You may download these slides from here:

`https://github.com/CU-BDA-2022/LectureResources`
(see the Zoom chat panel for a clickable link)

STSCI 4780/5780
Bayesian data analysis: Principles and practice

—

The big picture

Tom Loredo, CCAPS & SDS, Cornell University

2022-01-25

What is this course about?

For better or worse, statistical inference has provided an entirely new *style of reasoning*. The quiet statisticians have changed our world—not by discovering new facts or technical developments but by changing the ways we reason, experiment, and form our opinions about it.

—Ian Hacking, philosopher of science (in *Science '84*)

Two main styles of statistical reasoning

Statistics has not yet aged into a stable discipline with complete agreement on foundations. All the statisticians mentioned here [Pearson, Galton, Fisher] assumed that *the key to probability lay in the relative frequency with which different kinds of events occur*. . . . But what does that mean? Some say nothing, for probability is concerned with subjective degrees of belief, and that subjective approach only gives a *reasonable degree of certainty*. Work emanating from F. P. Ramsey in England, Bruno de Finetti in Italy, and L. J. Savage in the United States has turned such subjectivity into a serious scientific approach. Today we have vigorous, sometimes violent, disagreement on these matters, but perhaps battles about first principles are less important than the large-scale application of many competing methods.

—Ian Hacking (1984)

Bayesian and frequentist probability

Bayesian probability

Probability describes *uncertainty*, understood as a measure of how strongly evidence supports hypotheses about *the case at hand*

$\Rightarrow$ Bayesian statistics quantifies uncertainty using probability theory as a generalization of logic, providing an abstract measure of *strength of a data-based argument*. Argument strength is related to long-run performance, but not always in a simple way.

Frequentist probability

Probability describes *variability* and necessarily is *a property of an ensemble of outcomes/events* (*"frequentia"* = "crowd")

$\Rightarrow$ Frequentist statistics quantifies uncertainty via *long-run performance* of a procedure in repeated application across an ensemble

Long history of thought on uncertainty and frequency

A well-laid plan is always to my mind most profitable; even if it is thwarted later, the plan was no less good, and it is only *chance* that has baffled the design; but if fortune favor one who has planned poorly, then he has gotten only a prize of chance, and his plan was no less bad. . . .

. . . *in general* a well-laid plan leads to a happy issue.

—Herodotus, *Histories* (430 BC)

Foundations matter

Brad Efron, ASA President (2005)

> The 250-year debate between Bayesians and frequentists is unusual among philosophical arguments in actually having *important practical consequences*. . . . The physicists I talked with were really bothered by our 250 year old Bayesian-frequentist argument. Basically there's only one way of doing physics but there seems to be at least two ways to do statistics, and *they don't always give the same answers*. . . .
>
> Broadly speaking, Bayesian statistics dominated 19th Century statistical practice while the 20th Century was more frequentist. What's going to happen in the 21st Century?. . . I strongly suspect that statistics is in for a burst of new theory and methodology, and that this burst will feature a combination of Bayesian and frequentist reasoning. . . .

Roderick Little, ASA President's Address (2005)

Pragmatists might argue that good statisticians can get sensible answers under Bayes or frequentist paradigms; indeed maybe two philosophies are better than one, since they provide more tools for the statistician's toolkit. ... I am discomforted by this "inferential schizophrenia." Since *the Bayesian (B) and frequentist (F) philosophies can differ even on simple problems*, at some point decisions seem needed as to which is right. I believe our credibility as statisticians is undermined when we cannot agree on the fundamentals of our subject. ...

An assessment of strengths and weaknesses of the frequentist and Bayes systems of inference suggests that *calibrated Bayes*... captures the strengths of both approaches and provides a roadmap for future advances.

[*Calibrated Bayes* = Bayesian inference within a specified space of models + frequentist ideas for model checking; Andrew Gelman uses *"Bayesian data analysis"* similarly]

(see arXiv:1208.3035 [by TL] for discussion/references)

The weather forecaster

Joint Frequencies of
Actual & Predicted Weather

	Actual		
Prediction	Rain	Sun	
Rain	**1/4**	**1/2**	3/4
Sun	**0**	**1/4**	1/4
	1/4	3/4	

Forecaster is right only 50% of the time

Observer notes a prediction of ‘Sun’ *every day* (ignoring daily data) would be right 75% of the time, and applies for the forecaster’s job

Should the observer get the job?

