Static Single Assignment Form

Last Time

- Lattice theoretic frameworks for data-flow analysis

Today

- Program representations
- Static single assignment (SSA) form
  - Program representation for sparse data-flow
- Conversion to and from SSA

Next Time

- Reuse optimizations

Data Dependence

Definition

- Data dependences are constraints on the order in which statements may be executed

Types of dependences

- **Flow (true) dependence**: s_1 writes memory that s_2 later reads (RAW)
- **Anti-dependence**: s_1 reads memory that s_2 later writes (WAR)
- **Output dependences**: s_1 writes memory that s_2 later writes (WAW)
- **Input dependences**: s_1 reads memory that s_2 later reads (RAR)

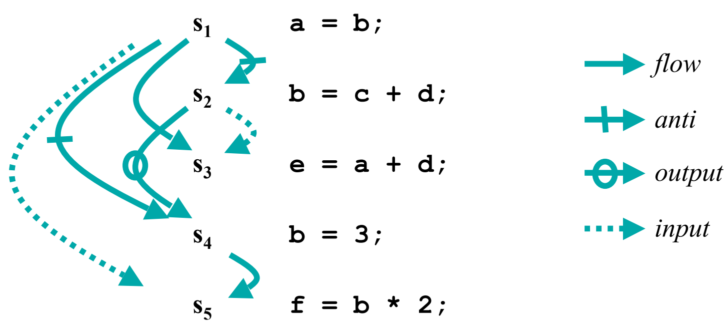
True dependences

- Flow dependences represent actual flow of data

False dependences

- Anti- and output dependences reflect reuse of memory, not actual data flow; can often be eliminated

Example



Representing Data Dependences

Implicitly

- Use variable defs and uses
- Pros: simple
- Cons: hides data dependence (analyses must find this info)

Def-use chains (du chains)

- Link each def to its uses
- Pros: explicit; therefore fast
- Cons: must be computed and updated, consumes space

Alternate representations

- *e.g.,* Static single assignment form (SSA), dependence flow graphs (DFG), value dependence graphs (VDG)

DU Chains

Definition

– du chains link each def to its uses

Example

s_1	`a = b;`
s_2	`b = c + d;`
s_3	`e = a + d;`
s_4	`b = 3;`
s_5	`f = b * 2;`

→ *du chain*

UD Chains

Definition

– ud chains link each use to its defs

Example

s_1	`a = b;`
s_2	`b = c + d;`
s_3	`e = a + d;`
s_4	`b = 3;`
s_5	`f = b * 2;`

→ *ud chain*

Role of Alternate Program Representations

Process

$$\begin{array}{ccccc} \text{Original Code (RTL)} & & & & \text{Optimized Code (RTL)} \\ \downarrow & & & & \uparrow \\ \text{SSA Code1} & \xrightarrow{\text{T1}} & \text{SSA Code2} & \xrightarrow{\text{T2}} & \text{SSA Code3} \end{array}$$

Advantage

- Allow analyses and transformations to be simpler & more efficient/effective

Disadvantage

- May not be “executable” (requires extra translations to and from)
- May be expensive (in terms of time or space)

Static Single Assignment (SSA) Form

Idea

- Each variable has only one static definition
- Makes it easier to reason about values instead of variables
- Similar to the notion of functional programming

