ANALOG-11.14.21

EE23BTECH11006 - Ameen Aazam*

Question : You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of

(a) The spring constant K
(b) The damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Solution :

- **Parameters:**

Parameter	Value(SI)	Description
x		Spring extension
x_0	0.15	Initial spring extension
x_p		Particular solution to x
x_n		Null solution to x
v	dx/dt	Velocity
a	d^2x/dt^2	Acceleration
g	9.8	Gravitational acc
m	750	Mass
k	mg/x_0	Spring constant
b		Damping constant
A_0		Initial Amplitude
A		Amplitude
w	$\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$	Angular frequency
ϕ		Initial phase
T	$2\pi\sqrt{\frac{m}{k}}$	Approximate time peiod
A_1, A_2, h, s		Constants
s_1, s_2		Values of s
F_{net}		Net force acting on the mass
F_1		Spring force
F_2		Damping force

- **Part-a:** Initially the automobile is in rest, so we can use,

$$mg = kx \quad (1)$$

$$\Rightarrow k = \frac{mg}{x} \quad (2)$$
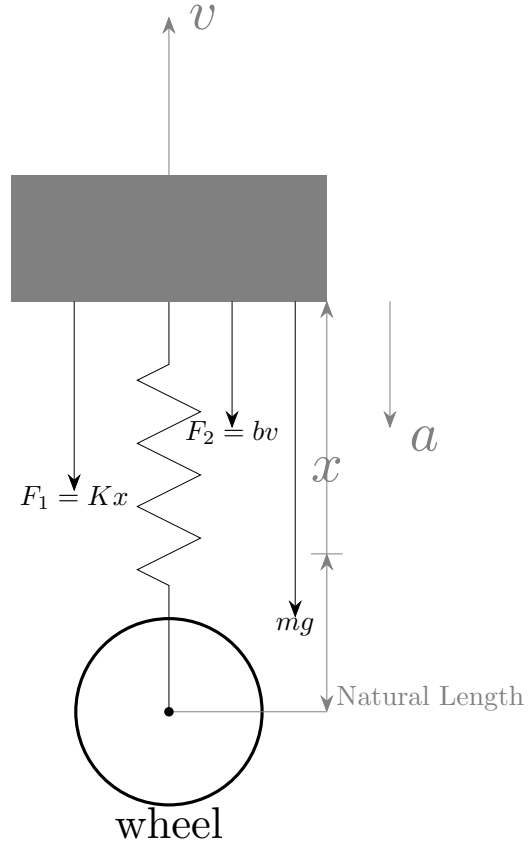
- **Part-b:**

Fig. 2. FBD of the damped oscillation system

Now, as the oscillation begins we would have the force equation from the FBD as,

$$F_{net} = F_1 + F_2 + mg \quad (3)$$

$$\Rightarrow F_{net} = Kx + bv + mg \quad (4)$$

$$\Rightarrow ma = kx + bv + mg \quad (5)$$

$$\Rightarrow -m\frac{d^2x}{dt^2} = kx + b\frac{dx}{dt} + mg \quad (6)$$

$$\Rightarrow \frac{d^2x}{dt^2} + \left(\frac{b}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = -g \quad (7)$$

So, x can be written as,

$$x = x_p + x_n \quad (8)$$

And a constant input implies a constant x_p. Putting x_p in (7) we get,

$$x_p = -\frac{mg}{k} \tag{9}$$

And for x_n consider the equation,

$$\frac{d^2x}{dt^2} + \left(\frac{b}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = 0 \tag{10}$$

x_n will be of the form,

$$x_n = he^{st}$$

using this in (11) we get,

$$s^2 + \left(\frac{b}{m}\right)s + \left(\frac{k}{m}\right) = 0 \tag{11}$$

$$\Rightarrow s_1 = -\frac{b}{2m} + \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \tag{12}$$

$$= -\frac{b}{2m} + jw \tag{13}$$

$$\Rightarrow s_2 = -\frac{b}{2m} - \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \tag{14}$$

$$= -\frac{b}{2m} + jw \tag{15}$$

So we have x as,

$$x = A_1 e^{s_1 t} + A_2 e^{s_2 t} - \frac{mg}{k} \tag{16}$$

$$= A_1 e^{\left(-\frac{b}{2m}+jw\right)t} + A_2 e^{\left(-\frac{b}{2m}-jw\right)t} - \frac{mg}{k} \tag{17}$$

$$= e^{-\frac{b}{2m}t}\left(A_1 e^{jwt} + A_2 e^{-jwt}\right) - \frac{mg}{k} \tag{18}$$

$$= e^{-\frac{b}{2m}t}\left[(A_1 + A_2)\cos wt + (A_1 - A_2)j\sin wt\right] - \frac{mg}{k} \tag{19}$$

$$= A_0 e^{-\frac{b}{2m}t}\sin(wt + \phi) - \frac{mg}{k} \tag{20}$$

From (20) we have the amplitudue A,

$$A = A_0 e^{-\frac{b}{2m}t} \tag{21}$$

$$\Rightarrow \frac{A_0}{2} = A_0 e^{-\frac{b}{2m}T} \tag{22}$$

$$\Rightarrow e^{\frac{b}{2m}T} = 2 \tag{23}$$

$$\Rightarrow \frac{b}{2m}2\pi\sqrt{\frac{m}{k}} = \ln 2 \tag{24}$$

$$\Rightarrow b = \frac{\sqrt{mk}\ln 2}{\pi} \tag{25}$$

Answer : Now substituting the values of the parameters we have

- **Part-a:** The spring constant, $k = 4.9 \times 10^4$N/m.
- **Pat-b:** The damping constant, $b = 1337.53$Kg/s.