Baye's Theorem

Let the events $A_1, A_2, \ldots \ldots, A_n$ represent a partition of the sample space S. Let B be any other event defined on S. if $P(A_i) \neq 0, i = 1, 2, \ldots., n$ and $P(B) \neq 0$ then

$$P(A_i/B) = \frac{P(A_i) \times P(B/A_i)}{\sum P(A_i) \times P(B/A_i)}$$

It can also be stated as,

$$P(A_i/B) = \frac{p_i p_i'}{p_1 p_1' + p_2 p_2' + p_3 p_3' + \cdots \ldots + p_n p_n'}$$

1. **There are in a bag three true coins and one false coin with head on both sides. A coin is chosen at random and tossed four times. If head occurs all the four times, what is the probability that the false coin was chosen and used?**

   Solution:

   $P(selecting\ true\ coin) = p_1 = \frac{3}{4}$

   $P(selecting\ false\ coin) = p_2 = \frac{1}{4}$

   $p_1' = P(getting\ all\ four\ heads\ with\ true\ coin) = \frac{1}{2}.\frac{1}{2}.\frac{1}{2}.\frac{1}{2} = \frac{1}{16}$

   $p_2' = P(getting\ all\ four\ heads\ with\ false\ coin) = 1.1.1.1 = 1$

   Required Probability $= \frac{p_2 p_2'}{p_1 p_1' + p_2 p_2'} = \frac{\frac{1}{4}.1}{\left(\frac{3}{4}\right)\left(\frac{1}{16}\right) + \left(\frac{1}{4}\right)(1)} = \frac{16}{19}$

2. **A coin is tossed. If it turns up heads two balls are drawn from urn A otherwise two balls are drawn from urn B. Urn A contains 3 black and 5 white balls. Urn B contains 7 black and one white ball. What is the probability that urn A was used, given that both balls drawn are black?**

   Solution:

   $p_1 = P(H) = \frac{1}{2}$

   $p_2 = P(T) = \frac{1}{2}$

   $p_1' = P(two\ black\ balls\ from\ A) = \frac{{}^3C_2}{{}^8C_2} = \frac{3}{28}$

   $p_2' = P(two\ black\ balls\ from\ B) = \frac{{}^7C_2}{{}^8C_2} = \frac{3}{4}$

   Required Probability $= \frac{p_1 . p_1'}{p_1 p_1' + p_2 p_2'} = \frac{\left(\frac{1}{2}\right)\left(\frac{3}{28}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{28}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)} = \frac{1}{8}$

3. A bag contains 7 red and 3 black balls and another bag contains 4 red and 5 black balls. One ball is transferred from the first bag to the second bag and then a ball is drawn from the second bag. If this ball happens to be red, find the probability that a black ball was transferred.

Solution:

$$p_1 = Probability\ of\ transferring\ black\ ball = \frac{3}{10}$$

$$p_2 = Probability\ of\ transferring\ red\ ball = \frac{7}{10}$$

$$p_1' = Probability\ of\ drawing\ a\ red\ ball\ if\ black\ was\ transferred = \frac{4}{10}$$

$$p_2' = Probability\ of\ drawing\ a\ red\ ball\ if\ red\ was\ transferred = \frac{5}{10}$$

Required Probability $= \frac{p_1 . p_1'}{p_1 p_1' + p_2 p_2'} = \frac{\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)}{\left(\frac{3}{10}\right)\left(\frac{4}{10}\right)+\left(\frac{7}{10}\right)\left(\frac{5}{10}\right)} = \frac{12}{47}$

4. In a certain test there are multiple choice questions. There are four possible answers to each question and one of them is correct. An intelligent student can solve 90% questions correctly by reasoning and for the remaining 10% questions he gives answers by guessing. A weak student can solve 20% questions correctly by reasoning and for the remaining 80% questions he gives answers by guessing. An intelligent students gets the correct answer, what is the probability that he was guessing?

Solution:

Consider the intelligent student,

$$p_1 = answer\ by\ reasoning = \frac{90}{100} = \frac{9}{10}$$

$$p_2 = answer\ by\ guessing = \frac{10}{100} = \frac{1}{10}$$

$$p_1' = answer\ is\ correct\ by\ reasoning = 1$$

$$p_2' = answer\ is\ correct\ by\ guessing = \frac{1}{4}$$

Required Probability $= \frac{p_2 p_2'}{p_1 p_1' + p_2 p_2'} = \frac{\left(\frac{1}{10}\right)\left(\frac{1}{4}\right)}{\left(\frac{9}{10}\right)(1)+\left(\frac{1}{10}\right)\left(\frac{1}{4}\right)} = \frac{1}{37}$

5. A man speaks truth 3 times out of 5. When a die is thrown, he states that it gave an ace. What is the probability that this event has actually happened?