	Actual	
Prediction	Rain	Sun
Rain	1/4	1/2
Sun	0	1/4

Forecaster: You'll never be in an unpredicted rain

Observer: You'll be in an unpredicted rain 1 day out of 4

Bayesian viewpoint

The value of an inference lies in its usefulness in the individual case

Long run performance is not an adequate criterion for assessing the usefulness of inferences

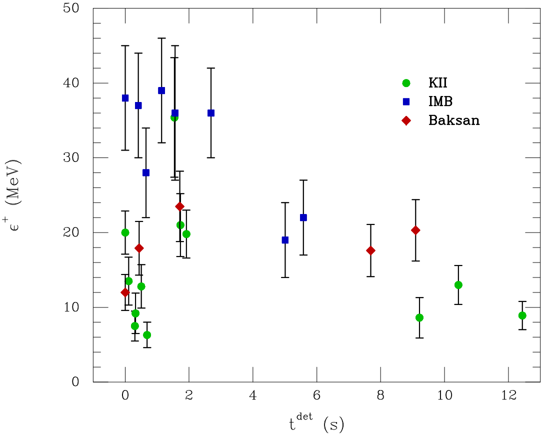
When long run performance is deemed important, it needs to be separately evaluated

Modeling SN 1987A Neutrino Data

Before 1987

Feb 1987

Arrival times and energies for SN 1987A νs



- Do we understand the basics of core collapse?
- Can we discriminate between prompt and delayed shock models?
- How does the nascent NS compare with predictions?
- What is the $\bar{\nu}_e$ rest mass?

So far a *unique event*. . .

Course GitHub organization (org)

`https://github.com/CU-BDA-2022/`

Main shared repos:

- `https://github.com/CU-BDA-2022/CourseInfo`
  Instructor & TA info, syllabus, etc.
- `https://github.com/CU-BDA-2022/LectureResources`
  Slides for lectures & supplementary info
- `https://github.com/CU-BDA-2022/LabResources`
  Jupyter notebooks and other resources for labs and assignments

All students should clone these repos.

Regularly run `git pull` in each one to get new/revised content.

Each registered student will maintain their own private repo within the course org, used to submit assignments and retrieve graded assignments.

The big picture

1. Statistics and scientific method

2. Models

3. Confidence intervals vs. credible intervals

Agenda

1. **Statistics and scientific method**
2. Models
3. Confidence intervals vs. credible intervals

Scientific method

Science is more than a body of knowledge; it is a way of thinking.
The method of science, as stodgy and grumpy as it may seem,
is far more important than the findings of science.
—Carl Sagan

Scientists *argue!*

Argument $\equiv$ Collection of statements comprising an act of reasoning from *premises* to a *conclusion*

A key goal of science: Explain or predict *quantitative measurements* (data!)

Data analysis: Constructing and appraising arguments that reason from data to interesting scientific conclusions (explanations, predictions)

The role of data

Data do not speak for themselves!

"No body of data tells us all we need to know about its own analysis."
— John Tukey, *EDA*

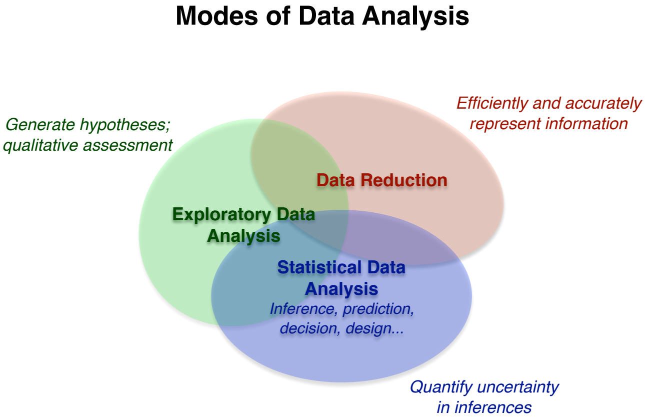
We don't just *tabulate* data, we *analyze* data

We gather data so they may speak for or against existing hypotheses, and guide the formation of new hypotheses

A key role of data in science is to be among the premises in scientific arguments

Data analysis

Building & Appraising Arguments Using Data



Inference: Learning models (populations, signal processes...) for the data generating process from observed data—just one of several interacting modes of analyzing data

