An Assessment of the Effectiveness of Daily Delta Hedging w/ Black-Scholes for Writers of European Option Contracts

Δ-Hedging

Amer Bellioum

June 2025

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1 The Concept of Hedging

1.1 The Necessity of Hedging Risk

Market-making financial institutions that sell option contracts seek to neutralise their exposure to market movements and instead earn structural revenues (e.g., bid-ask spreads). As such, they employ hedging strategies in an attempt to remain market-neutral.

Consider the situation faced by a financial institution at the expiration of a European call option contract it has sold. We initially assume that the institution has taken a *naked position* — meaning it has not purchased any of the underlying asset to hedge the option exposure. Two possible scenarios can arise at maturity:

1. **In-the-Money:** $S_T > K$
   The option is exercised. Since the institution is not holding the underlying asset, it must purchase the asset at the prevailing market price S_T and deliver it to the option buyer at the lower strike price K. This results in a loss of $S_T - K$ per option contract.

2. **Out-of-the-Money:** $S_T \leq K$
   The option expires worthless and is not exercised. In this case, the institution retains the premium V_0 received at initiation as pure profit, having taken no further action or incurred additional cost.

This naked position exposes the financial institution to **unbounded losses** if the option ends up deep in the money, as there is no limit to how high S_T can rise. Conversely, if the option expires out of the money, the institution retains the full premium. Thus, the seller is taking on significant *directional risk*, effectively betting that the underlying asset's price will remain below the strike price K.

One way the institution can mitigate the risk of unbounded losses is by purchasing the underlying asset at the time the option contract is initiated, at price S_0. This is known as taking a *covered position.* For simplicity, we assume the strike price is set at-the-money, such that $K = S_0$. Under this setup, two possible scenarios can occur at maturity:

1. **In-the-Money:** $S_T > K$
   The option is exercised. Since the institution already holds the underlying asset (purchased at price K) and must deliver it at the strike price K, there is no loss from the asset transaction itself. The institution retains the premium received and avoids any loss associated with acquiring the asset at a higher market price. When accounting for the time value of money, this position may even yield a modest real gain.

2. **Out-of-the-Money:** $S_T \leq K$
   The option expires worthless. However, the institution continues to hold the underlying asset, which is now worth less than the purchase price K. This results in a mark-to-market loss on the asset, partially offset by the premium received. While the downside risk remains, it is bounded and significantly less severe than in the case of a naked position.

While a covered call eliminates the unbounded losses associated with a naked call, it still carries significant directional risk. Under the assumption that the asset is purchased at the strike price $S_0 = K$, the institution remains exposed to the full downside if the asset price declines below S_0. The position is therefore still betting on the stock price rising. As such, adopting both a naked position and a covered call are both effectively wagers adopted by the financial institution on the future value of the asset and hence the option.

As stated prior, a dealer that writes options aims to earn structural revenues—bid–ask spreads, order-flow rebates, or the volatility risk premium. The purpose **is not** to speculate / bet on the direction of the market. As such, they want to **eliminate their exposure to changes in the value of the option** and secure profits purely made from market making.

1.2 What Is A Portfolio's Value Sensitive To?

The value of a financial option, denoted V, depends on several key market variables. These include:

1. The price of the underlying asset, S
2. The time to maturity, t
3. The volatility of the underlying asset, σ
4. The risk-free interest rate, r

Mathematically, this can be expressed as a multivariate function:

$$V = f(S, t, \sigma, r) \tag{1}$$

Assuming the underlying asset price, S, follows a stochastic process such as geometric Brownian motion, the total differential in the option's value is given by Itô's Lemma:

$$dV = \frac{\partial V}{\partial S}\,dS + \frac{\partial V}{\partial t}\,dt + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\,d\langle S\rangle + \frac{\partial V}{\partial \sigma}\,d\sigma + \frac{\partial V}{\partial r}\,dr \tag{2}$$

The partial derivatives of the option price with respect to key market variables are known as the **Greeks**. They quantify the option's sensitivity to the key parameters and are as follows:

- $\Delta = \frac{\partial V}{\partial S}$ — Sensitivity of the option value to changes in the underlying asset price.
- $\Gamma = \frac{\partial^2 V}{\partial S^2}$ — Sensitivity of delta to changes in the underlying asset price.
- $\Theta = \frac{\partial V}{\partial t}$ — Sensitivity of the option value to the passage of time.
- $\nu = \frac{\partial V}{\partial \sigma}$ — Sensitivity of the option value to changes in volatility.
- $\rho = \frac{\partial V}{\partial r}$ — Sensitivity of the option value to changes in the risk-free interest rate.

In theory, maintaining a constant portfolio value requires offsetting the option's sensitivities to all underlying market variables. In practice, however, only Delta (Δ) is typically hedged, which indirectly mitigates Gamma (Γ) exposure if done continuously (more on this later). In some cases, Vega (ν) may also be hedged to reduce sensitivity to changes in volatility. More commonly, though, volatility risk is priced into the option through a **volatility risk premium**—an additional cost that compensates the writer for bearing volatility exposure. The impact of the remaining sensitivities is generally far more limited and, as such, they are rarely hedged in practice.

Financial institutions typically only aspire to control the change in portfolio value with respect to the underlying asset price. That is, they seek to neutralise the portfolio's exposure to movements in S by setting the partial derivatives of Π with respect to S to zero. This implies:

$$\frac{\partial \Pi}{\partial S} = 0 \quad \text{(Delta neutrality)} \tag{3}$$

2 Delta Hedging

2.1 What Is Delta Hedging?

Delta hedging is a more sophisticated hedging strategy aimed at minimising the directional exposure faced by an option writer (theoretically eliminating it completely). Instead of choosing to hold the amount of the asset liable in the event of a payoff (as in a covered call — which would produce short directional risk), the amount of the asset held is mathematically geared to hold the change in the portfolio value with directional asset movements constant.

