HopeFOAM

(High Order Parallel Extensible CFD Software)

编程指南

版本 0.1

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2017 年 09 月

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关于 HopeFOAM

HopeFOAM 是一款在 OpenFOAM 基础上扩展了任意高阶有限元等数值方法的计算力学软件，由中国军事科学院国防科技创新研究员 Exercise 小组开发。该小组致力于开发针对大规模科学与工程计算的开源软件。HopeFOAM 具有以下特点：

- **高阶（High Order）**：在保留原有 OpenFOAM 有限体积离散方法之外，HopeFOAM 致力于融合更多高阶数值离散方法形成计算力学工具箱，其中 DGM（Discontinuous Galerkin Method，间断伽辽金方法）是第一个设计实现的高阶离散方法。
- **并行（Parallel）**：为了改进并行计算的性能及可扩展性，HopeFOAM 集成了多种并行计算工具集或软件以加速离散和计算过程。
- **可扩展（Extensible）**：在引入了高阶数值离散与高效并行计算的基础上，HopeFOAM 将为使用者的进一步应用功能开发提供一个可扩展的软件框架及便捷接口。
- **FOAM**：此版 HopeFOAM 是基于 2016 年 6 月 OpenFOAM-4.0 进行大幅扩展而成。

HopeFOAM-0.1 作为 HopeFOAM 的第一个公开发布版本，是基于 OpenFOAM-4.0 大幅扩展而成。广为人知的高阶数值离散方法 DGM（Discontinuous Galerkin Method，间断伽辽金方法）被设计实现于 HopeFOAM-0.x 系列中，关于该方法的相关知识可参考书籍：【Hesthaven J S, Warburton T. Nodal discontinuous Galerkin methods: algorithms, analysis, and applications[M]. Springer Science & Business Media, 2007.】

HopeFOAM-0.1 提供二维 DGM 离散及相关技术支持，主要包括数据结构、DGM 离散、求解器及相关工具，同时 PETSc 被集成用于求解线性方程系统。三维应用相关功能将在未来几个月的新版本中发布。

开发 HopeFOAM-0.1 的一个重要原则就是尽可能复用 OpenFOAM-4.0 的原始数据结构，并保持用户接口的一致性。因此，OpenFOAM 用户可基于以往经验通过较为直接的途径实现或使用对应高阶 DGM 求解器。同时，原有 OpenFOAM 支持的功能可以继续使用。

本手册是高阶并行可扩展开源 CFD 软件（HopeFOAM）0.1 版本编程指南。

1 代码结构

1.1 软件整体架构

HopeFOAM-0.1 整体基于 OpenFOAM-4.0 开发，原有 OpenFOAM-4.0 的 FVM(Finite volume method)方法软件架构如下图 1 所示。

