More on Fibonacci

this is ... so cool !!! ☺

Normally, the Fibonacci series is written like this :

$$1, 1, 2, 3, 5, 8, 13, \ldots$$

$n=1$ $n=2$ $n=3$

and f_n has a clear meaning for

$$n \geq 1, \; n \in \mathbb{N}$$

natural numbers.

But, you may recall that I started with Φ !

$$0, 1, 1, 2, \ldots$$

f_0 f_1 f_2 f_3

which is well-defined

$n \geq 0, \; n \in \mathbb{W}$

whole numbers.

Wait a moment! Could we ... possibly... extend this to negative numbers???

$z+(-1)=1 \quad y+1=0 \quad x+0=1$

$, \boxed{2}, \boxed{-1}, \boxed{1}, 0, 1, 1, 2, 3, 5, 8 \ldots$

Result

$\underbrace{13, -8, 5, -3, 2, -1, 1}_{\text{alternating } +/- \text{ signs!!}}, 0, 1, 1, 2, 3, 5, 8, \ldots$

- same $|f_n|$ on either side of zero!!

Q) what is $\lim \frac{f_{n+1}}{f_n}$?

Question : what ... $n \to \infty$ ...

answer $\to$ ϕ $(\approx 1.6 \ldots)$

Question what is $\lim_{n \to -\infty} \frac{f_{n+1}}{f_n}$?

$\ldots, 89, -55, 34, -21, 13, -8, 5, -3, 2, -1, 1, 0$

f_{-11} f_{-10} f_{-4} f_{-3} f_{-2} f_{-1}

So, if $n = -11$, then $n+1 = -10$

$$\frac{f_{n+1}}{f_n} = \frac{-55}{89}$$

$\to$ so, this is a ratio between -1 and Φ.

this tends to

as $n \to -\infty$,

$$\boxed{-\frac{1}{\phi}}$$!!

limit as $n \to +\infty$

Binet Formula:

$$f_n = \frac{(\phi)^n - \left(-\frac{1}{\phi}\right)^n}{\sqrt{5}}$$

limit as $n \to -\infty$!!!

First Cool Thing!!

This formula works perfectly well for negative values of n!

$$f_n = \frac{(\phi)^n - \left(-\frac{1}{\phi}\right)^n}{\sqrt{5}}$$

$$\underline{\underline{n \in \mathbb{Z}}}$$ ← integers!

I wonder ... what about non-integers, like, real numbers for example.

like .. what about

$$f_{0.5} \quad ???$$

Let's see ....

$$f_{0.5} = \frac{(\phi)^{0.5} - \left(-\frac{1}{\phi}\right)^{0.5}}{\sqrt{5}}$$

(Recall : $a^{0.5} = a^{\frac{1}{2}} = \sqrt{a}$)

$$\therefore \quad f_{0.5} = \frac{\sqrt{\phi} - \sqrt{-\phi}}{\sqrt{5}}$$

Square root of negative number !!

COMPLEX NUMBERS

$$(\sqrt{-1} = i)$$

$$\therefore \quad f_{0.5} = \frac{\sqrt{\phi} - \sqrt{\frac{1}{\phi}}\, i}{\sqrt{5}}$$

$$\boxed{f_{0.5} \approx 0.5689 - 0.3516\, i}$$

Task, in Python ...

Make a plot of

$$f_n \text{ vs. } n$$

for all real values of n between -30 and 30.

