

DISCRETE

EE23BTECH11006 - Ameen Aazam*

Question : Find the sum of the following APs:

- (a) 2, 7, 12, ... to 10 terms.
- (b) -37, -33, -29, ... to 12 terms.
- (c) 0.6, 1.7, 2.8, ... to 100 terms.
- (d) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms.

Solution: We have the general terms,

Input Parameters	Values	Description
$x(0)$	2, -37, 0.6, $\frac{1}{15}$	First term of AP
d	$x(1) - x(0)$	Common difference of AP
$x(n)$	$[x(0) + nd]u(n)$	General term of AP
$y(n-1)$		Sum to n terms of AP

TABLE 4
PARAMETERS

$$x(n) = [x(0) + nd]u(n) \quad (1)$$

Now for sum,

$$y(n) = x(n) * u(n) \quad (2)$$

$$Y(z) = X(z) U(z) \quad (3)$$

From (??), we get $Y(z)$ as,

$$Y(z) = \frac{x(0)}{(1 - z^{-1})^2} + \frac{dz^{-1}}{(1 - z^{-1})^3} \quad (4)$$

Now using contour integration for each case,

(a)

$$x(0) = 2 \quad (5)$$

$$d = 5 \quad (6)$$

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \quad (7)$$

$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{2z^{n-1}}{(1 - z^{-1})^2} + \frac{5z^{n-2}}{(1 - z^{-1})^3} \right) dz \quad (8)$$

(9) (b)

For R_1 we can observe that the pole has been repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{2z^{n-1}}{(z-1)^2} \right) \quad (10)$$

$$= 2(n+1) \quad (11)$$

For R_2 we can observe that the pole has been repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{5z^{n+1}}{(z-1)^3} \right) \quad (12)$$

$$= \frac{5(n)(n+1)}{2} \quad (13)$$

$$\Rightarrow R = R_1 + R_2 \quad (14)$$

Using (11) and (13),

$$y(n) = \frac{n+1}{2} (4 + 5n) u(n) \quad (15)$$

$$y(9) = 245 \quad (16)$$

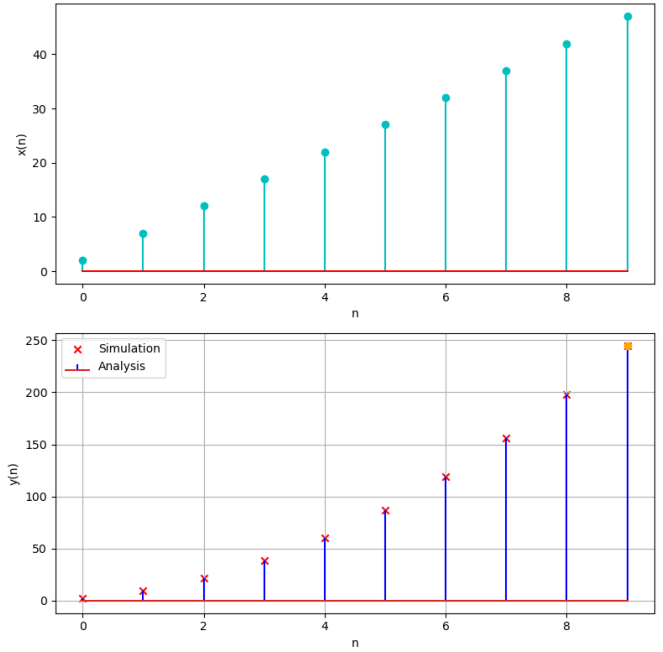


Fig. (a). 1st AP

For R_1 the pole has been repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{-37z^{n+1}}{(z-1)^2} \right) \quad (20)$$

$$= -37(n+1) \quad (21)$$

For R_2 the pole has been repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{4z^{n+1}}{(z-1)^3} \right) \quad (22)$$

$$= \frac{4(n)(n+1)}{2} \quad (23)$$

Using (21) and (23),

$$y(n) = \frac{n+1}{2} (-74 + 4n) u(n) \quad (24)$$

$$y(11) = -180 \quad (25)$$

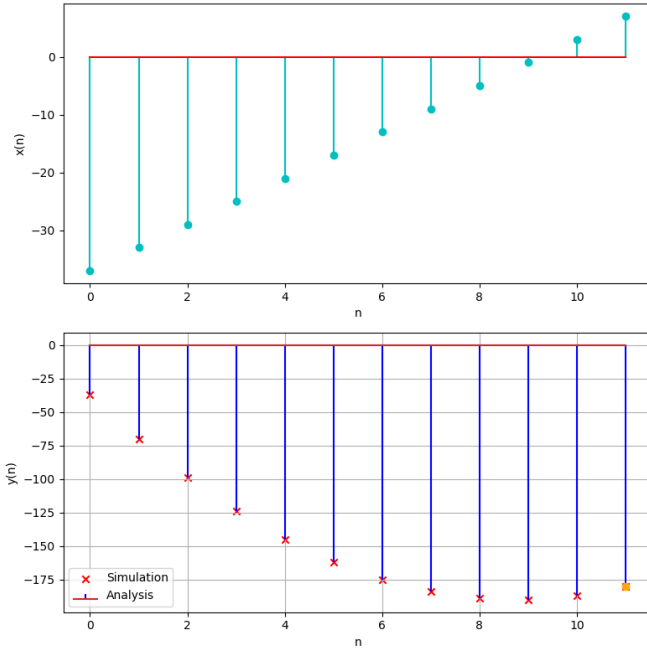


Fig. (b). 2nd AP

(c)

$$x(0) = 0.6 \quad (26)$$

$$d = 1.1 \quad (27)$$

$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{0.6z^{n-1}}{(1-z^{-1})^2} + \frac{1.1z^{n-2}}{(1-z^{-1})^3} \right) dz \quad (28)$$

