CHAPTER 4

Unit IV

Fuzzy Logic

Syllabus

4.1 Introduction to Fuzzy Set: Fuzzy set theory, Fuzzy set versus crisp set, Crisp relation & fuzzy relations, membership functions,

4.2 Fuzzy Logic: Fuzzy Logic basics, Fuzzy Rules and Fuzzy Reasoning

4.3 Fuzzy inference systems: Fuzzification of input variables, defuzzification and fuzzy controllers.

4.1 Introduction to Fuzzy Set

- Fuzzy logic was introduced by Prof. Lofti A. Zadeh in 1965.
- The word fuzzy means "Vagueness".
- Fuzzyness occurs when a boundary of a piece of information is not clear.
- He proposed a mathematical way of looking at the vagueness of the human natural language.

Why fuzzy set is required?

- Most of our traditional tools for formal modelling, reasoning and computing are crisp, deterministic and precise.
- While designing the system using classical set, we assume that the structures and parameters of the model are definitely known and there are no doubts about their values or their occurrence.
- But in real world there exists much fuzzy knowledge; knowledge that is vague, imprecise, uncertain, ambiguous, inexact or probabilistic in nature.
- There are two facts ;
  1. Real situations are very often not crisp and deterministic and they cannot be described precisely.
  2. The complete description of a real system often would require more detailed data than a human being could ever recognize simultaneously, process and understand.
- Because of these facts, modelling the real system using classical sets often do not reflect the nature of human concepts and thoughts which are abstract, imprecise and ambiguous.
- The classical (crisp) sets are unable to cope with such unreliable and incomplete information.
- We want our systems should also be able to cope with unreliable and incomplete information and give expert opinion.
- Fuzzy set theory has been introduced to deal with such unreliable, incomplete, vague and imprecise information.
- Fuzzy set theory is an extension to classical set theory where element have degree of membership.
- Fuzzy logic uses the whole interval between 0 (false) and 1 (true) to describe human reasoning.

4.2 Fuzzy Set vs. Crisp Set

- A classical set (or conventional or crisp set) is a set with a crisp boundary.

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- For example, a classical set A of real numbers greater than 6 can be expressed as

$$A = \{x \mid x > 6\}$$

Where there is a clear, unambiguous boundary '6' such that if x is greater than this number, then x belongs to the set A, otherwise x does not belong to the set.

- Although classical sets are suitable for various approximations and have proven to be an important tool for mathematics and computer science, they do not reflect the nature of human concepts and thoughts, which are abstract, imprecise and ambiguous.
- For example, mathematically we can express the set of all tall persons as a collection of persons whose height is more than 6 ft.

$$A = \{x \mid x > 6\}$$

Where A = "tall person" and x = "height".

- The problem with the classical set is that it would classify a person 6.001 ft. tall as a tall person, but a person 5.999 ft. tall as "not tall". This distinction is intuitively unreasonable.

The flaw comes from the sharp transition between inclusion and exclusion in a set.(Fig. 4.2.1)

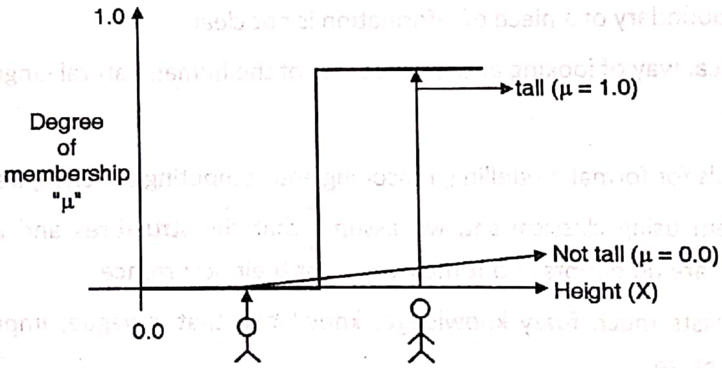

Fig. 4.2.1 : Sharp edged membership function for TALL

- Fuzzy logic uses the "degrees of truth" rather than the usual "true or false" (1 or 0) Boolean logic.
- Fuzzy logic includes 0 and 1 as extreme cases of truth but also includes the various states of truth in between so that, for example, the result of a comparison between two things could be not "tall" or "short" but "0.38 of tallness."

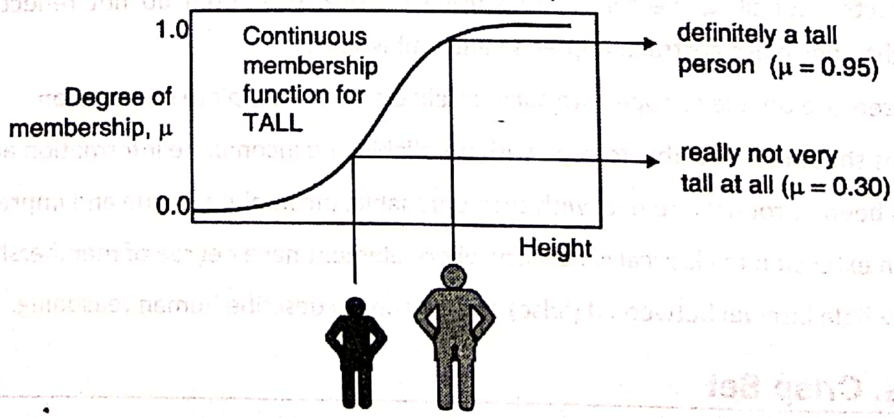

Fig 4.2.2 : Fuzzy membership function TALL

- As shown in Fig 4.2.2 the fuzzy logic defines smooth transition from 'not tall' to 'tall'. A person's height may now belong to both the groups "tall' and 'Not all' but now it will have the degree of membership associated with it for each group.
- A person has 0.30 membership in 'Not tall' group and '0.95' membership in 'tall' group, so definitely the person is categorized as a tall person.

Key differences between Fuzzy Set and Crisp Set

Sr. No.	Fuzzy Set	Crisp Set
1.	A fuzzy set follows the infinite-valued logic	A crisp set is based on bi-valued logic.
2.	A fuzzy set is determined by its indeterminate boundaries, there exists an uncertainty about the set boundaries	A crisp set is defined by crisp boundaries, and contain the precise location of the set boundaries.
3.	Fuzzy set elements are permitted to be partly accommodated by the set (exhibiting gradual membership degrees).	crisp set elements can have a total membership or non-membership.
4.	Fuzzy sets are capable of handling uncertainty and vagueness present in the data	Crisp set requires precise , complete and finite data.

4.3 Fuzzy Set Theory

4.3.1 Fuzzy Set : Definition

- If X is a collection of objects denoted generally by x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs :

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) \mid x \in X \}$$

Where $\mu_{\tilde{A}}(x)$ is called the Membership Function (MF) for the fuzzy set $\tilde{A}$.

- The MF maps each element of X to a membership grade between 0 and 1.
- If the value of membership function $\mu_{\tilde{A}}(x)$ is restricted to either 0 or 1, then $\tilde{A}$ is reduced to a classical set.

Note : Classical sets are also called ordinary sets, crisp sets, non fuzzy sets or just sets.

- Here, X is referred to as the **Universe of discourse** or simply the **Universe** and it may consist of discrete objects or continuous space.

4.3.2 Types of Universe of Discourse

1. **Fuzzy sets with a discrete non ordered universe**
   - Universe of discourse may contain discrete non-ordered objects.

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For example,

Let X = {San Francisco, Boston, Los Angeles} be the set of cities one may choose to live in.

The fuzzy set "desirable city to live in" may be described as follows:

A = {(San Francisco, 0.9), (Boston, 0.8), (Los Angeles, 0.6)}

– Here, the universe of discourse X is discrete and it contains non-ordered objects, in this case three big cities in United States.

2. Fuzzy sets with a discrete ordered universe

– Let X = {0, 1, 2, 3, 4, 5, 6} be the set of number of children a family may choose to have.

– Then, a fuzzy set "Sensible number of children in family" may be described as

$$\tilde{A} = \{(0, 0.1), (1, 0.3), (2, 0.7), (3, 1), (4, 0.7), (5, 0.3), (6, 0.1)\}$$

Here we have a discrete ordered universe X.

The MF is shown in Fig. 4.3.1.

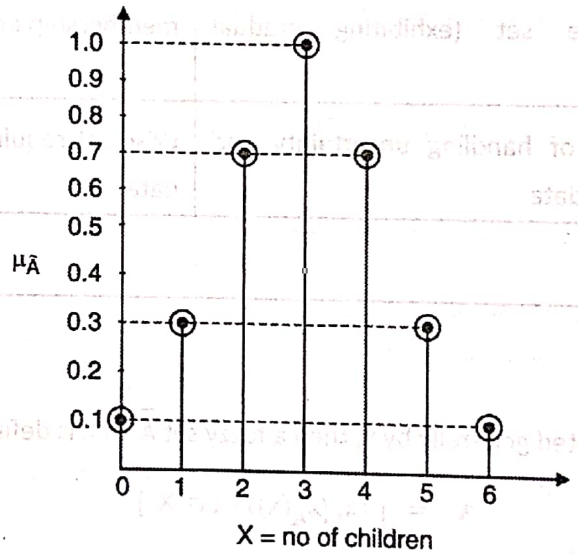

Fig. 4.3.1 : MF on a discrete universe

Note : Membership grades of the fuzzy set are subjective measures. (For example, the height 5'5" may be considered tall in Japan, but in Australia, it may be considered medium).

3. Fuzzy sets with a continuous universe

Let X = R^+ be the set of possible ages for human beings (Real numbers - continuous). Then the fuzzy set B = "about 50 years old" may be expressed as,

$$\tilde{B} = \{ (x, \mu_{\tilde{B}}(x)) \mid x \in X\}$$

Where, $$\mu_{\tilde{B}}(x) = \frac{1}{1 + \left(\frac{x - 50}{10}\right)^4}$$

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This is illustrated in Fig. 4.3.2.

