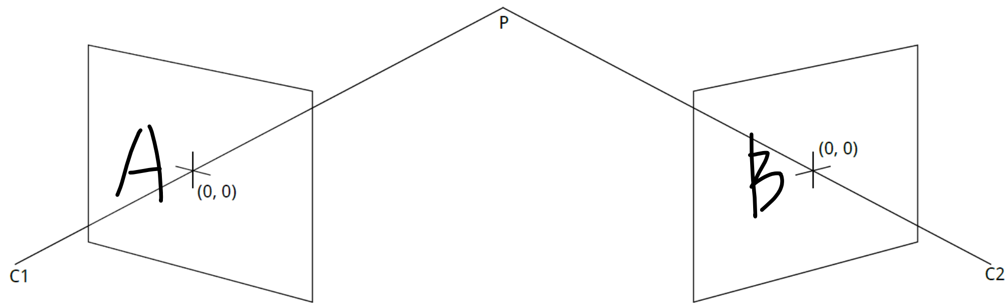
Q 1.1



Since point A and point B is the mapping of the same point P onto different image plane, there exists a fundamental matrix:

$$x_A{}^T F x_B = 0$$

As the coordinate of A can be represented as A(0,0,1) under homogeneous coordinate, and also B(0,0,1),

Then we have:

$$[0 \;\; 0 \;\; 1]\begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

Thus $F_{33} = 0$

Q 1.2

From the problem description, we know that the three vectors x, t, x' are coplanar, so there will be:

$$x'^{T}(R[t_X])x = 0$$
$$E = R[t_X]$$

Since it is pure translation so R is identity matrix: $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

And since the translation is paralleled to x axis so we can assume $t=[t_1, 0, 0]^T$

Its inner product should be:

$t_X = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$ and $E = R[t_X] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$

The epipolar line at second camera should be $l_B = Ex_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}\begin{bmatrix} x_A \\ y_A \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_A \end{bmatrix} = t_1 \begin{bmatrix} 0 \\ -1 \\ y_A \end{bmatrix}$

So the function expression for epipolar line2 only has parameter for y and z, so it is paralleled to x axis.

For epipolar line at first camera, $l_A = E^T x_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_1 \\ 0 & -t_1 & 0 \end{bmatrix}\begin{bmatrix} x_B \\ y_B \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ t_1 \\ -t_1 y_B \end{bmatrix} = t_1 \begin{bmatrix} 0 \\ 1 \\ -y_B \end{bmatrix}$

The function expression for epipolar line1 also only has parameter for y and z, so it is paralleled to x axis.

Thus epipolar lines in the two cameras are paralleled to the x-axis.

Q 1.3

For general purpose: $\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = K(R_i \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_i)$

At time1: $\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K(R_1 \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_1)$

At time2: $\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K(R_2 \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_2)$, thus $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = K^{-1}(\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2)R_2^{-1}$

Put them together: $\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K(R_1 \cdot K^{-1}(\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2)R_2^{-1} + t_1) = KR_1R_2^{-1}K^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - KR_1R_2^{-1}K^{-1}t_2 + Kt_1$

Thus $\begin{aligned} R_{rel} &= KR_1R_2^{-1}K^{-1} \\ t_{rel} &= -KR_1R_2^{-1}K^{-1}t_2 + Kt_1 \end{aligned}$

Essential matrix $E = R_{rel}[t_{rel\,X}] = KR_1R_2^{-1}K^{-1}[-KR_1R_2^{-1}K^{-1}t_2 + Kt_1]_X$

Fundamental matrix $F = (K^{-1})^T EK^{-1} = (K^{-1})^T R_{rel}[t_{rel\,X}]K^{-1}$

