Report for CS215 Assignment 3

AMEEYA RANJAN SETHY - 200050006

October 29, 2021

Question 1

A Maximum Likelihood Estimate

As per the question, it is known that $x_1, x_2, x_3, ..., x_n$ are drawn from a Gaussian Distribution $G(\mu, \sigma^2)$ where the variance σ^2 is known but the mean μ is unknown. We have already studied that,

$$\widehat{\mu}^{ML} = \frac{\Sigma_{i=0}^{N} x_i}{N}$$

B Gaussian Prior

As per the question, it is known that $x_1, x_2, x_3, ..., x_n$ are drawn from a Gaussian Distribution $G(\mu, \sigma^2)$ where the variance σ^2 is known but the mean μ is unknown. It is also given in the question that μ is known to be drawn from a Gaussian Distribution $G(\mu_0, \sigma_0^2)$ where $\sigma = 4$, $\sigma_0 = 1$ and $\mu_0 = 10.5$. The mean of the data is

$$\overline{x} = \frac{\Sigma_{i=0}^{N} x_i}{N}$$

$$\widehat{\mu}^{MAP1} = \frac{\overline{x}\sigma_0^2 + \frac{\mu_0 \sigma^2}{N}}{\sigma_0^2 + \frac{\sigma^2}{N}}$$

$$\widehat{\mu}^{MAP1} = \frac{\overline{x} \times 1 + 10.5 \times \frac{16}{N}}{1 + \frac{16}{N}}$$

C Uniform Prior

As per the question, it is known that $x_1, x_2, x_3, ..., x_n$ are drawn from a Gaussian Distribution $G(\mu, \sigma^2)$ where the variance σ^2 is known but the mean μ is unknown. It is also given in the question that μ is known to be drawn from a Uniform Distribution U(9.5,11.5) where $\sigma = 4$.

$$PosteriorPDF = \frac{P(x_1, x_2, ..., x_n|\mu)P(\mu)}{\int_{9.5}^{11.5} P(x_1, x_2, ..., x_n|\mu)P(\mu)d\mu} \quad for\ \mu \in (9.5, 11.5)$$

We know that $P(\mu) = 1\ for\ \mu \in (9.5, 11.5)$.So,

$$PosteriorPDF = \frac{P(x_1, x_2, ..., x_n|\mu)}{\int_{9.5}^{11.5} P(x_1, x_2, ..., x_n|\mu)d\mu} \quad for\ \mu \in (9.5, 11.5)$$

Maximum of the posterior within (9.5, 11.5) = maximum of P($x_1, x_2, ..., x_n|\mu$) within (9.5, 11.5). If the mode of the likelihood function lied within this then the mode of the posterior $\equiv$ ML estimate. Therefore, we have

