Report for CS215 Assignment 2

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Question No - 6

Part b

Let B represent the closest representation of original image A. So Both B and A can be written as a linear combination of the mean vector and the top 4 eigen vectors. For any vector P , Frobenius norm can be written as :

$$||P_{norm}|| = \sqrt{\Sigma_i \Sigma_j A_{ij}^2}$$

On reshaping B into column vector, we obtain vector 2 norm of row vector because change in basis does not change the norm of the vector and so we change the eigen basis of the covariance matrix. The basis changed to

$$v_1 = [1\ 0\ 0\ ....\ 0]^T_{19200\times 1}$$

$$v_2 = [0\ 1\ 0\ ....\ 0]^T_{19200\times 1}$$

$$v_{19200} = [0\ 0\ 0\ ....\ 1]^T_{19200\times 1}$$

Now $B = a_1u + a_2v_1 + a_3v_2 + a_4v_3 + a_5v_4$

As u is in the same domain, $u = \Sigma u_i v_i$ where u_i is a real number and $u_i = u.v_i$ and $J = \Sigma J_i v_i$.

Frobenius norm on matrix of difference between A and B is used.

$$\Delta = A - B$$

$$||\Delta_{frob}|| = (j_1 - u_1a_1 - a_2)^2 + (j_2 - u_2a_1 - a_3)^2 + (j_3 - u_3a_1 - a_4)^2 + (j_4 - u_4a_1 - a_5)^2 + \Sigma_{i=5}^{19200}(j_i - u_ia_i)^2$$

To make $||\Delta_{frob}||$ minimum, first 4 terms can be made 0 irresepective of choice of a_1, i.e.

$$a_2 = j_1 - u_1a_1$$

$$a_3 = j_2 - u_2a_1$$

$$a_4 = j_3 - u_3a_1$$

$$a_5 = j_4 - u_4a_1$$

On Differentiating the remaining expression, we get

$$\frac{d}{da}\Sigma_{i=5}^{19200}(j_i - u_ia_i)^2 = 0$$

$$\Sigma 2(j_i - u_ia_i)(-u_i) = 0$$

$$a_i = \frac{J.u - \Sigma_{i=1}^4 j_iu_i}{u.u - \Sigma_{i=1}^4 u_i^2} = \frac{\Sigma_{i=5}^{19200} u_ij_i}{\Sigma_{i=1}^5 u_i^2}$$

Hence we have got that the closed representation of B will be

$$B = a_1u + (j_1 - u_1a_1)v_1 + (j_2 - u_2a_1)v_2 + (j_3 - u_3a_1)v_3 + (j_4 - u_4a_1)v_4$$

as derived previously.

Part c

To generate a new set of images from Multi Variate Gaussian(MVG) from the given set of images, we can followe the steps given below :

MVG $X = \mu + AW$ where W is a vector consisting of independent and identically distributed variables from univariate Gaussian.

We can define covariance matrix $C = AA^T$ and μ = mean.

Now we can find a matrix V whose columns are corresponding eigen vectors for respective eigen values.

When covariance matrix C is given we can use $A = C^{0.5}$

Now W = randn(19200,1) and using A and W , we can find X.

By reshaping X, we can obtain the required new set of images. Now we can repeat this procedure 3 times to obtain 3 images that are distinct from given images and are representative of the data set.

Here C is a matrix of size 19200 $\times$ 19200 . So code was taking a lot of time. So I used SVD :

$$VSV^T = C$$

where V is a matrix with columns as corresponding eigen vectors for respective eigen values of C and [V,D] = eigs(C,10)

Here V_i is orthogonal means that any two eigen coumns i.e any 2 eigen vectors are mutually perpendicular.

