Report for CS215 Assignment 2

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October 12, 2021

Question 1

Part a

Let's consider the major axis of ellipse as P and the minor axis of ellipse as Q. Then the equation of the ellipse will be of the form : $(\frac{x}{P})^2 + (\frac{x}{P})^2 = 1$. Now the task is to generate points that are uniformly distributed inside the ellipse. This means all the points (x,y) that meet our requirements will satisfy the condition $(\frac{x}{P})^2 + (\frac{x}{P})^2 < 1$. It can also be seen that all the points will lie on some smaller ellipse and the smaller ellipse is formed by isotropic decrease in the dimensions of original ellipse. Now let's consider the major axis of the smaller ellipse as p and minor axis as q. To maintain the isotropic decrease in dimension, it should satisfy $\frac{p}{P} = \frac{q}{Q} = \lambda$. Let a point satisfying the conditions be represented by (x,y). Then

$$x = p * cos(\theta) = \lambda * P * cos(\theta)$$

$$x = q * sin(\theta) = \lambda * Q * sin(\theta)$$

As we are only considering the points inside the region bounded by the ellipse, so

$$0 < p < Pand0 < q < Q$$

and they should satisfy the equation:

$$(\frac{x}{P})^2 + (\frac{y}{Q})^2 < 1$$

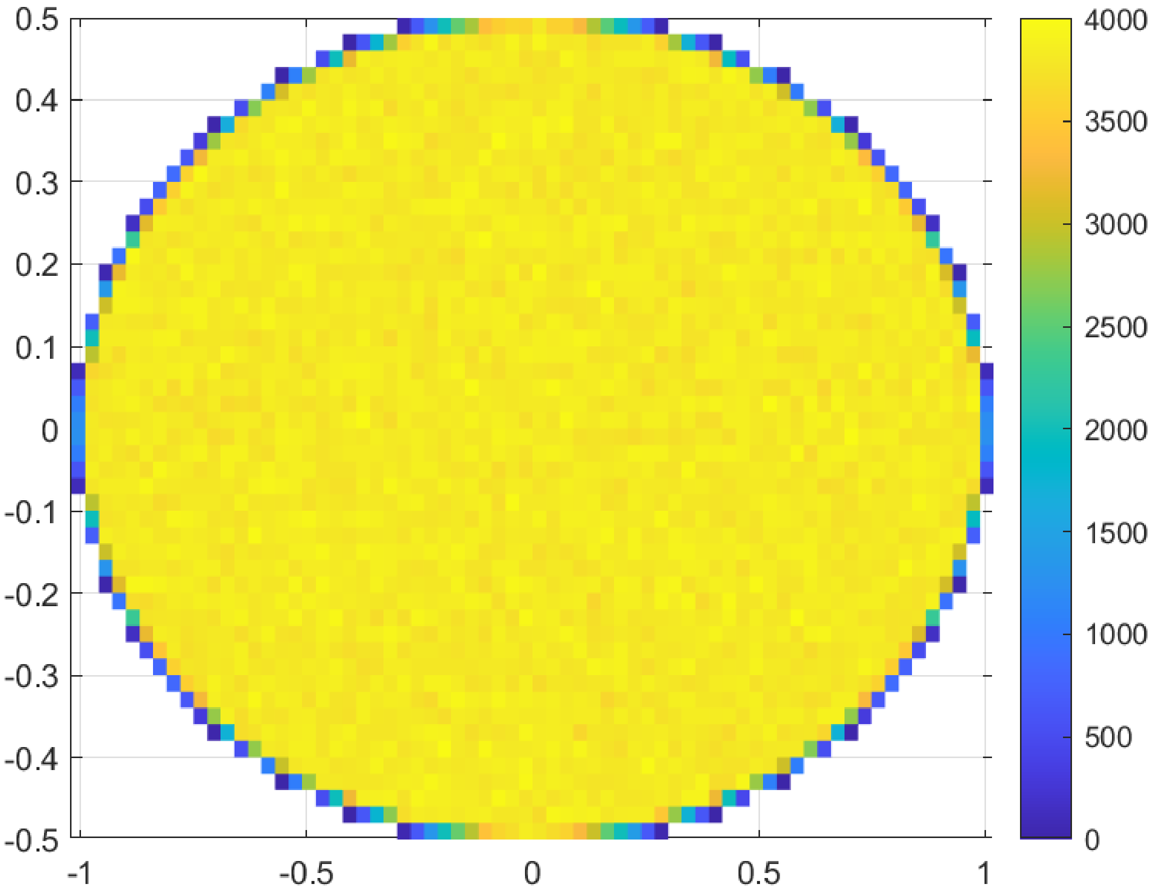
On solving the equation, we get $\lambda^2 < 1$. As we decrease both the major axis and minor axis by a factor λ, the area reduces by a factor of λ^2. This shows that λ^2 should be a random number uniformly distributed between 0 and 1. As $\lambda^2 = U(0,1)$, so $\lambda = \sqrt{U(0,1)}$ satisfies both equation 1 and 2. To maintain uniform distribution, we will choose θ uniformly between 0 and 2π. So finally

$$x = \sqrt{U(0,1)} * P * cos(\theta)$$

$$y = \sqrt{U(0,1)} * Q * sin(\theta)$$

repeated N times to generate N random points uniformly distributed in the region inside the ellipse.

