Probability Distribution.

A. Random Variable

Discrete R.V.

$X = -2, 0, 3, 11$

(Values are Given)

Continuous R.V.

$0 \leq x \leq 5$

(Interval is given)

B. Function

Prob. Density function

$f(x)$

Prob. Distribution function

$F(x)$

$$f(x) = \frac{d}{dx} F(x)$$

C. Probability Density function.

Discrete R.V.

x	0	1	2	3
$p(x)$	0.3	0.4	0.1	0.2

1) $\Sigma p(x) = 1$
2) $0 \leq p(x) \leq 1$

Continuous R.V.

$f(x) = 3x^2 \quad 0 \leq x \leq 1$

$$\int f(x)\,dx = \int_0^1 3x^2\,dx$$

$$= \cancel{3}\left(\frac{x^3}{\cancel{3}}\right)_0^1$$

$$= 1 - 0 = 1$$

D. Prob. Distribution function.

D.R.V.

x	0	1	2	3
$p(x)$	0.3	0.4	0.1	0.2
$F(x)$	0.3	0.7	0.8	1.0

C.R.V.

$$F(x) = \int_{-\infty}^{x} f(x)\,dx.$$

Mathematical Expectation.

Main formula:

$$\mu_r' = E(x^r) = \sum x^r p(x) = \int x^r f(x)\,dx.$$

A. Mean, variance, standard Deviation, mode.

1. Mean: put $r=1$ in main formula.

$$\mu_1' = E(X) = \sum x\,p(x) = \int x^1 f(x)\,dx$$

2. Variance: $\sigma^2 = V(x) =$

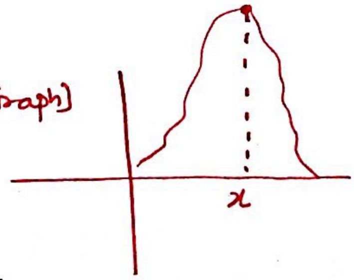
$$\sigma^2 = V(x) = E(X - E(x))^2 = \mu_2' - (\mu_1')^2$$

3. Standard Deviation:

$$\sigma = +\sqrt{\text{variance}}$$

4. Mode: [Heighest Point on Graph]

$$f'(x) = 0$$



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B. Moments.

Raw moment [about origin] μ_r'

$$\mu_r' = \sum x^r p(x) = \int x^r f(x)\,dx$$

$r=1$, $\mu_1' = \sum x\,p(x) = \int x f(x)\,dx$

$r=2$, $\mu_2' = \sum x^2 p(x) = \int x^2 f(x)\,dx$

$r=3$, $\mu_3' = \sum x^3 p(x) = \int x^3 f(x)\,dx$

$r=4$, $\mu_4' = \sum x^4 p(x) = \int x^4 f(x)\,dx$

Central moment [about mean] μ_r

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$\mu_4 = 1\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

Raw moment [about any Point]

$$\mu_{a(r)}' = E(X-a)^r$$

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C. Moment Generating Function

$$M_0(t) = \sum e^{tx} p(x) = \int e^{tx} f(x)\,dx$$

$$\left.\frac{d^r M_0(t)}{dt^r}\right|_{t=0} = \mu_r'$$

D. Properties.

1. $E(c) = c$
2. $E(ax) = a\,E(x)$
3. $V(c) = 0$
4. $V(ax) = a^2 V(x)$.

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Binomial Distribution.

(only two Probabilities p & q)

1. $p+q=1$
2. $P(x) = {}^nC_x\, p^x q^{n-x}$
3. Mean $= np$.
4. Variance $= npq$

Poisson Distribution

(prob is too small & samples are large)

1. $P(X=x) = \dfrac{e^{-m} m^x}{x!}$
2. Mean = Variance $= m$ $(n \times p)$

Normal Distribution

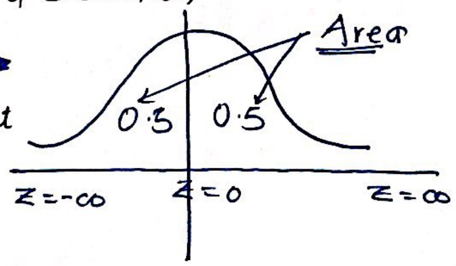
$$f(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$\mu \Rightarrow$ Mean

$\sigma \Rightarrow$ Standard Deviation

Normal curve $\Rightarrow$

- Symmetric about Y-axis.
- Total area $= 1$



Standard Normal variable: (Z)

$$Z = \frac{x-\mu}{\sigma}$$

$$Z = \frac{x-\mu}{\sigma/\sqrt{n}} \quad \text{(Central limit theorem)}$$

Note: For Binomial Distribution, Poisson Distribution, Normal Distribution

Final Ans

Find Prob.	Find Number
$= P(x)$	$= N \cdot P(x)$