Transformation to SSA

- Rename each definition
- Rename all uses reached by that assignment

Example

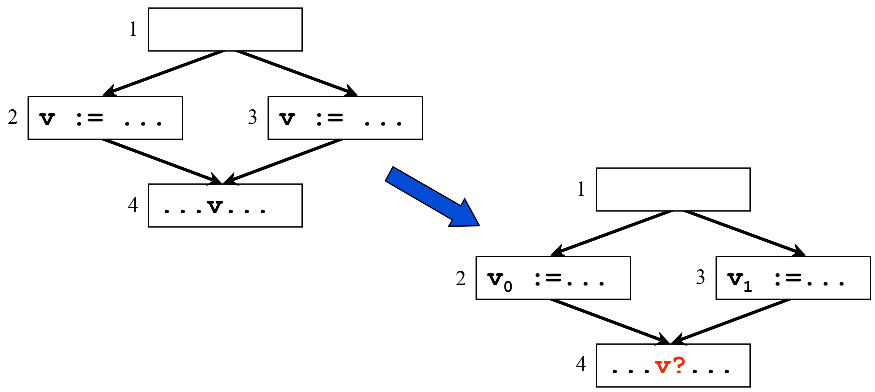
$$\begin{array}{lcl} \texttt{v := ...} & & \texttt{v}_0\texttt{ := ...} \\ \texttt{... := ... v ...} & & \texttt{... := ... v}_0\texttt{ ...} \\ \texttt{v := ...} & \Rightarrow & \texttt{v}_1\texttt{ := ...} \\ \texttt{... := ... v ...} & & \texttt{... := ... v}_1\texttt{ ...} \end{array}$$

What do we do when there’s control flow?

SSA and Control Flow

Problem

– A use may be reached by several definitions

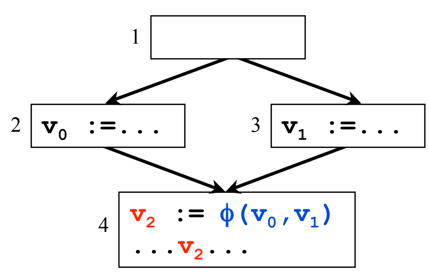


SSA and Control Flow (cont)

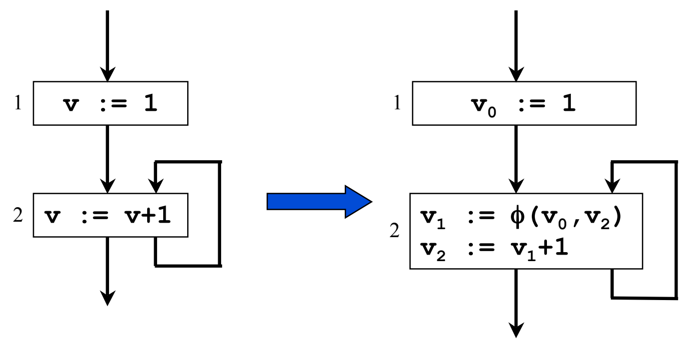
Merging Definitions

– ϕ-functions merge multiple reaching definitions

Example



Another Example



SSA vs. ud/du Chains

SSA form is more constrained

Advantages of SSA

- More compact
- Some analyses become simpler when each use has only one def
- Value merging is explicit
- Easier to update and manipulate?

Furthermore

- Eliminates false dependences (simplifying context)

```
for (i=0; i<n; i++)
     A[i] = i;
for (i=0; i<n; i++)
     print(foo(i));
```

Unrelated uses of `i` are given different variable names

SSA vs. ud/du Chains (cont)

Worst case du-chains?

```
switch (c1) {
case 1:     x = 1; break;
case 2:     x = 2; break;
case 3:     x = 3; break;
}
```

$x_4 = \phi(x_1, x_2, x_3)$

```
switch (c2) {
case 1:     y1 = x; break;
case 2:     y2 = x; break;
case 3:     y3 = x; break;
case 4:     y4 = x; break;
}
```

m defs and n uses leads to m×n du chains

Transformation to SSA Form

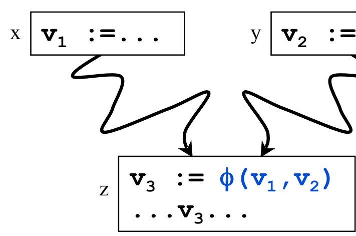
Two steps

- Insert φ-functions
- Rename variables

Where Do We Place ϕ-Functions?

