# Nord Quantique Internship summary

by Jean-Baptiste Bertrand

# Smilarity to my master's project

Simulate many GKP qubit using TN

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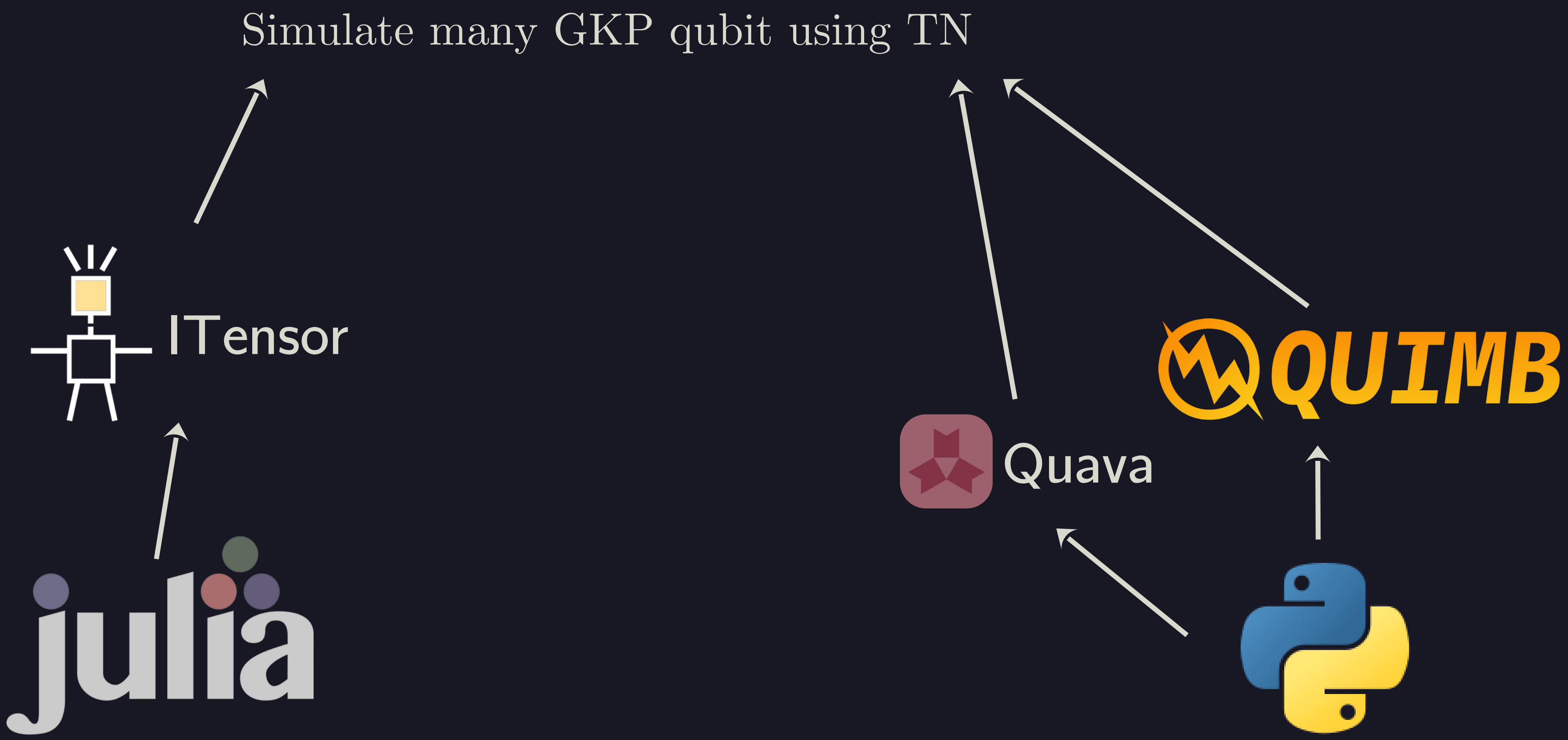
Simulate many GKP qubit using TN

ITensor

julia

# Smilarity to my master's project



# The main ideas of using Tensor Networks

A pictural representation introduced by Roger Penrose

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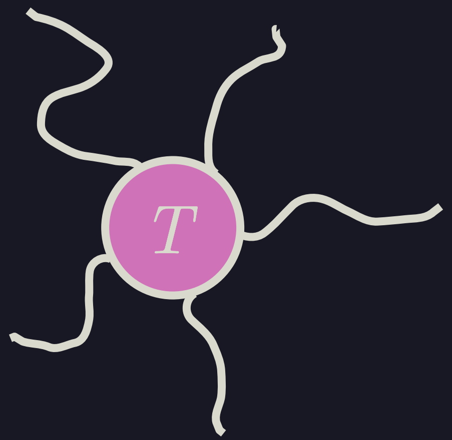
rank $n$ tensor $\rightarrow$ $n$ legged *blob*



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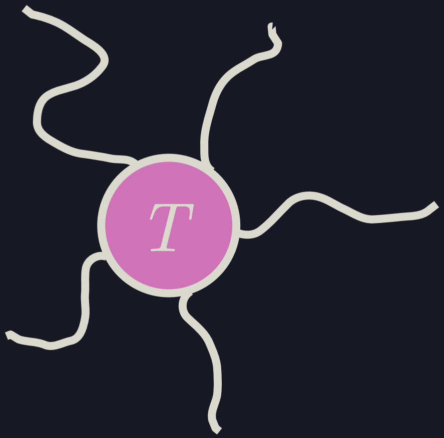


$\psi_i \longrightarrow$

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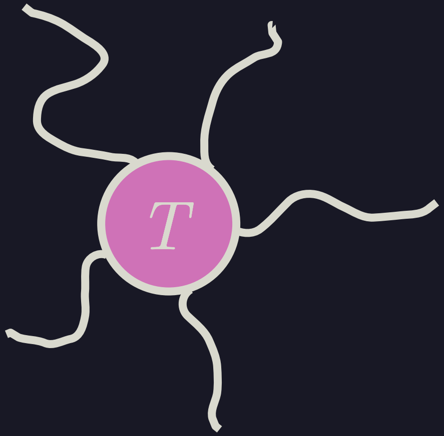


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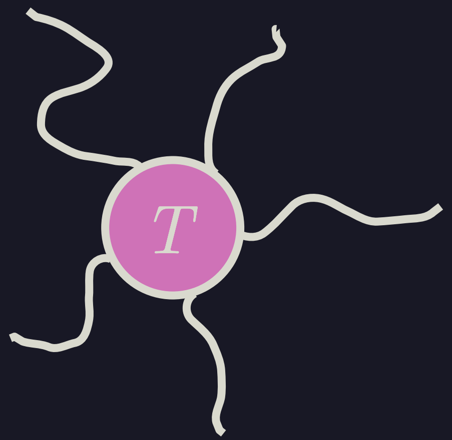
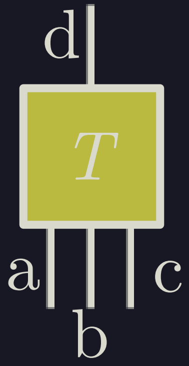



$\psi_i \longrightarrow$

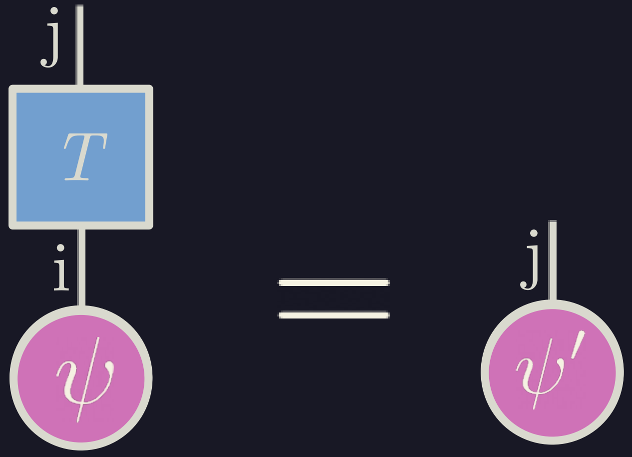

$A^i_j \longrightarrow$


$T^d_{abc} \longrightarrow$



# Examples

$$\sum_i A^i_j \psi_i \rightarrow A^i_j \psi_i \rightarrow$$



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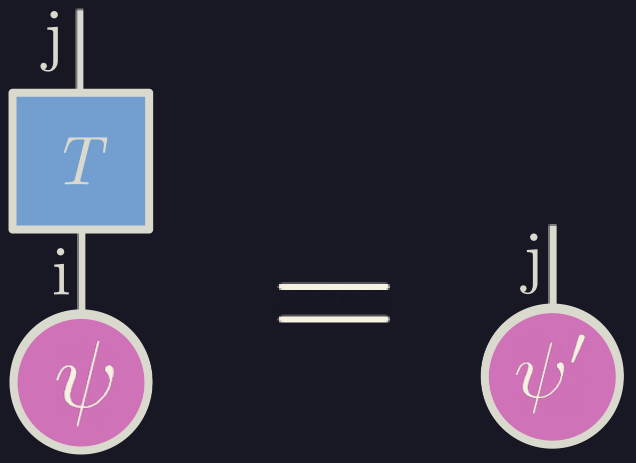
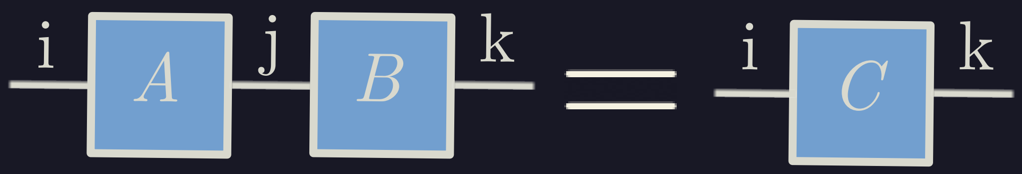



$$\sum_j A^i_j B^j_k \to A^i_j B^j_k \to$$



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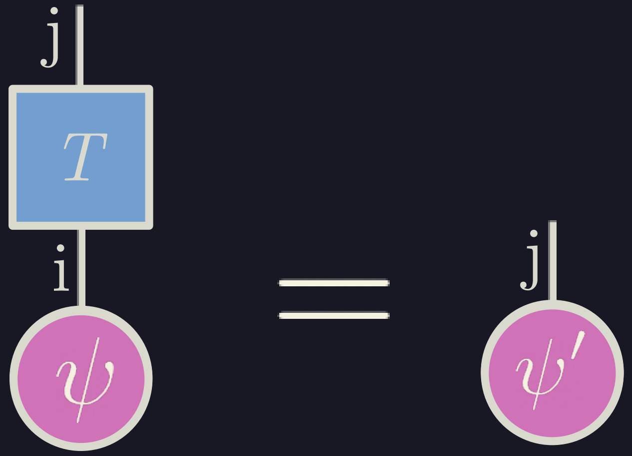


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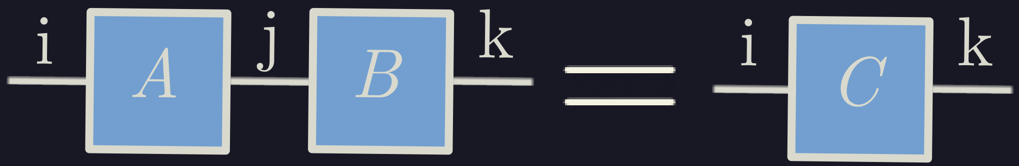
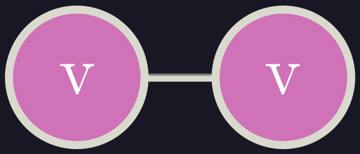



$$\vec{v} \cdot \vec{v} \to$$



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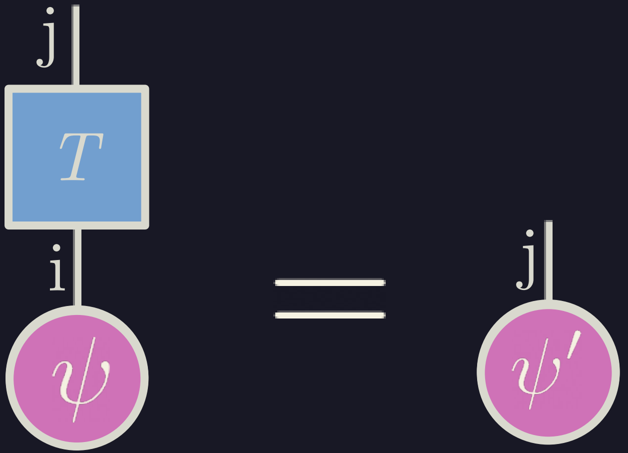