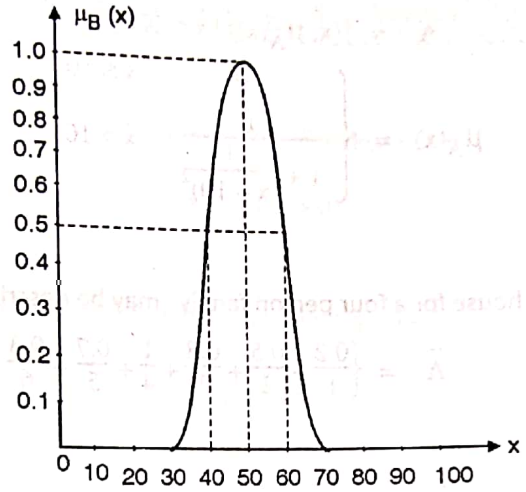

Fig. 4.3.2 : MF for "about 50 years old"

Table 4.3.1 shows $\mu_{\tilde{B}}(x)$ for some value of x.

Table 4.3.1: x and corresponding $\mu_{\tilde{B}}(x)$ for "about 50 years old"

X	40	42	45	48	50	52	53	55	56	58	60
$\mu_{\tilde{B}}(x)$	0.5	0.71	0.94	0.99	1	0.99	0.99	0.94	0.89	0.71	0.5

Note: Construction of a fuzzy set depends on two things:

1) The identification of a suitable universe of discourse and
2) Specification of an appropriate membership function.

The specification of membership function is **subjective**, which means that the membership functions specified for the same concept by different persons may vary considerably. Therefore, the subjectivity and non randomness of fuzzy sets is the primary difference between the fuzzy sets and probability theory.

4.3.3 Different Notations for Representing Fuzzy Sets

1) **Using ordered pairs :**

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

$$\text{E.g. Let } X = \{1, 2, 3, \ldots, 6\}$$

X is available types of houses described by X = "Number of bedrooms in a house" then comfortable house for four persons family is described using ordered pair as,

$$\tilde{A} = \{(1, 0.2), (2, 0.5), (3, 0.8), (4, 1), (5, 0.7), (6, 0.3)\}$$

2) **Using membership function :**

A fuzzy set can be represented by stating its membership function.

E.g. To represent "real numbers considerably larger than 10".

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We define,

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\},$$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x \leq 10 \\ \dfrac{1}{1+\dfrac{1}{(x-10)^2}}; & x > 10 \end{cases}$$

3) **Using + Notation :**

Fuzzy set for "comfortable type of house for a four person family" may be described as,

$$\tilde{A} = \left\{\frac{0.2}{1} + \frac{0.5}{2} + \frac{0.8}{3} + \frac{1}{4} + \frac{0.7}{5} + \frac{0.3}{6}\right\}$$

i.e. we define A as

$$\tilde{A} = \mu_{\tilde{A}}(x_1)/x_1 + \mu_{\tilde{A}}(x_2)/x_2 + \ldots..$$

$$= \sum_{i=1}^{n} \mu_{\tilde{A}}(x_i)/x_i$$

4) **Using Venn diagrams :**

Sometimes it is more convenient to give the graph that represents membership function.

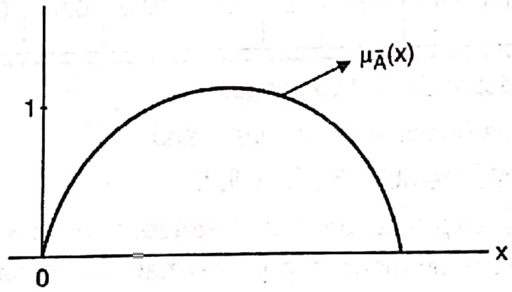

Fig. 4.3.3 : Representation of fuzzy set using Venn diagram

5) **Other notations :**

$$\tilde{A} = \{(3, 0.1) + (4, 0.3)\} + \ldots..\}$$

or

$$\tilde{A} = \left\{\frac{0.1}{3}, \frac{0.3}{4}, \frac{0.6}{5}, \ldots..\right\}$$

4.3.4 Linguistic Variables and Linguistic Values

- Suppose that X = "age". Then, we can define fuzzy sets "young", "middle aged" and "old" that are characterized by MFs $\mu_{young}(x)$, $\mu_{middle\ aged}(x)$ and $\mu_{old}(x)$.
- A linguistic variable ("age") can assume different linguistic values such as "young", "middle aged" and "old" in this case.
- Note that, the universe of discourse is totally covered by these MFs (MFs for young, middle aged and old) and transition from one MF to another is smooth and gradual.

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4.3.5 Important Terminologies related to Fuzzy Sets

Q. Define supports, core, normality, crossover points and α – cut for fuzzy set. (Dec. 11, 5 Marks)

1. **Support :**

A support of a fuzzy set $\tilde{A}$ is the set of all points x in X such that, $\mu_{\tilde{A}}(x) > 0$.

$$\text{Support}(\tilde{A}) = \{x \mid \mu_{\tilde{A}}(x) > 0\}$$

2. **Core / Nucleus :**

The core of a fuzzy set $\tilde{A}$ is the set of all points x in X such that $\mu_{\tilde{A}}(x) = 1$.

$$\text{Core } \tilde{A} = \{x \mid \mu_{\tilde{A}}(x) = 1\}$$

3. **Normality :**

A fuzzy set $\tilde{A}$ is normal if its core is non-empty. In other words there must be at least one point $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

4. **Crossover points :**

A cross over point of a fuzzy set $\tilde{A}$ is a point $x \in X$ at which $\mu_{\tilde{A}}(x) = 0.5$.

$$\text{Crossover}(\tilde{A}) = \{x \mid \mu_{\tilde{A}}(x) = 0.5\}$$

5. **Fuzzy singleton :**

A fuzzy set whose support is a single point in X with $\mu_{\tilde{A}}(x) = 1$ is called a fuzzy singleton.

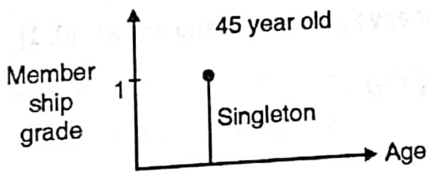

Fig.4.3.4: A fuzzy singleton

Fig. 4.3.5 shows three parameters (core, support and crossover points) of a fuzzy set.

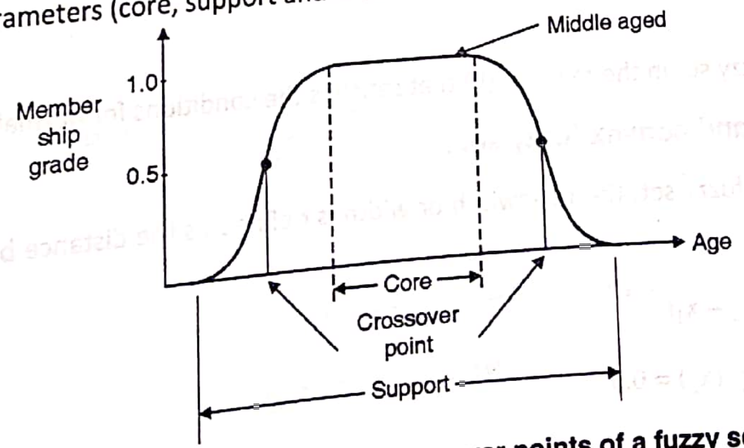

Fig. 4.3.5 : Core, Support and Crossover points of a fuzzy set

6. **α-cut :**

The α-cut or α-level set of a fuzzy set $\tilde{A}$ is a crisp set defined by,

$$A_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}$$

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7. Strong α-cut / strong α-level set :

Strong α-cut is defined by

$$A'_\alpha = \{x \mid \mu_{\tilde{A}}(x) > \alpha\}$$

Using the above notations, we can express support and core of a fuzzy set A as,

$$\text{Support}(\tilde{A}) = A'_0 \qquad \text{Here } \alpha = 0$$

$$\text{Core}(\tilde{A}) = A_1 \qquad \text{Here } \alpha = 1$$

8. Convexity :

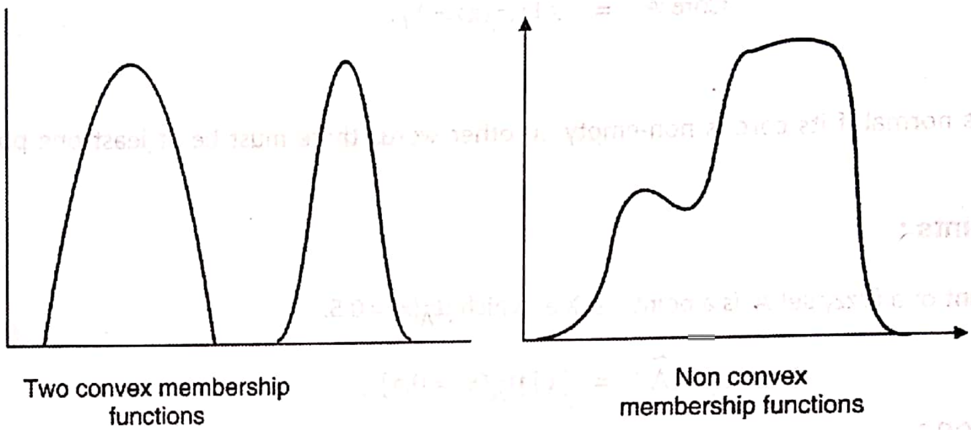

Fig. 4.3.6 : Convex and Non Convex MFs

A fuzzy set $\tilde{A}$ is convex if and only if for any x_1 and $x_2 \in X$ and any $\lambda \in [0, 1]$.