D_1 is a 4×4 diagonal matrix such that $D_1 = S(1:4, 1:4)$ and $S_1 = D_1^{0.5}$

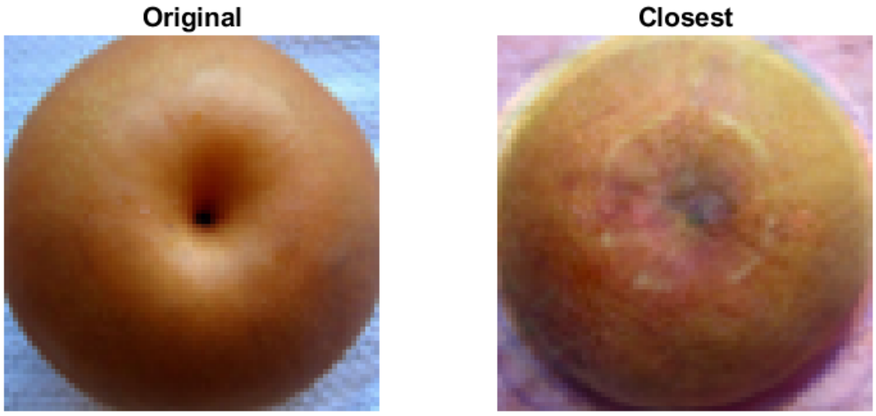
$V_1 = V(:, 1:4)$ is a matrix of first 4 columns of V. Hence we observe that

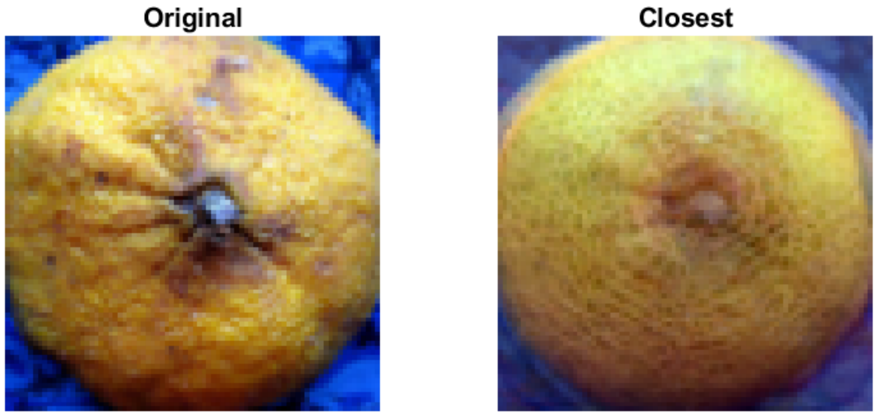
$$A = V_1 S_1 V_1^T$$

$$AA^T = V_1 S_1 V_1^T V_1 S_1^T V_1^T = V_1 S_1 S_1^T V_1^T = V_1 D_1 V_1^T$$

Hence we get A, V and proceed using the steps as mentioned above.

Original Image and Reconstructed image





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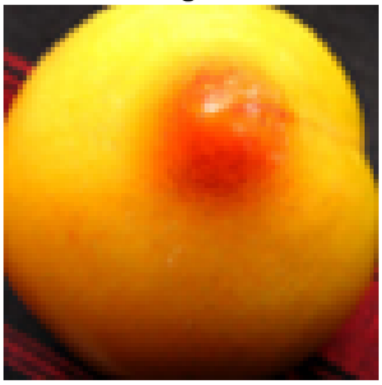


