

HopeFOAM

(High Order Parallel Extensible CFD Software)

User Guide

Version 0.1

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About HopeFOAM

HopeFOAM is a major extension of OpenFOAM to provide higher order finite element method and other numerical methods for computational mechanics. It is developed by Exercise Group, Innovation Institute for Defense Science and Technology, the Academy of Military Science (AMS), China. The Group aims at developing open source software packages for large scale computational science and engineering. HopeFOAM has following features:

- **High Order:** In addition to the finite volume discretization method used in OpenFOAM, HopeFOAM aims at integrating high-order discretization methods into the computational mechanics Toolbox, among which DGM (Discontinuous Galerkin Method) is the first one.
- **Parallel:** In order to improve the performance and scalability of parallel computing, parallel computational toolkits/software are integrated into HopeFOAM to accelerate the discretization and computational procedures.
- **Extensible:** By incorporating with the high order discretization and efficient parallel computing, HopeFOAM provides an extensible software framework for further development of application module and easy-to-use interfaces for developers.
- **FOAM:** The current version of HopeFOAM is a major extension of the OpenFOAM-4.0 released by the OpenFOAM Foundation on the 28th of June, 2016.

HopeFOAM-0.1 is the first publicly released version of HopeFOAM and developed by a major extension of OpenFOAM-4.0. The well-known high-order discretization method, Discontinuous Galerkin Method (DGM) is implemented in HopeFOAM-0.x. There are copious references about the method in Hesthaven and Warbuton's book:

【Hesthaven J S, Warburton T. Nodal discontinuous Galerkin methods: algorithms, analysis, and applications[M]. Springer Science & Business Media, 2007.】

HopeFOAM-0.1 provides 2D-DGM and related support. The major components include data structure, DGM discretization, solvers and related tools. PETSc is used for solving of linear systems of equations. 3D applications will be supported in a new release in a few months.

The guiding principle in the development of HopeFOAM-0.1 is to reuse the primitive

data structure of OpenFOAM-4.0 as much as possible and keep it consistent with the user interfaces. Thus, users of OpenFOAM could implement and adopt corresponding high order DGM solvers in a relatively straightforward way. The original OpenFOAM applications can be use normally.

This manual is the user guide of HopeFOAM-0.1, it provides a description of the basic operation of HopeFOAM through four tutorial exercises. The solver source code and command are located in the case directory, and all the cases are in the *tutorials/DG/2D* directory.

1 Compressible Isentropic Vortex

This tutorial describes how to pre-process, run, post-process a case involving inviscid compressible isentropic vortex in a two-dimensional square domain. The geometry is shown in Figure 1.1: $0\text{m} \leq x \leq 10\text{m}$, $-5\text{m} \leq y \leq 5\text{m}$ in 2D x-y coordinates.

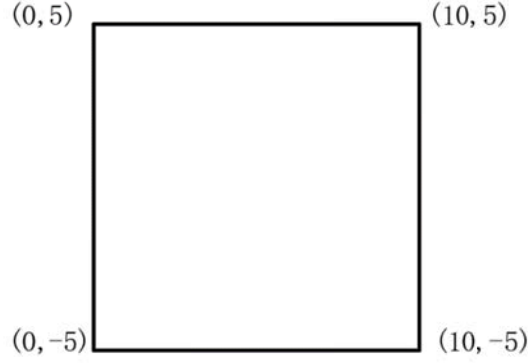


Figure 1.1 Geometry of the isentropic vortex

The analytical solutions of compressible isentropic vortex are

$$\begin{aligned} u &= u_0 - \beta e^{(1-r^2)} \frac{y - y_0}{2\pi}, \\ v &= v_0 + \beta e^{(1-r^2)} \frac{x - x_0 - t}{2\pi}, \\ \rho &= \left(1 - \left(\frac{\gamma - 1}{16\gamma\pi^2} \right) \beta^2 e^{2(1-r^2)} \right)^{\frac{1}{\gamma-1}}, \\ p &= \rho^\gamma, \end{aligned}$$

where $r = \sqrt{(x - t - x_0)^2 + (y - y_0)^2}$, $x_0 = 5$, $y_0 = 0$, $u_0 = 1$, $v_0 = 0$, $\beta = 5$, $\gamma = 1.4$, which indicates that the vortex moves from the center of the domain to the right at a speed of 1m/s .

1.1 Mesh generation

To run the `isentropicVortex` case, please change to the case directory firstly.

```
cd $FOAM_RUN/tutorials/DG/2D/isentropicVortex
```

The `isentropicVortex` domain is a unit square in the xy plane, all the boundaries are **Wall** types. Both structure and unstructured mesh format are supported by HopeFOAM. The mesh tool supplied by OpenFOAM, `blockMesh`, could be used directly to generate structure mesh for HopeFOAM by configuring *system/blockMeshDict*. The *blockMeshDict* entries for this case are shown below:

```

40 boundary
41 (
42     wall
43     {
44         type wall;
45         faces
46         (
47             (3 7 6 2)
48             (0 4 7 3)
49             (2 6 5 1)
50             (1 5 4 0)
51         );
52     }
53     frontAndBackPlanes
54     {
55         type empty;
56         faces
57         (
58             (0 3 2 1)
59             (4 5 6 7)
60         );
61     }
62 );

```

To generate the mesh, run the `blockMesh` utility by typing in the terminal:

```
blockMesh
```

Alternatively, mesh can also be imported from third-party mesh utilities, such as gambit, pointwise, fluent ICEM et al. In this case, we provide three unstructure fluent mesh files with different mesh scale: *vortex0256.msh*, *vortex1024.msh* and *vortex4096.msh*. Their corresponding characteristic lengths are h , $h/2$ and $h/4$ as shown in Figure 1.2. Mesh can be generated by typing in the terminal:

```
fluentMeshToFoam vortex0256.msh
```

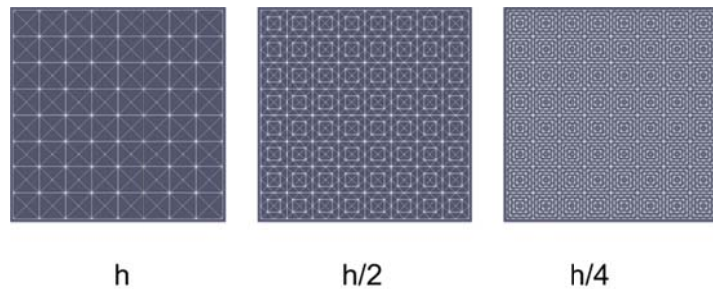


Figure 1.2 Meshes with different characteristic lengths

Users can refer to Chapter 5 of *OpenFOAM-User-Guide* for more information about mesh generation.

