034IN - FONDAMENTI DI AUTOMATICA - FUNDAMENTALS OF AUTOMATIC CONTROL
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Part V: Block Schemes

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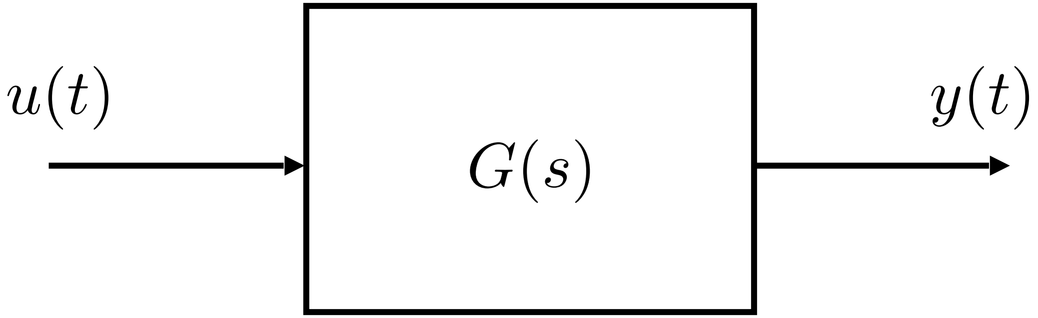
Motivations:

Block schemes are useful because they allow:

- **representing** – in a natural way – complex interconnected dynamic systems in terms of sub-systems interacting with each other.
- **emphasising** – in a graphical way – the interactions among the sub-systems
- **determining** – in an easy and systematic way – the overall (possibly complex) transfer function on the basis of the simpler transfer functions of the sub-systems

