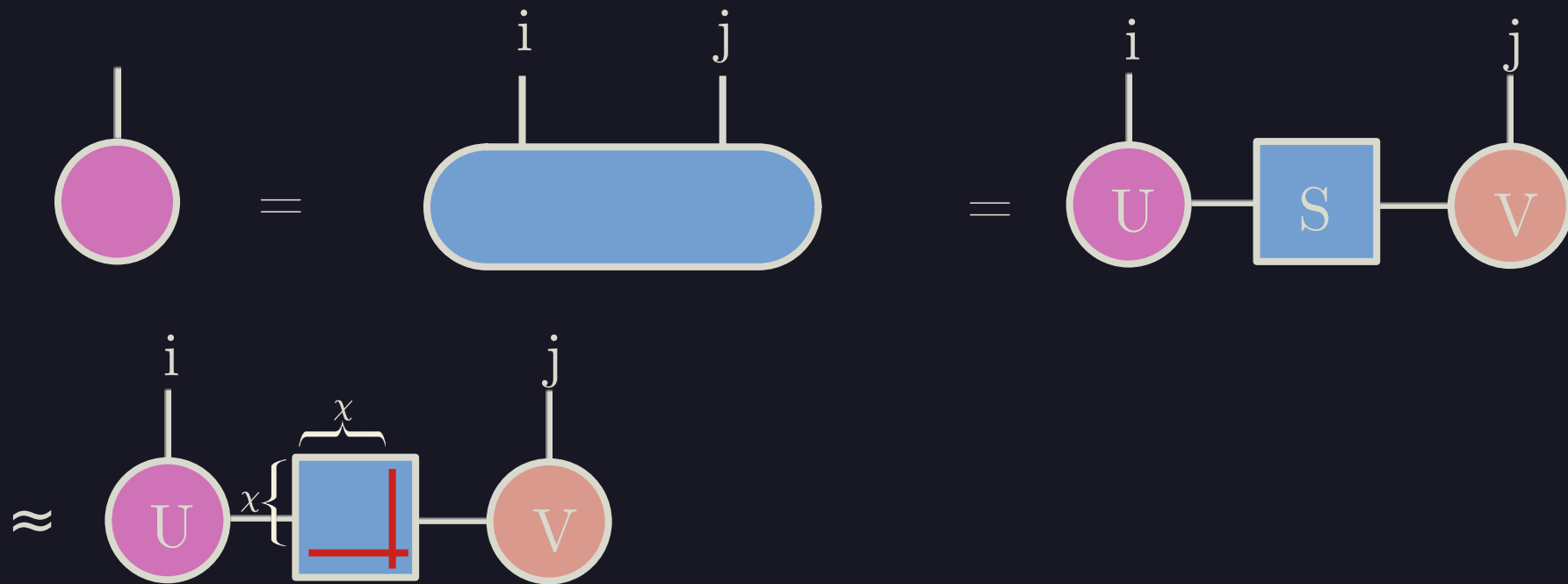
Hilbert space is a small(er) place!



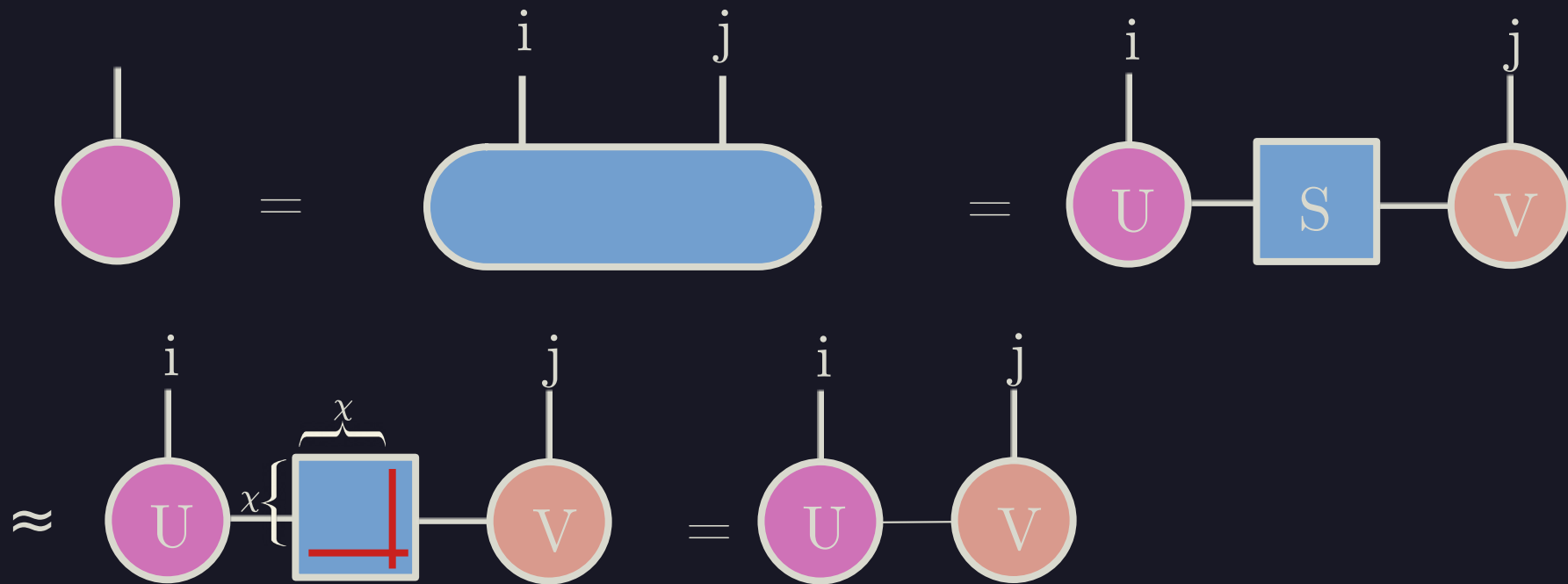
Hilbert space is a small(er) place!



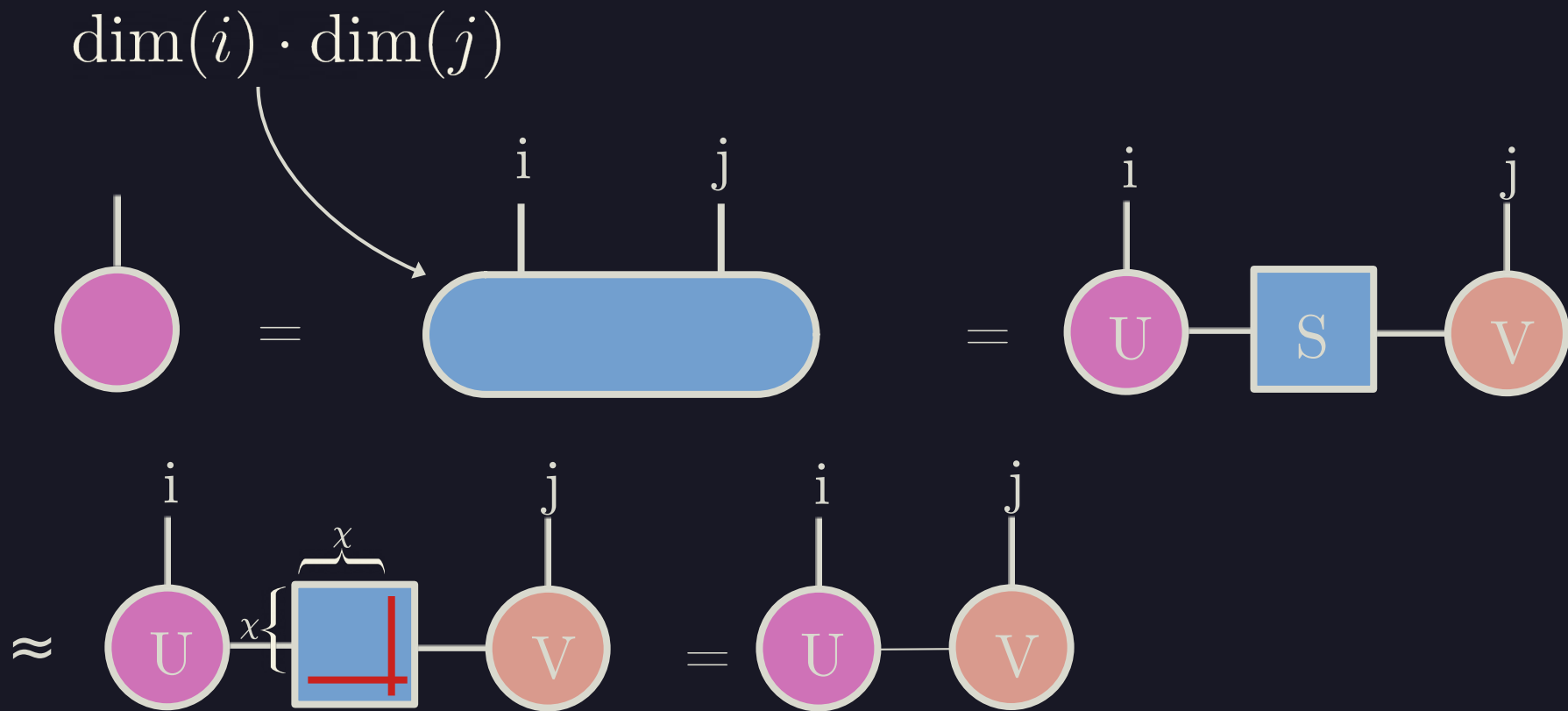
Hilbert space is a small(er) place!



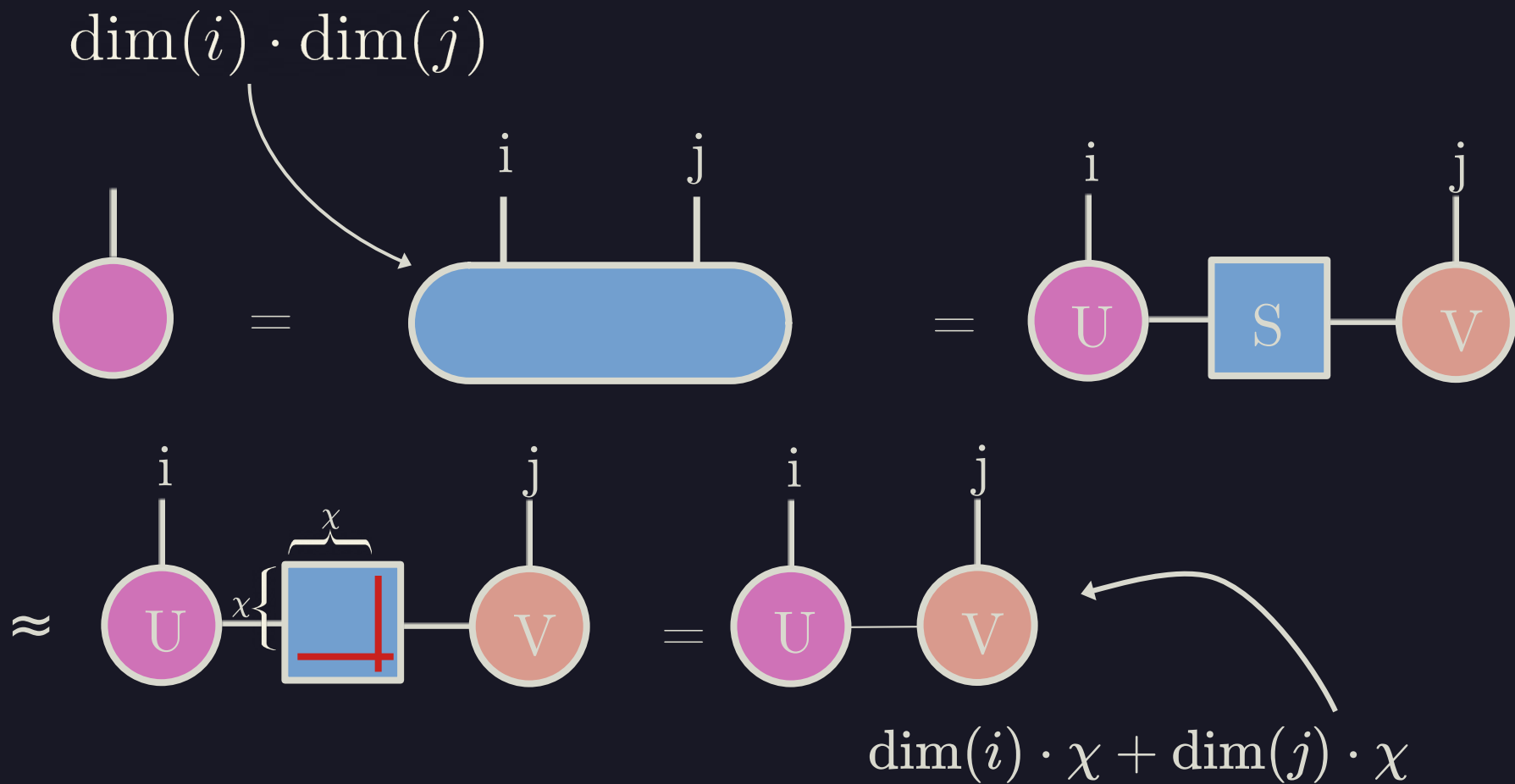
Hilbert space is a small(er) place!



