Binary Search

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Binary search :

This analysis is used to find average number of comparisons required for successful or unsuccessful search.

To find average number of comparisons

(i) Average number of comparison for successful

$$A.S(n) = \frac{IPL}{n} + 1$$

(ii) Average number of comparison for unsuccessful.

$$A.U(n) = \frac{EPL}{n+1}$$

IPL (Internal Path Length)

For Successful

EPL (External Path Length)

For Unsuccessful

Q. Find the average number of comparison for successful and unsuccessful search for array
-15, -6, 12, 18, 42, 64, 89, 95, 102, 118

Sol^n:

Index	1	2	3	4	5	6	7	8	9	10
Array	-15	-6	12	18	42	64	89	95	102	118
Comparison	3	2	3	4	1	3	4	2	3	4.

By using, mid = (low + high) / 2

① (1 + 10) / 2 = 5.5 ≈ 5

② (1 + 4) / 2 = 2.5 ≈ 2

(6 + 10) / 2 = 8

③ (1+1) / 2

(3+4) / 2 = 3.5 ≈ 3

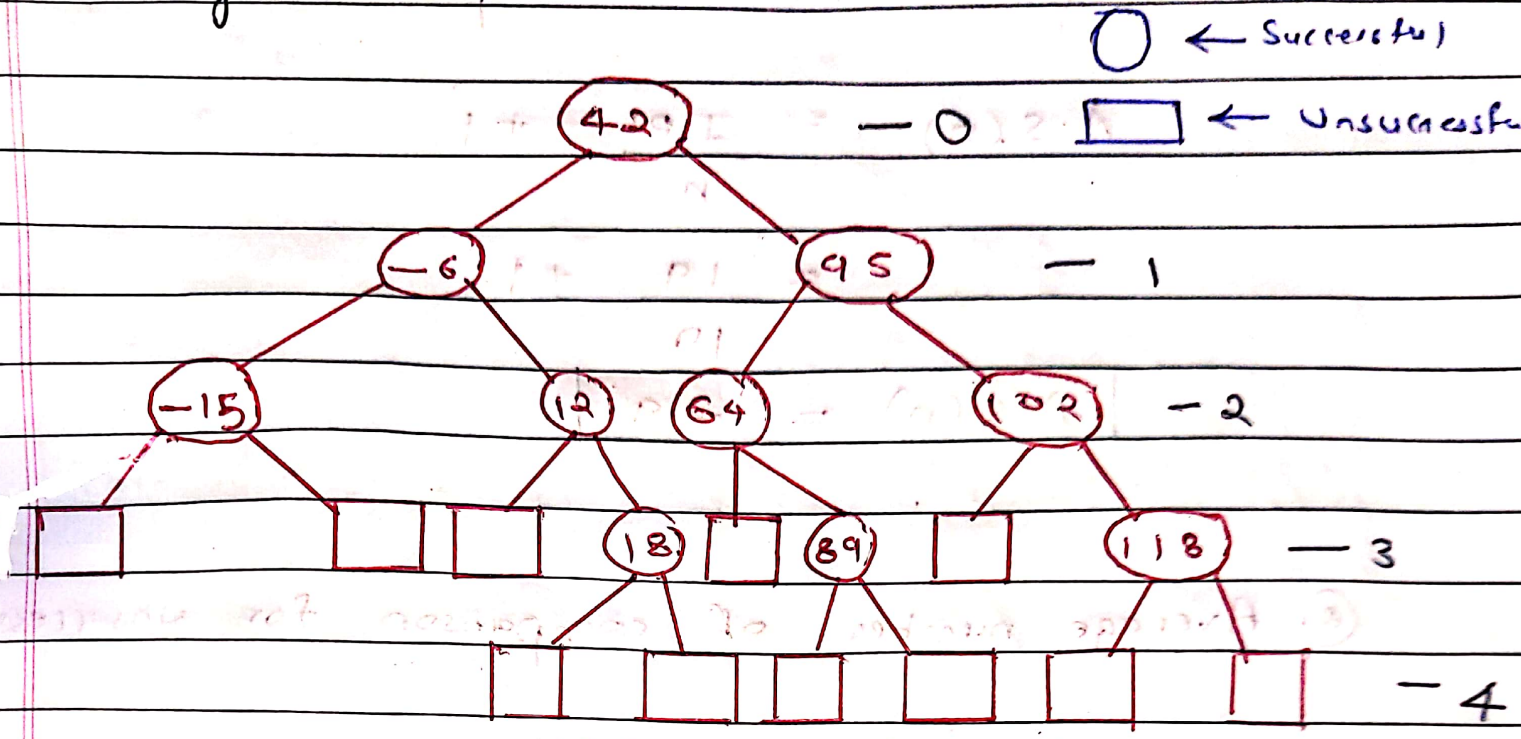