Agenda

1. Statistics and scientific method
2. **Models**
3. Confidence intervals vs. credible intervals

Models

model

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Dictionary

Search for a word

mod·el

/ˈmädl/

See definitions in:

All Art Mathematics

noun

1. a three-dimensional representation of a person or thing or of a proposed structure, typically on a smaller scale than the original.
   "a model of St. Paul's Cathedral"
   Similar: replica copy representation mock-up dummy imitation
2. a system or thing used as an example to follow or imitate.
   "the law became a model for dozens of laws banning nondegradable plastic products"
   Similar: prototype stereotype archetype type version style mold

Model

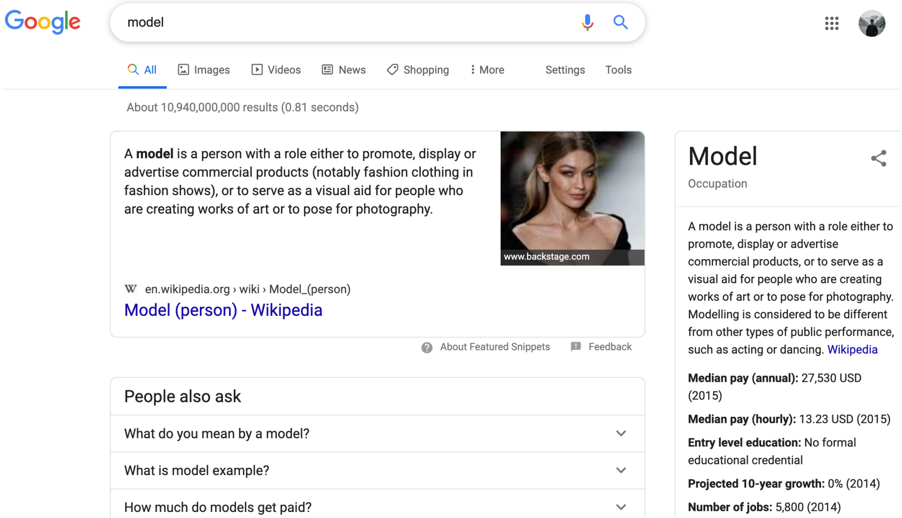
A model is an informative representation of an object, person or system. The term originally denoted the plans of a building in late 16th-century English, and derived via French and Italian ultimately from Latin modulus, a measure. Models can be divided into physical models and abstract models. Wikipedia

Feedback

See results about

Model
Occupation

Back in 2020. . .



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A **model** is a person with a role either to promote, display or advertise commercial products (notably fashion clothing in fashion shows), or to serve as a visual aid for people who are creating works of art or to pose for photography.
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en.wikipedia.org › wiki › Model_(person)
Model (person) - Wikipedia
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Model
Occupation
A model is a person with a role either to promote, display or advertise commercial products, or to serve as a visual aid for people who are creating works of art or to pose for photography. Modelling is considered to be different from other types of public performance, such as acting or dancing. Wikipedia
Median pay (annual): 27,530 USD (2015)
Median pay (hourly): 13.23 USD (2015)
Entry level education: No formal educational credential
Projected 10-year growth: 0% (2014)
Number of jobs: 5,800 (2014)

Wikipedia...

Model

From Wikipedia, the free encyclopedia

For other uses, see Model (person) and Model (disambiguation).

A **model** is an informative representation of an object, person or system. The term originally denoted the plans of a building in late 16th-century English, and derived via French and Italian ultimately from Latin *modulus*, a measure.

Models can be divided into physical models (e.g. a successful pupil as a role model for others in the school) and abstract models (e.g. mathematical expressions describing behavioural patterns). Abstract or conceptual models are central to philosophy of science,[1][2] as almost every scientific theory effectively embeds some kind of model of the physical or human sphere.

In commerce, "model" can refer to a specific design of a product as displayed in a catalogue or show room (e.g. Ford Model T), and by extension to the sold product itself.

Types of models include:

Physical model [edit]

A physical model is a physical representation of an object, such as a miniature aeroplane representing a real aeroplane.

