Learning to learn with backpropagation of Hebbian plasticity

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Abstract

How can we train systems that keep learning from their ongoing experience? In natural brains, continuous learning occurs largely through Hebbian synaptic plasticity. However, Hebbian plasticity is currently not amenable to the powerful optimization technique of gradient descent. Here we show how to compute analytical gradients in neural networks with Hebbian plastic connections. Using these expressions, we can use backpropagation to train not just the baseline *weights* of the connections, but also their *plasticity*. As a result, the networks "learn how to learn" in order to solve the problem at hand: backpropagation fashions connectivity and plasticity so that trained networks automatically extract relevant task features for each novel instance of a given task. We test the algorithm on various on-line learning tasks, including pattern completion with arbitrary patterns, one-shot learning, and reversal learning. The algorithm successfully learns how to learn the relevant associations from one-shot instruction, and fine-tunes the temporal dynamics of plasticity to allow for continual learning in response to changing environmental parameters. We conclude that backpropagation of Hebbian plasticity offers a powerful model for lifelong learning and meta-learning.

1 Introduction

Meta-learning, or "learning to learn", is the process of training systems to extract relevant information from their ongoing experience, and to use it appropriately to guide future behavior. As a result, the trained systems can solve generic *classes* of problems, because they can automatically acquire the contingent information necessary to solve each particular instance of the problem. This ability to form and exploit new memories from experience allows the systems to handle unpredictable or changing environments.

Recent research has produced several methods for meta-learning. A popular method is to use recurrent neural networks (RNNs), in which current activity can affect future input-output mappings (that is, effective connectivity); as a result, with an adequate training paradigm, RNNs can learn to extract specific features of their inputs to guide future responses to other inputs [3]. This principle can also be extended to "learn to reinforcement learn", training networks to automatically acquire behaviors that maximize a sparse, delayed reward signal [1, 8].

Another method is to include an explicit declarative memory in the system, that can be read from and written to in a fully differentiable fashion. This approach is used in Neural Turing Machines [2, 4] and Memory Networks [6].

In natural brains, however, long-term learning from ongoing experience occurs largely through synaptic plasticity, which generally follows the Hebbian principle: a cell that consistently contributes in making another cell fire will build a stronger connection to that cell. Note that this generic principle can be implemented in many different ways, including covariance learning, instar and outstar rules, BCM learning, etc. (see [7] and references therein).

Backpropagation can train neural networks to perform remarkably complex tasks. However, it is generally used to train fixed-weights networks, with no further changes in connectivity after training. While the methods described above allow some form of lifelong learning in backpropagation-trained networks, it would be useful to incorporate the powerful, well-studied principle of Hebbian plasticity in backpropagation training.

Here we derive analytical expressions for activity gradients in neural networks with Hebbian plastic connections. Using these expressions, we can use backpropagation to train not just the baseline *weights* of the connections, but also their *plasticity*. This allows backpropagation to "learn how to learn", in order to solve general types of problems with unpredictable features, rather than specific instances.

All software used for the present paper is available at `http://github.com/thomasmiconi`.

2 Networks with Hebbian synapses

We consider networks where the strength of each connection can vary according to Hebbian plasticity over the course of the network's lifetime. We will arrange things so that each network is fully specified by fixed parameters which determine both the baseline weight *and* the degree of plasticity of each connection. After training, these parameters are fixed and unchanging over the network's lifetime, but govern the way in which each connection changes over time, as a result of experience, according to Hebbian plasticity.

To model Hebbian plasticity, we maintain a time-dependent quantity for each connection in the network, which we call the *Hebbian trace* for this connection. As noted above, there are many possible expressions for Hebbian plasticity [7]. In this paper, we use the simplest stable form of Hebbian trace, namely, the running average of the product of pre- and post-synaptic activities. Thus, for a given target cell, the Hebbian trace associated with its k-th incoming connection is defined as follows:

$$Hebb_k(t) = (1-\gamma) * Hebb_k(t-1) + \gamma * x_k(t) * y(t) \tag{1}$$

where $y(t)$ is the activity of the post-synaptic cell, $x_k(t)$ is the activity of the pre-synaptic cell, and γ is a time constant. While other expressions of Hebbian plasticity are possible, this simple form turns out to be adequate for our present purposes and simplifies the mathematics.

