

HopeFOAM

(High Order Parallel Extensible CFD Software)

用户指南

版本 0.1

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关于 HopeFOAM

HopeFOAM 是一款在 OpenFOAM 基础上扩展了任意高阶有限元等数值方法的计算力学软件，由中国军事科学院国防科技创新研究院 Exercise 小组开发。该小组致力于开发针对大规模科学与工程计算的开源软件。HopeFOAM 具有以下特点：

- **高阶 (High Order):** 在保留原有 OpenFOAM 有限体积离散方法之外，HopeFOAM 致力于融合更多高阶数值离散方法形成计算力学工具箱，其中 DGM (Discontinuous Galerkin Method, 间断伽辽金方法) 是第一个设计实现的高阶离散方法。
- **并行 (Parallel):** 为了改进并行计算的性能及可扩展性，HopeFOAM 集成了多种并行计算工具集或软件以加速离散和计算过程。
- **可扩展 (Extensible):** 在引入了高阶数值离散与高效并行计算的基础上，HopeFOAM 将为使用者的进一步应用功能开发提供一个可扩展的软件框架及便捷接口。
- **FOAM:** 此版 HopeFOAM 是基于 2016 年 6 月 OpenFOAM-4.0 进行大幅扩展而成。

HopeFOAM-0.1 作为 HopeFOAM 的第一个公开发布版本，是基于 OpenFOAM-4.0 大幅扩展而成。广为人知的高阶数值离散方法 DGM (Discontinuous Galerkin Method, 间断伽辽金方法) 被设计实现于 HopeFOAM-0.x 系列中，关于该方法的相关知识可参考书籍：【Hesthaven J S, Warburton T. Nodal discontinuous Galerkin methods: algorithms, analysis, and applications[M]. Springer Science & Business Media, 2007.】

HopeFOAM-0.1 提供二维 DGM 离散及相关技术支持，主要包括数据结构、DGM 离散、求解器及相关工具，同时 PETSc 被集成用于求解线性方程系统。三维应用相关功能将在未来几个月的新版本中发布。

开发 HopeFOAM-0.1 的一个重要原则就是尽可能复用 OpenFOAM-4.0 的原始数据结构，并保持用户接口的一致性。因此，OpenFOAM 用户可基于以往经验通过较为直接的途径实现或使用对应高阶 DGM 求解器。同时，原有 OpenFOAM 支持的功能可以继续使用。

本手册作为 HopeFOAM-0.1 的使用指南，将通过 4 个案例教程指导读者练习 HopeFOAM-0.1 的基本使用方法。求解器源代码和执行命令位于案例目录下，所有算例均位于 `tutorials/DG/2D` 目录下。

1 可压熵涡流

本节介绍如何对二维可压缩无粘流体熵涡流问题算例进行前处理、运行和后处理。计算区域几何为 $0\text{m} \leq x \leq 10\text{m}$, $-5\text{m} \leq y \leq 5\text{m}$, 如图 1.1 所示。

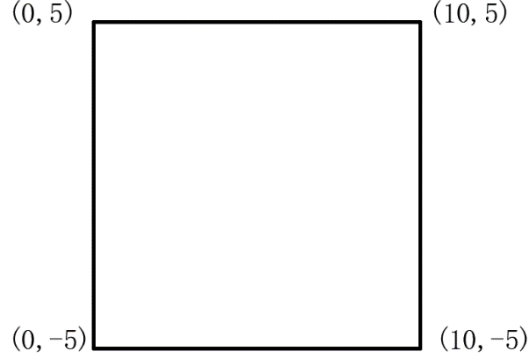


图 1.1 可压熵涡流几何

可压缩流体熵涡流问题的精确解为：

$$\begin{aligned} u &= u_0 - \beta e^{(1-r^2)} \frac{y - y_0}{2\pi}, \\ v &= v_0 + \beta e^{(1-r^2)} \frac{x - x_0 - t}{2\pi}, \\ \rho &= \left(1 - \left(\frac{\gamma - 1}{16\gamma\pi^2} \right) \beta^2 e^{2(1-r^2)} \right)^{\frac{1}{\gamma-1}}, \\ p &= \rho^\gamma, \end{aligned}$$

其中 $r = \sqrt{(x - t - x_0)^2 + (y - y_0)^2}$, $x_0 = 5$, $y_0 = 0$, $u_0 = 1$, $v_0 = 0$, $\beta = 5$, $\gamma = 1.4$, 表示涡流从计算区域中心以 1 米每秒的速度向右移动。

1.1 网格生成

切换到算例目录 isentropicVortex:

```
cd $FOAM_RUN/tutorials/DG/2D/isentropicVortex
```

本算例几何外形为 xy 平面的正方形，正方形边界设置为 wall。HopeFOAM 支持结构网格和非结构网格，一方面，可以使用 OpenFOAM 提供的 blockMesh 工具，通过配置 system/blockMeshDict 文件以生成结构网格命令生成网格。system/blockMeshDict 文件边界条件信息如下：

```
40 boundary
41 (
42     Wall
43     {
```

```

44         type wall;
45         faces
46         (
47             (3 7 6 2)
48             (0 4 7 3)
49             (2 6 5 1)
50             (1 5 4 0)
51         );
52     }
53     frontAndBackPlanes
54     {
55         type empty;
56         faces
57         (
58             (0 3 2 1)
59             (4 5 6 7)
60         );
61     }
62 );

```

通过上述方法生成网格，仅需要直接执行如下命令：

```
$ blockMesh
```

同时，HopeFOAM 也支持使用诸如 Gambit、Pointwise、Fluent ICEM 等第三方工具生成网格。在本案例下，我们提供了三套不同尺度的 Fluent 网格文件 vortex0256.msh, vortex1024.msh, vortex4096.msh，分别对应的特征尺度为 h , $h/2$, $h/4$ ，如图 1.2 所示。

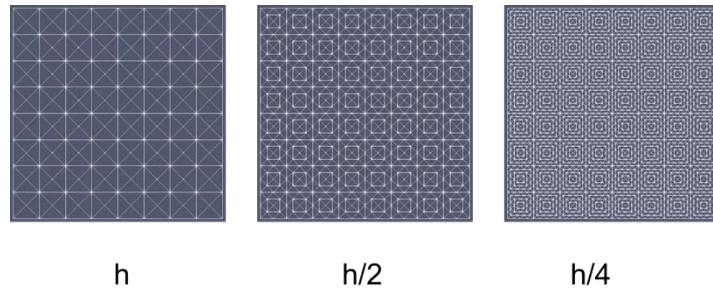


图 1.2 不同特征尺度网格

网格在算例目录下使用如下命令生成：

```
$ fluentMeshToFoam vortex0256.msh
```

用户可以查阅 OpenFOAM UserGuide 第五章详细了解网格相关内容。

1.2 初始条件和边界条件

HopeFOAM 中，诸如固定值、零梯度的简单边界条件可以通过配置场文件的 patch 类型指定。但是，isentropicVortex 算例的边界条件是通过用于表示解析解的复杂方程描述的。这里，我们将解析解编码在位于 dgEulerFoam 求解器目录的 setNonUniformInlet.H, setBoundaryValues.H 文件中。求解该问题的 Euler 方程可写成如下向量形式：

$$\frac{\partial \mathbf{q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = 0$$

$$\mathbf{q} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ E \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u(E + p) \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v(E + p) \end{pmatrix}$$

0 文件夹下包含 Ener, rho, rhoU, U, p 五个文件，上式中， ρ 为气体密度对应 rho 文件， (u, v) 为速度矢量的分量对应 U 文件， $(\rho u, \rho v)$ 动量矢量的分量对应于 rhoU 文件， p 为气体压力对应 p 文件， E 为气体能量对应 Ener 文件。其中 Ener, rho, rhoU 为原始变量，而 U 与 p 则通过如下方程导出：

$$E = \frac{p}{\gamma - 1} + \frac{\rho}{2}(u^2 + v^2)$$

边界和内部场的解析解可以通过上述方式给出，但 patch 的类型仍需明确。针对标量场 rho, p, Ener，它们的 internalField 内部场和 boundaryField 边界场类型指定如下：

```

19 internalField    uniform 0;
20
21 boundaryField
22 {
23     Wall
24     {
25         type          fixedValue;
26         value          uniform 0;
27     }
28
29     frontAndBackPlanes
30     {
31         type          empty;
32     }
33 }
```

矢量场 rhoU 和 U 的 internalField 内部场和 boundaryField 边界场类型指定为：

```

19 internalField    uniform (0 0 0);
20
21 boundaryField
22 {
23     Wall
24     {
25         type          fixedValue;
26         value          uniform (0 0 0);
27     }
28
29     frontAndBackPlanes
30     {
31         type          empty;
32     }
33 }
```

需要注意的是，上述文件的有效信息仅是指定 patch 的类型，其值并没有什么意义（值其实是通过之前头文件的形式直接写在代码中的）。

1.3 物理特性参数

isentropicVortex 算例需要指定的物理特性参数为气体类型无量纲常数 γ ，储存在 constant/transportProperties 字典文件，关键词为 gamma，如下所示：

```
18     gamma          gamma [0 0 0 0 0 0] 1.4;
19
20 // ***** //
```

1.4 离散方法和矩阵求解器设置

用户在 system 目录下的 dgScheme 文件中指定 DG 的离散格式，通量方法和限制器。与 OpenFOAM 类似，HopeFOAM 也支持 default 参数设置。这里，条目 godunovScheme 是用于对可压 NS 模拟设置通量与限制器的：

```
18 ddtSchemes
19 {
20     default          Euler;
21 }
22
23 gradSchemes
24 {
25     default          default none;
26     grad(p)          default none;
27     grad(gther_p)    default none;
28 }
29
30 godunovScheme
31 {
32     fluxScheme        Roe;
33     limiteScheme      Triangle;
34 }
35
```

dgSolution 文件指定逼近阶、残差以及其它算法控制。DG 子字典 meshDimension 网格维度设置为 2，baseOrder 逼近阶设置范围为 1-8。为测试高阶方法的高精度，solver 子字典的误差 tolerance 设置为 1e-12。

```
17 DG
18 {
19     meshDimension      2;
20     baseOrder          4;
21 }
22
23 solvers
24 {
25     dgrho
26     {
27         tolerance      1e-12;
28         relTol          0;
```

```

29     kspSolver      preonly;
30 }
31
32 rhoU
33 {
34     tolerance      1e-12;
35     relTol         0;
36     kspSolver      preonly;
37 }
38
39 Ener
40 {
41     tolerance      1e-12;
42     relTol         0;
43     kspSolver      preonly;
44 }
45
46 "rho(1|2|3)"
47 {
48     tolerance      1e-12;
49     relTol         0;
50     kspSolver      preonly;
51 }
52
53 "rhoU(1|2|3)"
54 {
55     tolerance      1e-12;
56     relTol         0;
57     kspSolver      preonly;
58 }
59
60 "Ener(1|2|3)"
61 {
62     tolerance      1e-12;
63     relTol         0;
64     kspSolver      preonly;
65 }
66 }
67
68 PISO
69 {
70     nCorrectors     2;
71     nNonOrthogonalCorrectors 0;
72     pRefCell        0;
73     pRefValue       0;
74 }
75
76 // *****

```

1.5 计算时间和控制

在 system 目录下 ControlDict 文件对时间步，输入输出时间进行控制。算例运算结束时，我们希望涡流没有穿过计算域，endtime 应小于 5s，我们设置结束时间为 2s。时间步长 deltaT 与网格大小和逼近阶有关，计算方法参考《Nodal Discontinuous Galerkin Methods Algorithms Analysis and Applications》6.4 节内容。

表 1.1 不同网格不同逼近阶 deltaT 设置

baseOrder	vortex0256	vortex1024	vortex4096
1	0.04	0.02	0.01
2	0.02	0.01	0.005
3	0.008	0.004	0.002
4	0.008	0.004	0.002
5	0.004	0.002	0.001
6	0.002	0.001	0.0005

1.6 运行算例

在计算案例之前，请在算例目录/求解器源代码文件夹下使用 `wmake` 生成求解器可执行文件。如同 `OpenFOAM` 程序一样，`HopeFOAM-0.1` 提供前置进程的方法运行求解器。

```
$ ./dgEulerFoamVortex
```

由于求解器源代码文件夹和执行命令都位于算例目录，它们不能重名，同时针对 `isentropicVortex` 算例修改了部分求解器代码，因此我们在标准执行命令后添加后缀 `Vortex`。

1.7 后处理

我们选择 `vortex1024.msh` 网格，`baseOrder=4` 案例进行计算。在结果写入时间步文件以后，使用命令

```
$ dgToVTK
```

生成 `VTK` 文件夹（`HopeFOAM` 支持高阶后处理），使用 `ParaView` 软件进行后处理。file>Open...选择生成的.vtk 文件，点击 OK。Properties 栏下点击 apply，选择 Coloring 下拉选框为 `rho`，`rhoU`，`Ener` 查看云图。

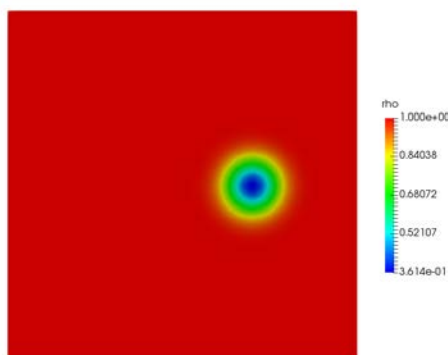


图 1.3 isentropicVortex 算例密度云图

更多使用方法参考 `OpenFOAM UserGuide` 第二章相关内容。

1.8 收敛阶计算

我们通过 eulererror.H 文件计算数值解精度。算例运行结束时，输出密度和动量数值解与精确解的误差：

rhoError: 8.807979526797244e-06

rhoUError: 1.865574862711117e-05

用户可以通过网格逐步加密实现相同逼近阶下收敛阶的计算。计算公式为：

$$\log_2\left(\frac{\varepsilon_h}{\varepsilon_{h/2}}\right)$$

ε_h 为特征尺度为 h 的密度或动量误差。

1.9 并行计算

针对计算量较大的问题，HopeFOAM 支持并行处理。与 OpenFOAM 提供 decomposePar 工具类似，首先配置 decomposeParDict 字典文件。numberOfSubdomains 指定计算区域划分的数量，通常等于处理器数量。simpleCoeffs 子字典中的向量 n 指定计算区域在 x, y, z 方向子区域的数量。保证 $n_x \times n_y \times n_z = \text{numberOfSubdomains}$ 。本算例几何为二维， $n_z = 1$ 。

```
26 numberOfSubdomains 12;  
27  
28 method simple;  
29  
30 simpleCoeffs  
31 {  
32     n      (3 4 1);  
33     delta  0.001;  
34 }
```

运行命令，将计算区域划分为 12 个子区域，算例目录生成 processor0 等 12 个文件。

```
$ dgDecomposePar
```

通过以下命令实现并行计算。

```
$ mpirun -np 12 ./dgEulerFoamVortex -parallel
```

计算完成之后通过以下命名将分割的场和网格合并：

```
$ dgReconstructPar
```

2 双马赫反射

双马赫反射与可压熵涡流使用同一求解器，类似地，我们修改了文件 `setNonUniformInlet.H`，`setBoundaryValues.H` 和 `eulererror.H` 文件。注意：进行双马赫反射模拟时器，`dgEulerFoam/dgEulerFoam.C` 文件中第 91 行和 118 行的注释符“//”被删除了，从而激活限制器。针对激波类问题模拟，为了模拟的稳定性，限制器是必不可少的。

```
91      Godunov.limite(rho1,rhoU1,Ener1);  
118     Godunov.limite(rho,rhoU,Ener);
```

保存文件，重新编译求解器。

```
$ wclean  
$ wmake
```

修改后的 `dgEulerFoam` 求解器文件夹保存在 `doubleMach` 算例目录下。

双马赫反射如图 2.1 所示，马赫数为 10 的激波向反射平面运动，激波与平面成 60 度夹角。计算区域几何为 $0 \leq x \leq 3.2$ ， $0 \leq y \leq 1$ ，如图 2.2 所示。气体类型无量纲常数 $\gamma=1.4$ 。初始时刻，马赫数为 10 的激波由 $(0.16667,0)$ ， $(0.74402,1)$ 两点连成的虚线表示。未扰动的气体（激波右侧） $\rho=1.4$ ， $p=1$ 。 $y=0$ 的 inlet 和 wall 边界为固定边界， $0 \leq x \leq 0.16667$ 的区域允许激波传播离开计算区域，wall 即为反射平面。 $y=1$ 的上部边界与时间有关，inlet 边界和 far 边界的交点向右移动，用来描述激波传播进入计算区域。当模拟时间等于 0.2 时，激波传播到计算区域的 outlet 边界附近。

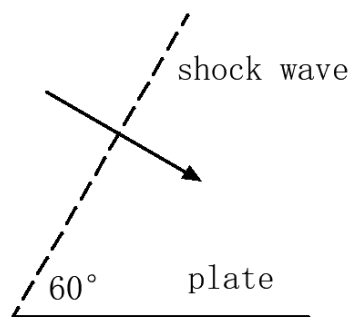


图 2.1 双马赫反射示意图

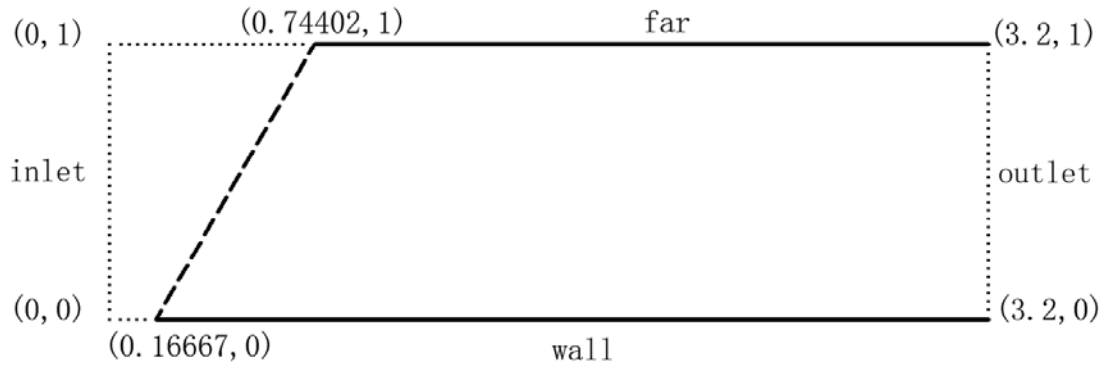


图 2.2 双马赫反射几何与边界设置

2.1 网格生成

我们在算例目录下提供 `doubleMach.msh` 网格文件，在算例目录使用如下命令生成网格：

```
$ fluentMeshToFoam doubleMach.msh
```

2.2 初始条件和边界条件

与 1.2 节所介绍情况类似，`doubleMach` 算例把初始条件和移动边界条件通过 `setNonUniformInlet.H`，`setBoundaryValues.H` 写入求解器 `dgEulerFoam`。0 文件夹下标量场 `rho`，`p`，`Ener` 的 `internalField` 内部场和 `boundaryField` 边界场统一指定为：

```
19 internalField    uniform 0;
20
21 boundaryField
22 {
23     wall
24     {
25         type        reflective;
26         value        uniform 0;
27     }
28
29     far
30     {
31         type        fixedValue;
32         value        uniform 0;
33     }
34
35     inlet
36     {
37         type        fixedValue;
38         value        uniform 0;
39     }
40
41     outlet
42     {
43         type        fixedValue;
44         value        uniform 0;
```

```

45     }
46
47     frontAndBackPlanes
48     {
49         type            empty;
50     }
51 }

```

矢量场 ρU , U 的 `internalField` 内部场和 `boundaryField` 边界场统一指定为:

```

19 internalField    uniform (0 0 0);
20
21 boundaryField
22 {
23     wall
24     {
25         type            reflective;
26         value            uniform (0 0 0);
27     }
28
29     far
30     {
31         type            fixedValue;
32         value            uniform (0 0 0);
33     }
34
35     inlet
36     {
37         type            fixedValue;
38         value            uniform (0 0 0);
39     }
40
41     outlet
42     {
43         type            fixedValue;
44         value            uniform (0 0 0);
45     }
46
47     frontAndBackPlanes
48     {
49         type            empty;
50     }

```

注意 `wall` 的边界条件设置为反射边界条件 `reflective`。

2.3 运行算例

在计算案例之前，请在算例目录/求解器源代码文件夹下使用 `wmake` 生成求解器可执行文件。本算例 `transportProperties` 和 `dgScheme` 与 `isentropicVortex` 算例一致。`dgSolution` 的子字典 `DG` 的关键字逼近阶 `baseOrder` 设置为 1。我们设置 `ControlDict` 文件中结束时间 `endTime` 为 0.2，时间步 `deltaT` 为 $1e-5$ 。

由于算例网格数量较多，我们建议采用并行计算。配置 `decomposeParDict` 参考 1.9 节。

划分计算区域：

```
$ dgDecomposePar
```

并行计算：

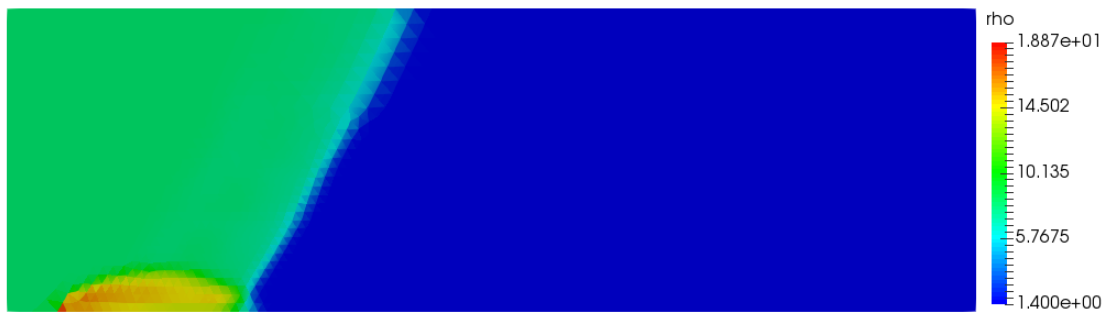
```
$ mpirun -np 12 ./dgEulerFoamDouble -parallel
```

合并场和网格：

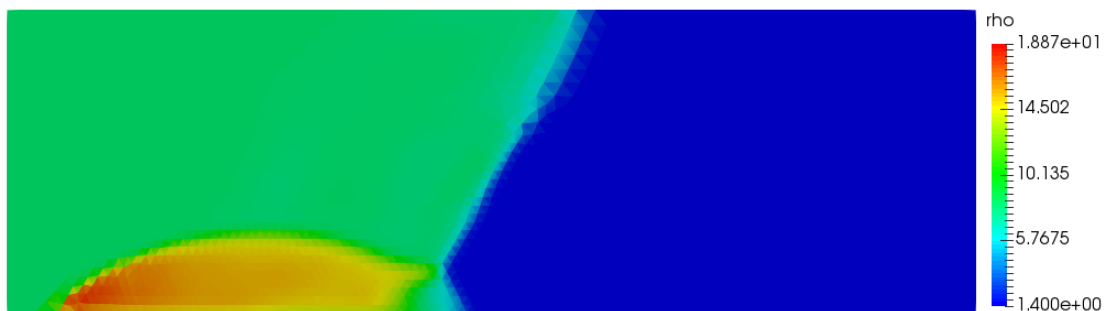
```
$ dgReconstructPar
```

2.4 后处理

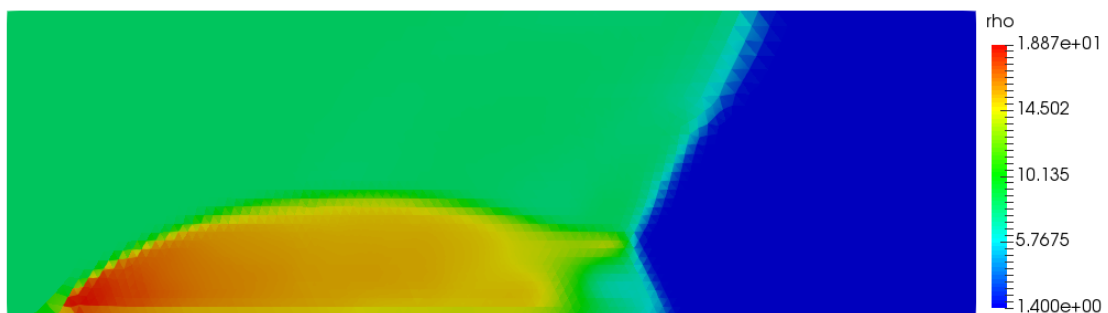
使用命令 `dgToVTK` 生成 VTK 文件夹，打开 ParaView 软件，file>Open...选择生成的.vtk 文件。Coloring 下拉选框为 `rho` 查看云图。点击 VCR controls 面板 play 查看不同时刻 `rho` 变化。



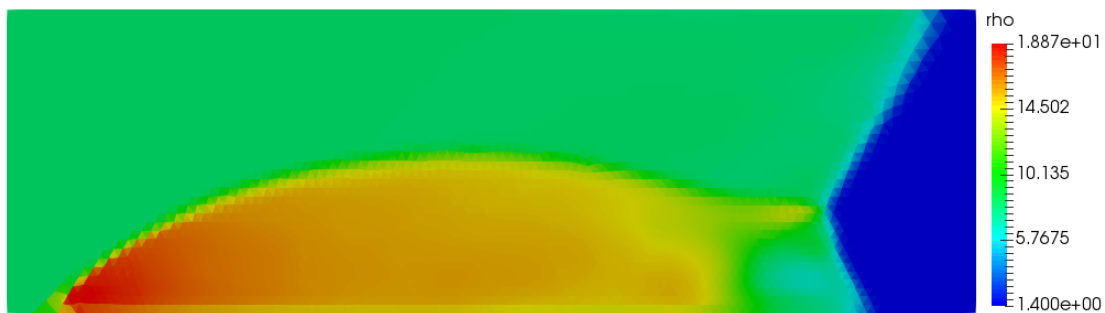
$t = 0.05$



$t = 0.1$



$t = 0.15$



$$t = 0.2$$

图 2.3 doubleMach 算例密度云图

激波问题需要使用限制器使计算稳定，当前版本的限制器为一阶限制器，不适用于高阶计算。我们提供加密网格 `doubleMachDense.msh`。计算更精细的流场。

3 不可压涡流

本算例为利用不可压涡流问题讲解不可压二维 Navier-Stokes 方程层流 dgChorinFoam 求解器的前处理，运行和后处理。计算区域几何如图 3.1 所示，有 4 个流入与 4 个流出边界。 $-0.5\text{m} \leq x \leq 0.5\text{m}$ ， $-0.5\text{m} \leq y \leq 0.5\text{m}$ 。

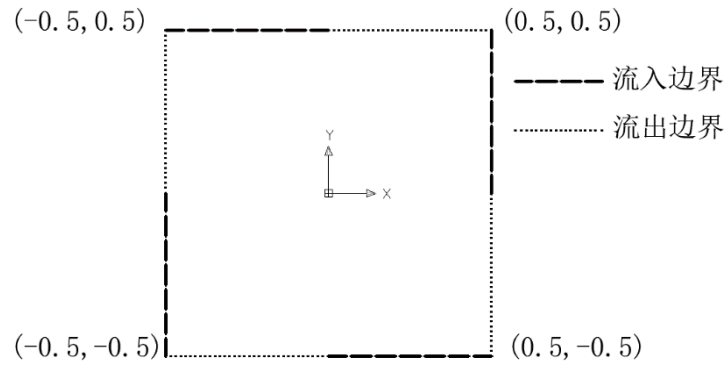


图 3.1 不可压涡流几何

3.1 网格生成

切换到算例目录 pearsonVortex，整个区域分为 4 个 block，xy 平面的 Block 结构如图 3.2 所示。用户可打开算例录下 system/blockMeshDict 文件查看。

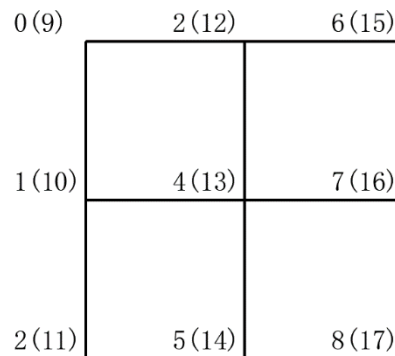


图 3.2 pearsonVortex 算例 block 结构

```

17 convertToMeters 1;
18
19 vertices
20 (
21     (-0.5 0.5 0)
22     (-0.5 0 0)
23     (-0.5 -0.5 0)
24     (0 0.5 0)
25     (0 0 0)
26     (0 -0.5 0)
27     (0.5 0.5 0)
28     (0.5 0 0)
29     (0.5 -0.5 0)
30     (-0.5 0.5 0.1)

```



```

31     (-0.5 0 0.1)
32     (-0.5 -0.5 0.1)
33     (0 0.5 0.1)
34     (0 0 0.1)
35     (0 -0.5 0.1)
36     (0.5 0.5 0.1)
37     (0.5 0 0.1)
38     (0.5 -0.5 0.1)
39 );
40
41 blocks
42 (
43     hex (0 1 4 3 9 10 13 12) (8 8 1) simpleGrading (1 1 1)
44     hex (1 2 5 4 10 11 14 13) (8 8 1) simpleGrading (1 1 1)
45     hex (3 4 7 6 12 13 16 15) (8 8 1) simpleGrading (1 1 1)
46     hex (4 5 8 7 13 14 17 16) (8 8 1) simpleGrading (1 1 1)
47 );
48
49 edges
50 (
51 );
52
53 boundary
54 (
55     inlet
56     {
57         type wall;
58         faces
59         (
60             (2 11 10 1)
61             (8 17 14 5)
62             (6 15 16 7)
63             (0 9 12 3)
64         );
65     }
66     outlet
67     {
68         type wall;
69         faces
70         (
71             (1 10 9 0)
72             (5 14 11 2)
73             (7 16 17 8)
74             (3 12 15 6)
75         );
76     }
77     frontAndBackPlanes
78     {
79         type empty;
80         faces
81         (
82             (1 0 3 4)
83             (10 13 12 9)
84             (2 1 4 5)
85             (11 14 13 10)
86             (5 4 7 8)
87             (14 17 16 13)
88             (4 3 6 7)
89             (13 16 15 12)
90         );
91     }
92 );
93
94 mergePatchPairs
95 (
96 );

```

输入如下命令生成网格：

```
$ blockMesh
```

3.2 初始条件和边界条件

与第 1.2 节类似，pearsonVortex 算例利用精确解设置速度和压力边界条件。通过头文件 setNonUniformInlet.H，setBoundaryValues.H 写入求解器 dgChorinFoam。

$$\begin{aligned}u &= -\sin(2\pi y)e^{-\nu 4\pi t}, \\v &= \sin(2\pi x)e^{-\nu 4\pi t}, \\p &= -\cos(2\pi x)\cos(2\pi y)e^{-\nu 8\pi t}.\end{aligned}$$

0 文件夹下 U 内部初始场和入口速度统一设置为 (0 0 0)。速度出口为 Neumann 边界条件，设置 outlet 边界的梯度值为 (1 1 0)。

```
19 internalField    uniform (0 0 0);
20
21 boundaryField
22 {
23     inlet
24     {
25         type        fixedValue;
26         value        uniform (0 0 0);
27     }
28
29     outlet
30     {
31         type        fixedGradient;
32         gradient     uniform (1 1 0);
33     }
34
35     frontAndBackPlanes
36     {
37         type        empty;
38     }
39 }
```

p 文件设置入口压力的边界条件为 zeroGradient，出口压力的边界条件设置为 fixedValue 0。

3.3 物理特性参数

pearsonVortex 算例需要指定的物理参数为运动粘度，储存在字典文件 transportProperties 中，输运模型选择牛顿流体 Newtonian，运动粘度关键词为 nu，本算例设置为 0.01。

```
17 transportModel    Newtonian;
18 nu                nu [ 0 2 -1 0 0 0 0 ] 0.01;
```

3.4 离散方法和矩阵求解器设置

system 目录下的 dgScheme 文件中离散格式和通量方法采用默认设置。dgSolution 文件设置与 isentropicVortex 案例类似, DG 子字典中 baseOrder 逼近阶设置范围为 1-8。

3.5 运行算例

在计算案例之前,请在算例目录/求解器源代码文件夹下使用 wmake 生成求解器可执行文件。逼近阶 baseOrder 设置为 2, ControlDict 文件关键词 endTime 为 0.1, deltaT 为 0.001, 运行算例:

```
$ ./dgChorinFoamVortex
```

3.6 后处理

使用命令 dgToVTK 生成 VTK 文件以后,使用 paraView 查看矢量图。1、pearsonVortex_*文件前眼睛图标活跃,选择 filter>Alphabetical>cell centers, 单击 Apply。2、cell center 文件前眼睛图标活跃,选择 filter>Alphabetical>Glyph,单击 Apply。3、进行设置: Glyph Source 选择 Arrow, Vectors 选择 U, scale Mode 中选 OFF, Coloring 选择 U, 单击 Apply。

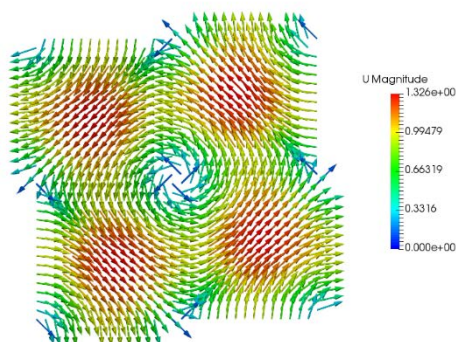


图 3.3 pearsonVortex 速度矢量图

3.7 收敛阶计算

与 isentropicVortex 算例类似，我们给出三套网格及其在不同逼近阶下 δT 的参考值来计算收敛阶。 δT 与特征尺度 h 成正比关系。算例运行结束以后，可获得计算误差。

表 3.1 不同网格不同逼近阶 δT 设置

baseOrder	vortexh	vortex0.5h	vortex0.25h
1	0.002500000	0.001250000	0.000625000
2	0.001100000	0.000555560	0.000285710
3	0.000625000	0.000322580	0.000161290
4	0.000400000	0.000204080	0.000103090
5	0.000285710	0.000142860	0.000071429
6	0.000208330	0.000105260	0.000052632

4 圆柱绕流

瞬态圆柱绕流和不可压涡流采用同一求解器，但是为配置圆柱绕流算例，我们修改 `pCorrectEquation.H`、`setNonUniformInlet.H`、`setparaT.H`、`pEqnCorrect.H` 和 `setBoundaryValues.H` 文件。同时在 `dgChorinFoam.C` 文件中的修改如下：

- 添加文件 `createTimeControls.H`

```
45    #include "createTimeControls.H"
```

- 每一时间步中参数设置

```
70    paraT4 = std::sin(pi*runTime.value()/8)/paraT3;
```

```
71    paraT3 = std::sin(pi*runTime.value()/8);
```

- `paraT3` 修改为 `paraT1`

```
86    shared_ptr<dg::Equation<scalar>> result1 =
```

```
make_shared<bCorrectEquation<scalar>>(dgm::laplacian(p), paraT1);
```

- 注释误差求解

```
116    // #include "chorinerror.H"
```

保存文件，重新编译求解器。

```
$ wclean
```

```
$ wmake
```

生产的 `dgChorinFoam` 求解器二进制文件保存在 `cylinder` 算例目录下。

瞬态圆柱绕流是依赖时间的二维管道流，在管道中间有一圆柱，圆柱位置偏离管道中心线。当流入速度由 0 逐渐形成抛物型流时，逐步形成涡流。算例的计算几何区域如图 4.1 所示。圆柱圆心位于原点 (0,0)，半径为 0.05。

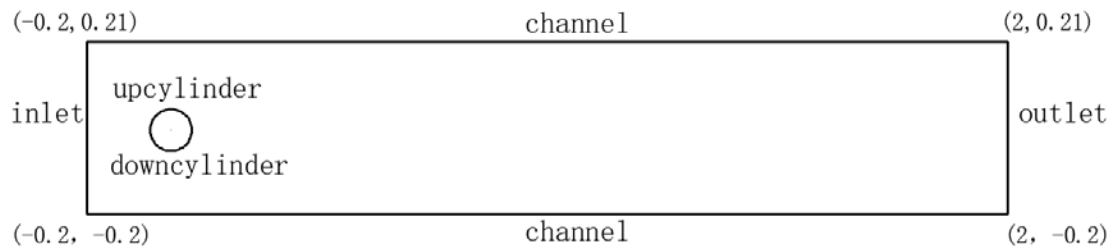


图 4.1 圆柱绕流几何与边界设置

$t=0$ 时流体速度为零，流入边界假设为抛物型流乘以时间正弦函数，模拟持续 8s。

$$u(x, y, t) = 0.41^{-2} \sin\left(\frac{\pi t}{8}\right) 6(y + 0.2)(0.21 - y),$$

$$v(x, y, t) = 0.$$

4.1 曲面边界

HopeFOAM 中引入了类 `arcPolyPatch` 以支持高阶曲面边界表示。曲面边界相关配置位于 `constant/polyMesh/ boundary` 文件中，下面以 `upcylinder` 为例讲解曲面边界配置：圆柱边界类型 `type` 选择圆弧 `arc`，`name` 指定曲面边界名称；重点通过 `code#{ #}` 内的弧形三维表达式分别描述参数 `u,v` 与圆弧的 `x,y,z` 坐标的关系，`u_Range` 指定参数 `u` 的取值范围为-0.5 到 0.5，`v_Range` 同理。

```

18 6
19 (
20     channel
21     {
22         type            wall;
23         inGroups        1(wall);
24         nFaces          128;
25         startFace       2666;
26     }
27     outlet
28     {
29         type            patch;
30         inGroups        1(wall);
31         nFaces          12;
32         startFace       2794;
33     }
34     inlet
35     {
36         type            patch;
37         inGroups        1(wall);
38         nFaces          16;
39         startFace       2806;
40     }
41     upcylinder
42     {
43         type            arc;
44         inGroups        1(wall);
45         nFaces          16;
46         startFace       2822;
47         name            codeup;
48         u_Range         (-0.5 0.5);
49         v_Range         (0 0);
50         code
51         #{
52             0.05*Foam::sin(Foam::constant::mathematical::pi*u),
53             0.05*Foam::cos(Foam::constant::mathematical::pi*u),
54             v
55         #};
56     }
57     downcylinder
58     {
59         type            arc;
60         inGroups        1(wall);
61         nFaces          16;
62         startFace       2838;
63         name            codedown;

```

```

64     u_Range      (-0.5 0.5);
65     v_Range      (0 0);
66     code
67     #{
68         0.05*Foam::sin(Foam::constant::mathematical::pi*u),
69         -0.05*Foam::cos(Foam::constant::mathematical::pi*u),
70         v
71     #};
72 }
73
74 frontAndBackPlanes
75 {
76     type          empty;
77     inGroups       1(empty);
78     nFaces         3680;
79     startFace      2854;
80 }
81 )

```

4.2 初始条件和边界条件

流入边界 inlet 速度场和压力场由相关文件设定,流出边界 outlet 速度设置为 zeroGradient。圆柱和管道固定壁面使用无滑移边界条件 fixedValue:

```

21 boundaryField
22 {
23     inlet
24     {
25         type          fixedValue;
26         value          uniform (0 0 0);
27     }
28
29     outlet
30     {
31         type          zeroGradient;
32     }
33
34     channel
35     {
36         type          fixedValue;
37         value          uniform (0 0 0);
38     }
39
40     upcylinder
41     {
42         type          fixedValue;
43         value          uniform (0 0 0);
44     }
45
46     downcylinder
47     {
48         type          fixedValue;
49         value          uniform (0 0 0);
50     }
51
52     frontAndBackPlanes
53     {
54         type          empty;
55     }
56 }

```

壁面的压力法相梯度为 0, 压力边界条件设置为 zeroGradient, 流出边界 outlet 压力设置为 0。

```

21 boundaryField
22 {
23     inlet
24     {
25         type            zeroGradient;
26     }
27
28     outlet
29     {
30         type            fixedValue;
31         value            uniform 0;
32     }
33
34     channel
35     {
36         type            zeroGradient;
37     }
38
39     upcylinder
40     {
41         type            zeroGradient;
42     }
43
44     downcylinder
45     {
46         type            zeroGradient;
47     }
48
49     frontAndBackPlanes
50     {
51         type            empty;
52     }
53 }

```

4.3 物理特性参数

使流体的雷诺数 $Re \approx 100$ ，在 constant/ transportProperties 文件中设置粘性系数 nu $\nu=10^{-3}$ 。

dgSolution 的子字典 DG 的关键字逼近阶 baseOrder 设置为 3。

圆柱绕流问题不存在精确解，定义圆柱的阻力与升力系数为圆柱体上总力在水平和垂直方向的分量，由公式给出：

$$C_d(t) = -\oint_{Cylinder} -p\hat{n}_x + \nu(\hat{n}_x 2\frac{\partial u}{\partial x} + \hat{n}_y(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y})) ds,$$

$$C_l(t) = -\oint_{Cylinder} -p\hat{n}_y + \nu(\hat{n}_y 2\frac{\partial v}{\partial y} + \hat{n}_x(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x})) ds,$$

其中 (\hat{n}_x, \hat{n}_y) 为圆柱表面外法向量。ControlDict 文件中结束时间 endTime 为 8，时间步 deltaT 为 2e-4，添加 forceCoeffs 计算升力、阻力系数功能。


```

18 application      dgChorinFoam;
19
20 startFrom        startTime;
21
22 startTime        0;
23
24 stopAt           endTime;
25
26 endTime          8;
27
28 deltaT           2.0E-04;
29
30 writeControl      timeStep;
31
32 writeInterval     200;
33
34 purgeWrite       0;
35
36 writeFormat       ascii;
37
38 writePrecision    6;
39
40 writeCompression  off;
41
42 timeFormat        general;
43
44 timePrecision     6;
45
46 runTimeModifiable false;
47
48 adjustTimeStep    no;
49
50 maxCo             0.9;
51
52 maxDeltaT         0.01;
53
54 functions
55 {
56     #include "forceCoeffs"
57 };

```

forceCoeffs 文件 rhoInf 设置密度参考值，magUInf 设置速度参考值，liftDir 和 dragDir 分别设置升力和阻力方向。pitchAxis 指定力矩参考轴。outputInterval 设置计算时间步。

```

11     type dgForceCoeffs;
12     functionObjectLibs ("libdgforces.so");
13     patches (upcylinder downcylinder);
14     log true;
15     pName p;
16     Uname U;
17     rho rhoInf;
18     rhoInf 1;
19     magUInf 1;
20     liftDir (0 1 0);
21     dragDir (1 0 0);
22     pitchAxis (0 0 -1);
23     CofR (0 0 0);
24     Aref 0.1; //2D example default height of the grid is 1
25     lRef 1;
26     outputControl    timeStep;
27     outputInterval 1;

```

4.4 运行算例及后处理

在计算案例之前，请在算例目录/求解器源代码文件夹下使用 `wmake` 生成求解器可执行文件。当前曲面边界不支持并行计算，在 `cylinder` 算例目录下串行计算：

```
$ ./dgChorinFoamCylinder
```

使用 Paraview 软件查看速度云图。

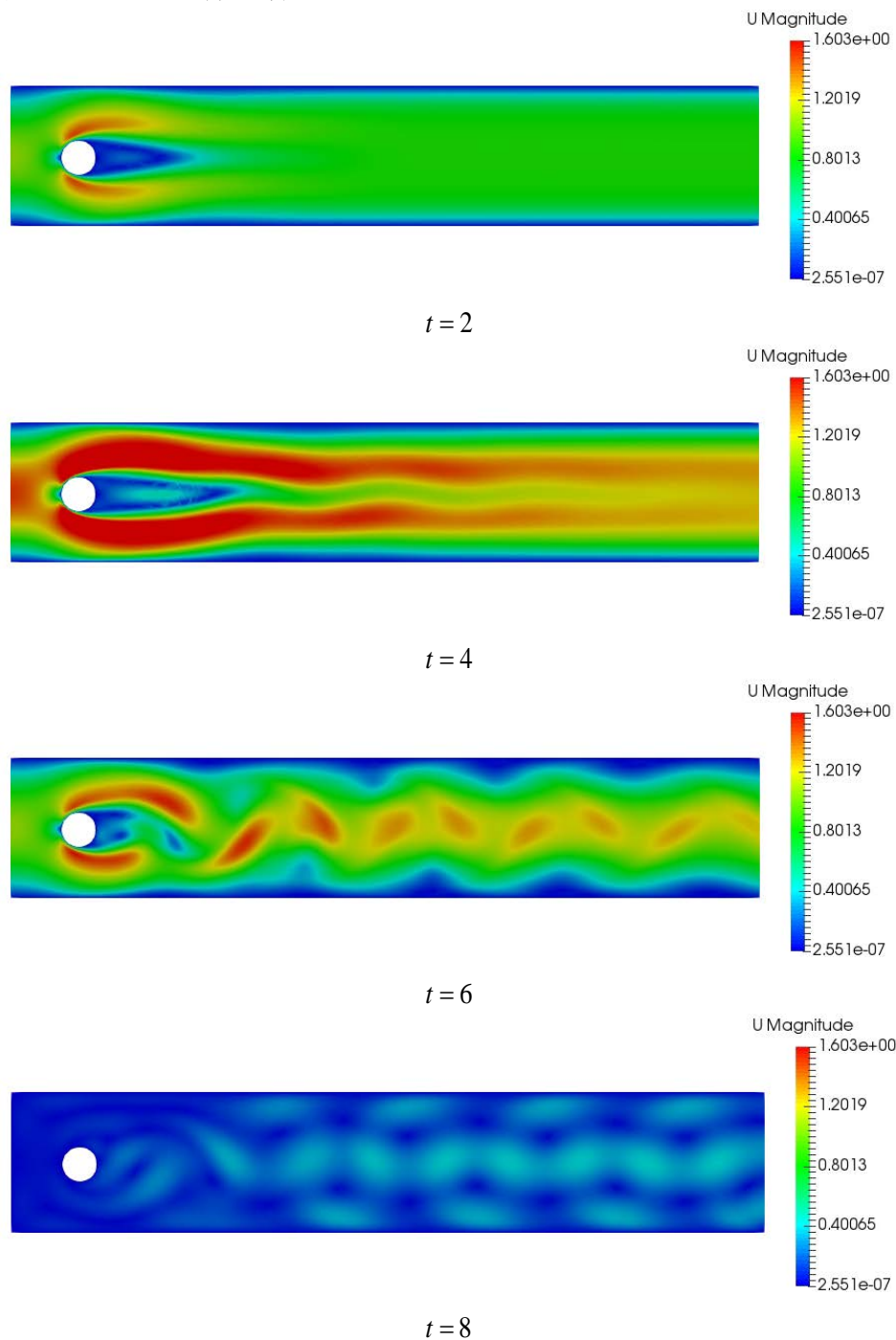


图 4.2 cylinder 算例速度云图

系数保存在 postProcessing/forceCoeffs/0/目录下的 .dat 文件中。

```
# dgForce coefficients
# liftDir      : (0.000000e+00 1.000000e+00 0.000000e+00)
# dragDir      : (1.000000e+00 0.000000e+00 0.000000e+00)
# pitchAxis    : (0.000000e+00 0.000000e+00 -1.000000e+00)
# magUInf      : 1.000000e+00
# lRef         : 1.000000e+00
# Aref         : 1.000000e-01
# CofR         : (0.000000e+00 0.000000e+00 0.000000e+00)
# Time         Cm          Cd          Cl          Cl(f)          Cl(r)
0.0002         3.304978e-08      -1.063947e+00  9.374337e-02  4.687172e-02  4.687165e-02
0.0004         -2.644742e-06       6.580886e+01 -2.782665e-01 -1.391359e-01 -1.391306e-01
0.0006         -1.471373e-06       -9.094215e+01  1.099570e-01  5.497701e-02  5.497995e-02
0.0008         6.263473e-06       -8.928743e+00  6.684385e-02  3.342819e-02  3.341566e-02
0.001          2.419367e-06       2.894768e+01 -1.229158e-02 -6.143369e-03 -6.148207e-03
0.0012         -3.171261e-06       1.875261e+01 -4.258361e-02 -2.129498e-02 -2.128864e-02
0.0014         -3.142487e-06       -1.222173e+00 -6.661137e-03 -3.333711e-03 -3.327426e-03
0.0016         -3.236623e-07       -7.782365e+00  3.629908e-02  1.814922e-02  1.814986e-02
0.0018         1.215003e-06       -4.201885e+00  1.098737e-02  5.494901e-03  5.492471e-03
```

Cm 为力矩系数，Cd 为阻力系数，Cl 为升力系数。