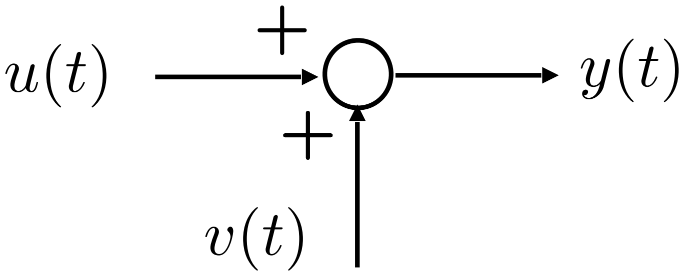
Basic Components of Block Schemes

- **Block**



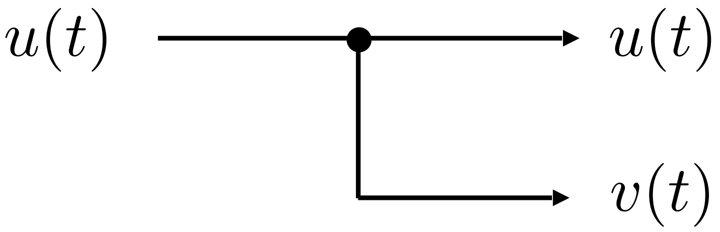
$$Y(s) = G(s)U(s)$$

- **Algebraic Summation**



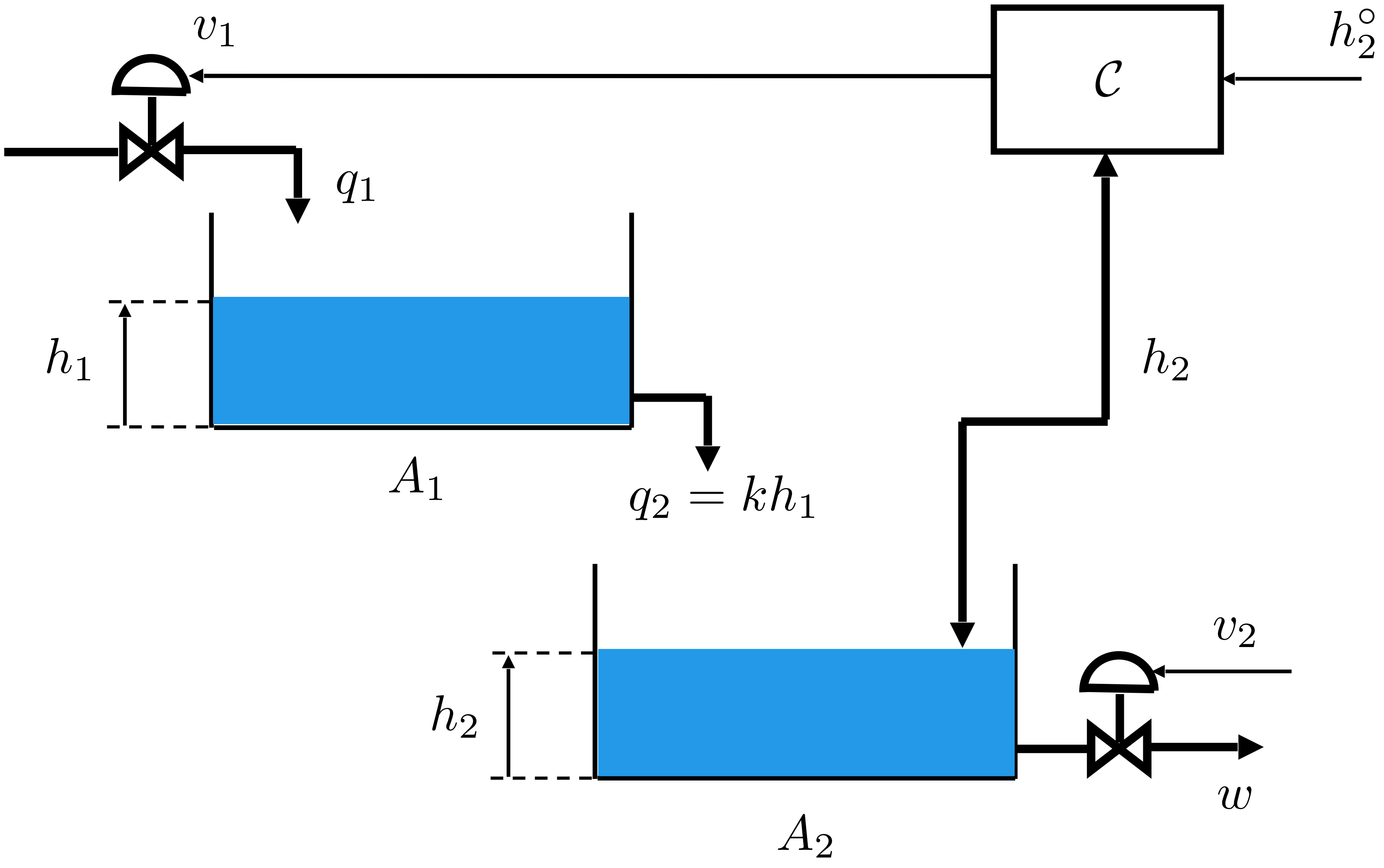
$$Y(s) = U(s) + V(s)$$

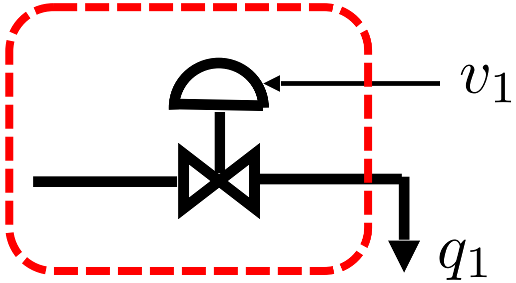
- **Interconnection Points**



$$U(s) = V(s)$$

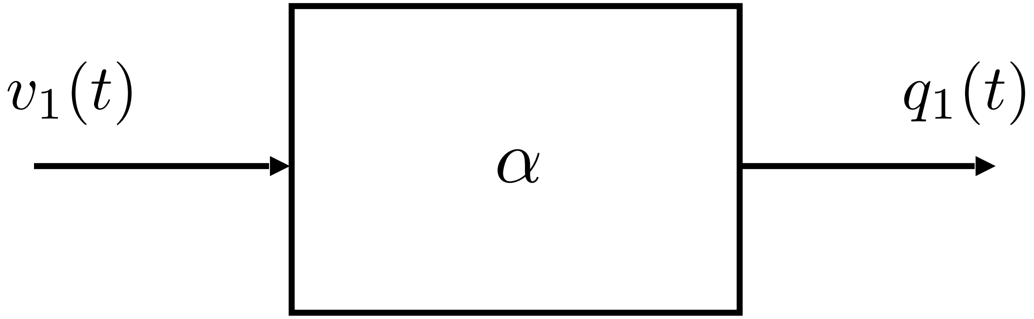
Example: Level Control of a Multi-Tank Hydraulic System





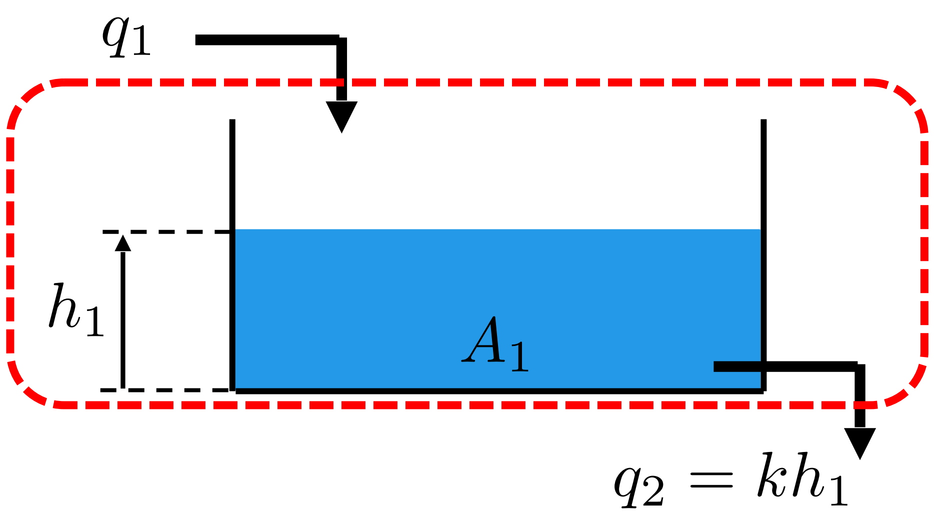
Consider an ideal electrically controlled valve such that:

$$q_1(t) = \alpha v_1(t)$$



$$Q_1(s) = \alpha V_1(s)$$

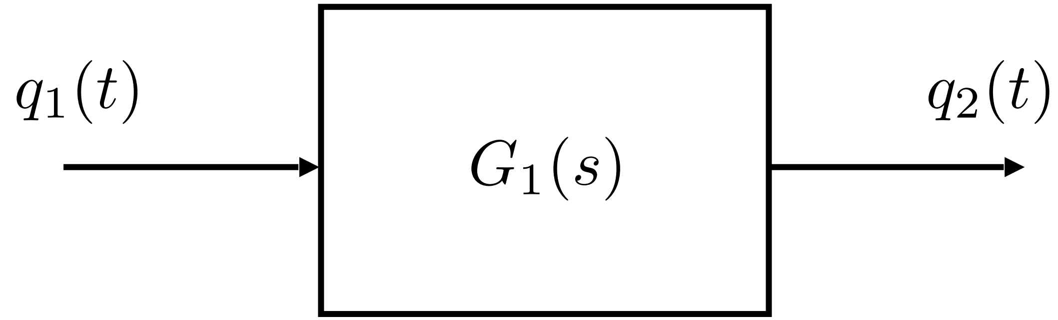
Example: Level Control of a Multi-Tank Hydraulic System (contd.)



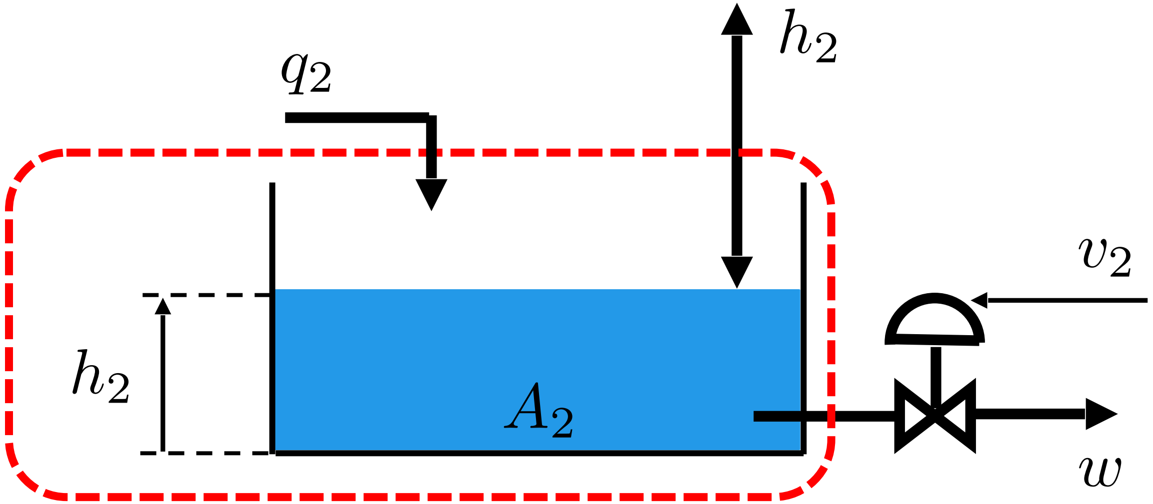
By the usual assumptions, the upper tank is modelled as:

$$\begin{cases} A_1\dot{h}_1 = q_1 - kh_1 \\ q_2 = kh_1 \end{cases}$$

$$G_1(s) = C(sI - A)^{-1}B + D = k(A_1 s + k)^{-1} = \frac{k}{A_1 s + k}$$



Example: Level Control of a Multi-Tank Hydraulic System (contd.)