Basic Rule

– If two distinct (non-null) paths x→z and y→z converge at node z, and nodes x and y contain definitions of variable v, then a ϕ-function for v is inserted at z



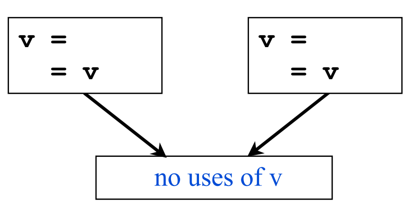
Approaches to Placing ϕ-Functions

Minimal

– As few as possible subject to the basic rule

Briggs-Minimal

– Same as minimal, except v must be live across some edge of the CFG



Briggs Minimal will not place a ϕ function in this case because v is not live across any CFG edge

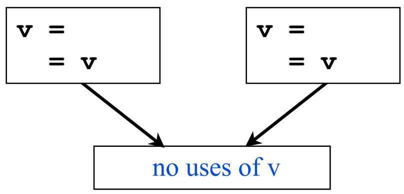
Approaches to Placing ϕ-Functions (cont)

Briggs-Minimal

– Same as minimal, except v must be live across some edge of the CFG

Pruned

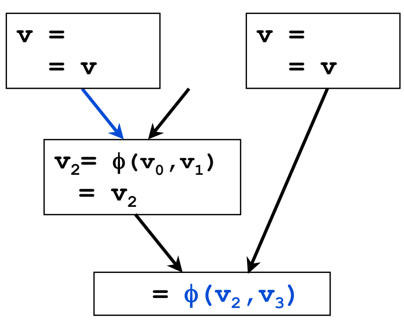
– Same as minimal, except dead ϕ-functions are not inserted



Will Pruned place a ϕ function at this merge?

What's the difference between Briggs Minimal and Pruned SSA?

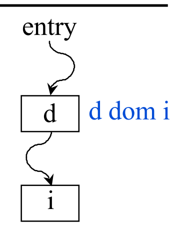
Briggs Minimal vs. Pruned



Briggs Minimal will add a ϕ function because v is live across the blue edge, but Pruned SSA will not because the ϕ function is dead

Why would we ever use Briggs Minimal instead of Pruned SSA?

Machinery for Placing ϕ-Functions



Recall Dominators

– d **dom** i if all paths from entry to node i include d
– d **sdom** i if d dom i and d≠i

Dominance Frontiers

– The **dominance frontier** of a node d is the set of nodes that are "just barely" not dominated by d; i.e., the set of nodes n, such that
  – d dominates a predecessor p of n, and
  – d does **not** strictly dominate n
– DF(d) = {n | ∃p∈pred(n), d dom p and d !sdom n}

Notational Convenience

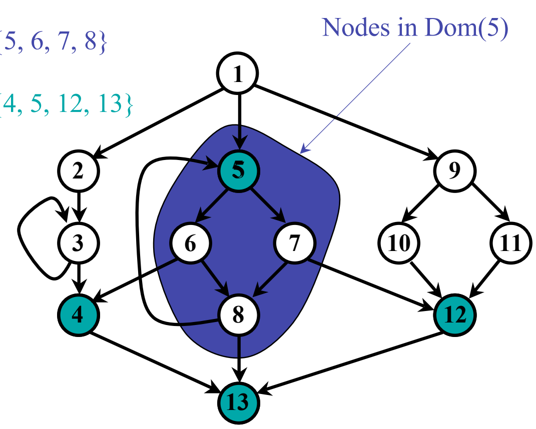
– $DF(S) = \cup_{s \in S} DF(s)$

Dominance Frontier Example

DF(d) = {n | ∃p∈pred(n), d dom p and d !sdom n}

Dom(5) = {5, 6, 7, 8}

DF(5) = {4, 5, 12, 13}



What's significant about the Dominance Frontier?