Let us consider that we label the amount of the asset held as Δ. The portfolio held is therefore the amount of the asset held, Δ, multiplied by the asset price (which represents our holdings), subtracted by the obligation to pay the option (which has value V). Letting Π represent the value of the portfolio, this is defined in Equation 4.

$$\Pi = \Delta \cdot S - V \tag{4}$$

For no directional exposure of our portfolio, the change in the portfolio value, Π, with respect to the underlying asset price, S, must equal zero. This condition is expressed in Equation 5:

$$\frac{\partial \Pi}{\partial S} = \Delta - \frac{\partial V}{\partial S} = 0 \tag{5}$$

Hence, the amount of stock to be held for a theoretically directionally neutral portfolio is given by Equation 6.

$$\Delta = \frac{\partial V}{\partial S} \tag{6}$$

By holding a quantity of the underlying asset that offsets changes in the option's value resulting from movements in the asset price, a direction-neutral position is effectively maintained. The resulting delta-hedged portfolio is thus expressed in Equation 7:

$$\Pi = \frac{\partial V}{\partial S} \cdot S - V \tag{7}$$

As the asset price and time evolve, both the value of the option, V, and its sensitivity to the asset price, $\frac{\partial V}{\partial S}$, are subject to change. To maintain a direction-neutral portfolio, the hedge ratio Δ must be continuously updated to match $\frac{\partial V}{\partial S}$. This is known as *dynamic hedging*, and in theory, continuous dynamic hedging is required to preserve directional neutrality at all times.

2.2 Limitations of Delta Hedging

2.2.1 Exposure to Volatility, Interest Rates, & Time

Delta hedging conditions eliminate the portfolio's sensitivity to small changes in the asset price. However, even the adoption of a theoretically continuous delta-hedging strategy still leaves the portfolio's value sensitive to *other factors*, such as time decay, volatility, and interest rates - as seen in Equation 8.

$$d\Pi \approx \frac{\partial \Pi}{\partial t}\, dt + \frac{\partial \Pi}{\partial \sigma}\, d\sigma + \frac{\partial \Pi}{\partial r}\, dr \tag{8}$$

For instance, changes in the volatility of the underlying can alter the portfolio's value. Without hedging against this risk, the option writer remains exposed to volatility fluctuations and potential losses. As noted earlier, volatility typically has the most significant impact among the unhedged factors, and this risk is often compensated for through a volatility risk premium.

2.2.2 Discretisation Error

Another source of error arises when one considers that, in practice, delta hedging cannot be performed continuously; instead, it is implemented at *discrete time intervals*. This means that the hedge ratio, or delta, is only updated at specific time points rather than continuously tracking the underlying asset. As a result, the failure to hedge against changes in Δ in the time interval between these updates may render the hedge exposed.

The portfolio may no longer remain directionally neutral during the interval. This discrepancy introduces what is known as *hedging error*. The resulting change in the portfolio's value between two discrete time points, t_i and t_{i+1}, *when we only consider the change due to asset price* is given by Equation 9.

$$\Pi_{i+1} - \Pi_i = \Delta_i(S_{i+1} - S_i) - (V_{i+1} - V_i) = \left.\frac{\partial V}{\partial S}\right|_i (S_{i+1} - S_i) - (V_{i+1} - V_i) \tag{9}$$

where:

- $\Delta_i = \left.\frac{\partial V}{\partial S}\right|_{t_i}$ is the option delta at time t_i – representing the amount of the asset bought.
- S_i and S_{i+1} are the underlying asset prices at t_i and t_{i+1}, respectively.
- V_i and V_{i+1} are the option values at t_i and t_{i+1}, respectively.

In order to assess the error introduced by the discrete nature of delta hedging, we consider the Taylor expansion of the option value V_{i+1} about V_i, expanding with respect to the underlying asset price S at time step i:

$$V_{i+1} = V_i + \left.\frac{\partial V}{\partial S}\right|_i (S_{i+1} - S_i) + \frac{1}{2} \left.\frac{\partial^2 V}{\partial S^2}\right|_i (S_{i+1} - S_i)^2 + \mathcal{O}((S_{i+1} - S_i)^3) \tag{10}$$

$$V_{i+1} - V_i = \left.\frac{\partial V}{\partial S}\right|_i (S_{i+1} - S_i) + \frac{1}{2} \left.\frac{\partial^2 V}{\partial S^2}\right|_i (S_{i+1} - S_i)^2 + \mathcal{O}((S_{i+1} - S_i)^3) \tag{11}$$

Here, all derivatives are evaluated at time t_i, i.e., at the asset price S_i. Upon substitution of these values into Equation 9, we prove that the following error occurs:

$$\Pi_{i+1} - \Pi_i = -\frac{1}{2} \left.\frac{\partial^2 V}{\partial S^2}\right|_i (S_{i+1} - S_i)^2 + \mathcal{O}((S_{i+1} - S_i)^3 \tag{12}$$

The hedging error is therefore a function of the option's convexity, $\frac{\partial^2 V}{\partial S^2}$. This is an intuitive result—when the sensitivity of delta is high (i.e., when Γ is large), delta is more prone to large changes. As a result, the hedge balanced at the previous Δ becomes increasingly inaccurate between rebalancing points when the deviation in Δ is large, leading to greater exposure and amplified hedging error.

2.2.3 Model Imperfections

Also one of the most important sources of error is the fact that the model used to value the option is not perfectly representative of reality. For context as to why this matters:

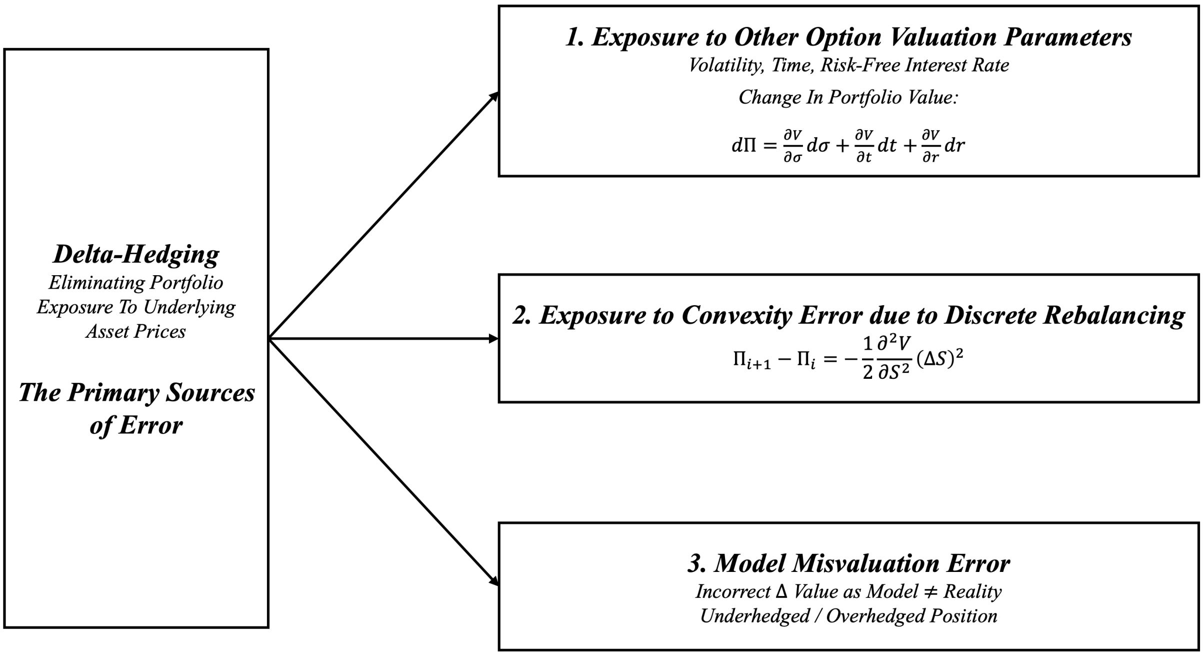
The PnL of an option writer that employs delta hedging to remove directional exposure can be given by:

$$\text{PnL} = \underbrace{V_0}_{\text{Option premium}} + \underbrace{\Delta_{T-1} S_T}_{\text{Final hedge value}} - \underbrace{\max(S_T - K,\, 0)}_{\text{Option payoff}} - \underbrace{\left(\Delta_0 S_0 + \sum_{t=1}^{T-1} (\Delta_t - \Delta_{t-1}) S_t\right)}_{\text{Cost of setting up and rebalancing hedge}} \tag{13}$$

V_0 and Δ_t *are determined through the application of a theoretical option pricing model.* In contrast, the realised payoff, hedging costs, and hedge values are governed by real market behaviour. In a world where the model perfectly captures the underlying asset dynamics—assuming constant volatility, and continuous rebalancing—the P&L would tend to zero. However, the practical dynamics of financial markets can cause models to diverge significantly from reality. These discrepancies give rise to a non-zero P&L.

Theoretical option pricing models can misprice the option for a number of reasons. Misestimating the asset's volatility and failing to capture how that volatility changes over time are typically some of the biggest sources of deviation—but they're not the only ones. If the option is mispriced, then the Δ used to hedge it is also incorrect, meaning the hedge doesn't fully cancel out the directional risk. As a result, model error ends up leaving you partially exposed to movements in the underlying.

2.2.4 Visualisation of Primary Error Sources



3 Black-Scholes Option Valuation Model

The Black–Scholes model provides closed-form expressions for the value and sensitivities (Greeks) options. For both European **call** and **put** options, the following equations apply:

$$C(S,t) = S\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) \tag{14}$$

$$P(S,t) = Ke^{-r(T-t)}\Phi(-d_2) - S\Phi(-d_1) \tag{15}$$

$$\Delta_{\text{call}} = \frac{\partial C}{\partial S} = \Phi(d_1) \tag{16}$$

$$\Delta_{\text{put}} = \frac{\partial P}{\partial S} = \Phi(d_1) - 1 = -\Phi(-d_1) \tag{17}$$

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{\partial^2 P}{\partial S^2} = \frac{\phi(d_1)}{S\sigma\sqrt{T-t}} \tag{18}$$

where:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Here, $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard normal distribution, and $\phi(\cdot)$ is the standard normal probability density function (PDF).

4 Quantifying the Effectiveness of Delta Hedging

As discussed, delta-hedging is a strategy designed to neutralise directional exposure to movements in the underlying asset. However, in practice, the portfolio remains exposed to other sources of risk—most notably volatility fluctuations, model mispricing, and discrete rebalancing errors. This project aims to **quantify** the extent to which the profit and loss (PnL) of a delta-hedged option writer deviates from the ideal scenario (i.e., zero PnL) across different levels of *moneyness* and *time to maturity*. It then seeks to diagnose the underlying drivers of these deviations.

By doing so, the project enables stakeholders to identify the option contract types where delta-hedging is less effective and to understand the root causes of the hedging error. This serves to inform mitigation strategies and option contract types where higher premiums are to be employed to offset the reduced hedging effectiveness. For example, if the ineffectiveness is driven by high gamma, this would indicate a need for more frequent rebalancing to improve hedge accuracy.

4.1 Delta-Hedging Simulations

The effectiveness of the delta-hedging strategy is achieved by back-testing the strategy on historical data for different ranges of moneyness and maturity times. This is done for both option valuation models such that an appreciation of the differences in applying both models in delta-hedging can be seen. A visualisation of the process can be observed in Figure 1.