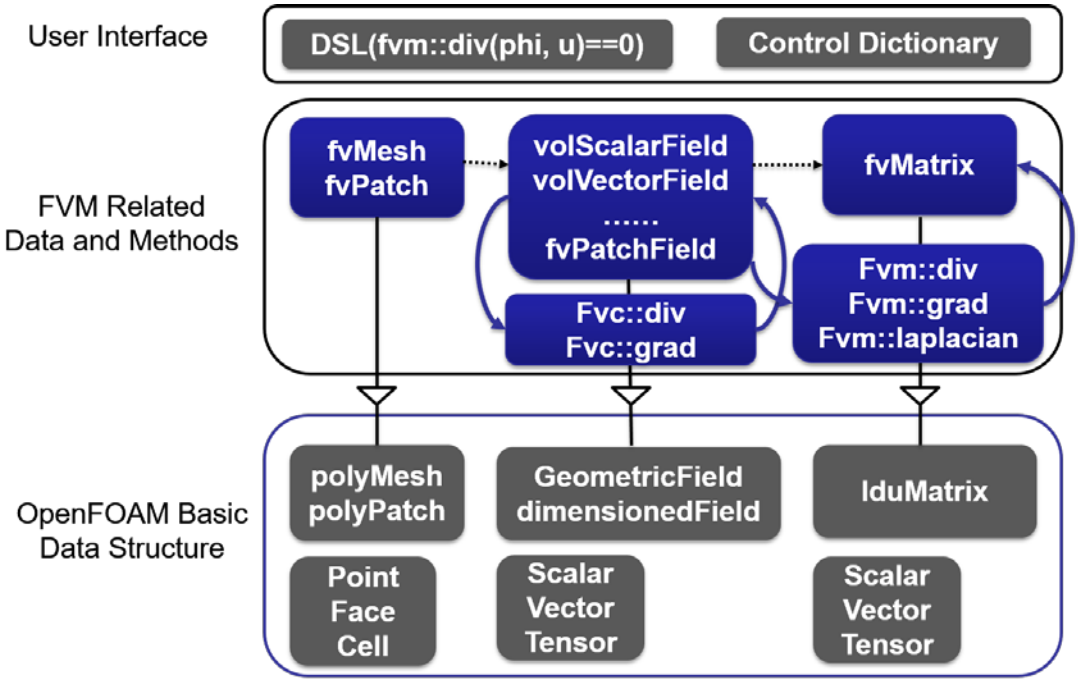

Fig.1 The origin FVM framework of OpenFOAM-4.0

在上述 OpenFOAM-4.0 架构中，自底向上主要分为三层：

- OpenFOAM Basic Data structure: 该层次的代码主要用于 CFD 计算的底层数据结构，包括网格、场数据、线性代数系统等。该层的数据结构与特定的离散方法(FVM,FEM 等)无关，主要目的是为普适的数值离散方法提供基础支持，能够支持大多数值离散方法开发。但在具体实现上，更偏向于 FVM 方法的高效支持。
- FVM Related Data and Methods: 该层次代码主要实现 FVM 方法，向下基于 OpenFOAM Basic data structure 开发 FVM 特有的数据结构，包括网格支持 FVM 的网格、场数据、线性系统等。同时还实现了 FVM 离散算子。
- User Interface: 为 CFD 用户提供 DSL 语言特性的使用接口。

基于上述框架，我们设计了 HopeFOAM-0.1 的软件框架，具体如图 2 所示。

该框架的在 OpenFOAM-4.0 的基础上增加了 DG(Discontinuous Galerkin)模块。该框架的设计遵循以下原则：

- Reuse the framework and data structure
- Keep consistent with the user interface
- Inherit the pre and post processing tools
- Add a separate DG source package
- High order DG discretization
- High performance
- High extensibility, runtime selection

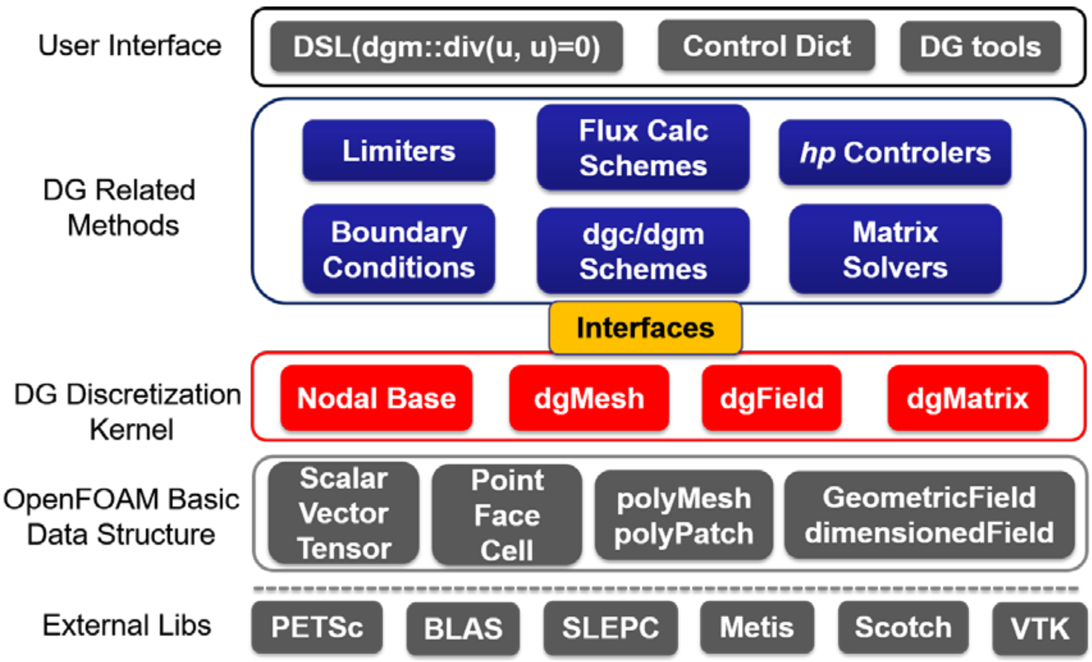

Fig.2 The Framework of HopeFOAM-0.1

与原有的 OpenFOAM-4.0 框架相比，新的 HopeFOAM 框架在没有更改原有 OpenFOAM 模块的前提下，进行了大幅度 DG 模块添加：

- External Libs：为提高求解性能和构建完整软件生态，引入了部分第三方软件库，包括：PETSc、BLAS、SLEPC 用于高效线性系统并行求解，Metis、Scotch 用于网格并行划分支持，VTK 用于数据可视化。
- OpenFOAM basic data structure: 保留并复用。
- DG Discretization Kernel: 该部分和其上层的 DG related data and methods 共同构成 HopeFOAM-0.1 中关于 DG 方法的核心。该部分主要提供 DG 方法的基函数、网格、场数据、线性系统等数据结构支持。

- DG related data and methods：该部分主要提供 DG 方法中的边界条件、显式/隐式离散算子、*hp* 自适应、通量计算、限制器等支持。
- User Interface：该部分主要为 CFD 用户提供 DG 计算接口，除常规微分算子以外，还添加了 DG control dict(DG 求解参数配置)和 DG tools 等支持。