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$$

or

$\tilde{A}$ is convex if all its α-level sets are convex.

9. Fuzzy numbers :

A fuzzy number $\tilde{A}$ is a fuzzy set in the real line (R) that satisfies the conditions for normality and convexity.

10. Bandwidth of normal and convex fuzzy set :

For a normal and convex fuzzy set, the bandwidth or width is defined as the distance between two unique crossover points.

$$\text{Width}(\tilde{A}) = |x_2 - x_1|$$

$$\text{Where } \mu_{\tilde{A}}(x_1) = \mu_{\tilde{A}}(x_2) = 0.5$$

11. Symmetry :

A fuzzy set $\tilde{A}$ is symmetric if its MF is symmetric around a certain point x = c,

$$\mu_{\tilde{A}}(c + x) = \mu_{\tilde{A}}(c - x) \quad \text{for all } x \in X$$

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12. **Open left, Open right and closed MFs :**

A fuzzy set $\tilde{A}$ is open left if,

$$\lim_{x \to -\infty} \mu_{\tilde{A}}(x) = 1 \text{ and}$$

$$\lim_{x \to +\infty} \mu_{\tilde{A}}(x) = 0$$

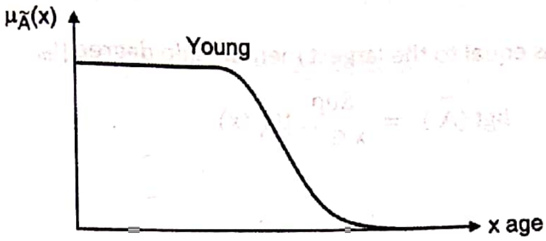

Fig. 4.3.7 : Open Left MF

A fuzzy set $\tilde{A}$ is open right if,

$$\lim_{x \to -\infty} \mu_{\tilde{A}}(x) = 0 \quad \text{and}$$

$$\lim_{x \to +\infty} \mu_{\tilde{A}}(x) = 1$$

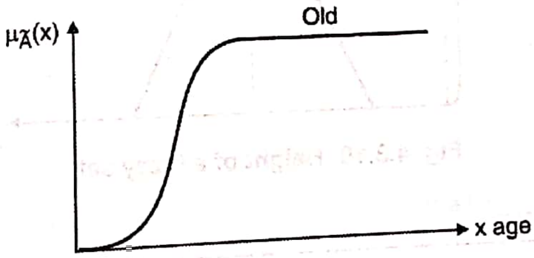

Fig. 4.3.8: Open right MF

A fuzzy set $\tilde{A}$ is closed if,

$$\lim_{x \to +\infty} \mu_{\tilde{A}}(x) = 0 \quad \text{and}$$

$$\lim_{x \to -\infty} \mu_{\tilde{A}}(x) = 0$$

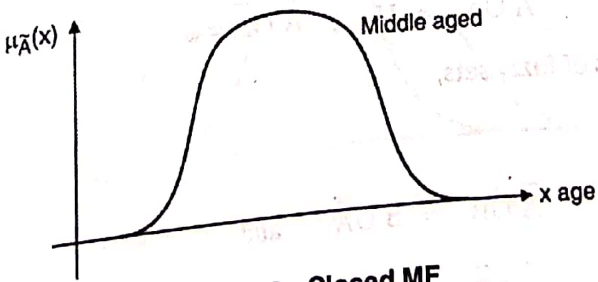

Fig. 4.3.9 : Closed MF

13. **Cardinality :**

Cardinality of a fuzzy set $\tilde{A}$ is defined as

$$|\tilde{A}| = \sum_{x \in X} \mu_{\tilde{A}}(x)$$

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14. Relative cardinality :

Relative cardinality of a fuzzy set $\tilde{A}$ is defined as,

$$\| \tilde{A} \| = \frac{|\tilde{A}|}{|X|}$$

15. Height of a fuzzy set :

The height of a fuzzy set $\tilde{A}$ in X, is equal to the largest membership degree μ_m

$$\text{hgt}(\tilde{A}) = \sup_{x \in X} \mu_{\tilde{A}}(x)$$

If hgt $(\tilde{A}) = 1$ then, $\tilde{A}$ is normal.

If hgt $(\tilde{A}) < 1$ then, $\tilde{A}$ is subnormal.

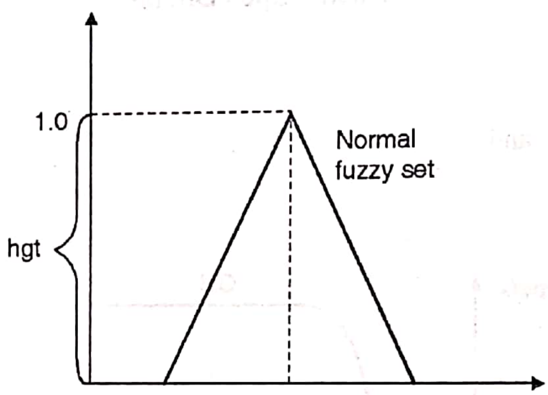

Fig. 4.3.10: Height of a fuzzy set

4.3.6 Properties of Fuzzy Sets

MU – Dec. 12

Q. State the different properties of fuzzy set. (Dec. 12, 8 Marks)

Fuzzy sets follow the same properties as crisp set except for the law of excluded middle and law of contradiction.

That is, for fuzzy set $\tilde{A}$

$$\tilde{A} \cup \bar{\tilde{A}} \neq U \quad ; \quad \tilde{A} \cap \bar{\tilde{A}} \neq \phi$$

The following are the properties of fuzzy sets,

1. Commutativity :

$$\tilde{A} \cup \tilde{B} = \tilde{B} \cup \tilde{A} \quad \text{and}$$

$$\tilde{A} \cap \tilde{B} = \tilde{B} \cap \tilde{A}$$

2. Associativity :

$$\tilde{A} \cup (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cup \tilde{C}$$

$$\tilde{A} \cap (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cap \tilde{C}$$

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3. **Distributivity :**

$$\tilde{A} \cup (\tilde{B} \cap \tilde{C}) = (\tilde{A} \cup \tilde{B}) \cap (\tilde{A} \cup \tilde{C})$$

$$\tilde{A} \cap (\tilde{B} \cup \tilde{C}) = (\tilde{A} \cap \tilde{B}) \cup (\tilde{A} \cap \tilde{C})$$

4. **Identity :**

$$\tilde{A} \cup \phi = \tilde{A} \quad ; \quad \tilde{A} \cup U = U$$

$$\tilde{A} \cap \phi = \phi \quad ; \quad \tilde{A} \cap U = \tilde{A}$$

5. **Involution :**

$$\bar{\bar{\tilde{A}}} = \tilde{A}$$

6. **Transitivity :**

If $\tilde{A} \subset \tilde{B} \subset \tilde{C}$, then $\tilde{A} \subset \tilde{C}$

7. **De Morgan's law :**

$$\overline{\tilde{A} \cup \tilde{B}} = \bar{\tilde{A}} \cap \bar{\tilde{B}}$$

$$\overline{\tilde{A} \cap \tilde{B}} = \bar{\tilde{A}} \cup \bar{\tilde{B}}$$

4.3.7 Operations on Fuzzy Sets

1. **Containment or Subset :**

Fuzzy set $\tilde{A}$ is contained in fuzzy set $\tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$ for all x.

$$\tilde{A} \subseteq \tilde{B} \Leftrightarrow \mu_{\tilde{A}}(x) \le \mu_{\tilde{B}}(x)$$

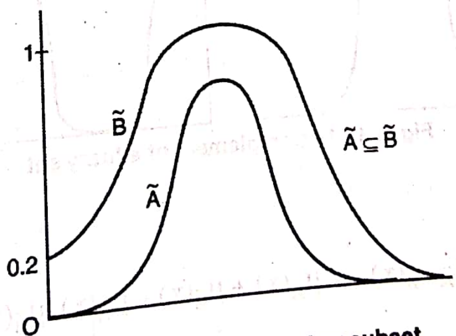

Fig. 4.3.11 : Containment or subset

2. **Union (Disjunction) :**

A union of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is a fuzzy set $\tilde{C}$, such that whose MF is,

$$\mu_{\tilde{C}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

Union of $\tilde{A}$ and $\tilde{B}$ is denoted by $(\tilde{A} \cup \tilde{B})$ or $(\tilde{A}$ or $\tilde{B})$

