Reuse Optimization

Last time

- Discussion (SCC)
- Loop invariant code motion
- Reuse optimization: Value numbering

Today

- More reuse optimization
  - Common subexpression elimination (CSE)
  - Partial redundancy elimination (PRE)

Common Subexpression Elimination

Idea

- Find common subexpressions whose *range* spans the same basic blocks and eliminate unnecessary re-evaluations
- Leverage available expressions

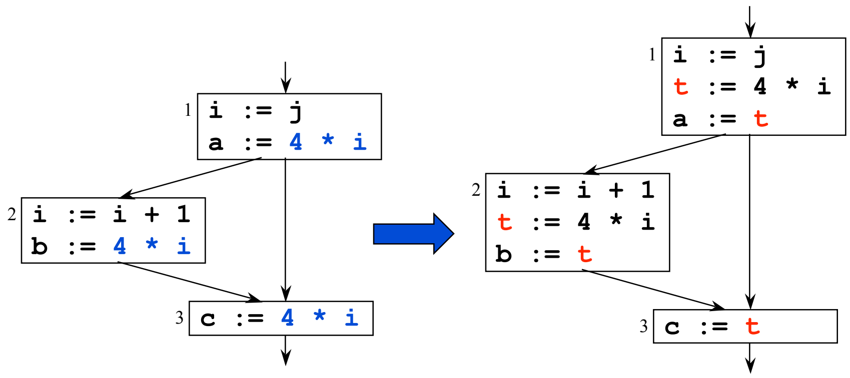
Recall available expressions

- An expression (*e.g.*, **`x+y`**) is **available** at node n if **every** path from the entry node to n evaluates **`x+y`**, and there are no definitions of **`x`** or **`y`** after the last evaluation along that path

Strategy

- If an expression is available at a point where it is evaluated, it need not be recomputed

CSE Example



Will value numbering find this redundancy?

- No; value numbering operates on values
- CSE operates on expressions

Another CSE Example

Before CSE

```
c := a + b
d := m & n
e := b + d
f := a + b
g := -b
h := b + a
a := j + a
k := m & n
j := b + d
a := -b
if m & n goto L2
```

Summary
11 instructions
12 variables
9 binary operators

After CSE

```
t1 := a + b
c := t1
t2 := m & n
d := t2
t3 := b + d
e := t3
f := t1
g := -b
h := t1
a := j + a
k := t2
j := t3
a := -b
if t2 goto L2
```

Summary
14 instructions
15 variables
4 binary operators

CSE Approach 1

Notation

– Avail(b) is the set of expressions available at block b
– Gen(b) is the set of expressions generated and not killed at block b

If we use e and e ∈ Avail(b)

– Allocate a new name n
– Search backward from b (in CFG) to find statements (one for each path) that most recently generate e
– Insert copy to n after generators
– Replace e with n

Problems

– Backward search for each use is expensive
– Generates unique name for each use
  – |names| ∝ |Uses| > |Avail|
  – Each generator may have many copies

Example

```
a  := b + c
t1 := a
t2 := a
e  := b1+ c
f  := b2+ c
```


CSE Approach 2

Idea

– Reduce number of copies by assigning a unique name to each unique expression

Summary

– ∀e Name[e] = unassigned
– if we use e and e ∈ Avail(b)
  – if Name[e]=unassigned, allocate new name n and Name[e] = n
    else n = Name[e]
  – Replace e with n
– In a subsequent traversal of block b, if e ∈ Gen(b) and Name[e] ≠ unassigned, then insert a copy to Name[e] after the generator of e

Problem

– May still insert unnecessary copies
– Requires two passes over the code

Example

```
a  := b + c
t1 := a
```

CSE Approach 3

Idea

- Don't worry about temporaries
- Create one temporary for each unique expression
- Let subsequent pass eliminate unnecessary temporaries

At an evaluation of e

- Hash e to a name, n, in a table
- Insert an assignment of e to n

At a use of e in b, if e ∈ Avail(b)

- Lookup e's name in the hash table (call this name n)
- Replace e with n

Problems

- Inserts more copies than approach 2 (but extra copies are dead)
- Still requires two passes (2nd pass is very general)

Extraneous Copies

Extraneous copies degrade performance

Let other transformations deal with them

- Dead code elimination
- Coalescing

Coalesce assignments to `t1` and `t2` into a single statement

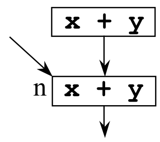
```
t1 := b + c
t2 := t1
```

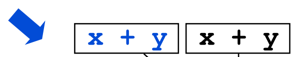
- Greatly simplifies CSE

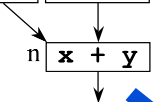
Partial Redundancy Elimination (PRE)

