Question 1

```
1  public static int add(int x, int y){
2    if (y == 0)
3      return x;
4    else
5      return 1 + add(x,y-1);
6  }
```

Base case:

Let's say X = 1, Y = 0.

add(X,0) will return the X value.

Let's say LHS = X + Y, RHS = *add(X, Y)*

And let X = 1, Y = 1.

LHS = 1 + 1 = 2

RHS = *1 + add(X, Y-1)*

Where *add(X, Y-1) = add(X, 0) = X = 1.*

So RHS = 1 + 1 = 2.

LHS = RHS.

As with each recursive call Y is decreased by 1, we will eventually reach the base case, therefore the algorithm terminates.

Induction Step:

Induction hypothesis: assume the algorithm is correct for a value Y = K > 0, such that the algorithm correctly returns K + X.

Will the algorithm then work for K + 1?

If Y = K + 1, line 5 of the code is:

1 + add(X, (K+1) – 1)

Where *add(X, (K+1) – 1) = add(X, K). 1 + add(X, K)* is the definition of *add(X, K+1).* This means that the algorithm will return *add(X, K+1).*

Therefore, by induction, *add(X, Y)* returns X + Y for all Y >= 0.