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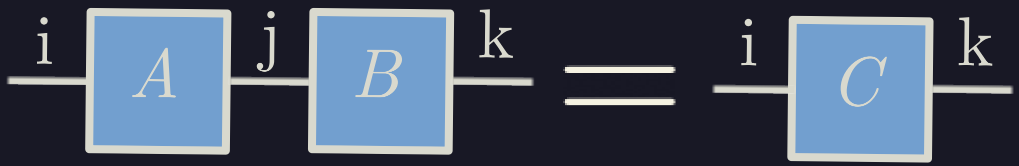


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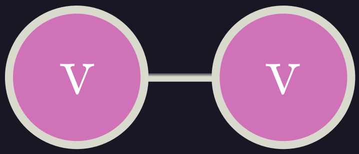
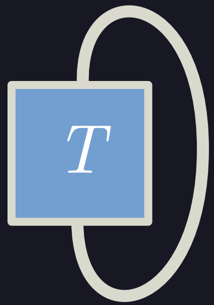



$$\mathrm{tr}(T) = T^i_i \rightarrow$$



The goal in is to use less memory via powerful approximation

# Singular Value Decomposition

- Similar to a diagonalisation
- Can be performed on any complex matrix, including rectangular ones

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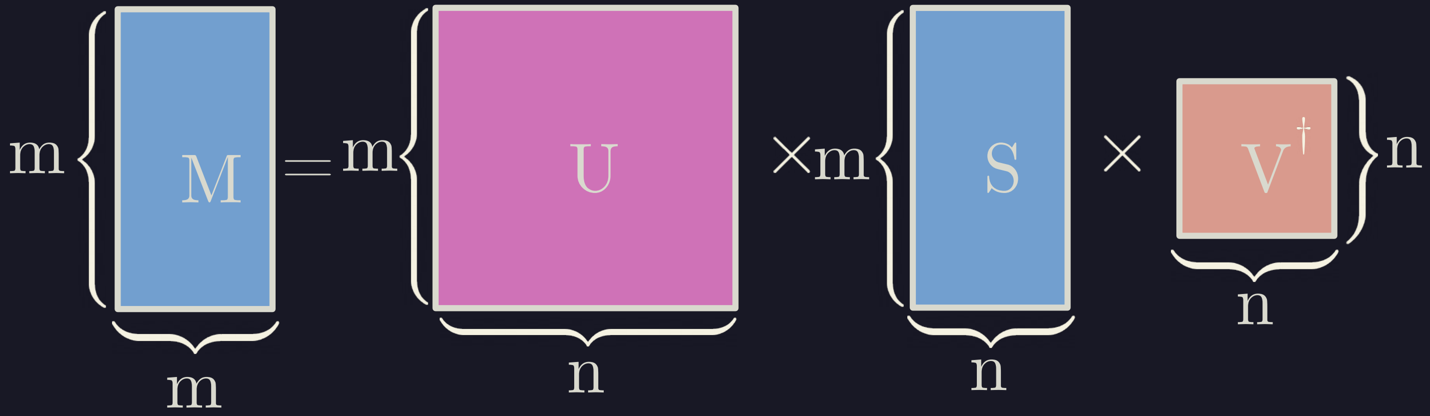


$$S = \begin{pmatrix} s_1 & 0 & 0 & \cdots \\ 0 & s_2 & 0 & \cdots \\ 0 & 0 & s_3 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

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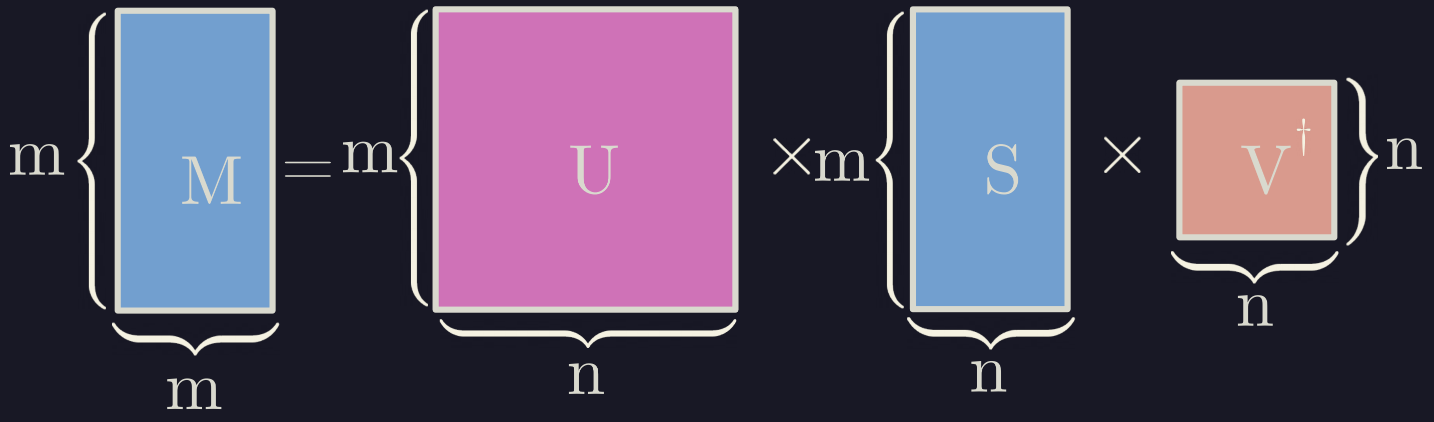


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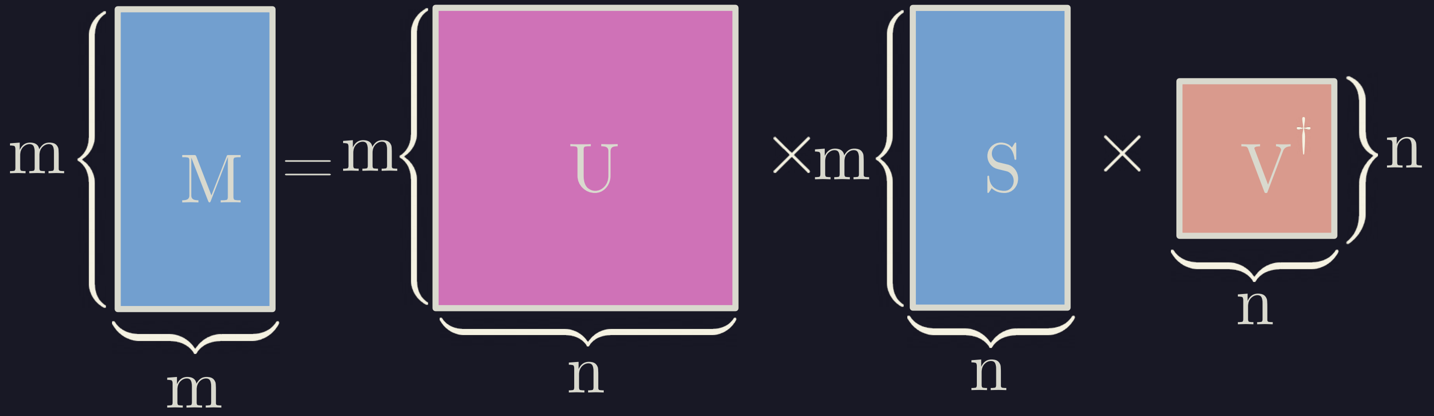



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- Uniquely defined when singular values are ordered in a specific way
- Singular values are always real and positive

# Splitting Hilbert space


$$\sum_i c_i |i\rangle \longrightarrow \sum_{\mu,\nu} c_{\mu,\nu} |\mu, \nu\rangle$$

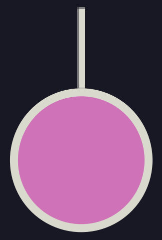
# Splitting Hilbert space


E.g. :

$$|5\rangle \rightarrow |1\rangle|0\rangle|1\rangle$$

$$\sum_i c_i |i\rangle \longrightarrow \sum_{\mu,\nu} c_{\mu,\nu} |\mu, \nu\rangle$$

# Hilbert space is a small(er) place!



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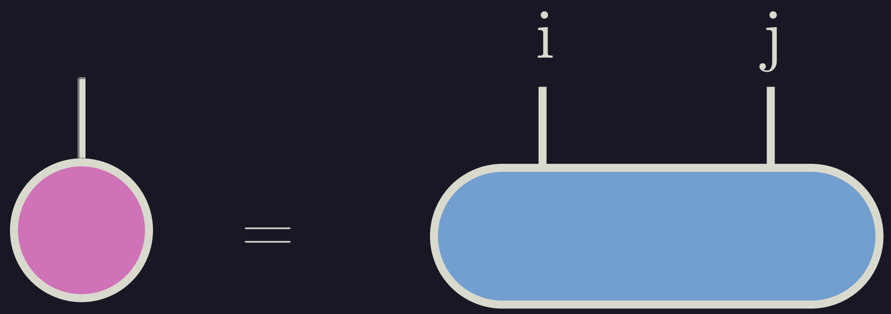

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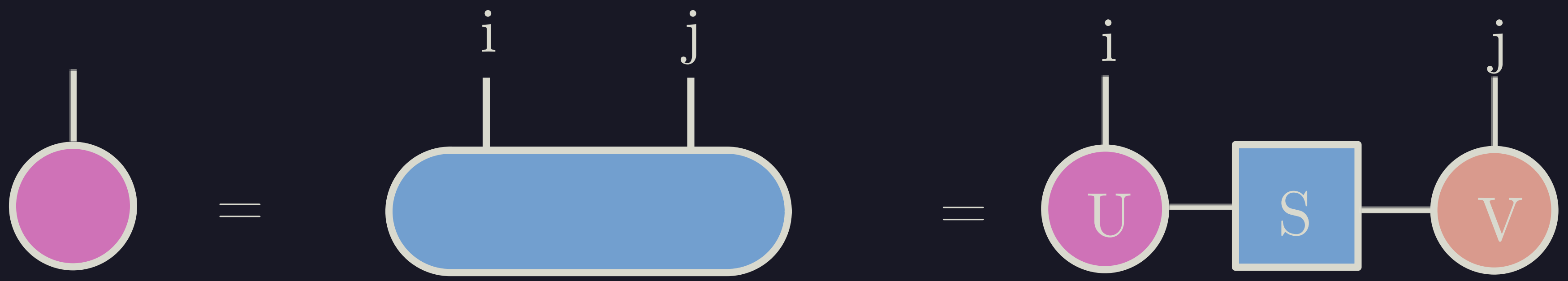

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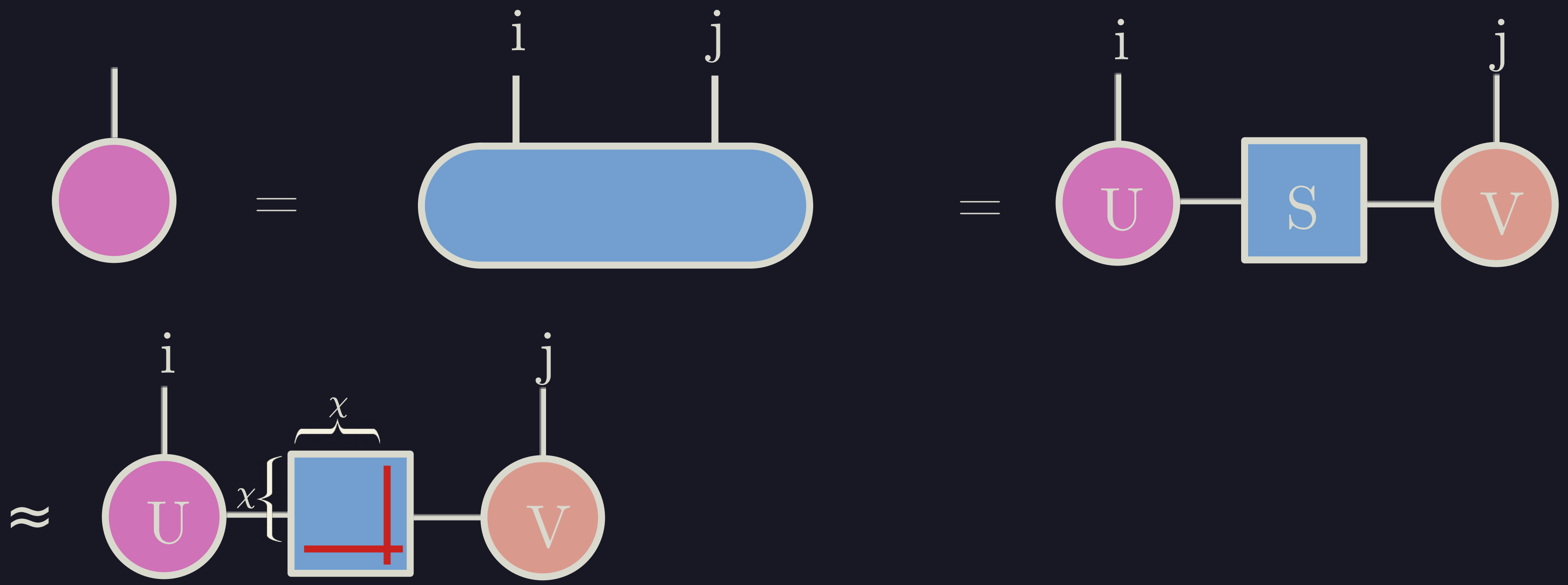

