Master Theorem

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General Format

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

where $a \geq 1$, $b > 1$ and $f(n)$ should +ve always.

Cases :

i] $f(n) < n^{\log_b a}$

$T(n) = \theta\left(n^{\log_b a}\right)$

ii] $f(n) = n^{\log_b a}$

$T(n) = \theta\left(n^{\log_b a} \log n\right)$

iii] $f(n) > n^{\log_b a}$

$T(n) = \theta \cdot f(n)$

Q. $T(n) = 4T(n/2) + n$

Soln:

Given : $a = 4$

$b = 2$

$f(n) = n$

$\therefore n^{\log_b a} = n^{\log_2 4} = n^2$

$\therefore f(n) < n^{\log_b a}$ Case I

$T(n) = \theta(n^{\log_b a})$

$\boxed{T(n) = \theta(n^2)}$

Q. $T(n) = 2T(n/2) + n$

Soln:

Given : $a = 2$

$b = 2$

$f(n) = n$

$\therefore n^{\log_b a} = n^{\log_2 2} = n$

$\therefore f(n) = n^{\log_b a}$ Case II

$T(n) = \theta(n^{\log_b a} \cdot \log n)$

$\boxed{T(n) = \theta(n \cdot \log n)}$

Q. $T(n) = 2T(n/2) + n^2$

Sol^n:

Given: $a = 2$

$b = 2$

$f(n) = n^2$

$\therefore n^{\log_b a} = n^{\log_2 2} = n$

$\therefore f() > n^{\log_b a}$ Case III

$T(n) = \theta\, f(n)$

$$\boxed{T(n) = \theta(n^2)}$$

Q. $T(n) = 4T(n/2) + n^3$

Sol^n:

Given: $a = 4$

$b = 2$

$f(n) = n^3$

$\therefore n^{\log_b a} = n^{\log_2 4} = n^2$

$\therefore f(n) > n^{\log_b a}$ Case III

$T(n) = \theta\, f(n)$

$$\boxed{T(n) = \theta(n^3)}$$

Q. $T(n) = T(9n/10) + n$

Soln:

$$T\left(\frac{9n/9}{10/9}\right) + n$$

$$T(n) = T\left(\frac{n}{1.1}\right) + n$$

$a = 1$

$b = 1.1$

$f(n) = n$

$\therefore n^{\log_b a} = \boxed{n^{\log_{1.1} 1} = n^0}$

$\therefore f(n) > n^{\log_b a}$ case III

$T(n) = \theta\, f(n)$

$$\boxed{T(n) = \theta(n)}$$

Q. $T(n) = 8T(n/4) + n^2$

Sol^n:

Given: $a = 8$

$b = 4$

$f(n) = n^2$

$\therefore n^{\log_b a} = n^{\log_4 8} = n^{3/2}$

$\therefore f(n) > n^{\log_b a}$ Case III

$T(n) = \theta\, f(n)$

$$\boxed{T(n) = \theta(n^2)}$$

Q. $T(n) = 4T(n/2) + n^3$

Sol^n:

Given: $a = 4$

$b = 2$

$f(n) = n^3$

$\therefore n^{\log_b a} = n^{\log_2 4} = n^2$

$\therefore f(n) > n^{\log_b a}$ Case III

$T(n) = \theta \cdot f(n)$

$$\boxed{T(n) = \theta(n^3)}$$

Q. $T(n) = 3T(2n/6) + n$

Soln:

$$3T\left(\frac{2n/2}{6/2}\right) + n$$

$$3T\left(\frac{n}{3}\right) + n$$

$a = 3$

$b = 3$

$f(n) = n$

$\therefore n^{\log_b a} = n^{\log_3 3} = n$

$\therefore f(n) = n^{\log_b a}$ Case II

$T(n) = \Theta(n^{\log_b a} \cdot \log n)$

$$\boxed{T(n) = \Theta(n \log n)}$$

Q. $T(n) = 8T\left(\frac{n}{4}\right) + n$

Soln:

Given: $a = 8$

$b = 4$

$f(n) = n$

$\therefore n^{\log_b a} = n^{\log_4 8} = n^{3/2}$

$\therefore f(n) < n^{\log_b a}$ Case I

$T(n) = \Theta(n^{\log_b a})$

$$\boxed{T(n) = \Theta(n^{3/2})}$$

Q. $T(n) = 16T(n/8) + n^2$

Soln:

Given: $a = 16$

$b = 8$

$f(n) = n^2$

$\therefore n^{\log_b a} = n^{\log_8 16} = n^{4/3}$

$\therefore f(n) > n^{\log_b a}$ Case III

$T(n) = \Theta\ f(n)$

$$\boxed{T(n) = \Theta(n^2)}$$