- Model (art), a person posing for an artist, e.g. a 15th-century criminal representing the biblical Judas in Leonardo da Vinci's painting *The Last Supper*
  - Model (person), a person who serves as a template for others to copy, often in the context of advertising commercial products
- Model (product), a particular design of a product offered by its manufacturer
  - Car model, a particular design of vehicle sold by a manufacturer
- Model organism (often shortened to *model*), a non-human species that is studied to understand biological phenomena present in other related organisms, e.g. a guinea pig starved of vitamin C to study scurvy, an experiment that would be immoral to conduct on a person
- Model (mimicry), a species that is mimicked by another species

Conceptual model [edit]

A conceptual model is a theoretical representation of a system, e.g. a set of equations attempting to describe the workings of the atmosphere for the purpose of weather forecasting.

- Conceptual model (computer science), a representation of entities and their relationships
- Mathematical model, a description of a system using mathematical concepts and language
- Economic model, a theoretical construct representing economic processes
- Statistical model, a mathematical model that usually specifies the relationship between one or more random variables and other non-random variables
- Model (CGI), a mathematical representation of any surface of an object in three dimensions via specialized software

Classes of models

- **Descriptive model:** Aims to describe how something *is*
  - *Phenomenological:* "Merely descriptive," e.g., fitting an ad hoc regression model describing correlations among variables
  - *Explanatory:* Typically causal; aka *process model*
  - *Predictive:* Description designed to forecast a closely related future/past/current outcome
- **Normative model:** Aims to describe how something *should be* (by some criteria)

model science

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A scientific model is **a physical and/or mathematical and/or conceptual representation of a system of ideas, events or processes**. Scientists seek to identify and understand patterns in our world by drawing on their scientific knowledge to offer explanations that enable the patterns to be predicted. Apr 24, 2020

https://www.education.vic.gov.au › Pages › scimodels

Scientific models - Department of Education and Training ...

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Scientific modelling - Wikipedia
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Jump to **Model**-based learning in education - **Model**–based learning in education, particularly in relation to learning **science** involves students creating **models** for **scientific** concepts in order to: Gain insight of the **scientific** idea(s); Acquire deeper understanding of the subject through visualization of the **model**; Improve ...
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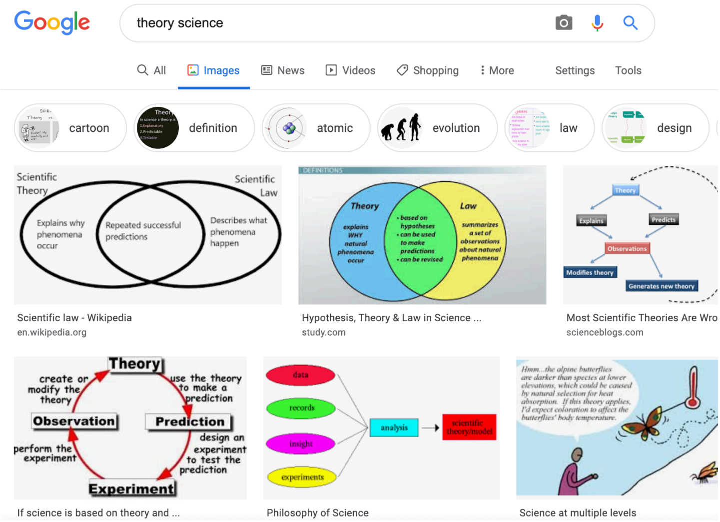

Theory (scientific): Principles/laws/desiderata guiding construction of scientific models modeling science instruction

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Scholarly articles for modeling science instruction

… framework and implications for **science instruction** - Chinn - Cited by 1853

Inquiry, **modeling**, and metacognition: Making **science** … - White - Cited by 1731

Modeling instruction: An effective model for **science** … - Jackson - Cited by 176

American Modeling Teachers Association – Fostering brilliant …

https://modelinginstruction.org/

AMTA is a professional organization of teachers, by teachers and for teachers who utilize **Modeling Instruction**(TM) in their **Science**, Technology, Engineering and Mathematics (STEM) teaching practice. Our mission is to provide professional development for teachers in the **Modeling** Method of **Instruction**, to provide ...

Synopsis of Modeling ... · Modeling Instruction ... · Middle School Storylines · News

Modeling Instruction in High School Physics

modeling.asu.edu/modeling-HS.html

Dec 14, 2015 - The program cultivates physics teachers as school experts on effective use of guided inquiry in **science** teaching, thereby providing schools and school districts with a valuable resource for broader reform. Program goals are fully aligned with National **Science** Education Standards. The **Modeling** Method ...