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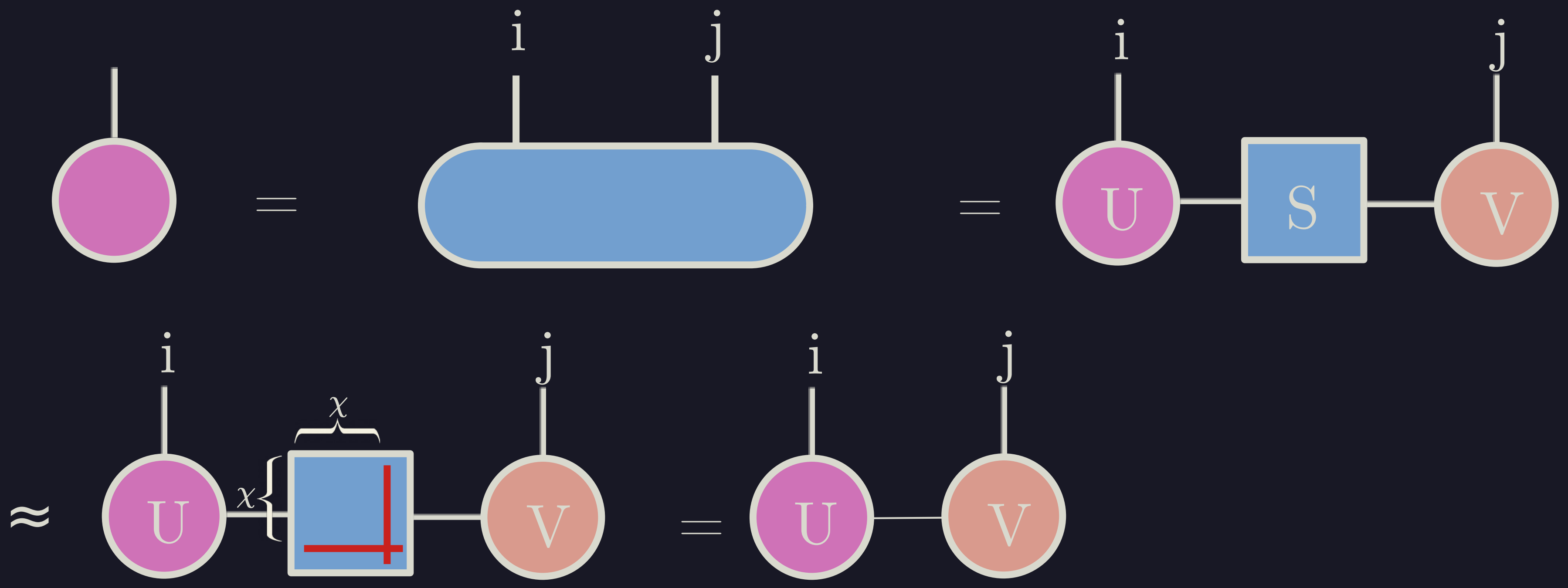

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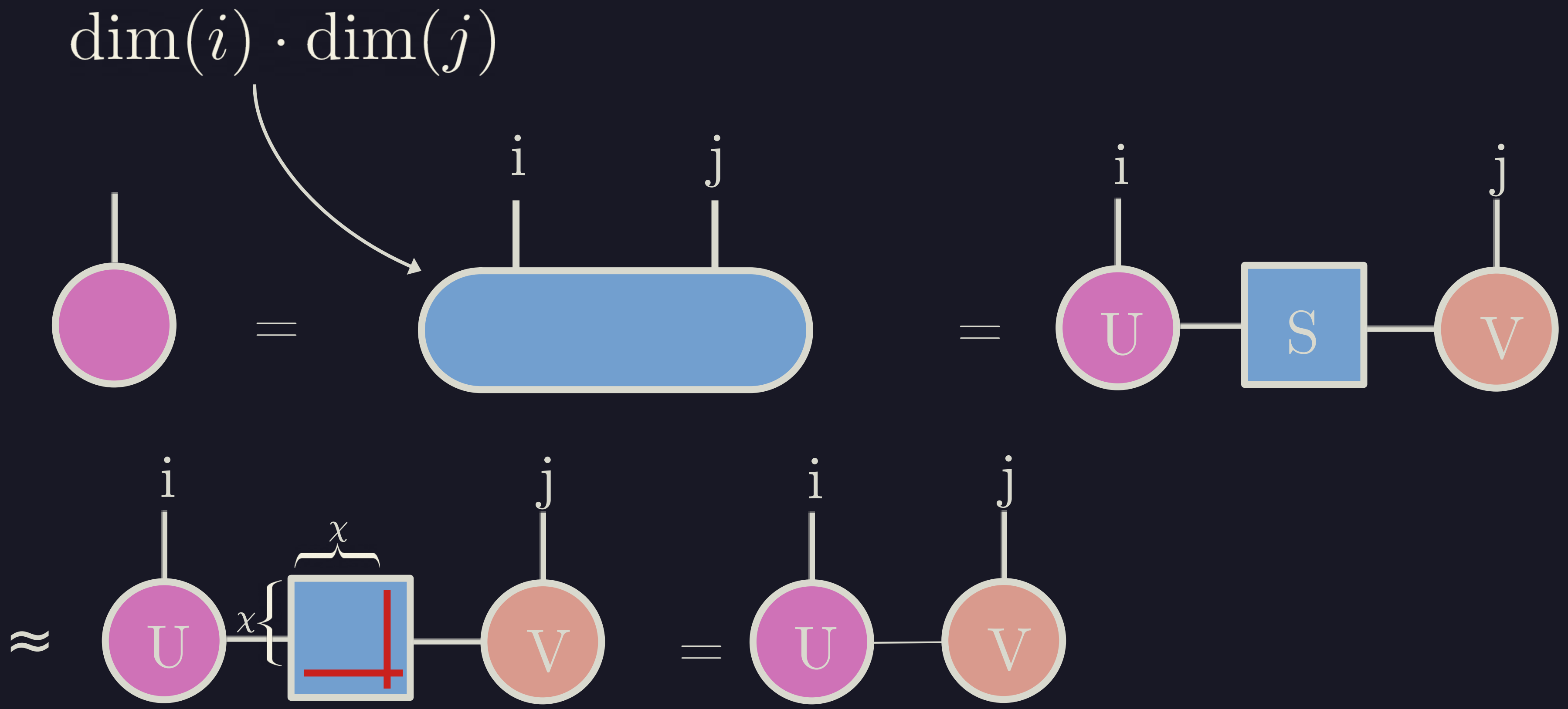

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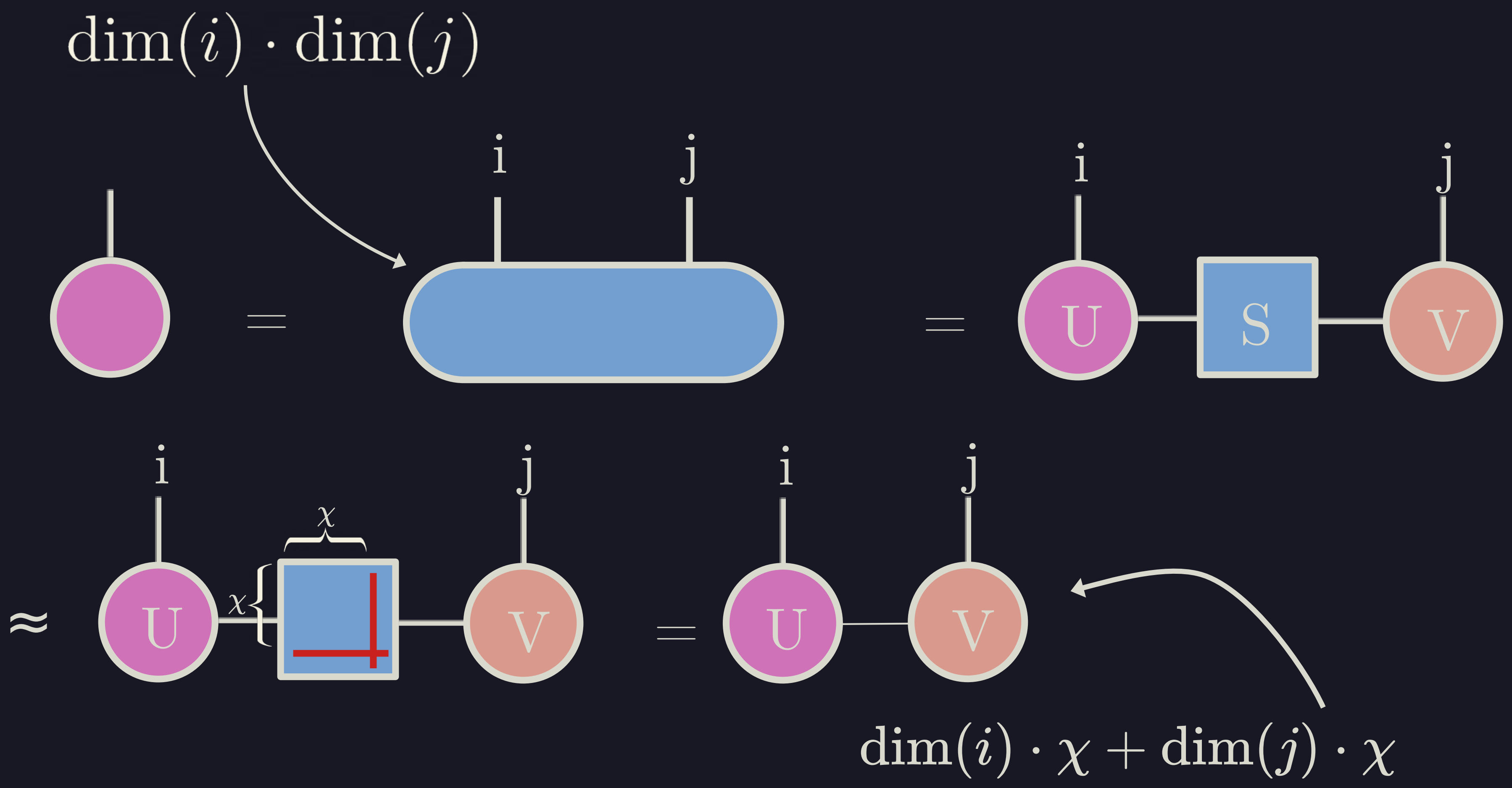