1.2 代码目录树状结构解析

HopeFOAM-0.1 的中 DG 核心目录结构及相应的功能如下：

```
|-- HopeFOAM-0.1
    |-- src
      --|-- DG: All DG modules
          |   |-- convectionSchemes : convection Schemes
          |   |   |-- convectionScheme: Abstract base for Convection Scheme
          |   |   `-- defaultConvectionScheme: default Convection Scheme
          |   |-- ddtSchemes: ddt Schemes
          |   |   |-- EulerDdtScheme: Euler Ddt Scheme
          |   |   `-- ddtScheme: Abstract base for ddt Scheme
          |   |-- dg: Namespace for Discontinuous Galerkin Method
          |   |-- dgSchemes: Selector class for DG Method differencing schemes
          |   |-- dgSolution: Selector class for DG solution solution
          |   |-- dgc: Namespace of functions to calculate explicit derivatives
          |   |-- dgm: Namespace of functions to implicit derivatives returning a matrix
          |   |-- divSchemes: class for div schemes
          |   |   |-- defaultDivScheme: Basic second-order div using face-gradients
          |   |   `-- divScheme: Abstract base class for div schemes
          |   |-- godunovFlux: Generic Godunov flux class
          |   |   |-- fluxSchemes: Abstract base class for flux Calculation schemes
          |   |   |   `-- scheme:
          |   |   |         `-- RoeFlux: Roe flux scheme class
          |   |   `-- limiteSchemes: Abstract base class for limiter
          |   |         `-- scheme
          |   |               `-- Trianglelimite: limite is only support triangle mesh
          |   |-- gradSchemes: class for grad Schemes
          |   |   |-- defaultGrad: class for default grad Schemes
          |   |   `-- gradScheme: Abstract base class for gradient schemes
          |   |-- laplacianSchemes: Basic second-order laplacian
          |       |       |-- defaultLaplacianScheme: second-order laplacian using face-gradients
          |   |   `-- laplacianScheme: Abstract base class for laplacian schemes
          |   `-- simpleFlux: flux calculation schemes
          |         |-- fluxCalcScheme: Abstract base class for flux calculation schemes
          |         `-- schemes
```

```
|                    |-- LFFlux: LF flux scheme class
|                    |-- averageFlux: average flux scheme class
|                    `-- noneFlux: zero flux scheme class
|-- Equation: composite design pattern to construct an equation
|-- Make
|-- cfdtools: include files for solvers
|      `-- general
|             `-- include
|-- dgMatrices: linear algebraic system
|      |-- denseMatrix: small dense matrix for the calculation of base functions
|      |-- dgLduMatrix: a general matrix class stored with LDU format
|      |-- dgMatrix: A special matrix type and solver
|      `-- dgScalarMatrix: A scalar instance of dgMatrix
|-- dgMesh: Mesh data for DG
|      |-- dgBoundaryMesh: Boundary Mesh for DG
|      |-- dgPatches: patch classes for DG
|      |      |-- basic: basic patchs classes for DG
|      |      |      |-- coupled: An abstract base class for patches that couple regions
|      |      |      `-- generic: DG variant of the genericPolyPatch
|      |      |-- constraint:
|      |      |      |-- arc: A arc patch
|      |      |      |-- empty: A patch which will not exist in the dgMesh
|      |      |      `-- processor: Processor patch
|      |      |-- derived
|      |      |      `-- wall: wall patch
|      |      `-- dgPatch: A DG patch using a polyPatch and a dgBoundaryMesh
|      |-- dgTree: A template class, the information is organized into a tree
structure
|      `-- polyPatchs:
|             `-- constraint
|                    `-- arc: arc front and back plane patch
|-- element
|      |-- baseFunctions
|      |      |-- baseFunction: Abstract class for Legendre base Function.
|      |      `-- straightBaseFunctions
|      |             |-- lineBaseFunction: standard Line Element
|      |             |-- quadrilateralBaseFunction:     standard quadrilateral Element
|      |             |-- tetrahedralBaseFunction：  standard Tetrahedral Element
|      |             `-- triangleBaseFunction：     standard Triangle Element
|      |-- dofAddressings
|      |      |-- dgDofAddressing：  Addressing rule for dof in dg method.
|      |      `-- dofAddressing：  Abstract class indicating the addressing rule for dof.
|      |-- gaussIntegration
|      |      |-- gaussIntegration：  Abstract class for gaussIntegration.
```

```
|       |       |-- gaussLineIntegration： gauss interpolation for Line Element
|       |       |-- gaussQuadrilateralIntegration：  gaussQuadrilateralIntegration
|       |       |-- gaussTetrahedralIntegration:  standard Tetrahedral Element
|       |       `-- gaussTriangleIntegration: standard Triangle Element
|       |-- physicalElementData: Contains the information for critical cell discretisation
|       |-- polynomials:
|       |       `-- Legendre: Abstract base class for standard Element.
|       |-- stdElement: including baseFunction and gauss integration Method
|       `-- stdElementSets: Hash Table from names to all the stdElemets
`-- fields: fields data for DG
        |-- GeometricDofField: GeometricField with Dof Addresing supported
        |       |-- GeometricDofSphericalTensorField: Spherical Tensor specific part
        |       |-- GeometricDofSymmTensorField: SymmTensor specific part
        |       `-- GeometricDofTensorField: Tensor specific part
        |-- dgFields: fields data for DG
        |-- dgGaussField: Field to store the data intepolated from GeometricDofField with gauss base
        `-- dgPatchFields: classes for DG Patch Fields
                |-- basic
                |       |-- calculated: calculated DgPatch Field
                |       |-- coupled: Abstract base class for coupled patches.
                |       |-- fixedGradient: supplies a fixed gradient condition
                |       |-- fixedValue: supplies a fixed value condition
                |       |-- mixed: class for 'mixed' type boundary
                |       |-- reflective: reflective boundary condition
                |       `-- zeroGradient: zero-gradient condition
                |-- constraint: classes for constraint DG Patch Fields
                |       |-- cyclic: enforces a cyclic condition between a pair of boundaries.
                |       |-- empty:
                |       |-- processor: processor communication across patches
                |       `-- slip: the vector is symmetry to the normal of the patch
                |-- derived: classes for derived DG Patch Fields
                |       |-- freestream: free-stream condition
                |       |-- freestreamPressure: free-stream condition for pressure
                |       |-- inletOutlet: generic outflow condition
                |       `-- noSlip: fixes the velocity to zero at walls
                `-- dgPatchField
```

