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DLI Accelerated Data Science Teaching Kit

Lecture 20.3 - SVD: Dimensionality Reduction, and Other Uses



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SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

U: document-concept similarity matrix

V: term-concept similarity matrix

Λ : diagonal elements: concept “strengths”

SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

Q: if **A** is the document-to-term matrix,
what is the similarity matrix **A^T A** ?

A: term-to-term ([m x m]) similarity matrix

Q: **A A^T** ?

A: document-to-document ([n x n]) similarity matrix

SVD properties

V are the eigenvectors of the *covariance matrix* $\mathbf{A}^T \mathbf{A}$

$$\mathbf{A}^T \mathbf{A} = (\mathbf{U} \Sigma \mathbf{V}^T)^T (\mathbf{U} \Sigma \mathbf{V}^T) = \mathbf{V} \Sigma^2 \mathbf{V}^T$$

U are the eigenvectors of the *Gram (inner-product) matrix* $\mathbf{A} \mathbf{A}^T$

$$\mathbf{A} \mathbf{A}^T = (\mathbf{U} \Sigma \mathbf{V}^T) (\mathbf{U} \Sigma \mathbf{V}^T)^T = \mathbf{U} \Sigma^2 \mathbf{U}^T$$

SVD is closely related to PCA, and can be numerically more stable.

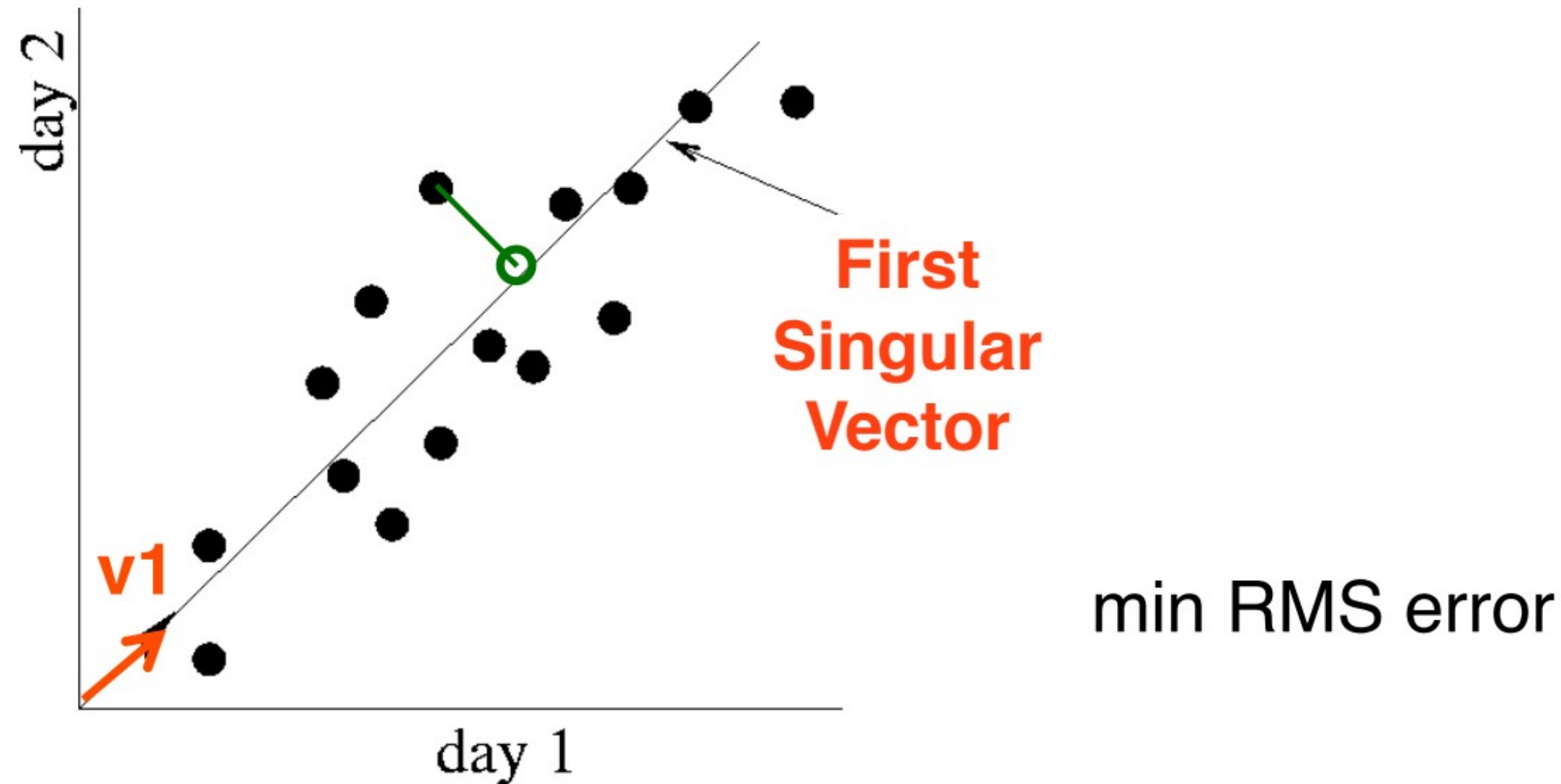
For more info, see:

