DISCRETE

EE23BTECH11006 - Ameen Aazam*

Question : Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots ar^{n-1}$ and $A, AR, AR^2, \ldots AR^{n-1}$ form a G.P., and find the common ratio.

Solution: General term of the n^{th}(n starts from 0) term of the 1^{st} G.P.,

Input Parameters	Values	Description
n		Independent Variable
a		First term of 1^{st} G.P.
r		Common ratio of 1^{st} G.P.
$x_1(n)$	$x_1(n) = ar^n u(n)$	General term of 1^{st} G.P.
$X_1(z)$		z-Transform of 1^{st} G.P.
A		First term of 2^{nd} G.P.
R		Common ratio of 2^{nd} G.P.
$x_2(n)$	$x_1(n) = AR^n u(n)$	General term of 2^{nd} G.P.
$X_2(z)$		z-Transform of 2^{nd} G.P.

TABLE 0
PARAMETERS

$$x_1(n) = ar^n u(n) \tag{1}$$

Now the sequence in the z domain would be,

$$X_1(z) = \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} \tag{2}$$

$$= \sum_{n=-\infty}^{\infty} ar^n u(n) z^{-n} \tag{3}$$

$$= a\left(1 + rz^{-1} + r^2 z^{-2} + r^3 z^{-3} + \ldots\right) \tag{4}$$

$$= \frac{a}{1 - rz^{-1}}, \quad |z| > r \tag{5}$$

General for the 2^{nd} G.P. is given as,

$$x_2(n) = AR^n u(n) \tag{6}$$

And the z-Transform,

$$X_1(z) = \sum_{n=-\infty}^{\infty} AR^n u(n) z^{-n} \tag{7}$$

$$= \frac{A}{1 - Rz^{-1}}, \quad |z| > R \tag{8}$$

Now taking the product will result in a sequence as,

$$y(n) = x_1(n) x_2(n) \tag{9}$$

$$= aA (rR)^n u(n) \tag{10}$$

z-Transform of the resulting sequence,

$$X_1(z) = \sum_{n=-\infty}^{\infty} aA (rR)^n u(n) z^{-n} \tag{11}$$

$$= \frac{aA}{1 - rRz^{-1}}, \quad |z| > rR \tag{12}$$

So the $(n-1)^{th}$ term would be,

$$y(n-1) = aA (rR)^{n-1} u(n-1) \tag{13}$$

So, form 10 and 13,

$$\frac{y(n)}{y(n-1)} = \frac{aA (rR)^n u(n)}{aA (rR)^{n-1} u(n)} \tag{14}$$

$$= rR \tag{15}$$

As we can see the ratio of any two consecutive terms, rR, is a constant. Which means the product of the corresponding terms of the two G.P.s results in another G.P. And the common ratio is rR.