Closest

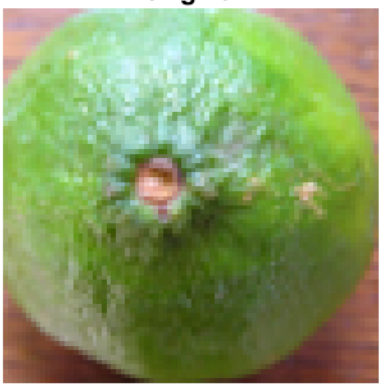
Original

Closest

Original

Closest

Original



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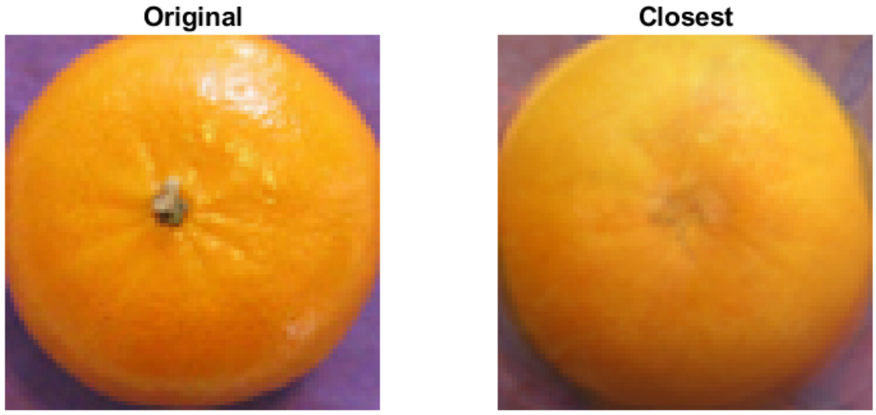
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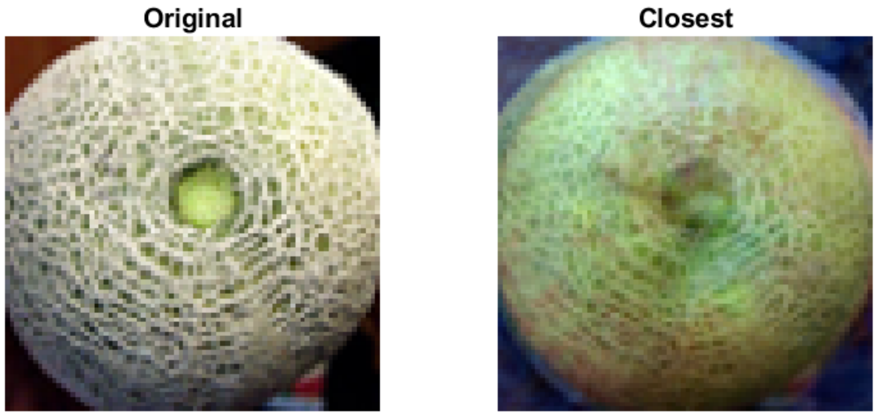
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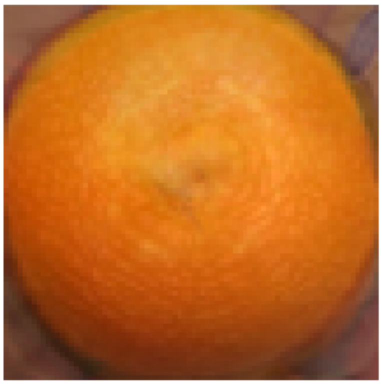
Closest

Original

Closest



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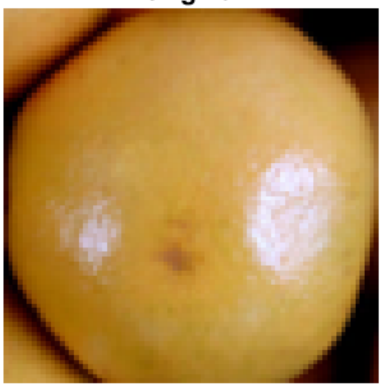

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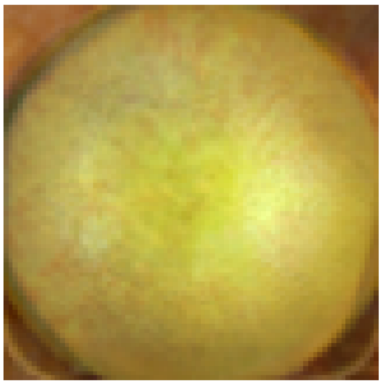

Original

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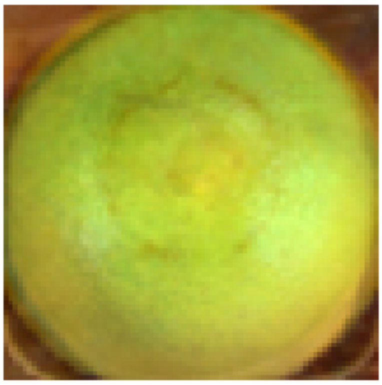