<http://math.stackexchange.com/questions/3869/what-is-the-intuitive-relationship-between-svd-and-pca>
Ian T. Jolliffe, Principal Component Analysis (2nd ed), Springer, 2002. Gilbert Strang, Linear Algebra and Its Applications (4th ed), Brooks Cole, 2005.

SVD - Interpretation #2

Find the best axis to project on.

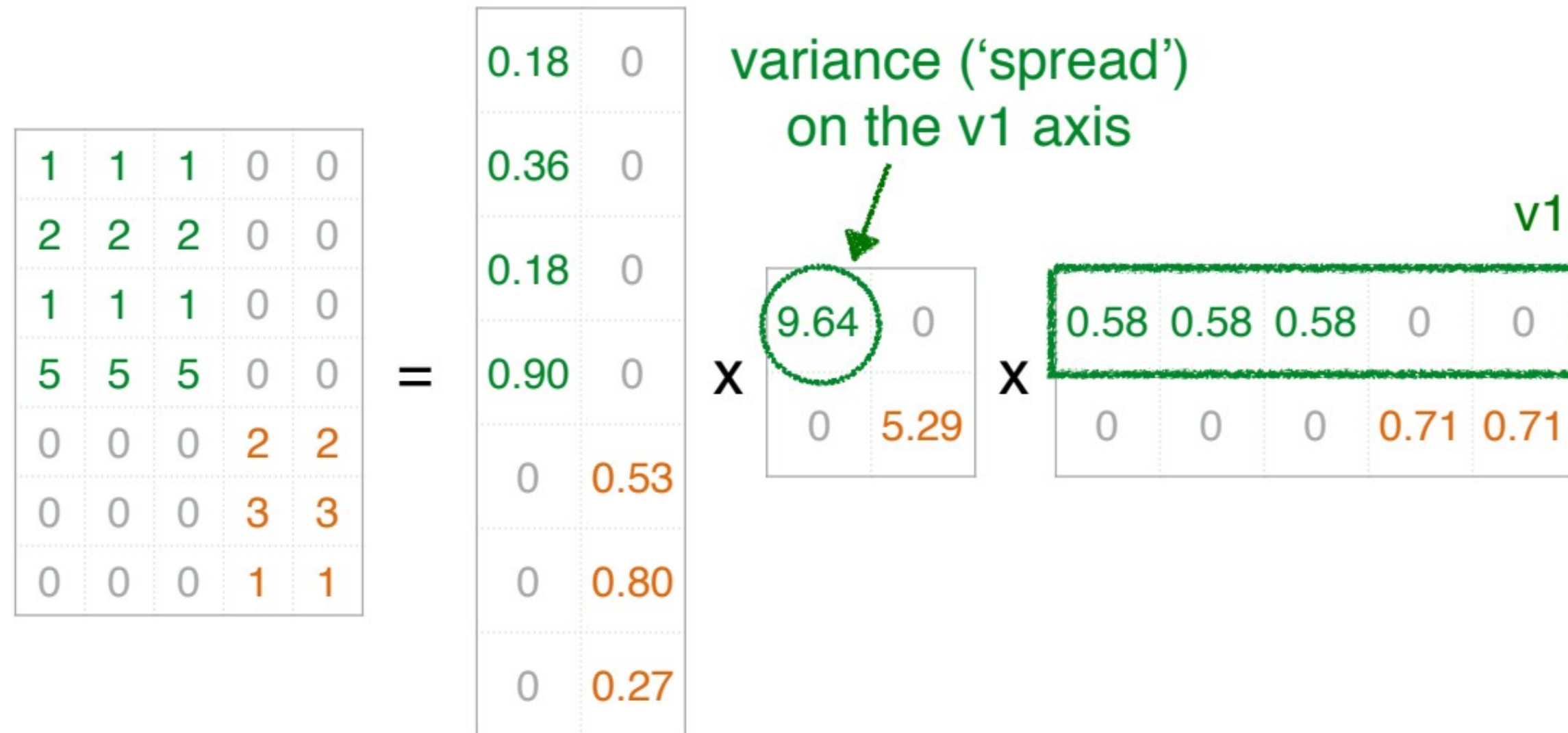
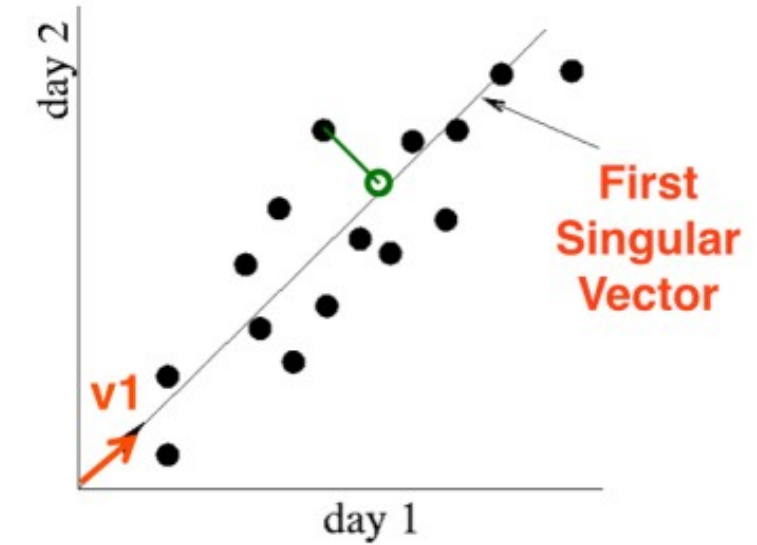
(‘best’ = min sum of squares of projection errors)