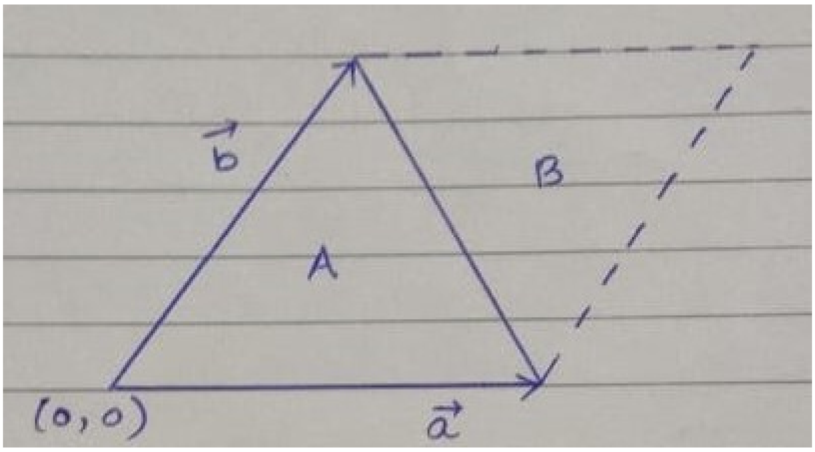
Part b



Code for part b is ques1b.m

Part c

The task is to generate points inside a triangle, Let first vertex be (0,0) and other two vertices be represented bu the vectors $\vec{a}$ and $\vec{b}$.



Now we will extend this triangle into a parallelogram represented by the dotted lines. We can easily generate uniform random points inside the parallelogram using uniform random generator where each point $P = \lambda * \vec{a} + \mu * \vec{b}$ where λ and μ belong to U(0,1) and these points lie inside the parallelogram. Now to convert the uniform distribution for parallelogram into a uniform distribution in triangle, we need to follow a simple procedure.

It can be easily seen that whenever $\lambda + \mu <= 1$, the point P lies inside the required triangle but when $\lambda + \mu > 1$, the point P lies outside the required triangle but inside the parallelogram. o whenever $\lambda + \mu > 1$, we will define

$$\lambda_{new} = 1 - \lambda$$

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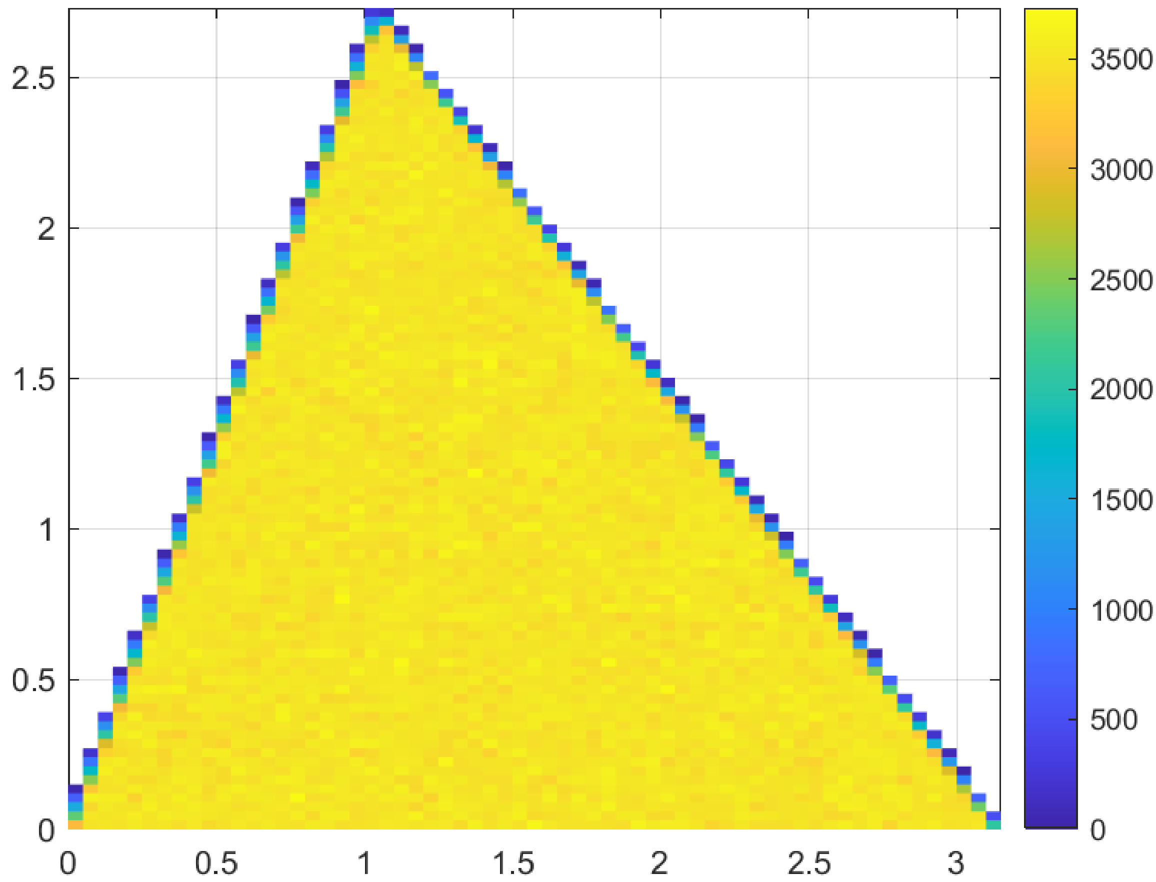
As $\lambda + \mu > 1$, it can be clearly seen that $\lambda_{new} + \mu_{new} < 1$, and the point P be represented by

$$P = \lambda_{new} * \vec{a} + \mu_{new} * \vec{b}$$

which will now lie inside our required triangle. And as λ and μ are uniform random variates, so are λ_{new} and μ_{new}. Actually this whole procedure is equivalent to a 180 degree rotation of the upper triangle about to the center of the parallelogram to get the required triangle. And as the distribution was uniform in both the triangles , so imposing one triangle over the other triangle will also lead to a uniform distribution.

Part d



Code for part d is ques1d.m