Q 1.4

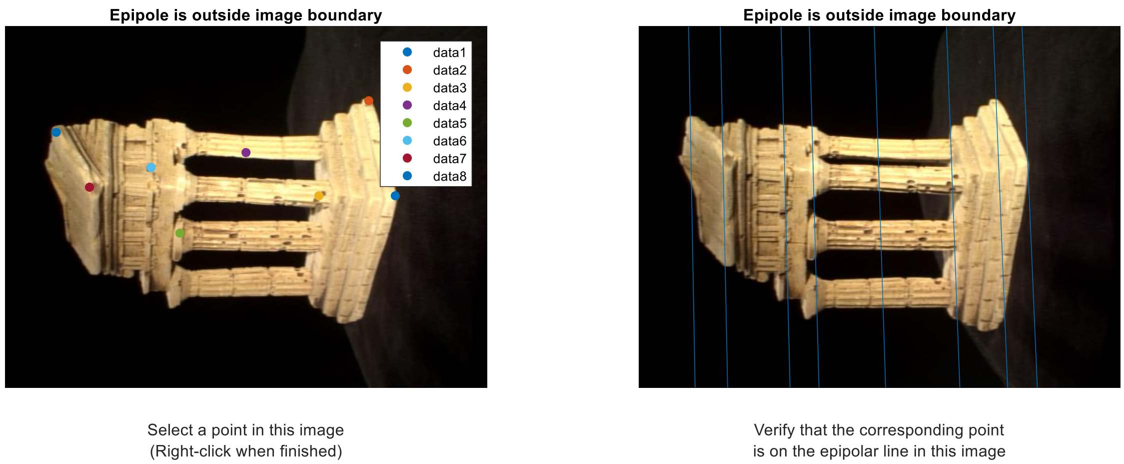
Since the object is flat, and all points on the object are of equal distance to the mirror, there will only be translation between the two image planes.

So $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$

As it has been discussed in question $E = R[t_X] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix}$

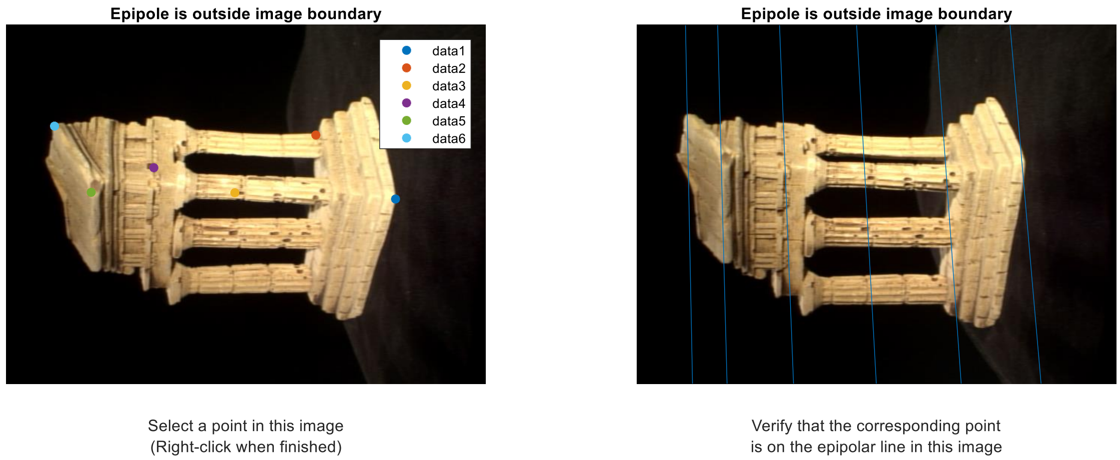
We can see the E is a skew-symmetric matrix, and $F = (K^{-1})^T EK^{-1}$, so F will also be a skew-symmetric matrix.