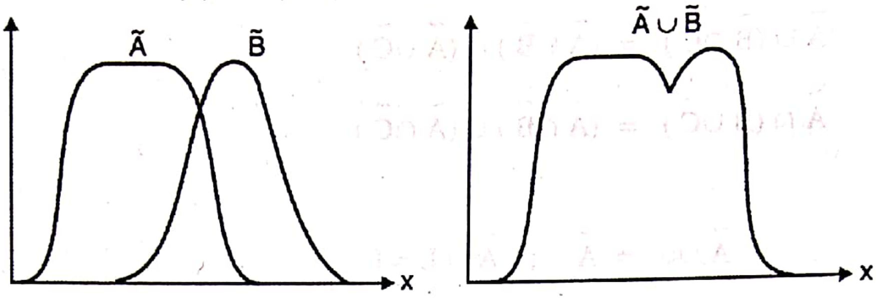

Fig. 4.3.12 : Union of two fuzzy sets

3. Intersection (Conjunction) :

The intersection of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is a fuzzy set $\tilde{C}$, such that whose MF is defined as

$$\mu_{\tilde{C}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$$

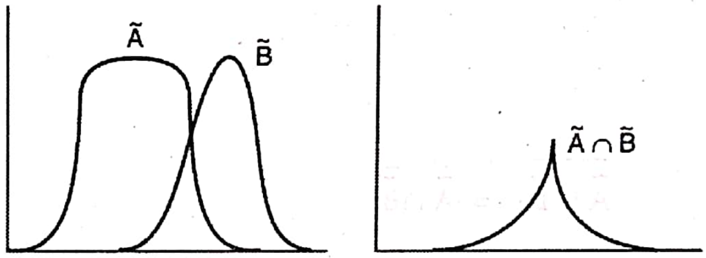

Fig. 4.3.13 : Intersection of two fuzzy sets

4. Complement (Negation) :

The complement of a fuzzy set $\tilde{A}$, denoted by $\bar{\tilde{A}}$ is defined as,

$$\mu_{\bar{\tilde{A}}}(x) = 1 - \mu_{\tilde{A}}(x)$$

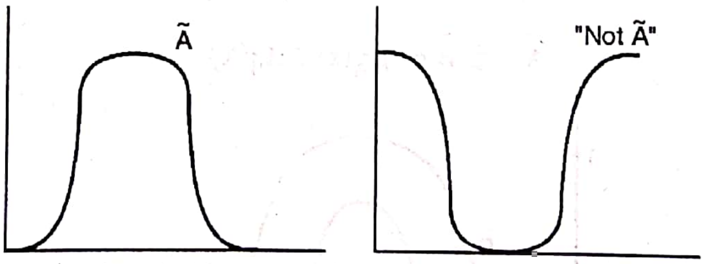

Fig. 4.3.14 : Complement of a fuzzy set

More operations of fuzzy sets

1. Algebraic sum :

$$\mu_{\tilde{A}+\tilde{B}}(x) = \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}}(x) . \mu_{\tilde{B}}(x)$$

2. Algebraic product :

$$\mu_{\tilde{A}\cdot\tilde{B}}(x) = \mu_{\tilde{A}}(x) . \mu_{\tilde{B}}(x)$$

3. Bounded sum :

$$\mu_{\tilde{A}\oplus\tilde{B}}(x) = \min[1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x)]$$

4. Bounded difference :

$$\mu_{\tilde{A}\odot\tilde{B}}(x) = \max[0, \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)]$$

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4.4 Crisp Relation and Fuzzy Relations

4.4.1 Crisp Relation

- An n-ary relation over $M_1, M_2, M_3, \ldots M_n$ is a subset of the Cartesian product $M_1 \times M_2 \times \ldots M_n$, where n = 2, the relation is a subset of the Cartesian product $M_1 \times M_2$. This is called a binary relation from M_1 to M_2.

Let X and Y be two universe and $X \times Y$ be their Cartesian product.

Then $X \times Y$ can be defined as,

$$X \times Y = \{(x, y) \mid x \in X, y \in Y\}$$

Every element in X is related to every element in Y.

- We can define characteristic function f that gives the strength of the relationship between the each element of X and Y.

$$f_{X \times Y}(x, y) = \begin{cases} 1, & (x, y) \in X \times Y \\ 0, & (x, y) \notin X \times Y \end{cases}$$

- We can represent the relation in the form of matrix.
- An n-dimensional relation matrix represents on n-ary relation.
- So, binary relation is represented by 2 dimensional matrices.

Ex: Consider the following two universe,

$X = \{a, b, c\}$, $Y = \{1, 2, 3\}$

- The Cartesian product - $X \times Y$ is,

$X \times Y = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$

From the above set, we may select a subset R, such that

$$R = \{(a, 1), (b, 2), (b, 3), (c, 1), (c, 3)\}$$

Then R can be represented in matrix form as,

R	1	2	3
a	1	0	0
b	0	1	1
c	1	0	1

The relation between set X and Y can also be represented as coordinate diagram as shown in Fig. 4.4.1.

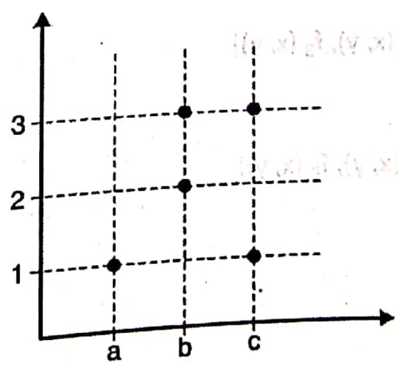

Fig. 4.4.1 : Co-ordinate diagram of a relation

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- The relation R can also be expressed by mapping representation as shown in Fig. 4.4.2.

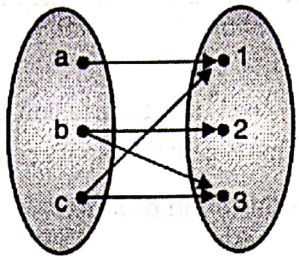

Fig. 4.4.2 : Mapping representation of a relation

- A characteristic function is used to assign values of relationship in the mapping of X × Y to the binary values and is given by,

$$f_{R\,(x,\,y)} = \begin{cases} 1 \ , & (x, y) \in R \\ 0 \ , & (x, y) \notin R \end{cases}$$

4.4.1(A) Cardinality of Classical Relation

- Let X and Y be two universe and n elements of X are related to m elements of y.
- Let the Cardinality of X is η_X and cardinality of Y is η_Y, then the cardinality of relation R between X and Y is,

$$\eta_{X \times Y} = \eta_X \times \eta_Y$$

- The Cardinality of the power set P (X × Y) is given as,

$$\eta_{P\,(X \times Y)} = 2^{(\eta_X \eta_Y)}$$

4.4.1(B) Operations on Classical Relations

- Let A and B be two separate relations defined on the Cartesian universe X × Y.
- Then the null relation defined as,

$$\phi_A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

And complete relation is defined as,

$$E_A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

- The following operations can be performed on two relations A and B.

1. **Union**

   $A \cup B \rightarrow f_{A \cup B}(x, y) : f_{A \cup B}(x, y) = \max [f_A(x, y), f_B(x, y)]$

2. **Intersection**

   $A \cap B \rightarrow f_{A \cap B}(x, y) : f_{A \cap B}(x, y) = \min [f_A(x, y), f_B(x, y)]$

3. **Complement**

   $\bar{A} \rightarrow f_{\bar{A}}(x, y) : f_{\bar{A}}(x, y) = 1 - f_{\bar{A}}(x, y)$

4. **Containment**

   $A \subset B \rightarrow f_A(x, y) : f_B(x, y) \leq f_B(x, y)$

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5. **Identity**

$\phi \rightarrow \phi_A$ and $X \rightarrow E_A$

4.4.1(C) Properties of Crisp (Classical) Relations

- The properties of classical set such as commutativity, associativity, involution, distributivity and idempotency hold good for classical relation also.
- Also De Morgan's law and excluded middle laws hold good for crisp relations.

4.4.1(D) Composition of Crisp Relations

- Composition is a process of combining two compatible binary relations to get a single relation.
- Let A be a relation that maps elements from universe X to universe Y.

  Let B be a relation that maps elements from universe Y to universe Z.
- The two binary relations A and B are said to be compatible if,

  $A \subseteq X \times Y$ and $B \subseteq Y \times Z$
- The composition between the two relations A and B can be denoted as A o B.

Ex. :

$$\text{Let } X = \{a_1, a_2, a_3\}$$
$$Y = \{b_1, b_2, b_3\}$$
$$Z = \{c_1, c_2, c_3\}$$

Let the relation A and B as,

$$A = X \times Y = \{(a_1, b_1), (a_1, b_2), (a_2, b_2)\}$$
$$B = Y \times Z = \{(b_1, c_1), (b_2, c_3), (b_3, c_2)\}$$

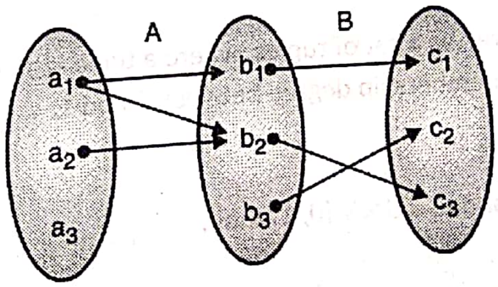

Fig. 4.4.3 : Illustration of relations A and B

Then A o B can be written as,

$$A \circ B = \{(a_1, c_1), (a_2, c_3), (a_2, c_3)\}$$

The representation of A and B in matrix form is given as,

$$A = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \; ; \; B = \begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \end{array} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

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Then, composition A o B is represented as,

$$A \circ B = \begin{array}{c|ccc} & c_1 & c_2 & c_3 \\ \hline a_1 & 1 & 0 & 1 \\ a_2 & 0 & 0 & 1 \\ a_3 & 0 & 0 & 0 \end{array}$$

There are two types of composition operations:

1. Max-min composition
2. Max-product composition

1. The max-min composition is defined as,

$$T = A \circ B$$

$$f_T(x, z) = \underset{y \in Y}{\vee} [f_A(x, y) \wedge f_B(y, z)]$$

2. The max-product composition is defined as,

$$T = A \circ B$$

$$f_T(x, z) = \underset{y \in Y}{\vee} [f_A(x, y) \cdot f_B(y, z)]$$

Note : In the above equations $\vee$ represents max operation, $\wedge$ represents min operation and $\cdot$ represents product operation.

Properties of composition operation

1. Associative : $(A \circ B) \circ C = A \circ (B \circ C)$
2. Commutative : $A \circ B \neq B \circ A$
3. Inverse : $(A \circ B)^{-1} = A^{-1} \circ B^{-1}$

4.4.2 Fuzzy Relations

In general, a relation can be considered as a set of tuples, where a tuple is an ordered pair. Similarly, a fuzzy relation is a fuzzy set of tuples i.e. each tuple has a membership degree between 0 and 1.

Definition:

Let U and V be continuous universe, and $\mu_R : U \times V$ [0, 1] then,

$$R = \int_{U \times V} \mu_R(u,v) / (u,v)$$

Is a binary fuzzy relation on $U \times V$.