The Hebbian trace is maintained automatically, independently of network parameters, for each connection. Given this Hebbian trace, the actual strength of the connection at time t is determined by two fixed parameters: a fixed weight w_k, which determines the "baseline", unchanging component of the connection; and a *plasticity parameter* α_k, which specifies how much the Hebbian trace influences the actual connection. More formally, the response y of a given cell can be written as a function of its inputs as follows:

$$y(t) = \tanh\left\{ \sum_{k \in inputs} [w_k x_k(t) + \alpha_k Hebb_k(t) x_k(t)] + b \right\} \tag{2}$$

where b is a bias parameter.

As a result, each connection possesses a fixed, unchanging "baseline" component withweight w (similar to standard neural network weights), and a plastic component with time-varying, experience-dependent weight $\alpha Hebb(t)$. Note that w and α specify the relative impact of fixed and plastic components in each connection: a connection can be either fully fixed ($\alpha = 0$, or fully plastic with no fixed/baseline component (if $w = 0$), or have both a fixed baseline and an additional plastic component (if both w and $alpha$ are nonzero).

The w and α coefficients are the ones that are optimized by backpropagation. By asjuting those, backpropagation can build networks with arbitrary combinations of plastic and fixed connections to learn novel instances of a given task, as described below.

3 Gradients

In order to use backpropagation, we must find the gradient of y over the w_k and α_k parameters. Importantly, these gradients will necessarily involve activities at previous times, since under plasticity activity at time t influences activity at future times $t+n$ due to its effects on Hebbian traces. Fortunately, these gradients turn out to have a simple, recursive form.

Temporarily omitting the $\tanh$ nonlinearity (see below), we get the following expressions:

$$\frac{\partial y(t_z)}{\partial w_k} = x_k(t_z) + \sum_{l \in inputs} [\alpha_l x_l(t_z) \sum_{t_u < t_z} (1-\gamma)\gamma^{t_z - t_u} x_l(t_u) \frac{\partial y(t_u)}{\partial w_k}] \tag{3}$$

$$\frac{\partial y(t_z)}{\partial \alpha_k} = x_k(t_z) Hebb_k(t_z) + \sum_{l \in inputs} [\alpha_l x_l(t_z) \sum_{t_u < t_z} (1-\gamma)\gamma^{t_z - t_u} x_l(t_u) \frac{\partial y(t_u)}{\partial \alpha_k}] \tag{4}$$

(See Appendix for a full derivation.)

These equations express the gradient of $y(t_z)$ as a function of the gradients of $y(t_z < t_u)$, that is, recursively.

In each of these equations, the summand over previous times $t_u < t_z$ is essentially the partial derivative of the Hebbian traces at time t_z with respect to either w_k (Eq. 3) or α_k (Eq. 4). Since the Hebbian trace is the exponential average of previous products of x and y, these partial derivatives turn out to be sums of the previous gradients of y over the corresponding parameter, multiplied by the concomitant activity of the input cell x_k (the γ terms account for the exponential decay of the running average). Thus, the gradient at time t_z is a function of (the weighted sum of) the gradients at times $t_u < t_z$.

Note that the sum is over the Hebbian traces of *all* inputs to y, not just the one associated with connection k for which we are computing the gradient. This is because the values of w_k and α_k, by affecting y, also influence the Hebbian traces of all other connections to y - which will in turn further affect y at later times. This effect must be accounted for in the above gradients.