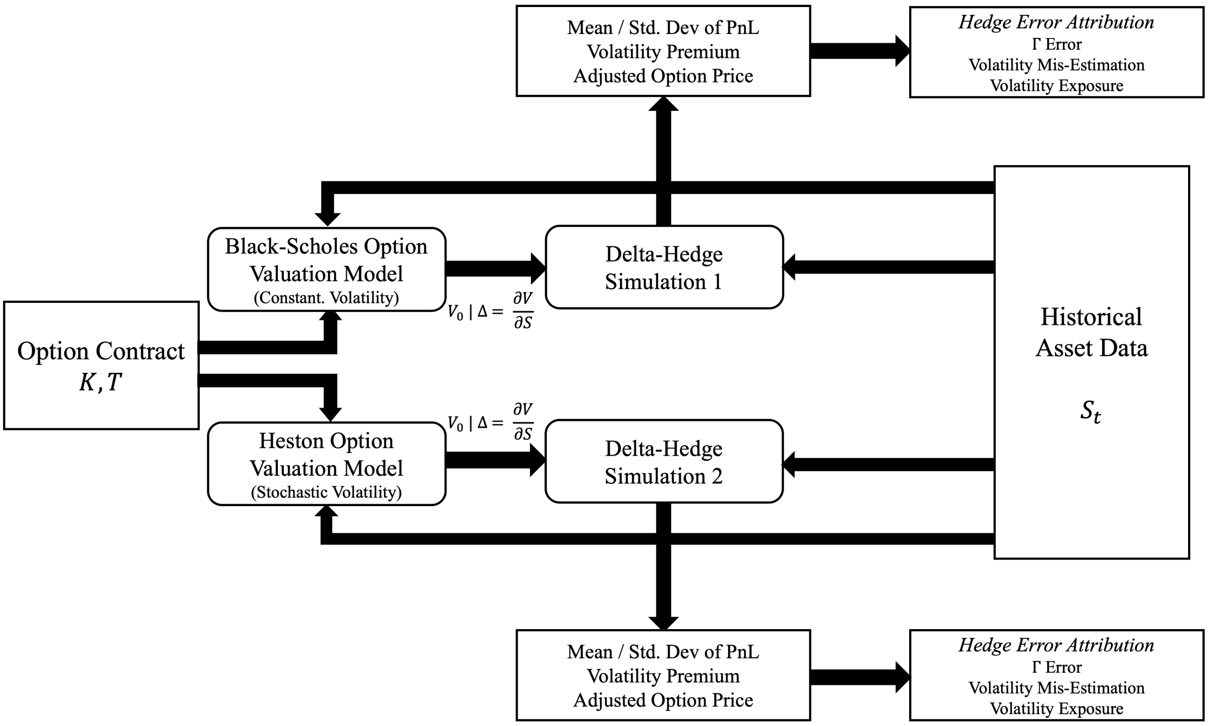


Figure 1: Flowchart of Methodology

4.1.1 Output 1 - PnL vs. Time

Before analysing the effectiveness of delta hedging in full, we first examine how the PnL evolves over time for a single, fixed option contract. This establishes a baseline for the typical variation in hedging outcomes over time and allows us to identify any specific periods where large deviations occur.

Performing this analysis across all contract combinations would be impractical. As such, we consider the basic circumstance of a short-dated, at-the-money (ATM) option to independently see how the delta-hedge performs across different timeframes and hence different market conditions.

4.1.2 Output 2 - Mean PnL, Std. Deviation, & Volatility Premium

The profit and loss (P&L) outcomes indicate the effectiveness of the delta hedge across different levels of moneyness and time to maturity for each option pricing model employed. The standard deviation reflects how much this effectiveness varies across time. Together, these metrics help identify which option contracts are riskier or less effective to hedge. This information can support various decisions, such as adjusting the premiums charged on certain contracts to compensate for increased hedging risk.

A premium on an options contract is computed as the sum of two components: (1) the absolute value of the mean P&L, which corrects for systematic underpricing, and (2) a risk adjustment term proportional to the standard deviation of the P&L, which captures the uncertainty associated with the hedge. This uncertainty reflects how drastically asset price paths and realised volatility differ across datasets: large price movements amplify gamma-related hedge errors, while changes in volatility impact vega exposure. To compensate for the additional risk in such high-variance scenarios, the volatility premium includes this risk-adjusted term.

$$\text{Volatility Premium} = \text{PnL} + \lambda \cdot \text{Std}(\text{PnL}) \tag{19}$$

Here, λ is a risk aversion parameter that reflects how much additional premium is required to compensate for variability in hedging outcomes. For 95% confidence, it is set at 1.645.

4.1.3 Output 3 - Diagnosis of Hedging Error & Realised Volatility

Another aim of the project is to diagnose the sources of variation in the hedged P&L for both the Black-Scholes model. As previously discussed, the observed discrepancies can be attributed to several key factors:

- **Volatility Misprediction:** Occurs when the assumed volatility used in pricing and computing hedge ratios deviates from the realised market volatility. This affects both the initial option price and the delta values used in rebalancing. It can be quantified as:

$$\text{Volatility Misprediction Error} = V_{\text{model}}(\sigma_{\text{realised}}) - V_{\text{model}}(\sigma_{\text{assumed}}) \tag{20}$$

  where $V_{\text{model}}(\cdot)$ is the model-implied option price under a given volatility input. With a larger volatility mispricing, the more incorrect the delta value we employ to hedge with is. A larger incorrect delta value means our hedge fails to be sufficiently hedged and is directionally exposed to market moves.

- **Gamma Error (Discretisation Error):** Arises due to infrequent hedge rebalancing in a portfolio exposed to gamma. It can be approximated over the hedging period by:

$$\text{Gamma Error} \approx \sum_{t=1}^{T-1} \frac{1}{2}\Gamma_t(\Delta S_t)^2 \tag{21}$$

  where Γ_t is the model-estimated gamma at time t, and ΔS_t is the change in the underlying asset price between t and $t-1$.

- **Residual Error:** Represents any remaining variation in P&L not explained by the above components. This includes vega errors, jump risk, or higher-order Greeks not included in the model.

4.2 Simulation Cases

The parameters of the project are tabulated in Table 1.