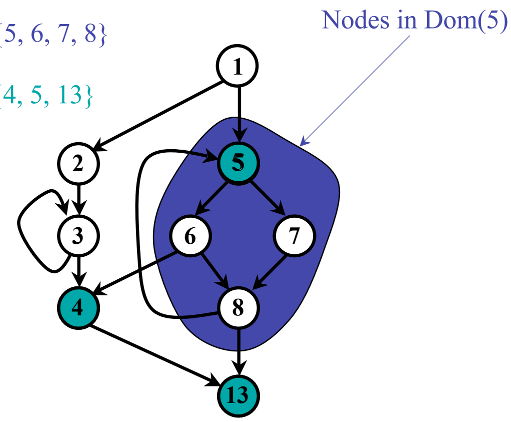
In SSA form, definitions must dominate uses

Dominance Frontier Example II

DF(d) = {n | ∃p∈pred(n), d dom p and d !sdom n}

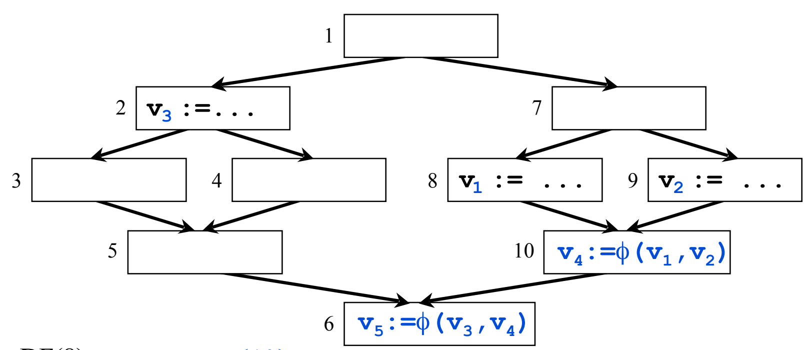
Dom(5) = {5, 6, 7, 8}

DF(5) = {4, 5, 13}



In this new graph, node 4 is the first point of convergence between the entry and node 5, so do we need a ϕ- function at node 13?

SSA Exercise

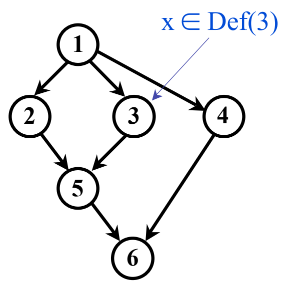


DF(8) = {10}
DF(9) = {10}
DF(2) = {6} DF(d) = {n | ∃p∈pred(n), d dom p and d !sdom n}
DF({8,9}) = {10}
DF(10) = {6}
DF({2,6,8,9,10}) = {6,10}

Dominance Frontiers Revisited

Suppose that node 3 defines variable x

DF(3) = {5}



Do we need to insert a ϕ- function for x anywhere else?

Yes. At node 6. Why?

Dominance Frontiers and SSA

Let

- $DF_1(S) = DF(S)$
- $DF_{i+1}(S) = DF(S \cup DF_i(S))$

Iterated Dominance Frontier

- $DF_\infty(S)$

Theorem

- If S is the set of CFG nodes that define variable v, then $DF_\infty(S)$ is the set of nodes that require ϕ-functions for v

Algorithm for Inserting ϕ-Functions

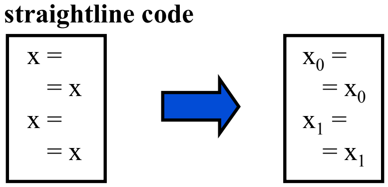
for each variable v
    WorkList ← ∅
    EverOnWorkList ← ∅
    AlreadyHasPhiFunc ← ∅
    for each node n containing an assignment to v   Put all defs of v on the worklist
        WorkList ← WorkList ∪ {n}
    EverOnWorkList ← WorkList
    while WorkList ≠ ∅
        Remove some node n from WorkList
        for each d ∈ DF(n)
            if d ∉ AlreadyHasPhiFunc   Insert at most one ϕ function per node
                Insert a ϕ-function for v at d
                AlreadyHasPhiFunc ← AlreadyHasPhiFunc ∪ {d}
                if d ∉ EverOnWorkList   Process each node at most once
                    WorkList ← WorkList ∪ {d}
                    EverOnWorkList ← EverOnWorkList ∪ {d}

Variable Renaming

Basic idea

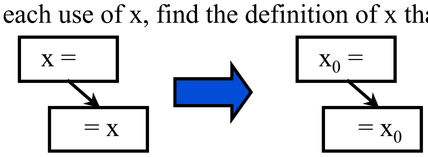
– When we see a variable on the LHS, create a new name for it
– When we see a variable on the RHS, use appropriate subscript

Easy for straightline code



Use a stack when there's control flow

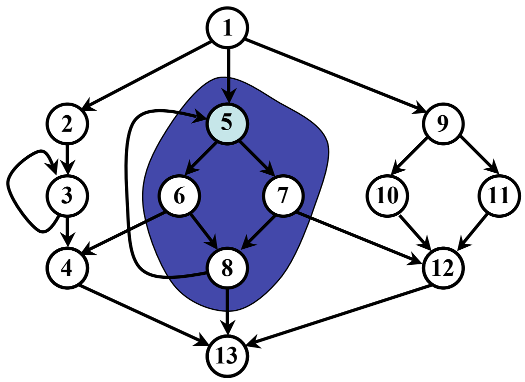
– For each use of x, find the definition of x that dominates it



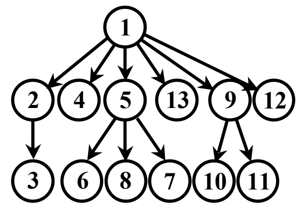
Traverse the dominance tree

Dominance Tree Example

The dominance tree shows the dominance relation



CFG



Dominance Tree

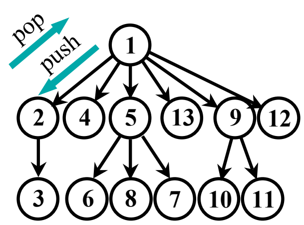
Variable Renaming (cont)

Data Structures

- Stacks[v] ∀v
  Holds the subscript of most recent definition of variable v, initially empty
- Counters[v] ∀v
  Holds the current number of assignments to variable v; initially 0

Auxiliary Routine

procedure GenName(variable v)
  i := Counters[v]
  push i onto Stacks[v]
  Counters[v] := i + 1

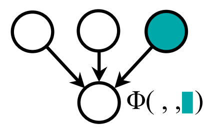


Use the Dominance Tree to remember the most recent definition of each variable

Variable Renaming Algorithm

Call Rename(entry-node)

procedure Rename(block b)
    for each ϕ-function p in b
        GenName(LHS(p)) and replace v with v_i, where i=Top(Stack[v])
    for each statement s in b (in order)
        for each variable $v \in$ RHS(s)
            replace v by v_i, where i = Top(Stacks[v])
        for each variable $v \in$ LHS(s)
            GenName(v) and replace v with v_i, where i=Top(Stack[v])
    for each $s \in$ succ(b)   (in CFG)
        j $\leftarrow$ position in s's ϕ-function corresponding to block b
        for each ϕ-function p in s
            replace the j^{th} operand of RHS(p) by v_i, where i = Top(Stack[v])
    for each $s \in$ child(b) (in DT)
        Rename(s)
    for each ϕ-function or statement t in b
        for each $v_i \in$ LHS(t)
            Pop(Stack[v])



$\}$ Recurse using Depth First Search

$\}$ Unwind stack when done with this node

Transformation from SSA Form

Proposal
- Restore original variable names (*i.e.*, drop subscripts)
- Delete all ϕ-functions

Complications
- What if versions get out of order? (simultaneously live ranges)

```
x0 =
x1 =
      = x0
      = x1
```

Alternative
- Perform dead code elimination (to prune ϕ-functions)
- Replace ϕ-functions with copies in predecessors
- Rely on register allocation coalescing to remove unnecessary copies

Concepts

Data dependences
- Three kinds of data dependences
- du-chains

Alternate representations

SSA form

Conversion to SSA form
- ϕ-function placement
  - Dominance frontiers
- Variable renaming
  - Dominance trees

Conversion from SSA form

Next Time

Assignments
- Project proposals due

Lecture
- Reuse optimizations