SOME REMARKS ON SURFACE ALGEBRAS I AND II

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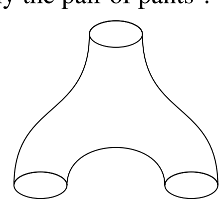
Abstract. Here we give a description of how to identify zeros of L-functions (with the prototype being ζ) with root systems of Lie groups. In particular we use the embedding $L^2(X) \hookrightarrow L^2(G)$ and $L^2(\mathbf{T}) \hookrightarrow L^2(G)$. We also give some remarks on relations to physics, in particular conformal field theory and string theory, and raise some questions about the physical and observable implications and the necessity for a new set of axioms for a yet to be defined quantum field theory.

Contents

1. Introduction 1
2. Group Representations without Groups 2
3. Remarks on L-functions and Semi-Invariants for Gentle Surface Algebras 3
4. Points Corresponding to $\overline{\mathbb{Q}}/\mathbb{Q}$ and $\mathbb{Q}^{ab}/\mathbb{Q}$ 5
5. Entanglement via Number Theory? 7
References 7

1. Introduction

Let $L^2(G)$ for $G = \mathbf{SL}_2$ be the square integrable functions over $\mathbf{SL}_2(\mathbb{A}_K)$ (over the *adeles* $\mathbb{A}_K$, or perhaps as a functor of points/algebraic group). Let $\mathfrak{g} = \mathfrak{sl}(2)$ be its Lie algebra. Let $\mathbf{T} = (K^*)^2$ be the torus in $\mathbf{SL}_2$ and $\mathfrak{t}$ its Lie algebra. Set $f : G \to \mathbb{C}$ a holomorphic function on G. Restricting $f|_{\mathbf{T}}$ to $\mathbf{T}$ give a *central character* and the *regular representations* have embedding $L^2(\mathbf{T}) \hookrightarrow L^2(G)$. Now, let $X = \mathbb{P}^1(\mathbb{C}) - \{0, 1, \infty\}$ and note that $L^2(X)$ embedds into $L^2(G)$. So, suppose f is holomorphic on $\mathbb{P}^1(\mathbb{C}) - \{0, 1, \infty\}$, which is topologically the pair of pants[1]:



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[1] For connections to conformal field theory see [Moore-Seiberg]. Pairs of pants will also be important for cobordisms mentioned later on in this note.

In terms of the roots and weights of the Gel'fand-Ponomarev algebra $K\langle x,y\rangle/\langle xy,yx\rangle$ [2] corresponding to the quiver path algebra with relations KQ/I where $I=\langle xy,yx\rangle$ we may find important information:

$$\begin{tikzcd} \bullet \arrow[loop left, "x"] \arrow[loop right, "y"] \end{tikzcd}$$

More generally, for Artin L-functions, we can do this for surface algebras as in [AS1] and using the noncommutative (universal) localization of Schofield [**?**], we can obtain the Leavitt path algebra of the surface algebras. Analytically completing the Leavitt path algebra we obtain the graph C^*-surface algebra.

Next, think of $f\in \mathcal{O}(X)$ (or $L^2(X)$, or $\mathcal{H}(X)$ (holomorphic)), and note $\mathcal{O}(X)_x\cong k[x]$ and $\mathcal{O}(X)_y\cong k[y]=k[1/x]$. We have the following pullback diagram:

$$\begin{tikzcd} k[x,y]/(xy) \arrow[r] \arrow[d] & k[y] \arrow[d] \\ k[x] \arrow[r] & k \cong_\lambda k \end{tikzcd}$$

where $\lambda\in k^*$. Since f is holomorphic on X, on any open disk $\mathbb{D}\cap\{0,1,\infty\}=\emptyset$ we can uniformly approximate f by polynomials, and on a punctured disk we can uniformly approximate by rational functions. This is important since we want to use polynomial and rational invariants from geometric invariant theory[3] of quivers to obtain approximations of such functions. Now, if

$$f=L(\chi,s)=\sum_{\mathbb{Z}}\frac{\chi(n)}{n^s}=\prod_{p\text{ prime}}\left(\frac{\chi(p)}{n^s}\right)^{-1}$$

