Agenda

1. Random Variables

Warmup: Viruses Sally graduated from Thims College and was immediately hired by a shop in downtown Northampton to help customers with computer-related problems. Lately, two different viruses have been bugging many customers (virus A and virus B). It is estimated that about 65% of the customers with virus problems are bothered by virus A and the remaining 35% by virus B. If the computer is infected by virus A, Sally has a 90% chance of fixing the problem. However, if the computer is infected by virus B, this chance is only 70%. Let A be the event that the computer is infected by virus A, and let F be the event that Sally fixed it. Find:

1. $\Pr(F^c|A)$
2. $\Pr(A \cap F^c)$
3. $\Pr(F)$
4. $\Pr(A|F)$

Random Variables Here is a brief guide to working with means and variances of random variables.

Quantity	Denoted	Formula	$a + bX$	$X + Y$	R
Mean	μ_X, $\mathbb{E}[X]$	$\sum_{i=1}^{n} p_i \cdot x_i$	$a + b\mu_X$	$\mu_X + \mu_Y$	`mean()`
Variance	σ_X^2, $Var[X]$	$\sum_{i=1}^{n} p_i \cdot (x_i - \mu_X)^2$	$b^2\sigma_X^2$	$\sigma_X^2 + \sigma_Y^2 + 2\rho_{XY}\sigma_X\sigma_Y$	`var()`

Table 1: Note: Here p_i is the probability that $X = x_i$. ρ_{XY} is the correlation between X and Y (Chapter 5).

Example On a roulette wheel there are 38 numbered slices: 18 red, 18 black, and two green (labelled 0 and 00). The ball lands in each slice with equal probability. A \$1 bet on red or black pays back \$1. A \$1 bet on a single number pays \$35. What is the expected value of these bets?

In-Class Problems

1. Consider the following card game with a well-shuffled deck of cards. If you draw a red card, you win nothing. If you get a spade, you win \$5. For any club, you win \$10 plus an extra \$20 for the ace of clubs.
   (a) Determine the probabilities for each amount you could win at this game. Also, find the expected winnings for a single game and the standard deviation of the winnings. Write out the full table.
   (b) What is the maximum amount you would be willing to pay to play this game? Explain.

2. The frequency table for the number of runs that the Smith softball team scored in 2012 is shown below. Estimate the expected value of runs scored by computing (by hand) the sample mean.

```
tally(~softball)

## softball
##  0  1  2  3  4  6 12 15
##  5  9  6  5  2  1  1  1
```

3. Suppose that in their final game, the team scored a season-high 16 runs. Estimate the new mean and variance (compute by hand).

4. What is the probability that a letter selected uniformly at random from the alphabet will be between B and E in alphabetical order (inclusive)? What about between E and N (inclusive)? Is this the same as the probability that the letter will be between B and N? Why or why not?

5. Suppose the mean gas bill for Nick's house is \$81.24 per month with standard deviation \$66.53, while the electric bill averages \$76.67 per month with a standard deviation of \$25.34. What is the mean and variance of Nick's combined utility bill, assuming that the gas and electric bill are independent?