(6+7) / 2 = 6.5 ≈ 6

(9+10) / 2 = 9.5 ≈ 9

④ 4, 7, 10

→ Index Numbers

Binary search Tree :



IPL : $0*1 + 1*2 + 2*4 + 3*3 + 4*0$

$= 0 + 2 + 8 + 9 + 0$

IPL = 19

EPL : $4*6 + 3*5 + 2*0 + 1*0 + 0*0$

$= 24 + 15 + 0 + 0 + 0$

EPL = 39

(1) Average number of comparison for successful

$$A.S(n) = \frac{I.PL}{n} + 1$$

$$= \frac{19}{10} + 1$$

$$\boxed{A.S(n) = 2.9}$$

(2) Average number of comparison for unsuccessful

$$A.U(n) = \frac{EPL}{n+1}$$

$$= \frac{39}{10+1}$$

$$= \frac{39}{11}$$

$$\boxed{A.U(n) = 3.5}$$

Complexity : $O(\log_2 n)$

$= O(\log_2 10)$

$= O(3.3)$

Q. Find average number of comparison for successful and unsuccessful search for array
-15, -6, 0, 7, 9, 23, 54, 82, 101, 112, 125, 131, 142, 151

Sol^:

Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Array	-15	-6	0	7	9	23	54	82	101	112	125	131	142	151
Comparison	3	4	2	4	3	4	1	4	3	4	2	4	3	4