Solution:

$$p_1 = prob\ of\ truth = \frac{3}{5}, p_2 = prob\ of\ lie = \frac{2}{5}$$

$$p_1' = prob\ of\ ace = \frac{1}{6},\ p_2' = prob\ of\ not\ ace = \frac{5}{6}$$

Required Probability $= \frac{p_1 . p_1'}{p_1 p_1' + p_2 p_2'} = \frac{\left(\frac{3}{5}\right)\left(\frac{1}{6}\right)}{\left(\frac{3}{5}\right)\left(\frac{1}{6}\right)+\left(\frac{2}{5}\right)\left(\frac{5}{6}\right)} = \frac{3}{13}$

6. **A lot of IC chips is known to contain 3% of defective chips. Each chip is tested before delivery but the tester is not completely reliable. It is known that:**
P(Tester says the chip is good/The chip is actually good)=0.95
P(Tester says the chip is good/The chip is actually defective)=0.96
If a tested chip us declared defective by the tester. What is the that it is actually defective?
Solution:
$p_1 = chip\ is\ defective = 0.03$
$p_1' = tester\ says\ defective\ and\ it\ is\ defective = 0.96$
$p_2 = chip\ is\ not\ defective = 0.97$
$p_2' = tester\ says\ defective\ and\ it\ is\ not\ defective = 0.05$
Required Probability $= \frac{p_1 . p_1'}{p_1 p_1' + p_2 p_2'} = \frac{(0.03)(0.96)}{(0.03)(0.96)+(0.97)(0.05)} = 0.37$

7. **A certain test for particular cancer is known to be 95% accurate. A person submits to the test and result is positive. Suppose that a person comes from a population of 1,00,000 where 2000 people suffer from disease. What can we conclude about the probability that person under test has particular cancer?**
[N17/IT/6M]
Solution:
$p_1 = prob\ a\ person\ has\ cancer = \frac{2000}{100000} = \frac{2}{100} = 0.02$
$p_2 = prob\ a\ person\ does\ not\ has\ cancer = 1 - 0.02 = 0.98$
$p_1' = test\ is\ positive\ when\ a\ person\ has\ cancer = \frac{95}{100} = 0.95$
$p_2' = test\ is\ positive\ when\ a\ person\ does\ not\ has\ cancer = 0.05$
Required Probability $= \frac{p_1 . p_1'}{p_1 p_1' + p_2 p_2'} = \frac{(0.02)(0.95)}{(0.02)(0.95)+(0.98)(0.05)} = \frac{19}{68}$

8. **In a bolt factory, machines A,B, C produce respectively 25%, 35% and 40%. Of their output 5%, 4% and 2% are defective. A bolt is drawn at random from a days production and is found defective. What is the probability that it was produced by machines A, B, C?**
Solution:
$p_1 = produced\ by\ A = 0.25$
$p_2 = produced\ by\ B = 0.35$
$p_3 = produced\ by\ C = 0.40$
$p_1' = defective\ by\ A = 0.05$
$p_2' = defective\ by\ B = 0.04$
$p_3 = defective\ by\ C = 0.02$

Defective obtained was produced by machine A,

Required Probability $= \frac{p_1 . p_1'}{p_1 p_1' + p_2 p_2' + p_3 p_3'} = \frac{25}{69}$

Defective obtained was produced by machine B,

Required Probability $= \frac{p_2 . p_2'}{p_1 p_1' + p_2 p_2' + p_3 p_3'} = \frac{28}{69}$

Defective obtained was produced by machine C,

Required Probability $= \frac{p_3 . p_3'}{p_1 p_1' + p_2 p_2' + p_3 p_3'} = \frac{16}{69}$

9. A box contains three biased coins A, B and C. the probability that a head will result when A is tossed is 1/3, when B is tossed is 2/3 and when C is tossed it is 3/4. If one of the coins is chosen and is tossed 3 times, head resulted twice and tail once. What is the probability that the coin chosen was A?

Solution:

$p_1 = P(choosing\ A) = \frac{1}{3}$

$p_2 = P(choosing\ B) = \frac{1}{3}$

$p_3 = P(choosing\ C) = \frac{1}{3}$

$p_1' = P(getting\ 2\ heads\ in\ 3\ tosses\ with\ A) = {}^3C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right) = \frac{2}{9}$

$p_2' = P(getting\ 2\ heads\ in\ 3\ tosses\ with\ B) = {}^3C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right) = \frac{4}{9}$

$p_3' = P(getting\ 2\ heads\ in\ 3\ tosses\ with\ C) = {}^3C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right) = \frac{27}{64}$

Required Probability $= \frac{p_1 . p_1'}{p_1 p_1' + p_2 p_2' + p_3 p_3'} = \frac{128}{627}$

10. A bag contains five balls, the colours of which are not known. Two balls were drawn from the bag and they were found to be white. What is the probability that all balls are white?

Solution:

Since two balls drawn are white, the bag may contain 2 white or 3 white or 4 white or 5 white balls.

$p_1 = P(bag\ conatins\ 2\ whites) = \frac{1}{4}$

$p_2 = P(bag\ conatins\ 3\ whites) = \frac{1}{4}$

$p_3 = P(bag\ conatins\ 4\ whites) = \frac{1}{4}$

$p_4 = P(bag\ conatins\ 5\ whites) = \frac{1}{4}$

$p_1' = P(drawing\ 2\ balls\ white\ when\ 2\ are\ white) = \frac{^2C_2}{^5C_2} = \frac{1}{10}$

$p_2' = P(drawing\ 2\ balls\ white\ when\ 3\ are\ white) = \frac{^3C_2}{^5C_2} = \frac{3}{10}$

$p_3' = P(drawing\ 2\ balls\ white\ when\ 4\ are\ white) = \frac{^4C_2}{^5C_2} = \frac{3}{5}$

$p_4' = P(drawing\ 2\ balls\ white\ when\ 5\ are\ white) = \frac{^5C_2}{^5C_2} = 1$

Required Probability $= \frac{p_4 p_4'}{p_1 p_1' + p_2 p_2' + p_3 p_3' + p_4 p_4'} = \frac{1}{2}$