

Week 2 Submission

Question 1

Problem: find a limit value l such that the cumulative existing booking values are maximal and less than N .

Input: a set of values corresponding to bookings, and the property's available time N .

Output: the adjusted set of bookings such that all values that were greater than l are now less than l .

Test with 5 values, $N = 100$:

20, 20, 20, 20, 20

Here we can set l to be the sum of the bookings/5. Easy, this is an ideal input.

30, 30, 30, 30, 30

Formula for sets of bookings with uniform values: sum of booking values/10 = y . Individual booking value/ y = l .

15, 25, 35, 45, 55

Sum = 175, obviously $175 > 100$. If we set l to 20, we get $15+20+20+20+20 = 95$. Not bad but if we make $l = 21$ then we get right up to 99. This is optimal as it is the best value of l that doesn't breach the N and has a maximal value.

So in this:

- Sum/10 = 17.5, the only value that we don't need to change is less than this. So, let's see if we can tick 15 off as a value that doesn't need to be changed; $N = 85$, set of bookings is 25, 35, 45, 55. $85/4 = 21.25$, take the int value and we have 21. $l = 21$
- So we have sum/10 = x , any values less than x don't need to be changed. Subtract the values smaller than x from N , then do $N/\text{number of values left} = l$. Make l an int so it rounds down.

25, 35, 15, 5, 90

- 5 and 15 are smaller than the sum/10 = 17. $N - 5 - 15 = 80$. 3 values left, $80/3 = 26.66$ so $l = 26$. $5+15+26+26+26 = 98 < 100$

What if there are 4 small values and 1 large one?

2, 2, 2, 2, 93.

- $X = 10.1$, 2's don't need to be changed, $N = 92$. One value left, so $l = 92/1 = 92$. $2+2+2+2+92 \leq 100$

$N = 69$, values are 26, 33, 84, 4, 7

- $X = \text{sum}/10 = 15.4 > 4, 7$. $N - 4 - 7 = 58$. $58/3 = 19.33$, $l = 19$. $19+19+19+4+7 = 68 \leq 69$

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makeTheValueLessThanTheLimitFunction(int bookings [], int N){  
    If sum of bookings is less than N  
        return / as the largest element in bookings  
    Sum of the booking values divided by 10 = x  
    M = values less than x subtracted from N  
    Y = number of values greater than x  
    //I needs to be an int so that it is rounded down to the nearest integer  
    Return int /= M/Y
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Question 2

a)

$$\begin{aligned} \text{a) } n^2 + n + 1 &= \Theta(n^3) \\ c_1 n^3 &\leq n^2 + n + 1 \leq c_2 n^3 \\ \text{choose } c_1 &= 1, n = 2, c_2 = 1 \\ 0 &\leq 8 \leq 4 \leq 8 \\ \text{For the given values, } 4 &\leq 8 \text{ is false, therefore } n^2 + n + 1 = \Theta(n^3) \text{ is disproven} \end{aligned}$$

b)

$$\begin{aligned} \text{b) If } f_1(n) &= O(g_1(n)) \text{ and } f_2 = O(g_2(n)), \text{ then } f_1(n) + f_2(n) = O(g_1(n) + g_2(n)) \\ \text{Say } g_1(n) &= n^2 + n \text{ and } g_2(n) = n^2 + 3n + 2 \\ f_1(n) &= O(g_1(n)) = O(n^2), \quad f_2(n) = O(g_2(n)) = O(n^2) \\ f_1(n) + f_2(n) &= O(g_1(n) + g_2(n)) = O(n^2 + n^2) = O(2n^2) = O(n^2) \end{aligned}$$

c)

$$\begin{aligned} \text{c) } \sum_{i=1}^n i &= O(n^3) \\ \sum_{i=1}^n i &= \frac{n(n-1)}{2} = \frac{n^2 - n}{2} \\ O\left(\frac{n^2 - n}{2}\right) &= O(n^2) \neq O(n^3) \therefore \text{disproven} \end{aligned}$$