Q 2.1

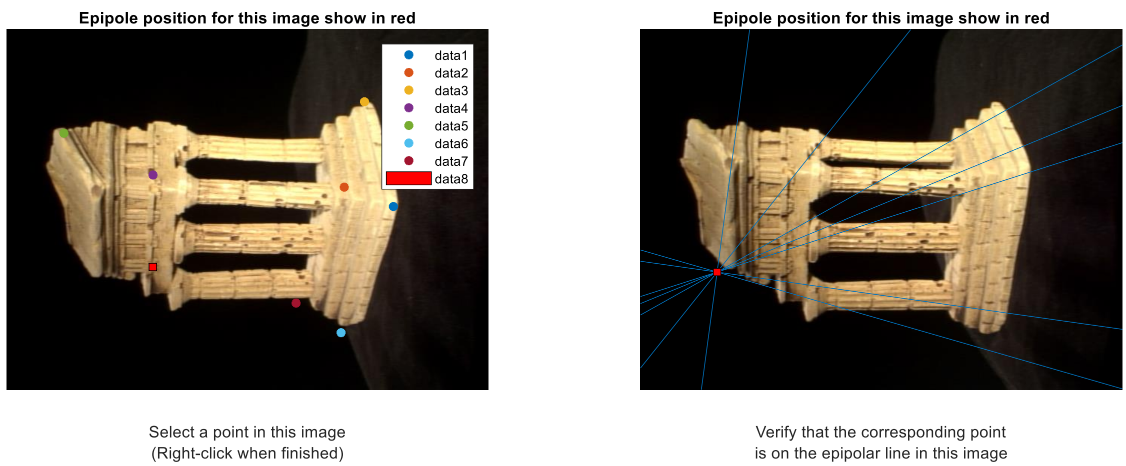


$$F = \begin{bmatrix} -0.0000 & -0.0000 & 0.0011 \\ -0.0000 & 0.0000 & -0.0000 \\ -0.0011 & 0.0000 & -0.0042 \end{bmatrix}$$

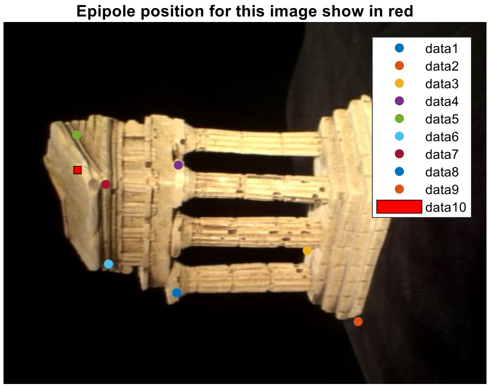
Q 2.2



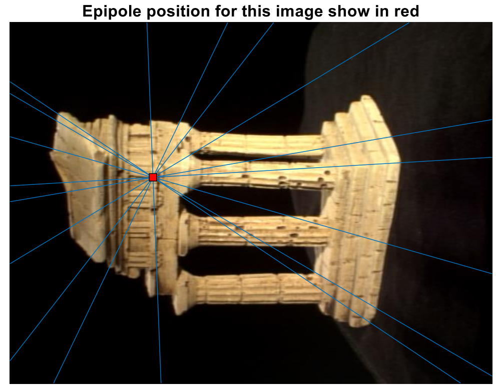
$$F1 = \begin{bmatrix} 0.0000 & -0.0000 & 0.0009 \\ -0.0000 & 0.0000 & -0.0000 \\ -0.0009 & 0.0001 & -0.0099 \end{bmatrix}$$



$$F2 = \begin{bmatrix} -0.0000 & 0.0000 & -0.0013 \\ -0.0000 & 0.0000 & -0.0002 \\ 0.0016 & -0.0016 & 0.2159 \end{bmatrix}$$



Select a point in this image
(Right-click when finished)



Verify that the corresponding point
is on the epipolar line in this image

$$F3 = \begin{bmatrix} -0.0000 & 0.0000 & -0.0005 \\ -0.0000 & 0.0000 & -0.0002 \\ 0.0007 & -0.0011 & 0.1385 \end{bmatrix}$$

Q 3.1

$$E = \begin{bmatrix} -0.0030 & -0.3035 & 1.6603 \\ -0.1374 & 0.0083 & -0.0512 \\ -1.6651 & -0.0125 & -0.0013 \end{bmatrix}$$

Q 3.2

$$A = \begin{bmatrix} \text{x1*C1(3,:)-C1(1,:)} \\ \text{y1*C1(3,:)-C1(2,:)} \\ \text{x2*C2(3,:)-C2(1,:)} \\ \text{y2*C2(3,:)-C2(2,:)} \end{bmatrix}$$

(x1, y1) are the points in im1, and (x2,y2) are the points in im2.