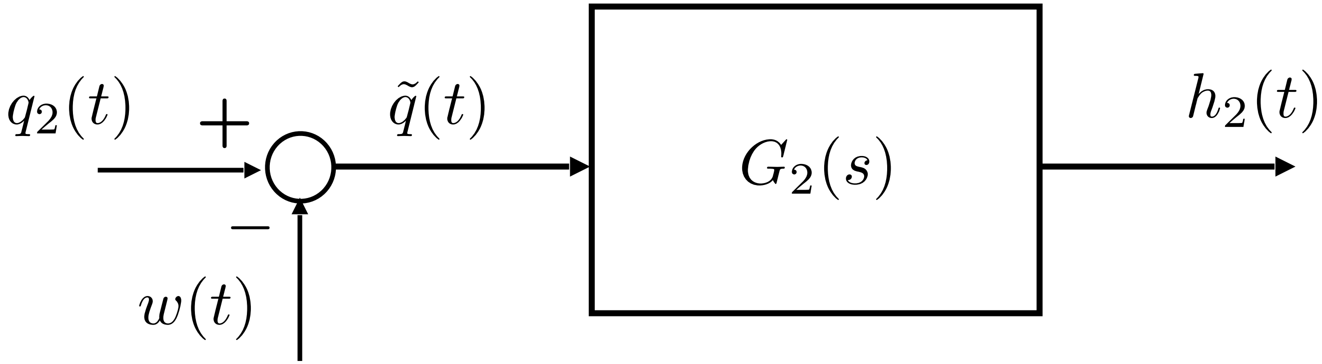


By the usual assumptions, the lower tank is modelled as:

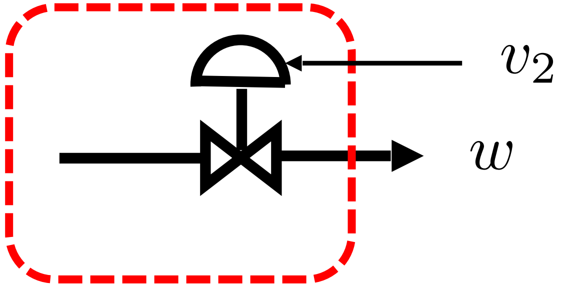
$$A_2\dot{h}_2 = \underbrace{q_2 - w}_{\tilde{q}}$$

$\tilde{q}$ "equivalent" input

$$H_2(s) = \frac{1}{sA_2}\left[Q_2(s) - W(s)\right] = G_2(s)\tilde{Q}(s)$$

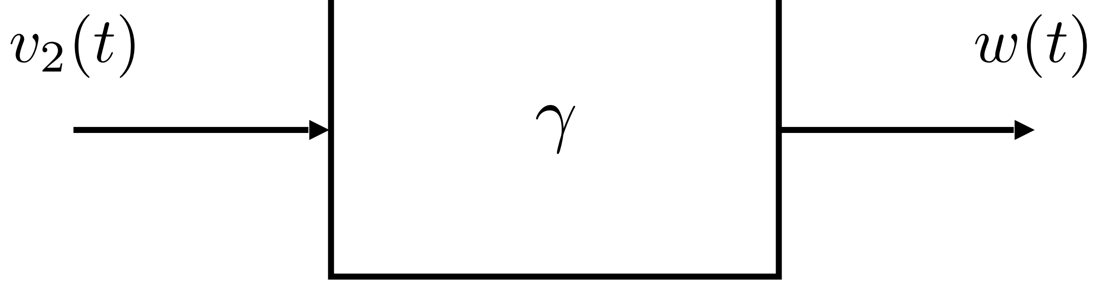


Example: Level Control of a Multi-Tank Hydraulic System (contd.)

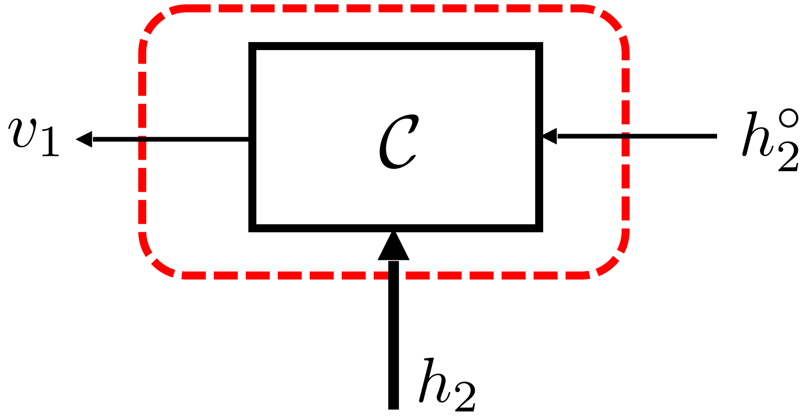


Consider an ideal electrically controlled pump such that:

