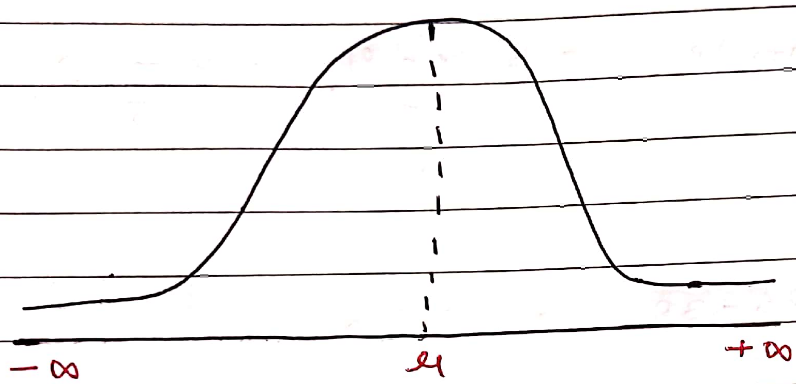
Normal Distribution

- A continuous random variable X is said to follow normal distribution with mean (μ) and standard deviation (σ), if its probability function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



$$-\infty < X < \infty$$
$$-\infty < \mu < \infty$$
$$\sigma > 0$$

- The curve representing the normal distribution is called normal curve
- The normal curve is 'Bell - shaped' and is symmetric about its mean.
- Probability of a normal random variable in an Interval

$$P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Q X is a normal variate with mean 30 and standard deviation 5. Find the probability that

(i) $26 \leq X \leq 40$

(ii) $X \geq 45$

(iii) $|X - 30| > 5$

Sol^n:

$\mu = 30$

$\sigma = 5$

Standard normal variate, $Z = \dfrac{X - \mu}{\sigma} = \dfrac{X - 30}{5}$

(i) $26 \leq X \leq 40$

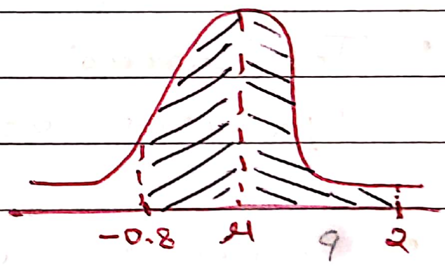
When $X = 26$, $Z = \dfrac{26 - 30}{5} = -0.8$

When $X = 40$, $Z = \dfrac{40 - 30}{5} = 2$

$\therefore P(26 \leq X \leq 40) = P(-0.8 \leq Z \leq 2)$

$= P(-0.8 \leq Z \leq 0) + P(0 \leq Z \leq 2)$

$= P(0 \leq Z \leq 0.8) + P(0 \leq Z \leq 2)$



$= 0.2881 + 0.4772$

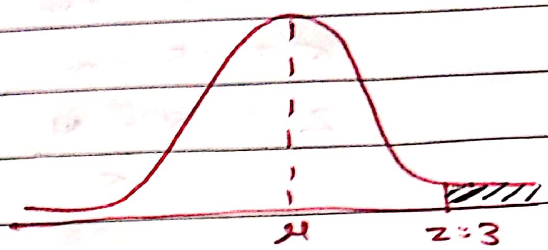
$= 0.7653$

(ii) $x \geq 45$

When $x = 45$, $z = \frac{45-30}{5} = 3$

$\therefore P(x \geq 45) = P(z \geq 3)$

$= 0.5 - P(0 \leq z \leq 3)$



$= 0.5 - 0.4987$

$= 0.0013$

(iii) $|x-30| \leq 5$

$25 \leq x \leq 35$

When $x = 25$, $z = -1$

When $x = 35$, $z = 1$

$|x-30| > 5$

$+$: $x - 30 > 5$, $x > 35$

$-$: $-(x-30) > 5$, $x - 30 < -5$, $x < 25$

$\therefore P(25 \leq x \leq 35) = P(-1 \leq z \leq 1)$

$= 2[P(0 \leq z \leq 1)]$

$= 2 \times 0.3413$

$= 0.6826$

$|x-30| \leq 5$

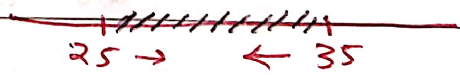
$+$: $x - 30 \leq 5$, $x \leq 35$

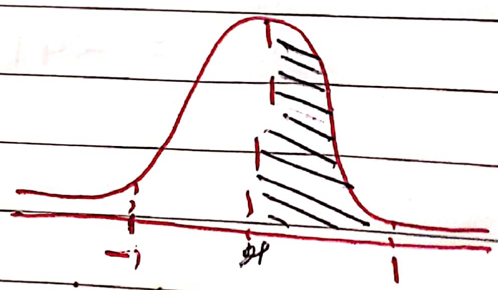
$-$: $-(x-30) \leq 5$, $x - 30 \geq -5$, $x \geq 25$