If U and V are discrete universe, then

$$R = \sum_{U \times V} \mu_R(u, v)/(u,v)$$

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We can express fuzzy relation R = U × V in matrix form as,

$$R = \begin{bmatrix} \mu_R(u_1, v_1) & \mu_R(u_1, v_2) & \dots & \mu_R(u_1, v_n) \\ \mu_R(u_2, v_1) & \mu_R(u_2, v_2) & \dots & \mu_R(u_2, v_n) \\ \vdots & & & \\ \mu_R(u_m, v_1) & \mu_R(u_m, v_2) & \dots & \mu_R(u_m, v_n) \end{bmatrix}$$

Where $U = \{u_1, u_2, u_3, \dots, u_m\}$ and

$V = \{v_1, v_2, v_3, \dots v_n\}$ are universe of discourse.

Ex. 4.4.1 : Given universe of discourse

U = {1,2,3} form a relation R where "x is approximately equal to y"

Then Relation R can be defined as,

$$R = \left\{\frac{1}{(1,1)} + \frac{1}{(2,2)} + \frac{1}{(3,3)} + \frac{0.8}{(1,2)} + \frac{0.3}{(1,3)} + \frac{0.8}{(2,1)} + \frac{0.8}{(2,3)} + \frac{0.3}{(3,1)} + \frac{0.8}{(3,2)}\right\}$$

Soln. : The membership function μ_R of this relation can be described as,

1. when x = y

$\mu_R(x, y) = 0.8$ { when $|x - y| = 1$

0.3 when $|x - y| = 2$

The matrix notation is,

		Y 1	2	3
	1	1	0.8	0.3
X	2	0.8	1	0.8
	3	0.3	0.8	1

N - ary fuzzy relation

It is possible to define n-ary fuzzy relations as fuzzy set of n-tuples. In general it is a relation of pairs.

$\mu_R(x_1, \dots, x_n)/(x_1, \dots, x_n)$;

4.4.2(A) Operations on Fuzzy Relation

MU - Ma

Q. Explain cylindrical extension and projection operations on fuzzy relation with example. (May 13, 5 Ma

Fuzzy relations are very important in fuzzy controller because they can describe interaction between variables.

Four types of operations can be performed on fuzzy relation.

(1) Intersection	(2) Union
(3) Projection	(4) Cylindrical extension

1. Intersection

- Let R and S be binary relations defined on X × Y. The intersection of R and S is defined by,

  $\forall (x, y) \in X \times Y : \mu_{R \cap S}(x, y) = \min(\mu_R(x, y), \mu_S(x, y))$

- Instead of the minimum, any T - Norm can be used.

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2. Union

- The union of R and S is defined as,

  $\forall (x, y) \in X \times Y : \mu_{R \cup S}(x, y) = \max(\mu_R(x, y), \mu_S(x, y))$.

Instead of maximum, any S – norm can be used.

- Given two relations R and S

R =

	y_1	y_2	y_3
x_1	0.3	0.2	0.1
x_2	0.4	0.6	0.1
x_3	0.2	0.3	0.5

S =

	y_1	y_2	y_3
x_1	0.4	0	0.1
x_2	1	0.2	0.8
x_3	0.3	0.2	0.4

- Then, using max operation.

$R \cup S$ =

0.4	0.2	0.1
1	0.6	0.8
0.3	0.3	0.5

- Suppose a simple s - norm

$S(a, b) = a + b - a \cdot b$ is used then,

$R \cup S$ =

0.58	0.2	0.19
1	0.68	0.84
0.44	0.44	0.7

- This operation is more optimistic than the max operation. All the membership degrees are at least as high as in the max operation.

Now, using min operation

$R \cap S$ =

0.3	0	0.1
0.4	0.2	0.1
0.2	0.2	0.4

- Suppose a simple T-norm $T(a, b) = \dfrac{a \cdot b}{a + b - a \cdot b}$ is used then,

$R \cap S$ =

0.20	0	0.1
0.4	0.17	0.10
0.13	0.13	0.28

- The above operation is more optimistic than the min operation. All the membership degrees are less than in the min operation.

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3. Projection

- The projection relation brings a ternary relation back to a binary relation, or a binary relation to a fuzzy set, or a fuzzy set to a single crisp value.

Ex. Consider the relation R as given below.

	y_1	y_2	y_3	y_4
x_1	0.8	1	0.1	0.7
x_2	0	0.8	0	0
x_3	0.9	1	0.7	0.8

Then the projection on X means that

- x_1 is assigned the maximum of the first row.
- x_2 is assigned the maximum of the second row.
- x_3 is assigned the maximum of the third row.

Thus,

$$\text{Proj. R on X} = \frac{1}{x_1} + \frac{0.8}{x_2} + \frac{1}{x_3}$$

Similarly

$$\text{Proj. R on Y} = \frac{0.9}{y_1} + \frac{1}{y_2} + \frac{0.7}{y_3} + \frac{0.8}{y_4}$$

4. Cylindrical Extension

- The projection operation is almost always used in combination with cylindrical extension.

Cylindrical extension is more or less opposite of projection. It converts fuzzy set to a relation.

Ex : Consider a fuzzy set,

$$A = \text{proj. of R on X} = 1/x_1 + 0.8/x_2 + 1/x_3.$$

- Its cylindrical extension on the domain $X \times Y$ is

ce (A) =

	y_1	y_2	y_3	y_4
x_1	1	1	1	1
x_2	0.8	0.8	0.8	0.8
x_3	1	1	1	1

Consider the fuzzy set

$$B = \text{proj of R on X} = \frac{0.9}{y_1} + \frac{0.8}{y_2} + \frac{0.7}{y_3} + \frac{0.8}{y_4}$$

ce (B) =

	y_1	y_2	y_3	y_4
x_1	0.9	0.8	0.7	0.8
x_2	0.9	0.8	0.7	0.8
x_3	0.9	0.8	0.7	0.8

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4.4.2(B) Properties of Fuzzy Relations

Let R, S and T be fuzzy relations defined on the universe X × Y. Then, the properties of fuzzy relations are stated below :

1. **Commutativity**

$$R \cup S = S \cup R$$
$$R \cap S = S \cap R$$

2. **Associativity**

$$R \cup (S \cup T) = (R \cup S) \cup T$$
$$R \cap (S \cap T) = (R \cap S) \cap T$$

3. **Distributivity**

$$R \cup (S \cap T) = (R \cup S) \cap (R \cup T)$$
$$R \cap (S \cup T) = (R \cap S) \cup (R \cap T)$$

4. **Idempotency**

$$R \cup R = R$$
$$R \cap R = R$$

5. **Identity**

$$R \cup \phi_R = R \ , \quad R \cap \phi_R = \phi_R$$
$$R \cup E_R = E_R \ , \quad R \cap E_R = R$$

Where ϕ_R and E_R are null relation (null matrix) and complete relation (unit matrix of all 1s) respectively.

6. **Involution**

$$\overline{\overline{R}} = R$$

7. **De-Morgan's law**

$$\overline{R \cap S} = \overline{R} \cup \overline{S}$$
$$\overline{R \cup S} = \overline{R} \cap \overline{S}$$

8. **Law of excluded middle** and **law of contradiction** are not satisfied.

$$\text{i.e. } R \cup \overline{R} \neq E_R$$
$$\text{and } R \cap \overline{R} \neq \phi_R$$

4.4.2(C) Fuzzy Composition

Composition operation can be used to combine two fuzzy relations in different product spaces.

There are two compositions that are used commonly.

1. Max - min composition
2. Max - product composition

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1. **Max - Min Composition**

- Let R_1 be a fuzzy relation defined on $X \times Y$.
- And R_2 be a fuzzy relation defined on $Y \times Z$.
- Then the max - min composition of two fuzzy relations R_1 and R_2 is denoted by R_1 o R_2 and defined as,

$$R_1 \circ R_2 = \{[(x, z), \max_{y \in Y} (\min (\mu_{R1}(x, y), \mu_{R2}(y, z))] \mid x \in X, y \in Y, z \in Z\}$$

OR
$$\mu_{R1 \circ R2} (x, z) = \max_{y \in Y} \{\min (\mu_{R1} (x, y), \mu_{R2}(y, z))\}$$

2. **Max - Product Composition**

The max - product composition is defined as,

$$R_1 \circ R_2 = \{[(x, z), \max_{y \in Y} (\mu_{R1}(x, y) . \mu_{R2} (y, z))] \mid x \in X, y \in Y, z \in Z\}$$

OR
$$\mu_{R1 \circ R2} (x, z) = \max_{y \in Y} \{\mu_{R1}(x, y) . \mu_{R2}(y, z)\}$$

The following are the properties of fuzzy composition. Assuming R, S and T are binary relations defined on $X \times Y$, $Y \times Z$ and $Z \times W$ respectively.

1. Associativity $\rightarrow$ R o (S o T) $\Rightarrow$ (R o S) o T
2. Monotonicity $\rightarrow$ $S \subseteq T \Rightarrow R \circ S \subseteq R \circ T$
3. Distributivity $\rightarrow$ R o (S $\cup$ T) $\Rightarrow$ (R o S) $\cup$ (R o T)
4. Inverse $\rightarrow$ $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

4.5 Membership Functions

MU - May 12, Dec. 12, Dec.13, Dec.14

Q. Explain the different Fuzzy membership function. **(Dec. 12, Dec. 14, 5/8 Marks)**

Q. Explain standard fuzzy membership functions. **(May 12, Dec. 13, 8 Marks)**

- One way to represent a fuzzy set is by stating its Membership Function (MF). MFs can be represented using any mathematical equation as per requirement or using one of the standard MFs available.
- There are several different standard MFs available.