The above expression omits the $\tanh$ nonlinearity: it really provides the gradient of the expression within the curly braces in Eq. 2, that is, the "raw" output (call it y_{raw}) provided by incoming excitation and biases. To obtain the full gradient $\frac{\partial y(t_z)}{\partial w_k}$, we simply rewrite y as $y = \tanh(y_{raw})$ and apply the chain rule: $\frac{\partial y}{\partial w_k} = \frac{\partial \tanh(y_{raw})}{\partial y_{raw}} \frac{\partial y_{raw}}{\partial w_k} = (1-y^2)\frac{\partial y_{raw}}{\partial w_k}$, where $\frac{\partial y_{raw}}{\partial w_k}$ is provided by Eq. 3 above (and similarly for $\frac{\partial y_{raw})}{\partial \alpha_k}$

4 Experiments

4.1 Applying BOHP: general description

In all tasks described below, lifetime experience is divided into *episodes*, each of which lasts for a certain number of timesteps. While all episodes involve the same generic class of task, the specific instantiation of the task varies from episode to episode; for example, when learning the general problem of pattern completion, the pattern to be completed will differ from one episode to the next. The goal of meta-learning (and of BOHP) is to train networks that can automatically and immediately acquire the specific features in each novel instance of a general task. For example, they can immediately learn the pattern to be completed with one instruction timestep, and also the proper connectivity to complete it from any partial input in subsequent timesteps of the episode.

At the beginning of each episode, all Hebbian traces are initialized to 0. Then, at each timestep, the network processes an input pattern and produces an output according to its current parameters, and synaptic plasticity occurs according to the inputs and outputs at each synapse (that is, the Hebbian traces are updated according to Equation 1). Furthermore, errors and gradients are also computed and accumulated. At the end of each episode, the errors and gradients accumulated during the episode are used to update network parameters (weights and plasticity coefficients) according to error

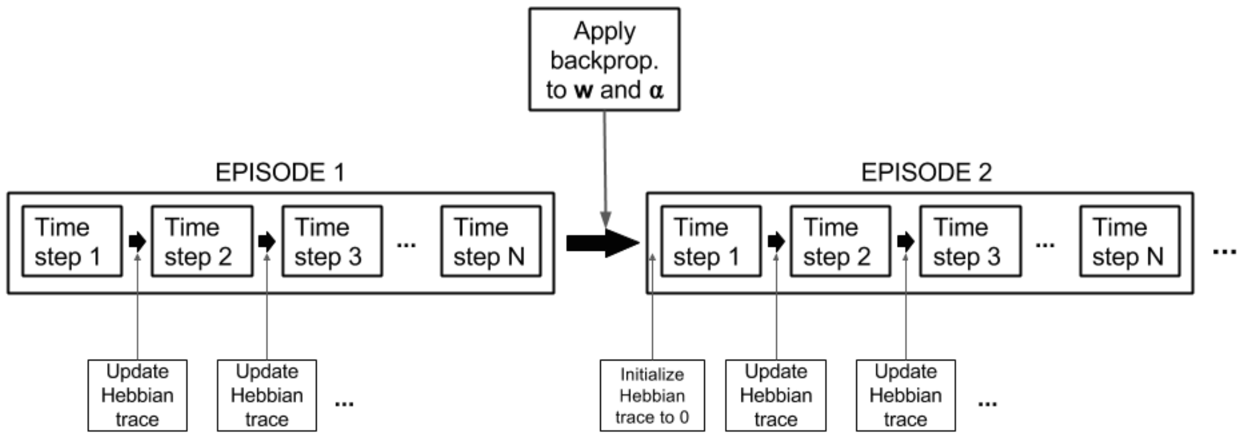
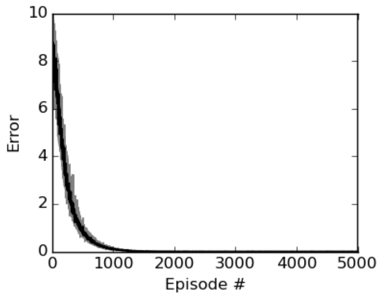
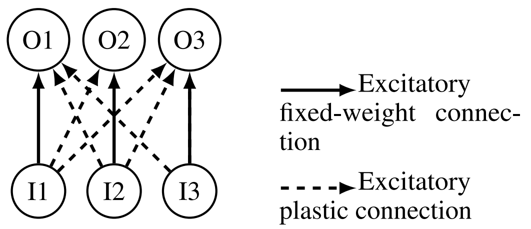