Table 1: Project Strategy Overview

Component	**Details**
Underlying Assets	META, GOOG, AMZN
Parameters Varied	Moneyness (S_0/K), Time to Maturity (T)
Moneyness Levels	0.7 - 1.3
Time to Maturity	0.5 Months - 2 Years
Rebalancing Frequency	Daily
Historical Timeframe	2017–2023
Analysis Method	Historical Simulation (Rolling Windows every 10 Days)
Output Metrics	Profit and Loss (PnL)
	Standard Deviation of PnL

The type of asset underlying the option specifically being non-dividend paying, large cap U.S. equities is a measure taken to isolate the impact of different asset types on the effectiveness of the strategy. Differently behaved underlying assets, such as assets with a low market cap, assets that are dividend-paying, and so on may exhibit vastly different characteristics. Hence, in order to isolate the impact of these extraneous factors and investigate the variation of the effectiveness purely in terms of controlled variables, the aforementioned asset class is chosen for analysis such that the characteristics of the asset classes are consistent in their behaviour. Following this research, an extension to the investigation of other asset types can be conducted.

Algorithm 1 PnL Surface Generation Procedure

Data: Historical Price series for each asset
Result: Mean and Std. Dev of PnL across moneyness and maturity configurations

for *each asset* **do**
  Load historical price data
  for *each maturity* T *in* $\{15, 30, 45, \ldots, 720\}$ *days* **do**
    for *each moneyness* S_0/K *in* $\{0.7, 0.8, \ldots, 1.3\}$ **do**
      Generate rolling windows of size T
      for *each rolling window* **do**
        Construct option contract with strike and maturity Simulate delta-hedge strategy with daily rebalancing Record final PnL for the window
      Compute mean and standard deviation of PnL for this configuration

5 Results

5.1 Pricing with the Black-Scholes Model

5.1.1 Fixed Option Contract: PnL vs. Time

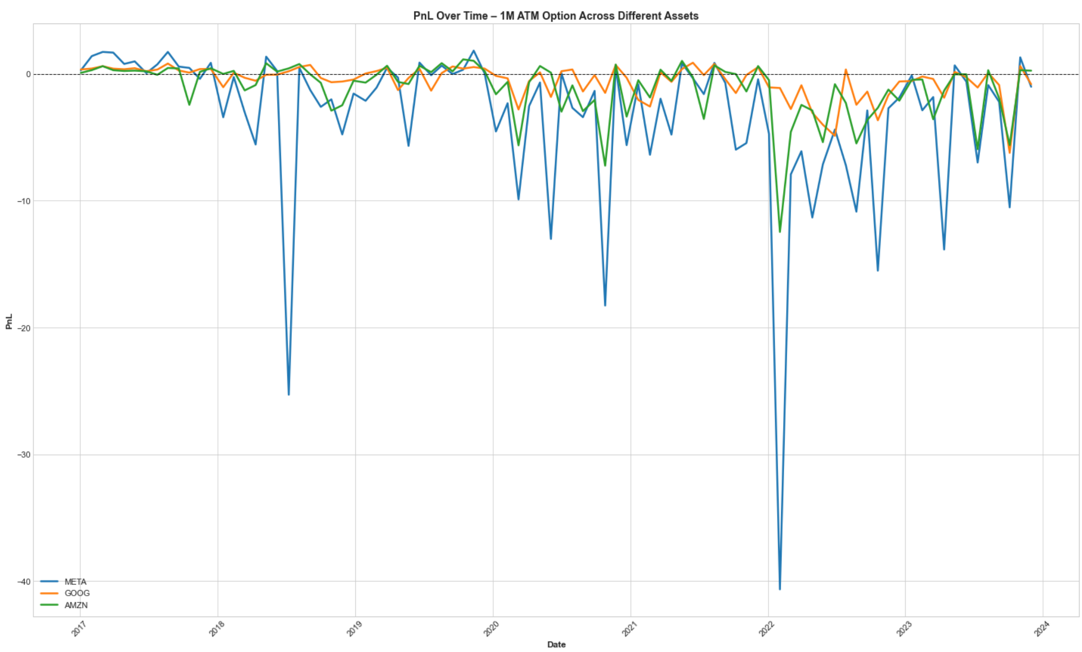


Figure 2: PnL vs. Time for short-maturity, ATM options across different underlying assets.

5.1.2 Mean and Standard Deviation of PnL

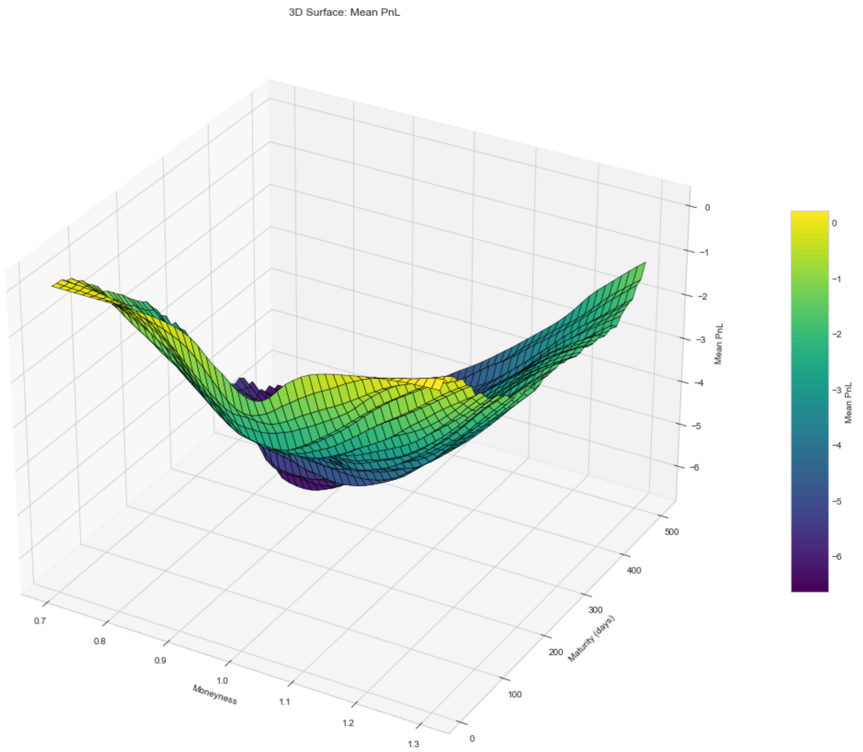


Figure 3: Mean PnL surface across moneyness and time to maturity using the Black-Scholes model.

