DISCRETE

EE23BTECH11006 - Ameen Aazam*

Question : Find the sum of the following APs:
(a) $2, 7, 12, \ldots$ to 10 terms.
(b) $-37, -33, -29, \ldots$ to 12 terms.
(c) $0.6, 1.7, 2.8, \ldots$ to 100 terms.
(d) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.

Solution: We have the general terms,

Input Parameters	Values	Description
$x(0)$	$2, -37, 0.6, \frac{1}{15}$	First term of AP
d	$x(1) - x(0)$	Common difference of AP
$x(n)$	$[x(0) + nd]u(n)$	General term of AP
$y(n-1)$		Sum to n terms of AP

TABLE 4
Parameters

$$x(n) = [x(0) + nd]\, u(n) \tag{1}$$

Now for sum,

$$y(n) = x(n) * u(n) \tag{2}$$
$$Y(z) = X(z)\, U(z) \tag{3}$$

From (??), we get $Y(z)$ as,

$$Y(z) = \frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3} \tag{4}$$

Now using contour integration for each case,
(a)

$$x(0) = 2 \tag{5}$$
$$d = 5 \tag{6}$$
$$y(n) = \frac{1}{2\pi j} \oint_C S(z)\, z^{n-1} dz \tag{7}$$
$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{2z^{n-1}}{(1 - z^{-1})^2} + \frac{5z^{n-2}}{(1 - z^{-1})^3} \right) dz \tag{8}$$
$$\tag{9}$$

For R_1 we can observe that the pole has been repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{2z^{n+1}}{(z-1)^2} \right) \tag{10}$$
$$= 2(n+1) \lim_{z \to 1} (z^n) \tag{11}$$
$$= 2(n+1) \tag{12}$$

For R_2 we can observe that the pole has been repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{5z^{n+1}}{(z-1)^3} \right) \tag{13}$$
$$= \frac{5(n+1)}{2} \lim_{z \to 1} \frac{d}{dz} (z^n) \tag{14}$$
$$= \frac{5(n+1)(n)}{2} \lim_{z \to 1} \left(z^{n-1} \right) \tag{15}$$
$$= \frac{5(n)(n+1)}{2} \tag{16}$$
$$\implies R = R_1 + R_2 \tag{17}$$

Using (12) and (16),

$$R = 2(n+1) + \frac{5(n)(n+1)}{2} \tag{18}$$

Finally,

$$y(n) = 2(n+1)\, u(n) + 5 \left(\frac{n(n+1)}{2} \right) u(n) \tag{19}$$
$$= \frac{n+1}{2} (4 + 5n)\, u(n) \tag{20}$$
$$y(9) = 245 \tag{21}$$

(b)

$$x(0) = -37 \tag{22}$$
$$d = 4 \tag{23}$$
$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{-37z^{n-1}}{(1 - z^{-1})^2} + \frac{4z^{n-2}}{(1 - z^{-1})^3} \right) dz \tag{24}$$
$$\tag{25}$$

For R_1 the pole has been repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{-37z^{n+1}}{(z-1)^2} \right) \tag{26}$$
$$= -37(n+1) \lim_{z \to 1} (z^n) \tag{27}$$
$$= -37(n+1) \tag{28}$$