1.2 Boundary and initial conditions

Simple boundary conditions (`fixedValue`, `zeroGradient`) can be imposed by configure patch types in field file for HopeFOAM. However, boundary of

IsentropicVortex is described by complex formulations standing for analytical solutions. Herein, the analytical solutions are coded in *setNonUniformInlet.H* and *setBoundaryValues.H* files, located in *dgEulerFoam* solver directory. The governing equations can be written as follows:

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = 0$$

$$\mathbf{q} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ E \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(E + p) \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(E + p) \end{pmatrix}$$

The initial fields are stored in the *0* sub-directory. File *rho* sets initial values and boundary conditions of density ρ , file *rhoU* corresponds to the momentum vector $(\rho u, \rho v)$, file *Ener* corresponds to the energy E , file *U* corresponds to the velocity vector (u, v) , file *p* corresponds to the pressure p . *rho*, *rhoU* and *Ener* are primitive variables, while *U* and *p* are derived by formulation:

$$E = \frac{p}{\gamma - 1} + \frac{\rho}{2}(u^2 + v^2).$$

Besides the value of boundary and internal fields, which are set by analytical solution, the patches type need to be specified. For scalar fields *rho*, *p*, *Ener*, *internalField* and *boundaryField* are set as follows:

```

19 internalField    uniform 0;
20
21 boundaryField
22 {
23     wall
24     {
25         type        fixedValue;
26         value        uniform 0;
27     }
28
29     frontAndBackPlanes
30     {
31         type        empty;
32     }
33 }
34
35 //***** //
```

While the entries of vector fields *U*, *rhoU* for *internalField* and *boundaryField* are set as follows:

```

19 internalField    uniform (0 0 0);
20
21 boundaryField
22 {
23     wall
24     {
25         type            fixedValue;
26         value            uniform (0 0 0);
27     }
28
29     frontAndBackPlanes
30     {
31         type            empty;
32     }
33 }
34
35 //*****

```

It should be noted that, the useful information above is patch type entries, while values are meaningless.

1.3 Physical properties

Dimensionless gas constant γ must be specified for *isentropicVortex* case. It could be configured in the file *transportProperties* under the *constant* sub-directory. The keyword for gas constant is *gamma*, and below is an example for the dictionary:

```

18     gamma            gamma [ 0 0 0 0 0 0 0 ] 1.4;
19
20 // *****

```

1.4 Discretization and solver settings

Discontinuous galerkin(DG) method discretization, flux and limiter schemes are specified in the file *dgSchemes* under *system* directory. Compared with the OpenFOAM-4.0, HopeFOAM-0.1 keeps the default parameter settings of discretization schemes. Entry *godunovScheme* is designed for compressible NS simulation, controlling flux and limiter scheme.

```

18 ddtSchemes
19 {
20     default            Euler;
21 }
22
23 gradSchemes
24 {
25     default            default none;
26     grad(p)            default none;
27     grad(gther_p)      default none;
28 }
29
30 godunovScheme
31 {
32     fluxScheme          Roe;
33     limiterScheme        Triangle;
34 }

```

Specification of the DG polynomial order, solver tolerances and other algorithm configurations are stored in the *dgSolution* dictionary. The *dgSolution* dictionary offers a sub-dictionary “DG”, which contains the configurations of dimension and order. In current version, baseOrder can be set ranging from 1 to 8.

```

17 DG
18 {
19     meshDimension      2;
20     baseOrder           4;
21 }
22
23 solvers
24 {
25     dgrho
26     {
27         tolerance       1e-12;
28         relTol           0;
29         kspSolver        preonly;
30     }
31
32     rhoU
33     {
34         tolerance       1e-12;
35         relTol           0;
36         kspSolver        preonly;
37     }
38
39     Ener
40     {
41         tolerance       1e-12;
42         relTol           0;
43         kspSolver        preonly;
44     }
45
46     "rho(1|2|3)"
47     {
48         tolerance       1e-12;
49         relTol           0;
50         kspSolver        preonly;
51     }
52
53     "rhoU(1|2|3)"
54     {
55         tolerance       1e-12;
56         relTol           0;
57         kspSolver        preonly;
58     }
59
60     "Ener(1|2|3)"
61     {
62         tolerance       1e-12;
63         relTol           0;
64         kspSolver        preonly;
65     }
66 }
67
68 PISO
69 {
70     nCorrectors          2;
71     nNonOrthogonalCorrectors 0;
72     pRefCell              0;
73     pRefValue             0;
74 }
75
76 // ***** //
```


1.5 Control

The information relating to the control of the solution procedure, including time step, start time and end time, is stored in the *system/controlDict* dictionary. For this case, the `startTime` is set to 0. To guarantee the vortex not pass through this domain, end time should be less than 5s. The `writeInterval` is set to 25. Time step `deltaT` is decided by the mesh size and the order approximation to satisfy the CFL restriction (the calculation method refer to *Nodal Discontinuous Galerkin Methods Algorithms Analysis and Applications* Section 6.4). The *controlDict* entries are written as follows:

```
18 application      dgEulerFoam;
19
20 startFrom         startTime;
21
22 startTime         0;
23
24 stopAt            endTime;
25
26 endTime           2;
27
28 deltaT            0.008;
29
30 writeControl       timeStep;
31
32 writeInterval      25;
33
34 purgeWrite        0;
35
36 writeFormat        ascii;
37
38 writePrecision     16;
39
40 writeCompression   off;
41
42 timeFormat         general;
43
44 timePrecision      6;
45
46 runTimeModifiable true;
47
48 adjustTimeStep     no;
49
50 maxCo              1;
51
52 maxDeltaT          1;
53
54 // ***** //
```

A stable `deltaT` configuration for various base order and mesh size is given in Table 1.1.