Partial Redundancy

– An expression (*e.g.*, **`x+y`**) is **partially redundant** at node n if **some** path from the entry node to n evaluates **`x+y`**, and there are no definitions of **`x`** or **`y`** between the last evaluation of **`x+y`** and n





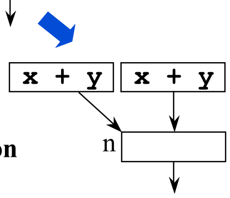


Elimination

– Discover partially redundant expressions

– Convert them to fully redundant expressions

– Remove redundancy



PRE subsumes CSE and loop invariant code motion

Loop Invariance Example

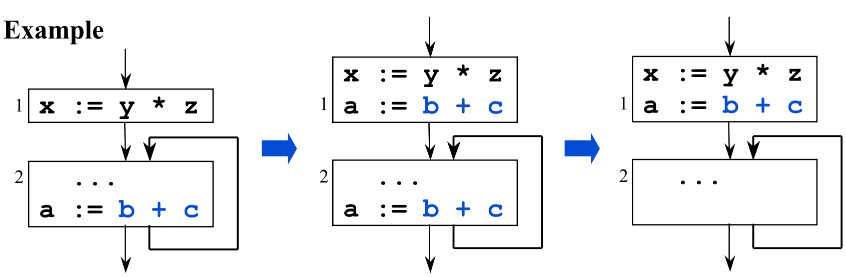
PRE removes loop invariants

– An invariant expression is partially redundant

– PRE converts this partial redundancy to full redundancy

– PRE removes the redundancy

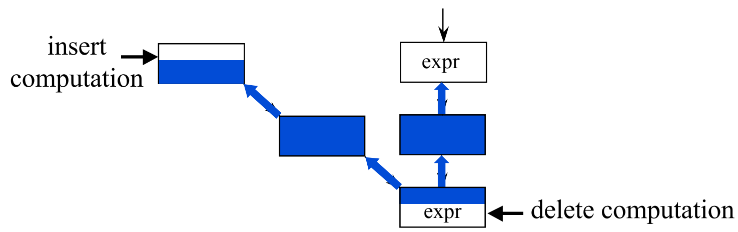
Example



Implementing PRE

Big picture

- Use local properties (**availability** and **anticipability**) to determine where redundancy can be created within a basic block
- Use global analysis (data-flow analysis) to discover where partial redundancy can be converted to full redundancy
- Insert code and remove redundant expressions



Local Properties

An expression is locally **transparent** in block b if its operands are not modified in b

An expression is locally **available** in block b if it is computed at least once and its operands are not modified after its last computation in b

An expression is locally **anticipated** if it is computed at least once and its operands are not modified before its first evaluation

Example

```
a := b + c
d := a + e
```

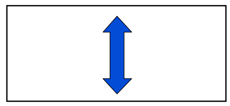
Transparent: {`b + c`}
Available: {`b + c, a + e`}
Anticipated: {`b + c`}

Local Properties (cont)

How are these properties useful?

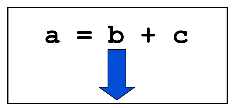
– They tell us where we can introduce redundancy

Transparent



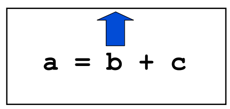
The expression can be redundantly evaluated anywhere in the block

Available



The expression can be redundantly evaluated anywhere after its last evaluation in the block

Anticipated



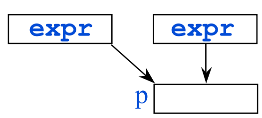
The expression can be redundantly evaluated anywhere before its first evaluation in the block

Global Availability

Intuition

– Global availability is the same as Available Expressions

– If e is globally available at p, then an evaluation at p will create redundancy along all paths leading to p



Flow Functions

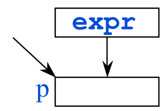
$$\text{available\_in}[n] = \bigcap_{p \in pred[n]} \text{available\_out}[p]$$

$$\text{available\_out}[n] = \text{locally\_available}[n] \cup (\text{available\_in}[n] \cap \text{transparent}[n])$$

(Global) Partial Availability

Intuition

- An expression is partially available if it is available along some path
- If e is partially available at p, then ∃ a path from the entry node to p such that the evaluation of e at p would give the same result as the previous evaluation of e along the path



Flow Functions

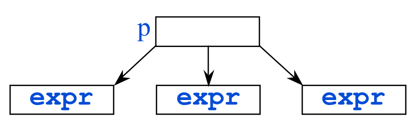
$$\text{partially\_available\_in}[n] = \bigcup_{p \in pred[n]} \text{partially\_available\_out}[p]$$

$$\text{partially\_available\_out}[n] = \text{locally\_available}[n] \cup (\text{partially\_available\_in}[n] \cap \text{transparent}[n])$$

