Binomial Distribution

Probability of r success is

$$P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}$$

${}^nC_0 = 1$
${}^nC_1 = n$
${}^nC_n = 1$

$n \rightarrow$ No. of repeated trials
$p \rightarrow$ Probability of a success
$q \rightarrow$ Probability of a failure

$$p + q = 1$$

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

Mean, $m = np$

Variance $= npq$

Standard deviation $= \sqrt{npq}$

Note: When the experiment be repeated 'N' times, the frequency of r success is $N\ {}^nC_r \cdot p^r\ q^{n-r}$.

Q. The probability that a pen manufacturing by a company will be defective is 1/10. If 12 such pens are manufactured. Find the probability that

(a) Exactly two will be defective

(b) None will be defective

(c) At least two will be defective

Solⁿ:

Total no. of pens, $n = 12$

Probability of a defective pen, $p = 1/10 = 0.1$

Probability of a non-defective pen, $q = 1 - p = 0.9$

(a) Probability that exactly two will be defective

$$P(2) = {}^{12}C_2 (0.1)^2 (0.9)^{12-2}$$

$$= \frac{12!}{2!\,10!} (0.1)^2 (0.9)^{10}$$

$$= \frac{12 \times 11 \times \cancel{10!}}{2 \times 1 \times \cancel{10!}} (0.1)^2 (0.9)^{10}$$

$$= 0.2301$$

(b) Probability that none will be defective

$$P(0) = {}^{12}C_0 (0.1)^0 (0.9)^{12}$$

$$= 1 \times 1 \times (0.9)^{12}$$

$$= 0.2824$$

(c) Probability that at least two will be defective

$$P(2) + P(3) + P(4) + \ldots\ldots + P(12) = 1 - [P(0) + P(1)]$$

$$= 1 - [{}^{12}C_0 (0.1)^0 (0.9)^{12} + {}^{12}C_1 (0.1)^1 (0.9)^{11}]$$

$$= 1 - [0.2824 + 12 (0.1)^1 (0.9)^{11}]$$

$$= 1 - [0.2824 + 0.3766]$$

$$= 0.3410$$

Q. In a sampling, a large no. of parts manufactured by a machine, the mean no. of defectines in a sample of 20 in 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts.

Sol^n:

Year no. of defectives, $m = 2 = np$

Here, $n = 20 \Rightarrow p = \frac{2}{20} = 0.1$

The probability of a defective part, $p = 0.1$

The probability of non-defective part, $q = 1 - p$
$= 0.9$

Probability of at least 3 defective parts in a sample of 20 $= P(3) + P(4) + P(5) + \dots P(20)$

$$= 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - [\,{}^{20}C_0 (0.1)^0 (0.9)^{20}$$

$$+ {}^{20}C_1 (0.1)^1 (0.9)^{19}$$

$$+ {}^{20}C_2 (0.1)^2 (0.9)^{18}\,]$$

$$= 1 - [\,1 \times 1 \times (0.9)^{20}$$

$$+ 20 \times (0.1) (0.9)^{19}$$

$$+ \frac{20!}{2!\,18!} (0.1)^2 (0.9)^{18}\,]$$

$$= 1 - \left[(0.9)^{20} + 2 \times (0.9)^{19} + \frac{20 \times 19 \times 18!}{2 \times 18!} (0.1)^2 (0.9)^{18} \right]$$

$$= 0.323$$

Hence, number of samples having at least 3 defectives out of 1000 samples

$$= 1000 \times 0.323$$

$$= \underline{323}$$

Q. In 256 sets of 12 tosses of a coin. How many cases one can expect 8 heads and 4 tails.

Sol^n:

$P(H) = 0.5$

$P(T) = 0.5$

By Binomial distribution

Probability of 8H and 4T in 12 trials is

$P(8) = {}^{12}C_8 \ (0.5)^8 \ (0.4)^4$

$= \dfrac{12!}{8! \ 4!} \ (0.5)^{12}$

$= \dfrac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 4 \times 3 \times 2 \times 1} \cdot (0.5)^{12}$