Table1.1 DeltaT of different meshes and different order approximations

baseOrder	vortex0256	vortex1024	vortex4096
1	0.04	0.02	0.01
2	0.02	0.01	0.005
3	0.008	0.004	0.002
4	0.008	0.004	0.002
5	0.004	0.002	0.001
6	0.002	0.001	0.0005

1.6 Running the code

The dgEulerFoam solver is executed by entering the case directory and typing:

```
$ ./dgEulerFoamVortex
```

1.7 Post-processing

vortex1024.msh is used and baseOrder is set to 4. After computational results are written to time directories, the *VTK* folder could be generated by typing in the terminal:

```
$ dgToVTK
```

, which is a high order post-processing utility released within HopeFOAM. The results can be viewed by ParaView, clicking file>Open... to choose .vtk files, as shown in Figure1.3.

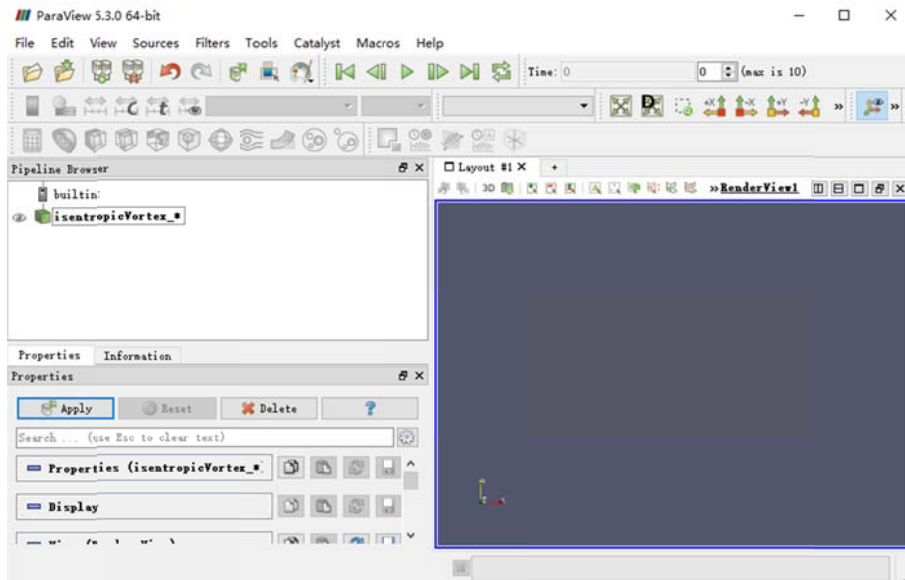


Figure1.3 ParaView window

To view contour plots as shown in Figure 1.4, click apply button and select rho, rhoU, Ener from the Coloring menu.

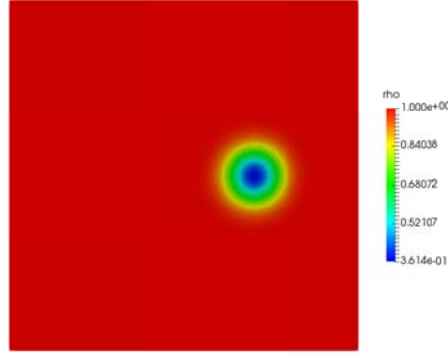


Figure1.4 Density in the isentropicVortex case

Please refer to Chapter 2 of OpenFOAM-User-Guide for more information about Post-processing.

1.8 Computing convergence rate

At the end of the simulation, the errors in L2 norm of numerical solution are calculated by *dgEulerFoam /eulererror.H*.

```
rhoError: 8.807979526797244e-06
```

```
rhoUError: 1.865574862711117e-05
```

The rate of convergence can be calculated with a sequence of nested refined meshes, as defined by:

$$\log_2\left(\frac{\mathcal{E}_h}{\mathcal{E}_{h/2}}\right)$$

where \mathcal{E}_h is the density or momentum error of the mesh with characteristic length h .

1.9 Running in parallel

For large scale problems, parallel processing supported by HopeFOAM could significantly reduce the computational time.

The configuration dictionary for parallel processing named *decomposeParDict* locates in the *system* directory. The first entry is *numberOfSubdomains*, which specifies the number of decomposed subdomains. Specifically, *numberOfSubdomains* should be consistent to the number of processors used to run the solver. Within the *simpleCoeffs* sub-dictionary, the vector *n* specifies the

number of subdomains in x, y and z directions. The number of subdomains specified by $n_x * n_y * n_z$ should equal to `numberOfSubdomains`. If the geometry is two-dimensional, we should set $n_z = 1$.

```

26 numberOfSubdomains 12;
27
28 method simple;
29
30 simpleCoeffs
31 {
32     n          (3 4 1);
33     delta      0.001;
34 }
35 // ***** //
```

After running `dgDecomposePar`, the geometry and associated fields are decomposed into 12 pieces. The case directory generates 12 folders from *processor0* to *processor11*. Parallel simulation will start by typing:

```
$ mpirun -np 12 ./dgEulerFoamVortex -parallel
```

Once the case has finished running, the decomposed fields and mesh are reassembled for post-processing using the `dgReconstructPar` utility. Simply execute it from the command line:

```
$ dgReconstructPar
```

2 Double Mach Reflection

Double Mach reflection uses the same solver as isentropic vortex. For the double Mach reflection case simulation, we modify the *setNonUniformInlet.H*, *setBoundaryValues.H*, and *eulererror.H* files. Notice that different from solver in IsentropicVortex case, comment annotation of line 91 and 118 in *dgEulerFoam/dgEulerFoam.C* are removed to active the limiter. For problems with shocks, limiter is required to stabilize the simulation.

```
91      Godunov.limite(rho1,rhoU1,Ener1);  
118     Godunov.limite(rho,rhoU,Ener);
```

Recompile the solver by typing the following command in terminal window:

```
$ wclean  
$ wmake
```

The source code of the modified *dgEulerFoam* solver and *dgEulerFoamDouble* command are located in the *tutorials/DG/2D/doubleMach* case directory.

This case involves a Mach 10 shock in the air ($\gamma=1.4$) with an initial angle of 60° to the reflecting wall. The schematic diagram is shown in Figure 2.1.