Beautiful visualization explaining PCA:
<http://setosa.io/ev/principal-component-analysis/>

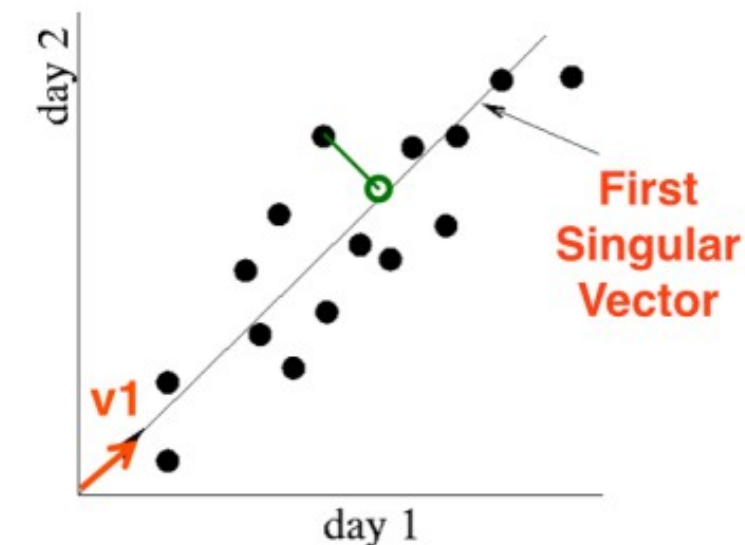
SVD - Interpretation #2

$$\mathbf{A} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^T$$



SVD - Interpretation #2

$U \Lambda$ gives the **coordinates** of the points in the projection axis



$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

The matrix $\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$ is labeled $v1$ in green.