Types of Membership Functions

1. Increasing MFs (T Function)
2. Decreasing MF (L function)
3. Triangular MF ($\wedge$ function)
4. Trapezoidal MFs (π function)
5. Gaussian MFs
6. Generalized bell MF / Cauchy MF
7. Sigmoidal MFs

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1. Increasing MFs (T Function)

An increasing MF is specified by two parameters (a, b) as follows:

$$T(x\,;a,b) = \begin{cases} 0 & ;\ x \le a \\ (x-a)/(b-a) & ;\ a \le x \le b \\ 1 & ;\ x \ge b \end{cases}$$

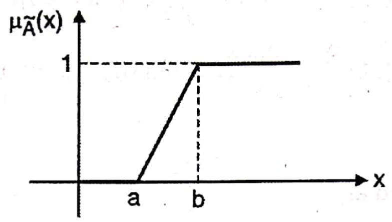

Fig. 4.5.1: Increasing MF

2. Decreasing MF (L function)

A decreasing MF is specified by two parameters (a, b) as follows :

$$L(x\,;a,b) = \begin{cases} 1 & ;\ x \le a \\ (b-x)/(b-a) & ;\ a \le x \le b \\ 0 & ;\ x \ge b \end{cases}$$

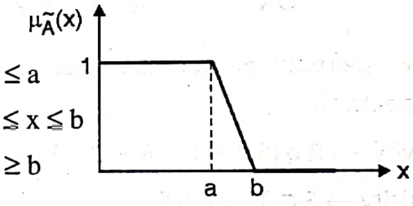

Fig. 4.5.2 : Decreasing MF

3. Triangular MF (∧ function)

– A triangular MF is specified by three parameters (a, b, c) as follows:

$$\wedge(x\,;a,b,c) = \begin{cases} 0 & ;\ x \le a \\ (x-a)/(b-a) & ;\ a \le x \le b \\ (c-x)/(c-b) & ;\ b \le x \le c \\ 0 & ;\ x \ge c \end{cases}$$

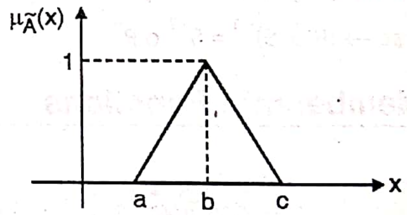

Fig. 4.5.3 : Triangular MF

4. Trapezoidal MFs (π function)

– A trapezoidal MF is specified by four parameters (a, b, c, d) as follows :

$$\text{Trapezoid}(x\,;a,b,c,d) = \begin{cases} 0 & ;\ x \le a \\ (x-a)/(b-a) & ;\ a \le x \le b \\ 1 & ;\ b \le x \le c \\ (d-x)/(d-c) & ;\ c \le x \le d \\ 0 & ;\ x \ge d \end{cases}$$

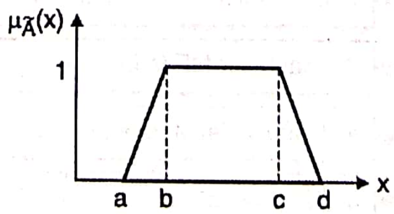

Fig. 4.5.4 : Trapezoid MF

– An alternative expression using min and max can be given as,

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$$\text{trapezoid}(x\,;a,b,c,d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right)$$

- The parameters {a, b, c, d} (with a < b < c < d) determine the x coordinates of the four corners of the trapezoidal MF.
- Here, if b = c then trapezoidal MF reduces to triangle MF.
- Since two MFs triangular and trapezoidal are composed of straight line segment, they are not smooth at the corner points specified by the parameters. However due to the simple formulae and computational efficiency, they are used extensively.
- Some **smooth** and **non-linear** MFs (Gaussian and Generalized Bell) are discussed below :

5. Gaussian MFs

A Gaussian MF is specified by two parameters {c,σ}.

$$\text{Gaussian}(x\,;c,\ \sigma) = e^{-1/2\left(\frac{x-c}{\sigma}\right)^2}$$

c represents MFs center and

σ determines MFs width.

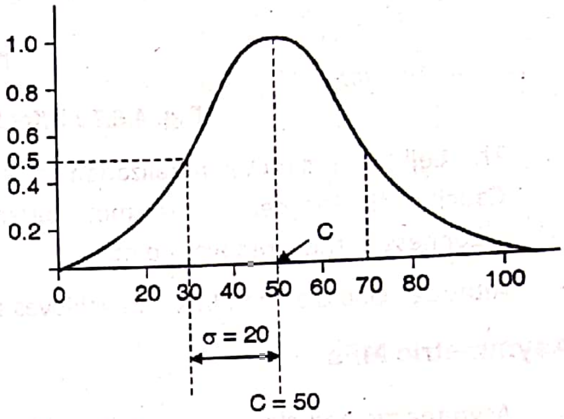


Fig. 4.5.5 : Gaussian (x; 50, 20) MF

6. Generalized bell MF / Cauchy MF

- A generalized bell MF (or bell MF) is specified by three parameters (a, b, c).

$$\text{bell}(x\,;a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

- A desired generalized bell MF can be obtained by a proper selection of the parameters a, b, c.

  c specifies the center of a bell MF

  a specifies the width of a bell MF

  and b determines the slope at the crossover points.

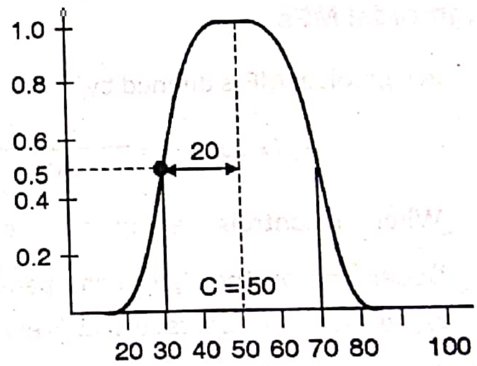
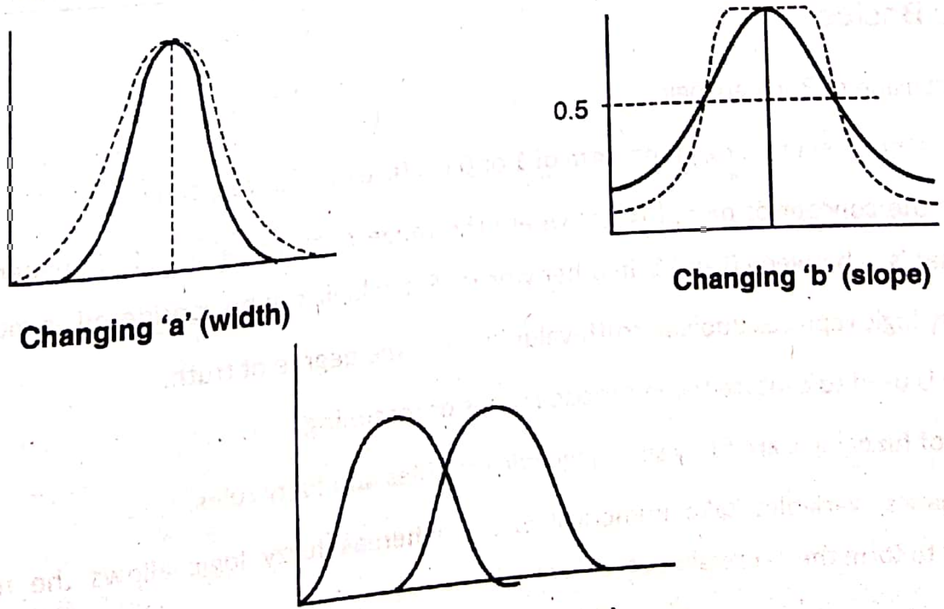


Fig. 4.5.6: Bell (x ; 20, 4, 50)

- Fig. 4.5.7 illustrates the effect of changing these parameters on the shape of the curve.



Changing 'a' (width)

Changing 'b' (slope)

Changing 'c' (centre)

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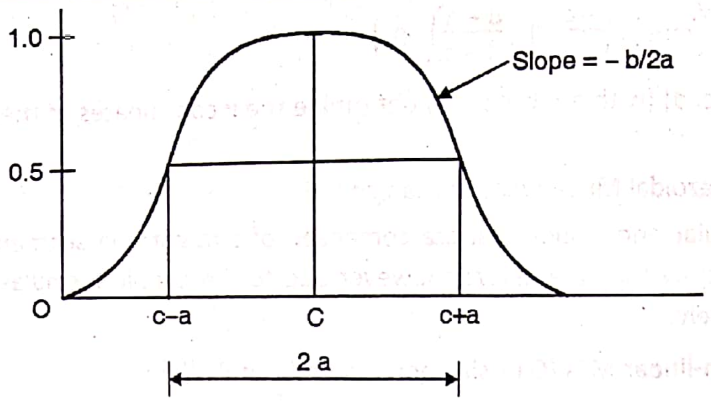

Fig. 4.5.7 : Effect of change of different parameters in Bell MF

- The bell MF is direct generalization of Cauchy distribution used in probability theory; so it is also referred to as the Cauchy MF. The bell MF has more parameter than Gaussian MF, so it has more degree of freedom to adjust the steepness at the crossover point.
- Although Gaussian and bell MFs achieves smoothness, they are unable to specify asymmetric MFs.

Asymmetric MFs

Asymmetric and close MFs can be achieved by using either the absolute difference or the product of two sigmoidal functions.

Sigmoidal MFs

- A sigmoidal MF is defined by,

$$\text{sig}(x\,;\,a, c) = \frac{1}{1 + \exp[-a(x-c)]}$$

Where, a controls the slope at the crossover point $x = c$.

