Homework 1

Q1

A student answers a multiple choice examination with two questions that have four possible answers each. Suppose that the probability that the student knows the answer to a question is 0.65 and the probability that the student guesses is 0.35. If the student guesses, the probability of guessing the correct answer is 0.25. The questions are independent, that is, knowing the answer on one question is not influenced by the other question.

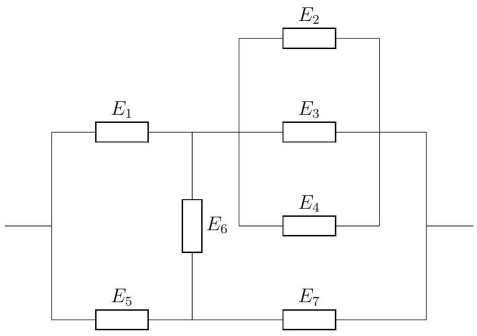
a) What is the probability that both questions will be answered correctly?

b) Suppose both questions were answered correctly. What is the probability that the student really knew the correct answer to both questions?

c) How would you generalize the above from 2 to n questions, that is, what are answers to (a) and (b) if the test has n independent questions? What happens to probabilities in (a) and (b) if $n \to \infty$.

Q2

A circuit S consisting of seven independent elements $E_1, \ldots, E_7$ is connected as shown:



The elements are operational during time interval T with probabilities

	E_1	E_2	E_3	E_4	E_5	E_6	E_7
Probability of working (p)	0.5	0.4	0.1	0.6	0.9	0.8	0.7

a) Find the probability that the circuit is operational during time interval T.

b) If the circuit was found operational at the time T, what is the probability that the element E_6 was operational.

Q3

A machine has four independent components, three of which fail with probability $q = 1 - p$, and one with probability 0.5. The machine is operational as long as at least three components are working.

a) What is the probability that the machine will fail? Evaluate this probability for $p = 0.75$.

b) If the machine failed, what is the probability that the component which fails with probability 0.5 actually failed?

c) Suppose that after the machine fails, a diagnostic test is performed to determine whether the component which fails with probability 0.5 actually failed. The test is 90% accurate when this component fails (sensitivity) and 80% accurate when it does not fail (specificity). Compute the posterior probability that this component failed, given that the diagnostic test indicates a failure.