Here's the expression in slides, which is the same.

Concatenate the 2D points from both images

$$\begin{bmatrix} y\boldsymbol{p}_3^\top - \boldsymbol{p}_2^\top \\ \boldsymbol{p}_1^\top - x\boldsymbol{p}_3^\top \\ y'\boldsymbol{p}_3'^\top - \boldsymbol{p}_2'^\top \\ \boldsymbol{p}_1'^\top - x'\boldsymbol{p}_3'^\top \end{bmatrix} \boldsymbol{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A}\boldsymbol{X} = \mathbf{0}$$

Q 3.3

Squared error:

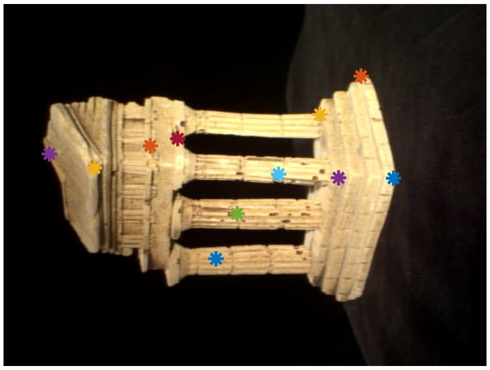
Error1 = 93.1082064310258

Error2 = 93.1082161135894

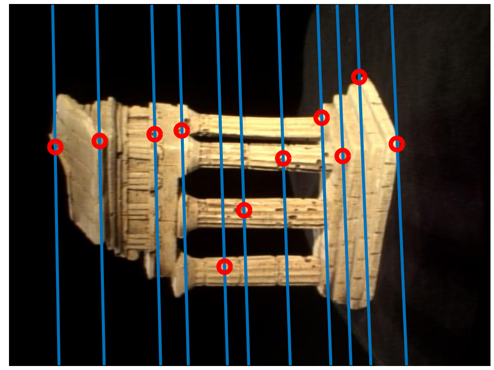
Picked C2 matrix:

$$C2 = 1.0\mathrm{e}{+}03 * \begin{bmatrix} -1.5200 & 0.0648 & -0.2974 & -0.0218 \\ 0.0354 & 1.5424 & -0.0957 & -1.5055 \\ -0.0000 & 0.0001 & -0.0010 & 0.0001 \end{bmatrix}$$