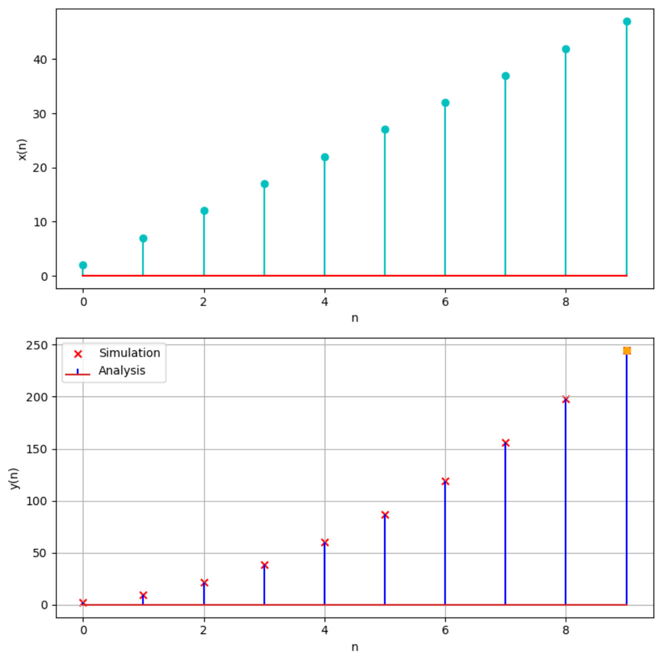

Fig. (a). $1st$ AP

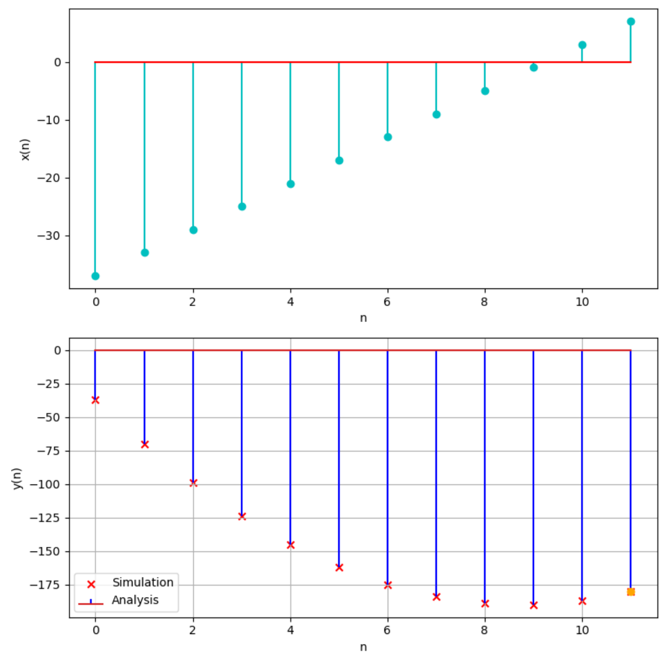

Fig. (b). $2nd$ AP

For R_2 the pole has been repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{4z^{n+1}}{(z-1)^3} \right) \tag{29}$$

$$= \frac{4(n+1)}{2} \lim_{z \to 1} \frac{d}{dz} (z^n) \tag{30}$$

$$= \frac{4(n+1)(n)}{2} \lim_{z \to 1} \left(z^{n-1} \right) \tag{31}$$

$$= \frac{4(n)(n+1)}{2} \tag{32}$$

Using (28) and (32),

$$y(n) = \frac{n+1}{2} (-74 + 4n) u(n) \tag{33}$$

$$y(11) = -180 \tag{34}$$

(c)

$$x(0) = 0.6 \tag{35}$$

$$d = 1.1 \tag{36}$$

$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{0.6z^{n-1}}{(1 - z^{-1})^2} + \frac{1.1z^{n-2}}{(1 - z^{-1})^3} \right) dz$$

(37)

(38)

For R_1 the pole is repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{0.6z^{n+1}}{(z-1)^2} \right) \tag{39}$$

$$= 0.6(n+1) \lim_{z \to 1} (z^n) \tag{40}$$

$$= 0.6(n+1) \tag{41}$$

For R_2 the pole is repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{1.1z^{n+1}}{(z-1)^3} \right) \tag{42}$$