$$w(t) = \gamma v_2(t)$$

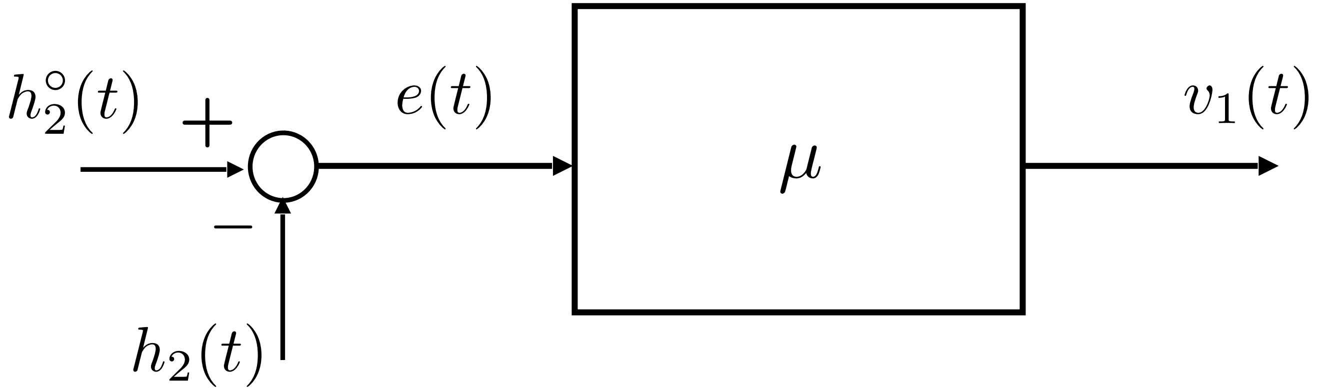


$$W(s) = \gamma V_2(s)$$

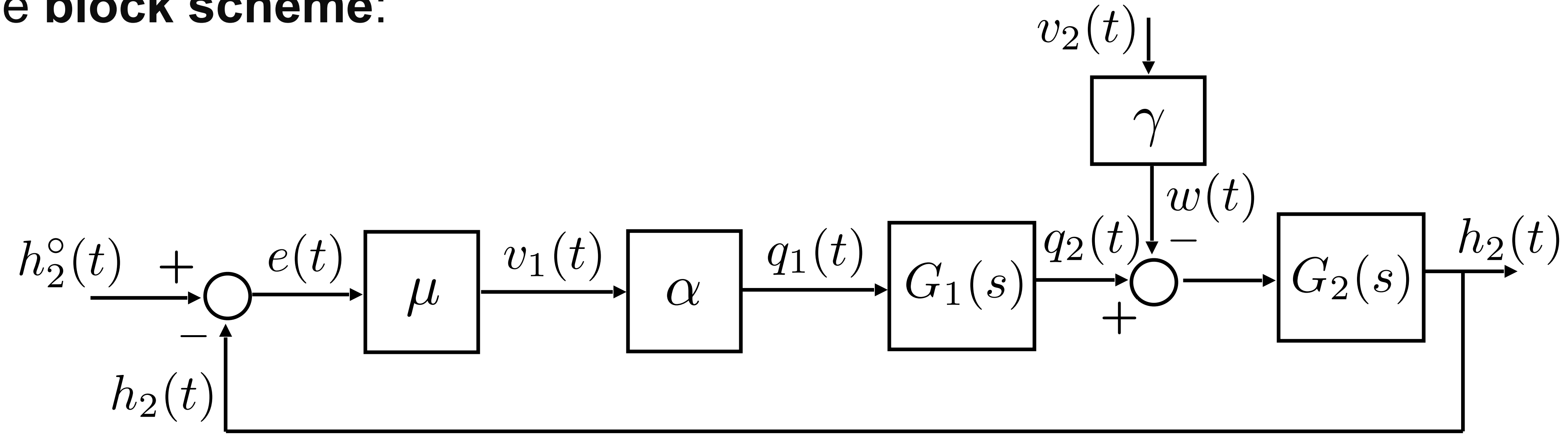


Consider a **proportional** controller:

$$v_1(t) = \mu \, \underbrace{[h_2^\circ(t) - h_2(t)]}_{e(t)}$$

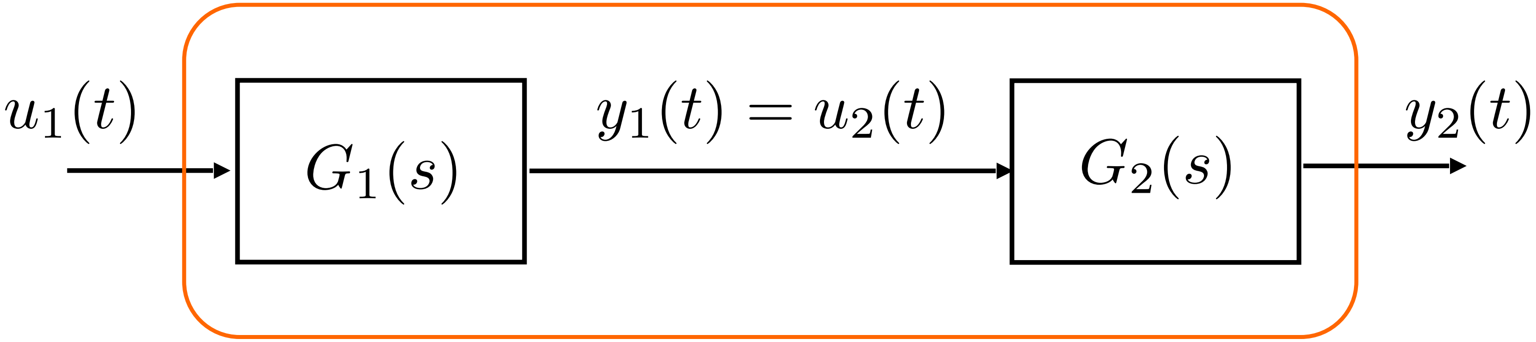


Hence, the original **logical/functional scheme** can be represented by the **block scheme**:



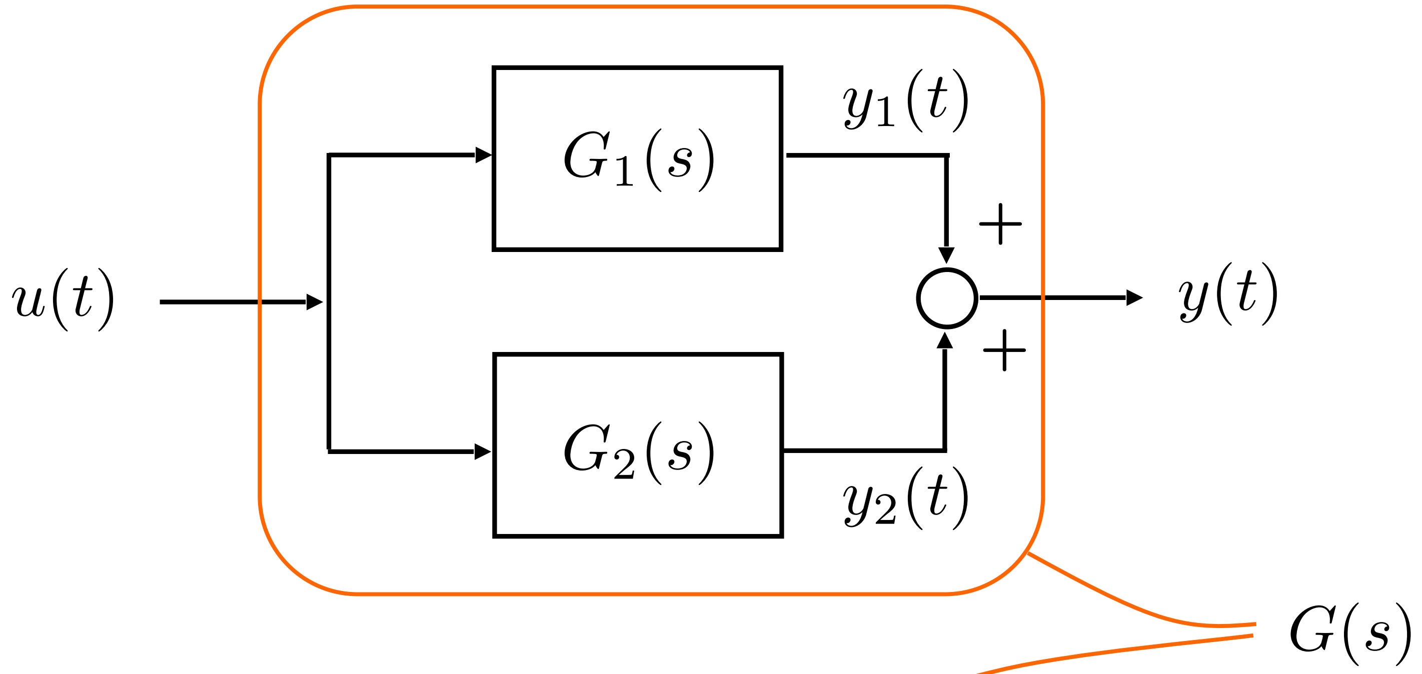
The availability of the **block scheme makes it easy to compute all kinds of transfer functions** and **input/output trajectories** such as:

- T.F. from the input $h_2^\circ(t)$ to the output $h_2(t)$
- T.F. from the input $v_2(t)$ to the output $h_2(t)$
- T.F. from the input $v_2(t)$ to the output $e(t)$



$$Y_2(s) = G_2(s)U_2(s) = G_2(s)Y_1(s) = \overbrace{\boxed{G_2(s)G_1(s)}}^{G(s)}U_1(s)$$

Block Schemes - Parallel Interconnections

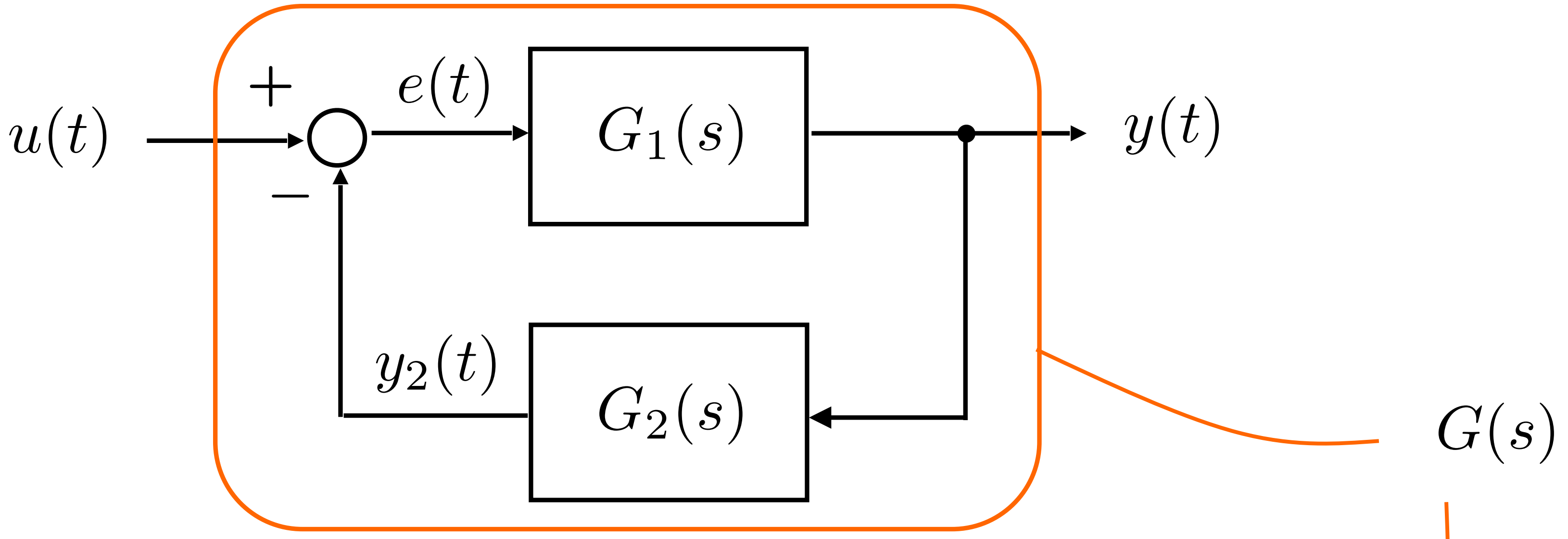


$$
\begin{aligned}
Y(s) &= Y_1(s) + Y_2(s) = G_1(s)U(s) + G_2(s)U(s) \\
&= \boxed{[G_1(s) + G_2(s)]}\, U(s)
\end{aligned}
$$

Of course, if, for example, $y(t) = y_1(t) - y_2(t)$ then $G(s) = G_1(s) - G_2(s)$ and analogous results can be obtained in other similar cases

