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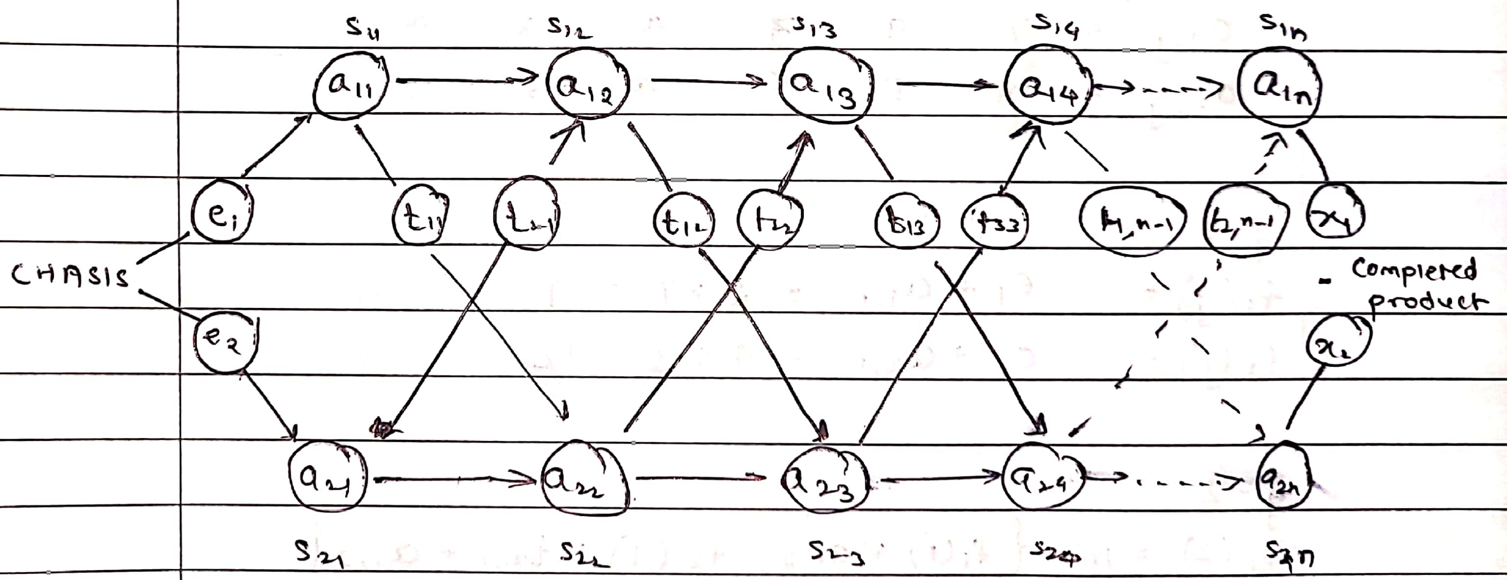
FRIDAY

Assembly line scheduling problem

- It is a manufacturing problem to find fastest way through a factory to complete the product.
- Here, Assuming 2 assembly line, each line has n number of stations. Each station carry some specific work. J^{th} station on a line i, is denoted as $S_{i,j}$ and the assembly time at that station is denoted as $a_{i,j}$.
- An automobile chasis enters the factory line and goes on to the line i (where $i = 1$ or 2). Taking e_i time, after going through J^{th} station on line the chasis goes onto the ($J+1$ station) on either line.
- There is no transfer cost if it stay on same line but it takes time $t_{i,j}$ if it transfer on another line. After the n^{th} station product will be ready and exists cost will be added x_i
- The problem is to determine which stations are selected from line 1 to line 2 to complete the product and that too the minimum time.



$$f^*(n) = \min\{f_1(n) + x_1,\ f_2(n) + x_2\}$$

$$f_1(j) = \begin{cases} e_1 + a_{1,1} & \text{if } j = 1 \\ \min\{ f_1(j-1) + a_{1,j},\ f_2(j-1) + t_{2,j-1} + a_{1,j} \} & \text{otherwise.} \end{cases}$$

$$f_2(j) = \begin{cases} e_2 + a_{2,1} & \text{if } j = 1 \\ \min\{ f_2(j-1) + a_{2,j},\ f_1(j-1) + t_{1,j-1} + a_{2,j} \} & \text{otherwise} \end{cases}$$

Q. To find the minimum assembly line for the given data.

2 assembly line, 6 station on each line

$e_1 = 2$, $e_2 = 4$, $x_1 = 3$, $x_2 = 2$

Transfer time from line 1 to line 2.