- Depending on the sign of the parameter a, a sigmoidal MF is open right or open left and thus is appropriate for representing concepts such as "very large" or "very negative".
- They are widely used as the activation function in artificial neural networks.

4.6 Fuzzy Logic

4.6.1 Fuzzy Logic Basics

- Fuzzy logic is an extension of Boolean logic.
- In Boolean logic we express everything in the form of 1 or 0 i.e. true or false respectively.
- Fuzzy logic handles the concept of partial truth, where the range of truth value is in between completely true and completely false, that is in between 0 and 1. In other words , Fuzzy logic can be considered as multi-valued logic.
- In other words, fuzzy logic replaces Boolean truth-values with some degree of truth.
- This degree of truth is used to capture the imprecise modes of reasoning.
- The basic elements of fuzzy logic are fuzzy sets, linguistic variables and fuzzy rules.
- Usually in mathematics, variables take numerical values whereas fuzzy logic allows the non-numeric linguistic variables to be used to form the expression of rules and facts.

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- The linguistic variables are words, specifically adjectives like "small," "little," "medium," "high," and so on. A fuzzy set is a collection of couples of elements.

Linguistic Variables and Linguistic Values

- A **linguistic variable** is a **variable** whose values are words or sentences in a natural or artificial language.
- Consider the variable X = "age".
- A linguistic variable ("age") can assume different linguistic values such as "young", "Middle aged", "Mature" and "old" in this case.
- Then, 'age' can be considered as a linguistic variable whose values can be "young", "Middle aged" "Mature" and "old" and these values can be characterized by MFs $\mu_{young}(x)$, $\mu_{middle\ aged}(x)$, $\mu_{mature}(x)$and $\mu_{old}(x)$.
- The universe of discourse is totally covered by these MFs (MFs for young, middle aged and old) and transition from one MF to another is smooth and gradual.

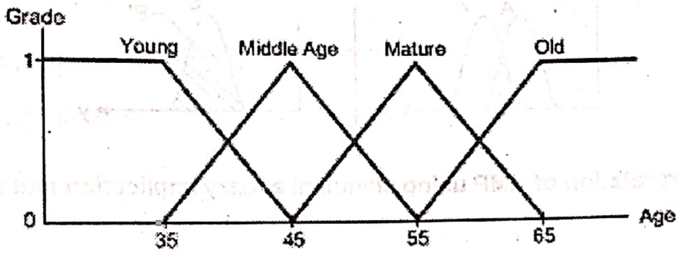

Fig 4.6.1 : Linguistic variable "age" as membership functions

4.6.2 Fuzzy Rules and Fuzzy Reasoning

- Fuzzy inference is the process of obtaining a new knowledge from an existing knowledge.
- To perform inference, knowledge must be represented in some form.
- Fuzzy logic uses IF –THEN rules to represent its knowledgebase.
- The basic rule of inference in traditional two-valued logic is **modus ponens**, according to which, we can infer the truth of a proposition B from the truth of A and the implication A → B.

  Ex. : If A is identified with - "the tomato is red" and B with "the tomato is ripe" then if it is true that "the tomato is red" it is also true that "the tomato is ripe".

  i.e. Premise 1 (fact) X is A

  Premise 2 (rule) if X is A then Y is B

  Consequence (conclusion) : Y is B

- However, in most of the human reasoning, modus ponens is employed in an approximate manner.

  For e.g. : if we have the same implication rule, "if the tomato is red, then it is ripe" and we know that, "the tomato is more or less red" then we may infer that "the tomato is more or less ripe"

  Premise 1 (fact) X is A′

  Premise 2 (rule) if X is A then Y is B

  Conclusion Y is B′

  Where A′ is close to A and

  B′ is close to B

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– When A, B, A′ and B′ are fuzzy sets of appropriate universes, the inference procedure is called **"approximate reasoning"** or **fuzzy reasoning**, it is also called Generalized Modus Ponens (GMP).

Definition : Approximate reasoning / fuzzy reasoning

Let A, A′ and B be fuzzy sets of X, X and Y respectively. Assume that the fuzzy implication A → B is expressed as a fuzzy relation R on X × Y. Then the fuzzy set B induced by "x is A" (fact) and the fuzzy rule "if x is A then y is B" is defined by,

$$\mu_{B'}(y) = \max \min [\mu_{A'}(x), \mu_R(x, y)] = V_x [\mu_{A'}(x) \wedge \mu_R(x, y)]$$

$$\text{or} \quad \beta' = A' o (A \rightarrow B)$$

if x is A, y is B

a. Single rule with single antecedent

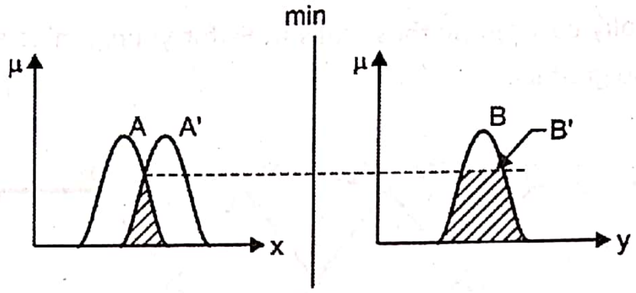

Fig. 4.6.2 : Graphic interpretation of GMP using Mamdani's fuzzy implication and max-min composition

Here $\mu_{B'}$ can be defined as ;

$$\mu_{B'}(y) = [V_x (\mu_{A'}(x) \wedge \mu_A(x))] \wedge \mu_B(y)$$

$$= w \wedge \mu_B(y)$$

Thus, we first find the degree of match w which is the maximum of $\mu_{A'}(x) \wedge \mu_A(x)$.

Then the MF of resulting B′ is equal to the MF of B clipped by w.

b. Single rule with multiple antecedent

A fuzzy if then rule with two antecedents can be written as,

"if x is A and y is B then z is C"

Premise 1 (fact)	:	x is A′ and y is B′
Premise 2 (rule)	:	If x is A and y is B then z is C
Consequence	:	z is C′

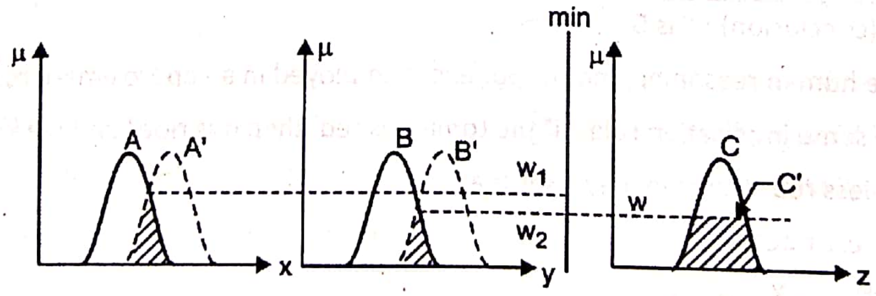

Fig. 4.6.3 : Approximate reasoning for multiple antecedents

$$\mu_{C'}(z) = \underbrace{\{V_x [\mu_{A'}(x) \wedge \mu_A(x)]\}}_{w_1} \wedge$$

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$\{ \vee_y [\mu_{B'}(y) \wedge \mu_B(y)]\} \wedge \mu_C(z)$

$\underbrace{\qquad\qquad\qquad\qquad}_{w_2}$

$$= \underbrace{(w_1 \wedge w_2)}_{\text{firing strength}} \wedge \mu_C(z)$$

- When w_1 and w_2 are the maxima of the MFs of $A \cap A'$ and $B \cap B'$ respectively.
- Thus, w_1 denotes the degree of compatibility between A and A′, similarly for w_2.
- Since the antecedent parts of the fuzzy rule is constructed using **and** connective, $w_1 \wedge w_2$ is called **firing strength or degree of fulfilment of the fuzzy rule.**
- The firing strength represents the degree to which the antecedent part of the rule is satisfied.
- The MF of the resulting C′ is equal to the MF of clipped by the firing strength w (when $w = w_1 \wedge w_2$)

c. Multiple rules with multiple antecedents

- The GMP problem for multiple rules with multiple antecedents can be written as,

Premise 1 (fact)	:	x is A′ and y is B′
Premise 2 (rule 1)	:	If x is A_1 and y is B_1 then z is C_1
Premise 3 (rule 2)	:	If x is A_2 and y is B_2 then z is C_2
Consequence	:	z is C′

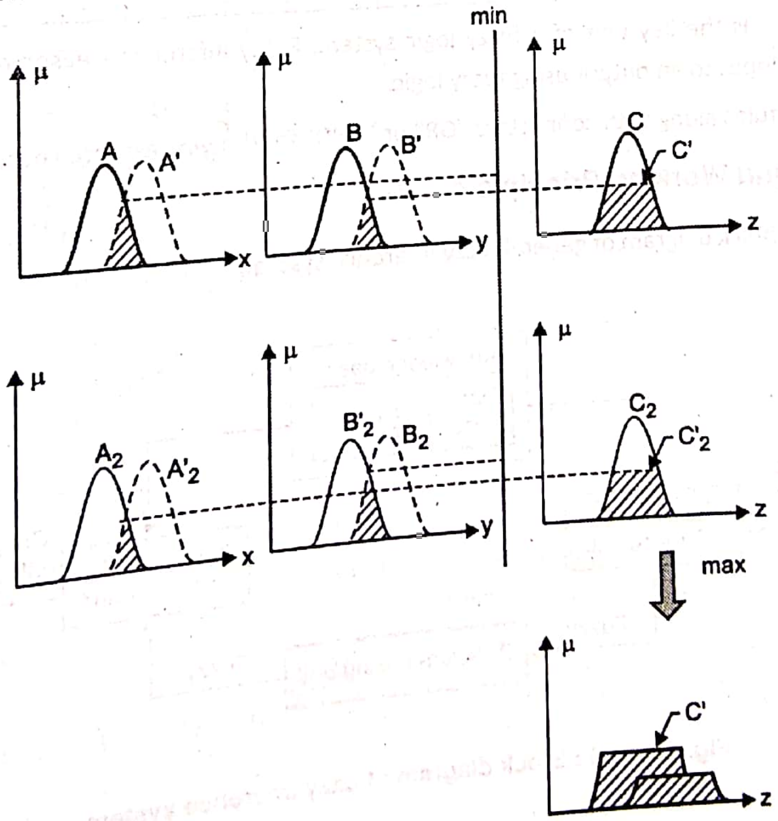

Fig. 4.6.4 : Fuzzy reasoning for multiple rules with multiple antecedents

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- Here C_1' and C_2' are the inferred fuzzy sets for rule 1 and rule 2 respectively.
- When a given fuzzy rule assumes the form "if x is A or y is B" then firing strength is given as the maximum of degree of match on the antecedent part for a given condition.