SVD - Interpretation #2

More details

Q: how exactly is dim. reduction done?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

More details

Q: how exactly is dim. reduction done?

A: set the smallest singular values to zero:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.9 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

The diagram illustrates the SVD decomposition of a 7x5 matrix. The original matrix is decomposed into three matrices: a 7x2 matrix of singular values, a 2x2 matrix of singular vectors, and a 2x5 matrix of singular vectors. The singular values are 9.64, 5.9, and 0. The singular vectors are 0.58, 0.71, and 0. The singular value 5.9 is highlighted with a red 'X' and a red box, indicating it is the smallest singular value and is being set to zero for dimensionality reduction.

SVD - Interpretation #2

More details

Q: how exactly is dim. reduction done?

A: set the smallest singular values to zero:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.9 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

The diagram illustrates the SVD decomposition of a 7x5 matrix. The original matrix is shown on the left. It is decomposed into three matrices: a 7x2 matrix of left singular vectors (U), a 2x2 matrix of singular values (Σ), and a 2x5 matrix of right singular vectors (V). The singular values are 9.64 and 5.9. The right singular vectors are 0.58, 0.58, 0.58, 0.71, and 0.71. Red boxes and 'X' marks highlight the process of dimensionality reduction: the smallest singular value (5.9) is being set to zero, and the corresponding right singular vectors (0.71, 0.71) are being set to zero.

SVD - Interpretation #2

More details

Q: how exactly is dim. reduction done?

A: set the smallest singular values to zero:

1	1	1	0	0
2	2	2	0	0
1	1	1	0	0
5	5	5	0	0
0	0	0	2	2
0	0	0	3	3
0	0	0	1	1

 $=$

0.18	
0.36	
0.18	
0.90	
0	
0	
0	

 \times

9.64	0
0	

 \times

0.58	0.58	0.58	0	0

SVD - Interpretation #2

More details

Q: how exactly is dim. reduction done?

A: set the smallest singular values to zero:

1	1	1	0	0
2	2	2	0	0
1	1	1	0	0
5	5	5	0	0
0	0	0	2	2
0	0	0	3	3
0	0	0	1	1

~

1	1	1	0	0
2	2	2	0	0
1	1	1	0	0
5	5	5	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

SVD - Complexity

$O(n*m*m)$ or $O(n*n*m)$ (whichever is less)

Faster version, if just want singular values
or if we want first k singular vectors
or if the matrix is sparse [Berry]

No need to write your own!
Available in most linear algebra packages
(LINPACK, matlab, Splus/R,
mathematica ...)

Case Study

How to do queries with LSI?

Case Study

How to do queries with LSI?

For example, how to find documents with 'data'?

The diagram illustrates the LSI query process for finding documents with 'data'. It shows a matrix of documents (CS and MD) and a query vector, followed by the resulting dot products.

Document Matrix:

	data	info	retrieval	brain	lung
CS docs	1	1	1	0	0
2	2	2	2	0	0
1	1	1	1	0	0
5	5	5	5	0	0
MD docs	0	0	0	2	2
0	0	0	3	3	3
0	0	0	1	1	1

Query Vector (data):

0.18	0
0.36	0
0.18	0
0.90	0
0	0.53
0	0.80
0	0.27

Intermediate Results:

9.64	0
0	5.29

0.58	0.58	0.58	0	0
0	0	0	0.71	0.71

Case Study

How to do queries with LSI?