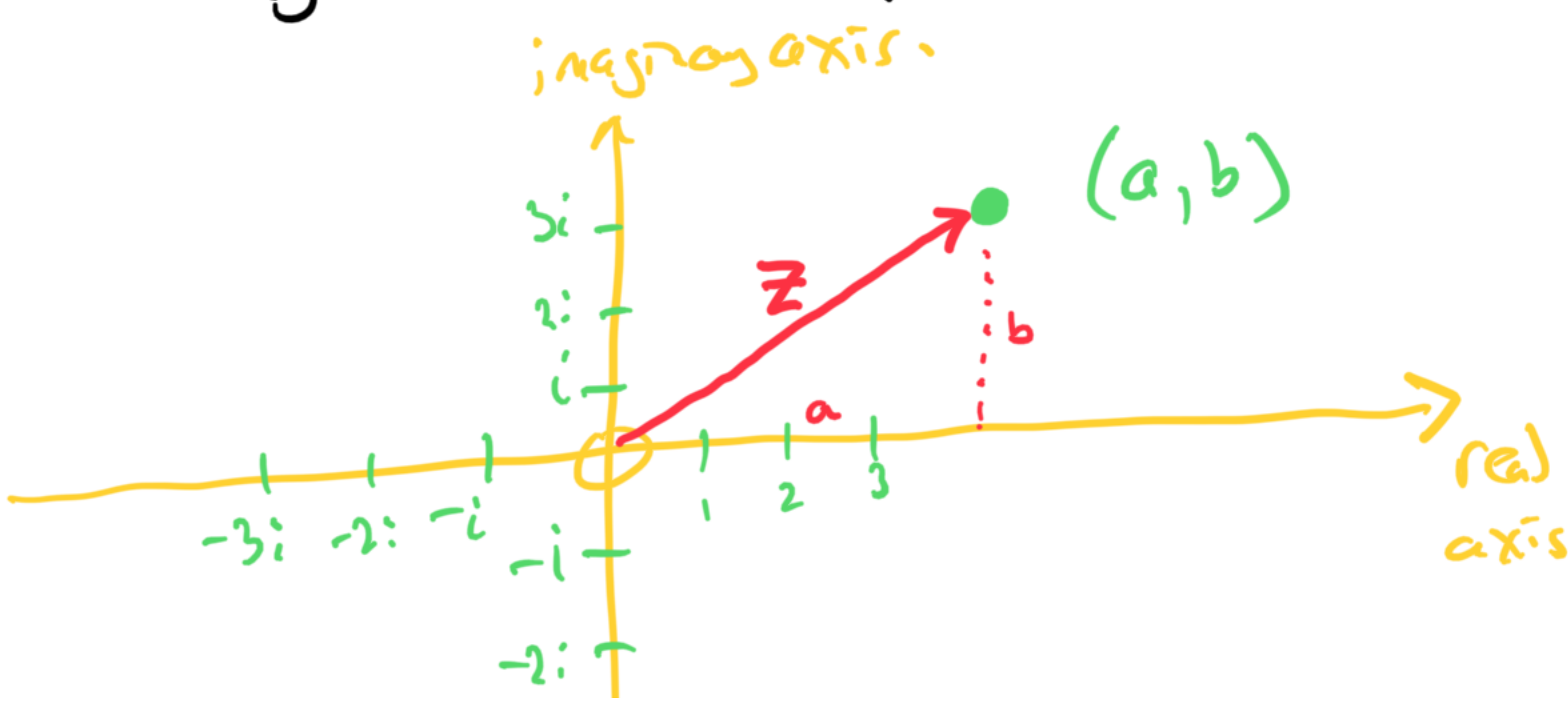
Complex Numbers in Python

The general mathematical notation for a complex number is:

$$Z = a + ib$$

where $a \equiv$ the real part of Z

$b \equiv$ the complex part of Z



$$|z| = \sqrt{a^2 + b^2}$$

In Python.

```
import cmath

a = 4
b = 3
z = complex(a, b)

print(f'The real part of z is {z.real}')

print(f'The imaginary part of z is {z.imag}')

w = cmath.polar(z)
```

← convert to polar coordinates!


```python
print(f'The size and phase of z is {w}')
```

Can also do this:

use j like in EE !!

```python
z2 = 4 + 3j

w2 = cmath.polar(0+1j)

print(w2)
```

N.B. The angle is given in **radians** !!!

convert to rectangular coords !!

```python
z3 = cmath.rect(w2[0], w2[1])

print(z3)
```

'''

see complex.py !!

So, now we are ready to go!

<u>Step 1</u>: Grab the f_n formula from previous examples.

(Small idiosyncratic point... Decimal and complex numbers don't play nice together, so let's just use regular floating pt., and keep the numbers small for now)

<u>Step 2</u>: Write the code to make the plotting arrays!

Cool thing # 2 → Python just handles the complex number stuff automagically! 

Plot things.

Step 3 :

1. title, x axis, y axis labels

2. Plot the "normal" fibonacci values as distinct points.

3. Plot x and y axes :

plt. axvline (x=0) ← vertical line

plt. axhline (y=0) ← horizontal line.