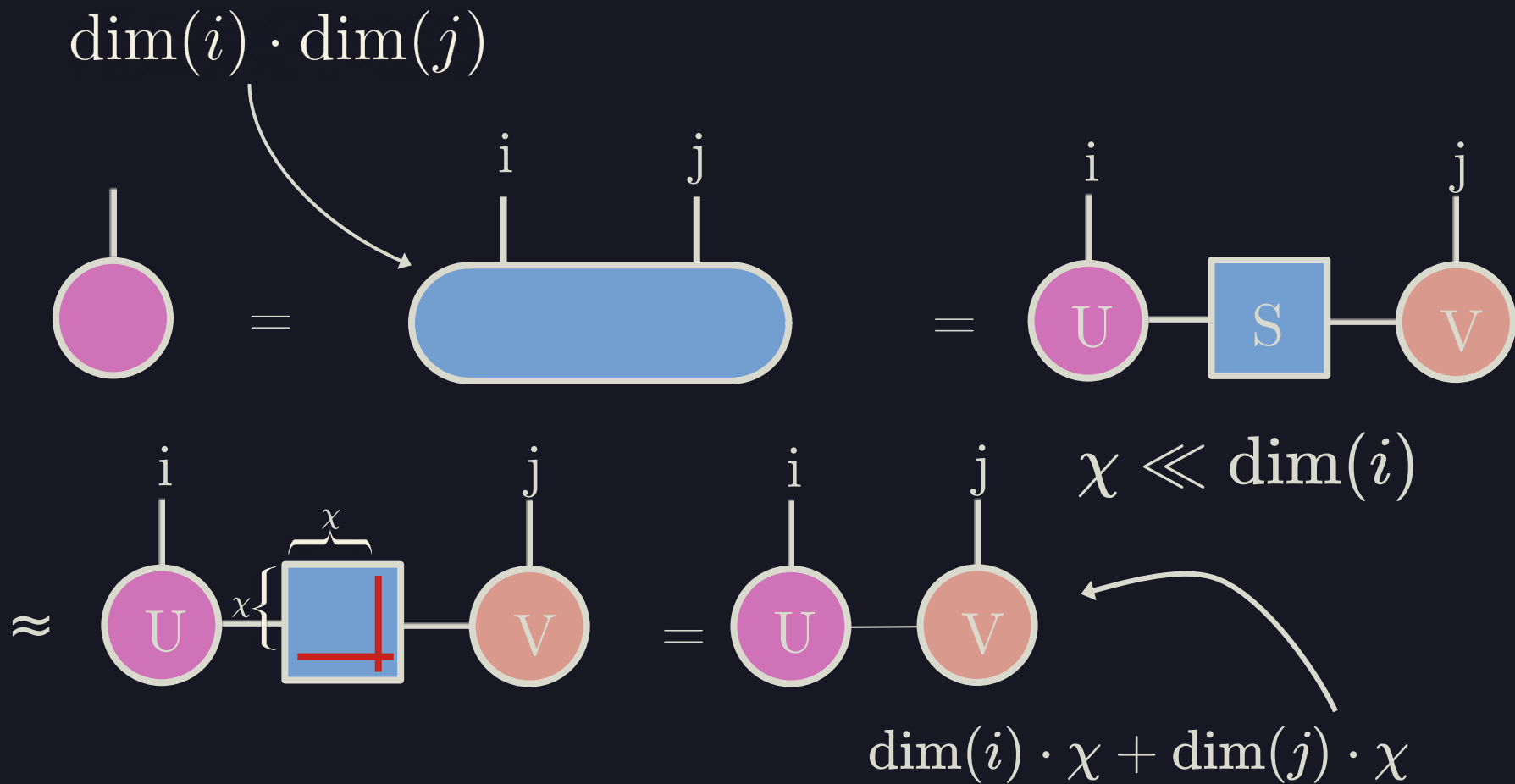
Hilbert space is a small(er) place!



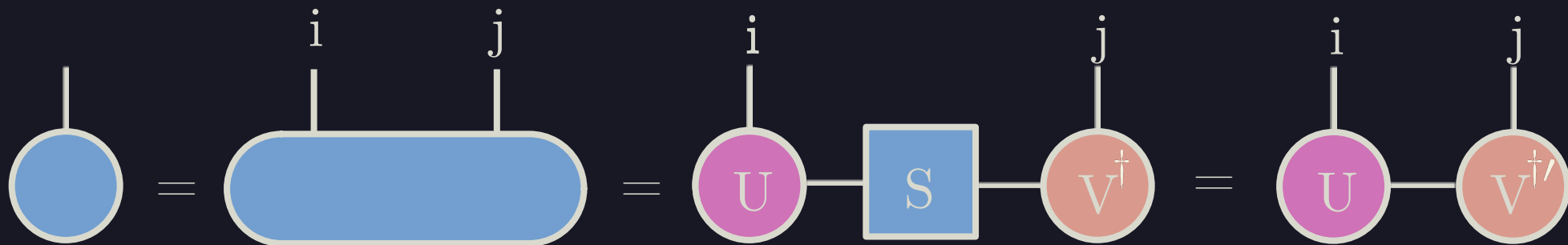
Hilbert space is a small(er) place!



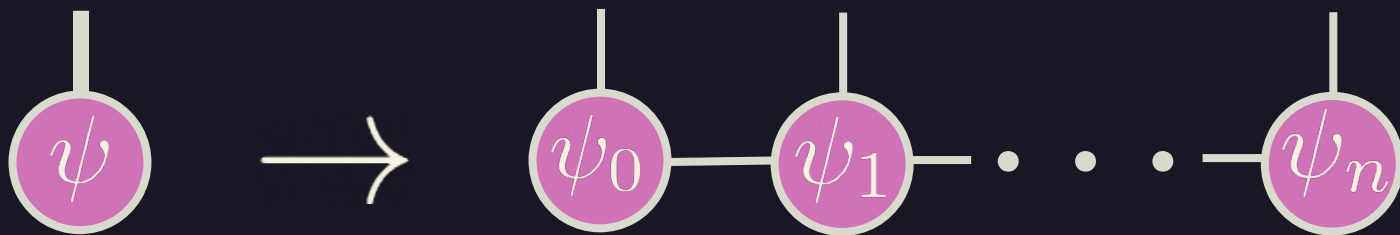
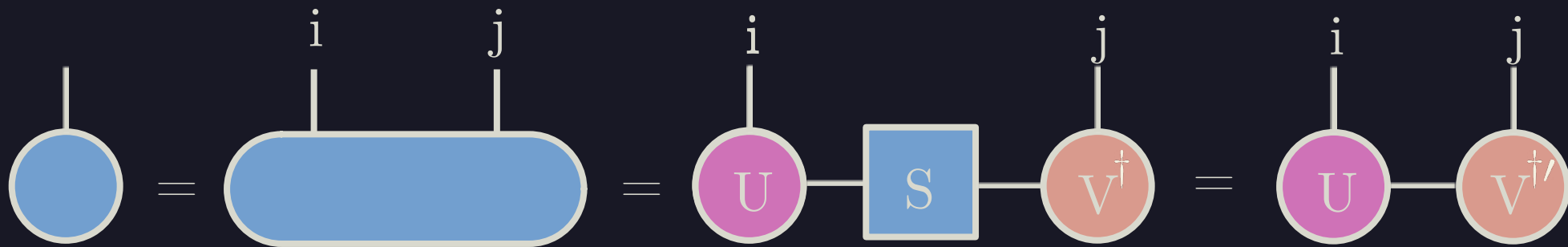
Hilbert space is a small(er) place!



Matrix Product States/Tensor Trains

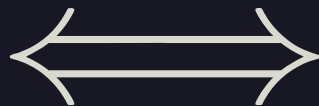


Matrix Product States/Tensor Trains



Systems that are not too entangled

Schmidt
decomposition



SVD

Systems that are not too entangled

Schmidt
decomposition \longleftrightarrow SVD

Singular values are the coefficients of Schmidt decomposition!

Systems that are not too entangled

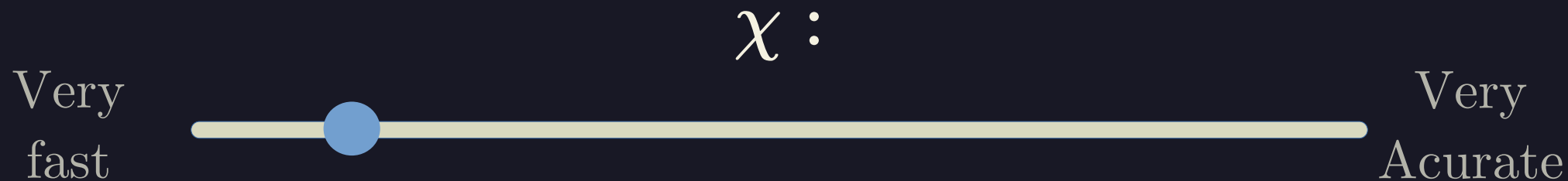
Schmidt
decomposition \longleftrightarrow SVD

Singular values are the coefficients of Schmidt decomposition!

The size of the bond is the same thing as the number of
coefficients in Schmidt decomposition!

Systems that are no *too* entangled

χ is a slider for the amount of entanglement you represent



Using Tensor Network to do simulations

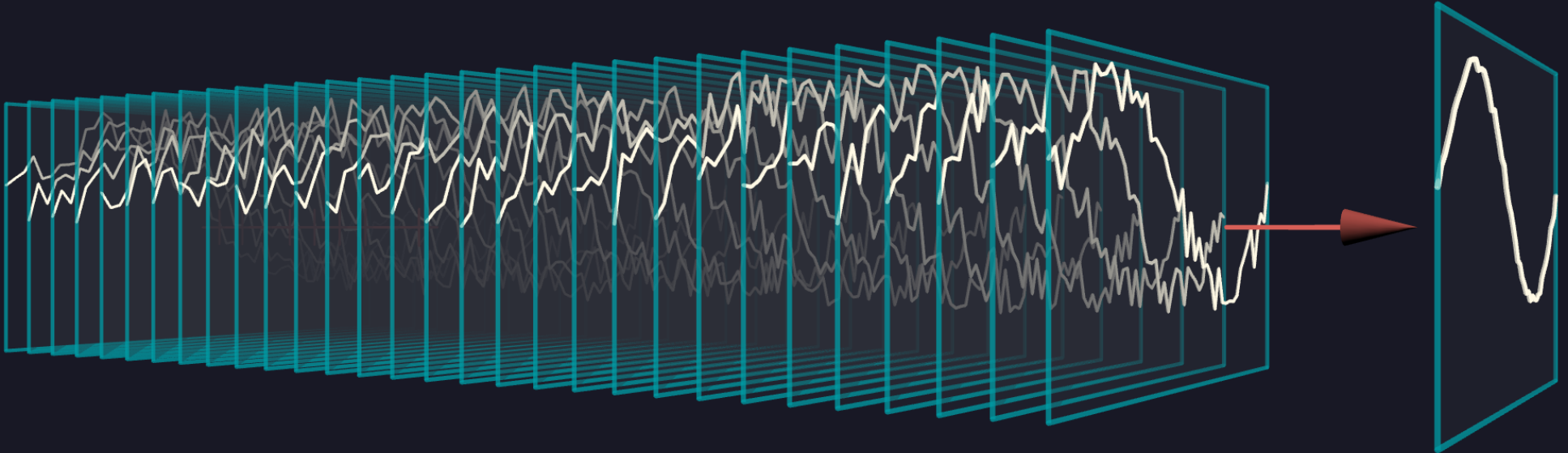
Tensor Networks are good at representing quantum circuits,
not necessarily quantum systems