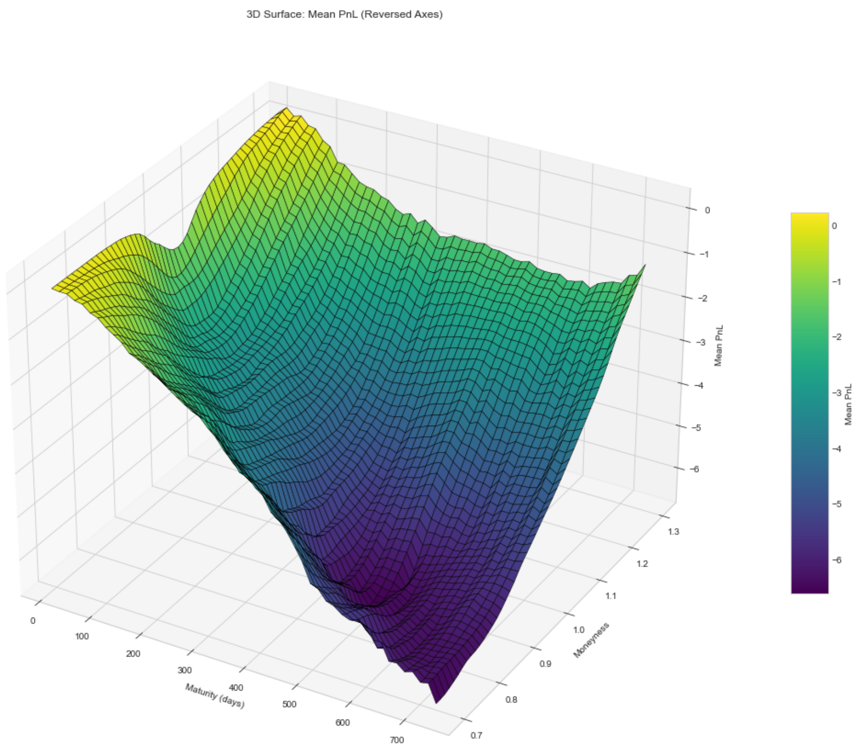


Figure 4: Alternative perspective of the mean PnL surface.

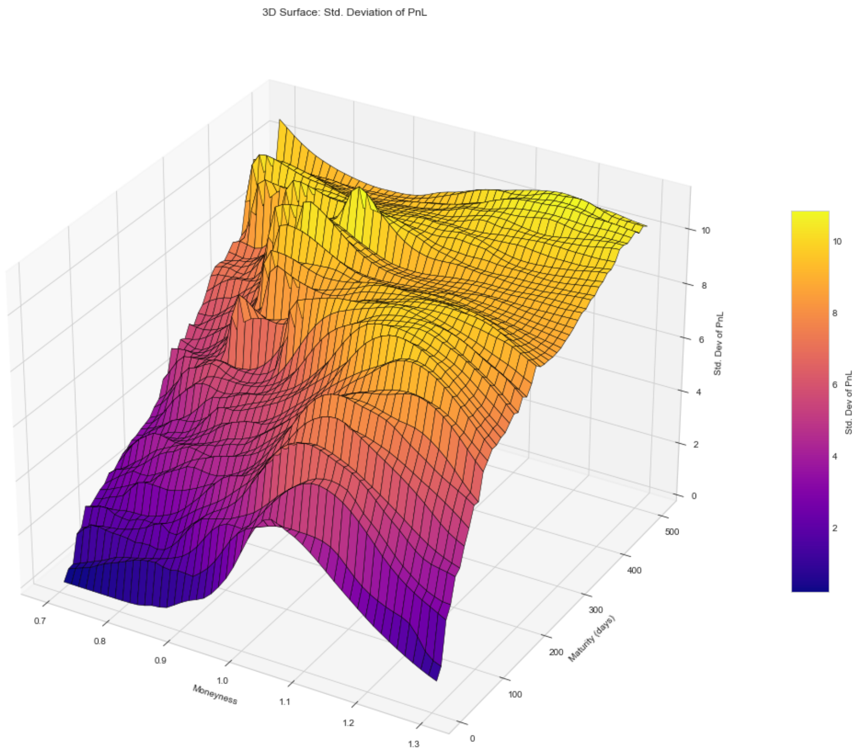


Figure 5: Standard deviation of PnL across moneyness and maturity.

5.1.3 Volatility Premiums from Black-Scholes Delta Hedging

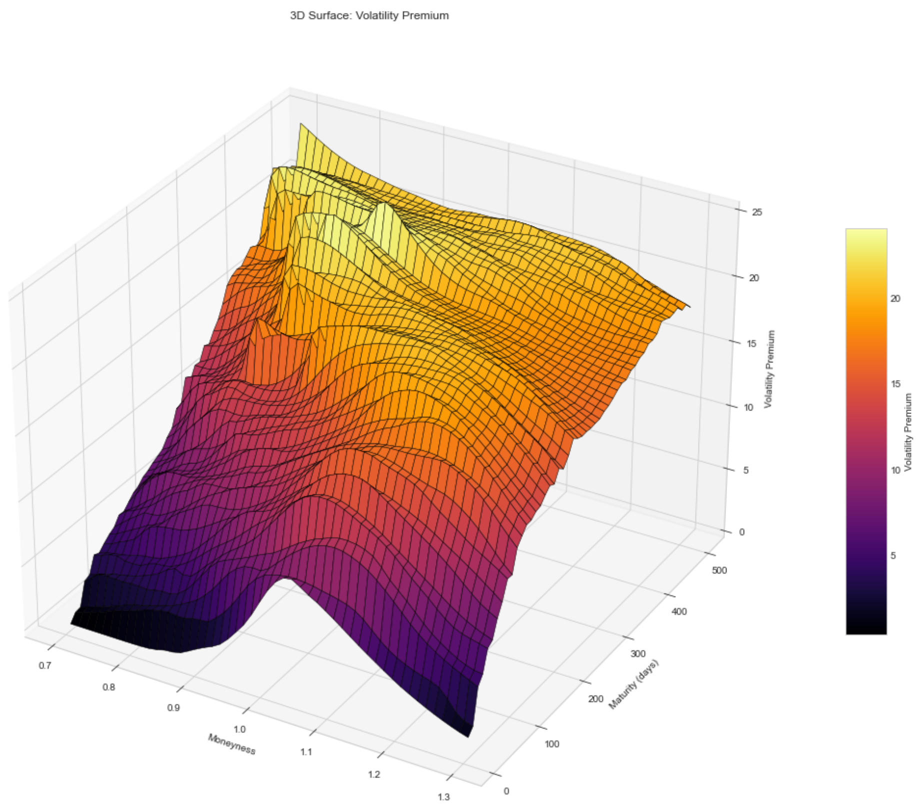


Figure 6: Volatility premium surface.