By using, mid = (low + high) / 2

① (1 + 14) / 2 = 7.5 ≈ 7

② (1 + 6) / 2 = 3.5 ≈ 3

(8 + 14) / 2

③ (1 + 2) / 2 = 1.5 ≈ 1

(4 + 6) / 2 = 5

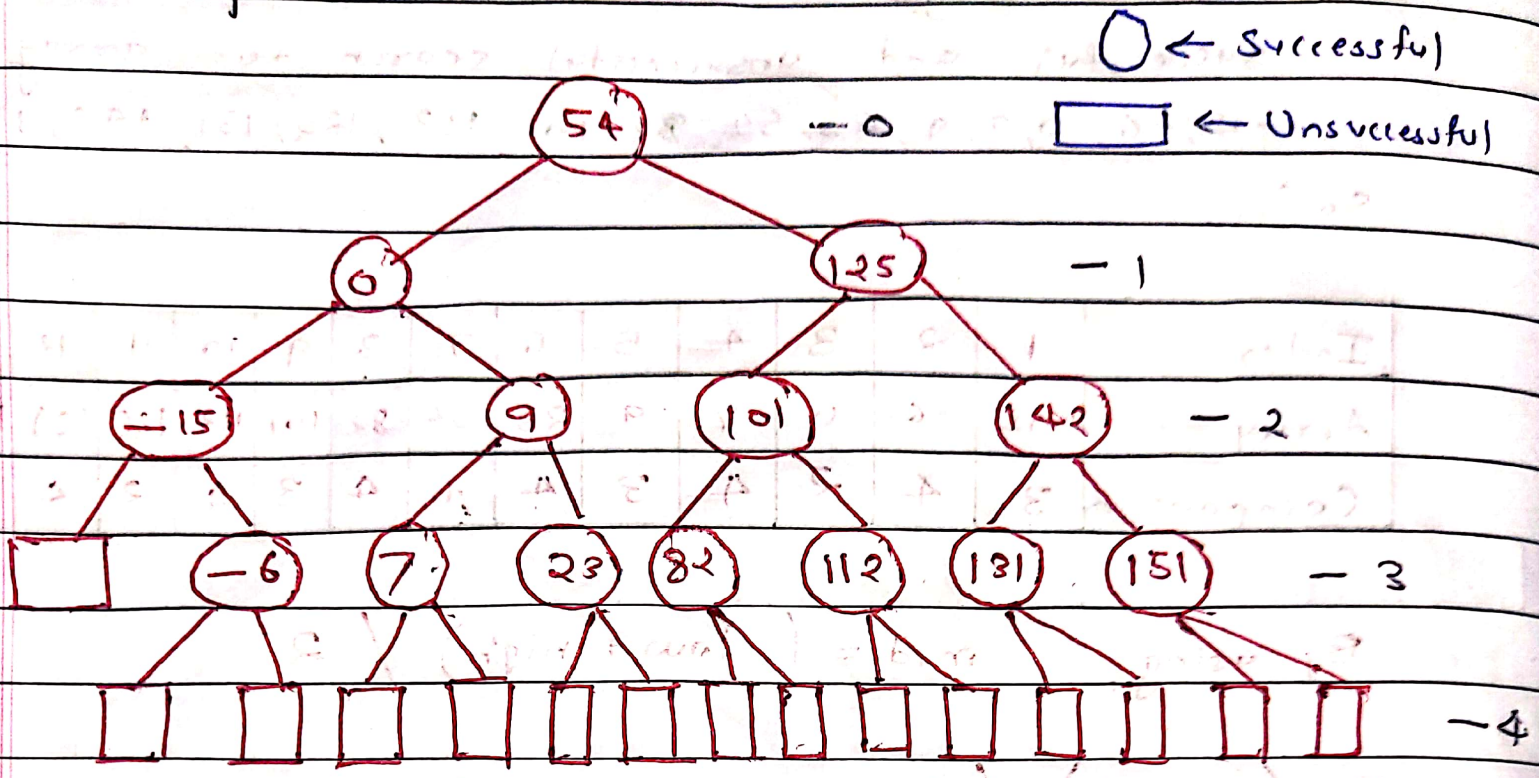
(8 + 10) / 2 = 9

(12 + 14) / 2 = 13

④ 2, 4, 6, 8, 10, 12, 14

↳ Index numbers

Binary Search Tree



IPL : $0*1 + 1*2 + 2*4 + 3*7 + 4*0$

$= 0 + 2 + 8 + 21$

IPL = 31.

EPL : $4*14 + 3*1 + 2*0 + 1*0 + 0*0$

$= 56 + 3 + 0 + 0 + 0$

EPL = 59

① Average number of comparison for successful

$$A.S(n) = \frac{IPL}{n} + 1$$

$$= \frac{31}{14} + 1$$

$$\boxed{A.S(n) = 3.2}$$

② Average number of comparison for unsuccessful

$$A.U(n) = \frac{EPL}{n+1}$$

$$= \frac{59}{14+1}$$

$$= \frac{59}{15}$$

$$\boxed{AU(n) = 3.9}$$

Complexity : $O(\log_2 n)$

$= O(\log_2 14)$

$= O(3.8)$

Q. Find average number of comparison for successful and unsuccessful search for array

-12, -7, 28, 40, 54, 64, 83, 120, 145

Sol^n:

Index	1	2	3	4	5	6	7	8	9
Array	-12	-7	28	40	54	64	83	120	145
Comparison	3	2	3	4	1	3	2	3	4