~~sesolve~~
~~mesolve~~

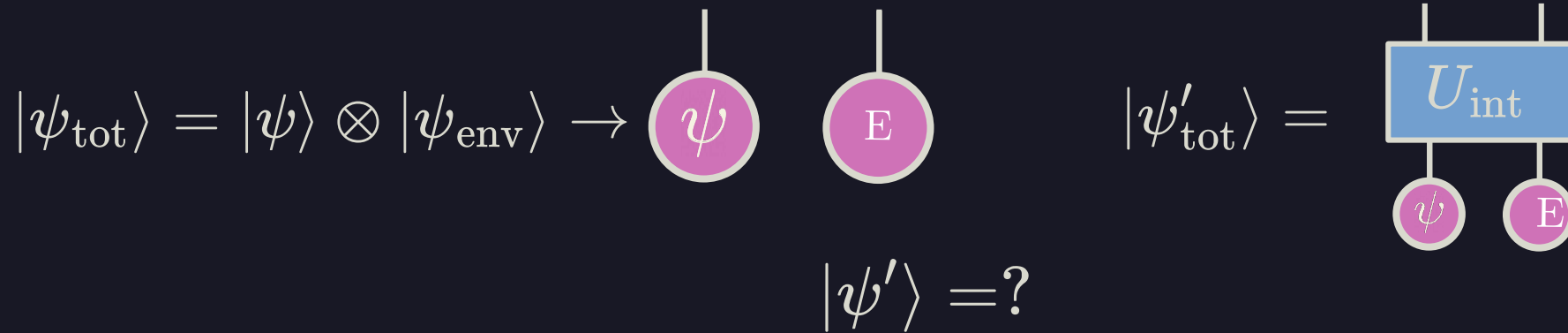
time-evolving block
decimation



Using Monte Monte Carlo simulation



How to simulate interaction with the environment

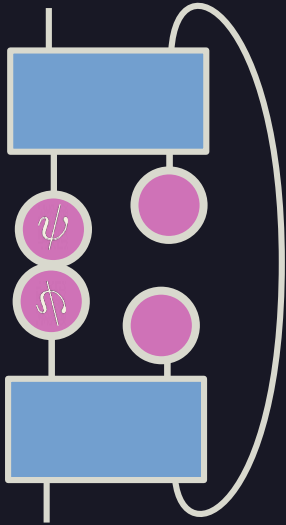


How to simulate interaction with the environment

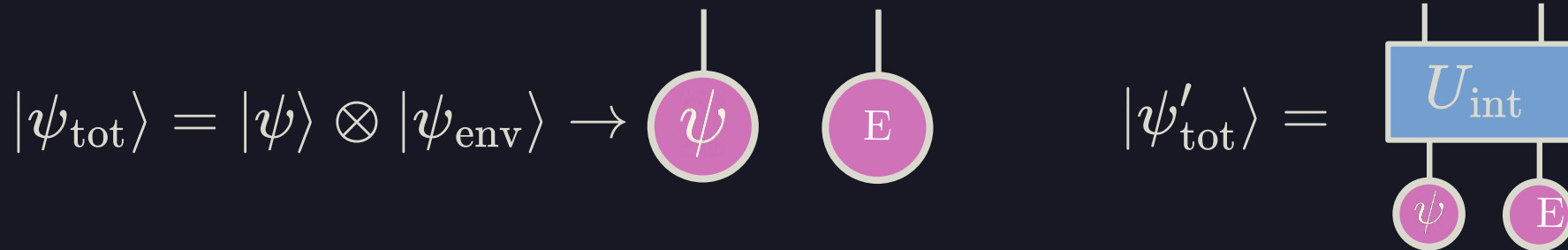
$$|\psi_{\text{tot}}\rangle = |\psi\rangle \otimes |\psi_{\text{env}}\rangle \rightarrow \begin{array}{c} \text{---} \\ | \psi \rangle \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | E \rangle \\ \text{---} \end{array}$$

$$|\psi'_{\text{tot}}\rangle = \begin{array}{c} \text{---} \quad \text{---} \\ \boxed{U_{\text{int}}} \\ \text{---} \quad \text{---} \\ | \psi \rangle \quad | E \rangle \end{array}$$

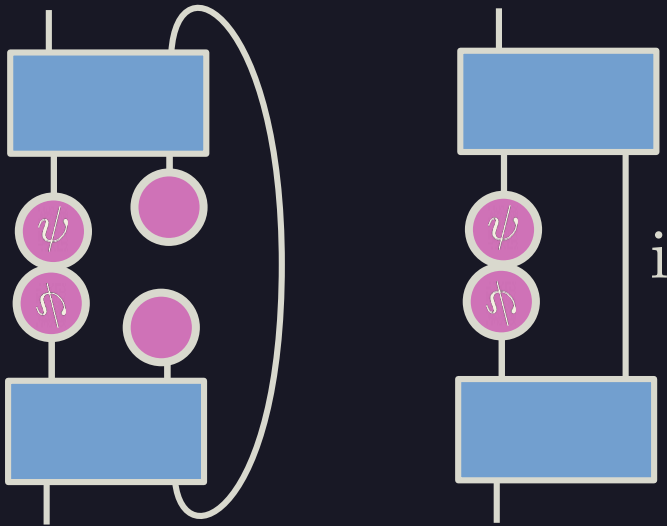
$$|\psi'\rangle = ?$$



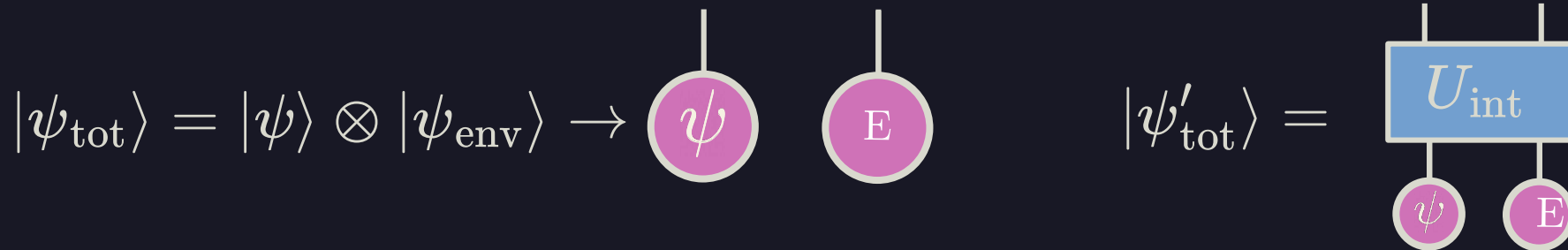
How to simulate interaction with the environment



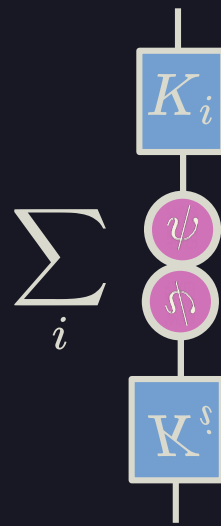
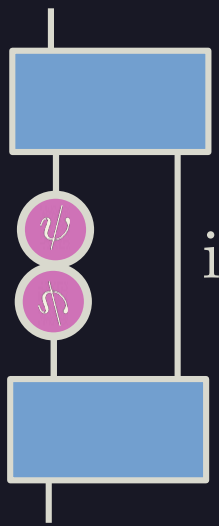
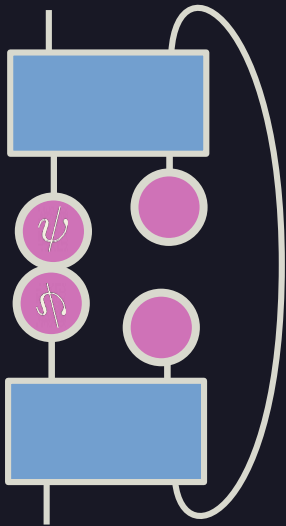
$$|\psi'\rangle = ?$$



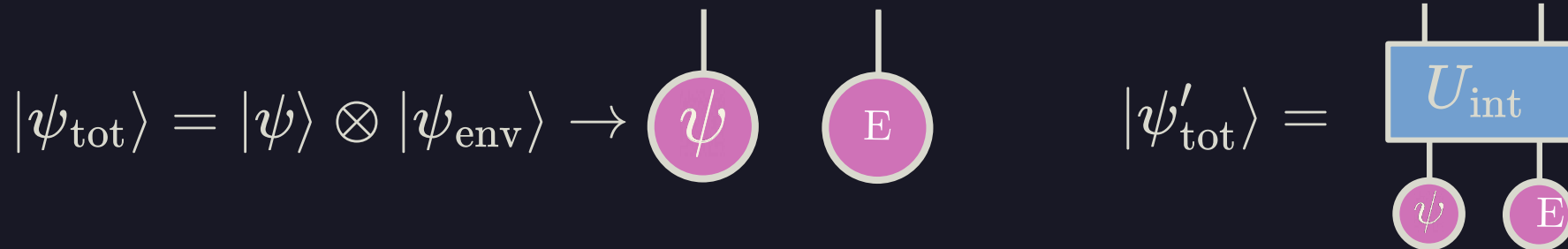
How to simulate interaction with the environment



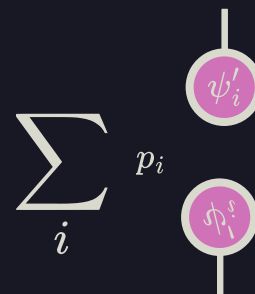
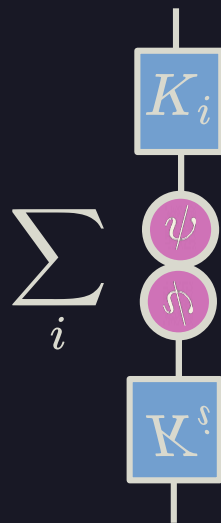
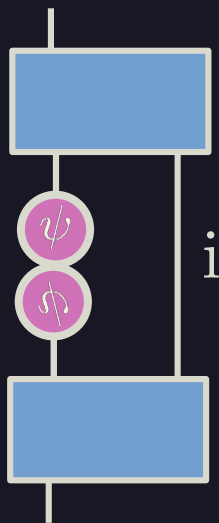
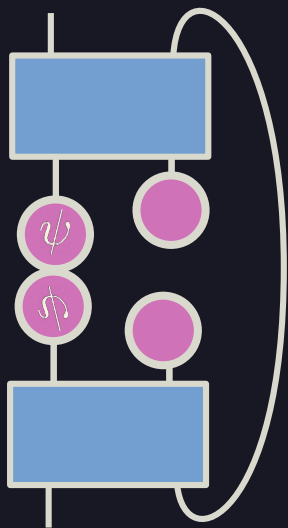
$$|\psi'\rangle = ?$$



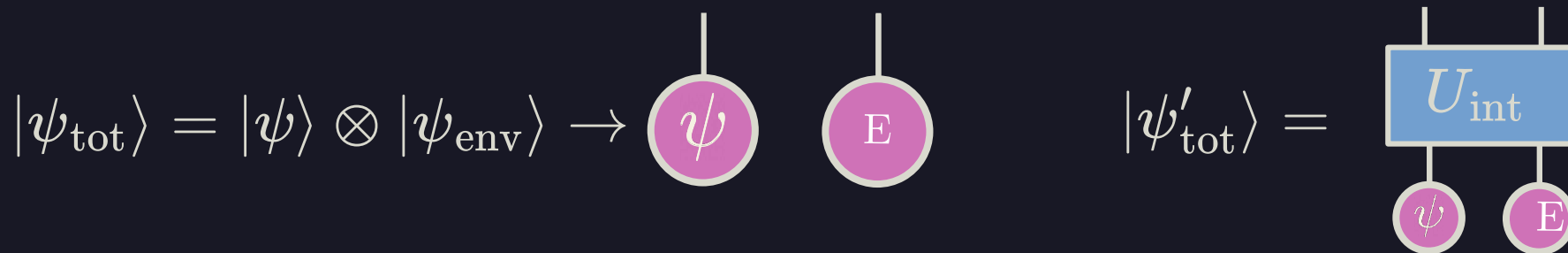
How to simulate interaction with the environment