5.1.4 Attribution of Hedging Errors under Black-Scholes Assumptions

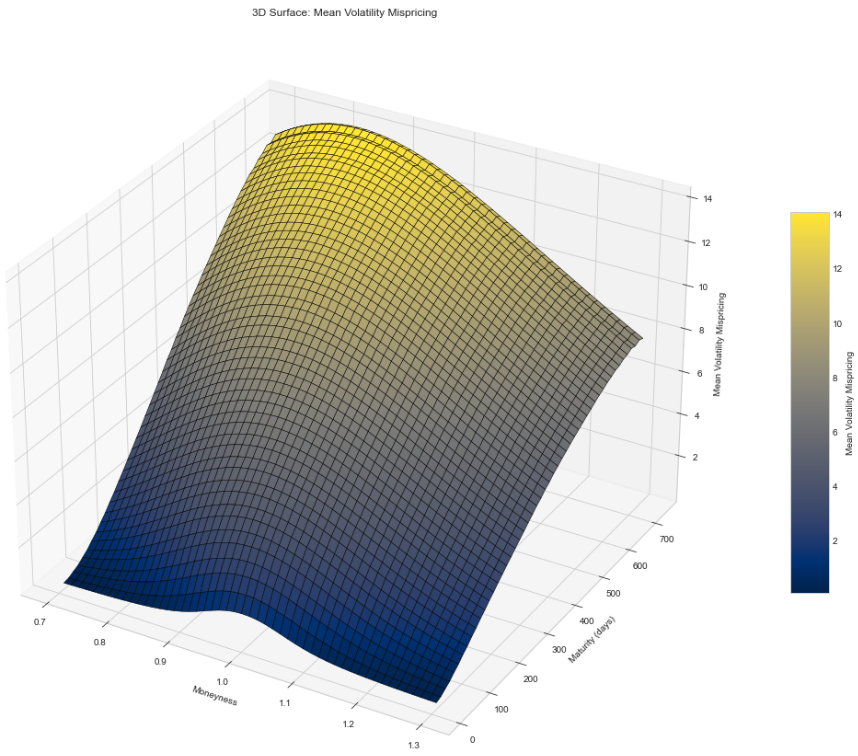


Figure 7: Volatility mispricing surface.

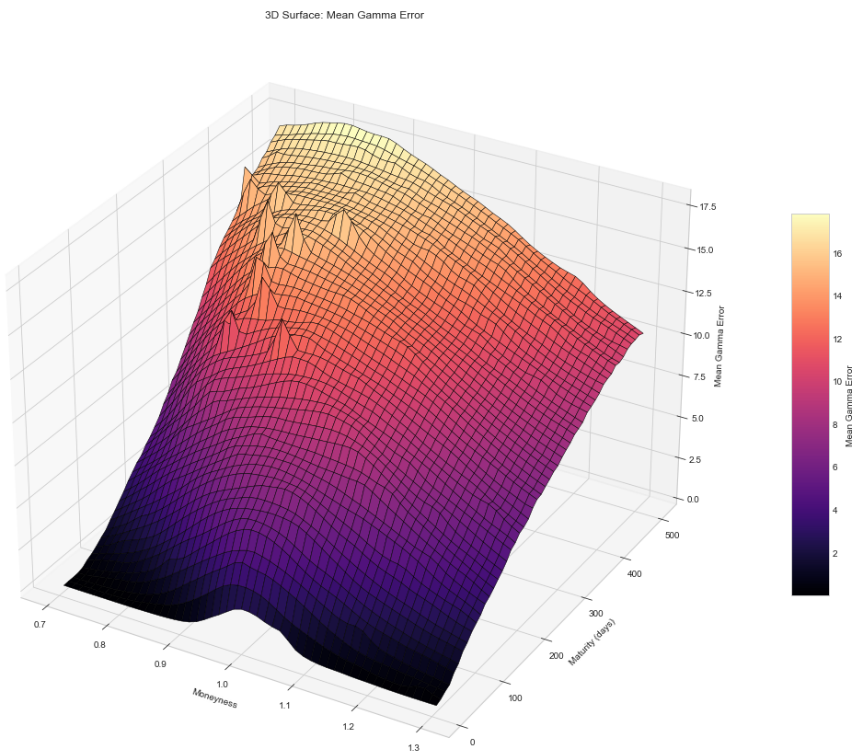


Figure 8: Gamma error surface.

6 Conclusions

6.1 Delta Hedging v/ Time v/ Asset for ATM Contracts

Figure 2 highlights that *META experiences the greatest PnL loss and variation across the time-frames measured.* Losses per option contract exceeded \$40 in 2018 and during the COVID-19 period. The especially large losses and significant variability indicate that **attempting to delta-hedge option contracts on META stock via the Black–Scholes model with daily rebalancing is ineffective**. A more suitable approach would be to use a *stochastic volatility model* to capture META's changing volatility, compute Δ, and hedge more effectively.

In contrast, the other assets seem to exhibit much more stability in their behaviour - suggesting that the delta-hedge is much more effective for option contracts with these underlying assets. However, they also experience notable deviations from a PnL of zero during turbulent market conditions.