2 关键数据结构

2.1 网格数据结构:dgMesh

DG 离散基于网格数据结构 dgMesh 完成，Fig.1 描述了 dgMesh 相关数据结构关系。dgPolyMesh 保存网格几何信息，dgScheme 与 dgSolution 分别保存算例中同名的配置文件的信息，physicalElementData 保存 DG 离散时所需的基函数和相关系数矩阵。

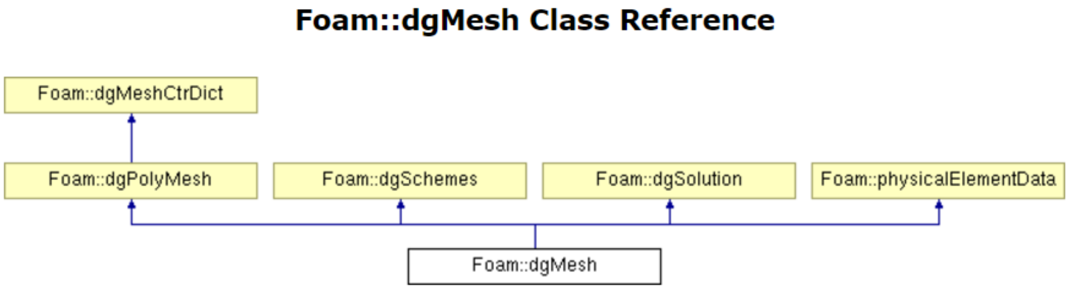

Fig.1 网格相关数据结构类图

dgMeshCtrDict & dgPolyMesh：完成算例目录下配置文件 dgSolution 的读取，dgSolution 中用户定义了网格的空间维数和 DG 的离散阶数；dgPolyMesh 继承自 OpenFOAM 基础网格数据结构 polyMesh，结合 dgMeshCtrDict 提供的空间维数生成 DG 离散的树状网格。polyMesh 中网格信息可用 Fig.2 表示，dgPolyMesh 中网格信息被组织成和特殊的树状结构（对应 DG 中的 dgTree<Type>数据结构），在 dgPolyMesh 中 Cell 和 Face 都被组织成这种形式，分别对应成员数据 cellTree_ 和 faceTree_，该树状结构可用 Fig.3 表示，dgTree<Type>上层为一个指针数组，每一个指针指向一个 dgTreeUnit<Type>对象，通过在 dgTreeUnit<Type>中装入 Cell 和 Face 完成网格信息存储。

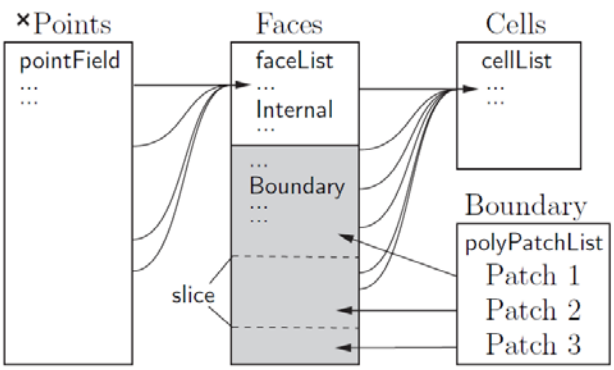

Fig.2 polyMesh 网格几何信息

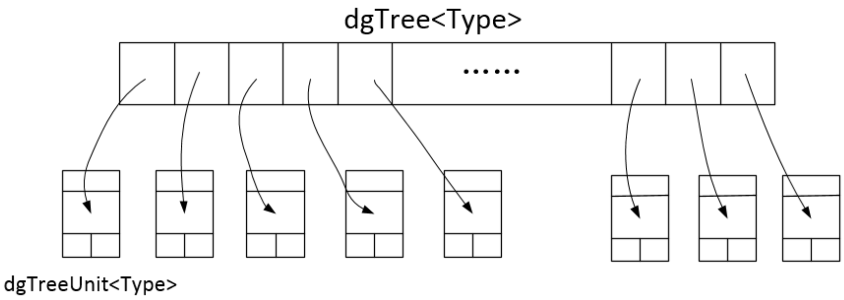

Fig.3 特殊树状结构

dgScheme & dgSolution：dgSolution 用于完成 DG 求解方法选择，dgScheme 则完成 DG 离散算子离散方法的选择，dgMesh 作为其子类可以保证对场的操作能够通过 dgMesh 获取相关信息。