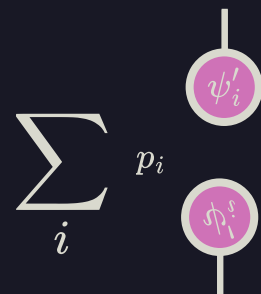
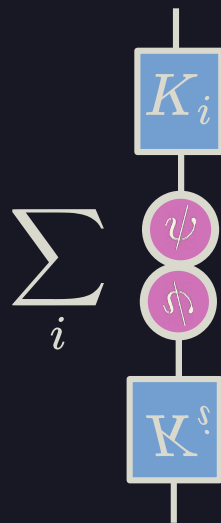
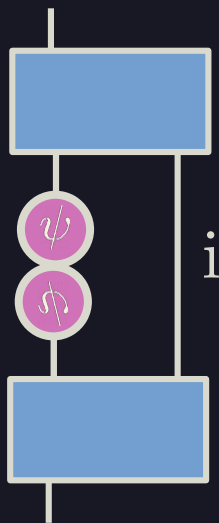
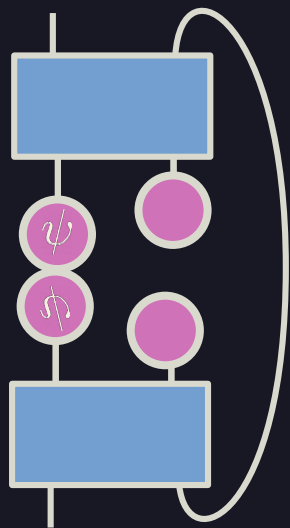
$$|\psi'\rangle = ?$$



How to simulate interaction with the environment



$$|\psi'\rangle = ?$$

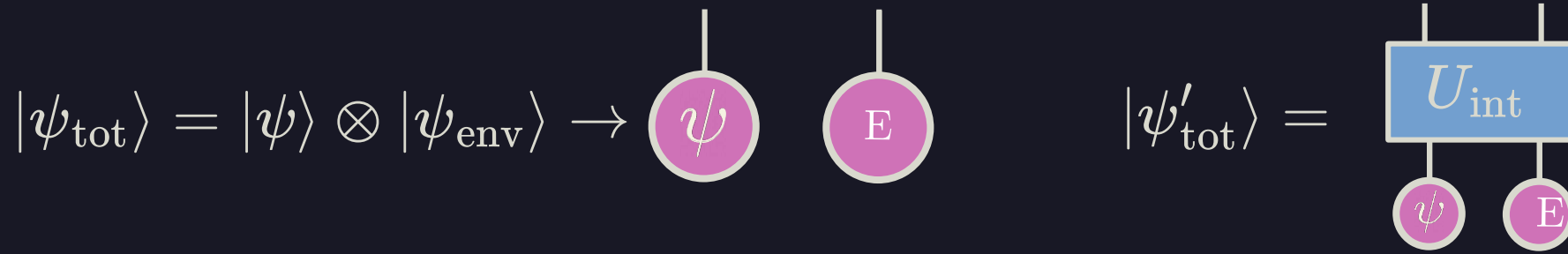


$|\psi'_i\rangle$ with probability p_i

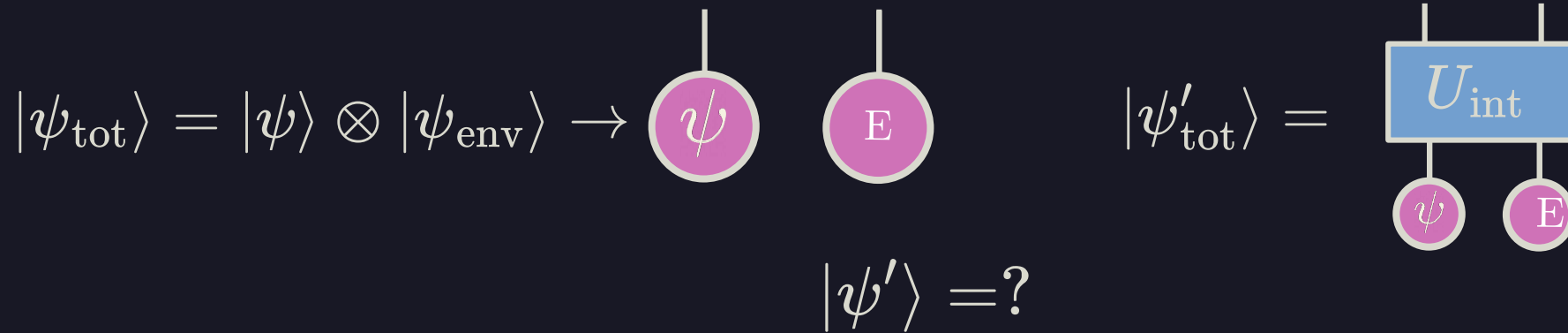
How to simulate interaction with the environment

$$|\psi_{\text{tot}}\rangle = |\psi\rangle \otimes |\psi_{\text{env}}\rangle \rightarrow \begin{array}{c} | \\ \textcircled{\psi} \end{array} \quad \begin{array}{c} | \\ \textcircled{E} \end{array}$$

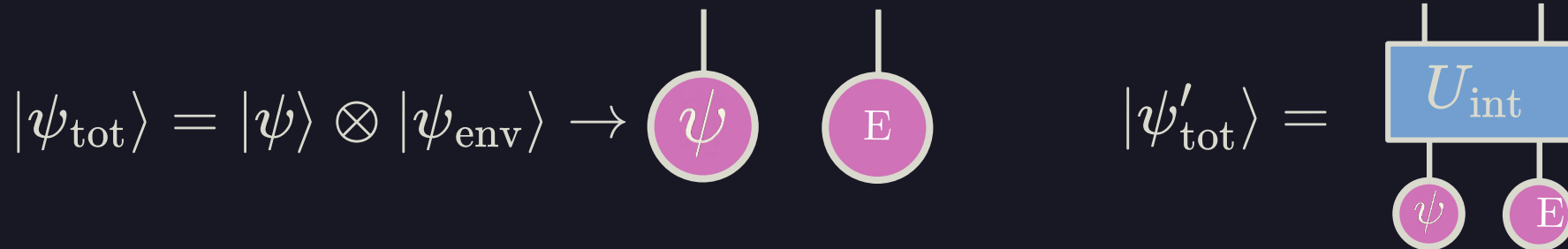
How to simulate interaction with the environment



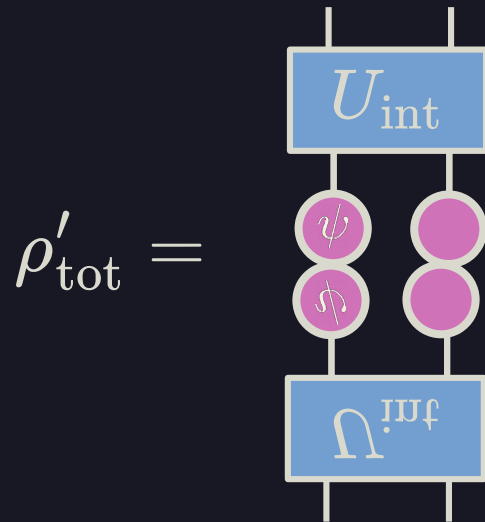
How to simulate interaction with the environment



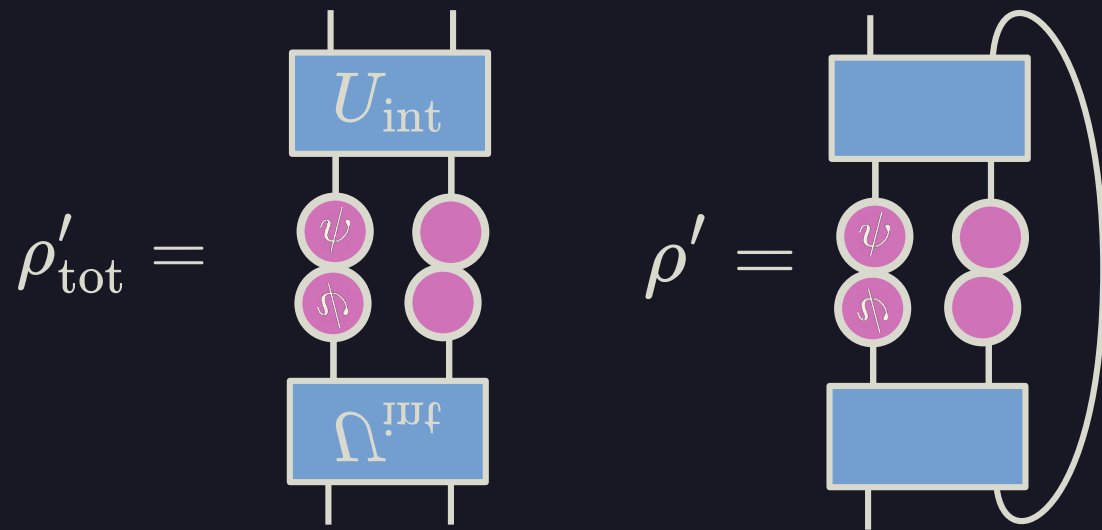
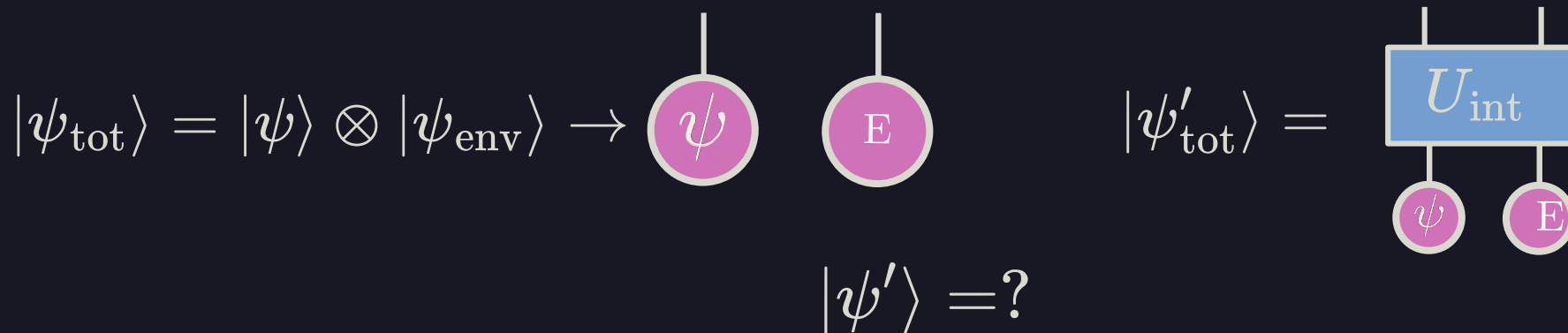
How to simulate interaction with the environment



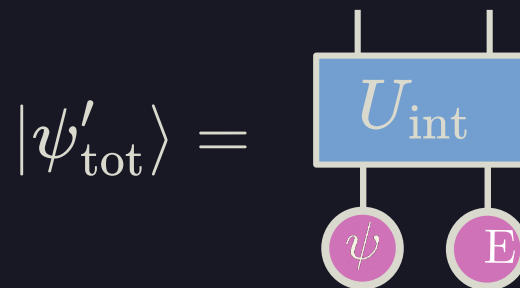
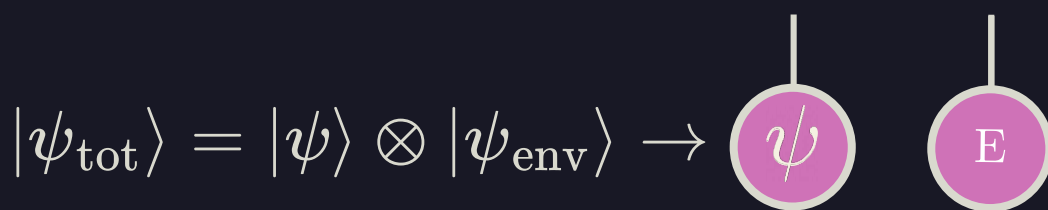
$$|\psi'\rangle = ?$$