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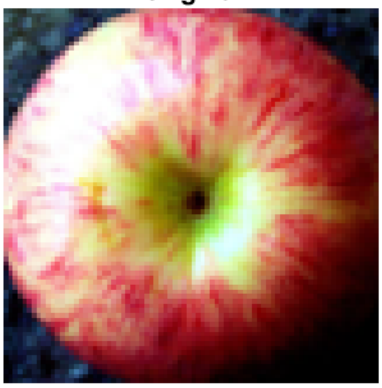

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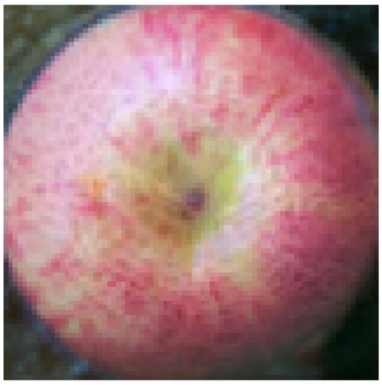

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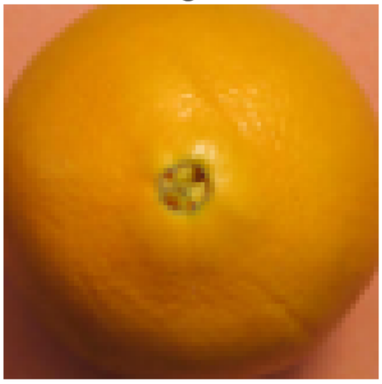

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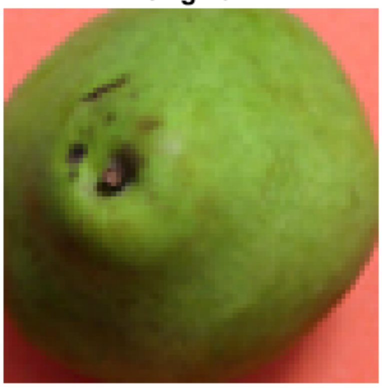