physicalElementData：保存 DG 离散过程相关数值基础，在 1.3 小节会详细介绍。

2.2 场数据结构：GeometricDofField

GeometricDofField 是保存基于 dgMesh 场数据的模板类，继承自 OpenFOAM 数据结构 GeometricField，三个模板参数的介绍如下：

- Type：表示场数据的基本数据类型，如 label、vector 等；
- PatchField：表示边界场的数据类型，在 GeometricDofField 中该参数只能为 dgPatchField<Type>，dgPatchField 为 DG 方法对应的边界场数据结构，用于保存边界面上 DG 离散点的场值，与 OpenFOAM 中 fvPatchField 的功能相似。
- GeoMesh：表示离散网格的类型，在 GeometricDofField 中该参数只能为 dgMesh。

GeometricDofField 中包含以下成员数据：

- boundaryField_：该数据保存边界网格离散点的场值，该数据结构并未单独定义，是通过在 GeometricField 中定义的 Boundary 内部类完成，可通过成员函数 boundaryField()和 boundaryFieldRef()获取；
- timeIndex_：label 数据类型，用于保存模拟求解过程中时间步的索引值，以确定保存的场值是否为当前时间步所得，从而确定成员数据 field0Ptr_是否需要更新；
- field0Ptr_：GeometricDofField 类指针，存储上一个时间步的场值；
- fieldPrevIterPtr_：GeometricDofField 类指针，存储迭代过程中上一个迭代步的场值。

➢ gaussField_：dgGaussField 类指针，指向基于 GeometricDofField 生成的高斯场，其中存储的是基于原离散点及场值生成的高斯点的场值。

2.3 基函数数据结构：physicalElementData

基函数是 DG 数值离散的基础，在这个过程中，包含三个主要步骤：1)基于 dgPolyMesh 中网格的几何信息及离散阶数进行插点，形成 DG 高阶离散所需的高阶网格；2）几何单元向标准单元映射；3）基于标准单元基函数完成数值离散。

高阶离散是通过使用更高阶数的多项式对流场中的场值进行拟合，因此所需要的流场信息也会增加，即增加高阶点，以二维三角形单元为例，Fig.4 表示基于 dgPolyMesh 的网格插点得到 DG 三阶离散所需网格的过程。OpenFOAM 网格中的 Face 和 Cell 插入高阶点后，分别对应 DG 框架中的 physicalFaceElement 类与 physicalCellElement 类，所得高阶离散网格信息同样被存储为特殊的树状结构 dgTree<Type>，physicalElementData 中的 faceElementsTree_与 cellElementsTree_分别为面树和单元树。

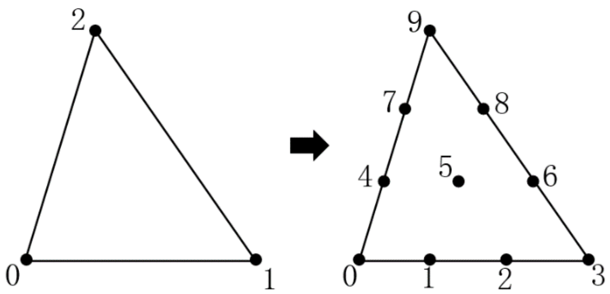

Fig.4 三角形单元三阶插点过程

以二维空间下三角形 Cell 为例，Fig.5 表示一般直边三角形与到标准单元的映射，通过坐标系变化将所有单元映射成标准单元，唯一不同的是每个单元的映射关系Ψ，后续的数值积分则基于标准单元展开，具体数值原理的讲解请参阅[1]。单元的映射信息被保存在 physicalCellElement 类中。