Figure 1: General process of meta learning. During each episode, synaptic plasticity occurs according to the current values of parameters α and w for each connection. At the end of eac episode, the errors and gradients accumulated during the episode are used to update every w and α. Hebbian traces (plastic weights) are reset to 0 before each new episode.



(a) Training error


O1
O2
O3
I1
I2
I3
Excitatory fixed-weight connection
Excitatory plastic connection

(b) Typical trained network

Figure 2: Results for the pattern completion experiment. (a) Mean absolute error per timestep over each episode, for mutually exclusive stimuli. The dark line indicates median error over 20 runs, while shaded areas indicate interquartile range. For the last 500 episodes, training is halted and parameters are frozen. (b) Schema of a typical network after training. Only 3 elements shown for clarity (actual pattern size: 8 elements).

backpropagation and gradient descent. The whole process iterates for a fixed number of episodes. For the last 500 episodes, training stops and networks parameters are frozen.

All source code for these experiments is available at `http://github.com/thomasmiconi`.

4.2 Pattern completion

To test the BOHP method, we first apply it to a task for which Hebbian learning is known to be efficient, namely, pattern completion. The network is composed of an input and an output layer, each having N neurons. In every episode, the network is first exposed to a random binary vector of length N with at least one nonzero element. This binary vector represents the pattern to be learned. Then, at the next timestep, a partial pattern containing only one of the non-zero bits of the pattern (all other bits set to 0) is presented. The task of the network is to produce the full pattern in the output layer. The error for each episode is the Manhattan distance between the network's output at the second time step and the full pattern (network response during the first step is ignored).

The algorithm quickly and reliably learns to solve the task (Figure 2). The final networks after training exhibit the expected pattern: each input node sends one strong, fixed connection to the corresponding output node, as well as one plastic connection to every output node. As a result, during pattern presentation, each non-zero input develops a strong connection to every non-zero output due to Hebbian learning, ensuring successful pattern completion on the second step when one of the nonzero inputs is stimulated.

4.3 One-shot learning of arbitrary patterns

In this task, at each episode, the network must learn to associate each of two random binary vectors with its label. The labels are simply two-element vectors set to 01 for one of the vectors, and 10 for the other. Importantly, learning is one-shot: at the first timestep, the input consists of the first pattern, suffixed with label 01; and at the second timestep, the input vector is the second pattern, suffixed with label 10. These are the only times the labels are presented as inputs: at all other times, the input is one of the patterns, suffixed with the neutral suffix 00, and the network's output must be the label associated with the current pattern.

Patterns are random vectors of N elements, each having value 1 or -1, with at least one position differing between the two patterns to be learned ($N = 8$ for all experiments). The networks have an input layer ($N + 2$ nodes), a hidden layer (2 nodes), and an output layer (2 nodes). For simplicity, only the first layer of weights (input-to-hidden) can have plasticity. The final layer implements softmax competition between the nodes. Each episode lasts 20 timestep, of which only the first two contain the expected label for each pattern. We use cross-entropy loss between the output values and the expected label at each time step, except for the 2-step learning period during which network output is ignored.