For example, how to find documents with 'data'?

A: map query vectors into 'concept space' – how?

The diagram illustrates the process of mapping query vectors into a concept space using Latent Semantic Indexing (LSI). It shows a matrix of document counts for five concepts (data, info, retrieval, brain, lung) across two sets of documents (CS docs and MD docs). The matrix is then multiplied by a series of matrices to produce a final vector.

	data	info	retrieval	brain	lung
CS docs	1	1	1	0	0
CS docs	2	2	2	0	0
CS docs	1	1	1	0	0
CS docs	5	5	5	0	0
MD docs	0	0	0	2	2
MD docs	0	0	0	3	3
MD docs	0	0	0	1	1

The matrix is multiplied by a series of matrices to produce a final vector:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

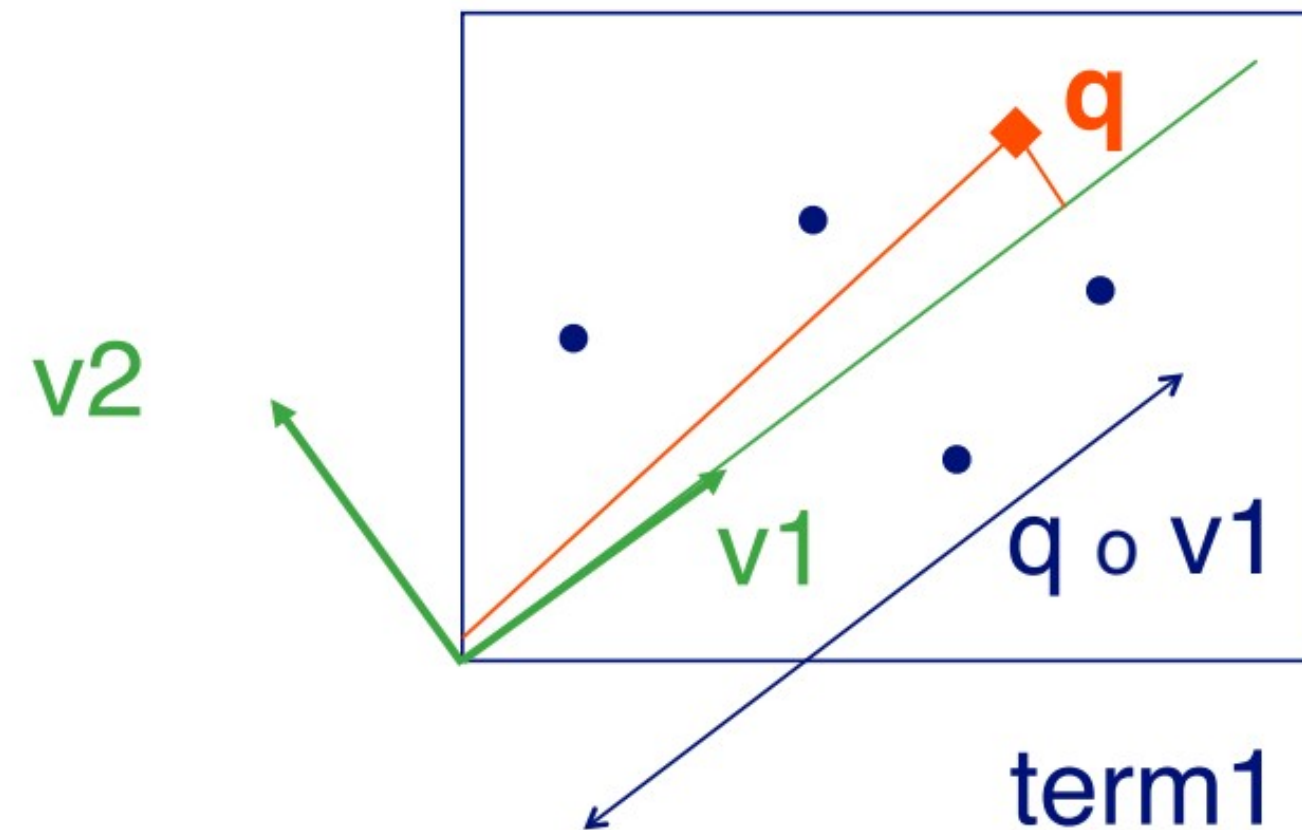
Case Study

How to do queries with LSI?