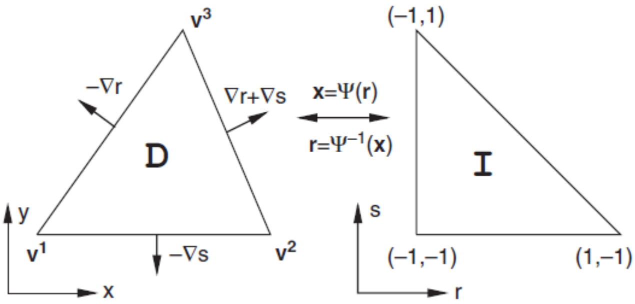

Fig.5 三角形单元映射过程

标准单元的基函数通过的 physicalElementCell 的 baseFuntion_访问，基函数部分的类图如 Fig.6 所示。

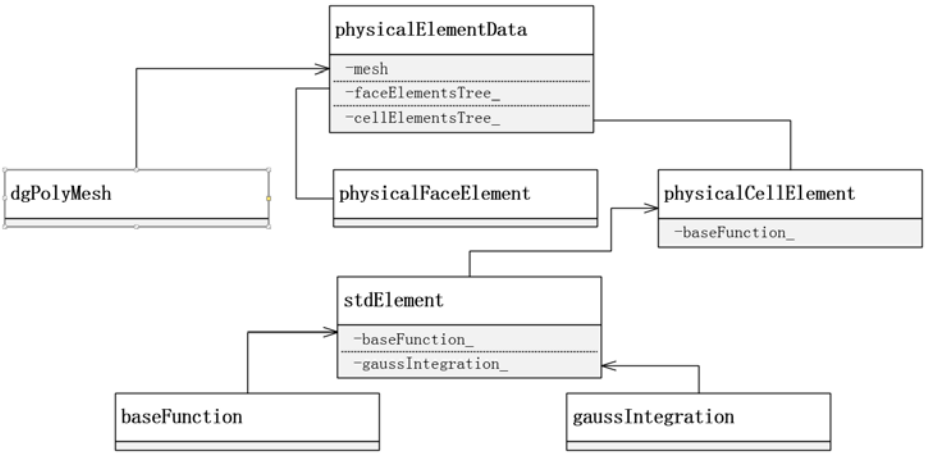

Fig.6 基函数类图

2.4 矩阵数据结构：dgMatrix

数值离散方法对连续偏微分方程离散后得到线性方程组，DG 离散所得的线性方程组的存储于求解由 dgMatrix 类负责，DG 框架矩阵部分的数据结构类图如 Fig.7，这样的数据结构与 DG 离散后的线性方程组系数矩阵的性质相关，系数矩阵为稀疏矩阵，denseMatrix 类用于存储该稀疏矩阵中的小型稠密块，dgLduMatrix 类将所有稠密块组装形成最终的系数矩阵，保存在 dgMatrix 类的 mat_中，ksp_与 mat_都属于外部依赖包 PETSc 的数据结构，用于求解线性方程组。

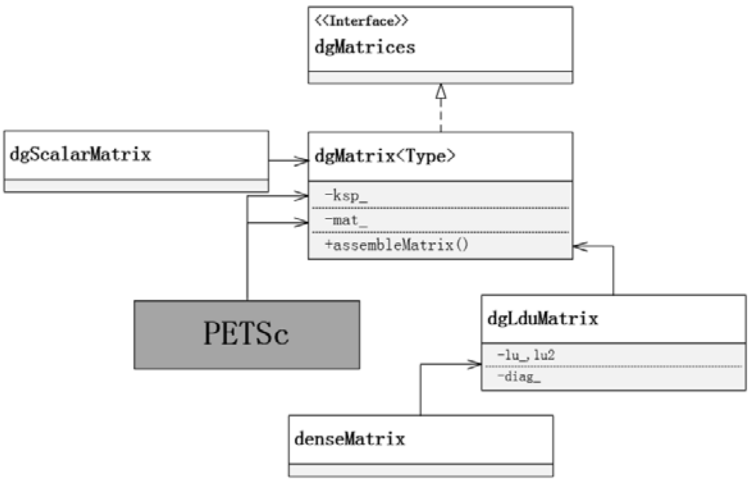

Fig.7 矩阵部分类图

3 典型离散与求解过程

3.1 DG 数值原理简介

本小节将以一个简单偏微分方程为例，通过介绍对其使用 DG 进行数值离散的过程，从而阐明 DG 数值离散原理。

DG 作为一种高阶有限元方法，相比于普通高阶有限元法最大的特点是在 Cell 与 Cell 之间引入间断，从而能够将有限元求解过程中的残差积分过程约束在元内部，从而对任意的场值函数 u 在任意元 D_k 内部，其 N 阶多项式拟合函数 u_h^k 满足

$$u_h^k = \sum_{i=1}^{N} \hat{u}_i^k \cdot \varphi_i^k \tag{1}$$

考虑方程

$$\frac{\partial u}{\partial x} = 0 \tag{2}$$

根据残量与任意基函数 $\varphi_j^k(j = 1, 2, \cdots, N)$ 正交的性质，并在 D_k 内对式(2)两侧积分可得

$$\begin{aligned} 0 &= \int_{D_k} \frac{\partial u_h^k}{\partial x} \cdot \varphi_j^k dx \\ &= -\int_{D_k} u_h^k \cdot \partial \varphi_j^k dx + \oint_{D_k} \vec{\boldsymbol{n}} \cdot u_h^{k*} \cdot \varphi_j^k dx,\ (j = 1, 2, \cdots, N) \end{aligned} \tag{3}$$