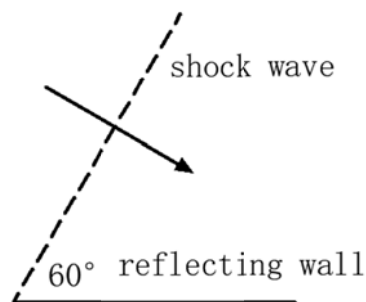


Figure 2.1 Double Mach reflection schematic diagram

The two-dimensional domain is shown in Figure 2.2 with $0 \leq x \leq 3.2$ and $0 \leq y \leq 1$. A Mach 10 shock is set up with an initial position of $x=0.16667$ on the lower boundary. The shock makes an angle of 60° to the x axis and extends to the top of the domain at $y=1$. Special fixed boundary conditions are set up for *inlet* and *wall* patches at $y=0$, and $0 \leq x \leq 0.16667$ allowing the shock to propagate off the domain. The reflecting wall named *wall* patch lies along the bottom of the problem domain, beginning at $x=0.16667$. For the upper boundary $y=1$ a time-dependent

boundary is imposed to allow the shock to propagate onto the domain as though it extended to infinity. The values along the top boundary including `inlet` and `far` patches are set to describe the exact motion of the initial Mach 10 shock. The simulation is run until $t = 2$, by which time the shocks should be nearly at the `outlet` patch. The undisturbed air ahead of the shock has a density of 1.4 and a pressure of 1.

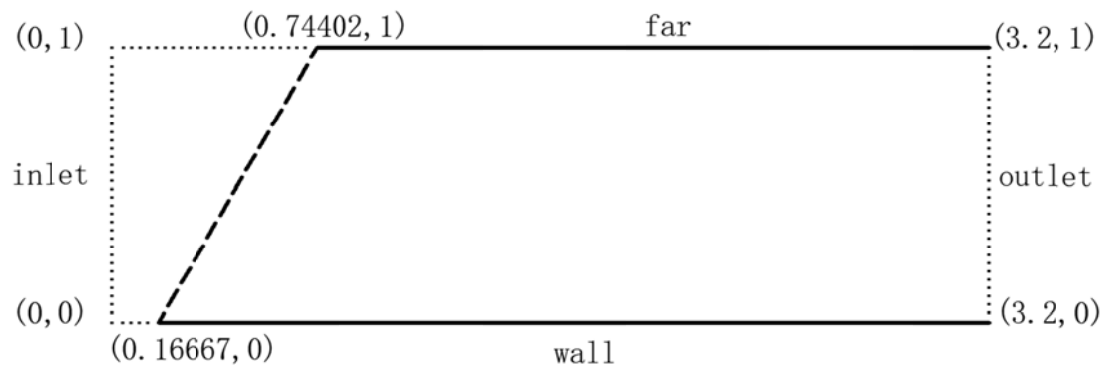


Figure 2.2 Geometry and boundary of Double Mach reflection

2.1 Mesh Generation

We provide a *doubleMach.msh* mesh file under `doubleMach` case directory, the mesh is generated by typing in the terminal:

```
$ fluentMeshToFoam doubleMach.msh
```

2.2 Boundary and initial conditions

Similar with Section 1.2, the boundary and initial conditions are programmed into `dgEulerFoam` solver, we add *setNonUniformInlet.H* and *setBoundaryValues.H* files to `dgEulerFoam` solver source code. The entries for the scalar files *rho*, *p*, *Ener* in `0` sub-directory are configured as follows:

```
19 internalField uniform 0;
20
21 boundaryField
22 {
23     wall
24     {
25         type            reflective;
26         value            uniform 0;
27     }
28
29     far
30     {
31         type            fixedValue;
32         value            uniform 0;
```

```

33     }
34
35     inlet
36     {
37         type            fixedValue;
38         value           uniform 0;
39     }
40
41     outlet
42     {
43         type            fixedValue;
44         value           uniform 0;
45     }
46
47     frontAndBackPlanes
48     {
49         type            empty;
50     }
51 }
52
53 //*****

```

The entries for the vector fields U and ρU are shown as below SSA zx:

```

19 internalField    uniform (0 0 0);
20
21 boundaryField
22 {
23     wall
24     {
25         type            reflective;
26         value           uniform (0 0 0);
27     }
28
29     far
30     {
31         type            fixedValue;
32         value           uniform (0 0 0);
33     }
34
35     inlet
36     {
37         type            fixedValue;
38         value           uniform (0 0 0);
39     }
40
41     outlet
42     {
43         type            fixedValue;
44         value           uniform (0 0 0);
45     }
46
47     frontAndBackPlanes
48     {
49         type            empty;
50     }
51 }
52
53 //*****

```

Note that the wall patch are given a reflective boundary condition.

2.3 Running the code

Configurations of *constant/transportProperties* and *system/dgSchemes* dictionaries are set the same as *isentropicVortex* case. Ensure that the *baseOrder* in the *dgSolution* dictionary is set to 1. In the *system/controlDict* directory, set *deltaT* to $1e-5$ and *endTime* to 0.2.

This case typically takes a few hours when running in serial. To decrease the running time, edit *system/decomposeParDict* as referred to section 1.9. Then decompose the mesh and fields:

```
$ dgDecomposePar
```

Then run in parallel:

```
$ mpirun -np 12 ./dgEulerFoamDouble -parallel
```

At last, reassemble the fields and mesh data:

```
$ dgReconstructPar
```

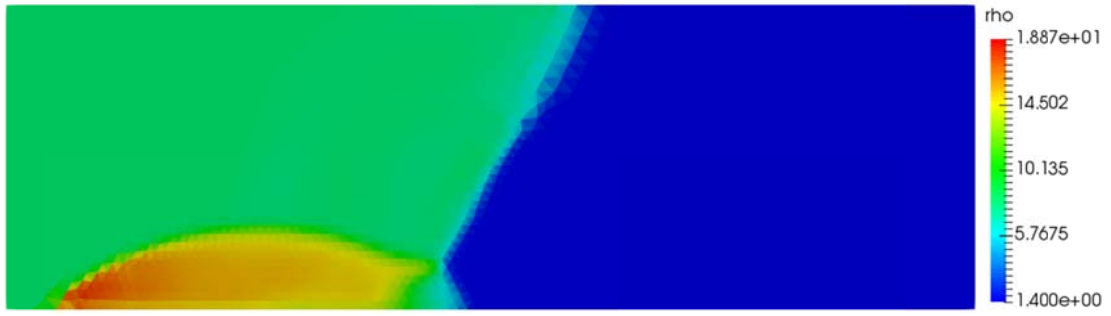
2.4 Post-processing

Use the command *dgToVTK* to generate the *VTK* folder. Then open the *ParaView* software and click the button *file>Open...* to choose *.vtk* files. To view density contour plots as shown in Figure 2.3, please open the *Properties* panel, select *rho* from the *Coloring* menu. Then click *play* in *VCR* controls panel to view density at different times.