$P\{|x-30| > 5\} = 1 - P\{|x-30| \leq 5\}$

$= 1 - 0.6826$

$= 0.3174$





Q. The mean height of 500 students is 151 cm and standard deviation is 15 cm. Assuming that heights are normally distributed. Find how many student's height lie between 120 and 155 cm

Solⁿ:

$\mu = 151$ cm

$\sigma = 15$ cm

$z = \frac{x - \mu}{\sigma} = \frac{x - 151}{15}$

When $x = 120$, $z = \frac{120 - 151}{15} = -2.06$

When $x = 155$, $z = \frac{155 - 151}{15} = 0.26$

$P(120 \leq x \leq 155) = P(-2.06 \leq z \leq 0.26)$

$= P(-2.06 \leq z \leq 0) + P(0 \leq z \leq 0.26)$

$= P(0 \leq z \leq 2.06) + P(0 \leq z \leq 0.26)$

$= 0.4803 + 0.1026$

$= 0.5829$

Hence, number of student's heights lie between 120 and 155 cm

$= 0.5829 \times 500$

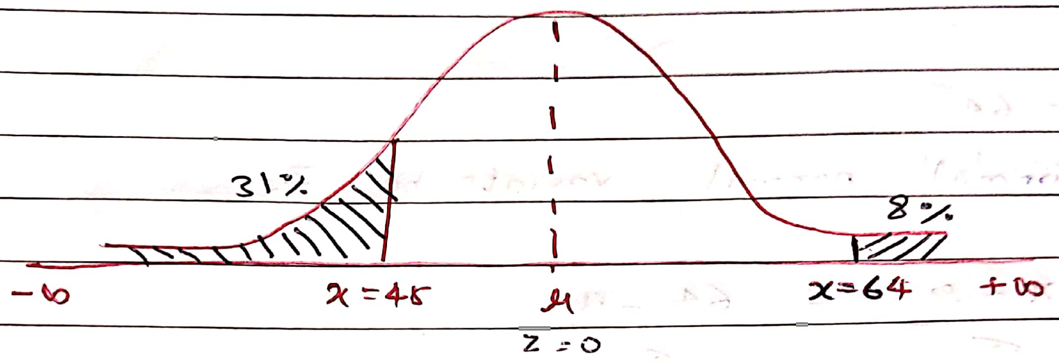
$= 291.45$

≈ 291 students

Q. In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation of distribution

Sol^n:

Let mean $= \mu$ and standard deviation $= \sigma$



When $x = 45$,

Let the standard normal variate be z_1

$$\Rightarrow z_1 = \frac{x-\mu}{\sigma} = \frac{45-\mu}{\sigma}$$

$$\int_{-\infty}^{z_1} \phi(z)\,dz = 0.31$$

$$\Rightarrow \int_{-\infty}^{0} \phi(z)\,dz - \int_{z_1}^{0} \phi(z)\,dz = 0.31$$