$= 0.1208$

$\therefore$ The expected no. of such cases in 256 sets

$= 256 \times P(8)$

$= 256 \times 0.1208$

$= 30.9$

≈ 31

Q. If the mean and variance of binomial distribution are 4 and 2 respectively.
Find the probability of
(a) Exactly 2 success
(b) Less than 2 success
(c) At least 2 success

Sol^n:

$np = 4$

$npq = 2$

$\frac{npq}{np} = q = \frac{2}{4} = 0.5 \Rightarrow p = 1-q = 0.5$

$np = 4 \Rightarrow n = \frac{4}{1/2} = 8 \Rightarrow n = 8$

(i) Probability of exactly 2 success

$P(2) = {}^8C_2 \, (0.5)^2 \, (0.5)^6 = \frac{7}{64}$

(ii) Probability of less than 2 success

$P(0) + P(1) = {}^8C_0 \, (0.5)^0 \, (0.5)^8 + {}^8C_1 \, (0.5)^1 \, (0.5)^8$

$= \frac{9}{256}$

(iii) Probability of at least 2 success

$P(2) + P(3) + P(4) + \dots + P(8) = 1 - [P(0) + P(1)]$

$= 1 - \frac{9}{256} = \frac{256-9}{256} = \frac{247}{256}$

Q. The probability that a bomb dropping a plane will strike the target is $1/5$.
If 6 bombs are dropped. Find the probability
(a) Exactly 2 strike the target
(b) At least 2 will strike the target

Soln:

Probability of striking the target, $p = 0.2 = 1/5$.

Probability of not striking the target, $q = 1-p = 0.8$

$n = 6$

(a) Probability that exactly two will strike the target

$$P(2) = {}^6C_2 (0.2)^2 (0.8)^4 = 0.245$$

(b) Probability that at least two will strike the target

$$P(2) + P(3) + P(4) + P(5) + P(6) = 1 - [P(0) + P(1)]$$

$$= 1 - [\,{}^6C_0 (0.2)^0 (0.8)^6 + {}^6C_1 (0.2)^1 (0.8)^5\,]$$

$$= 0.345$$

Q. If the chance that one of the 10 telephone lines is busy at an instant is 0.2

(a) What is the chance that 5 of the lines are busy?

(b) What is the probability that all the lines are busy?

Sol^n:

Probability of 1 telephone busy out of 10, $p = 0.2$

Probability of no telephone busy out of 10, $q = 1 - p$
$= 0.8$

(a) Probability that 5 of the lines are busy

$$P(5) = {}^{10}C_5 (0.2)^5 (0.8)^5 = 0.0265$$

(b) Probability that all the lines are busy

$$P(10) = {}^{10}C_{10} (0.2)^{10} (0.8)^{10-10}$$

$$= 1 \times (0.2)^{10} \times 1$$

$$= 1.024 \times 10^{-7}$$

Q. Out of 800 families with 5 children equally how many would you expect to have
(a) 3 boys
(b) 5 girs
(c) Either 2 or 3 boys
Assume equal probabilities for boys and girls.

Soln:

$p = 0.5$ $\quad$ $p \rightarrow$ Probability of a boy

$q = 0.5$ $\quad$ $q \rightarrow$ Probability of a girl

(a) Probability of 3 boys in 800 families

$$= {}^5C_3 \ (0.5)^3 \ (0.5)^2 \times 800 = 250$$

(b) Probabilities of 5 girls in 800 families

$$= {}^5C_5 \ (0.5)^5 \ (0.5)^0 \times 800 = 25$$

(c) Probability of either 2 or 3 boys in 800 families

$$= \left[{}^5C_2 \ (0.5)^2 \ (0.5)^3 + {}^5C_3 \ (0.5)^3 \ (0.5)^2 \right] \times 800$$

$$= 500$$

Q. The following data are the no. of seeds germinating out of 10 on damp filter paper for 80 sets of seeds. fit a binomial distribution of these data and compare the theoretical frequency with the actual ones.