Block Schemes - Feedback Interconnections

- **Negative** feedback case



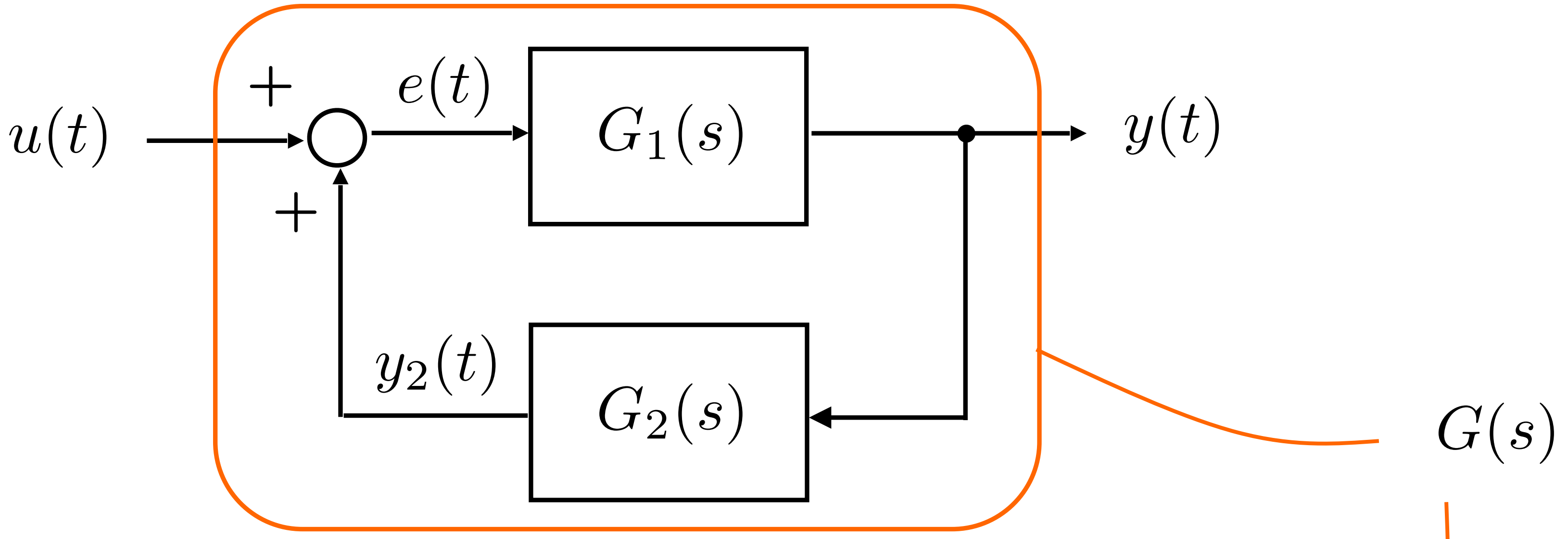
$$
\begin{aligned}
Y(s) \quad &= G_1(s)E(s) = G_1(s)\left[U(s) - Y_2(s)\right] \\
&= G_1(s)\left[U(s) - G_2(s)Y(s)\right]
\end{aligned}
$$

$$Y(s)\left[1 + G_1(s)G_2(s)\right] = G_1(s)U(s)$$

$$Y(s) = \boxed{\frac{G_1(s)}{1 + G_1(s)G_2(s)}}\,U(s)$$

Block Schemes - Feedback Interconnections

- **Positive** feedback case

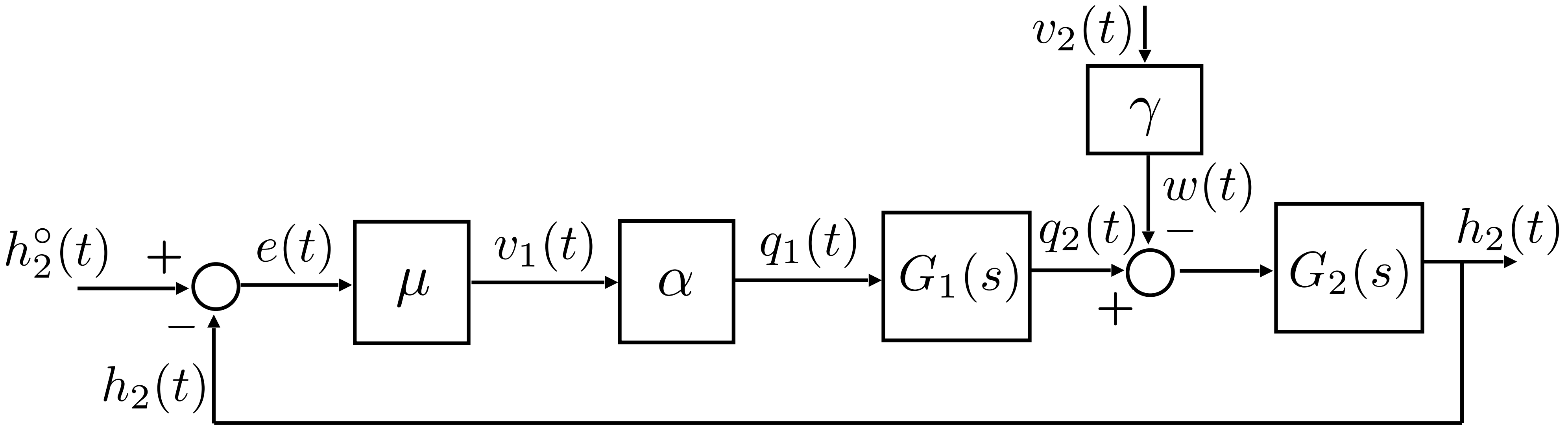


$$
\begin{aligned}
Y(s) &= G_1(s)E(s) = G_1(s)\left[U(s) + Y_2(s)\right] \\
&= G_1(s)\left[U(s) + G_2(s)Y(s)\right]
\end{aligned}
$$

$$\Rightarrow \quad Y(s)\left[1 - G_1(s)G_2(s)\right] = G_1(s)U(s)$$

$$\Rightarrow \quad Y(s) = \boxed{\frac{G_1(s)}{1 - G_1(s)G_2(s)}}\, U(s)$$

Example: Level Control of a Multi-Tank Hydraulic System (contd.)



- The T.F. $F_1(s)$ from the input $h_2^\circ(t)$ to the output $h_2(t)$ can be computed by setting the other inputs to zero (in this case $v_2 = 0$):

$$F_1(s) = \frac{H_2(s)}{H_2^\circ(s)} = \frac{\mu\alpha G_1(s)G_2(s)}{1+\mu\alpha G_1(s)G_2(s)}$$

- The T.F. $F_2(s)$ from the input $v_2(t)$ to the output $h_2(t)$ can be computed by setting the other inputs to zero (in this case $h_2^\circ = 0$):

$$F_2(s) = \frac{H_2(s)}{V_2(s)} = \frac{-\gamma G_2(s)}{1+\mu\alpha G_1(s)G_2(s)}$$