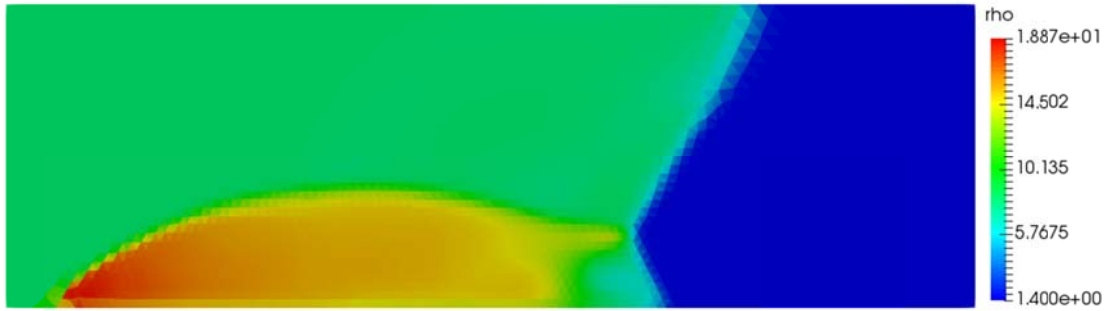
The limiter in *HopeFOAM-0.1* is first order and does not support higher order calculations. We provide encryption mesh file *doubleMachDense.msh* for fine flow field.



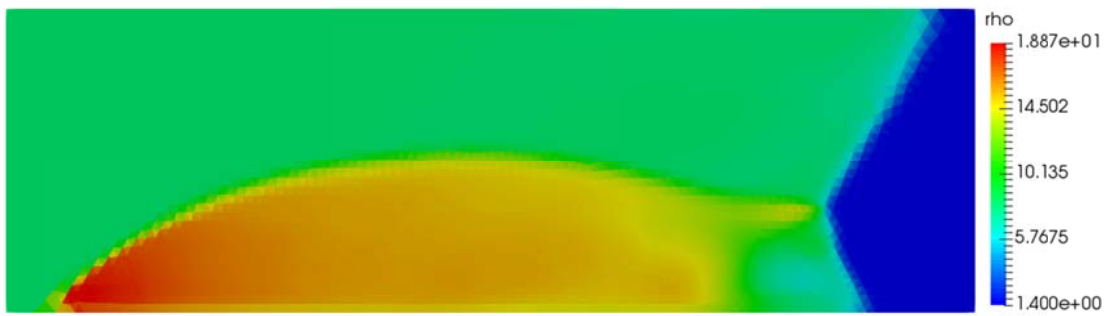
$t = 0.05$



$t = 0.1$



$t = 0.15$



$t = 0.2$

Figure 2.3 Density in the doubleMach case

3 Incompressible vortex

This tutorial describes how to pre-process, run, post-process a case involving incompressible vortex in a two-dimensional square domain. Two-dimensional Navier-Stokes equations for viscous laminar flow are implemented into a HopeFOAM solver named dgChorinFoam.

In this particular case there are four inflow and four outflow regions, as shown in Figure 3.1: $-0.5\text{m} \leq x \leq 0.5\text{m}$, $-0.5\text{m} \leq y \leq 0.5\text{m}$.

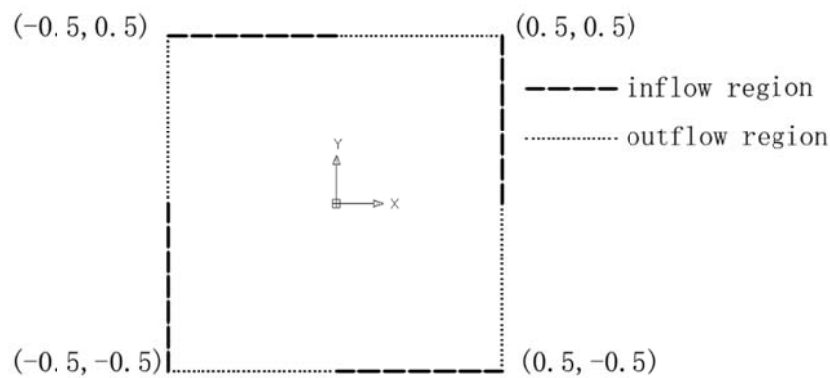


Figure 3.1 Geometry of the incompressible vortex

3.1 Mesh generation

After changing to the pearsonVortex case directory, the mesh divides to 4 blocks, the block structure is shown in Figure 3.2.