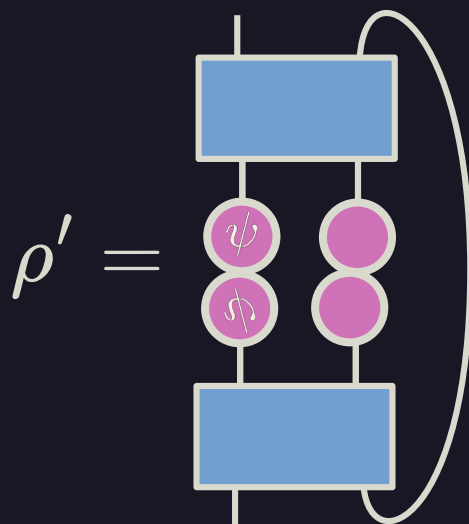
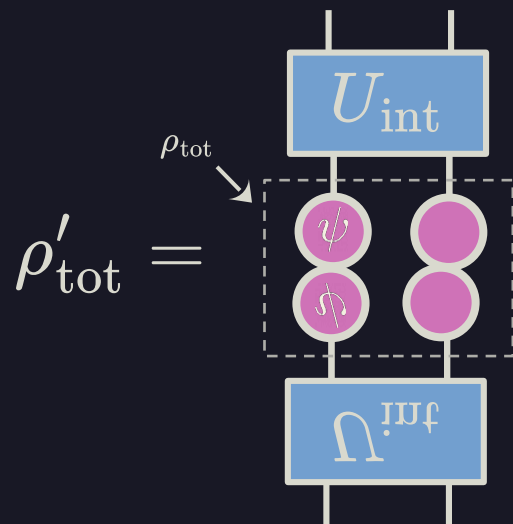
How to simulate interaction with the environment



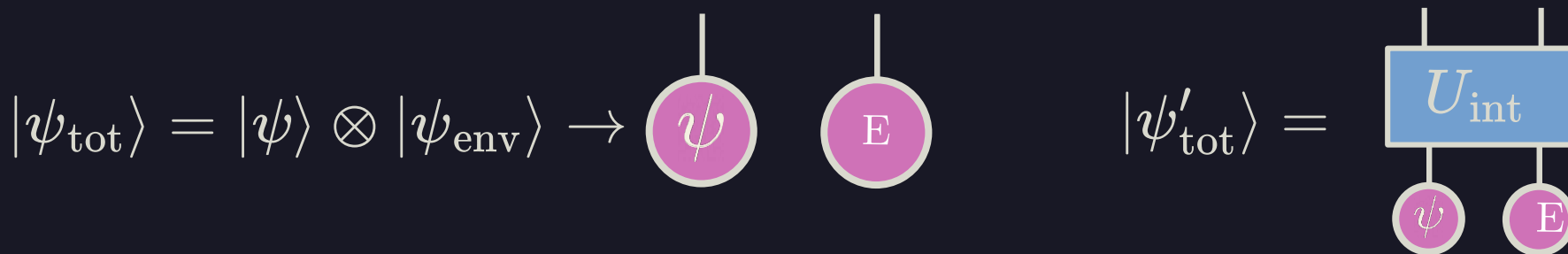
How to simulate interaction with the environment



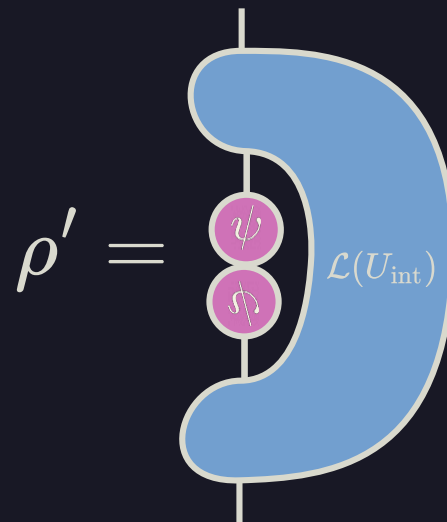
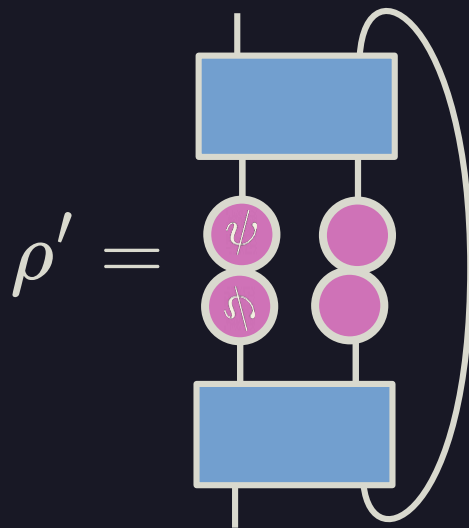
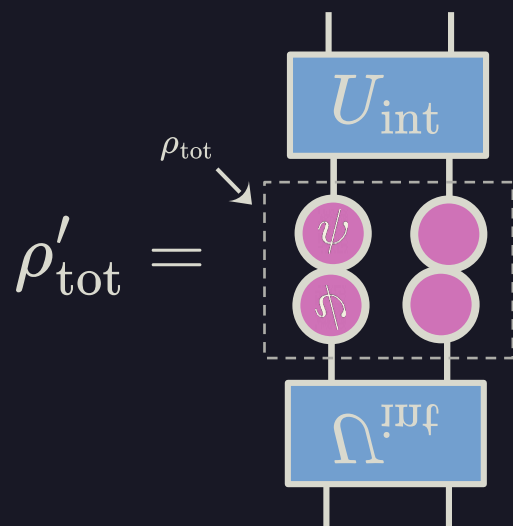
$|\psi'\rangle = ?$



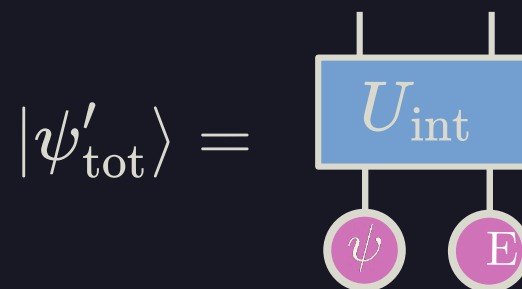
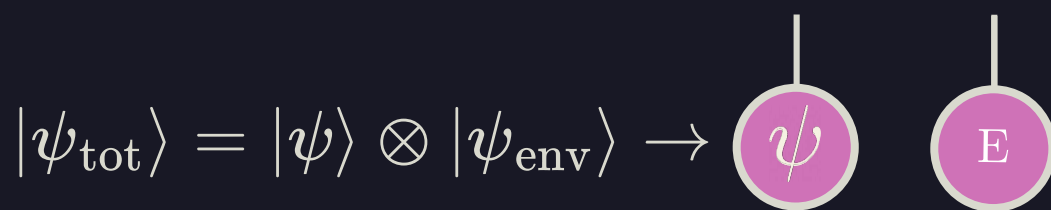
How to simulate interaction with the environment



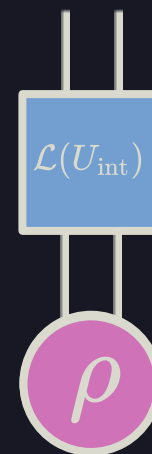
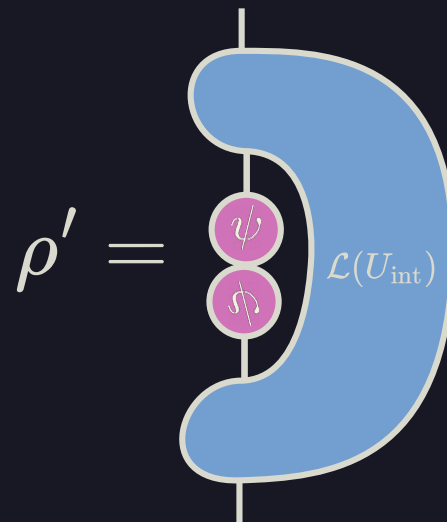
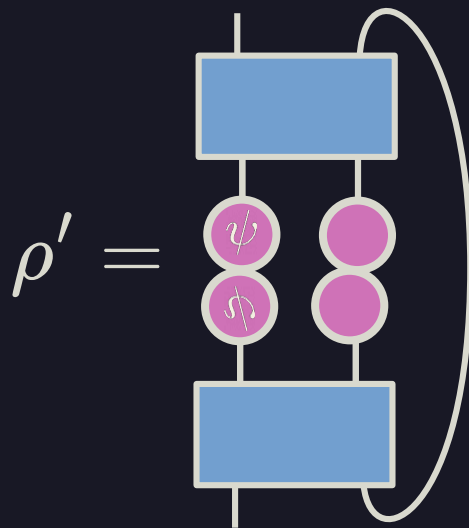
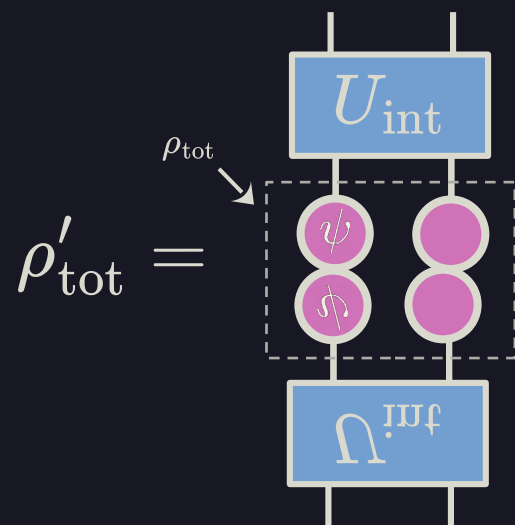
$$|\psi'\rangle = ?$$



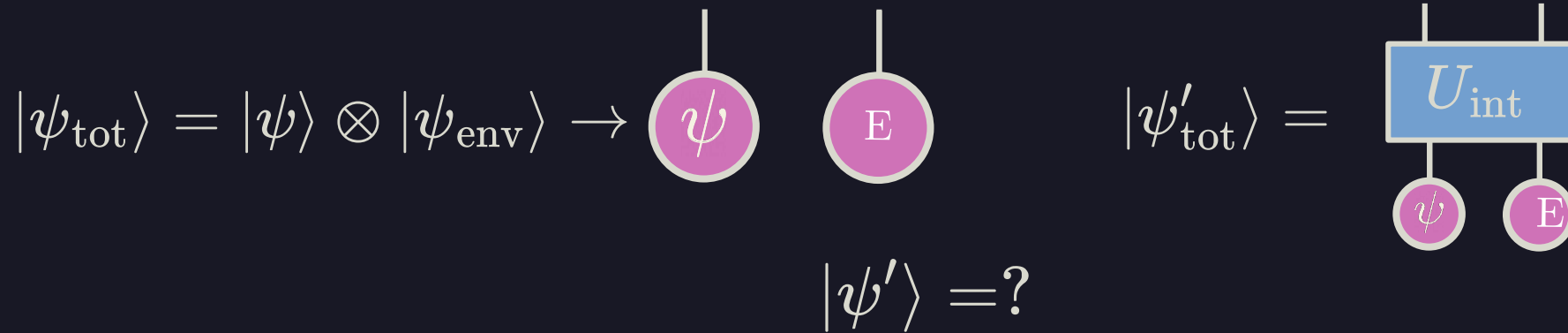
How to simulate interaction with the environment



$|\psi'\rangle = ?$



How to simulate interaction with the environment

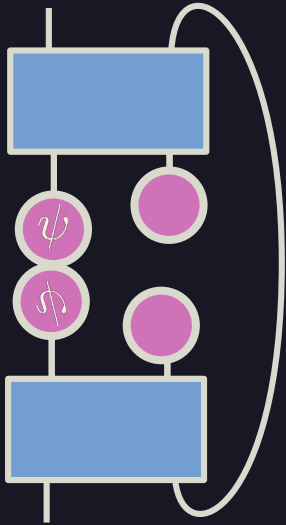


How to simulate interaction with the environment

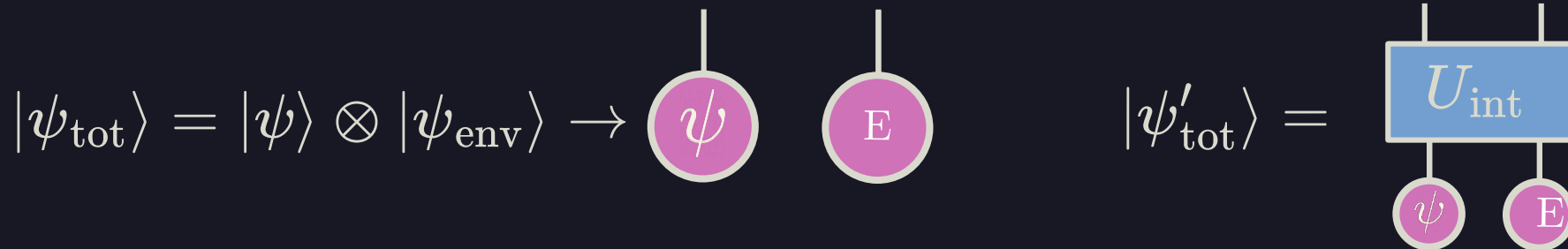
$$|\psi_{\text{tot}}\rangle = |\psi\rangle \otimes |\psi_{\text{env}}\rangle \rightarrow \begin{array}{c} \text{---} \\ | \psi \rangle \\ \text{---} \end{array} \quad \begin{array}{c} \text{---} \\ | E \rangle \\ \text{---} \end{array}$$

$$|\psi'_{\text{tot}}\rangle = \begin{array}{c} \text{---} \quad \text{---} \\ \boxed{U_{\text{int}}} \\ \text{---} \quad \text{---} \\ | \psi \rangle \quad | E \rangle \end{array}$$

