

ANALOG-11.14.21

EE23BTECH11006 - Ameen Aazam*

Question : You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of

- The spring constant K
- The damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Solution :

1) Input Parameters:

| Parameter | Value(SI) | Description |
|-----------|-----------|----------------------------|
| x_0 | 0.15 | Initial spring compression |
| m | 750 | Mass |
| g | 9.8 | Gravitational acc |
| k | mg/x_0 | Spring constant |
| b | | Damping constant |

2) Intermediate Parameters:

| Parameter | Value(SI) | Description |
|------------|-------------------|-------------------|
| x | | Spring Extension |
| F_1 | kx | Spring Force |
| F_2 | $b \frac{dx}{dt}$ | Damping Force |
| s | | Complex Frequency |
| s_1, s_2 | | Values of s |

- Initially the automobile is in rest, so we can use,

$$mg = kx_0 \quad (1)$$

$$\Rightarrow k = \frac{mg}{x_0} \quad (2)$$

- Now, as the oscillation begins, from the Fig. 4 we net force on the mass as,

$$F = F_1 + F_2 + mg u(t) \quad (3)$$

$$\Rightarrow -m \frac{d^2 x(t)}{dt^2} = kx(t) + b \frac{dx(t)}{dt} + mg u(t) \quad (4)$$

$$\Rightarrow \frac{d^2 x(t)}{dt^2} + \left(\frac{b}{m}\right) \frac{dx(t)}{dt} + \left(\frac{k}{m}\right) x(t) = -g u(t) \quad (5)$$

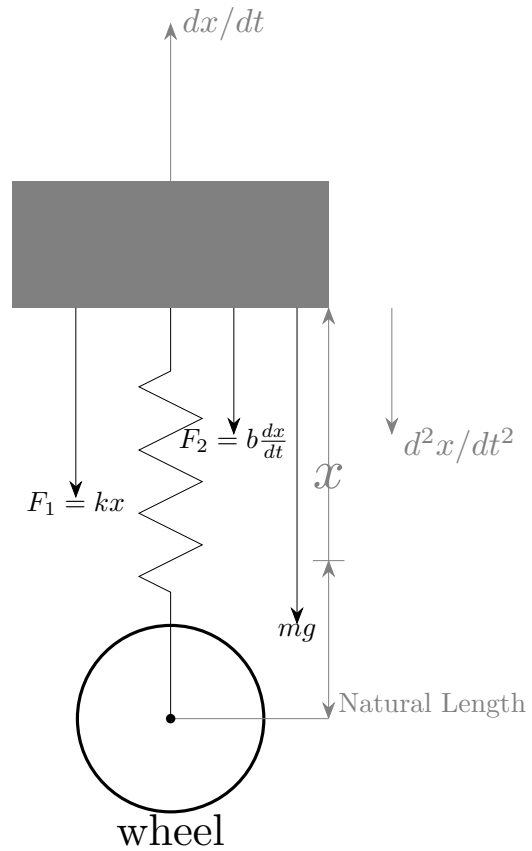


Fig. 4. FBD of the damped oscillation system

Now, taking the Laplace transform on both sides,

$$s^2 X(s) + \frac{b}{m} s X(s) + \frac{k}{m} X(s) = -\frac{g}{s} \quad (6)$$

$$\Rightarrow X(s) = -\frac{g}{s \left(s^2 + \frac{b}{m} s + \frac{k}{m} \right)} \quad (7)$$

$$\Rightarrow X(s) = -\frac{g}{s(s - s_1)(s - s_2)} \quad (8)$$

Where

$$s_1 = -\frac{b}{2m} + \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \quad (9)$$

$$s_2 = -\frac{b}{2m} - \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \quad (10)$$

From (8) we get,

$$\Rightarrow X(s) = \frac{g}{(s_1 - s_2)} \left[\frac{1}{s_2(s - s_2)} - \frac{1}{s_1(s - s_1)} \right] + \frac{g}{s_1 s_2} \left(\frac{1}{s} \right) \quad (11)$$

Now again taking the inverse Laplace transform we have,

$$x(t) = \frac{g}{s_1 s_2} u(t) + \frac{g}{(s_1 - s_2)} \left[\frac{1}{s_2} e^{s_2 t} - \frac{1}{s_1} e^{s_1 t} \right] u(t) \quad (12)$$

$$\Rightarrow x(t) = \left[\sqrt{\left(\frac{mg}{k} \right)^2 + \left(\frac{gb}{2mk} \right)^2} e^{-bt/2m} \sin \left(\sqrt{\frac{k}{m} - \left(\frac{b}{2m} \right)^2} t + \tan^{-1} \left(\frac{2mg \sqrt{\frac{k}{m} - \left(\frac{b}{2m} \right)^2}}{gb} \right) \right] + \frac{mg}{k} \right] u(t) \quad (13)$$

(Substituting the values of s_1 and s_2 from (9) and (10))

From (13) we have the amplitude after one

time period T ,

$$\frac{1}{2} \sqrt{\left(\frac{mg}{k} \right)^2 + \left(\frac{gb}{2mk} \right)^2} = \sqrt{\left(\frac{mg}{k} \right)^2 + \left(\frac{gb}{2mk} \right)^2} e^{-bT/2m} \quad (14)$$

$$\Rightarrow e^{\pi b / \sqrt{mk}} = 2 \quad (15)$$

$$\Rightarrow b = \frac{\sqrt{mk} \ln 2}{\pi} \quad (16)$$

Answer : Now substituting the values of the parameters from Table 1 we have

- a) The spring constant, $k = 4.9 \times 10^4 \text{N/m}$.
- b) The damping constant, $b = 1337.53 \text{Kg/s}$.

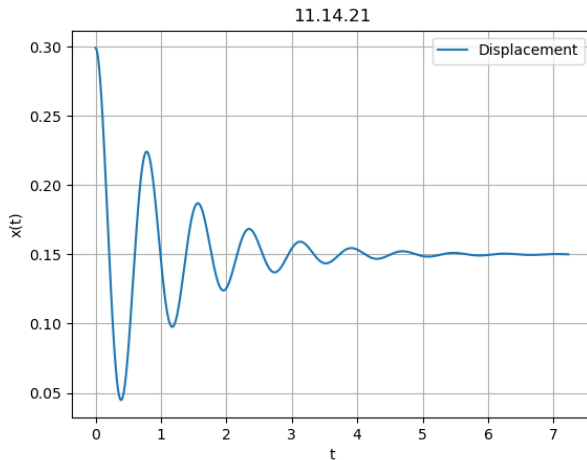


Fig. 4. Displacement Vs. Time Graph