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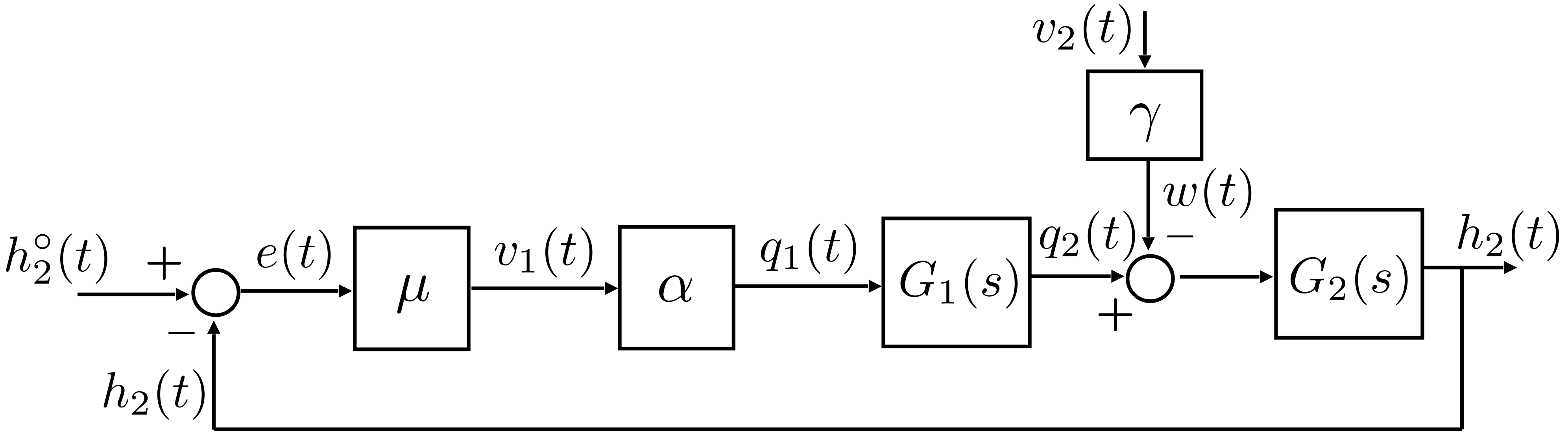
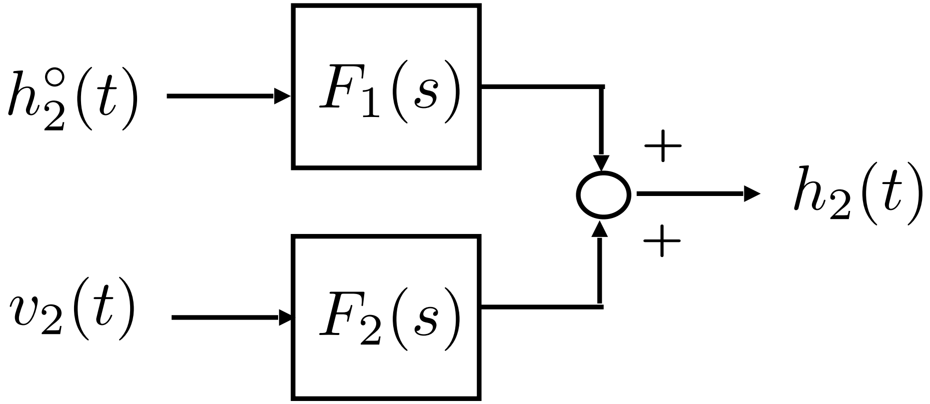
Due to **linearity**, the interconnected system:



is **equivalent** to:



Substituting the expressions of $G_1(s), G_2(s)$:

$$G_1(s) = \frac{k}{A_1 s + k} \; ; \quad G_2(s) = \frac{1}{sA_2}$$

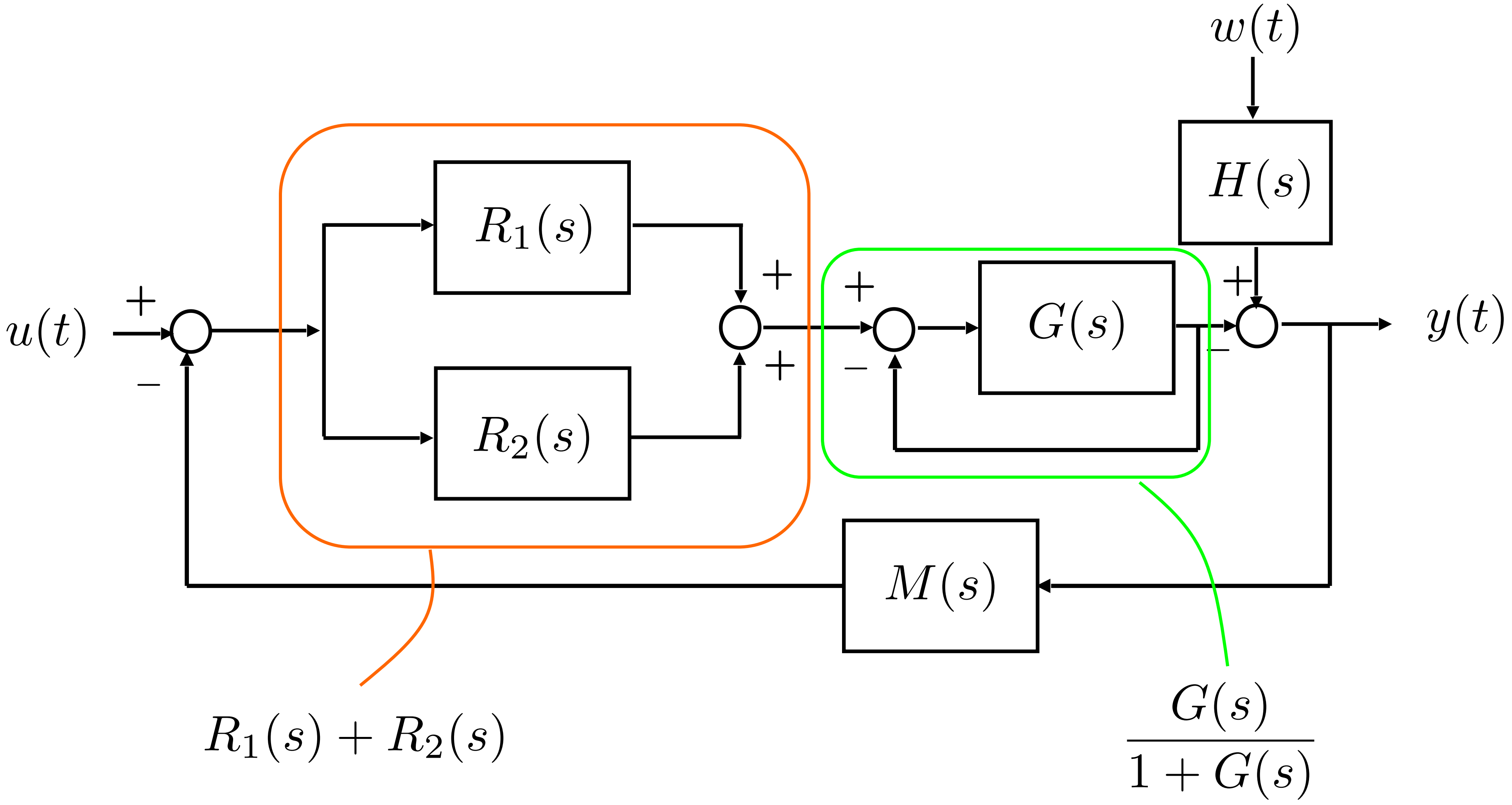
we obtain:

$$F_1(s) = \frac{\mu\alpha G_1(s) G_2(s)}{1 + \mu\alpha G_1(s) G_2(s)} = \frac{\dfrac{\alpha\mu k}{sA_2(A_1 s + k)}}{1 + \dfrac{\alpha\mu k}{sA_2(A_1 s + k)}} = \frac{\alpha\mu k}{\boxed{sA_2(A_1 s + k) + \alpha\mu k}}$$

$$F_2(s) = \frac{-\gamma G_2(s)}{1 + \mu\alpha G_1(s) G_2(s)} = \frac{-\dfrac{\gamma}{sA_2}}{1 + \dfrac{\alpha\mu k}{sA_2(A_1 s + k)}} = -\frac{\gamma(A_1 s + k)}{\boxed{sA_2(A_1 s + k) + \alpha\mu k}}$$

The fact that the **denominators** of $F_1(s), F_2(s)$ **are the same** is general and always holds except (possibly) in the presence of common factors

Another Example



Another Example (contd.)