For example, how to find documents with 'data'?

A: map query vectors into 'concept space', using **inner product** (cosine similarity) with each 'concept' vector v_i

$$\mathbf{q} = \begin{array}{c} \text{data} \\ \text{info} \\ \text{retrieval} \\ \text{brain} \\ \text{lung} \end{array} \begin{array}{|c|c|c|c|c|} \hline 1 & 0 & 0 & 0 & 0 \\ \hline \end{array}$$

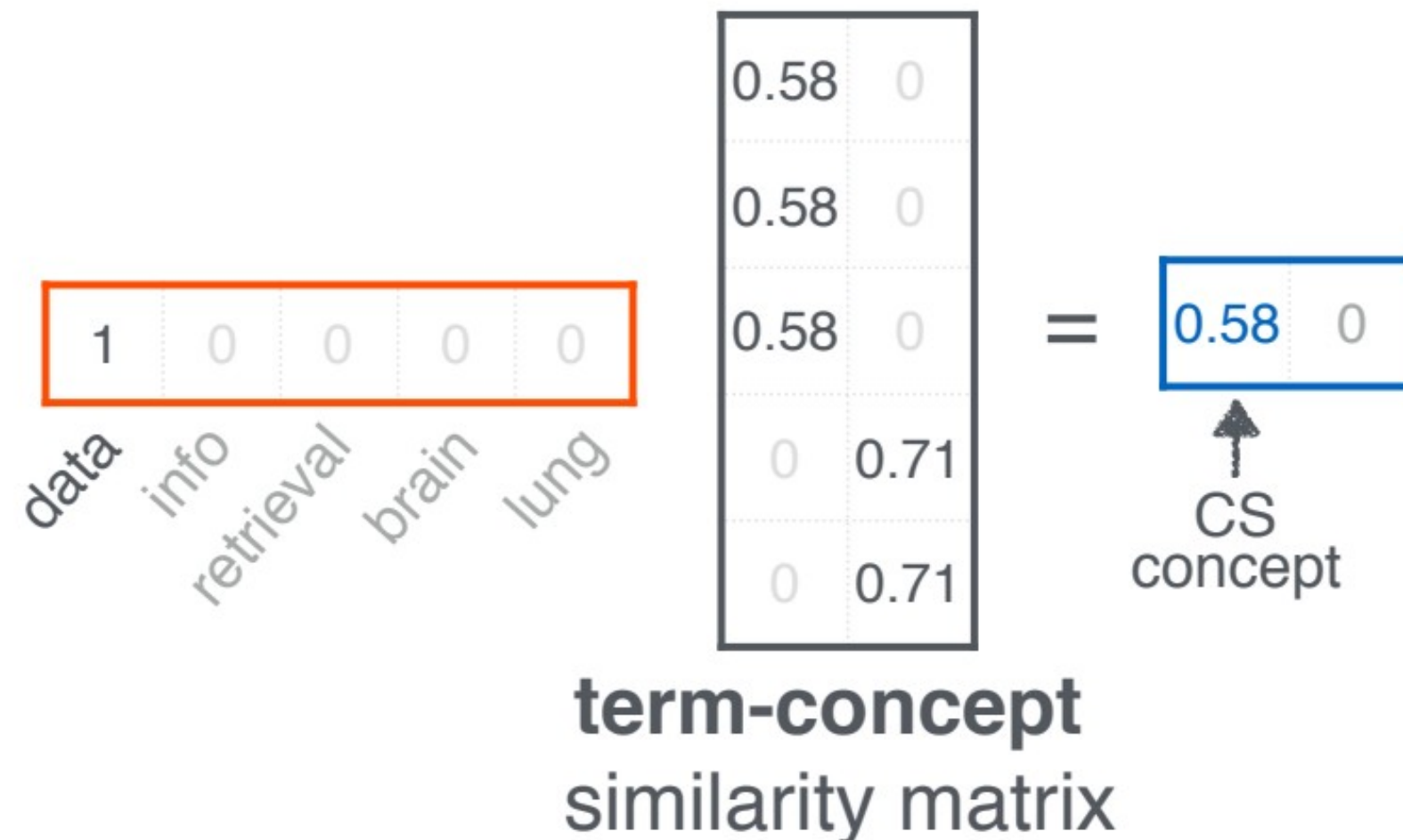


Case Study

How to do queries with LSI?

Compactly, we have:

$$\mathbf{q} \mathbf{V} = \mathbf{q}_{\text{concept}}$$



Case Study

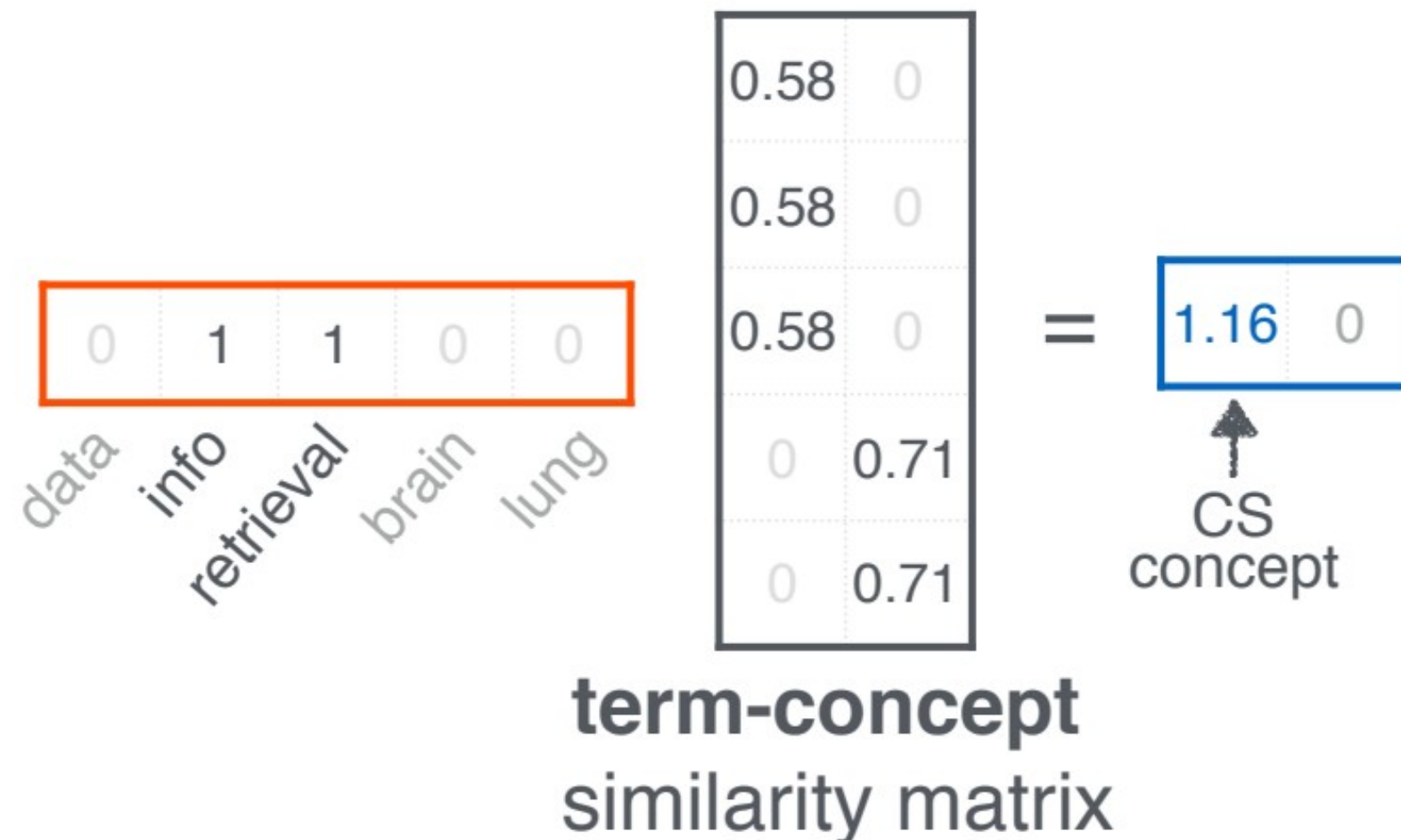
**How would the document
(‘information’, ‘retrieval’) be handled?**