we can obtain f as such an invariant via the constructions of [AS1] and [CCKW][4]. The zero locus of any partial sum or product approximating such an f using polynomial or rational invariants is then a toric variety since it is invariant under $\mathbf{T}$ and these zero loci converge to the zero locus of f. This construction performed for surface algebras in general shows that Artin L-functions are holomorphic according to [CN1, CN2, CN3]. Ratios of L-functions also provide interesting results. The Grothendieck group given by weights (corresponding to dimension vectors of quivers) can be realized as $g-f$ where

$$P_i^{f_i}\to P_i^{g_i}$$

$g=(g_1,...,g_k)$, $f=(f_1,...,f_k)$ gives a minimal projective presentation of $k[x,y]/(xy)\cong k\langle x,y\rangle/\langle xy,yx\rangle$-module. Traces and determinants must be done as Fredholm determinants and using trace class operators on $L^2(X)$ to get the appropriate invariant functions.

2. Group Representations without Groups

According to [GR] and references therein, we know for k an algebraically closed field of characteristic $p>0$, and G a finite group, that blocks of the group algebra $k[G]$ with cyclic defect groups are *Morita-equivalent* to algebras arising from Brauer trees. The algebras arising from Brauer trees are also known to be stably equivalent to symmetric Nakayama algebras by the same paper. Such algebras arise as the local cyclic quiver of a vertex with half-edges coming from a dessin given by

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[2]or surface algebras in general

[3]Note this is closely connected to the universal localizations of [NRS].

[4]See also [CC] and [C].

a constellation $[\sigma, \alpha, \phi]$, when the local cycle is truncated by some zero relations (for example some power of the shift operator as described in [AS1]).

Now, thinking of the quiver in the same way as in [GR], i.e. as indicating permutations of roots of unity as one might encounter from the action of a Galois group G on a cyclic field extension $\mathbb{Q}(\zeta)/\mathbb{Q}$, we may think of this (transitive) action as defining some quotient space identifying the roots. In this case, at least if $k = \mathbb{C}$, we get a noncommutative space which can be identified with a noncommutative algebra of functions $\bigoplus_{i \in O} k[G_i] \otimes_k M_{n_i}(\mathbb{C})$ as described on pg. 81 of [K]. This should give us a connection to the fundamental group(oid) corresponding to the surface algebra according to [SW]. This was briefly mentioned in [AS1] and its followup [AS2], but was never elaborated on or explained. The idea also relied heavily on [Hi][5].

In any case, this construction is somewhat different from the conventions in the literature on dessins. In particular, black vertices of a clean dessin are generally considered the preimage of 0 in a Belyi covering $f : \Sigma \to \mathbb{P}^1(\mathbb{C})$, and the half edges are understood to have white vertices that represent the preimage of 1. In this construction the role of the color label on the vertices are reversed. This of course is a matter of convention, and in more general constructions with arbitrary constellations $[g_1, g_2, ..., g_n]$ and coverings using hypermaps for example, we have a different convention, and in any case, such a change could be accounted for by a Möbius transformation.

3. Remarks on L-functions and Semi-Invariants for Gentle Surface Algebras

According to the Clay Mathematics Institute's website, *"The Riemann hypothesis has become a central problem of pure mathematics, and not just because of its fundamental consequences for the law of distribution of prime numbers. One reason is that the Riemann zeta function is not an isolated object, rather is the prototype of a general class of functions, called L-functions, associated with algebraic (automorphic representations) or arithmetical objects (arithmetic varieties); we shall refer to them as global L-functions. They are Dirichlet series with a suitable Euler product, and are expected to satisfy an appropriate functional equation and a Riemann hypothesis. The factors of the Euler product may also be considered as some kind of zeta functions of a local nature, which also should satisfy an appropriate Riemann hypothesis (the so-called Ramanujan property). The most important properties of the algebraic or arithmetical objects underlying an L-function can or should be described in terms of the location of its zeros and poles, and values at special points."*

...they go on to say...

"Not a single example of validity or failure of a Riemann hypothesis for an L-function is known up to this date. The Riemann hypothesis for $\zeta(s)$ does not seem to be any easier than for Dirichlet L-functions (except possibly for non-trivial real zeros), leading to the view that its solution may require attacking much more general problems, by means of entirely new ideas."