$t_{1,1} = 2$, $t_{1,2} = 3$, $t_{1,3} = 1$, $t_{1,4} = 8$, $t_{1,5} = 4$

Transfer time from line 2 to line 1

$t_{2,1} = 2$, $t_{2,2} = 1$, $t_{2,3} = 3$, $t_{2,4} = 2$, $t_{2,5} = 1$

Station time on line 1

$a_{1,j}$ =	7	9	3	4	8	4
$a_{2,j}$ =	8	5	6	4	5	7

$j = 1$

$f_1(j) = e_1 + a_{1,1} = 2 + 7 = 9$

$f_2(j) = e_2 + a_{2,1} = 4 + 8 = 12$

$j = 2$

$f_1(2) = \min\{ f_1(1) + a_{1,2},\ f_2(1) + t_{2,1} + a_{1,2} \}$

$= \min\{ 9 + 9,\ 12 + 2 + 9 \}$

$= \min\{ 18,\ 23 \}$

$= 18$

$$f_2(2) = \min\{ f_2(1) + a_{2,2} ,\ f_1(1) + t_{1,1} + a_{2,2} \}$$
$$= \min\{ 12 + 5 ,\ 9 + 2 + 5 \}$$
$$= \min\{ 17 ,\ 16 \}$$
$$= 16$$

$j = 3$

$$f_1(3) = \min\{ f_1(2) + a_{1,3} ,\ f_2(2) + t_{2,2} + a_{1,3} \}$$
$$= \min\{ 18 + 3 ,\ 16 + 1 + 3 \}$$
$$= \min\{ 21 ,\ 20 \}$$
$$= 20$$

$$f_2(3) = \min\{ f_2(2) + a_{2,3} ,\ f_1(2) + t_{1,2} + a_{2,3} \}$$
$$= \min\{ 16 + 6 ,\ 18 + 3 + 6 \}$$
$$= \min\{ 22 ,\ 27 \}$$
$$= 22$$

$j = 4$

$$f_1(4) = \min\{ f_1(3) + a_{1,4} ,\ f_2(3) + t_{2,3} + a_{1,4} \}$$
$$= \min\{ 20 + 4 ,\ 22 + 3 + 4 \}$$
$$= \min\{ 24 ,\ 29 \}$$
$$= 24$$

$$f_2(4) = \min\{ f_2(3) + a_{2,4} ,\ f_1(3) + t_{1,3} + a_{2,4} \}$$
$$= \min\{ 22 + 4 ,\ 20 + 1 + 4 \}$$
$$= \min\{ 26 ,\ 25 \}$$
$$= 25$$

$j = 5$

$$f_1(5) = \min\{ f_1(4) + a_{1,5},\ f_2(4) + t_{2,4} + a_{1,5} \}$$
$$= \min\{ 24 + 8,\ 25 + 2 + 8 \}$$
$$= \min\{ 32,\ 35 \}$$
$$= 32$$

$$f_2(5) = \min\{ f_2(4) + a_{2,5},\ f_1(4) + t_{1,4} + a_{2,5} \}$$
$$= \min\{ 25 + 5,\ 24 + 8 + 5 \}$$
$$= \min\{ 30,\ 37 \}$$
$$= 30$$

$j = 6$

$$f_1(6) = \min\{ f_1(5) + a_{1,6},\ f_2(5) + t_{2,5} + a_{1,6} \}$$
$$= \min\{ 32 + 4,\ 30 + 1 + 4 \}$$
$$= \min\{ 36,\ 35 \}$$
$$= 35$$

$$f_2(6) = \min\{ f_2(5) + a_{2,6},\ f_1(5) + t_{1,5} + a_{2,6} \}$$
$$= \min\{ 30 + 7,\ 32 + 4 + 7 \}$$
$$= \min\{ 37,\ 43 \}$$
$$= 37$$

$$f = \min\{ f_1(6) + x_1,\ f_2(6) + x_2 \}$$
$$= \min\{ 35 + 3,\ 37 + 2 \}$$
$$= \min\{ 38,\ 39 \}$$
$$= 38$$

Path:

$e_1 - S_{11} - S_{22} - S_{13},\ S_{14},\ S_{25} - S_{16} - x_1$