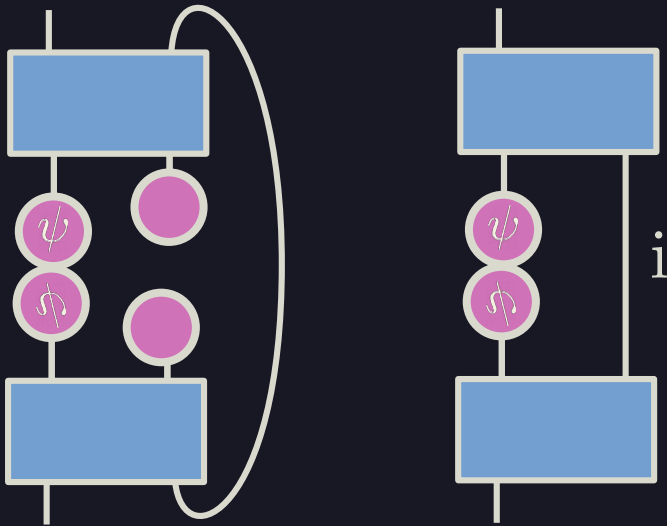
$$|\psi'\rangle = ?$$



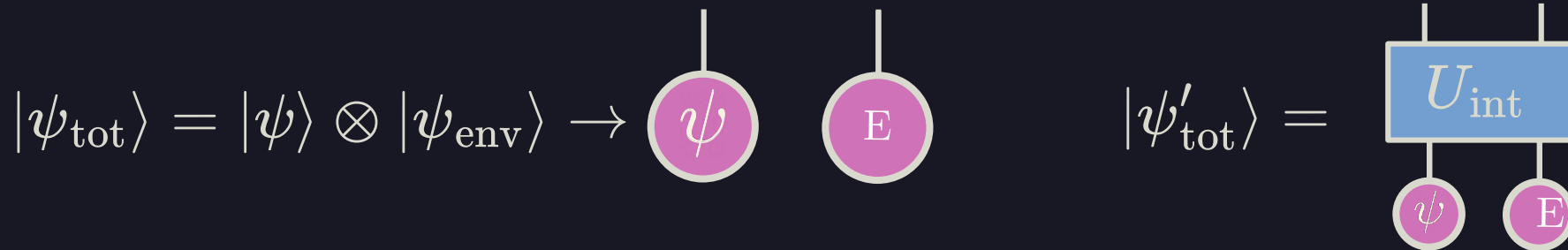
How to simulate interaction with the environment



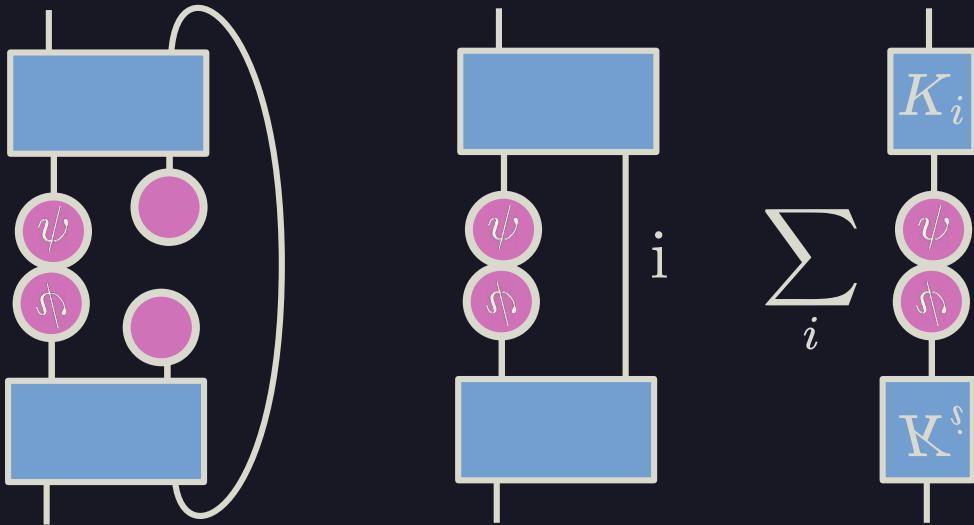
$$|\psi'\rangle = ?$$



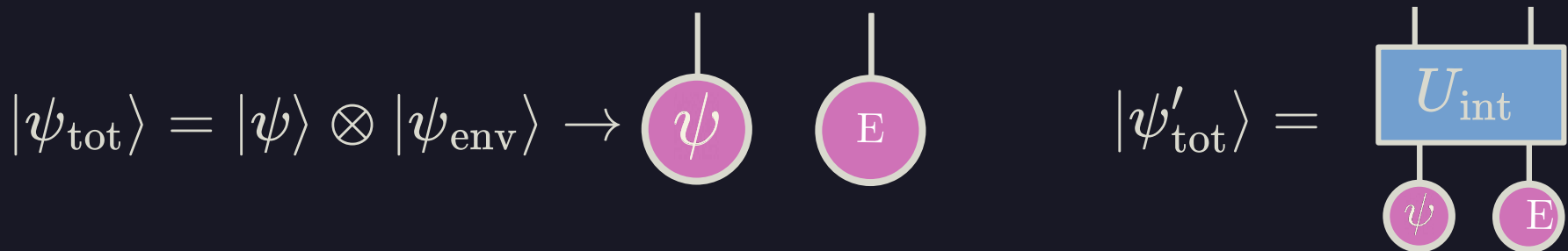
How to simulate interaction with the environment



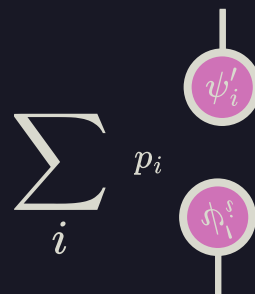
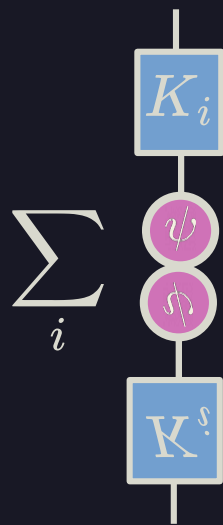
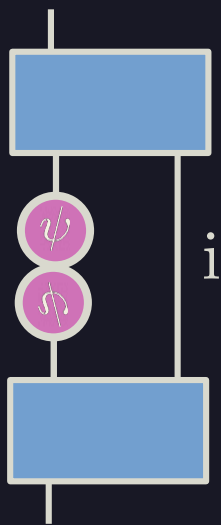
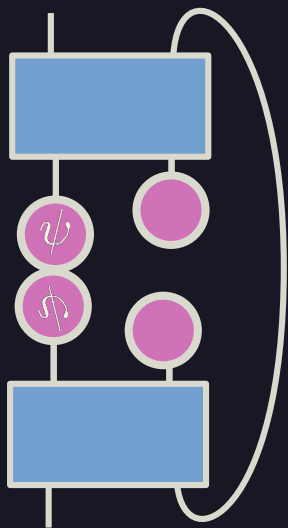
$$|\psi'\rangle = ?$$



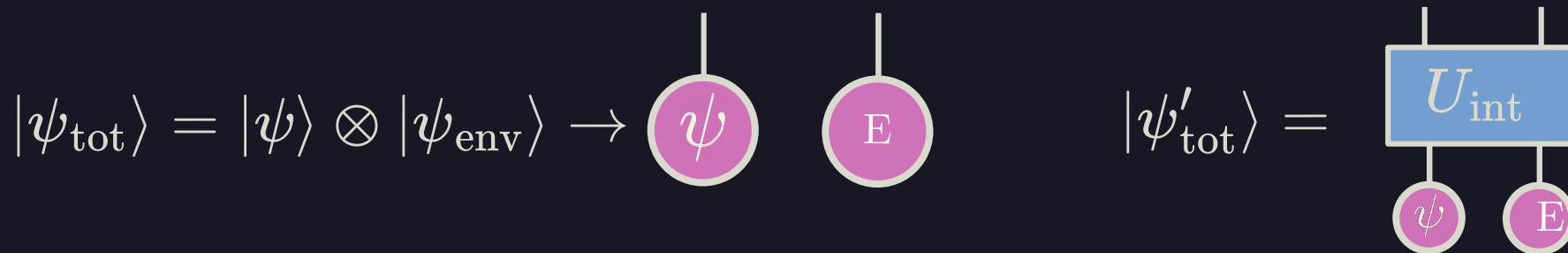
How to simulate interaction with the environment



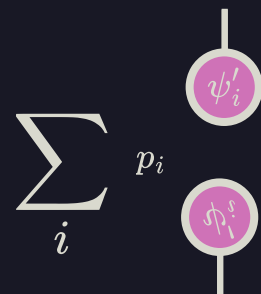
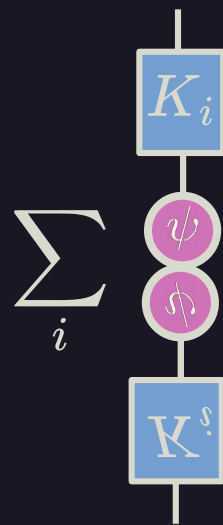
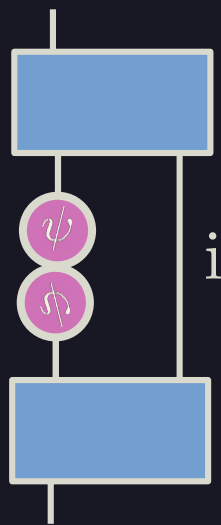
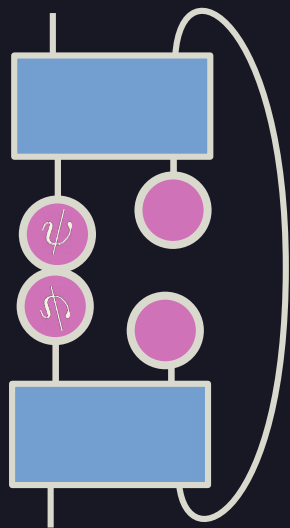
$$|\psi'\rangle = ?$$