Now, we can say: From "Surface algebras I", and using semi-invariants of gentle surface algebras, one knows the zero locus of L-functions, since L-functions are rational invariants, which have

[5] According to the preface of Higgins' book [Hi], "the use of groupoids was confined to a small number of pioneering articles, notably by Ehresmann [12] and Mackey [57] (see references in Higgins' book), which were largely ignored by the mathematical community. Indeed groupoids were generally considered at that time not to be a subject for serious study. It was argued by several well-known mathematicians that group theory sufficed for all situations where groupoids might be used, since a connected groupoid could be reduced to a group and a set." The author goes on to state connections to Serre's notes on groups acting on trees, which are compiled into the book [S1], and which served as some inspiration for [AS1] and [AS2], but was never explicitly used in the papers.

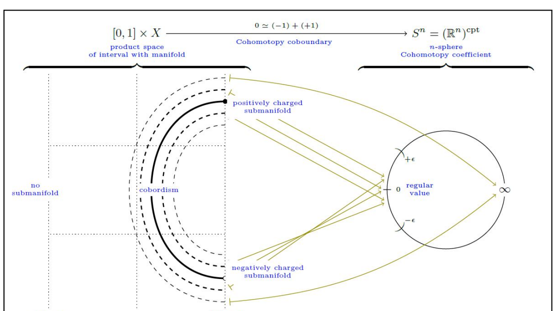
local factors expressed in terms ratios of semi-invariants (really just reciprocals of polynomial invariants given by Lebruyn-Procesi, Ringel, Schofield, Carroll-Chindris, and Caroll-Chindris-Kinser-Weyman).

This description (as in Carroll's thesis, Carroll-Chindris, and Carroll-Chindris-Kinser-Weyman) gives a full explicit description of the semi-group rings of semi-invariants as toric varieties, then from this one gets an explicit description of the rational invariants, and one has a very complete understanding of the geometry of the moduli spaces of the semi-stable regular modules (which include the semi-simple modules of interest for determining the determinants for the local factors of Artin L-functions in the (arithmetic) Langlands Program, which subsumes the Dirichlet L-functions).

This gives a significant amount of information and understanding of L-functions, and gives very explicit ways of computing them combinatorially via representation theory as in the papers mentioned. Being holomorphic and applying Hurwitz and Runge as mentioned before in order to get a uniform approximation of the global L-function via a limit of the local factors then gives the desired description of the zeros for the global L-functions.

Now, this at the very least, without anymore thinking or pondering, gives a description of the zeros of the L-functions, locally and globally. It gives explicit ways of computing "actual" approximations of the global information via the local factors by putting these things into a computer (by someone better at programming than me). The constructions at the local level are all very combinatorial and use things like the Littlewood-Richardson rule coming from tensor products of Schur functors.

This certainly addresses the above two paragraphs from CMI for Artin L-functions in general, and in my opinion is one of the features of the theory that makes it so attractive both mathematically and to physics. The fact that Artin L-functions arise in such a natural, physical way, and the implications in terms of geometric phases of quantum systems, behavior of (arithmetic) quantum chaos and phase transitions/topological order, observables (and non-observables) implied by the center of the surface algebras, and who knows what else, seems thrilling to me. It seems to require a whole new set of axioms for quantum field theory and especially new versions of conformal field theory, the AdS/CFT correspondence, and new string theories. However, I do not claim to completely understand the specific case of zeta itself. It seems as though some quotient is involved related to cobordisms though,



Brane/anti-brane pair creation & annihilation
is exhibited, under Hypothesis H, by normally framed cobordism.

In particular, if we allow zeta cycles as defined by [Consani-Connes] to represent paths on a Riemann surface which define modules (or indecomposables in the bounded derived category $\mathcal{D}^b(A)$, of a gentle surface algebra A) we get something like the following mentioned on [nLab1], *"A central insight connecting algebraic topology with mathematical physics is that a strong monoidal functor on a category of cobordisms with values in something like the category Vect (with its standard tensor product of modules) may be thought of as a formalized incarnation of what in physics is called a topological quantum field theory*