For R_1 the pole is repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{0.6z^{n+1}}{(z-1)^2} \right) \quad (29)$$

$$= 0.6(n+1) \quad (30)$$

For R_2 the pole is repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{1.1z^{n+1}}{(z-1)^3} \right) \quad (31)$$

$$= \frac{1.1(n)(n+1)}{2} \quad (32)$$

Using (30) and (32),

$$y(n) = \frac{n+1}{2} (1.2 + 1.1n) u(n) \quad (33)$$

$$y(99) = 5505 \quad (34)$$

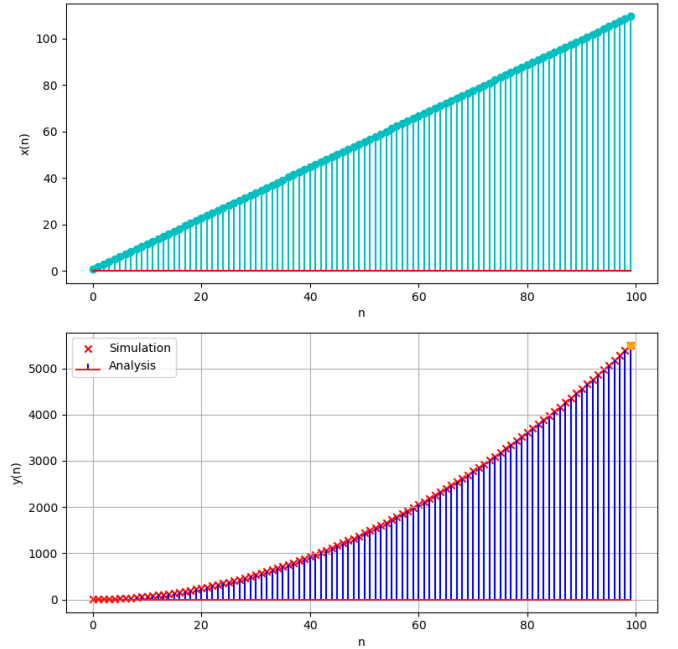


Fig. (c). 4th AP

(d)

$$x(0) = \frac{1}{15} \quad (35)$$

$$d = \frac{1}{60} \quad (36)$$

$$y(n) = \frac{1}{2\pi j} \oint_C \left(\frac{\frac{1}{15}z^{n-1}}{(1-z^{-1})^2} + \frac{1.1z^{n-2}}{(1-z^{-1})^3} \right) dz \quad (37)$$

For R_1 the pole is repeated twice.

$$R_1 = \frac{1}{(1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \frac{\frac{1}{15}z^{n+1}}{(z-1)^2} \right) \quad (38)$$

$$= \frac{1}{15}(n+1) \quad (39)$$

For R_2 the pole is repeated thrice.

$$R_2 = \frac{1}{(2)!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{\frac{1}{60} z^{n+1}}{(z-1)^3} \right) \quad (40)$$

$$= \frac{(n)(n+1)}{120} \quad (41)$$

Using (39) and (41),

$$y(n) = \frac{n+1}{2} \left(\frac{2}{15} + \frac{n}{60} \right) u(n) \quad (42)$$

$$y(10) = 1.65 \quad (43)$$

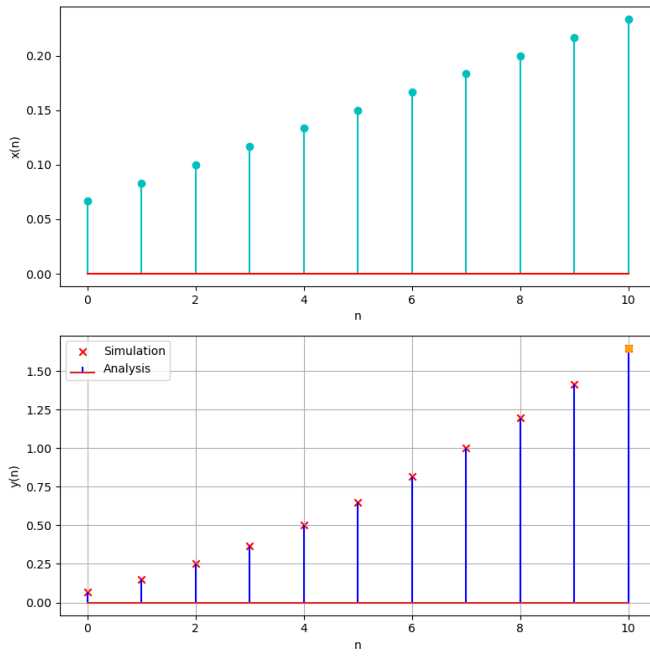


Fig. (d). 4th AP