Q 4.1

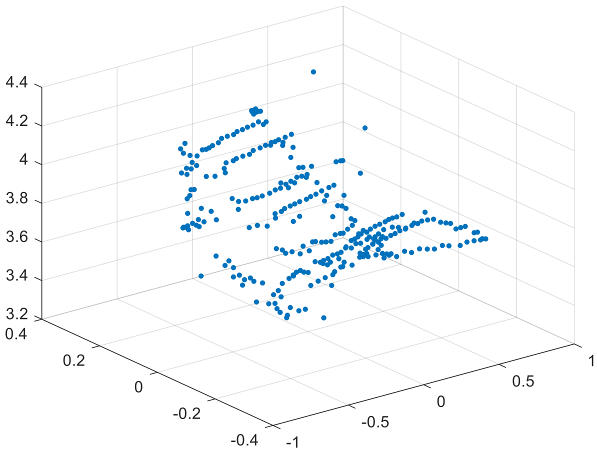


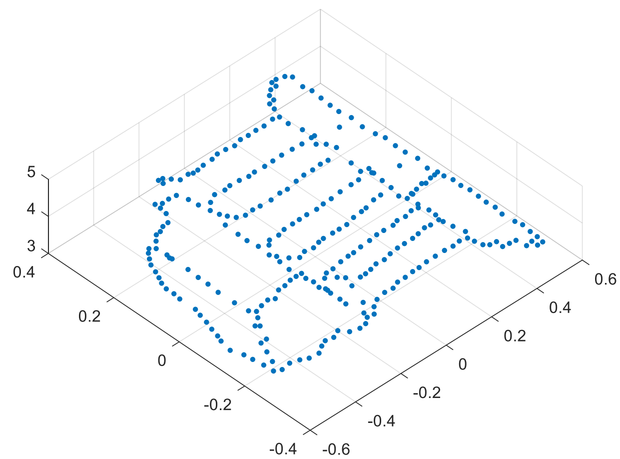
Select a point in this image
(Right-click when finished)

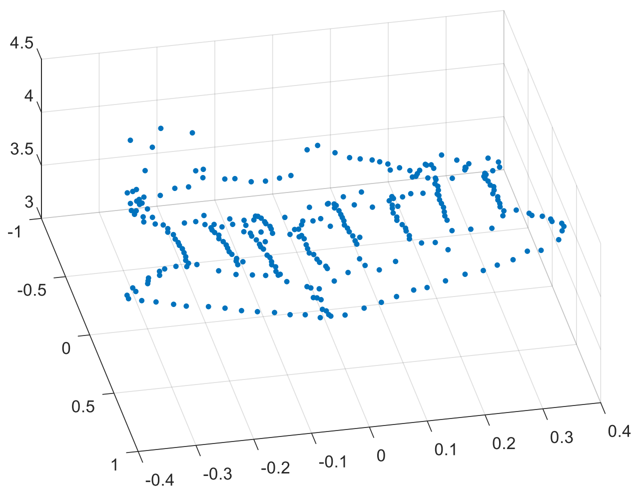


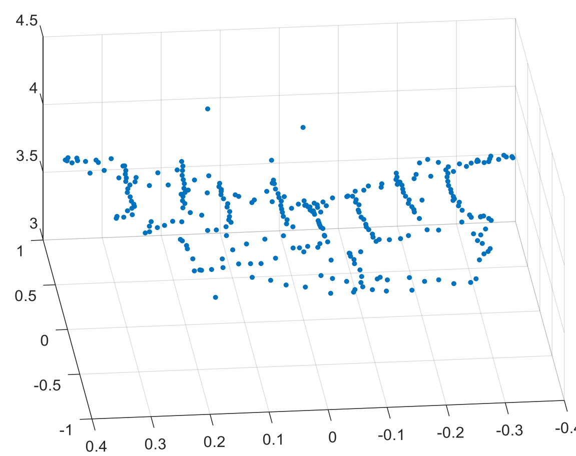
Verify that the corresponding point
is on the epipolar line in this image