$$Z : \mathbf{Bord}_n^{\sqcup} \to \mathbf{Vect}{\otimes}$$

Here one thinks of a cobordism Σ as a piece of spacetime (or worldvolume) of dimension n, and of the $(n-1)$-dimensional manifolds that this goes between as a piece of space (or brane). A functor Z as above is then thought of as sending each $(n-1)$-dimensional space X to its space of quantum states $Z(X)$ and each spacetime Σ between X_{in} and X_{out} to a linear map"

$$Z(\Sigma) : Z(X_{in}) \to Z(X_{out}).$$

In particular, for a module (or indecomposable in the bounded derived category $\mathcal{D}^b(A)$, of a gentle surface algebra A) coming from considering a (closed) zeta cycle as closed vibrating strings on the Riemann surface given by a dessin, (or covering of a zeta cycle), we get something like an X_{in} boundary of a pair of pants, and another for X_{out}. The cobordism Σ is then the pair of pants surface between them swept out over a time coordinate, which corresponds to a linear map $Z(\Sigma)$ between modules (or indecomposable in $\mathcal{D}^b(A)$), and which are completely classified in terms of paths on the Riemann surface of the dessin of the surface algebra[6]. The above picture should then correspond to the dessin with a single black vertex, connected to two white vertices (on for $+1$ and one for -1, each connected to the black vertex 0 by a unique edge, and then taking a quotient by one of the loops at one of the white vertices (or perhaps to a single black vertex connected to a single white vertex as in the cohomotopy image above). This relates the representation theory of the corresponding path algebra to the representation theory of $\mathbf{SL}(2,\mathbb{R})$ and to the Lorentz group as mentioned on pg. 13 of [Gnedin].

4. Points Corresponding to $\overline{\mathbb{Q}}/\mathbb{Q}$ and $\mathbb{Q}^{ab}/\mathbb{Q}$

One would now like to do a few more things. Computing a resolution of the gentle surface algebras by projectives, along with its center, the Koszul complex, and Hochschild cohomology is something often of interest in noncommutative geometry. Some of this work has been initiated in [CSS].

We might also think about vertices with infinitely many half edges corresponding to the maximal abelian extension $\mathbb{Q}^{ab}/\mathbb{Q}$, one half edge for each root of unity. We might also consider points on S^1 corresponding to irrational angles θ and allowing points with half-edges corresponding to the dense subset generated by an irrational rotation algebra for such θ (which was mentioned in [AS1] and [AS2]. Allowing points corresponding to the algebraic closure $\overline{\mathbb{Q}}/\mathbb{Q}$ and the all important Galois group $\mathbf{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}$ would seem to necessitate including such points.

The question then arises, what should the algebra of matrices/operators (not necessarily observables) be over such vertices? For $\overline{\mathbb{Q}}/\mathbb{Q}$ one would expect an infinite version of the matrix algebras already mentioned in [AS1] and [AS2]. For irrational θ one would expect the algebra to at least contain some kind of noncommutative torus $UV = e^{2\pi i\theta}VU$, perhaps tensored with some ring such as Laurent series or power series, or even the coordinate rings of some varieties, as would be suggested by [BD1, BD2, BD3, BD4], or algebras of functions on some manifold. We also note that for noncommutative tori generated by the $n\times n$ t'Hooft clock and shift operators generalizes the 2×2 Pauli matrices used in quantum computing on qubits, and are very closely related to the matrix algebras

[6] see for example [CPS] and [ALP], as well as the thesis [Gnedin].

defined in [AS1] and [AS2] (which uses generalized shift operators to obtain U and function field versions of U):

$$U = \begin{pmatrix} 1 & & & & \\ & e^{2\pi i/n} & & & \\ & & \ddots & & \\ & & & e^{2\pi i(n-2)/n} & \\ & & & & e^{2\pi i(n-1)/n} \end{pmatrix}, \quad V = \begin{pmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & & 1 \\ 1 & & & & 0 \end{pmatrix}$$

