Week 7 Submission

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$T(n) = 4T(n/3) + n = \Theta(n^{\log_3 4})$

a) $T(n) = 4T(n/3) + cn$, where $c > 0$

Recursive call:	Tree	Level sum	# operations per level
$T(n)$	cn	cn	$1 = 4^0$
$T(n/3)$	$cn/3 \quad cn/3 \quad cn/3 \quad cn/3$	$4n/3$	$4 = 4^1$
$T(n/9)$	$cn/9$	$16n/9$	$16 = 4^2$
$\vdots$		$\vdots$	
$T(n/3^i)$		$4^i n/3^i$	4^i

$i = \log_3 n$. The subproblem size for a node at depth i is $n/3^i$

Thus, the subproblem size hits $n=1$ when $n/3^i = 1$. The tree has $\log_3 n + 1$ levels, and the cost at each level is $\frac{4^i}{3^i} n$.

At depth $\log_3 n$, there are 4^i operations where $i = \log_3 n$, $\therefore$

$4^{\log_3 n} = n^{\log_3 4}$ nodes, each costing $T(1)$

$\therefore \Theta(n^{\log_3 4})$

b) Show $T(n) \le cn^{\log_3 4}$

Assume guess is true for $m<n$. Must prove:

$$T(n) \le cn^{\log_3 4}, \text{ assuming } T(m) \le cm^{\log_3 4} \quad \forall\, m<n$$

If $m = n/3 < n$:

$$T(n) = 4T(n/3) + n \le 4\left[c(n/3)^{\log_3 4}\right] + n$$

$$= \left(\frac{4cn}{3}\right)^{\log_3 4} + n$$

$$= \frac{4cn^{\log_3 4}}{3^{\log_3 4}} + n \qquad = cn^{\log_3 4} + n$$

But we want to prove $T(n) \le cn^{\log_3 4}$, not $T(n) \le cn^{\log_3 4} + n$

$\therefore$ This proof fails

c) Make the guess: $T(n) \le cn^{\log_3 4} - 3n$

$$T(n) = 4T(n/3) + n \le 4\left[c(n/3)^{\log_3 4} - n\right] + n$$

$$= cn^{\log_3 4} - 4n + n$$

$$= cn^{\log_3 4} - 3n$$