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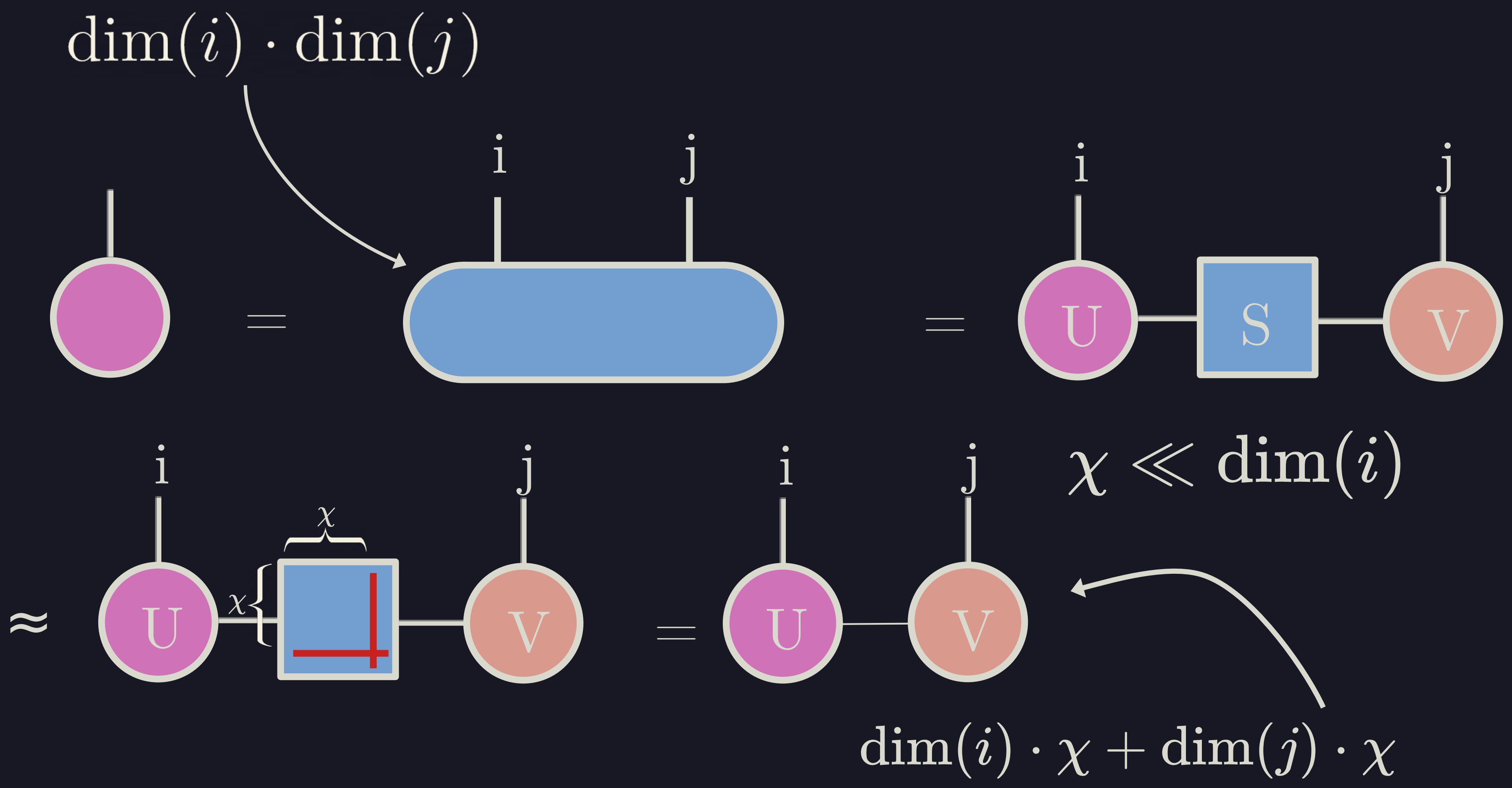
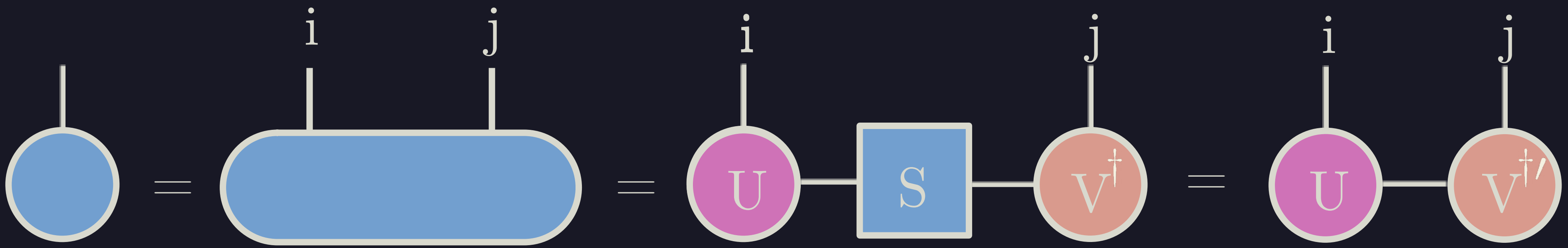


# Matrix Product States/Tensor Trains



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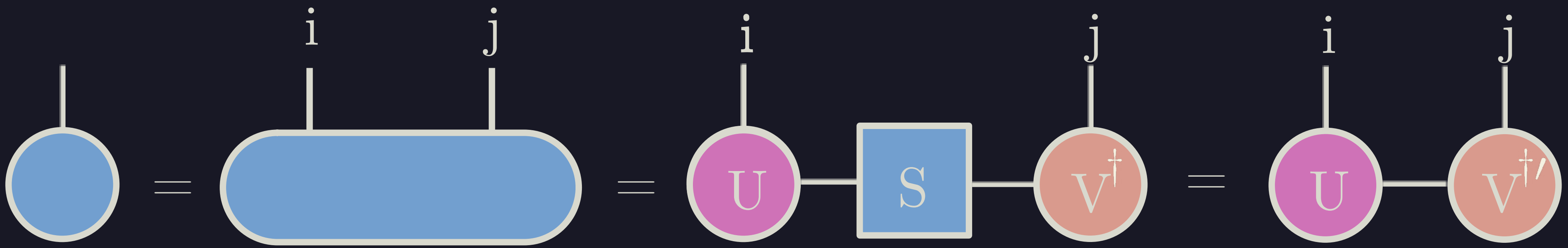


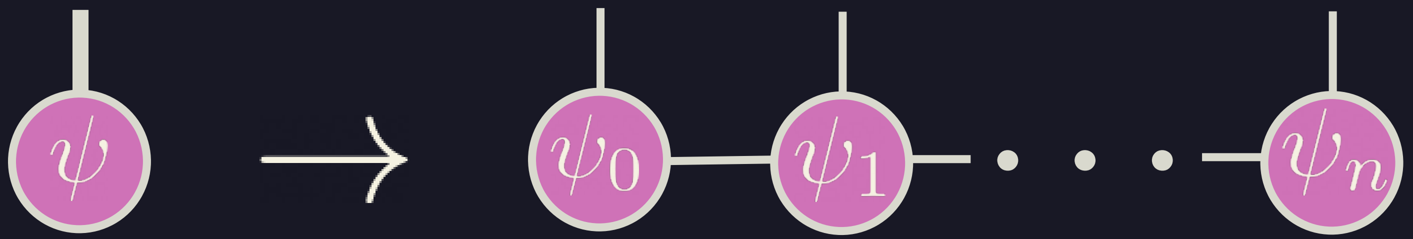

# Systems that are no *too* entengled

**Schmidt decomposition** $\iff$ **SVD**

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$$\textbf{Schmidt decomposition} \iff \textbf{SVD}$$

Singular values are the coefficient of Schmidth decomposition!

# Systems that are no *too* entengled

**Schmidt decomposition** $\Longleftrightarrow$ **SVD**

Singular values are the coefficient of Schmidth decomposition!

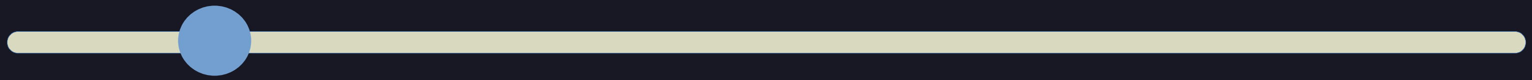
The size of the bond is the same thing as the number of coefficient in Schmidt decomposition!

# Systems that are no *too* entangled

$\chi$ is a slider for the amount of entanglement you represent

$$\chi :$$

Very fast



Very Acurate

# Using Tensor Network to do simulations

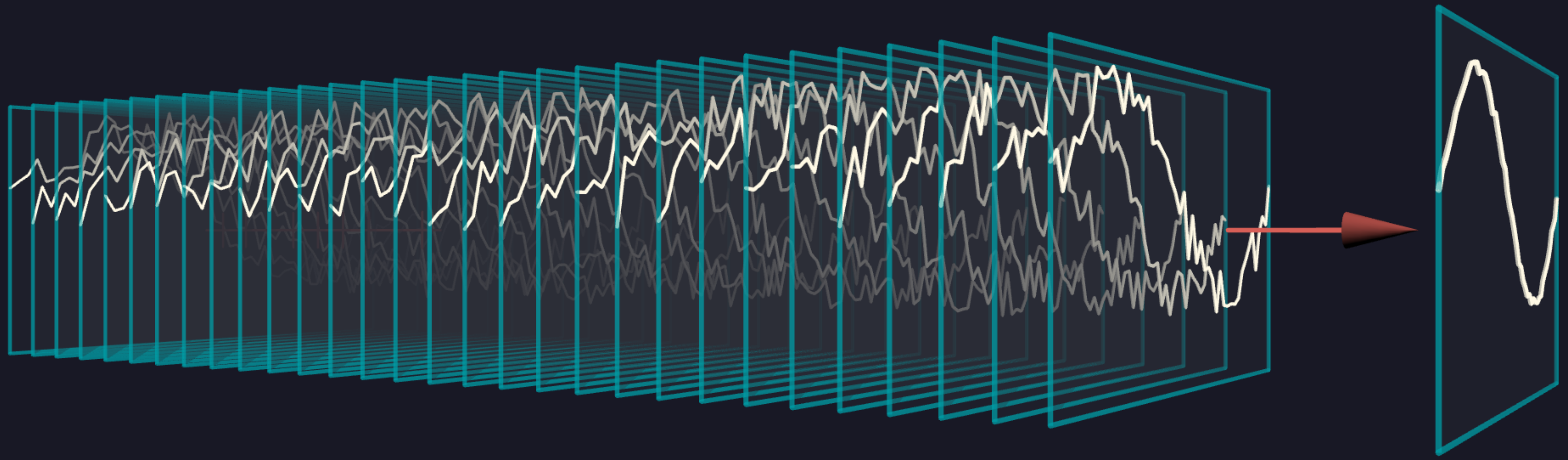
Tensor Networks are good at representing quantum circuits, not necessararaly quantum systems

~~`sesolve`~~

~~`mesolve`~~

time-evolving block decimation

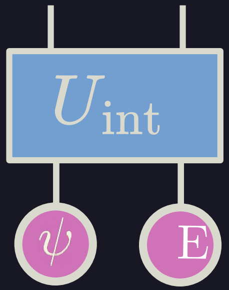
Using Monte Monte Carlo simulation



# How to simulate interaction with the environement

$|\psi_{\mathrm{tot}}\rangle = |\psi\rangle \otimes |\psi_{\mathrm{env}}\rangle \rightarrow$


$|\psi'_{\mathrm{tot}}\rangle =$




$$|\psi'\rangle = ?$$

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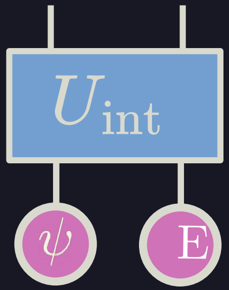


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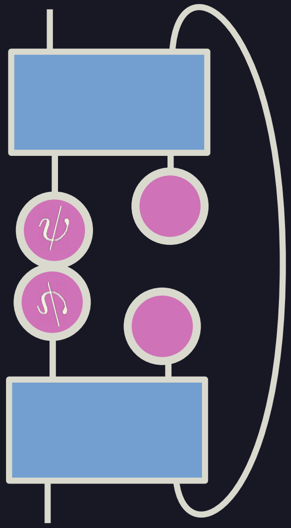

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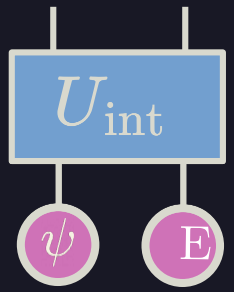


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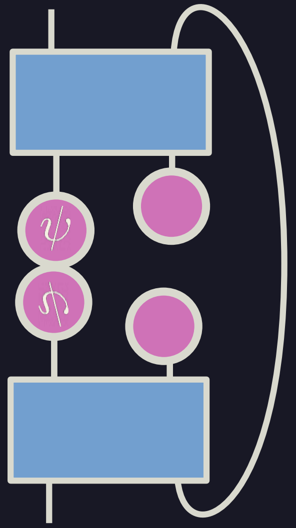


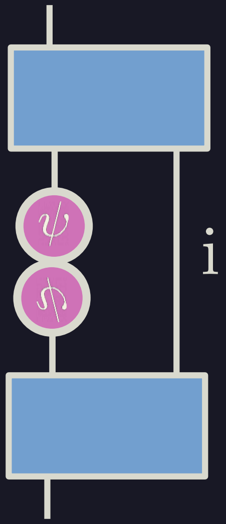

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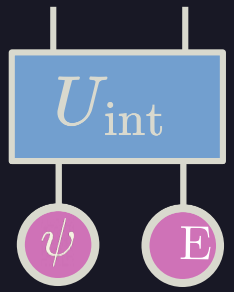