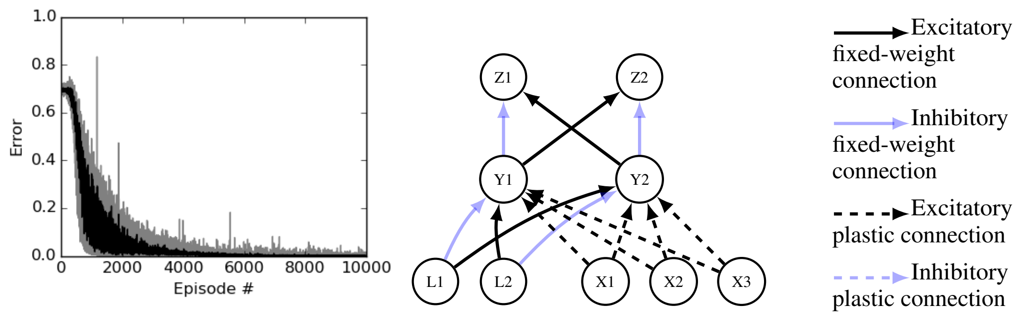
Again, the algorithm reliably learns to solve the task (Figure 3). The trained networks are organized in such a way that one hidden node learns the pattern with label 01, and the other learns the pattern associated with label 10: they receive strong, fixed (positive and negative) connections from the label bits, but receive only strong plastic connections (with zero fixed-weight connections) from the pattern bit. This allows each node to be imprinted to the corresponding pattern. The weights between hidden and top layer ensures that the top two nodes produce the adequate label.

Note that some networks displayed a somewhat different pattern where all connections between pattern nodes and hidden nodes have negative plasticity coefficients. We discuss this configuration in the next subsection.

4.4 Reversal learning

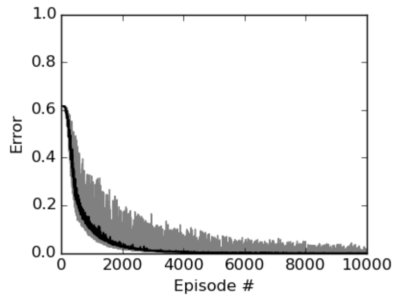
Previous experiments show that the algorithm can train networks to learn fast associations of environmental inputs. But can it also teach networks to adapt to a changing environment - that is, to perform continual learning over their lifetime?

To test this, we adapt the previous one-shot learning task into a continual learning task: halfway through each episode, we invert the two patterns, so that the pattern previously associated with label

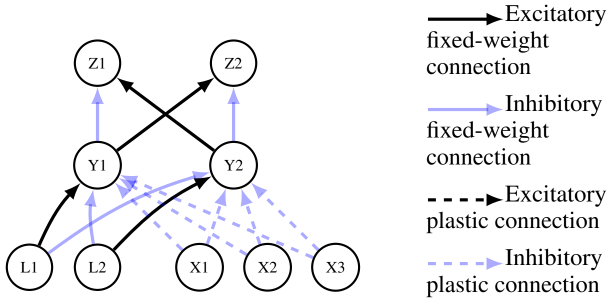


(a) Training error (b) Typical trained network

Figure 3: Results for the one-shot learning experiment. (a) Median absolute error per timestep over each episode. Conventions are as in Figure 2. (b) Schema of a typical network after training. In addition to the label nodes L1 and L2, only 3 pattern elements shown for clarity (actual pattern size: 8 elements). See text for details.



(a) Training error


Z1
Z2
Y1
Y2
L1
L2
X1
X2
X3
Excitatory fixed-weight connection
Inhibitory fixed-weight connection
Excitatory plastic connection
Inhibitory plastic connection

(b) Typical trained network

Figure 4: Results for the reversal learning experiment. Conventions are as in Figure 2. (a) Mean absolute error per timestep over each episode. (b) Schema of a typical network after training. Notice the negative plasticity connections from the pattern nodes to the hidden layer. See text for details.

01 is now associated with label 10, and vice-versa. We show each of the pattern with its updated label once. Then we resume showing input patterns with neutral, 00 suffixes, and expect the network's output to be the adequate new label for each input pattern.