Modeling Instruction Program

modeling.asu.edu/

The ASU **Modeling Instruction** program and MNS degree program meet the following needs. The National **Science** Education Standards (NRC, 1996) emphasize that "coherent and integrated programs" supporting "lifelong professional development" of **science** teachers are essential for significant reform. "The conventional ...

[PDF] **Modeling Instruction: An Effective Model for Science Education - Eric**

https://files.eric.ed.gov/fulltext/EJ851867.pdf

by J Jackson - Cited by 175 - Related articles

Introduction. **Modeling Instruction** is an evolving, research-based program for high school **science** education reform that was supported by the National. **Science** Foundation (NSF) from. 989 to 2005. The name **Modeling**. **Instruction** expresses an emphasis on the construction and application of conceptual **models** of physical.

“Modeling is the main activity of scientists. . . ” — David Hestenes

Modeling

Modeling Theory aims at elucidating the roles of **models** and **modeling** in scientific knowledge and practice. It provides a foundation for a realist philosophy of science and a constructivist teaching methodology. Modeling pedagogy and its implementation in physics teaching is addressed at the Modeling Instruction Program site.

Conceptual Modeling in physics, mathematics and cognitive science

Abstract. Scientific thinking is grounded in the evolved human ability to freely create and manipulate mental models in the imagination. This modeling ability enabled early humans to navigate the natural world and cope with challenges to survival. Then it drove the design and use of tools to shape and control the environment. Spoken language facilitated the sharing of mental models in cooperative activities like hunting and in maintaining tribal memory through storytelling. The evolution of culture accelerated with the invention of written language, which enabled creation of powerful symbolic systems and tools to think with. That includes deliberate design of mathematical tools that are essential for physics and engineering . A mental model coordinated with a symbolic representation is called a conceptual model. Conceptual models provide symbolic expressions with meaning. This essay proposes a Modeling Theory of cognitive structure and process. Basic definitions, principles and conclusions are offered. Supporting evidence from the various cognitive sciences is sampled. The theory provides the foundation for a science pedagogy called Modeling Instruction, which has been widely applied with documented success and recognized most recently with an Excellence in Physics Education award from the American Physical Society.

D. Hestenes, *SemiotiX*, November 2015.
http://semioticon.com/semiotix/2015/11/.

Modeling Theory for Math and Science Education

Abstract. Mathematics has been described as the science of patterns. Natural science can be characterized as the investigation ofpatterns in nature. Central to both domains is the notion of model as a unit of coherently structured knowledge. Modeling Theory is concerned with models as basic structures in cognition as well as scientific knowledge. It maintains a sharp distinction between mental models that people think with and conceptual models that are publicly shared. This supports a view that cognition in science, math, and everyday life is basically about making and using mental models. We review and extend elements of Modeling Theory as a foundation for R&D in math and science education.

D. Hestenes, Modeling Theory for Math and Science Education, In R. Lesh, P. Galbraith, C. Hines, A. Hurford (eds.) *Modeling Students' Mathematical Competencies* (New York: Springer, 2010).

Scientific models

What is a scientific model?

- A model is a surrogate
- A model is an idealization, a partial representation
- A model is often an abstraction
- A model is provisional—a consequence of idealiziation
- Many models are quantitative—*mathematical models*
- ...

Bayesian inference

- Bayesian inference uses probability theory to *quantify the strength of data-based arguments about mathematical models* (i.e., a more abstract view than restricting PT to describe variability in repeated "random" experiments)
- Uses probability theory as a *normative model* for reasoning in the presence of uncertainty
- A different approach to *all* statistical inference problems (i.e., not just another method in the list: BLUE, linear regression, least squares/χ^2 minimization, maximum likelihood, ANOVA, survival analysis . . .)
- Focuses on *deriving consequences of modeling assumptions* rather than *devising and calibrating procedures*

Bayesian data analysis (BDA): Using Bayesian ideas across various data analysis tasks—not just inference, but also prediction, decision, design, EDA, data reduction. . .

Frequentist vs. Bayesian hypothesis appraisal

"The data D_{obs} support conclusion C . . . "

Frequentist appraisal

"C was selected with a procedure that's right 95% of the time over a set $\{D_{hyp}\}$ that includes D_{obs}."