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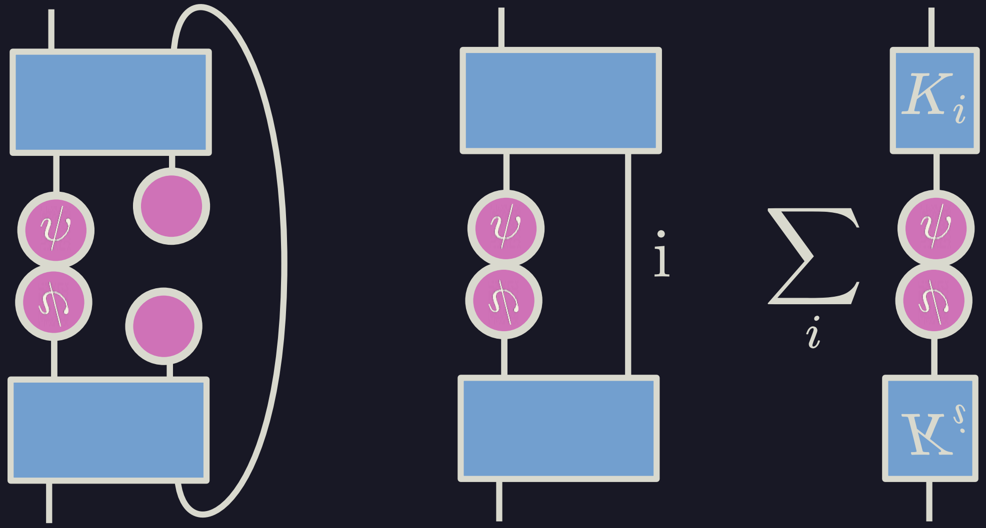

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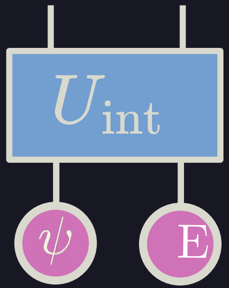


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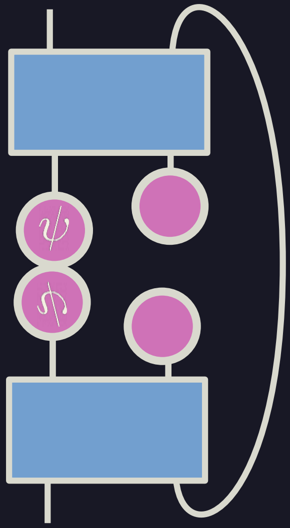

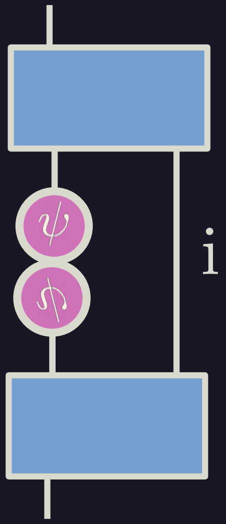


$$\sum_i$$


$$\sum_i p_i$$

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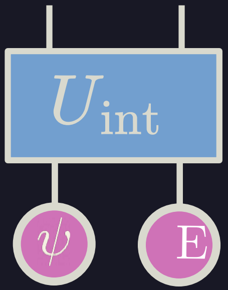
$|\psi'_{\text{tot}}\rangle =$ 



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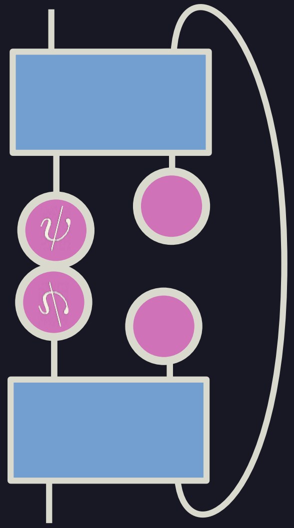

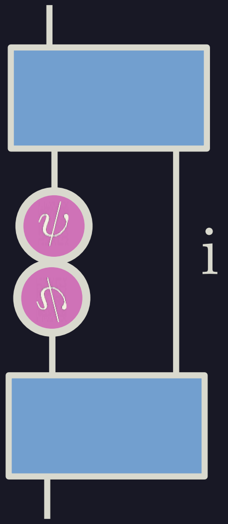



$\sum_i$ 



$\sum_i p_i$ 



$|\psi'_i\rangle$ with probability $p_i$

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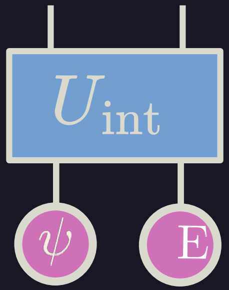

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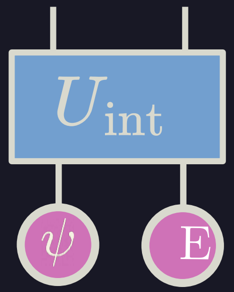


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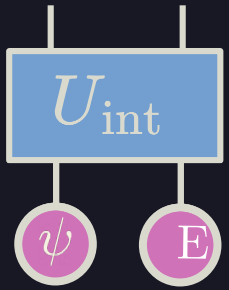


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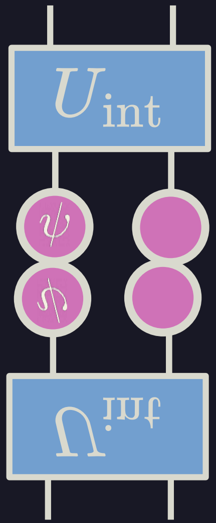

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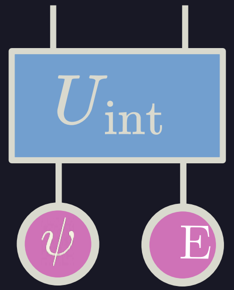
$|\psi'_{\mathrm{tot}}\rangle =$ 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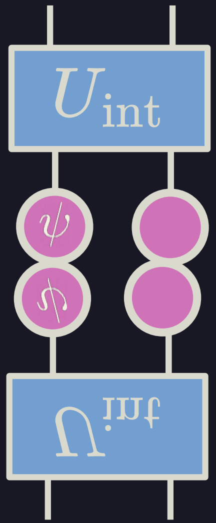
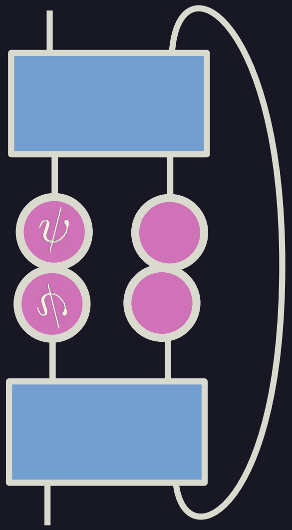
$\rho'_{\mathrm{tot}} =$ 


$\rho' =$ 

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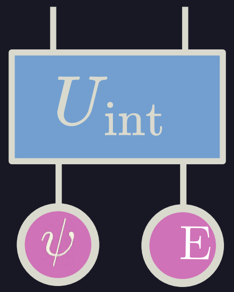


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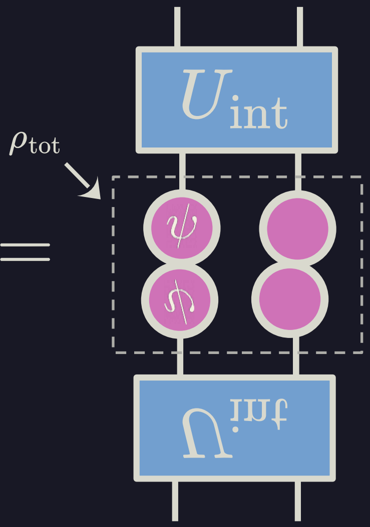


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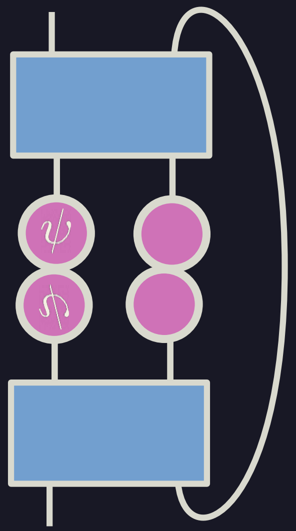

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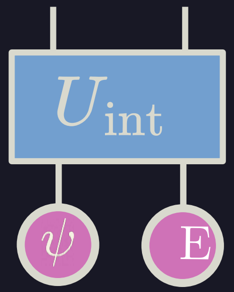
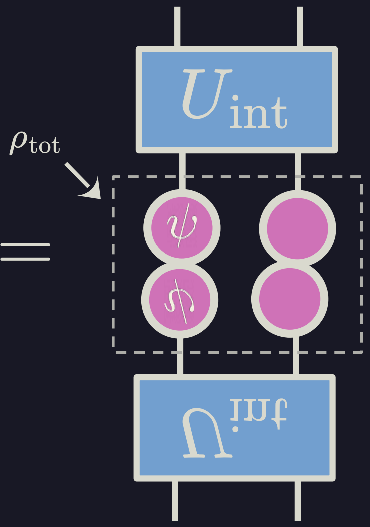
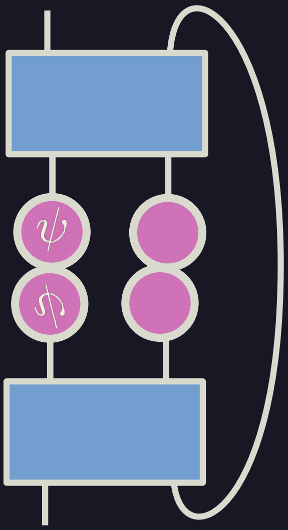
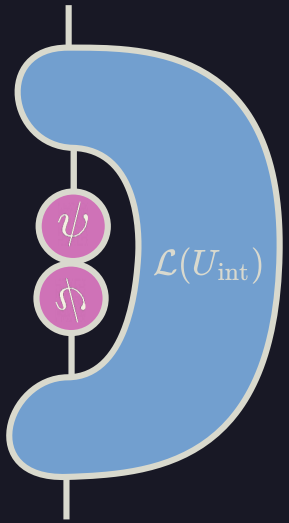
$|\psi'_{\text{tot}}\rangle =$ 


$$|\psi'\rangle = ?$$

$\rho'_{\text{tot}} =$ 


$\rho' =$ 


$\rho' =$ 

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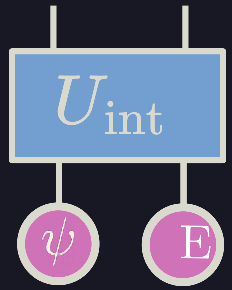
$|\psi'_{\text{tot}}\rangle =$ 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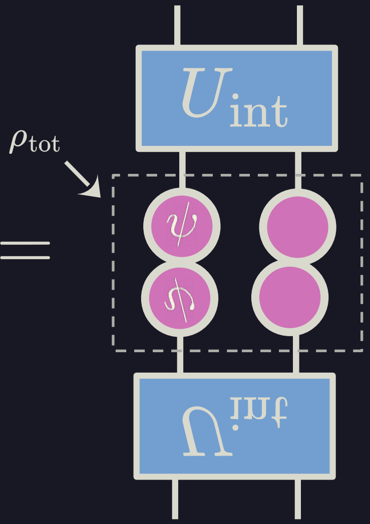
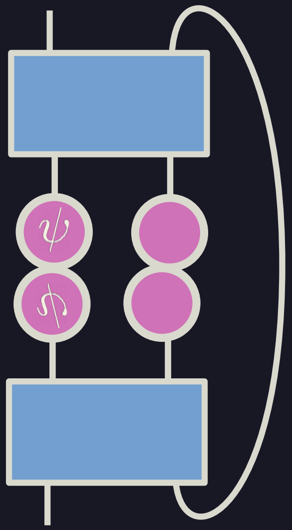
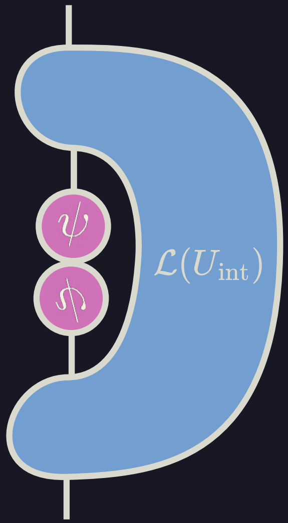
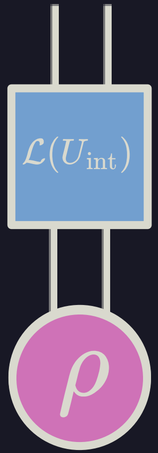
$\rho'_{\text{tot}} =$ 