The algorithm also successfully learns to solve this problem (Figure 4). The networks are somewhat similar to the ones obtained in one-shot learning, but with an important difference: connections from pattern input to hidden nodes now consistently have *negative* plasticity coefficients. While some networks trained for the one-shot learning task also had this feature, all networks trained for reversal learning consistently show it. This seems to be a crucial feature for reversal learning, because clipping plasticity coefficients to positive values prevents learning in this task while still allowing successful learning in the one-shot task (data not shown). What does reversal learning seem to require negative plasticity?

Negative plasticity implies that hidden layer firing will be anti-correlated to the presence of the imprinted stimulus. This in itself is unlikely to have any effect, since the effect can be negated by switching signs in the output layer or label node inputs. However, negative plasticity also has the important consequence of making the Hebbian traces *decrease* over time, rather than increase as they would if plasticity coefficient were positive.

It is well-known that Hebbian plasticity creates a positive feedback: correlation between input and output increases the connection weight, which in turn increases the correlation in firing, etc. This would pose a problem for one-shot reversal learning, because by the time the new patterns are shown, the existing associations would be too strong to be erased in a single timestep. However, with negative plasticity coefficients, the opposite is true: the large Hebbian trace created on initial imprinting becomes self-decreasing due to a *negative* feedback loop. As a result, the Hebbian traces created by the first association decrease over time. This has little effect on ongoing responses, since the output nonlinearities will magnify even small differences to produce the correct responses; for the same reason, one-shot learning (in which there is no reversal) is mostly indifferent to the sign of plasticity coefficients, since the episodes are short enough that the final Hebbian traces will always be large enough to support correct choice. However, in the case of reversal learning, decreasing Hebbian traces are vital: when the second association is shown, the existing Hebbian traces are now small enough to be completely erased (and indeed reversed) in a single presentation, which would not be the case under positive plasticity and increasing Hebbian traces.

In short, the BOHP method has not only determined which connections must be plastic to learn an association; it can also develop a precise fine-tuning of the temporal dynamics of this plasticity, by modulating the sign of plasticity coefficients. This remarkable result confirms the potential of BOHP to deal with temporal dynamics in environmental learning.

5 Conclusions and future work

In this paper we have introduced a method for designing networks endowed with Hebbian plasticity through gradient descent and backpropagation. This method allows the network to "learn how to learn" in order to solve a problem with unpredictable parameters. The method successfully solved simple learning tasks, including one-shot and reversal learning.

In this expository we only use a very simple form of Hebbian plasticity, namely, the running average of the product between pre- and post-synaptic activities. However, there are other possible formulations of Hebbian plasticity, such as covariance learning (mean-centering pre- and post-synaptic responses), instar and outstar rules, or BCM learning. These can be implemented in BOHP by rewriting the gradient equations appropriately, which might expand the capacities of BOHP. However, as shown above, the simple Hebbian plasticity used here can already produce fast, reliable lifelong learning.

Furthermore, the networks shown here use fixed plasticity constants to determine the strength of plasticity. However, in biological brains, plasticity is modulated over time by various neuromodulators (especially dopamine, acetylcholine and norepinephrine), which are themselves under neural control. Thus, biological brains can decide not just where, but *when* to apply plasticity, which is crucial for learning complex behaviors. While neuromodulation has been used in neural networks built by evolutionary methods [5], the methods described in this paper could be extended to allow backpropagation to design modulable-plasticity networks.

In conclusion, we suggest that backpropagation of Hebbian plasticity is an efficient way to endow neural networks with long-term memories and lifelong learning abilities.

Appendix

Here we provide a derivation of the gradients of output cell response y at a given timestep t_z with regard to the α coefficient of an incoming connection k (where input activity of the pre-synaptic neuron at this connection is denoted by x_k).