其中 $u_h^{k*} = f(u^{k-}, u^{k+})$，由此可将其转为体积分与面积分的和。

将式(1)代入式(3)，使用矩阵表示相应的系数可得

$$\begin{aligned} \int_{D_k} u_h^k \cdot \varphi_j^k dx &= \int_{D_k} \sum_{i=1}^{N} \hat{u}_i^k \cdot \varphi_i^k \cdot \varphi_j^k dx \\ &= \sum_{i=1}^{N} \hat{u}_i^k \int_{D_k} \varphi_i^k \cdot \varphi_j^k dx,\ (j = 1, 2, \cdots, N) \end{aligned} \tag{4}$$

$$\int_{D_k} u_h^k \cdot \partial \varphi_j^k dx = \int_{D_k} \sum_{i=1}^{N} \hat{u}_i^k \cdot \varphi_i^k \cdot \partial \varphi_j^k dx$$

$$=\sum_{i=1}^{N}\hat{u}_i^k\int_{D_k}\varphi_i^k\cdot\partial\varphi_j^k dx,\ (j=1,2,\cdots,N) \tag{5}$$

将式(4)与(5)代入(3)便可得到以$\hat{u}_i^k$ $(i=1,2,\cdots,N)$为未知数的线性方程组，由此完成偏微分方程(2)的数值离散。

3.2 离散算子介绍

DG 框架的离散算子是通过使用 DG 数值方法对偏微分方程中的差分算子进行数值离散所得，HopeFOAM-0.1 中支持 DG 的离散算子如 Tab.1。

Tab.1 Discretisation of PED terms in DG of HopeFOAM-0.1

Term description	Implicit/ Explicit	Text Expression	dgm::/dgc:: function
Time derivative	Imp/Exp	$\frac{\partial\phi}{\partial t}$	ddt(phi)
Convection	Imp/Exp	$\nabla\cdot(U\phi)$	div(U,phi)
Divergence	Exp	$\nabla\cdot\phi$	div(phi)
Gradient	Exp	$\nabla\phi$	grad(phi)
Laplacian	Imp	$\nabla^2\phi$	laplacian(phi)
Source	Imp	ϕ	Sp(phi)

phi:dg<Type>Field

U:Field of velocity

3.3 离散系统架构解析

DG 在上层离散算子接口部分与 OpenFOAM 相似（Fig.8），使用 dgm 与 dgc 分别作为隐式离散与显式离散的接口，根据离散形式通过 dgm 与 dgc 访问不同离散算子的接口，从而访问不同的离散算子。

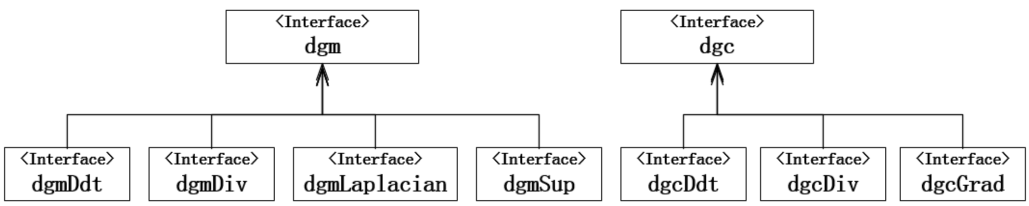

Fig.8 离散系统上层接口结构

通过离散算子接口访问离散算子的功能函数，由 DG 框架网格的按照单元组织的性质，决定了数值离散过程也按照单元组织，Equation 类用于保存离散过程的中间结果，EquationAdd 类提供 Equation 相加功能，EquationMul 类提供 Equation 与常数相乘功能，EquationEqual 类提供两个 Euqation 相等的运算功能。

各个离散算子的组织结构上相似，本节以一阶时间离散算子 ddt 为例，解析 DG 离散算子的组织结构，Fig.9 为 ddt 相关数据结构类图。通过 ddt 算子的显式接口 dgcDdt 和隐式接口 dgmDdt 中的 ddt() 成员函数访问 EquationDdtSchemeScheme<Type>类中的 dgcNew()与 dgmNew()函数，从而分别产生 EquationEulerDgcDdtScheme<Type>类和 EquationEulerDgmDdtScheme<Type>类的对象，通过调用 calculateCell()函数分别调用一阶时间离散算子的功能函数 dgcDdtCalculateCell()函数和 dgmDdtCalculateCell()函数，完成基于单个单元的一阶时间离散。其余算子的数据结构与之类似。