By using, mid = (low + high) / 2

① (1+9)/2 = 5

② (1+4)/2 = 2.5 ≈ 2

(6+9)/2 = 7.5 ≈ 7

③ (1+1)/2 = 1

(3+4)/2 = 3.5 ≈ 3

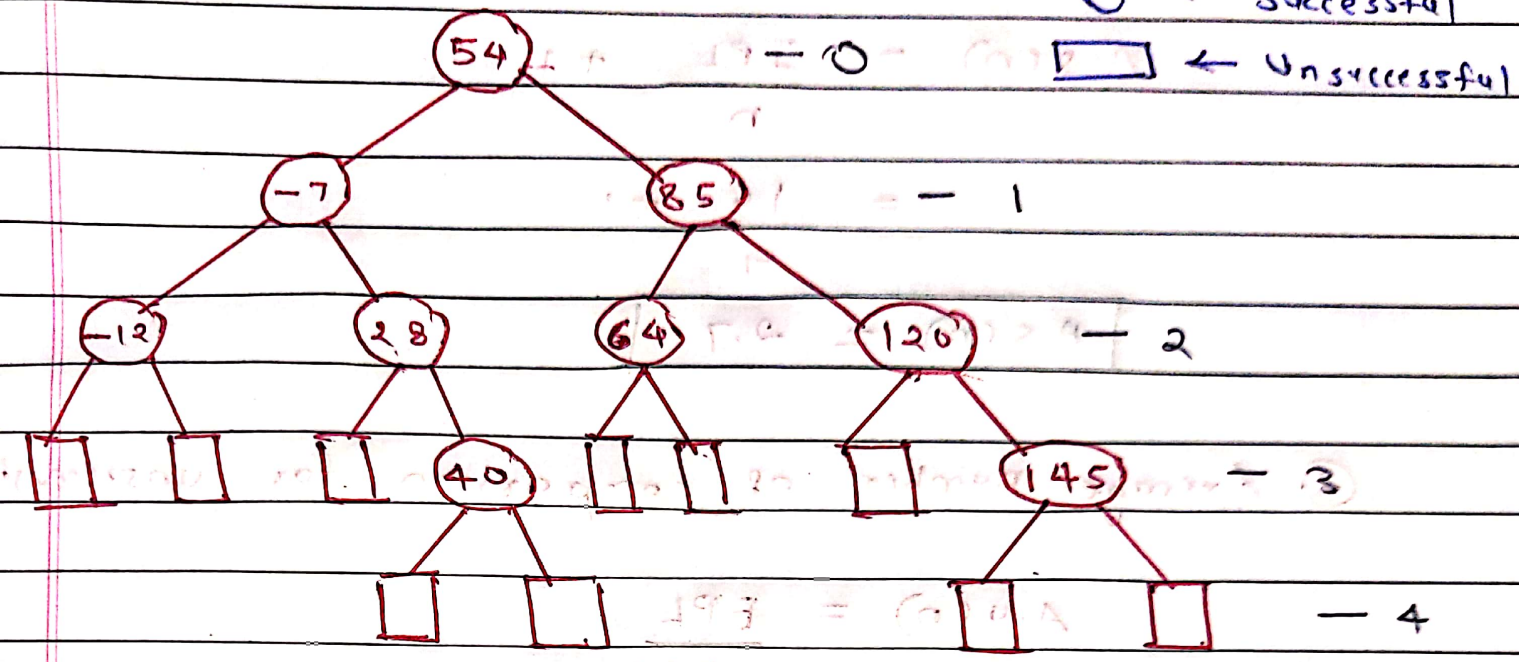
(6+6)/2 = 6

(8+9)/2 = 8.5 ≈ 8

④ 4, 9

↳ Index numbers

Binary Search Tree



IPL : $0 * 1 + 1 * 2 + 2 * 4 + 3 * 2 + 4 * 0$

$= 0 + 2 + 8 + 6 + 0$

IPL = 16

EPL : $4 * 4 + 3 * 6 + 2 * 0 + 1 * 0 + 0 * 0$

$= 16 + 18 + 0 + 0 + 0$

EPL = 34

(1) Average number of comparison for successful

$$A.S(n) = \frac{IPL}{n} + 1$$

$$= \frac{16}{9} + 1$$

$$\boxed{A.S(n) = 2.7}$$

(2) Average number of comparison for unsuccessful.

$$A.U(n) = \frac{EPL}{n+1}$$

$$= \frac{34}{9+1}$$

$$= \frac{3.4}{10}$$

$$\boxed{Au(n) = 3.4}$$

Complexity: $O(\log_2 n)$

$= O(\log_2 9)$

$= O(3.16)$