$\rho' =$ 


$\rho' =$ 

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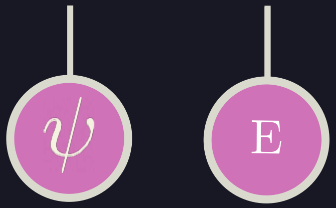


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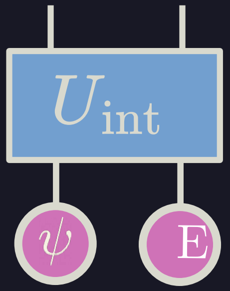


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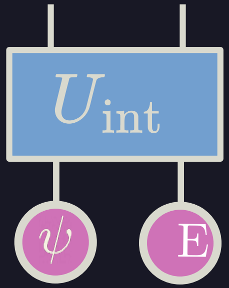


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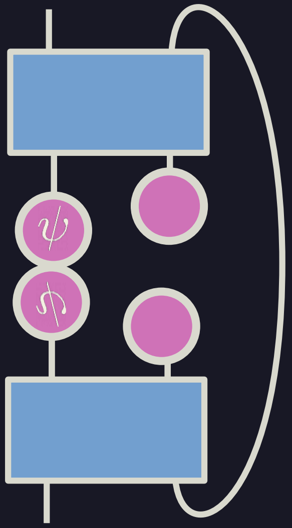

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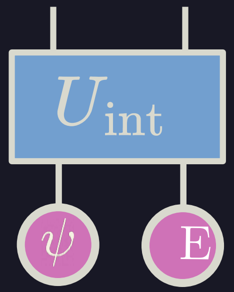


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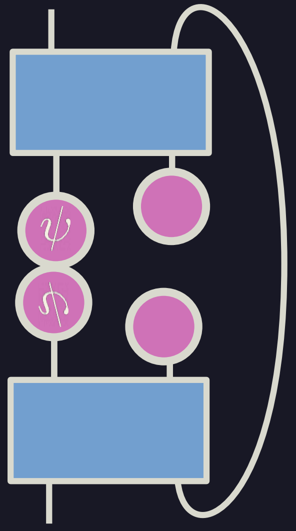


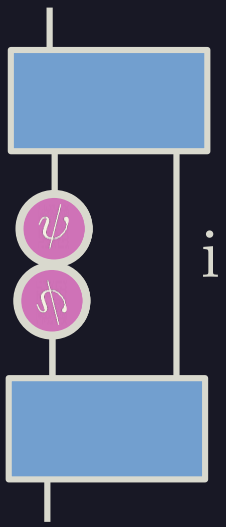

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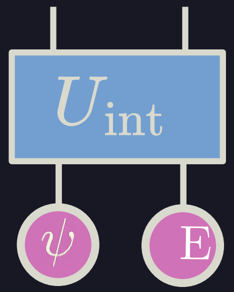
$|\psi'_{\text{tot}}\rangle =$ 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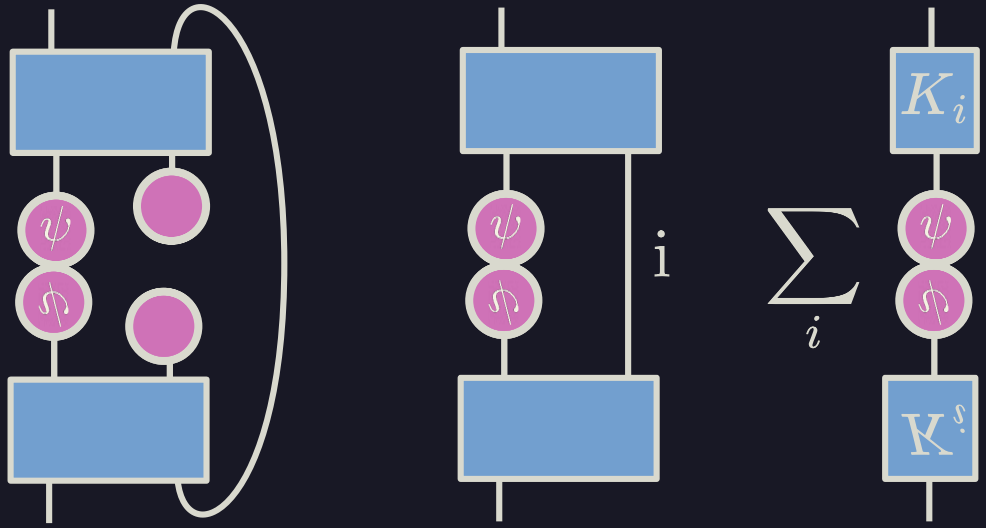

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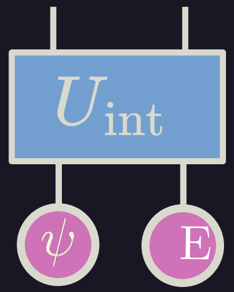


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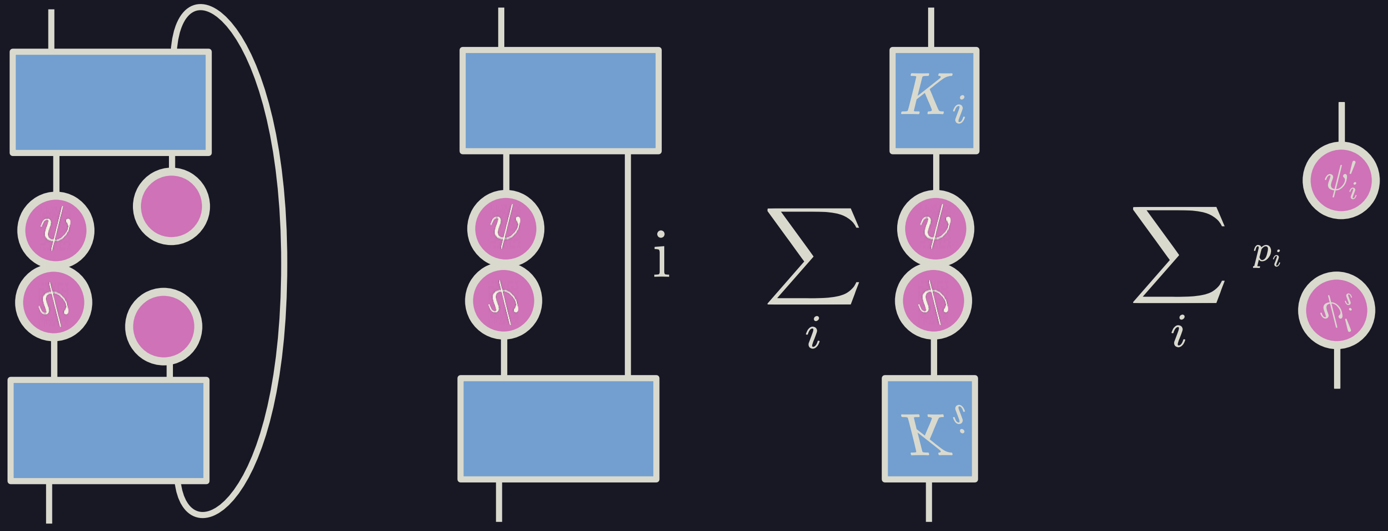

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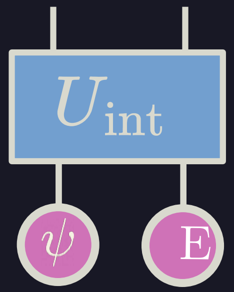


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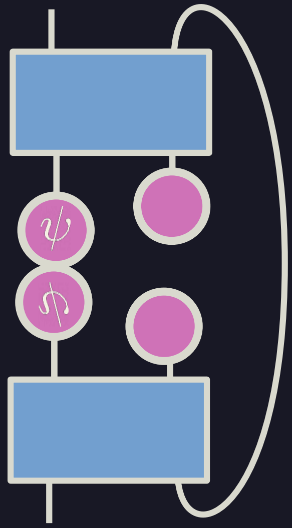
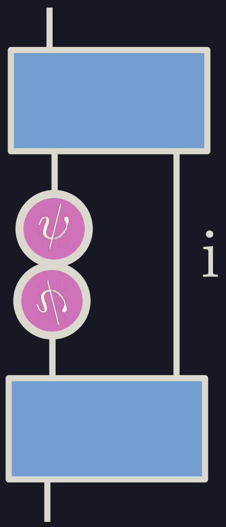


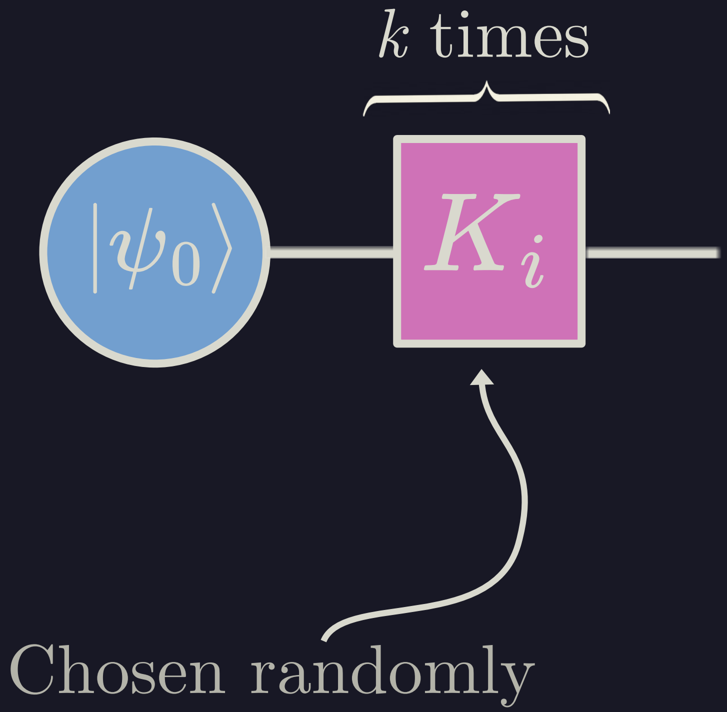




$$\sum_i$$


$$\sum_i p_i$$


$|\psi'_i\rangle$ with probability $p_i$

# Example: photon loss



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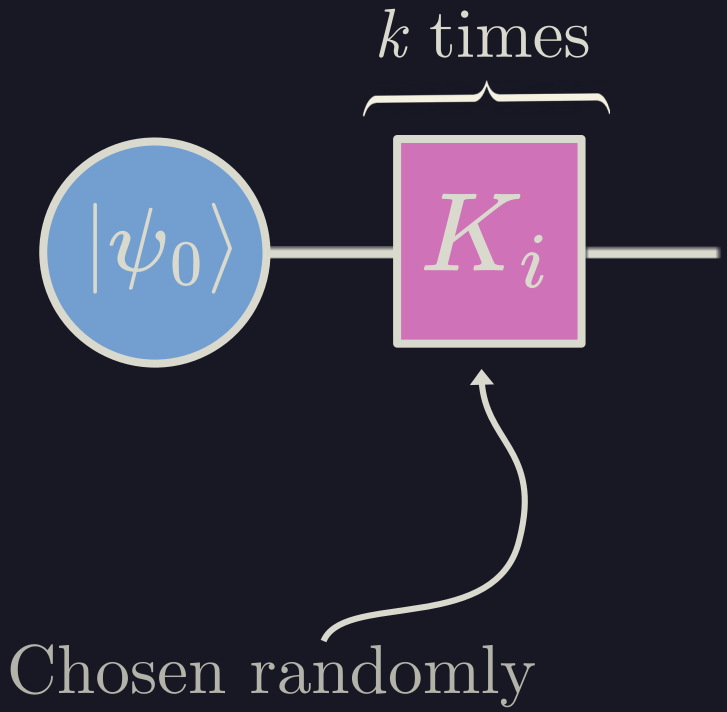