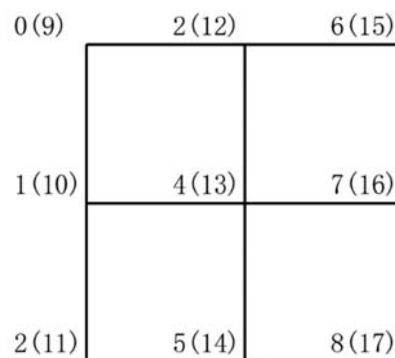


Figure 3.2 Block structure for incompressible vortex

The *blockMeshDict* entries for this case are as follows:

```
17 convertToMeters 1;
18
19 vertices
20 (
21     (-0.5 0.5 0)
22     (-0.5 0 0)
```

```

23     (-0.5 -0.5 0)
24     (0 0.5 0)
25     (0 0 0)
26     (0 -0.5 0)
27     (0.5 0.5 0)
28     (0.5 0 0)
29     (0.5 -0.5 0)
30     (-0.5 0.5 0.1)
31     (-0.5 0 0.1)
32     (-0.5 -0.5 0.1)
33     (0 0.5 0.1)
34     (0 0 0.1)
35     (0 -0.5 0.1)
36     (0.5 0.5 0.1)
37     (0.5 0 0.1)
38     (0.5 -0.5 0.1)
39 );
40
41 blocks
42 (
43     hex (0 1 4 3 9 10 13 12) (8 8 1) simpleGrading (1 1 1)
44     hex (1 2 5 4 10 11 14 13) (8 8 1) simpleGrading (1 1 1)
45     hex (3 4 7 6 12 13 16 15) (8 8 1) simpleGrading (1 1 1)
46     hex (4 5 8 7 13 14 17 16) (8 8 1) simpleGrading (1 1 1)
47 );
48
49 edges
50 (
51 );
52
53 boundary
54 (
55     inlet
56     {
57         type wall;
58         faces
59         (
60             (2 11 10 1)
61             (8 17 14 5)
62             (6 15 16 7)
63             (0 9 12 3)
64         );
65     }
66     outlet
67     {
68         type wall;
69         faces
70         (
71             (1 10 9 0)
72             (5 14 11 2)
73             (7 16 17 8)
74             (3 12 15 6)
75         );
76     }
77     frontAndBackPlanes
78     {
79         type empty;
80         faces
81         (
82             (1 0 3 4)
83             (10 13 12 9)
84             (2 1 4 5)
85             (11 14 13 10)
86             (5 4 7 8)
87             (14 17 16 13)
88             (4 3 6 7)
89             (13 16 15 12)
90         );

```

```

91     }
92 );
93
94 mergePatchPairs
95 (
96 );
97
98 // *****

```

Mesh is generated by typing in the terminal:

```
$ blockMesh
```

3.2 Boundary and initial conditions

Similar with Section 1.2, *pearsonVortex* case uses analytical solution to set initial values and boundary conditions, which is implemented by adding *setNonUniformInlet.H* and *setBoundaryValues.H* files to *dgChorinFoam* solver source code.

$$\begin{aligned}
 u &= -\sin(2\pi y)e^{-v^4\pi t}, \\
 v &= \sin(2\pi x)e^{-v^4\pi t}, \\
 p &= -\cos(2\pi x)\cos(2\pi y)e^{-v^8\pi t}.
 \end{aligned}$$

Since the boundary and initial conditions are programmed into *dgChorinFoam* solver, the field value in the *0/U* file is meaningless. Here, *internalField* and *inlet* patch are simply initialized as uniform zero, which is expressed by 3 vector components (0 0 0). Neumann boundary condition for the velocity at the outflow boundary is assumed by a *fixedGradient* condition with a value of uniform (1 1 0).

```

17 dimensions      [0 1 -1 0 0 0 0];
18
19 internalField    uniform (0 0 0);
20
21 boundaryField
22 {
23     inlet
24     {
25         type      fixedValue;
26         value      uniform (0 0 0);
27     }
28
29     outlet
30     {
31         type      fixedGradient;
32         gradient   uniform (1 1 0);
33     }
34
35     frontAndBackPlanes
36     {
37         type      empty;
38     }
39 }

```

```

40
41 // ***** //

```

Similarly, we set the pressure field in the `O/p` file. Neumann boundary condition for the pressure on the `inlet` patch are set as `zeroGradient`. At the `outlet` patch, a zero pressure is assumed.

3.3 Physical properties

The physical properties for the `pearsonVortex` case is stored in the `constant/transportProperties` dictionary. The keyword for kinematic viscosity is `nu`. The keyword `transportModel` represents fluid type and is set as `Newtonian`. The `transportProperties` entries for this case are as follows:

```

17 transportModel Newtonian;
18 nu              nu [ 0 2 -1 0 0 0 0 ] 0.01;
19
20 // ***** //

```

3.4 Discretisation and solver settings

Keep the default configurations for the discretization schemes in `dgSchemes` dictionary under `system` directory. The `dgSolution` dictionary contains a sub-dictionary `DG`, which specifies dimension and order approximation. The dimension of the mesh `meshDimension` is set to 2. The order approximation `baseOrder` can be set ranging from 1 to 8. The `solver` tolerance should be set as 10^{-12} to ensure high order convergence rate.

3.5 Running the code

To stabilize the computation, we set `deltaT` to 0.001 and `endTime` to 0.1 in `ControlDict` dictionary, while `baseorder` is 2. Run the case by typing in the terminal:

```
$ ./dgChorinFoamVortex
```

3.6 Post-processing

After generating `VTK` files by the command `dgToVTK`, we use `paraView` to plot the vectors of the flow velocity.

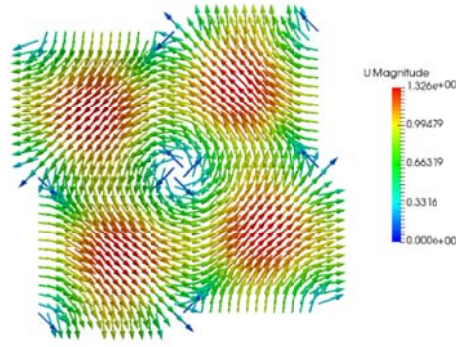


Figure 3.3 Velocities in the pearsonVortex case

3.7 Computing rate of convergence

Similar to the isentropicVortex example, we give three sets of meshes and their reference values for Δt at different order approximation to calculate the rate of convergence, as shown in Table 3.1. Δt is proportional to characteristic length h . At the end of the simulation, the errors of numerical solution are outputted.

Table3.1 Δt of different meshes and different order approximations

baseOrder	h	$0.5h$	$0.25h$
1	0.002500000	0.001250000	0.000625000
2	0.001100000	0.000555560	0.000285710
3	0.000625000	0.000322580	0.000161290
4	0.000400000	0.000204080	0.000103090
5	0.000285710	0.000142860	0.000071429
6	0.000208330	0.000105260	0.000052632

4 Unsteady flow around cylinder

Unsteady flow around cylinder in channel uses the same solver as the incompressible vortex case. To simulate this case, we modify the *setNonUniformInlet.H*, *setparaT.H*, *pEqnCorrect.H* and *setBoundaryValues.H* files. This case requires to modify the *dgEulerFoam/dgEulerFoam.C* file as follow:

- Add *createTimeControls.H* file

```
45    #include "createTimeControls.H"
```

- Set parameters at each time step

```
70    paraT4 = std::sin(pi*runTime.value()/8)/paraT3;
```

```
71    paraT3 = std::sin(pi*runTime.value()/8);
```

- paraT3 modified as paraT1

```
86    shared_ptr<dg::Equation<scalar>> result1 =
        make_shared<bCorrectEquation<scalar>>(dgm::laplacian(p), paraT1);
```

- Add line 116 annotation comment `//`:

```
116    // #include "chorinerror.H"
```

Recompile the solver and generated binary file *dgChorinFoamCylinder* is located in the *cylinder* case directory.

