DISCRETE

EE23BTECH11006 - Ameen Aazam*

Question : Find the sum of the following APs:

(a) $2, 7, 12, \ldots$ to 10 terms.
(b) $-37, -33, -29, \ldots$ to 12 terms.
(c) $0.6, 1.7, 2.8, \ldots$ to 100 terms.
(d) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.

Solution: We have the general terms,

Input Parameters	Values	Description
$x(0)$	$2, -37, 0.6, \frac{1}{15}$	First term of AP
d	$x(1) - x(0)$	Common difference of AP
$x(n)$	$[x(0) + nd]u(n)$	General term of AP
$y(n-1)$		Sum to n terms of AP

TABLE 4
PARAMETERS

$$x(n) = [x(0) + nd]\, u(n) \tag{1}$$

Now for sum,

$$y(n) = x(n) * u(n) \tag{2}$$
$$Y(z) = X(z)\, U(z) \tag{3}$$

From (**??**), we get $Y(z)$ as,

$$Y(z) = \frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3} \tag{4}$$

Now using contour integration for each case,

(a)

$$x(0) = 2 \tag{5}$$
$$d = 5 \tag{6}$$
$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)\, z^{n-1} dz \tag{7}$$
$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{2z^{n-1}}{(1 - z^{-1})^2} + \frac{5z^{n-2}}{(1 - z^{-1})^3} \right) dz \tag{8}$$
$$\tag{9}$$

For R_1 we can observe that the pole has been repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 \frac{2z^{n+1}}{(z-1)^2} \right) \tag{10}$$
$$= 2(n+1) \tag{11}$$

For R_2 we can observe that the pole has been repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{5z^{n+1}}{(z-1)^3} \right) \tag{12}$$
$$= \frac{5(n)(n+1)}{2} \tag{13}$$
$$\implies R = R_1 + R_2 \tag{14}$$

Using (11) and (13),

$$y(n) = \frac{n+1}{2} (4 + 5n)\, u(n) \tag{15}$$
$$y(9) = 245 \tag{16}$$

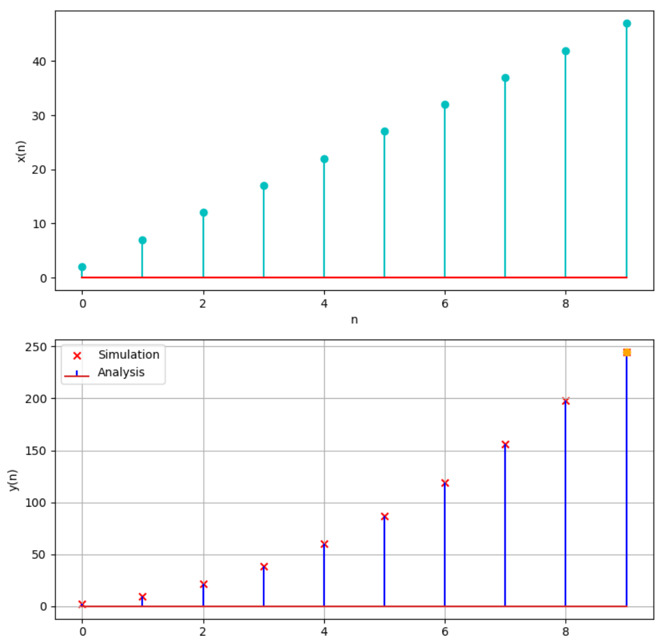

Fig. (a). 1st AP

(b)

$$x(0) = -37 \tag{17}$$
$$d = 4 \tag{18}$$
$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{-37z^{n-1}}{(1 - z^{-1})^2} + \frac{4z^{n-2}}{(1 - z^{-1})^3} \right) dz \tag{19}$$

For R_1 the pole has been repeated twice.

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{-37z^{n+1}}{(z-1)^2}\right) \tag{20}$$

$$= -37\,(n+1) \tag{21}$$

For R_2 the pole has been repeated thrice.

$$R_2 = \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{4z^{n+1}}{(z-1)^3}\right) \tag{22}$$

$$= \frac{4\,(n)\,(n+1)}{2} \tag{23}$$

Using (21) and (23),

$$y\,(n) = \frac{n+1}{2}\,(-74+4n)\,u\,(n) \tag{24}$$

$$y\,(11) = -180 \tag{25}$$

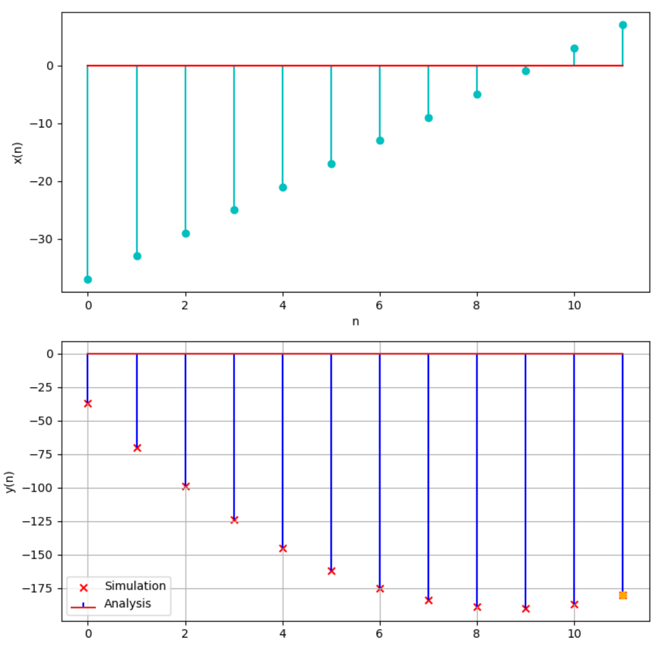

Fig. (b). *2nd* AP

(c)

$$x\,(0) = 0.6 \tag{26}$$

$$d = 1.1 \tag{27}$$

$$y\,(n) = \frac{1}{2\pi j}\oint_C\left(\frac{0.6z^{n-1}}{(1-z^{-1})^2}+\frac{1.1z^{n-2}}{(1-z^{-1})^3}\right)dz \tag{28}$$

For R_1 the pole is repeated twice.