Probabilities are properties of *procedures*, not of particular results

Bayesian appraisal

"The strength of the chain of reasoning from the model and D_{obs} to C is 0.95, on a scale where 1= certainty."

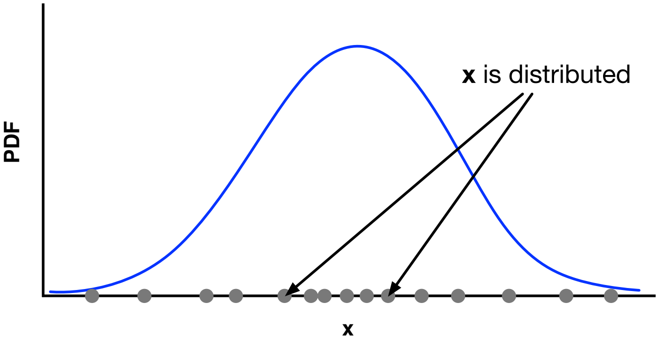
Probabilities are associated with *specific, observed data*

Long-run performance must be separately evaluated (and is often good by frequentist criteria)

Interpreting PDFs

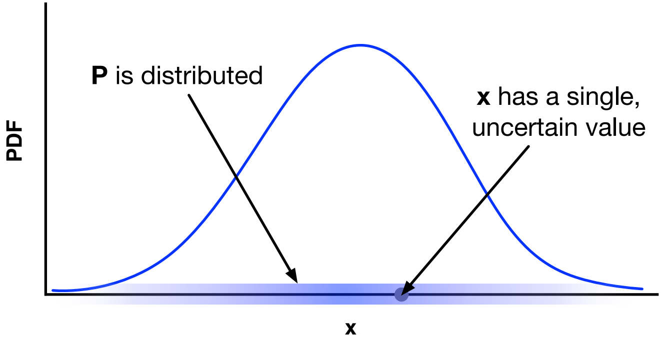
Frequentist

Probabilities are always (limiting) rates/proportions/frequencies that *quantify variability* in a sequence of trials. $p(x)$ describes how the *values of* x would be distributed among infinitely many trials:



Bayesian

Probability *quantifies uncertainty* in an inductive inference. $p(x)$ describes how *probability* is distributed over the possible values x might have taken in the single case before us:



Agenda

1. Statistics and scientific method
2. Models
3. **Confidence intervals vs. credible intervals**

A Simple (?) confidence region

Problem

Estimate the location (mean) of a Gaussian distribution from a set of samples $D = \{x_i\}$, $i = 1$ to N

Report a *point estimate*, and a *region* summarizing the uncertainty

Model

$$p(x_i|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x_i-\mu)^2}{2\sigma^2}\right]$$

$$\text{Equivalently,} \quad x_i \sim \mathcal{N}(\mu,\sigma^2)$$

Here assume σ is *known*; we are uncertain about μ

Classes of variables

- μ is the unknown we seek to estimate—the *parameter*. The *parameter space* is the space of possible values of μ—here the real line (perhaps bounded). *Hypothesis space* is a more general term.
- A particular set of N data values $D = \{x_i\}$ is a *sample*. The *sample space* is the N-dimensional space of possible samples.

Standard inferences

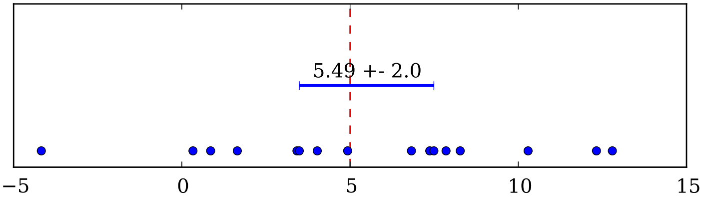
Let $\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$.

- "Standard error" (rms error) is $\sigma/\sqrt{N}$
- "1σ" interval: $\bar{x} \pm \sigma/\sqrt{N}$ with conf. level CL $= 68.3\%$
- "2σ" interval: $\bar{x} \pm 2\sigma/\sqrt{N}$ with CL $= 95.4\%$

Some simulated data

Take $\mu = 5$ and $\sigma = 4$ and $N = 16$, so $\sigma/\sqrt{N} = 1$

What is the CL associated with this interval?