This case simulates a time-dependent two-dimensional flow through a channel, with a cylinder placed off the centerline of the channel. It begins to shed vortices as soon as the parabolic inflow is ramped up from its zero initial condition. The domain is shown in Figure 4.1, the center of cylinder locates at origin (0,0) and its radius is 0.05.

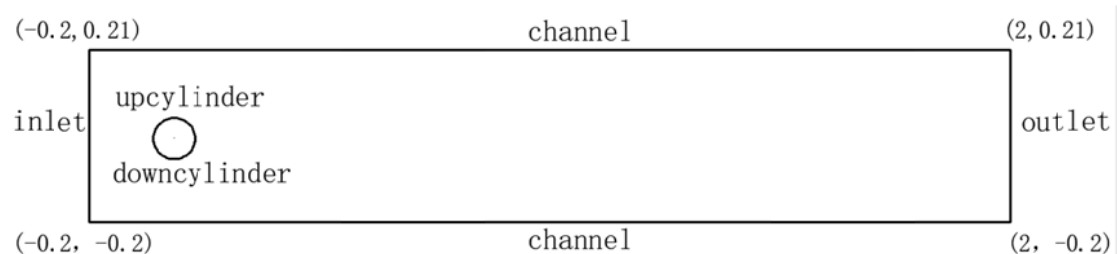


Figure 4.1 Geometry and boundary of unsteady flow around cylinder

At initial time $t=0$, zero flow is assumed. The simulations ends at $t=8$. The inflow boundary conditions for this case are given by a parabolic inflow profile, modulated by a sine function depending on time as:

$$u(x, y, t) = 0.41^{-2} \sin\left(\frac{\pi t}{8}\right) 6(y + 0.2)(0.21 - y),$$

$$v(x, y, t) = 0.$$

4.1 High order curved boundary

Class `arcPolyPatch` is introduced in HopeFOAM for supporting high-order representation of curved boundary. Detailed configuration locates in the *constant/polyMesh/boundary* dictionary. We take `upcylinder` as an example to illustrate how to configure curved boundary. Boundary type entry `type` is set to `arc` and name specifies the surface boundary name. The parametric equation of the 2D curved boundary is marked with notation `# { # }`, which describes the relationship between x, y, z coordinates and parameter u, v . `u_Range` specifies that the value of parameter u is in the range of `[-0.5 0.5]`.

```

18 6
19 (
20     channel
21     {
22         type            wall;
23         inGroups        1(wall);
24         nFaces          128;
25         startFace       2666;
26     }
27     outlet
28     {
29         type            patch;
30         inGroups        1(wall);
31         nFaces          12;
32         startFace       2794;
33     }
34     inlet
35     {
36         type            patch;
37         inGroups        1(wall);
38         nFaces          16;
39         startFace       2806;
40     }
41     upcylinder
42     {
43         type            arc;
44         inGroups        1(wall);
45         nFaces          16;
46         startFace       2822;
47         name            codeup;
48         u_Range         (-0.5 0.5);
49         v_Range         (0 0);
50         code
51         #{

```



```

52             0.05*Foam::sin(Foam::constant::mathematical::pi*u),
53             0.05*Foam::cos(Foam::constant::mathematical::pi*u),
54             v
55         #};
56     }
57     downcylinder
58     {
59         type            arc;
60         inGroups        1(wall);
61         nFaces          16;
62         startFace       2838;
63         name            codedown;
64         u_Range         (-0.5 0.5);
65         v_Range         (0 0);
66         code
67         #{
68             0.05*Foam::sin(Foam::constant::mathematical::pi*u),
69             -0.05*Foam::cos(Foam::constant::mathematical::pi*u),
70             v
71         #};
72     }
73
74     frontAndBackPlanes
75     {
76         type            empty;
77         inGroups        1(empty);
78         nFaces          3680;
79         startFace       2854;
80     }
81 )
82
83 //*****

```

4.2 Boundary and initial conditions

Similar with Section 1.2, velocity field and pressure field of inlet patch are set by formulas. Natural boundary conditions for the velocity on the outlet patch is set to `fixedGradient`. No slip boundary conditions are imposed on the `upcylinder`, `downcylinder` and channel walls by setting `fixedValue` type with value of `uniform (0 0 0)`.

```

21 boundaryField
22 {
23     inlet
24     {
25         type            fixedValue;
26         value            uniform (0 0 0);
27     }
28
29     outlet
30     {
31         type            zeroGradient;
32     }
33
34     channel
35     {
36         type            fixedValue;
37         value            uniform (0 0 0);
38     }
39
40     upcylinder

```

```

41  {
42      type          fixedValue;
43      value          uniform (0 0 0);
44  }
45
46  downcylinder
47  {
48      type          fixedValue;
49      value          uniform (0 0 0);
50  }
51
52  frontAndBackPlanes
53  {
54      type          empty;
55  }
56 }
57
58 //*****

```

The pressure of the boundary upcylinder, downcylinder and channel are set as zeroGradient condition. The outlet patch is assumed as zero pressure boundary.

```

21 boundaryField
22 {
23     inlet
24     {
25         type          zeroGradient;
26     }
27
28     outlet
29     {
30         type          fixedValue;
31         value          uniform 0;
32     }
33
34     channel
35     {
36         type          zeroGradient;
37     }
38
39     upcylinder
40     {
41         type          zeroGradient;
42     }
43
44     downcylinder
45     {
46         type          zeroGradient;
47     }
48
49     frontAndBackPlanes
50     {
51         type          empty;
52     }
53 }
54
55 //*****

```

4.3 Physical properties

This case runs with a Reynolds number of 100. The kinematic viscosity ν is configured by the *constant/transportProperties* dictionary with a value of 0.001. Order approximation *baseOrder* is set to 3 in *system/dgSolution* dictionary.

