Question 1:

a)

Recursive call:	Tree:	Level sum:	Operations per level:
$T(n)$	n^2	n^2	$1 = 4^0$
$T(n/2^2)$	$\frac{n^2}{(2^2)^2}$ $\frac{n^2}{(2^2)^2}$ $\frac{n^2}{(2^2)^2}$ $\frac{n^2}{(2^2)^2}$	n^2	$4 = 4^1$
		$\vdots$	$\vdots$
$T(n/2^3)$	$\frac{n^2}{(2^3)^2} = \left(\frac{n}{2^i}\right)^2$	n^2	4^i

Base case at $T(1) = 1$, $1 = \frac{n^2}{(2^i)^2}$, so $\log_2 n^2 = 2i$

$= 2\log_2 n = 2i$

$\log_2 n = i$

~~$\sum_{i=0}^{\log_2 n} n^2 \rightarrow \underbrace{n^2 + n^2 + \dots + n^2}_{\log_2 n \text{ times}} = n^2(\underbrace{1+1+\dots 1}_{\log_2 n \text{ times}})$~~

~~$\sum_{i=0}^{\log_2 n} n^2 = \log_2 n^2$~~

~~$= 2\log_2 n$~~

~~$= n, \therefore O(n)$~~

Operations per level $= 4^i$, so $4^{\log_2 n} = n^{\log_2 4}$

$= n^2$

so $\sum_{i=0}^{\log_2 n} n^2 = \log n^2$

$= 2\log_2 n$

$= n, \therefore O(n)$

Question 2

Base case, $n=1$, $x=1$

let's say LHS $= x^n$, RHS $= exp(x,n)$

LHS $= 1^1 = 1$, RHS $=$ return $x = 1$

LHS = RHS $\therefore$ base case true

As each recursive call @ line 4, we need a base case here as well.

Say $x=2$, $n=2$,
LHS $= 2^2 = 4$

RHS: $y = exp(2, 2/2) = 2$
@ line 6: $y \times y = 4$

LHS = RHS, $\therefore$ True
We can observe that line 4 will reach a base case, $\therefore$ the algorithm terminates

Induction step: assume the algorithm is correct for a value $z \geq 1$ ~~st.~~ and $z > n$.
If we have then line 4 is:
$\hookrightarrow exp(x, z)$

$y_1 = exp(x, z)$, where $exp(x, z) = exp(x, \frac{z}{2}) * y_2$
(Assume y_2 is a multiple of 2).
Then $y_1 = y_2 * y_2$ or $exp(x, z/2) * y_2 * x$

As $exp(x, \frac{z}{2})$ is the definition of $exp(x, z)$, we can conclude that

$exp(x, n) = x^n$ for all $n >= 1$

Question 3

a) Let P be purchases

$P_1 = \{100, 90, 80\}$, discount $= \$80 = d_1$

$P_2 = \{70, 60, 50\}$, $d_2 = \$50$

$P_3 = \{40, 30, 20\}$, $d_3 = \$20$

$P_4 = \{10\}$

Buying in one Transaction $= 550 - 10 = 540$

With the above method, we save $d_1 + d_2 + d_3 = \$150$ and spend $400

b) The greedy choice is to purchase in lots of 3 items where each lot is the maximal possible set. This would result in the maximal discount as the discount for each transaction would be the smallest item in each maximal set.

Question 4.

a)

		4	1	3	6	7	8
	0	1	2	3	4	5	6
4	1	0	1	2	3	4	5
4	2	1	1	2	3	4	5
9	3	2	2	2	3	4	5
4	4	3	3	3	3	4	5
8	5	4	4	4	4	4	4
9	6	5	5	5	5	5	5
5	7	6	6	6	6	6	6
6	8	7	7	7	7	7	7

b)

4	4	9	4	8	9	5	6
	4		1	3	6	7	8
4	4	9	4	8	9	5	6
4			1	3	6	7	8
4	4	9	4	8	9	5	6
	4	1		3	6	7	8
4	4	9	4	8	9	5	6
4		1		3	6	7	8
4	4	9	4	8	9	5	6
4	1			3	6	7	8
4	4	9	4	8	9	5	6
	4	1	3		6	7	8
4	4	9	4	8	9	5	6
4		1	3		6	7	8
4	4	9	4	8	9	5	6
4	1		3		6	7	8
4	4	9	4	8	9	5	6
4	1	3			6	7	8
4	4	9	4	8	9	5	6
	4	1	3	6		7	8