Closest

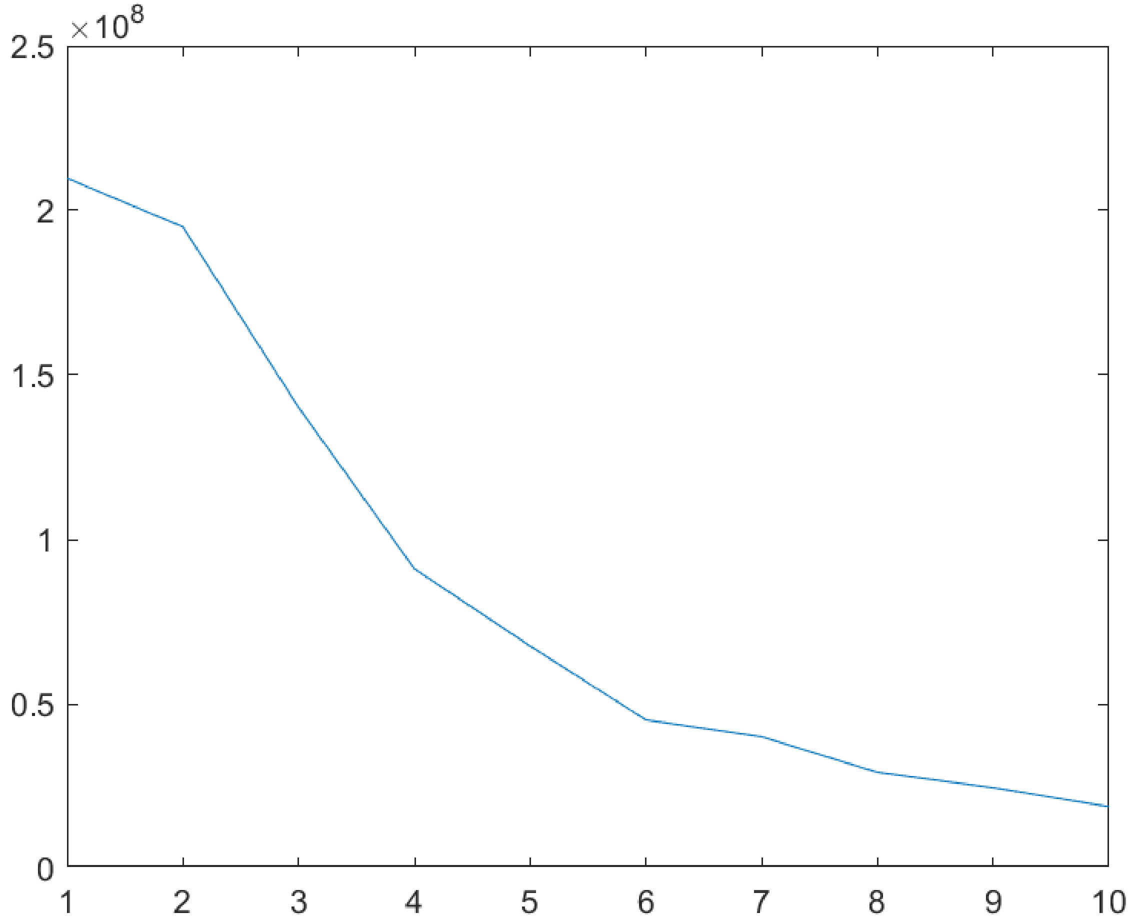
Original

Closest

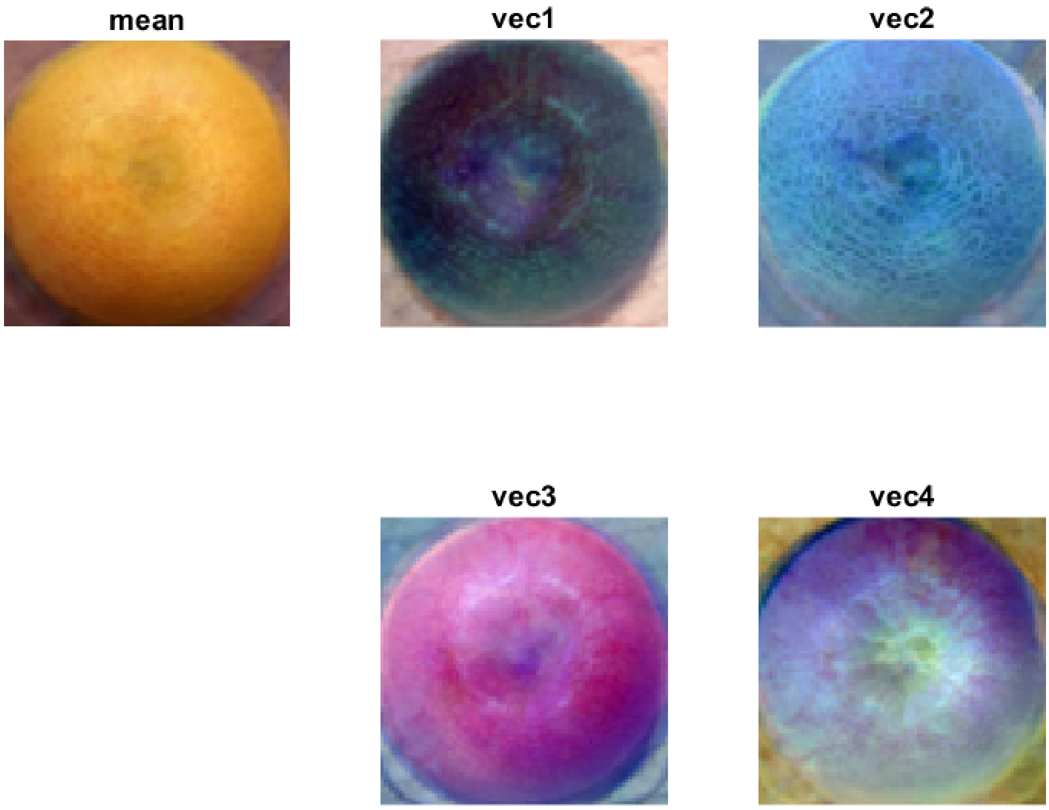
Original

Closest

Graph of top 10 eigenvalues



Images of mean and top 4 eigenvectors



New set of generated images