There is no closed-form analytical solution for this problem. Instead, we compute the drag and lift coefficients for the cylinder, which are defined by:

$$C_d(t) = -\oint_{Cylinder} -p\hat{n}_x + \nu(\hat{n}_x 2\frac{\partial u}{\partial x} + \hat{n}_y(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})) ds,$$

$$C_l(t) = -\oint_{Cylinder} -p\hat{n}_y + \nu(\hat{n}_y 2\frac{\partial v}{\partial y} + \hat{n}_x(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})) ds,$$

where (\hat{n}_x, \hat{n}_y) is the outward normal along the cylinder surface.

In the *controlDict* directory, the *forceCoeffs* is specified in the functions sub-dictionary and *deltaT* is set to 2E-4. The *controlDict* entries are presented as follows:

```

18 application      dgChorinFoam;
19
20 startFrom         startTime;
21
22 startTime         0;
23
24 stopAt            endTime;
25
26 endTime           8;
27
28 deltaT            2.0E-04;
29
30 writeControl       timeStep;
31
32 writeInterval      200;
33
34 purgeWrite         0;
35
36 writeFormat        ascii;
37
38 writePrecision      6;
39
40 writeCompression  off;
41
42 timeFormat         general;
43
44 timePrecision       6;
45
46 runtimeModifiable false;
47
48 adjustTimeStep     no;
49
50 maxCo              0.9;
51
52 maxDeltaT          0.01;
53
54 functions

```

```

55 {
56     #include "forceCoeffs"
57 };
58
59 //*****

```

The calculation of drag and lift coefficients is specified in `system/forceCoeffs` dictionary. `rhoInf` refers to the reference density and `magUInf` specifies reference velocity. Both of them are set to 1 for convenience. `liftDir` and `dragDir` respectively indicate the direction of lift and drag force.

```

9 forceCoeffs
10 {
11     type dgForceCoeffs;
12     functionObjectLibs ("libdgforces.so");
13     patches (upcylinder downcylinder);
14     log true;
15     pName p;
16     Uname U;
17     rho rhoInf;
18     rhoInf 1;
19     magUInf 1;
20     liftDir (0 1 0);
21     dragDir (1 0 0);
22     pitchAxis (0 0 -1);
23     CofR (0 0 0);
24     Aref 0.1; //2D example default height of the grid is 1
25     lRef 1;
26     outputControl   timeStep;
27     outputInterval 1;
28 }
29
30 // *****

```

4.4 Running the code and post-processing

Currently, HopeFOAM-0.1 does not support parallel computing with curved boundaries. Run the code by typing in the terminal:

```
$ ./dgChorinFoamCylinder
```

Use `paraView` to plot nephogram of the flow velocity. Coefficients are stored in `.dat` file in `postProcessing/forceCoeffs/0/directory`.



$t=2$

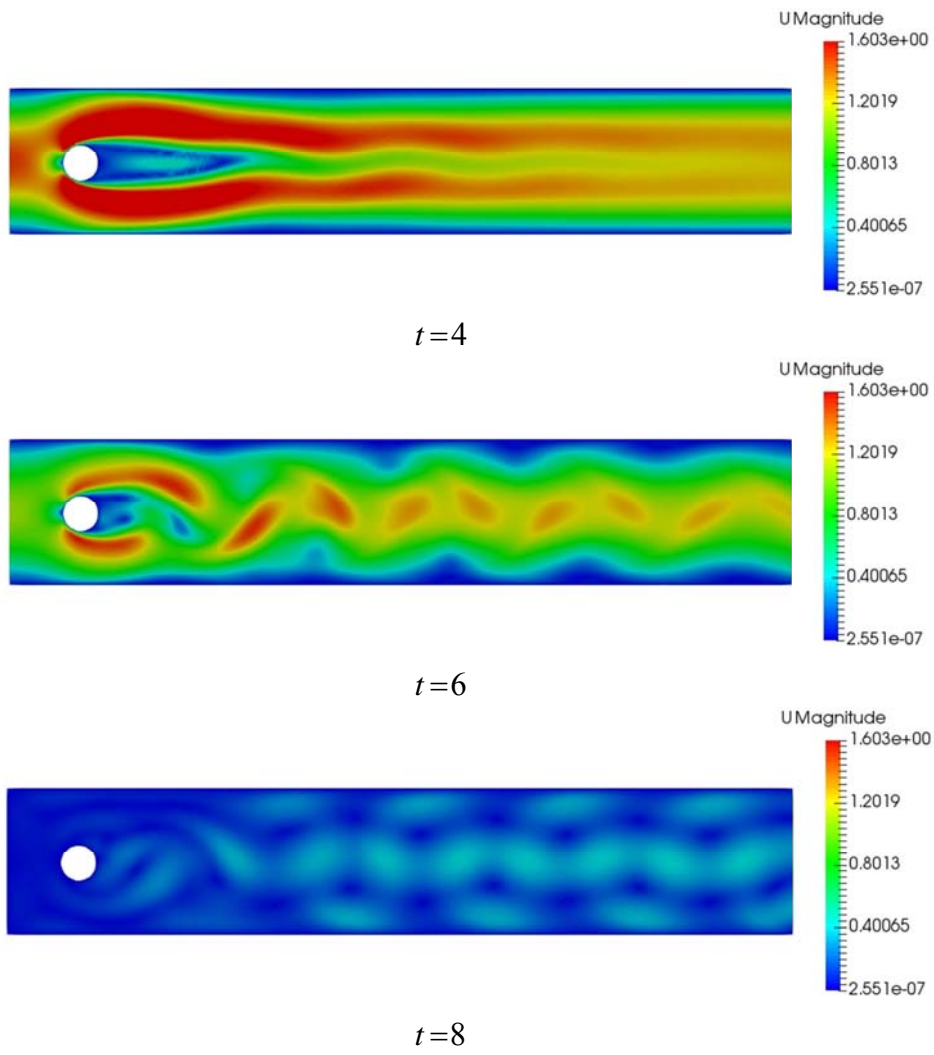


Figure 4.2 Velocity in cylinder case