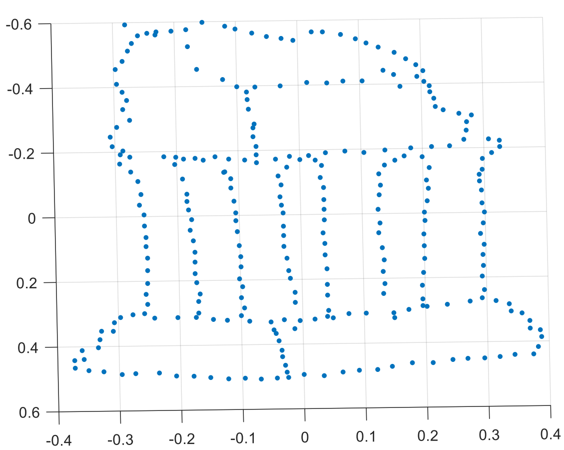
Q 4.2

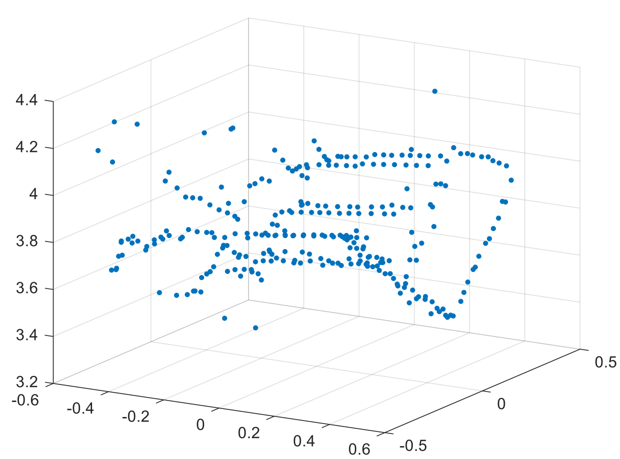

4.4
4.2
4
3.8
3.6
3.4
3.2
0.4
0.2
0
-0.2
-0.4
-1
-0.5
0
0.5
1


5
4
3
0.4
0.2
0
-0.2
-0.4
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


4.5
4
3.5
3
-1
-0.5
0
0.5
1
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4

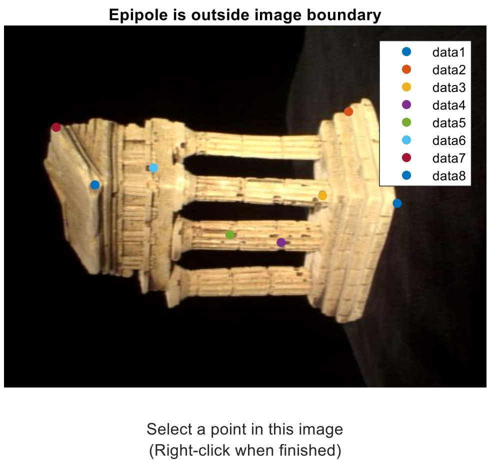

4.5
4
3.5
3
1
0.5
0
-0.5
-1
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4

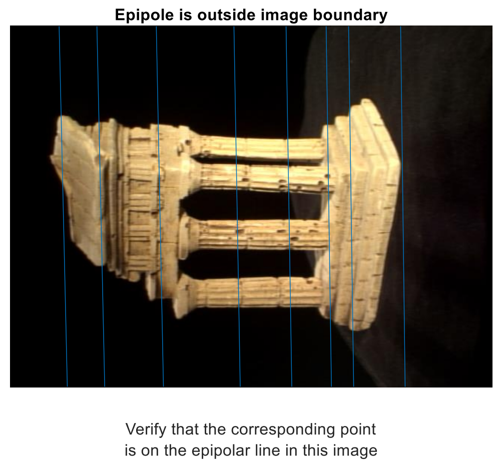

-0.6
-0.4
-0.2
0
0.2
0.4
0.6
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4


4.4
4.2
4
3.8
3.6
3.4
3.2
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
-0.5
0
0.5

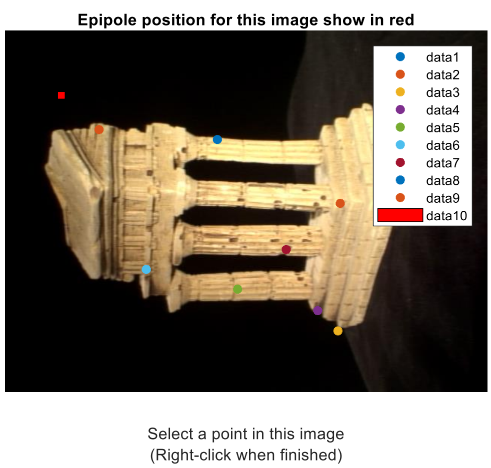
Q 5.1

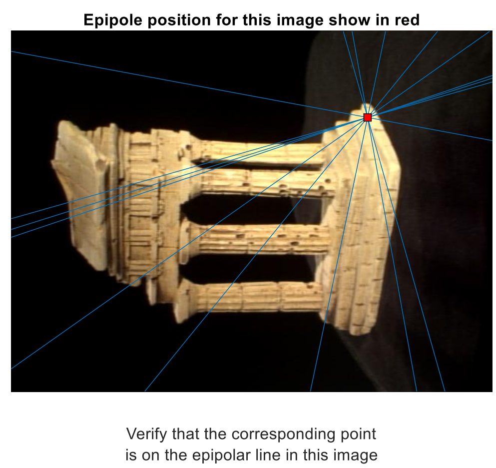
Using RANSAC, eight-points :





Only using eight-points:





Explain:

For RANSAC, I used similar strategy as used in last assignment.