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{0.6z^{n+1}}{(z-1)^2}\right) \tag{29}$$

$$= 0.6\,(n+1) \tag{30}$$

For R_2 the pole is repeated thrice.

$$R_2 = \frac{1}{(2)!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{1.1z^{n+1}}{(z-1)^3}\right) \tag{31}$$

$$= \frac{1.1\,(n)\,(n+1)}{2} \tag{32}$$

Using (30) and (32),

$$y\,(n) = \frac{n+1}{2}\,(1.2+1.1n)\,u\,(n) \tag{33}$$

$$y\,(99) = 5505 \tag{34}$$

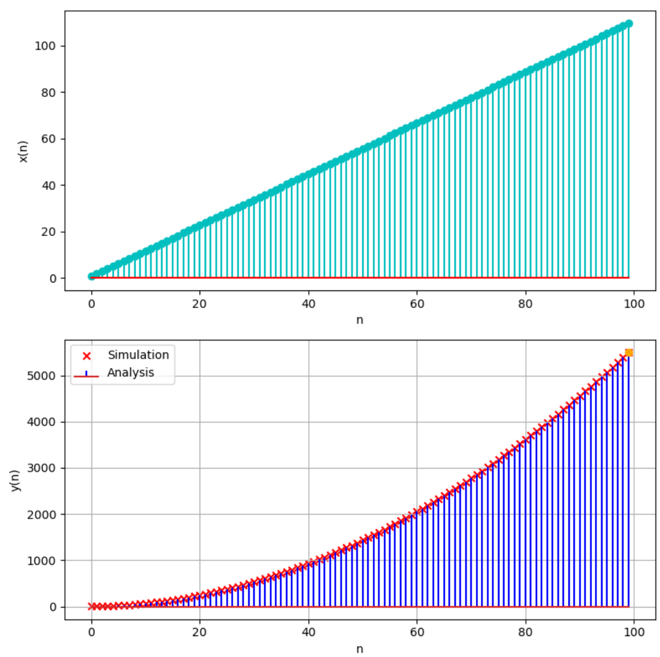

Fig. (c). *4th* AP

(d)

$$x\,(0) = \frac{1}{15} \tag{35}$$

$$d = \frac{1}{60} \tag{36}$$

$$y\,(n) = \frac{1}{2\pi j}\oint_C\left(\frac{\frac{1}{15}z^{n-1}}{(1-z^{-1})^2}+\frac{1.1z^{n-2}}{(1-z^{-1})^3}\right)dz \tag{37}$$

For R_1 the pole is repeated twice.

$$R_1 = \frac{1}{(1)!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2\frac{\frac{1}{15}z^{n+1}}{(z-1)^2}\right) \tag{38}$$

$$= \frac{1}{15}\,(n+1) \tag{39}$$

For R_2 the pole is repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{\frac{1}{60} z^{n+1}}{(z-1)^3} \right) \tag{40}$$

$$= \frac{(n)(n+1)}{120} \tag{41}$$

Using (39) and (41),

$$y(n) = \frac{n+1}{2} \left(\frac{2}{15} + \frac{n}{60} \right) u(n) \tag{42}$$

$$y(10) = 1.65 \tag{43}$$

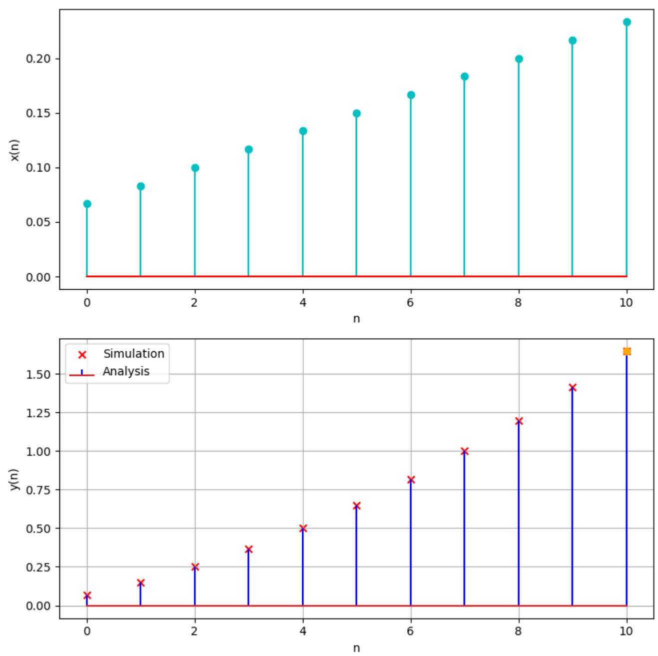

Fig. (d). 4*th* AP