

Agenda

1. Conditional Probability

In-Class Problems

Conditional Probability Suppose there are three two-layer cakes at a bakery: one is vanilla on both layers, one is chocolate on both layers, and the other is vanilla on one layer and chocolate on the other layer. You cut into only the first layer of a randomly selected cake and take a peek. The first layer is vanilla. What is the probability that when you cut into the second layer, it will also be vanilla? Let's simulate it with R!

```
bakery <- data.frame(layer_one = c("V", "V", "C"), other_layer = c("V", "C", "C"))
library(dplyr)
n <- 100000
sim <- bakery %>%
  sample_n(size = n, replace = TRUE) %>%
  mutate(peek = sample(c("order_one", "order_two"), size = n, replace = TRUE),
         peek = ifelse(peek == "order_one",
                       as.character(layer_one), as.character(other_layer)),
         full_cut = ifelse(peek == "order_one",
                           as.character(other_layer), as.character(layer_one)))
sim %>%
  filter(peek == "V") %>%
  summarize(peek_vanilla = n(), full_cut_vanilla = sum(full_cut == "V")) %>%
  mutate(peek_vanilla_pct = peek_vanilla / n, full_cut_pct = full_cut_vanilla / peek_vanilla)
```

In-Class Problems

1. **Joint and conditional probabilities** $\Pr(A) = 0.3, \Pr(B) = 0.7$

- (a) Can you compute $\Pr(A \cap B)$ if you only know $\Pr(A)$ and $\Pr(B)$?
- (b) Assuming that events A and B arise from independent random processes,
 - i. what is $\Pr(A \cap B)$?
 - ii. what is $\Pr(A \cup B)$?
 - iii. what is $\Pr(A|B)$?
- (c) If we are given that $\Pr(A \cap B) = 0.1$, are the random variables giving rise to events A and B independent?
- (d) If we are given that $\Pr(A \cap B) = 0.1$, what is $\Pr(A|B)$?
- (e) In its monthly report, the local animal shelter states that it currently has 24 dogs and 18 cats available for adoption. Eight of the dogs and 6 cats are male. Find each of the following conditional probabilities if an animal is selected at random.
 - i. The pet is male, given that it's a cat.
 - ii. The pet is a cat, given that it's female
 - iii. The pet is female, given that it's a dog.

2. **Global warming** A 2010 Pew Research poll asked 1,306 Americans “From what you’ve read and heard, is there solid evidence that the average temperature on earth has been getting warmer over the past few decades, or not?”. The table below shows the distribution of responses by party and ideology, and the counts have been replaced with a relative frequencies.

		<i>Response</i>			Total
		Earth is warming	Not warming	Don’t Know Refuse	
<i>Party and Ideology</i>	Conservative Republican	0.11	0.20	0.02	0.33
	Mod/Lib Republican	0.06	0.06	0.01	0.13
	Mod/Cons Democrat	0.25	0.07	0.02	0.34
	Liberal Democrat	0.18	0.01	0.01	0.20
Total		0.60	0.34	0.06	1

- What is the probability that a randomly chosen respondent believes the earth is warming or is a liberal Democrat?
 - What is the probability that a randomly chosen respondent believes the earth is warming given that he is a liberal Democrat?
 - What is the probability that a randomly chosen respondent believes the earth is warming given that he is a conservative Republican?
 - Does it appear that whether or not a respondent believes the earth is warming is independent of their party and ideology? Explain your reasoning.
 - What is the probability that a randomly chosen respondent is a moderate/liberal Republican given that he does not believe that the earth is warming?
3. **It’s never lupus** Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease. The test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease.
- There is a line from the Fox television show *House*, often used after a patient tests positive for lupus: “It’s never lupus.” Do you think there is truth to this statement? Use appropriate probabilities to support your answer.