x:	0	1	2	3	4	5	6	7	8	9	10
f:	6	20	28	12	8	6	0	0	0	0	0

Soln:

$n = 10 \qquad \Sigma f_i = 80 = N$

Mean for grouped data.

$$m = np = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0 + 20 + 56 + 36 + 32 + 30 + 0}{80}$$

$$= 2.175$$

$$p = \frac{m}{n} = \frac{2.175}{10} = 0.2175$$

$$q = 1 - p = 0.7825$$

The binomial distribution to be fitted is

$$N(q-p)^n = 80(0.7825 + 0.2175)^{10}$$

Formula: $P(r) = {}^nC_r \cdot p^r \cdot q^{n-r} \times N$

For $r = 0$, $P(0) = {}^{10}C_0 (0.2175)^0 (0.7825)^{10} \times 80 = 6.88$

For $r = 1$, $P(1) = {}^{10}C_1 (0.2175)^1 (0.7825)^9 \times 80 = 19.13$

For $r = 2$, $P(2) = {}^{10}C_2 (0.2175)^2 (0.7825)^8 \times 80 = 23.93$

For $r = 3$, $P(3) = {}^{10}C_3 (0.2175)^3 (0.7825)^7 \times 80 = 17.74$

For $r = 4$, $P(4) = {}^{10}C_4 (0.2175)^4 (0.7825)^6 \times 80 = 8.63$

For $r = 5$, $P(5) = {}^{10}C_5 (0.2175)^5 (0.7825)^5 \times 80 = 2.87$

For $r = 6$, $P(6) = {}^{10}C_6 (0.2175)^6 (0.7825)^4 \times 80 = 0.66$

For $r = 7$, $P(7) = {}^{10}C_7 (0.2175)^7 (0.7825)^3 \times 80 = 0.10$

For $r = 8$, $P(8) = {}^{10}C_8 (0.2175)^8 (0.7825)^2 \times 80 = 0.01$

For $r = 9$, $P(9) = {}^{10}C_9 (0.2175)^9 (0.7825)^1 \times 80 = 6.81 \times 10^{-4}$

For $r = 10$, $P(10) = {}^{10}C_{10} (0.2175)^{10} (0.7825)^0 \times 80 = 1.89 \times 10^{-5}$

Q. Fit a binomial distribution for the following data and compare the rhetorical frequencies with the actual ones

x →	0	1	2	3	4	5
f →	2	14	20	34	22	8

Sol^n:

$n = 5$ $\quad \Sigma f_i = 100 = N$

Mean for grouped data

$$m = np = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{0 + 14 + 40 + 102 + 88 + 40}{100}$$

$$= 2.84$$

$p = 0.57$

$q = 1 - p = 0.43$

The binomial distribution to be fitted in

$N(q-p)^n = 100(0.43 + 0.57)^5$

Formula: $P(r) = {}^nC_r \cdot p^r \cdot q^{n-r} \times N$

For $r = 0$, $P(0) = {}^5C_0 (0.57)^0 (0.43)^5 \times 100 = 1.47$

For $r = 1$, $P(1) = {}^5C_1 (0.57)^1 (0.43)^4 \times 100 = 9.74$

For $r = 2$, $P(2) = {}^5C_2 (0.57)^2 (0.43)^3 \times 100 = 25.83$

For $r = 3$, $P(3) = {}^5C_3 (0.57)^3 (0.43)^2 \times 100 = 34.24$

For $r = 4$, $P(4) = {}^5C_4 (0.57)^4 (0.43)^1 \times 100 = 22.69$

For $r = 5$, $P(5) = {}^5C_5 (0.57)^5 (0.43)^0 \times 100 = 6.01$