$$\Rightarrow 0.5 - \int_{z_1}^{0} \phi(z)\,dz = 0.31$$

$$\Rightarrow \int_{z_1}^{0} \phi(z)\,dz = 0.19$$

From table, we get $z_1 = -0.5$

$$-0.5 = \frac{45-\mu}{\sigma}$$

$$\Rightarrow -0.5\sigma = 45-\mu$$

$$\Rightarrow \mu - 0.5\sigma = 45 \quad \text{—— (1)}$$

When $x = 64$

Let the standard normal variate be z_2,

$$\Rightarrow z_2 = \frac{x-\mu}{\sigma} = \frac{64-\mu}{\sigma}$$

$$\int_{z_2}^{\infty} \phi(z)\,dz = 0.08$$

$$\Rightarrow \int_0^{\infty} \phi(z)\,dz - \int_0^{z_2} \phi(z)\,dz = 0.08$$

$$\Rightarrow 0.5 - \int_0^{z_2} \phi(z)\,dz = 0.08$$

$$\Rightarrow \int_0^{z_2} \phi(z)\,dz = 0.42$$

From table, we get $z_2 = 1.4$

$$\Rightarrow 1.4 = \frac{64-\mu}{\sigma}$$

$$\Rightarrow 1.4\sigma = 64-\mu$$

$$\Rightarrow \mu + 1.4\sigma = 64 \quad \text{—— (2)}$$

From eqⁿ 1 and eqⁿ 2

$$\mu - 0.5\sigma = 45 \quad - (1)$$

$$\mu + 1.4\sigma = 64 \quad - (2)$$

$$\Rightarrow \mu = 50 \text{ and } \sigma = 10$$

Q. Find the eq^n of normal probability curve that may be filled to the following data

x	:	0	1	2	3	4	5
f	:	13	23	34	15	11	4

Sol^n:

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} \quad \text{and}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

x	x^2	f	fx	fx^2
0	0	13	0	0
1	1	23	23	23
2	4	34	68	136
3	9	15	45	135
4	16	11	44	176
5	25	4	20	100
		$\Sigma f = 100$	$\Sigma fx = 200$	$\Sigma fx^2 = 570$

$$\therefore \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{200}{100} = 2 = \mu$$

$$SD = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2} = \sqrt{\frac{570}{100} - \left(\frac{200}{100}\right)^2}$$

$$= \sqrt{5.7 - 4} = \sqrt{1.7}$$

$$= 1.3 = \sigma$$

The eq^n of normal curve is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{(1.3)\sqrt{2\pi}} \cdot e^{\frac{-(x-2)^2}{2(1.3)^2}}$$

$$y = \frac{1}{(1.3)\sqrt{2\pi}} \cdot e^{\frac{-(x-2)^2}{3.38}}$$

This is the required equation.

Q. In a test on 2000 electric bulbs, it found that the life of a particular make, was normally distributed with an average life of 2040 hrs and SD of 60 hrs. Estimate the no. of bulbs likely to bum for

(i) more than 2150 hrs

(ii) Less than 1950 hrs

(iii) More than 1920 hrs and but less than 2160 hrs

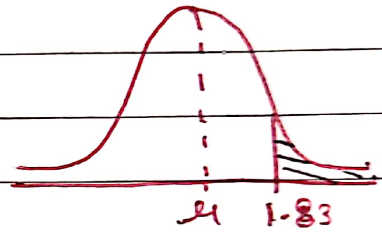
Soln:

$\mu = 2040$

$\sigma = 60$

(i) For $x = 2150$

$$Z = \frac{x - \mu}{\sigma} = \frac{2150 - 2040}{60} = 1.83$$



Area against $Z = 1.83$

From table $= 0.4664$

Required area $= 0.5 - 0.4664$

$= 0.0336$

$\therefore$ The number of bulbs likely to burn for more than 2150 hrs

$= 0.0336 \times 2000$

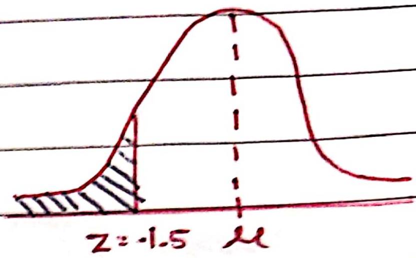
$= 67.2$

≈ 67

(ii) For $x = 1950$

$$z = \frac{x - \mu}{\sigma} = \frac{1950 - 2040}{60} = -1.5$$

Area against $z = -1.5$



From table $= 0.4232$

Required area $= 0.5 - 0.4232$

$= 0.0768$

∴ The number of bulbs likely to burn for less than 1950 hrs

$= 0.0768 \times 2000$

$= 153.6$

≈ 153

(iii) When $x = 1920$, $z = \frac{1920 - 2040}{60} = -2$

When $x = 2160$, $z = \frac{2160 - 2040}{60} = 2$

$P(-2 \le z \le 2) = 2P(0 \le z \le 2)$

$= 2 \times 0.4772$

$= 0.9544$

∴ The number of bulbs likely to burn for more than 1920 hrs and but less than 2160 hrs

$= 0.9544 \times 2000$

$= 1908.8$

≈ 1908