Some simulated data

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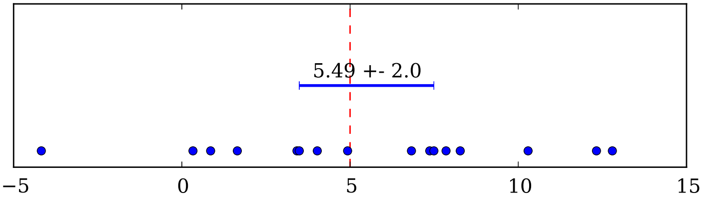
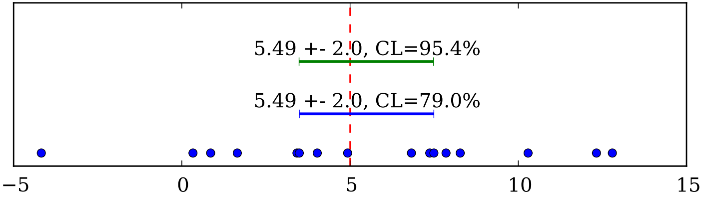


The (frequentist) confidence level for this interval is 79.0%

Two intervals

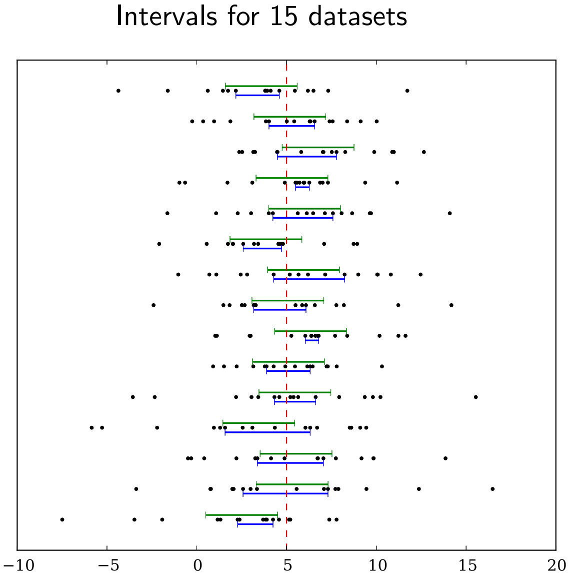


- Green interval: $\bar{x} \pm 2\sigma/\sqrt{N}$
- Blue interval: Let $x_{(k)} \equiv k$'th order statistic
  Report $[x_{(6)}, x_{(11)}]$ (i.e., leave out 5 outermost each side)

Moral

> *The confidence level is a property of the* ***procedure****, not of the particular interval reported for a given dataset*

Performance of intervals



Bayesian credible interval for a normal mean

Suppose we have a sample of $N = 5$ values x_i, with

$$x_i \sim N(\mu, 1)$$

We want to estimate μ, including some *quantification of uncertainty* in the estimate: an interval *with a probability attached*

Frequentist approaches: method of moments, BLUE, least-squares/χ^2, maximum likelihood

Bayesian approach focuses on the full *likelihood function*:

$$\begin{aligned}
\mathcal{L}(\mu) &= p(\{x_i\}|\mu) \\
&= \prod_i \frac{1}{\sigma\sqrt{2\pi}} e^{-(x_i-\mu)^2/2\sigma^2}; \qquad \sigma = 1 \\
&\propto e^{-\chi^2(\mu)/2}
\end{aligned}$$

Likelihood to probability via Bayes's theorem

Recall the likelihood, $\mathcal{L}(\mu) \equiv p(D_{\text{obs}}|\mu)$, is a probability for the observed data, but *not* for the parameter μ

Convert likelihood to a probability distribution over μ via *Bayes's theorem*:

$$\begin{aligned} p(A, B) &= p(A)p(B|A) \\ &= p(B)p(A|B) \\ \to p(A|B) &= p(A)\frac{p(B|A)}{p(B)}, \quad \text{Bayes's th.} \end{aligned}$$

$$\Rightarrow p(\mu|D_{\text{obs}}) \quad \propto \quad \pi(\mu)\mathcal{L}(\mu)$$

$p(\mu|D_{\text{obs}})$ is called the *posterior probability distribution*

Requires a prior probability density, $\pi(\mu)$, often taken to be constant over the allowed region if there is no significant information available (or sometimes constant wrt some reparameterization motivated by a symmetry in the problem)

