Poisson Distribution

Page No.:
Date:

Poisson Distribution

$n \to$ Large

Probability of r success

$$P(r) = \frac{m^r \cdot e^{-m}}{r!}$$

$m = np$

$\downarrow$

mean

Q. If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad injection

Soln:

$n = 2000 \qquad p = 0.0001$

$\therefore \; m = np = 2000 \times 0.001 = 2$

Probability that more than 2 will get a bad reaction by poisson distribution

$P(3) + P(4) + P(5) + \ldots\ldots + P(2000)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{m^0 . e^{-m}}{0!} + \frac{m^1 . e^{-m}}{1!} + \frac{m^2 . e^{-m}}{2!}\right]$

$= 1 - e^{-m}\left[1 + 2 + \cancel{4}^{2}/\cancel{2}_{1}\right]$

$= 1 - \frac{5}{e^m}$

$= 1 - \frac{5}{e^2}$

$= 0.3233$

Q. Fit a poisson distribution to set of observations

x :	0	1	2	3	4
f :	122	60	15	2	1

Solⁿ:

$$P(r) = \frac{m^r \cdot e^{-m}}{r!}$$

Mean for grouped data

$$m = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0 + 60 + 30 + 6 + 4}{200} = 0.5$$

Theoretical frequency for r success is

$$N \times \frac{m^r \cdot e^{-m}}{r!} = \frac{200 \times (0.5)^r \cdot e^{-(0.5)}}{r!}$$

$e^{-0.5} = 0.6065$

where $r = 0, 1, 2, 3, 4$

For $r = 0$, $P(0) = \frac{200 \times (0.5)^0 (0.6065)}{0!} = 121.3$

For $r = 1$, $P(1) = \frac{200 \times (0.5)^1 (0.6065)}{1!} = 60.65$

For $r = 2$, $P(2) = \frac{200 \times (0.5)^2 (0.6065)}{2!} = 15.16$

For $r = 3$, $P(3) = \frac{200 \times (0.5)^3 (0.6065)}{3!} = 2.52$

For $r = 4$, $P(4) = \frac{200 \times (0.5)^4 (0.6065)}{4!} = 0.31$

Hence the theoretical frequencies fitted by poisson distribution

x	$\rightarrow$	0	1	2	3	4
f	$\rightarrow$	121.3	60.65	15.16	2.52	0.31

Q. A manufacturer knows that the condensor he makes contain on an average 1% defective. He packs them in boxes of 100. What is the probability that a box picked at random will contain 3 or more defective condensor.

Soln:

$$p = 0.01 = \frac{1}{100}, \quad n = 100$$

$$m = np = 100 \times \frac{1}{100} = 1$$

By poisson distribution, the probability that a box picked at random will contain 3 or more defective condensors

$$P(r) = \frac{m^r \cdot e^{-m}}{r!}$$

$$P(3) + P(4) + P(5) + \ldots\ldots + P(100)$$

$$= 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left[\frac{1^0 \cdot e^{-m}}{0!} + \frac{1^1 \cdot e^{-m}}{1!} + \frac{1^2 \cdot e^{-m}}{2!}\right]$$

$$= 1 - e^{-1}\left[1 + 1 + 1/2\right]$$

$$= 1 - \frac{1}{e}\left[\frac{5}{2}\right]$$

$$= 1 - \frac{5}{2e}$$

$$= 0.0803$$

Q. In a certain factory turning out razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use poisson distribution to calculate the approximate number of packets containing no defective, one defective and 2 defective blades respectively in a consignment of 10,000 packets.

Soln:

$P = 0.002$ $\quad$ $n = 10$

Mean, $m = np = 10 \times 0.002 = 0.02$

$e^{-0.02} = 0.9802$

By poisson distribution, $P(r) = \frac{m^r \cdot e^{-m}}{r!}$

(i) Probability of no defective blade $= P(0) = \frac{m^0 e^{-m}}{0!} = 0.9802$

∴ No. of packets containing zero defective blades

$= 0.9802 \times 10000 = 9802$

(ii) Probability of one defective blade $= P(1) = \frac{m^1 \cdot e^{-m}}{1!}$

$= \frac{(0.02)^1 (0.9802)}{1!} = 0.019604$

∴ No. of packets containing one defective blades

$= 0.019604 \times 10000 = 196$ (Approx)

(iii) Probability of two defective blades $= P(1) = \frac{m^2 \cdot e^{-m}}{2!}$

$= \frac{(0.02)^2 (0.9802)}{2!} = 1.9604 \times 10^{-4}$

∴ No. of packets containing two defective blades

$= 1.9604 \times 10^{-4} \times 10000 = 1.96$

≈ 2 (Approx)

Q. The frequency of accidents per shift in a factory is as shown in following table

Accidents per shift :	0	1	2	3	4
Frequency :	180	92	24	3	1

Calculate the mean no. of accidents per shift and corresponding poisson distribution and compare with actual observations.

Solⁿ:

$$\text{Mean, } m = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0+92+48+9+4}{\underbrace{300}_{N}} = 0.57$$

$$e^{-m} = e^{-0.57} = 0.6005$$

The poisson distribution frequency for corresponding $r = 0, 1, 2, 3, 4$ accidents per shift.

$$P(r) = \frac{N \cdot m^r \cdot e^{-m}}{r!} \quad \text{where } r = 0, 1, 2, 3, 4$$

For $r = 0$, $P(0) = \frac{300 \times (0.51)^0 \, (0.6005)}{0!} = 180.15$

For $r = 1$, $P(1) = \frac{300 \times (0.51)^1 \, (0.6005)}{1!} = 91.87$

For $r = 2$, $P(2) = \frac{300 \times (0.51)^2 \, (0.6005)}{2!} = 23.42$

For $r = 3$, $P(3) = \frac{300 \times (0.51)^3 \, (0.6005)}{3!} = 3.98$

For $r = 4$, $P(4) = \frac{300 \times (0.51)^4 \, (0.6005)}{4!} = 0.5$

Accidents per shift :	0	1	2	3	4
Frequency :	180.15	91.87	23.42	3.98	0.5

Q. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a poisson distribution with mean 1.5. Calculate the probability of day (i) On which there is no demand
(ii) On which demand is refused

Soln:

$m = 1.5 = np$

$e^{-m} = 0.2231$

By poisson distribution, $\frac{m^r \cdot e^{-m}}{r!}$

(i) Probability on which there is no demand

$= P(0) = \frac{m^0 \cdot e^{-m}}{0!} = \frac{(1.5)^0 \, (0.2231)}{0!} = 0.2231$

(ii) Since the firm has two cars which it hires out day by day so the probability on which there is no demand

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\frac{m^0 \cdot e^{-m}}{0!} + \frac{m^1 \cdot e^{-m}}{1!} + \frac{m^2 \cdot e^{-m}}{2!}\right]$

$= 1 - \frac{1}{e^m}\left[1 + 1.5 + 1.125\right]$

$= 1 - \frac{3.625}{e^{1.5}}$

$= 0.1911$