How to simulate interaction with the environment

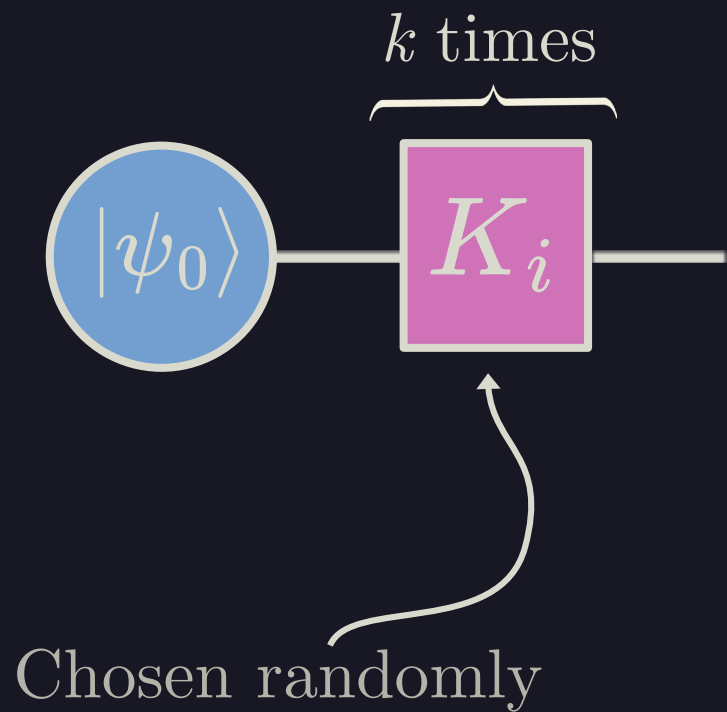


$$|\psi'\rangle = ?$$

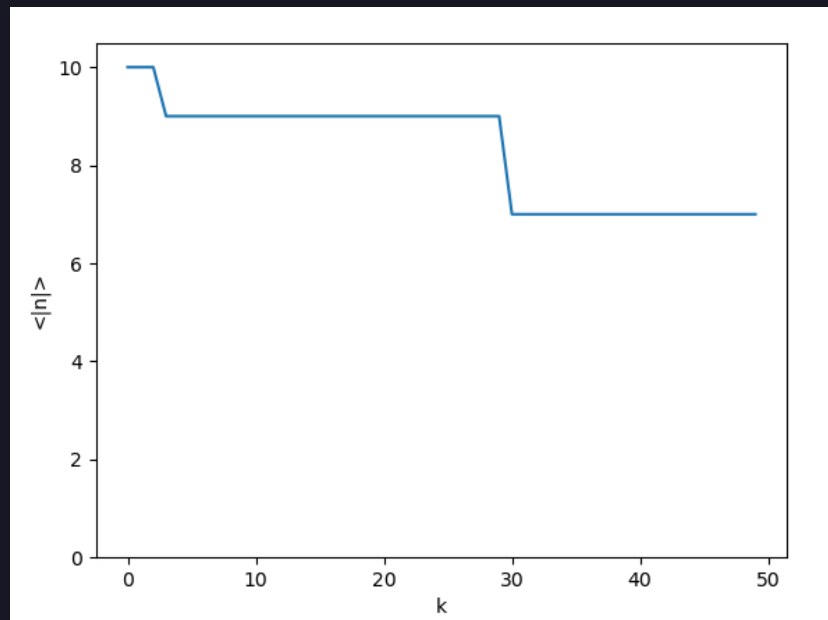
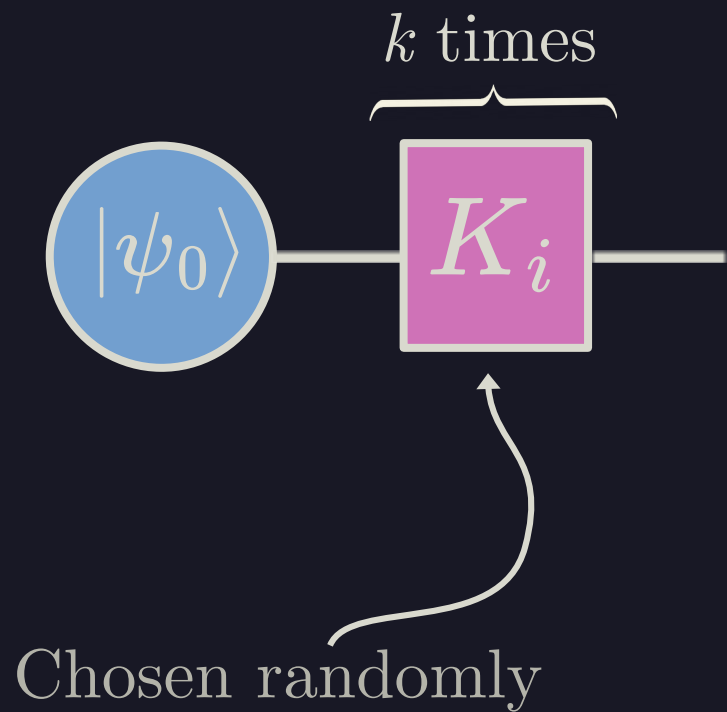


$|\psi'_i\rangle$ with probability p_i

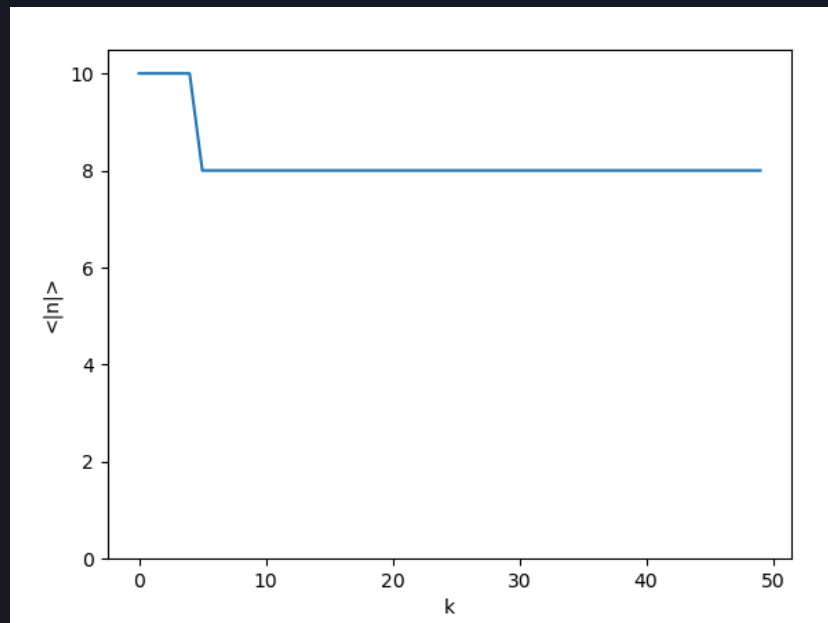
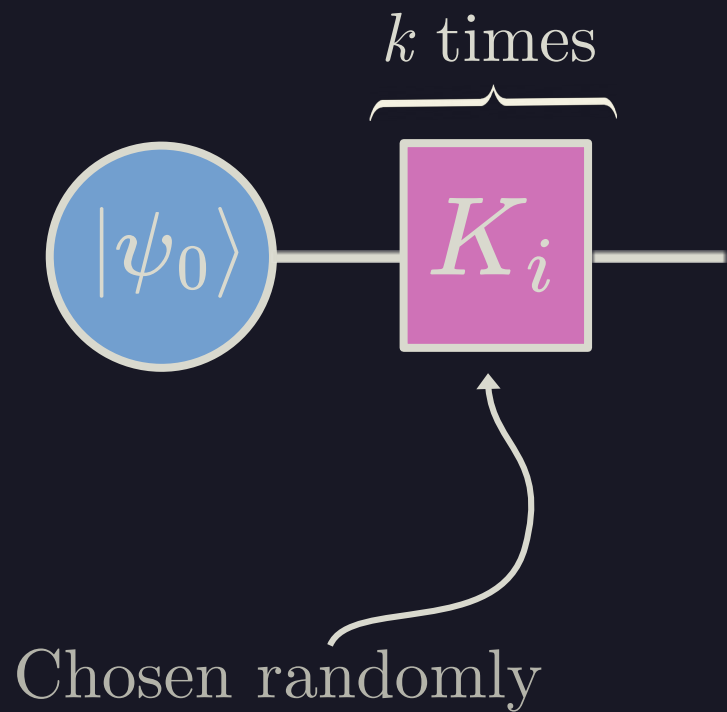
Example: photon loss



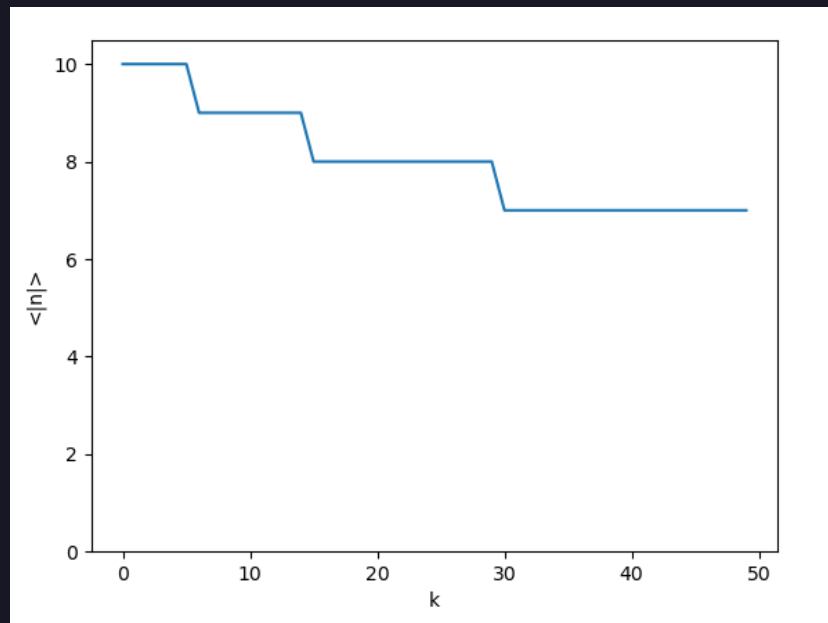
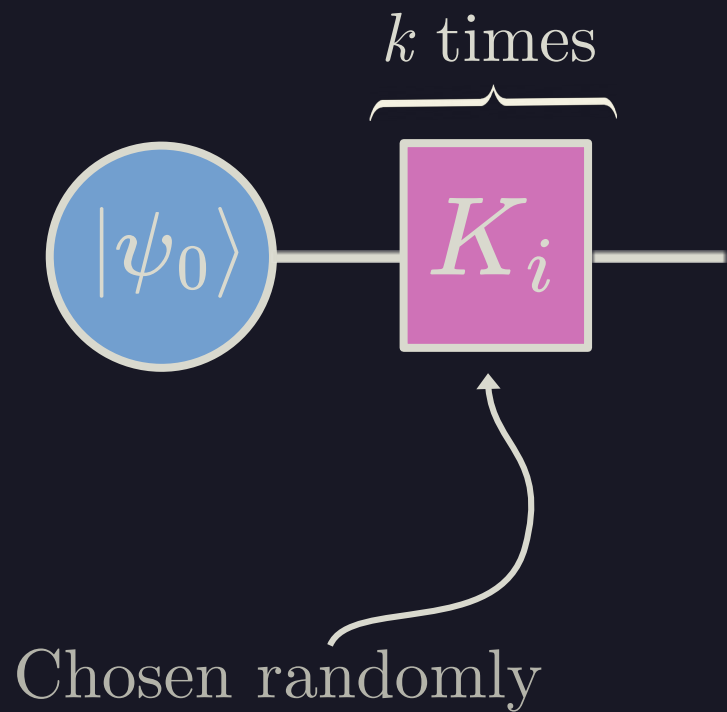
Example: photon loss



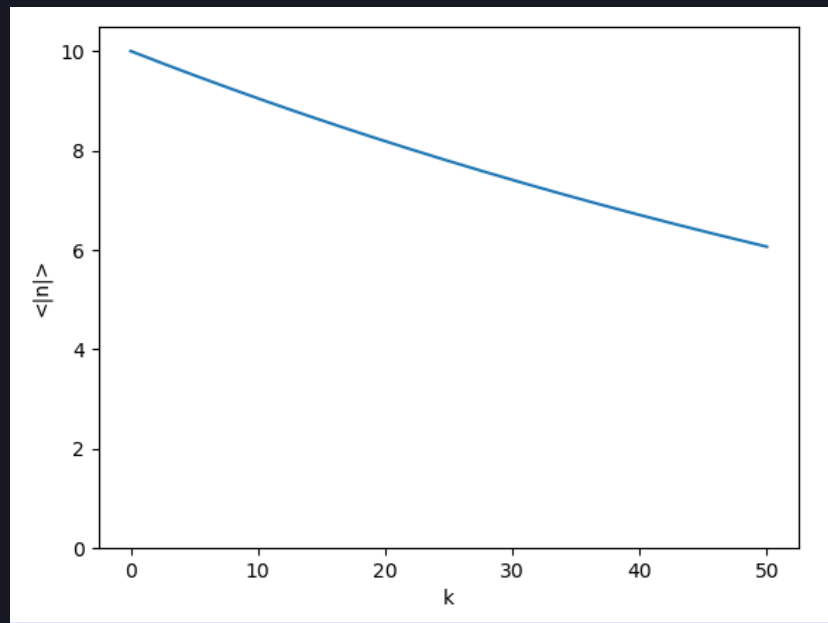
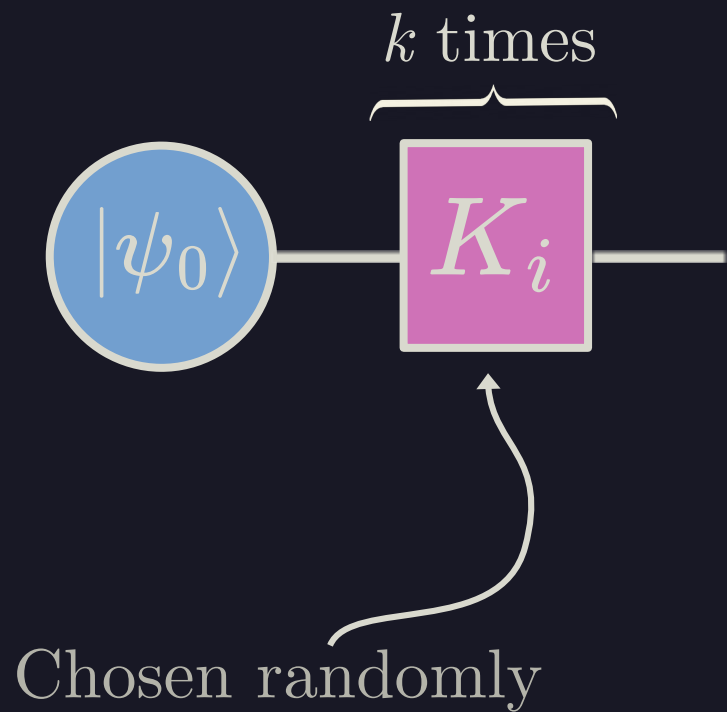
Example: photon loss