Global Anticipability

Intuition

- If e is globally anticipated at p, then an evaluation of e at p will make the next evaluation of e redundant along all paths from p



Flow Functions

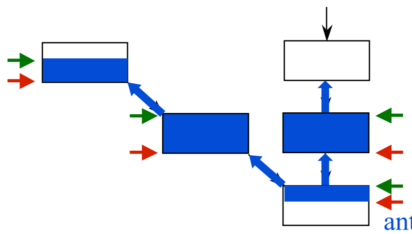
$$\text{anticipated\_out}[n] = \bigcap_{s \in succ[n]} \text{anticipated\_in}[s]$$

$$\text{anticipated\_in}[n] = \text{locally\_anticipated}[n] \cup (\text{anticipated\_out}[n] \cap \text{transparent}[n])$$

Global Possible Placement

Goal

- Convert partial redundancies to full redundancies
- Possible Placement uses a backwards analysis to identify locations where such conversions can take place
  - e ∈ ppin[n] can be placed at entry of n
  - e ∈ ppout[n] can be placed at exit of n



Start with locally anticipated expressions

Push Possible Placement backwards as far as possible

Global Possible Placement (cont)

The placement will create a redundancy on every edge out of the block

Flow Functions

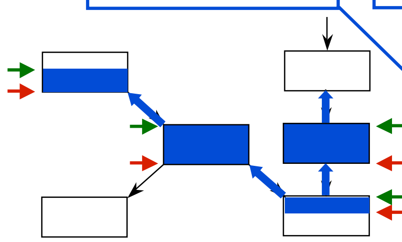
Will turn partial redundancy into full redundancy

$$\text{ppout}[n] = \bigcap_{s \in \text{succ}[n]} \text{ppin}[s]$$

$$\text{ppin}[n] = \boxed{\text{anticipated\_in}[n]} \cap \boxed{\text{partially\_available\_in}[n]} \cap \boxed{(\text{locally\_anticipated}[n]} \cup \boxed{(\text{ppout}[n] \cap \text{transparent}[n]))}$$

Middle of chain

This block is at the beginning of a chain

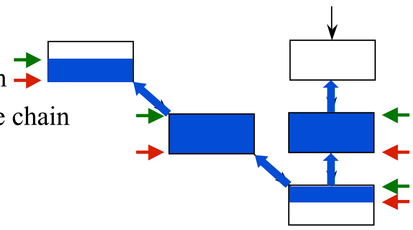


Updating Blocks

Intuition

– Perform insertions at top of the chain

– Perform deletion at the bottom of the chain

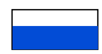


Functions

– delete[n] = ppin[n] ∩ locally_anticipated[n]

– insert[n] = ppout[n]

∩ (¬ppin[n] ∪ ¬ transparent[n])



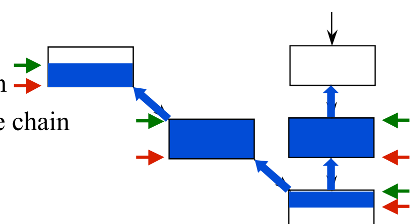
∩ ¬available_out[n]   Don't insert it where it's fully redundant

Updating Blocks (cont)

Intuition

– Perform insertions at top of the chain

– Perform deletion at the bottom of the chain



Functions   ¬ ppout[n] ?   No

– delete[n] = ppin[n] ∩ ~~locally_anticipated[n]~~

– insert[n] = ppout[n]

∩ (¬ppin[n] ~~∪ ¬ transparent[n])~~   Can we omit this clause?

∩ ¬available_out[n]

Example

B1: `a := b + c` **B2:** `b := b + 1`

B3: `a := b + c`

	B1	B2	B3
transparent	{b+c}		{b+c}
locally_available	{b+c}		{b+c}
locally_anticipated	{b+c}	{b+1}	{b+c}
available_in			
available_out	{b+c}		{b+c}
partially_available_in			{b+c}
partially_available_out	{b+c}		{b+c}
anticipated_out	{b+c}	{b+c}	
anticipated_in	{b+c}	{b+1}	{b+c}
ppout	{b+c}	{b+c}	
ppin			{b+c}
insert		{b+c}	
delete			{b+c}

Comparing Redundancy Elimination

Value numbering
- Examines values not expressions
- Symbolic
- Knows nothing about algebraic properties (1+x = x+1)

CSE
- Examines expressions

PRE
- Examines expressions
- Subsumes CSE and loop invariant code motion
- Simpler implementations are now available

Constant propagation
- Requires that values be statically known

PRE Summary

What's so great about PRE?

- A modern optimization that subsumes earlier ideas
- Composes several simple data-flow analyses to produce a powerful result
  - Finds earliest and latest points in the CFG at which an expression is anticipated

Next Time

Lecture

- Alias analysis