Gaussian problem posterior distribution

For the Gaussian example, a bit of algebra ("complete the square") gives:

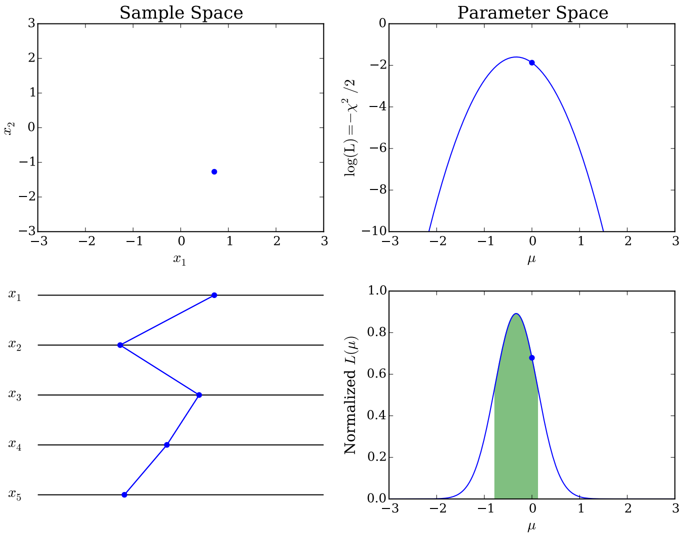
$$
\begin{aligned}
\mathcal{L}(\mu) &\propto \prod_i \exp\left[-\frac{(x_i-\mu)^2}{2\sigma^2}\right] \\
&\propto \exp\left[-\frac{(\mu-\bar{x})^2}{2(\sigma/\sqrt{N})^2}\right]
\end{aligned}
$$

The likelihood is Gaussian in μ

Flat prior $\rightarrow$ posterior density for μ is $\mathcal{N}(\bar{x}, \sigma^2/N)$

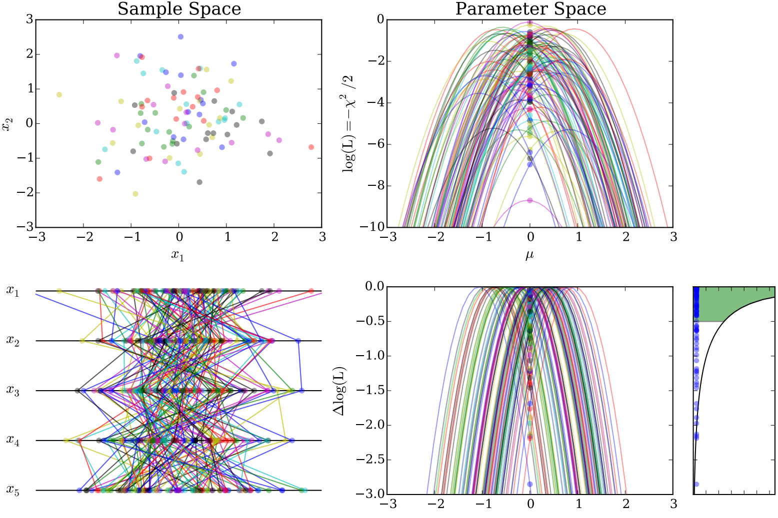
Bayesian credible region

Normalize the likelihood for the observed sample; report the region that includes 68.3% of the normalized likelihood



Confidence region via likelihood

Likelihoods for 100 simulated data sets, $\mu = 0$



When They'll Differ

Both approaches report $\mu \in [\bar{x} - \sigma/\sqrt{N}, \bar{x} + \sigma/\sqrt{N}]$, and assign 68.3% to this interval (with different meanings)

This matching is a *coincidence*!

When might results differ? ($\mathcal{F}$ = frequentist, $\mathcal{B}$ = Bayes)

- If $\mathcal{F}$ procedure doesn't use likelihood directly
- If $\mathcal{F}$ procedure properties depend on params (nonlinear models, need to find pivotal quantities)
- If likelihood shape varies strongly between datasets (conditional inference, ancillary statistics, recognizable subsets)
- If there are extra uninteresting parameters (nuisance parameters, corrected profile likelihood, conditional inference)
- If $\mathcal{B}$ uses important prior information

Also, for a different task—comparison of parametric models—the approaches are qualitatively different (significance tests & info criteria vs. Bayes factors)