First we simply write out the full expression for y, from Equation 2 (again, we initially omit the $\tanh$ nonlinearity):

$$\frac{\partial y(t_z)}{\partial \alpha_k} = \frac{\partial}{\partial \alpha_k}[\sum_{l \in inputs} w_l x_l(t_z) + \sum_{l \in inputs} \alpha_l Hebb_l(t_z) x_l(t_z)]$$

The first summand on the right-hand side denotes the inputs to y from incoming connections through the fixed weights; since this term does not depend on α in any way, we can remove it from the gradient computation.

The second summand denotes inputs through plastic connections. The cases for $l = k$ and $l \neq k$ must be handled differently, since we are differentiating with regard to α_k:

$$\frac{\partial y(t_z)}{\partial \alpha_k} = \frac{\partial}{\partial \alpha_k}[\sum_{l \neq k} \alpha_l Hebb_l(t_z) x_l(t_z) + \alpha_k Hebb_k(t_z) x_k(t_z)] \quad (5)$$

$$= \sum_{l \neq k}[\frac{\partial}{\partial \alpha_k}(\alpha_l x_l(t_z) Hebb_l(t_z))] + \frac{\partial}{\partial \alpha_k}[\alpha_k Hebb_k(t_z) x_k(t_z)] \quad (6)$$

With regard to α_k, the derivative in the first right-hand-side term has the form $d(Const * f(\alpha_k))/d\alpha_k$, since only the $Hebb_l(t_z)$ depends on α_k (indirectly through y), while neither α_l nor $x_l(t_z)$ does. By contrast, the second right-hand-side term has the form $d(Const * \alpha_k * f(\alpha_k))/d\alpha_k$, so we must develop it using the identity $(xf(x))' = xf'(x) + f(x)$. Therefore:

$$\frac{\partial y(t_z)}{\partial \alpha_k} = \sum_{l \neq k}[\alpha_l x_l(t_z) \frac{\partial}{\partial \alpha_k} Hebb_l(t_z)] + x_k(t_z)(\alpha_k \frac{\partial}{\partial \alpha_k} Hebb_k(t_z) + Hebb_k(t_z))$$

Replacing the $Hebb(t)$ terms by their full expression as the accumulated product of x and y (Eq. 1), we get:

$$\frac{\partial y(t_z)}{\partial \alpha_k} = \sum_{l \neq k} [\alpha_l x_l(t_z) \frac{\partial}{\partial \alpha_k} \sum_{t_u < t_z} (1-\gamma)\gamma^{tz-tu} x_l(t_u) y(t_u))]$$

$$+ x_k(t_z)[\alpha_k \frac{\partial}{\partial \alpha_k} \sum_{t_u < t_z} (1-\gamma)\gamma^{tz-tu} x_k(t_u) y(t_u) + Hebb_k(tz)] \quad (7)$$

$$= \sum_{l \neq k} [\alpha_l x_l(t_z) \sum_{t_u < t_z} (1-\gamma)\gamma^{tz-tu} x_l(t_u) \frac{\partial}{\partial \alpha_k} y(t_u)]$$

$$+ x_k(t_z)[\alpha_k \sum_{t_u < t_z} x_k(t_u)(1-\gamma)\gamma^{tz-tu} \frac{\partial}{\partial \alpha_k} y(t_u) + Hebb_k(tz)] \quad (8)$$

$$= \sum_{l \in inputs} [\alpha_l x_l(t_z) \sum_{t_u < t_z} (1-\gamma)\gamma^{t_z - t_u} x_l(t_u) \frac{\partial}{\partial \alpha_k} y(t_u)] + x_k(t_z) Hebb_k(tz) \quad (9)$$

where in the last equation above, l runs over all incoming connections to y, including k. This recursive gradient equation is identical to Eq. 4.

Eq. 3 is derived much in the same manner (though slightly simpler since we do not need to use the $(xf(x))' = xf'(x) + f(x)$ identity). For future applications to many-layers networks, equations for the gradient $\frac{\partial y(t_z)}{\partial x_k}$ are easily obtained with a similar derivation.

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