I set a relatively large iteration number which is 2000 to ensure it can find the best F.

In each iteration, 8 pair of points were randomly selected to compute F. Then eppipolar lines can be calculated, we get different a, b and c for all 140 matching points, The we can test them in im2 using a*x2 +b*y2 + c = 0.

The error matrix will be a 140*1 matrix with the value of a*x2 +b*y2 + c

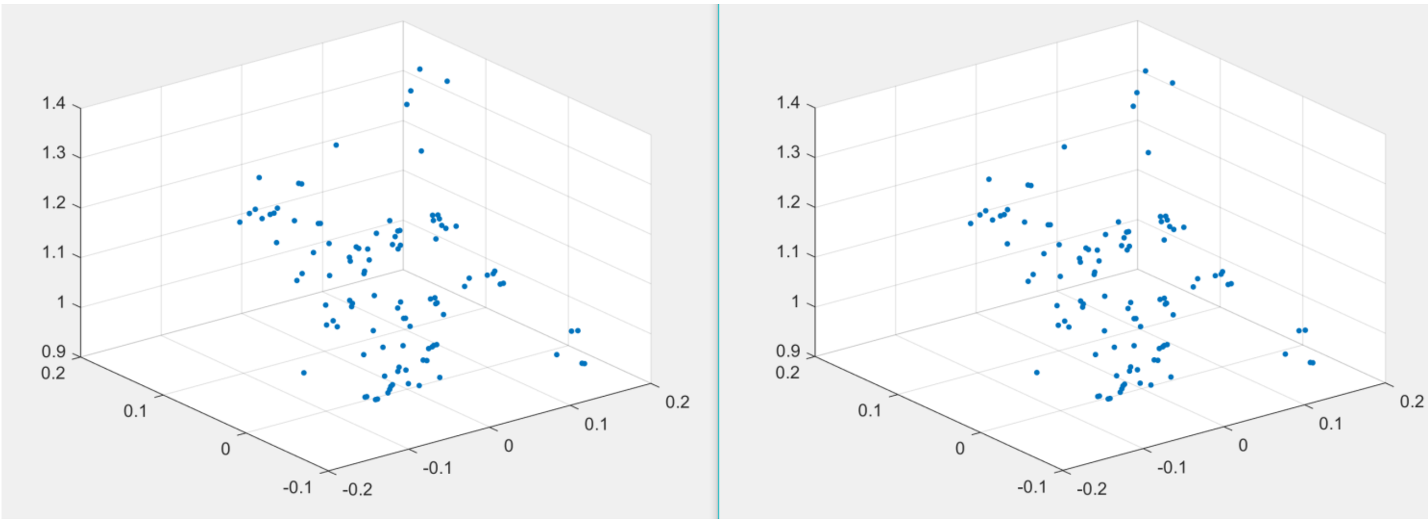
If the point is perfectly matched, a*x2 +b*y2 + c should be zero, also zero should be zero.

I set this error=0.008, in this way there is 106 inliers, which is about 75% of all data.

Q 5.3

Sum of reprojection error before optimization: 2.2867e+04

Sum of reprojection error after optimization: 7.1167e+03



Left: before optimizing

Right: after optimizing

Q 6.1

I used a software named 3DF Zephyr, and I also download the full data set of “temple” which is in the link of the Youtube video.

Here is the video: https://www.youtube.com/watch?v=3FP1bw96WCQ (my youtube)

Here is the outcome: