

CONTEXT**System_Ctx****CONSTANTS**

S

TIME

sigma

AXIOMS

axm1 : S=RReal×RReal

axm2 : TIME=RRealPlus

axm3 : $\text{sigma} \in \text{RRealPlus} \wedge \text{sigma} \neq \text{Rzero} \Rightarrow \text{gt}$ **END**

CONTEXT**Thoerems****AXIOMS**

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axm1 :  $\forall a, b, c, d. a \mapsto b \in \text{leq} \wedge c \mapsto d \in \text{leq} \Rightarrow \text{plus}(a \mapsto c) \mapsto \text{plus}(b \mapsto d) \in \text{leq}$ 
axm2 :  $\forall a, b, c, d. \text{Rzero} \mapsto a \in \text{leq} \wedge \text{Rzero} \mapsto b \in \text{leq} \wedge \text{Rzero} \mapsto c \in \text{leq} \wedge \text{Rzero} \mapsto d \in \text{leq} \wedge a \mapsto b \in \text{leq} \wedge c \mapsto d \in \text{leq} \Rightarrow \text{times}(a \mapsto c) \mapsto \text{times}(b \mapsto d) \in \text{leq}$ 
axm3 :  $\forall a, b, c. a \mapsto b \in \text{leq} \wedge b \mapsto c \in \text{leq} \Rightarrow a \mapsto c \in \text{leq}$ 
axm4 :  $\forall a, b. a \in \text{RReal} \wedge b \in \text{RReal} \Rightarrow$ 
       $\text{minus}(\text{times}(a \mapsto a) \mapsto \text{times}(b \mapsto b)) = \text{times}(\text{plus}(a \mapsto b) \mapsto \text{minus}(a \mapsto b))$ 
axm5 :  $\forall a. a \in \text{RReal} \Rightarrow \text{uminus}(a) = \text{minus}(\text{Rzero} \mapsto a)$ 
       $\forall a. a \in \text{RReal} \Rightarrow$ 
       $a = \text{plus}(\text{times}(\text{divide}(\text{Rone} \mapsto \text{Rtwo}) \mapsto a)$ 
axm6 :  $\mapsto \text{times}(\text{divide}(\text{Rone} \mapsto \text{Rtwo}) \mapsto a)$ 
       $)$ 
       $\forall a, b. a \in \text{RReal} \wedge b \in \text{RReal} \wedge \text{times}(a \mapsto b) \in \text{RRealStar}$ 
axm7 :  $\Rightarrow \text{inverse}(\text{times}(a \mapsto b)) = \text{times}(\text{inverse}(a) \mapsto \text{inverse}(b))$ 

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END

MACHINE**System_M****SEES****System_Ctx****Thoerems****VARIABLES****t****plantV****INVARIANTS****inv1** : $t \in \text{TIME}$ **inv2** : $\text{plantV} \in \text{Closed2Closed}(\text{Rzero}, t) \leftrightarrow S$ **EVENTS****INITIALISATION** \triangleq **STATUS****ordinary****BEGIN****act1** : $t := \text{Rzero}$ **act2** : $\text{plantV} := \{\text{Rzero}\} \rightarrow S$ **END****Progress** \triangleq **STATUS****ordinary****BEGIN****act1** : $t : | t' \in \text{TIME} \wedge (t \mapsto t' \in \text{lt} \wedge \text{minus}(t' \mapsto t) \mapsto \text{sigma} \in \text{geq})$ **END****Plant** \triangleq **STATUS****ordinary****ANY****e****plant1****WHERE****grd1** : $e \in \text{DE}(S)$ **grd2** : $\text{Solvable}(\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}), e)$ $\text{plant1} \in \text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}) \rightarrow S \wedge$ **grd3** : $\text{AppendSolutionBAP}(e,$ $\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}),$ $\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}), \text{plant1})$ **THEN****act1** : $\text{plantV} := \text{plantV} \circ \text{plant1}$ **END****END**

CONTEXT

EventTriggered_Ctx

EXTENDS

System_Ctx

SETS

EXEC

CONSTANTS

safe

evt_trig

ctrl

plant

prg

f_evol

f_evol_plantV

evade_value

AXIOMSaxm1 : safe $\in (S \times \mathbb{RReal}) \rightarrow \text{BOOL}$ axm2 : evt_trig $\in (S \times \mathbb{RReal}) \times \mathbb{RReal} \rightarrow \text{BOOL}$

axm3 : partition(EXEC, {ctrl},{plant},{prg})

axm4 : f_evol $\in \mathbb{RReal} \rightarrow S$ axm5 : f_evol_plantV $\in (\mathbb{RReal} \rightarrow (\text{TIME} \times S \rightarrow (\mathbb{RReal} \times \mathbb{RReal})))$ axm6 : $\forall \text{ctrlV} \cdot \text{ctrlV} \in \mathbb{RReal} \Rightarrow (\text{f_evol_plantV}(\text{ctrlV}) =$
 $(\lambda t \mapsto \text{plantV} \cdot t \in \text{TIME} \wedge \text{plantV} \in S \mid \text{f_evol}(\text{ctrlV})))$ axm7 : evade_value $\subseteq \mathbb{RReal} \wedge \text{evade_value} \neq \emptyset$ **END**

MACHINE

EventTriggered_M

REFINES

System_M

SEES

EventTriggered_Ctx

VARIABLES

t
 plantV
 ctrlV
 exec

INVARIANTS

inv1 : ctrlV \in RReal
 inv2 : exec \in EXEC
 inv3 : $\text{exec} \neq \text{plant} \Rightarrow \text{dom}(\text{plantV}) = \text{Closed2Closed}(\text{Rzero}, t)$
 inv4 : $\text{exec} = \text{plant} \Rightarrow t \notin \text{dom}(\text{plantV})$

EVENTS**INITIALISATION** \triangleq

extended

STATUS

ordinary

BEGIN

act1 : $t = \text{Rzero}$
 act2 : $\text{plantV} : \in \{\text{Rzero}\} \rightarrow S$
 act3 : ctrlV \in RReal
 act4 : $\text{exec} = \text{ctrl}$

END**Progress** \triangleq **STATUS**

ordinary

REFINES

Progress

ANY

t1

WHERE

grd1 : $\text{exec} = \text{prg}$
 grd2 : $t1 \in \text{TIME} \wedge (t \mapsto t1 \in \text{lt} \wedge \text{minus}(t1 \mapsto t) \mapsto \text{sigma} \in \text{geq})$
 grd3 : $\text{ctrlV} \notin \text{evade_value} \Rightarrow \text{evt_trig}(\text{plantV}(t) \mapsto \text{minus}(t1 \mapsto t) \mapsto \text{ctrlV}) = \text{TRUE}$

THEN

act1 : $t = t1$
 act2 : $\text{exec} = \text{plant}$

END**Plant** \triangleq **STATUS**

ordinary

REFINES

Plant

ANY

plant1

WHERE

grd1 : $\text{exec} = \text{plant}$
 grd2 : $\text{plant1} \in \text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}) \rightarrow S$
 grd3 : $\text{ode}(\text{f_evol_plantV}(\text{ctrlV}), \text{plant1}(t), t) \in \text{DE}(S)$
 grd4 : $\text{Solvable}(\text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}), \text{ode}(\text{f_evol_plantV}(\text{ctrlV}), \text{plant1}(t), t))$
 grd5 : $\text{AppendSolutionBAP}(\text{ode}(\text{f_evol_plantV}(\text{ctrlV}), \text{plant1}(t), t), \text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}), \text{Closed2Closed}(\text{Rzero}, t) \setminus \text{dom}(\text{plantV}), \text{plant1})$

WITHe : $e = \text{ode}(\text{f_evol_plantV}(\text{ctrlV}), \text{plant1}(t), t)$ **THEN**act1 : $\text{plantV} = \text{plantV} \leftarrow \text{plant1}$

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    act2 : exec:=ctrl
END

Ctrl_normal ≐
STATUS
  ordinary
ANY
  nrml_value
WHERE
  grd1 : exec = ctrl
  grd2 : nrml_value∈RReal
  grd3 : nrml_value≠ evade_value ⇒safe(plantV(t)⇒nrml_value) = TRUE
THEN
  act1 : ctrlV :=nrml_value
  act2 : exec := prg
END

Ctrl_evade ≐
STATUS
  ordinary
ANY
  evade_val
WHERE
  grd1 : exec = ctrl
  grd2 : evade_val∈evade_value
THEN
  act1 : ctrlV:= evade_val
  act2 : exec := prg
END

END

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CONTEXT**TimeTriggered_Ctx****EXTENDS****EventTriggered_Ctx****CONSTANTS**

epsilon

safeEpsilon

AXIOMS**axm1** : $\text{epsilon} \in \text{TIME} \wedge \text{Rzero} \Rightarrow \text{epsilon} \in \text{leq} \wedge \text{sigma} \Rightarrow \text{epsilon} \in \text{leq}$ **axm2** : $\text{safeEpsilon} \in (\text{S} \times \text{RReal}) \rightarrow \text{BOOL}$ **END**

MACHINE

TimeTriggered_M

REFINES

EventTriggered_M

SEES

TimeTriggered_Ctx

VARIABLES

t
 plantV
 ctrlV
 exec

EVENTS**INITIALISATION** \triangleq

extended

STATUS

ordinary

BEGIN

act1 : $t := Rzero$
act2 : $plantV : \in \{Rzero\} \rightarrow S$
act3 : $ctrlV : \in RReal$
act4 : $exec := ctrl$

END**Progress_time** \triangleq **STATUS**

ordinary

REFINES

Progress

ANY

t1

WHERE

grd1 : $exec = prg$
grd2 : $t1 \in TIME \wedge t \mapsto t1 \in lt \wedge minus(t1 \mapsto t) \mapsto sigma \in geq \wedge$
 $minus(t1 \mapsto t) \mapsto epsilon \in leq$
grd3 : $ctrlV \notin evade_value \Rightarrow evt_trig(plantV(t) \mapsto minus(t1 \mapsto t) \mapsto ctrlV) = TRUE$

THEN

act1 : $t := t1$
act2 : $exec := plant$

END**Plant** \triangleq

extended

STATUS

ordinary

REFINES

Plant

ANY*plant1***WHERE**

grd1 : $exec = plant$
grd2 : $plant1 \in Closed2Closed(Rzero, t) \setminus dom(plantV) \rightarrow S$
grd3 : $ode(f_evol_plantV(ctrlV), plant1(t), t) \in DE(S)$
grd4 : $Solvable(Closed2Closed(Rzero, t) \setminus dom(plantV),$
 $ode(f_evol_plantV(ctrlV), plant1(t), t))$
grd5 : $AppendSolutionBAP(ode(f_evol_plantV(ctrlV), plant1(t), t),$
 $Closed2Closed(Rzero, t) \setminus dom(plantV),$
 $Closed2Closed(Rzero, t) \setminus dom(plantV), plant1)$

THEN

act1 : $plantV := plantV \sim plant1$
act2 : $exec := ctrl$

END**Ctrl_normal_time** \triangleq

extended


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STATUS
  ordinary
REFINES
  Ctrl_normal
ANY
  nrml_value
WHERE
  grd1 : exec = ctrl
  grd2 : nrml_value ∈ ℝReal
  grd3 : nrml_value ≠ evade_value ⇒ safe(plantV(t) ↦ nrml_value) = TRUE
  grd4 : nrml_value ≠ evade_value ⇒ safeEpsilon(plantV(t) ↦ nrml_value) = TRUE
THEN
  act1 : ctrlV := nrml_value
  act2 : exec := prg
END

Ctrl_evade ≐
  extended
STATUS
  ordinary
REFINES
  Ctrl_evade
ANY
  evade_val
WHERE
  grd1 : exec = ctrl
  grd2 : evade_val ∈ evade_value
THEN
  act1 : ctrlV := evade_val
  act2 : exec := prg
END

END

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