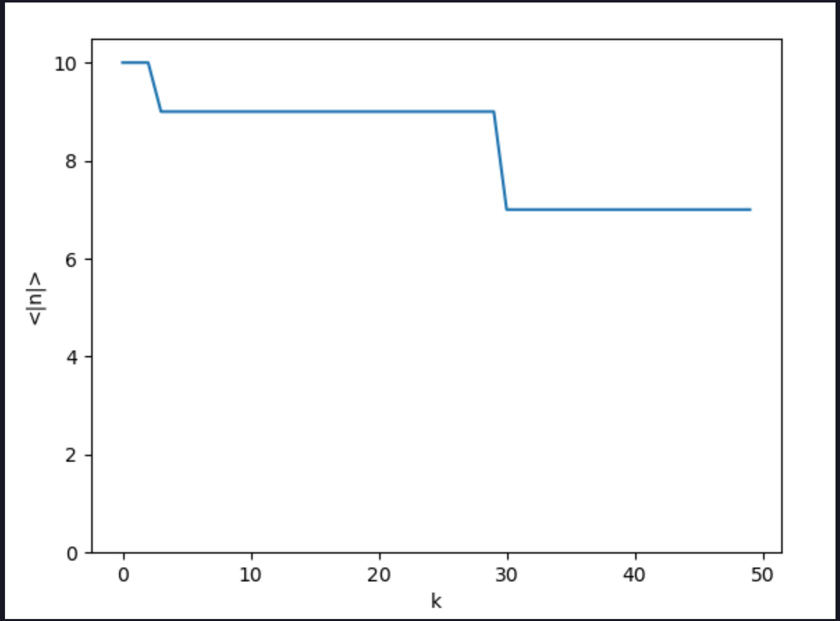

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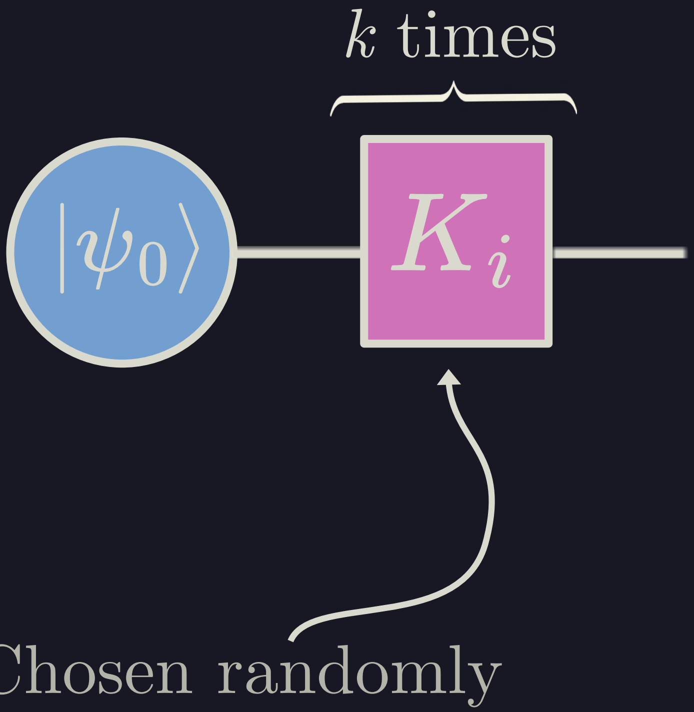


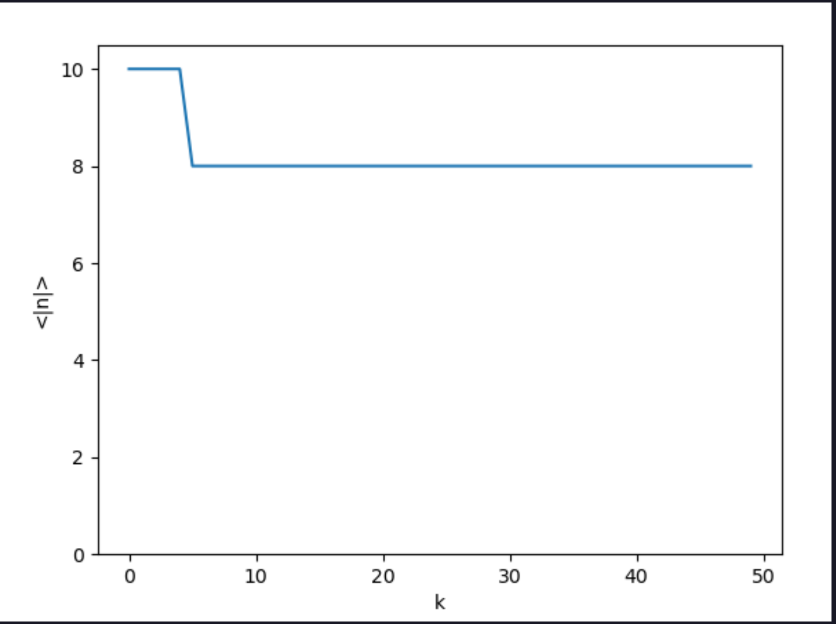

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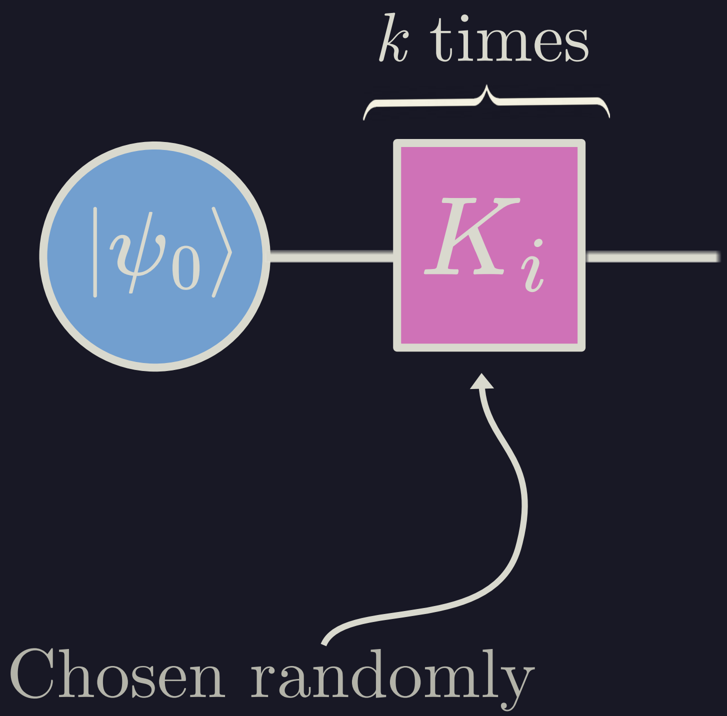


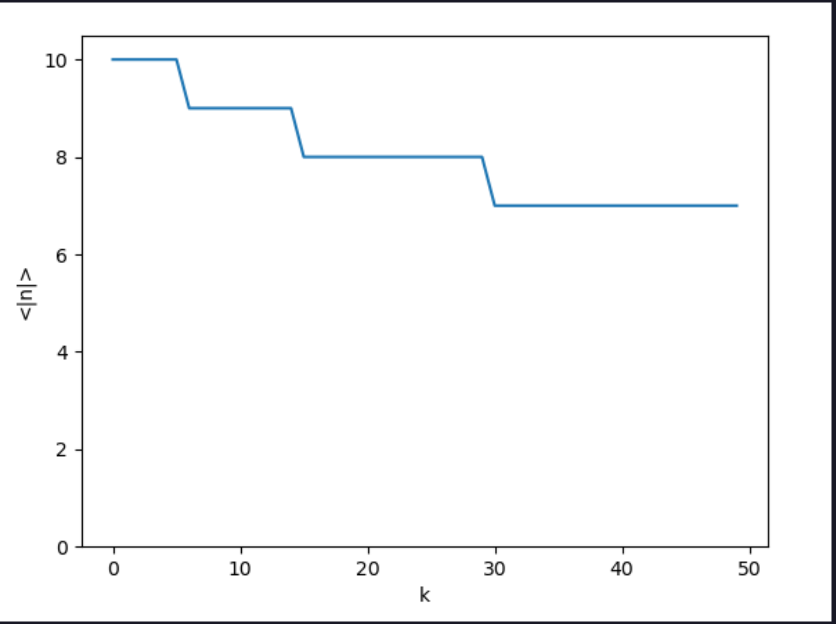

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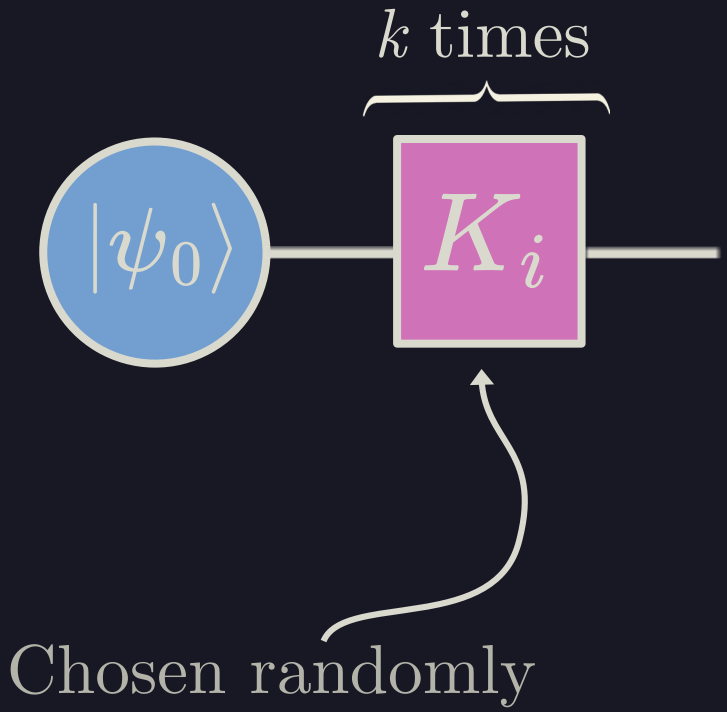