$$\widehat{\mu}^{MAP2} = min(max(9.5, \widehat{\mu}^{ML}), 11.5)$$

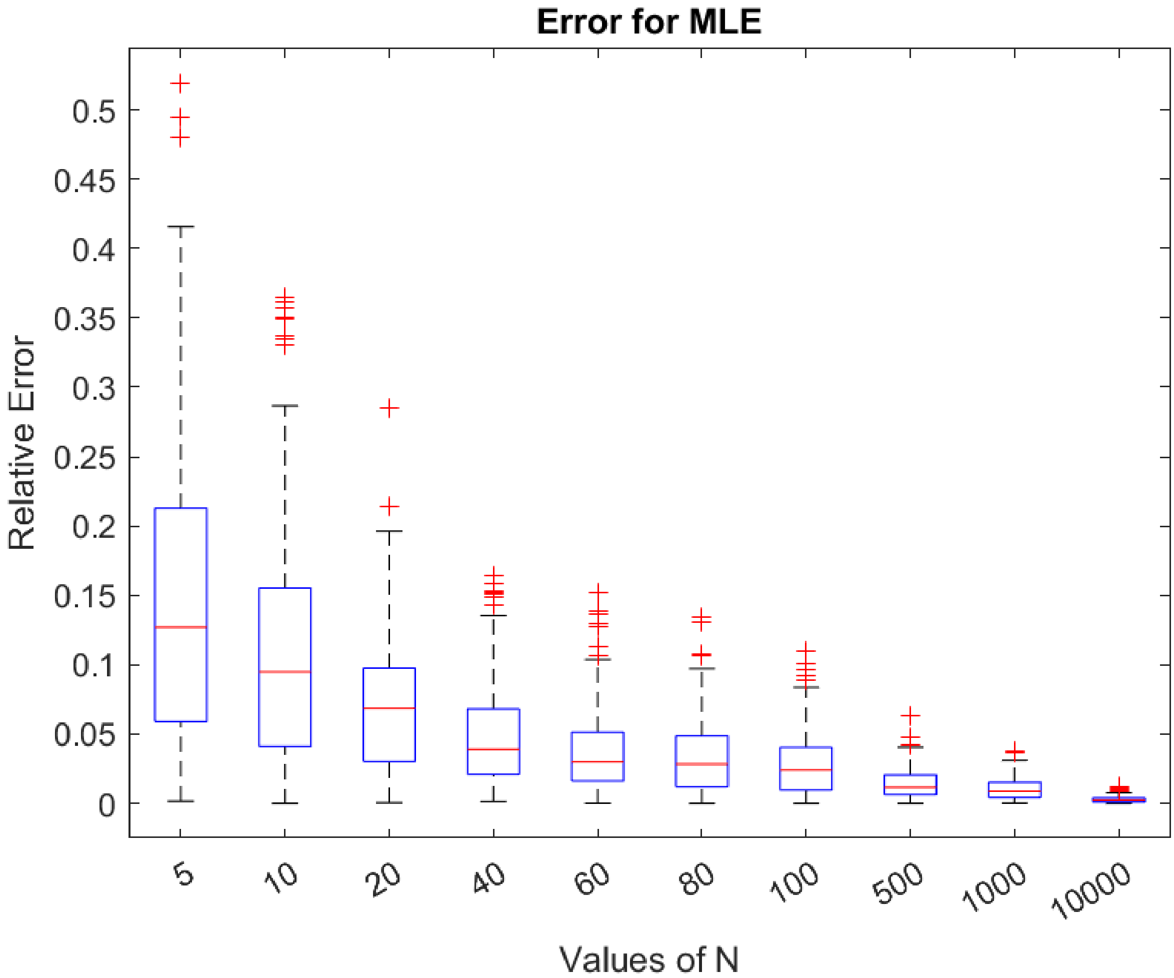
where

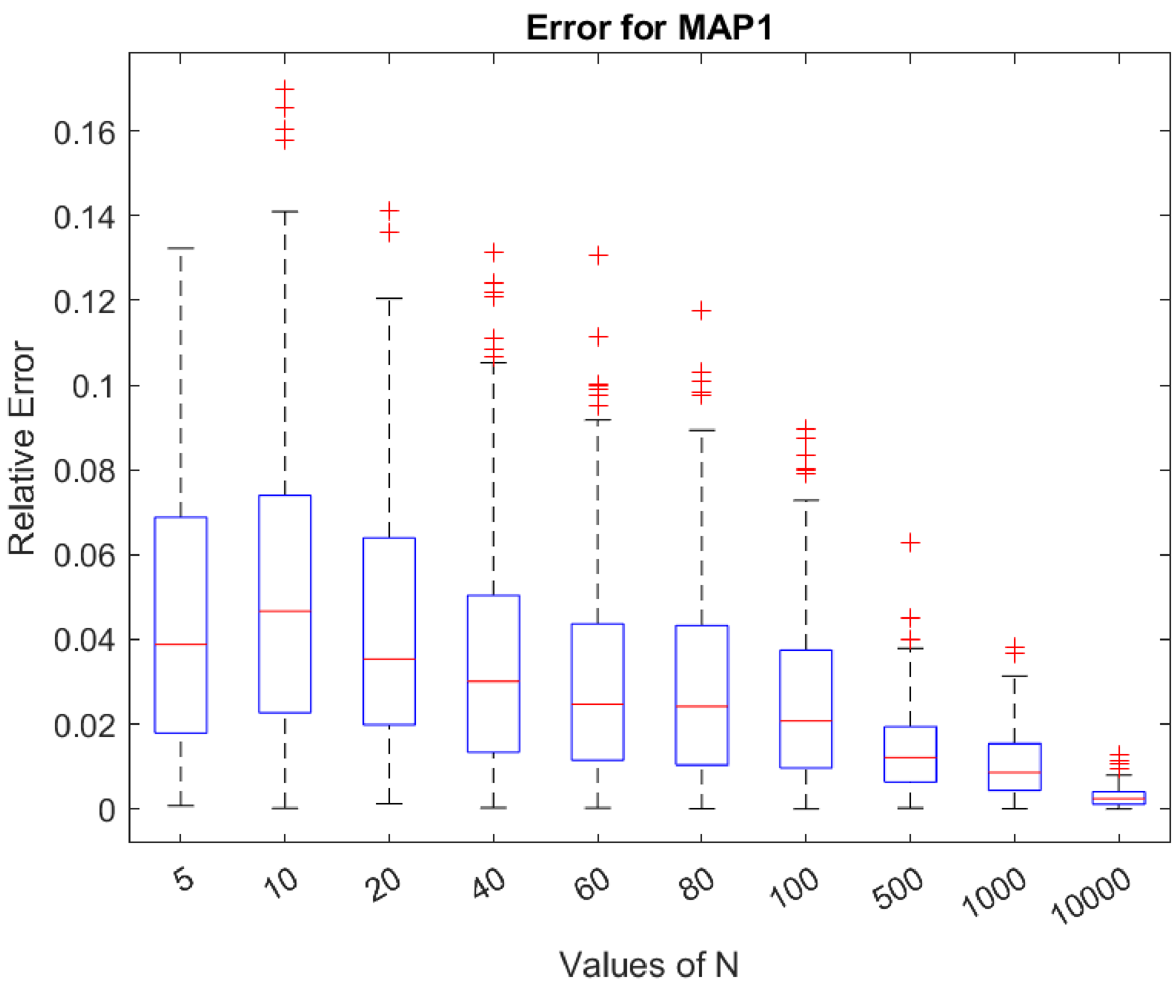
$$\widehat{\mu}^{ML} = \frac{\Sigma_{i=0}^{N} x_i}{N}$$