Case Study

How would the document ('information', 'retrieval') be handled?

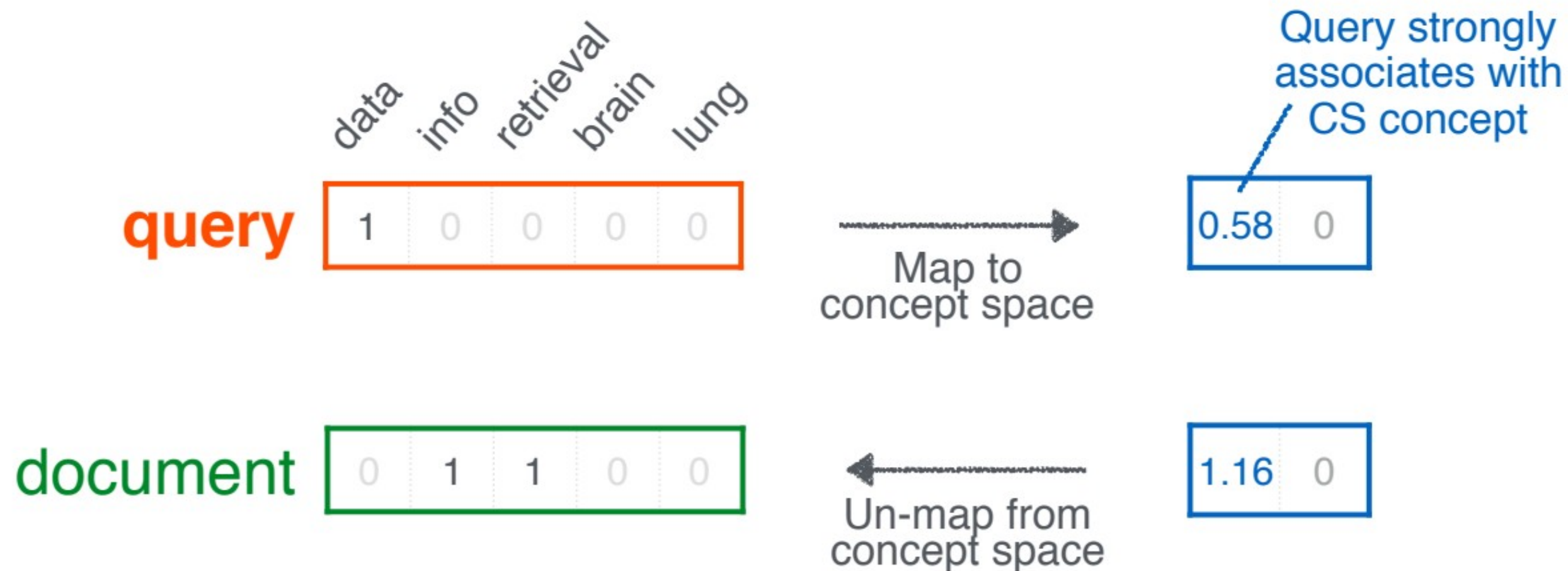
SAME!

$$\mathbf{d} \mathbf{V} = \mathbf{d}_{\text{concept}}$$



Case Study Observation

Document ('information', 'retrieval') will be retrieved by **query** ('data'), even though it does not contain 'data'!!





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Thank You