So, it would make sense to use the irrational rotation algebras (i.e. noncommutative tori) as mentioned on pg. 24 §3.1 of [Marcolli]. This is defined as the universal C^*-algebra $C^*(U, V)$, generated by two unitary operators U and V satisfying

$$UV = e^{2\pi i\theta} VU.$$

It can be realized as a subalgebra of bounded operators in $\mathcal{H} = L^2(S^1)$, with the circle $S^1 = \mathbb{R}/\mathbb{Z}$. What of allowing adjoining roots for multiple values of θ though? Or of *all* θ? Would one wish to restrict attention to computable θ? Should we restrict according to Planck's constant or assumptions based on the discreteness or continuity of space? I personally advocate for no such restrictions. How might one then glue such points together to obtain a "surface"? What of the relative tensor products arising in the noncommutative algebra of functions on the surface coming from the spacial fibre products given by the surface algebra? The surface algebra would necessarily have some noncommutative/directed topology corresponding to the (infinite) quiver with relations on the surface.[7]

Assuming we have suitable operator algebras for such points with infinitely many half-edges, and some gluing giving a fibre product of operator algebras, we should then of course think of how one defines and understands strings (of operators or observables) on the surface, both closed and open, and the string theory that arises from it, as well as the cobordism theory that arises by considering the surfaces swept out by such strings (assuming a time coordinate for a (2+1)-dimensional CFT). How might we then define the module category of this surface algebra, and can we give a correspondence between strings/paths on the surface we have built and modules over this surface algebra as is the case with gentle surface algebras coming from traditional dessins? Can we obtain any insight using inverse Galois theory and the covering theory of gentle surface algebras (which is well understood)?

We know in the case of classical dessins, we have some generalization of a lattice gauge theory or gauge theory on a cellularly embedded graph, which is studied in the context of string theory, condensed matter physics, and surface codes in quantum computing (though typically only with regular lattices rather than arbitrary graphs on surfaces, and the generalization to arbitrary dessins,

[7]This was originally approached using methods developed in [Hi] when writing [AS1] and [AS2] as much of the literature on covering theory for the representation theory of quivers and linear categories is very reminiscent of this exposition. One might well use noncommutative geometry of Connes for such a "surface" implied by a "*generalized dessin*" glued together from such vertices with infinitely many half-edges. As is mentioned above in §2 *Group Representations without Groups*, referencing noncommutative algebras of functions of the form $\bigoplus_{i\in O} k[G_i]\otimes_k M_{n_i}(\mathbb{C})$ as described on pg. 81 of [K], we would want some suitable generalization for such points, or some generalizations of the matrix/operator algebras discussed in [AS1] and [AS2]. Here though, we are dealing with the much more complicated Galois groups $\mathbf{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ and/or $\mathbf{Gal}(\mathbb{Q}^{ab}/\mathbb{Q})$, so generalizing the algebras mentioned by Khalkhali, [AS1], and [AS2] does not seem straightforward. However, we must note that understanding particular directions from one point on the boundary of a disk to another (corresponding to a vertex with half-edges), or to a neighboring disk glued to it, given by arrows of the quiver on the surface, has such seemingly complicated implications about the center and spectral properties of the surface algebras, that new theory needs to be developed. There may be some results from noncommutative geometry on flows that may be useful in this endeavour.

infinite dessins, or tesselations of the upper half plane or Poincare disk by triangle groups for example, seems underrepresented in the literature). What of the case of vertices with infinitely many half-edges though? What might such a generalized "dessin" tell us about $\mathbf{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$?

5. Entanglement via Number Theory?

The implications on our understanding of entanglement in quantum systems coming from the gluing via fibre products, and the relative tensor products of (local) operator algebras of functions on this spacial fibre product of algebras seems to imply a good explanation of nonlocality and phenomena such as the EPR-paradox. The fact that entanglement should be explained by number theory to me is nothing short of absurdly interesting. Having something like the action of α in a constellation $[\sigma, \alpha, \phi]$ give a description of propagating photons in a quantum mechanical system, or of a string propagating in a 2D-CFT, and using covering theory/Hurwitz theory and the permutations σ and ϕ to further describe such propagation (or perhaps more general permutations g_i in more general constellations $[g_1, ..., g_n]$ if we are not concerned with dessins and Belyi maps, but rather moduli spaces) gives a way of understanding locality/nonlocality of entanglement and seems to indicate new measure of entanglement entropy. The relative tensor products implied by the fibre products allows us to move certain coefficients through the tensor products or neighboring local operator algebras. What does this imply physically? It seems related to geometric phase of quantum systems, and may have implications in terms of the complexity theory of these systems when viewed through the lens of quantum information theory.

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