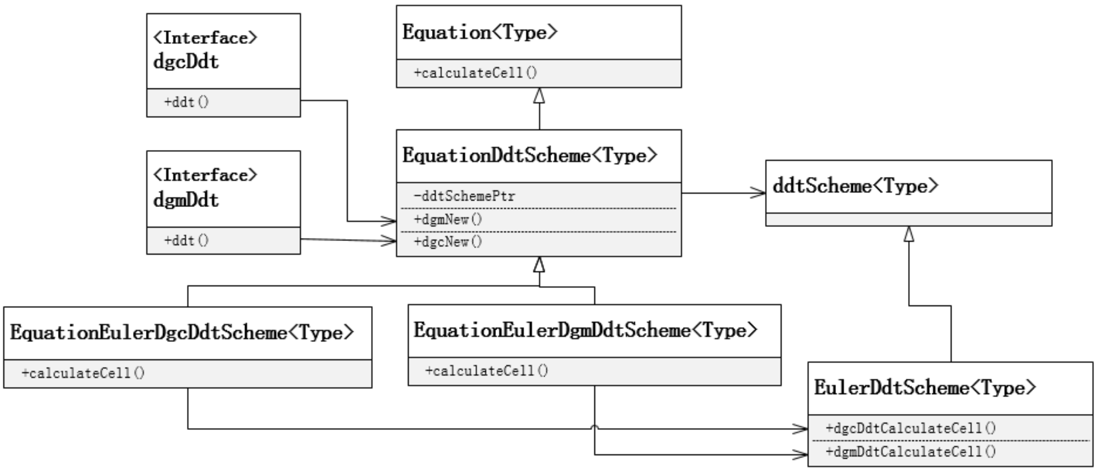

Fig.9 ddt 离散相关数据结构类图

DG 框架中通量的计算位于 simpleFlux 和 godunovFlux 目录下，simpleFlux 下包括 averageFlux，LFflux，noneFlux 三种简单的通量计算方式，godunovFlux 为较为复杂的通量计算方法，该版本提供 DG 的 Roe 通量计算，在该文件夹下的 limiter 文件夹下为限制器功能，该版本仅提供一个一阶的限制器。

4 HopeFOAM 典型算例开发：圆柱绕流

4.1 问题描述

圆柱绕流是一个依赖于时间的二维渠道流，在渠道中间有一圆形的障碍物，此圆柱放置在偏离渠道中心线处，因此，一旦由零初始条件慢慢形成抛物型流时，它就开始产生涡流。这个案例于 90 年代中期提出，是一个标准测试问题。此问题的空间区域如 Fig.10 所示。

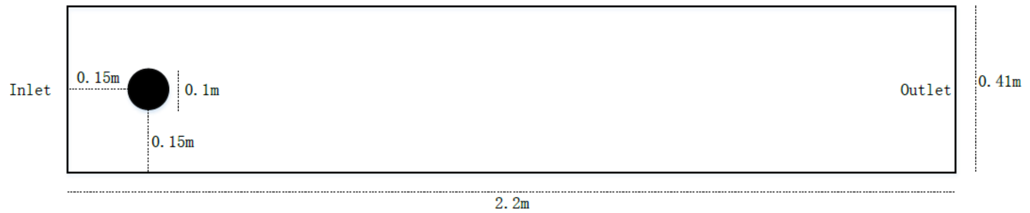

Fig.10 圆柱绕流计算域示意图

数值模拟时间为 8s，初始时刻 t=0，流入边界 Inlet 增加抛物型流，并乘上时间的正弦函数，其表达式如式(6)。

$$
\begin{aligned}
&u(x,y,t)=0.41^{-2}\sin(\frac{\pi t}{8})6(y+0.2)(0.21-y),\\
&v(x,y,t)=0.
\end{aligned}
\tag{6}
$$

4.2 求解器开发

4.2.1 DG 内核依赖声明

使用 DG 离散方法需要引入头文件 dgCFD.H。此外还需引入如下头文件：

- setRootCase.H：用来初始化相关系统参数；
- createTime.H：创建时间控制对象 runTime；
- createField.H：该头文件为用户自行定义，用于定义求解过程中需要使用的场数据及其他数据；
- createMesh.H：该头文件构造数值模拟的网格对象，如果使用 DG 方法，则需构造 dgMesh 对象；
- createTimeControl.H：该头文件引入后允许用户通过定义算例下 system/conrolDict 中的配置参数来调节 runTime。

4.2.2 场值初始化与边界面场值更新

在 OpenFOAM 中，场值初始化与边界面场值更新可以通过用户在算例目录下的配置文件中配置边界类型完成，但由于目前 DG 框架所支持的边界类型有限，对于初值复杂以及边界场值随时间变化的算例，需要在求解器中编写代码进行赋值。在本算例中，场值初始化在 setNonUniformInlet.H 文件对入口边界的速度场进行了初始化，边界面场值在 setBoundaryValues.H 中进行更新，每一个时间步结束都会调用该文件更新边界场值。

4.2.3 物理方程描述

物理方程描述 DG 框架仍然沿用 OpenFOAM 的描述风格，只改变了离散的命名空间(namspace)，dgc 为隐式离散空间，dgm 为显式离散空间。以式(7)为例

$$U^{n+1} + \nabla p^{n} = U^{n} \tag{7}$$

其中上标 n 表示时间步，n+1 表示当前时间步，n 表示上一个时间步，因此U^{n+1}为未知量，p^{n}与U^{n}为上一个时间步的值，即为已知量。其使用在求解器中的表示如式(8)

$$dgm::Sp(U) + dgc::grad(p) == U \tag{8}$$

对方程的求解通过调用 dg::solveEquation()函数实现，如(9)

$$dg::solveEquation(dgm::Sp(U) + dgc::grad(p) == U) \tag{9}$$

参考文献

[1] Hesthaven J S, Warburton T. Nodal discontinuous Galerkin methods: algorithms, analysis, and applications[M]. Springer Science & Business Media, 2007.