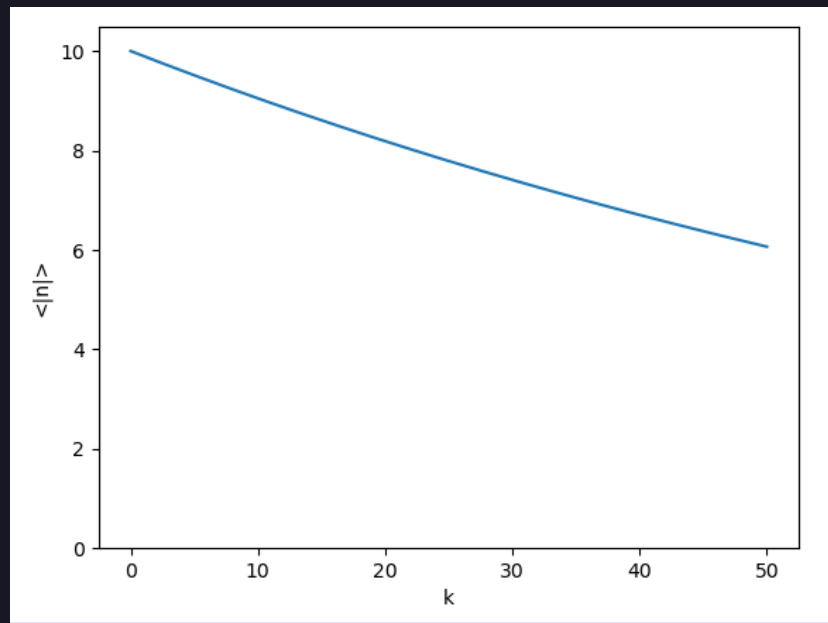
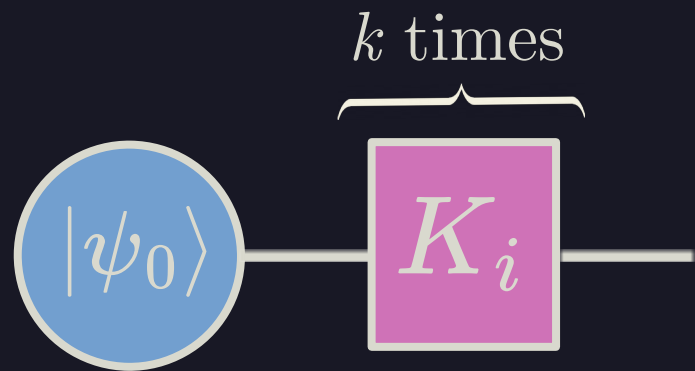
Example: photon loss



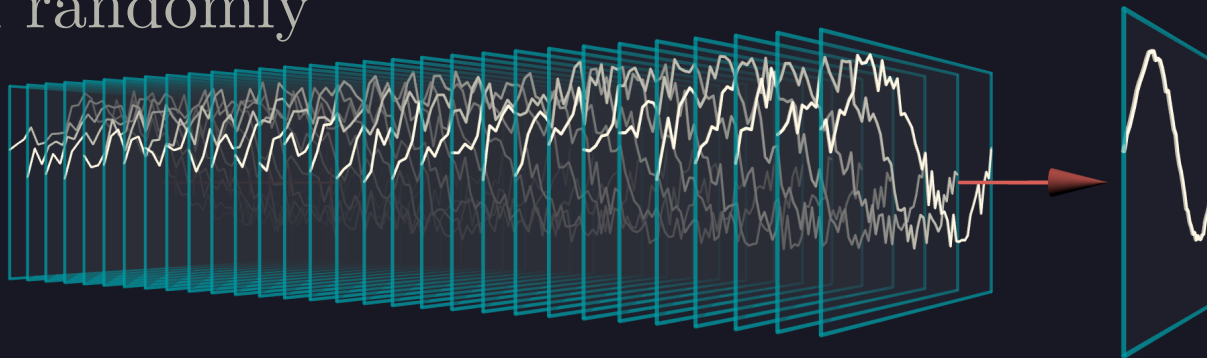
Example: photon loss



Example: photon loss



Chosen randomly

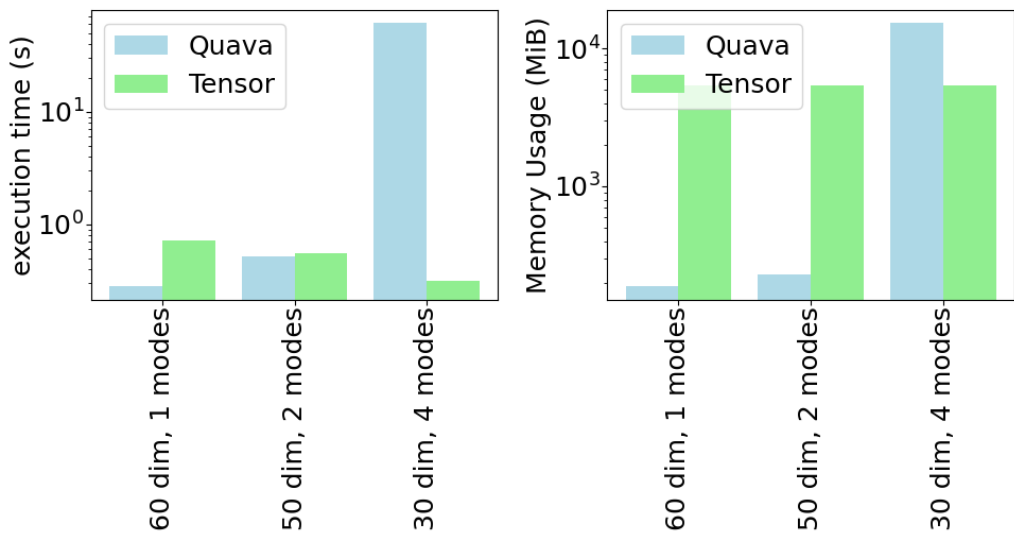


More specific goal

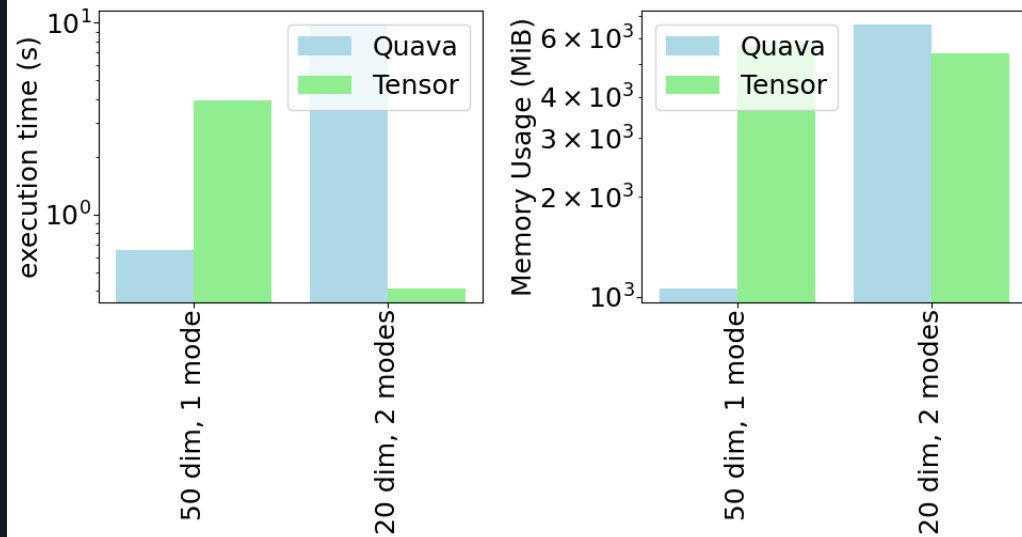
- Build a tensor network based framework that can be built upon and serve as backend for quantum simulation
- Build on top of already defined states, operators and protocols
- Run multimode cavity state preparation simulation using the framework to compare with current Nord Quantique software (Quava) and compare the performances of both methods

Performance results

GKP state preparation



Tesseract state preparation

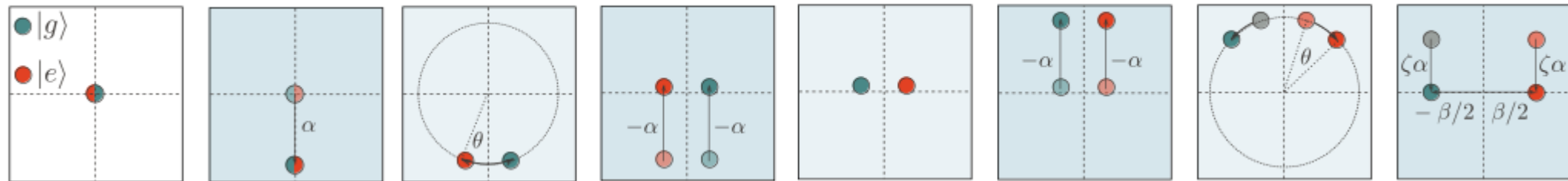


GKP and Tessaract state preperation

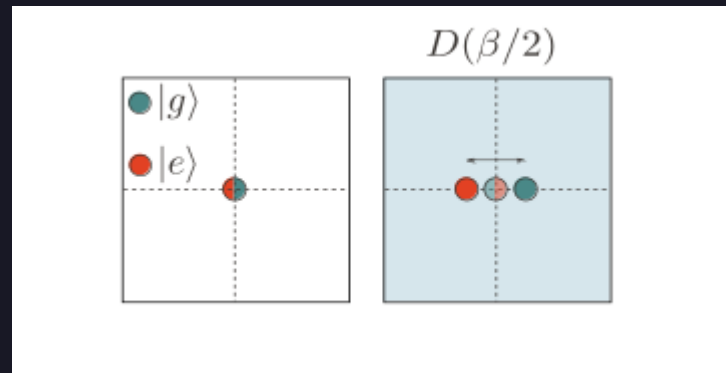
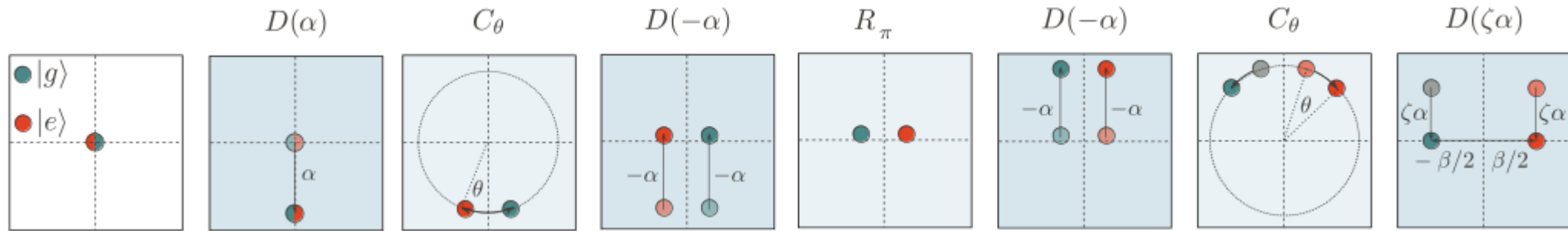


Transmon ECD based stabilisation

Transmon ECD based stabilisation



Transmon ECD based stabilisation



Transmon ECD based stabilisation

