Week 12 Submission

• Partition Problem: given a multiset S of positive integers, can you partition them into two subsets S_1 and S_2 such that the sum of the numbers in S_1 is equal to the sum of the numbers in S_2.

• Subset Sum Problem: given a multiset S of positive integers, is there a subset of S that sums up to k?

(a) Show that both problems are in NP.
The Partition Problem is in NP as we solve the problem by guessing what the two partitions are, and then verify that the sum of each partition equal the same value. However, this guessing step cannot be completed in polynomial time, so the Partition Problem is in NP.

To show that the Subset Sum Problem is in NP, suppose that we want to solve a subset sum where the set has size s and suppose it takes n bits to record. To use a dynamic programming approach and store these values in a table would take roughly $2^{n/s}$ columns, so the table is exponentially large and therefore cannot be populated in polynomial time.

(b) If you know that the subset sum Problem is NP-complete, how do you show that the partition problem is also NP-complete? Make sure you get the direction of the reduction right.
(Sorry for the word-vomit Daniel)
In the case of NP-complete, every problem in NP can be reduced to problems in NP-complete. As we have shown that both problems are in NP in part (a), we just need to show that the Partition Problem is NP-complete. We define a decision problem to be NP-complete if it is NP and NP-Hard. A problem G is NP-Hard if there is an NP-Complete problem H, and H can be reduced to G in polynomial time. As we are assuming in this question that the Subset Sum Problem is NP-Complete, and as we will show how the Partition Problem can be reduced to the Subset Sum Problem in part (c), we can prove that the Partition Problem is NP-Complete by reducing a problem in NP.

We can therefore say Partition Problem $\leq_p$ Subset Problem.

(c) Show how you can reduce the partition problem into the subset-sum problem.

Recall that the Subset Sum Problem is defined as:

> *Given a set S of integers and a target k, find a subset R ⍰ S such that the elements of R add up to k.*

If the Partition Problem has a solution, then there are subsets P_1 and P_2 such that P_1=P_2. If we were to make the 'k' value for the Subset Sum Problem equal to P_1 or P_2, we would be able to find a solution to the Subset Sum Problem using the Partition Problem.

Therefore, we have Partition Problem $\leq_p$ Subset Problem.