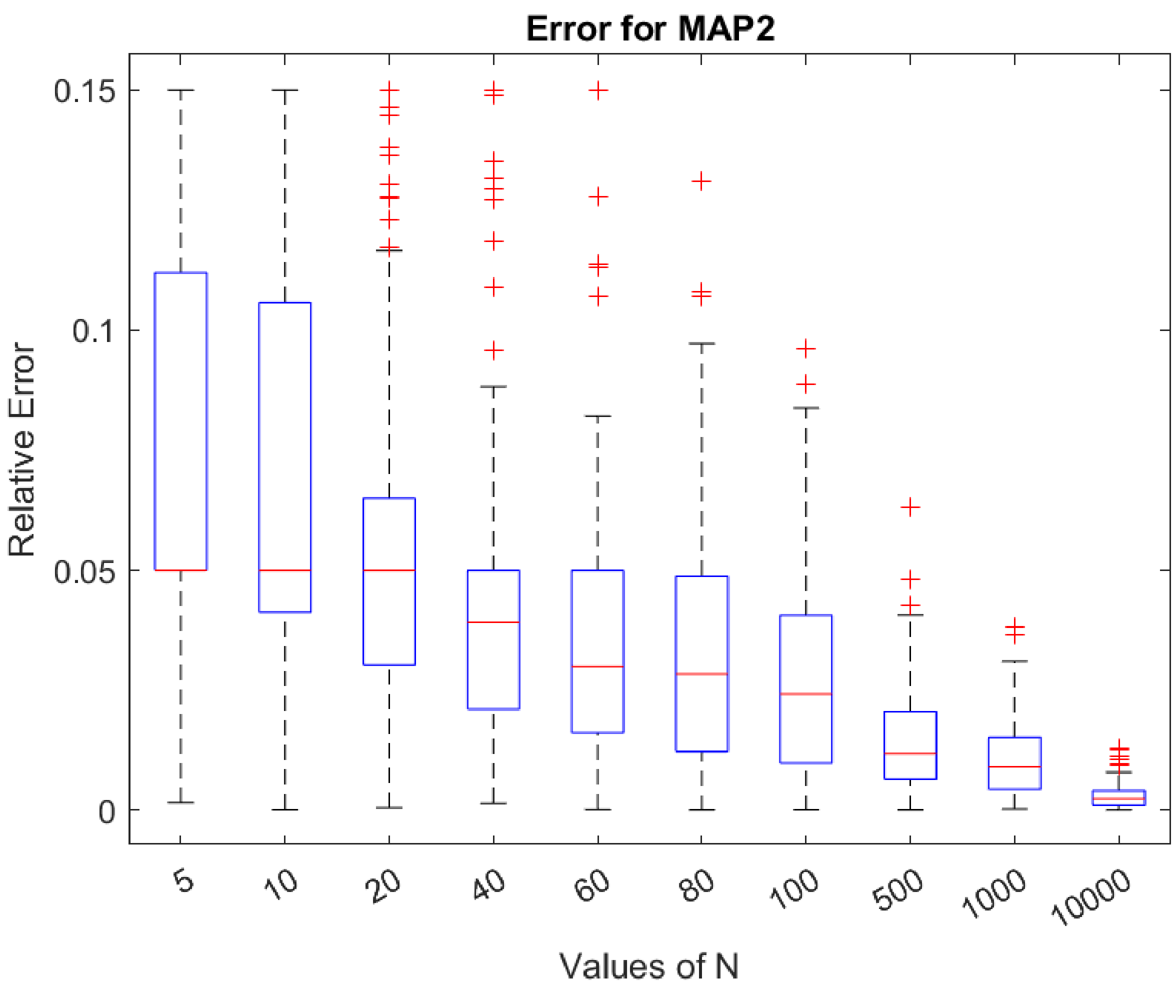
D Interpretation of BoxPlots

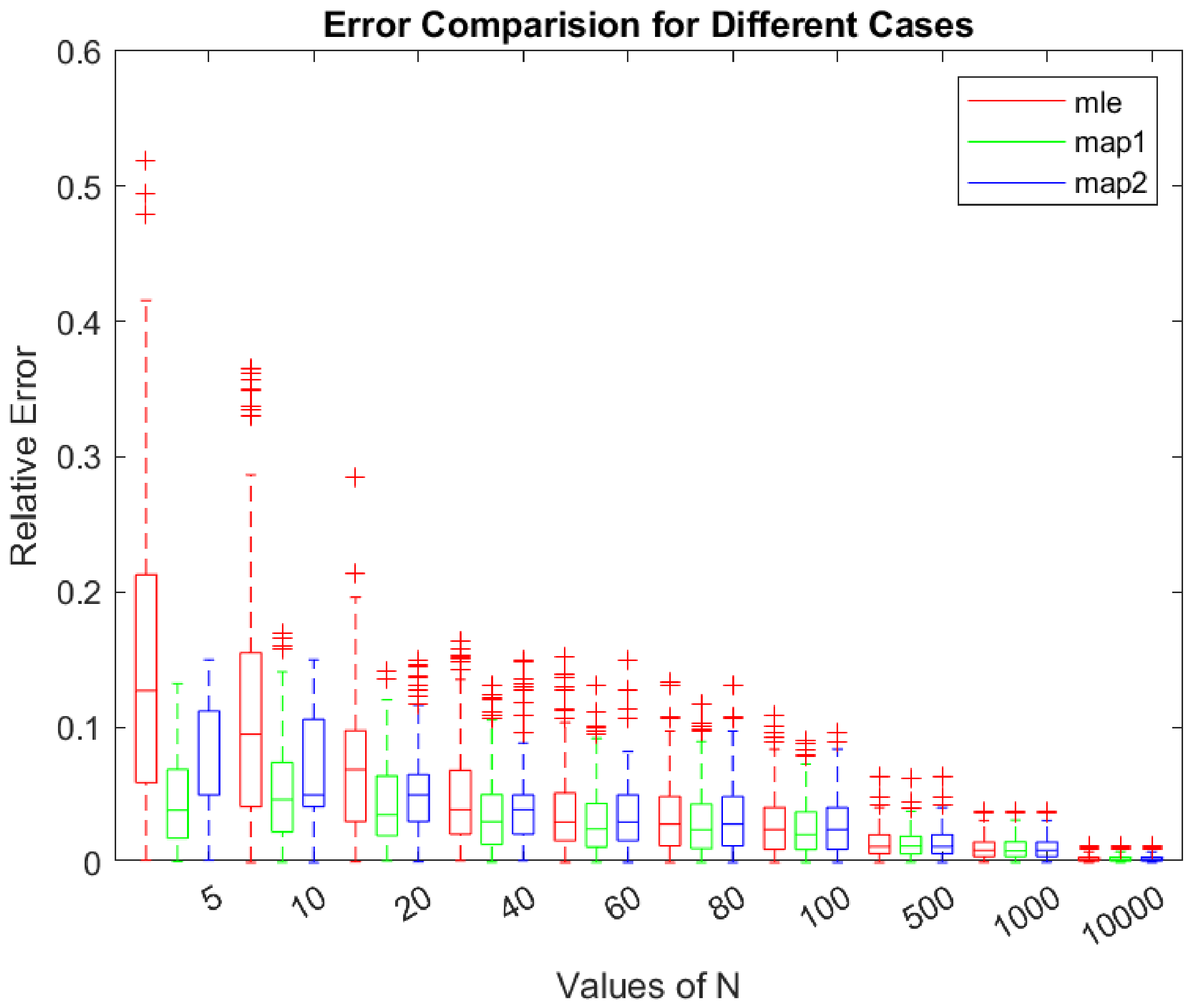
- As the value of N keeps on increasing, the error keeps on decreasing and approaches to zero in all of the three cases.

- The second estimate, i.e. the MAP estimate with a Gaussian Prior is the most preferable since it has the smallest error values even for small values of N. Since in practice, we only have finite data, this estimate should be a good choice. Also, for large values of N it converges to the ML estimate and hence it possesses all the desired asymptotic properties(consistency, asymptotic normality, efficiency) of the ML estimate.




Error for MAP1
Relative Error
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
5
10
20
40
60
80
100
500
1000
10000
Values of N


Error for MAP2
Relative Error
0.15
0.1
0.05
0
5
10
20
40
60
80
100
500
1000
10000
Values of N


Error Comparision for Different Cases
mle
map1
map2
Relative Error
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
20
40
60
80
100
500
1000
10000
Values of N