Hence, the T.F. from the inputs $u(t), w(t)$ and the output $y(t)$ are:

$$F_1(s) = \frac{Y(s)}{U(s)} = \frac{-[R_1(s) + R_2(s)]\dfrac{G(s)}{1+G(s)}}{1 - [R_1(s) + R_2(s)]\dfrac{G(s)}{1+G(s)}M(s)}$$

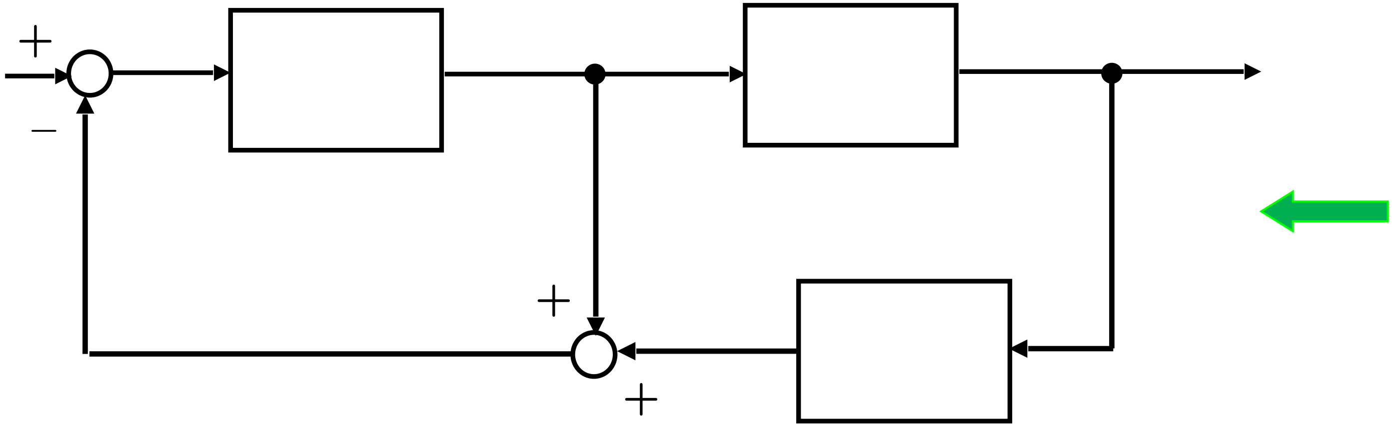
$$F_2(s) = \frac{Y(s)}{W(s)} = \frac{H(s)}{1 - [R_1(s) + R_2(s)]\dfrac{G(s)}{1+G(s)}M(s)}$$

A Systematic Block-Scheme Reduction Procedure

- There are cases where it is not immediate (or it is not possible) to use the series, parallel, and feedback equivalent transfer functions to reduce an interconnected scheme into an equivalent simpler one

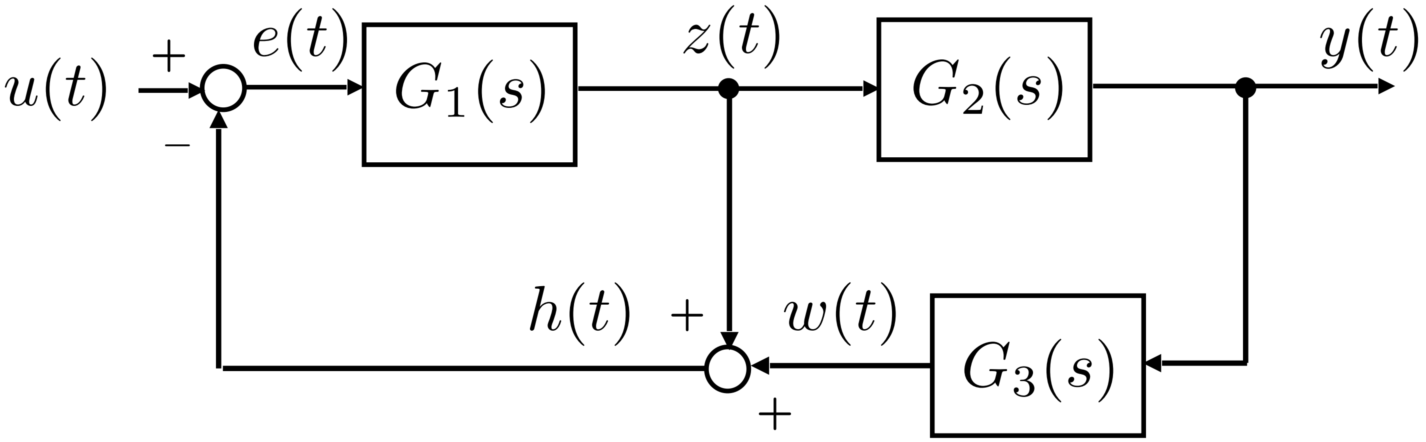
- A **systematic procedure** can be used in these cases

  For example:



No evidence of series, parallel or feedback interconnections can be singled out in this interconnected system

Example of Systematic Block-Scheme Reduction Procedure



$$
\begin{cases}
E = U - H \\
Z = G_1 E \\
Y = G_2 Z \\
H = Z + W \\
W = G_3 Y
\end{cases}
$$

$$E = U - Z - W = U - G_1 E - G_3 Y$$

$$E(1 + G_1) = U - G_3 Y$$

$$E = \frac{1}{1 + G_1}(U - G_3 Y)$$

$$Y = \frac{G_1 G_2}{1 + G_1}(U - G_3 Y)$$

$$Y(s) = \frac{G_1(s) G_2(s)}{1 + G_1(s) + G_1(s) G_2(s) G_3(s)} U(s)$$