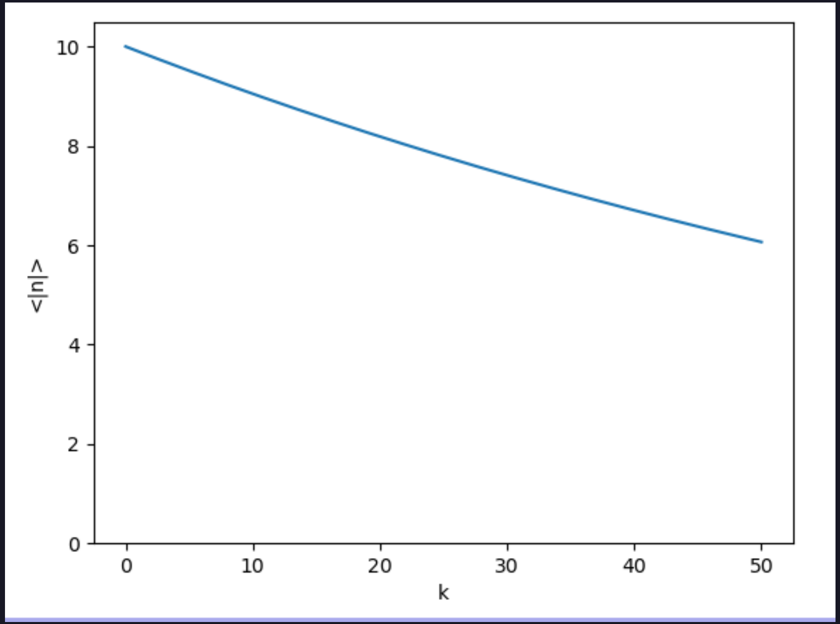

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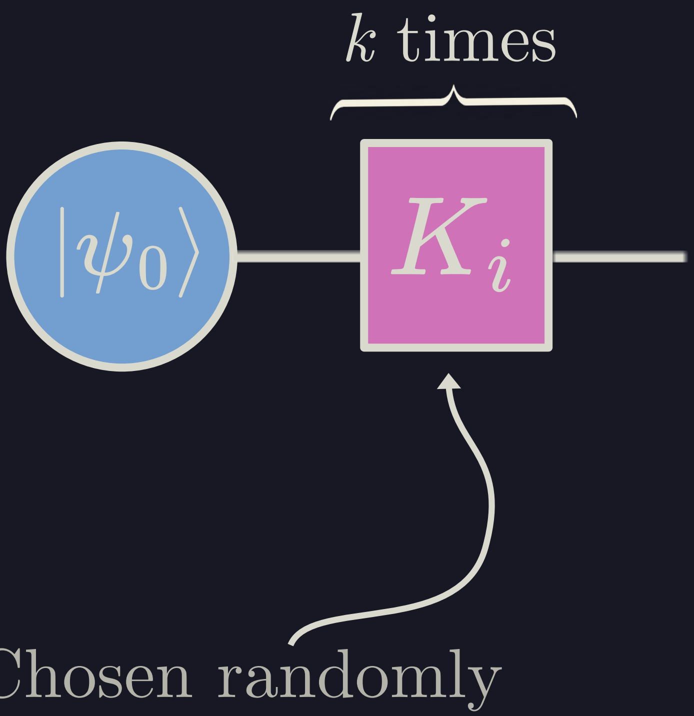
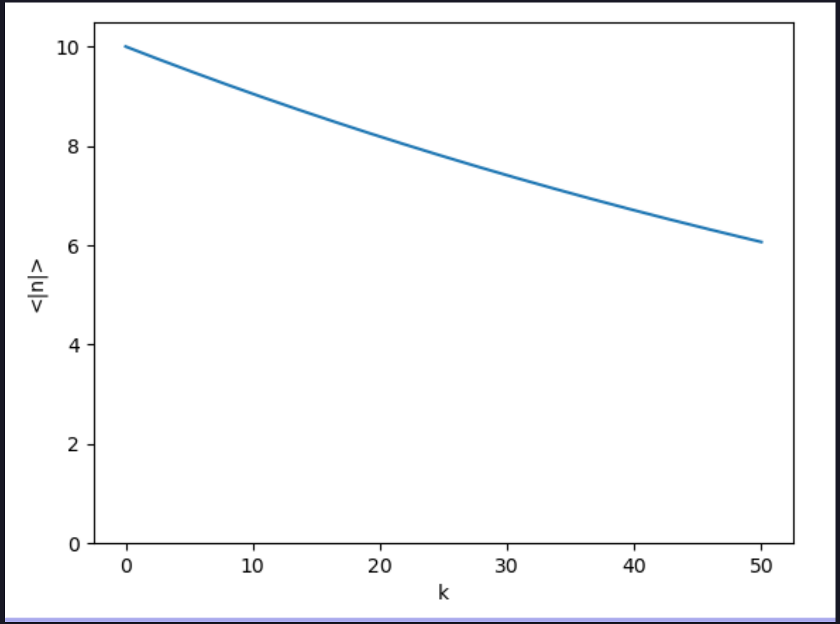
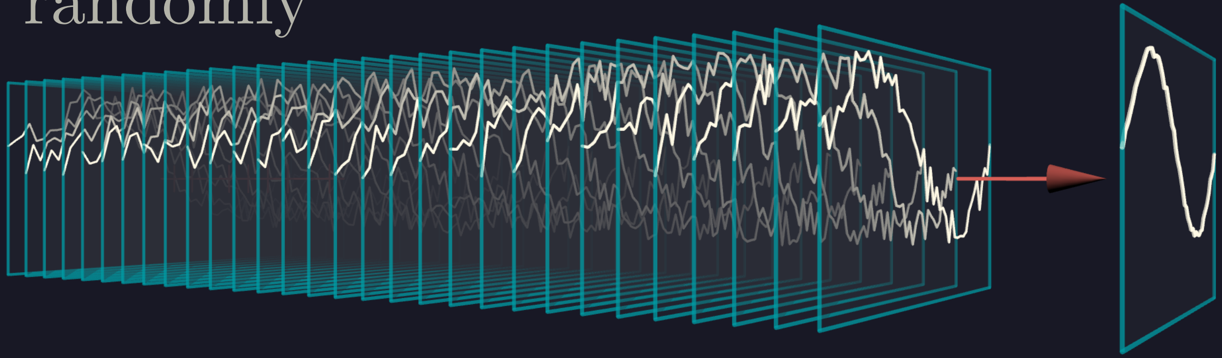


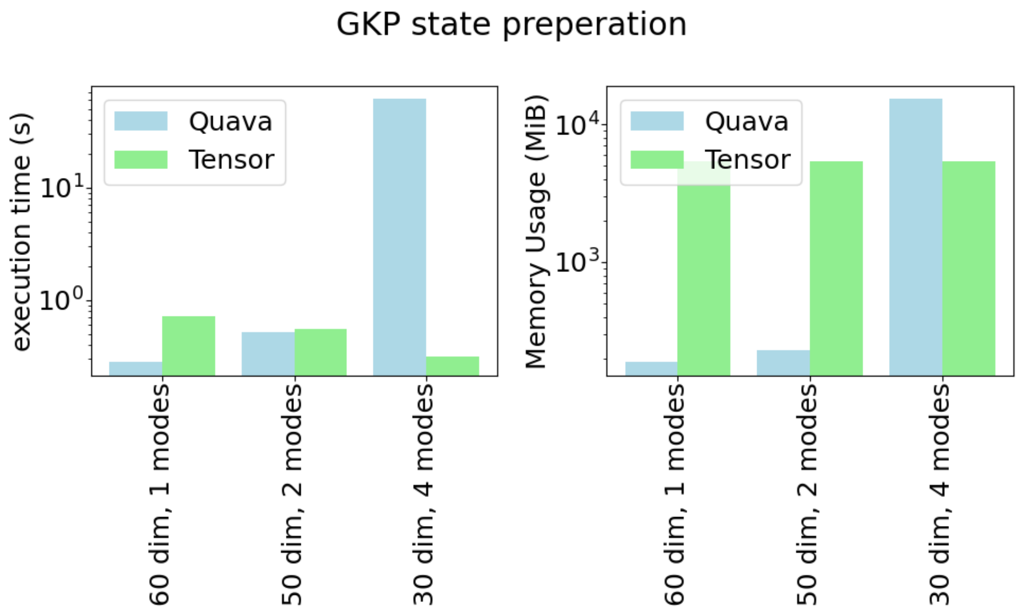
# More specific goal

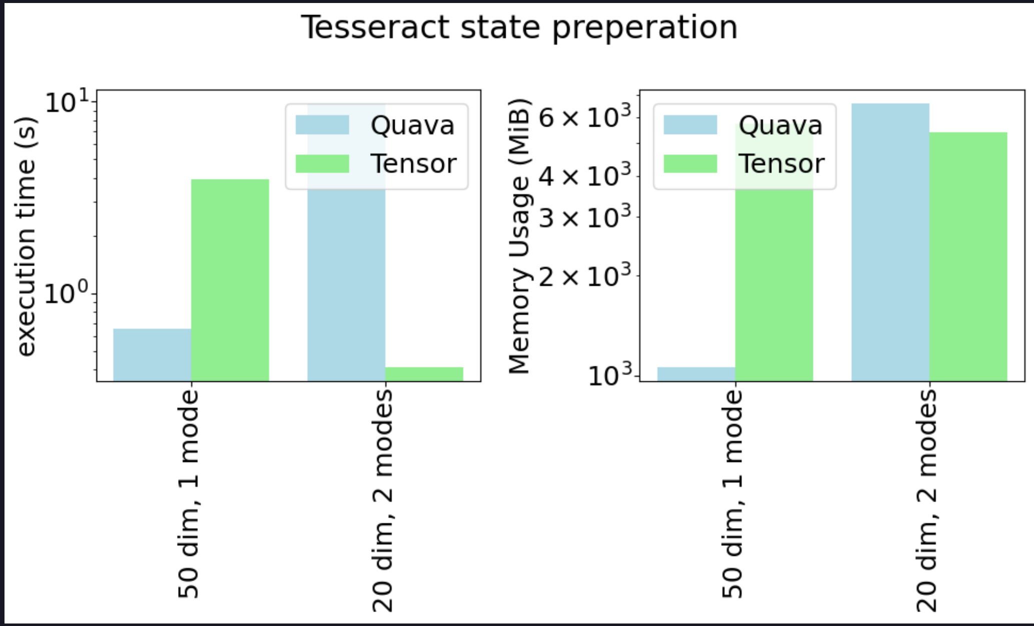
- Build a tensor network based framework that can be built upon and serve as backend for quantum simulation

- Build on top of already defined states, operators and protocols

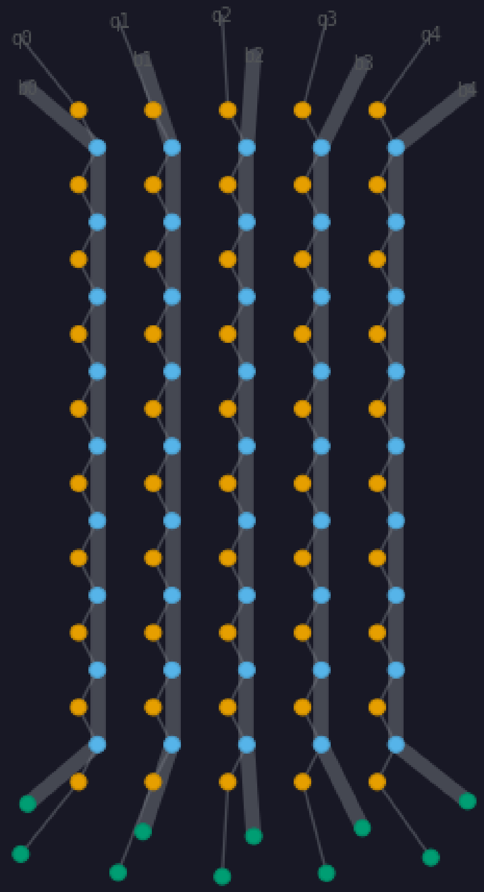
- Run multimode cavity state preperation simulation using the framwork to compare with current Nord Quantique software (Quava) and compare the performances of both methods

# Performance results



# GKP and Tessaract state preperation



# Transmon ECD based stabilisation

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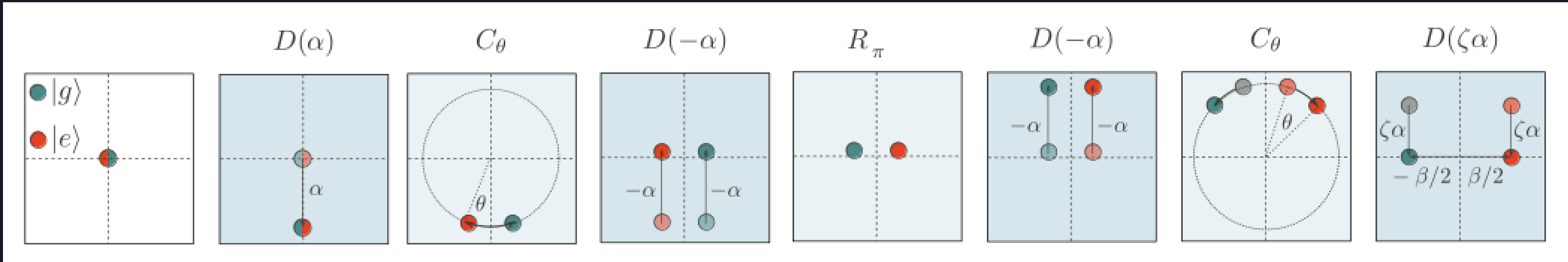

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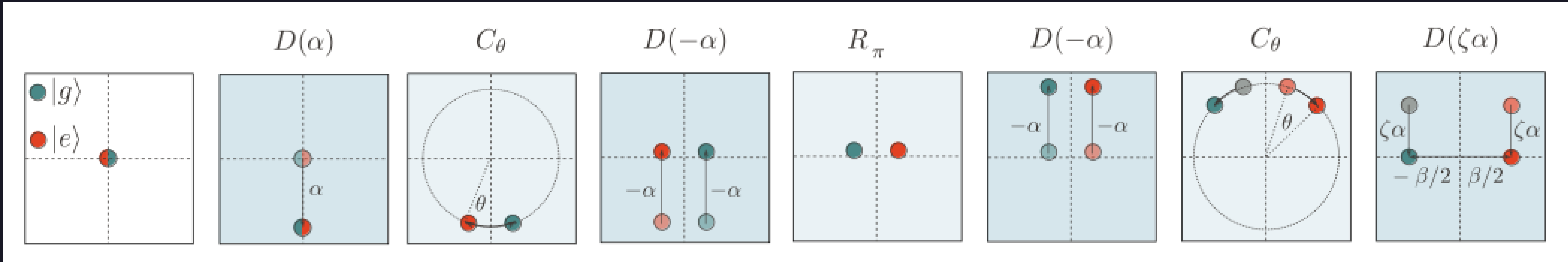


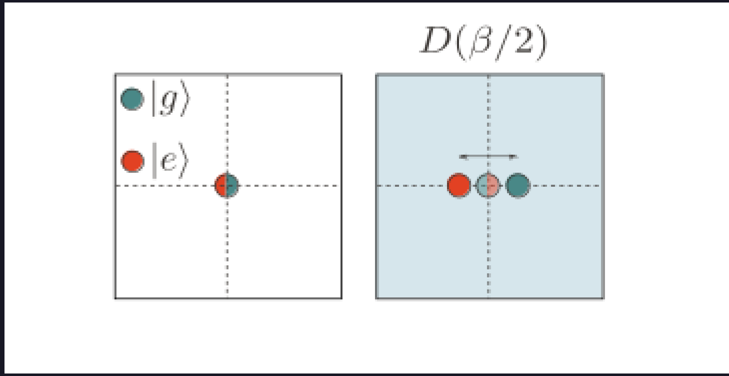

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