$$= \frac{1.1(n+1)}{2} \lim_{z \to 1} \frac{d}{dz} (z^n) \tag{43}$$

$$= \frac{1.1(n+1)(n)}{2} \lim_{z \to 1} \left(z^{n-1} \right) \tag{44}$$

$$= \frac{1.1(n)(n+1)}{2} \tag{45}$$

Using (41) and (45),

$$y(n) = \frac{n+1}{2} (1.2 + 1.1n) u(n) \tag{46}$$

$$y(99) = 5505 \tag{47}$$

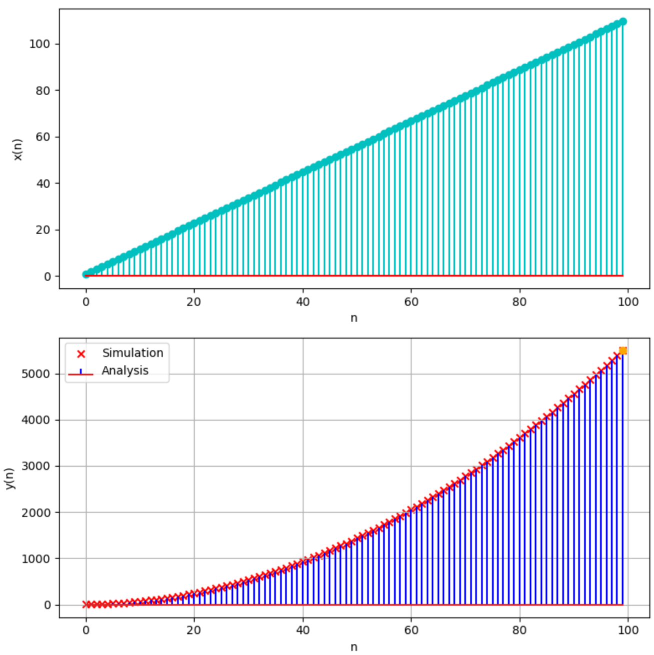

Fig. (c). $4th$ AP

(d)

$$x(0) = \frac{1}{15} \tag{48}$$

$$d = \frac{1}{60} \tag{49}$$

$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{\frac{1}{15} z^{n-1}}{(1 - z^{-1})^2} + \frac{1.1 z^{n-2}}{(1 - z^{-1})^3} \right) dz \tag{50}$$

$$\tag{51}$$

For R_1 the pole is repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{\frac{1}{15} z^{n+1}}{(z-1)^2} \right) \tag{52}$$

$$= \frac{1}{15} (n+1) \lim_{z \to 1} (z^n) \tag{53}$$

$$= \frac{1}{15} (n+1) \tag{54}$$

For R_2 the pole is repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{\frac{1}{60} z^{n+1}}{(z-1)^3} \right) \tag{55}$$

$$= \frac{\frac{1}{60}(n+1)}{2} \lim_{z \to 1} \frac{d}{dz} (z^n) \tag{56}$$

$$= \frac{\frac{1}{60}(n+1)(n)}{2} \lim_{z \to 1} \left(z^{n-1} \right) \tag{57}$$

$$= \frac{\frac{1}{60}(n)(n+1)}{2} \tag{58}$$

Using (54) and (58),

$$y(n) = \frac{n+1}{2} \left(\frac{2}{15} + \frac{n}{60} \right) u(n) \tag{59}$$

$$y(10) = 1.65 \tag{60}$$

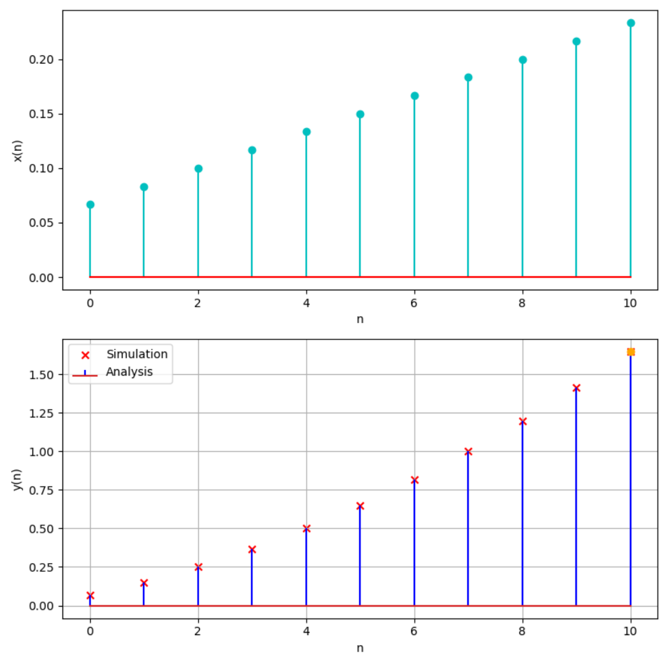

Fig. (d). $4th$ AP