The variability and deviation from the desired PnL being especially pronounced during major market events such as the COVID-19 crisis suggests that delta-hedging using the Black–Scholes model becomes particularly ineffective during periods of elevated market volatility. In such environments, the assumption of constant volatility breaks down most severely, leading to significant hedge errors and increased PnL losses.

6.2 Effectiveness of Delta Hedging vs. Moneyness

For a fixed time to maturity, Figure 4 suggests that the mean PnL exhibits a roughly parabolic relationship with moneyness. High in-the-money (ITM, moneyness > 1) and out-of-the-money (OTM, moneyness < 1) contracts appear to have the most effective delta hedges, as indicated by lower PnL losses. This effectiveness progressively deteriorates as the contracts approach at-the-money (ATM, moneyness ≈ 1), where the PnL losses are highest.

We know that option contracts that are at the money exhibit the highest convexity. The extreme sensitivity of delta to the underlying asset price means that the high-convexity ATM contracts are more subject to mis-pricings and gamma error. This can be verified through observation of the variation of volatility mispricing and gamma error graphs - which peak, for a fixed time to maturity, for ATM options.

To improve the effectiveness of delta hedging for ATM contracts, a natural remedy is to increase the rebalancing frequency of the hedge. This would reduce discretisation error due to gamma and thereby help to mitigate the associated PnL loss. Practical implementation, however, may be constrained by transaction costs incurred during frequent re-hedging.

6.3 Effectiveness of Delta Hedging vs. Time to Maturity

As time to maturity increases, the effectiveness of delta hedging deteriorates, with progressively larger drops in PnL. This decline occurs at a faster rate for at-the-money (ATM) options and at a slower rate for deep ITM or OTM contracts—a trend likely attributable to the fact that the higher convexity amplifies both volatility and discretisation-based errors – as discussed prior. However, **all moneyness levels experience some degree of PnL degradation with increasing maturity.**

The general degradation of PnL with increasing time to maturity for all moneyness levels is likely attributed to the fact that errors continue to accumulate with a longer option contract life. The mispricing of the option's Δ over longer periods of time leads to compounding portfolio exposure for longer periods of time - which can accumulate and lead to greater PnL deviation. Similarly, the gamma error can also accumulate over long periods of time - with the compounding effect causing it to be of greater detriment than for short-dated options.

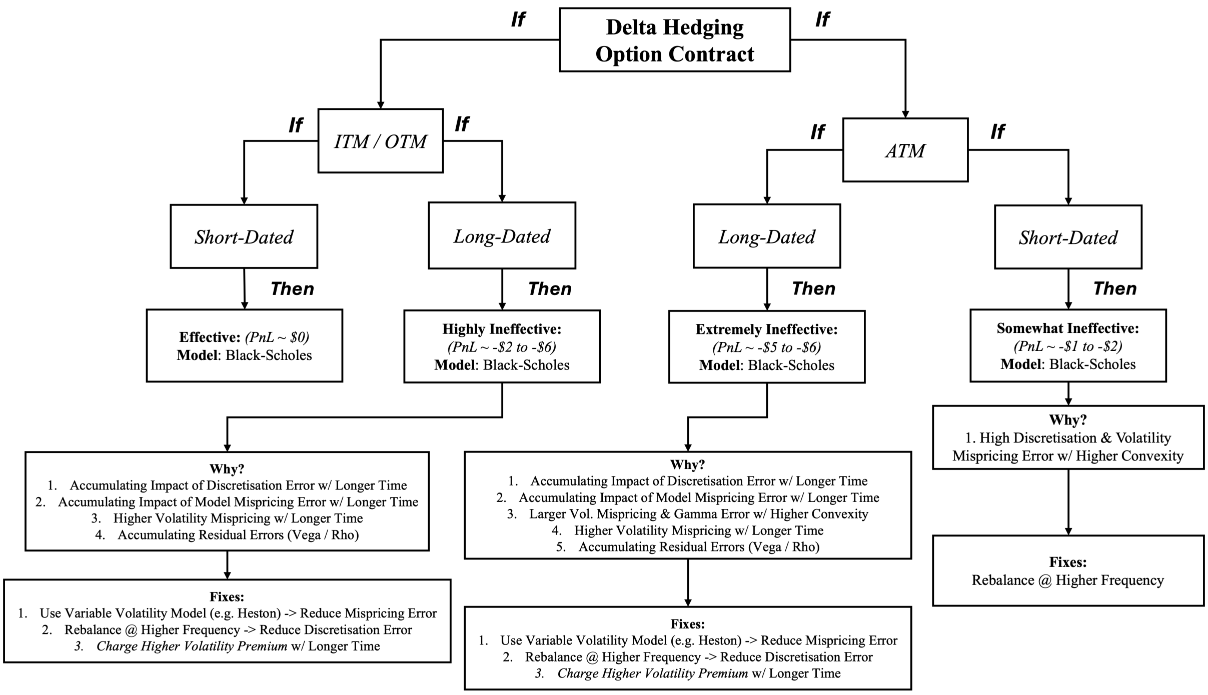
As well as increasing error accumulation, the volatility mispricing itself is amplified with longer maturity. Not only is volatility mispricing's impact of delta compounded over time, the volatility is mispriced to a greater extent - as seen by the volatility mispricing surface showcasing a larger deviation with time. As more time elapses, the volatility of the underlying is more likely to experience changes so with long-dated options, they are more likely to suffer from mispricing eventually - causing large deviations.

These also explain the increase in standard deviation with time to maturity. With the errors accumulating, the hedge becomes more path-sensitive and thus exhibits larger fluctuations. As such, the higher standard deviation reinforces that longer-dated options experience less effective hedges.

As such, when delta-hedging with longer-dated contracts, switching to a stochastic-volatility framework (e.g., the Heston model) is advisable. Treating volatility as a state variable allows the hedge to adapt to the market's changing vol dynamics, reducing model error and improving overall hedge quality.

Another possible remedy is to charge a larger volatility premium for longer-dated option contracts to account for the increased risks (greater standard deviation) and reduced hedging effectiveness associated with them.

6.4 Visualisation of Effectiveness vs. Contract Parameters



6.5 Total Visual Summarisation