Ex.

If x is A_1 or y is B_1 then z is C_1.

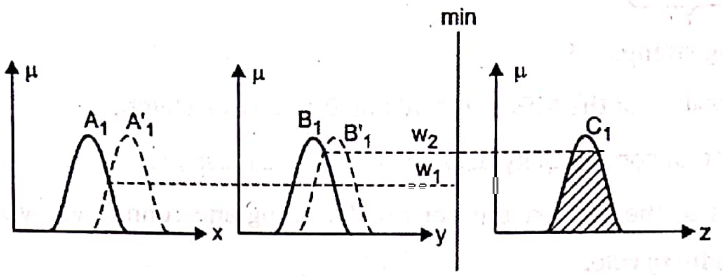

Fig. 4.6.5

- In the above example, because two antecedents are connected using **or**, we take maximum of w_1 and w_2 as a firing strength.
- Since $w_2 > w_1$, we take w_2 as a firing strength and then we apply **min** implication operator on the output MF C_1.

4.7 Fuzzy Inference Systems

MU - May 12, May 13, Dec. 15

Q.	Explain the three types of Fuzzy Inference Systems in detail.	**(May 12, 10 Marks)**
Q.	Compare Mamdani and Sugeno fuzzy models.	**(May 13, 10 Marks)**
Q.	Write short note on Fuzzy inference system.	**(Dec. 15, 10 Marks)**

- Fuzzy Inference System is the key unit of a fuzzy logic system. Fuzzy inference (reasoning) is the actual process of mapping from a given input to an output using fuzzy logic.
- It uses the "IF...THEN" rules along with connectors "OR" or "AND" for drawing essential decision rules.

4.7.1 Construction and Working Principle of FIS

- Fig. 4.7.1(a) shows the block diagram of general fuzzy inference system.

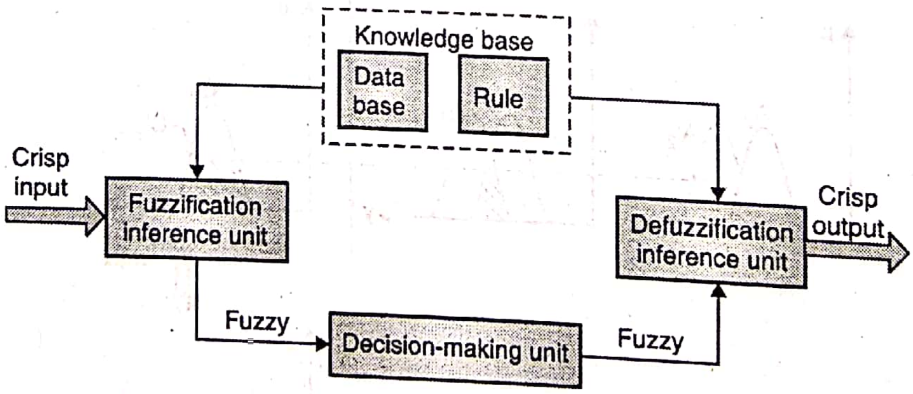

Fig. 4.7.1 (a) : Block diagram : Fuzzy Inference system

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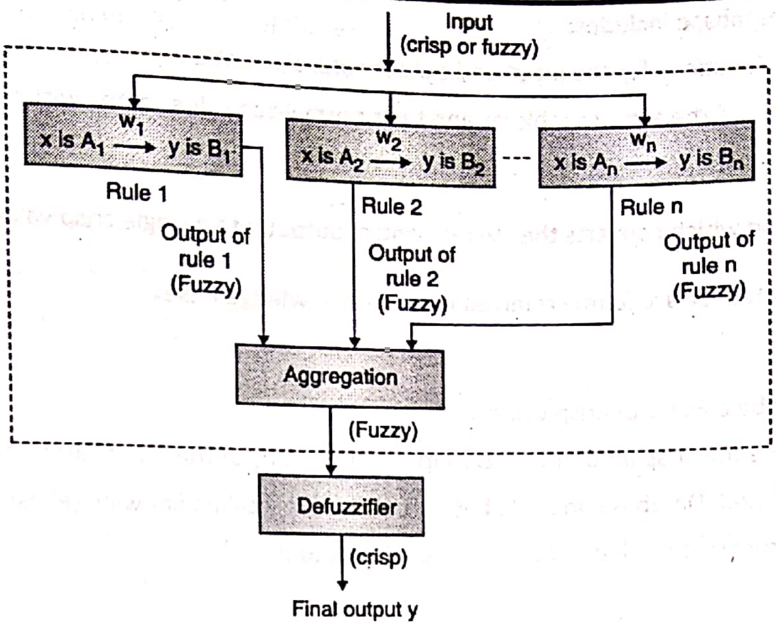

Fig 4.7.1(b) : Fuzzy Inference using If-Then rules

- As shown in Fig. 4.7.1(a), FIS involves five important modules.

1. Fuzzification Inference Unit (FU)
2. Decision making / Inferencing Unit
3. Rule Base
4. Data Base
5. Defuzzification Inference Unit (DU)

1. Fuzzification Inference Unit

This block performs a fuzzification which converts a crisp input in to a fuzzy set. Here we need to decide the proper fuzzification strategy.

2. Decision making/Inferencing Unit

- The basic function of the inference unit is to compute the overall value of the control output variable based on the individual contribution of each rule in the rule base.
- The output of the fuzzification module representing the crisp input is matched to each rule-antecedent.
- The degree of match of each rule is established. Based on this degree of match, the value of the control output variable in the rule-consequent is modified. The result is, we get the "clipped" fuzzy set representing the control output variable.
- The set of all clipped control output values of the matched rules represent the overall fuzzy value of control output.

3. A rule base

It contains a number of fuzzy IF-THEN rules.

4. A database

- Data Base defines the membership functions of the fuzzy sets used in the fuzzy rules.

- The information in the database includes:
  - Fuzzy Membership Functions for the input and output control variables
  - The physical domains of the actual problems and their normalized values along with the scaling factors.

5. **Defuzzification Unit**

- It performs defuzzification which converts the overall control output into a single crisp value.
- The **rule base** and the **database** are jointly referred to as the **knowledge base.**

Working

The input to the FIS may be a Fuzzy or crisp value.

1. Fuzzification Unit converts the crisp input into fuzzy input by using any of the fuzzification methods.
2. The next, rule base is formed. Database and rule base are collectively called knowledgebase.
3. Finally, defuzzification process is carried out to produce crisp output.

Methods of FIS

- The most important two types of fuzzy inference method are :

1) Mamdani FIS
2) Sugeno FIS

- Mamdani fuzzy inference is the most commonly seen inference method. This method was introduced by Mamdani and Assilian (1975).
- Another well-known inference method is the so- called Sugeno or Takagi–Sugeno–Kang method of fuzzy inference process. This method was introduced by Sugeno (1985). This method is also called as TS method.
- The main difference between the two methods lies in the consequent of fuzzy rules.

1. Mamdani FIS

- Mamdani FIS was proposed by Ebahim Mamdani in the year 1975 to control a steam engine and boiler combination.
- To compute the output of this FIS given the inputs, six steps has to be followed.
  1. Determining a set of fuzzy rules.
  2. Fuzzifying the inputs using the input membership functions.
  3. Combining the fuzzified inputs according to the fuzzy rules to establish a rule strength (Fuzzy Operations).
  4. Finding the consequence of the rule by combining the rule strength and the output membership function (implication).
  5. Combining the consequences to get an output distribution (aggregation).
  6. Defuzzifying the output distribution (this step is only if a crisp output (class) is needed).

Fuzzy Rule Composition in Mamdani Model

- In Mamdani FIS, The fuzzy rules are formed using IF-THEN statements and AND/OR connectives.
- The consequent of the rule can be obtained in two steps.
  - By computing the strength of each rule
  - By clipping the output membership function at the rule strength.

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- The outputs of all the fuzzy rules are then combined to obtain the aggregated fuzzy output. Finally, defuzzification is applied on to the aggregated fuzzy output to obtain a crisp output value.
- Consider two inputs, two rule Mamdani fuzzy inference system.
- Assume two inputs are crisp value x and y.
- Assume the following two rules :

Rule 1 : if x is A_1 and y is B_1 then z is C_1

Rule 2 : if x is A_2 and y is B_2 then z is C_2

- Fig. 4.7.2 (a) shows Mamdani fuzzy inference system using **min – max** decomposition.
- Fig. 4.7.2 (a) illustrates a procedure of deriving overall output ***z*** when presented with two crisp inputs x and y. In the above Mamdani inference system, we have used **min** as T - norm and **max** as T - conorm operators.
- The T-norm operator is used for inferencing antecedent part of the rule. And co-norm operator used to aggregate outputs resulting form of each rule.
- Mamdani model also supports **max - product** composition to derive overall output z. Here the **algebraic product** is used as T-norm operator and **max** is used as T-conorm operator.

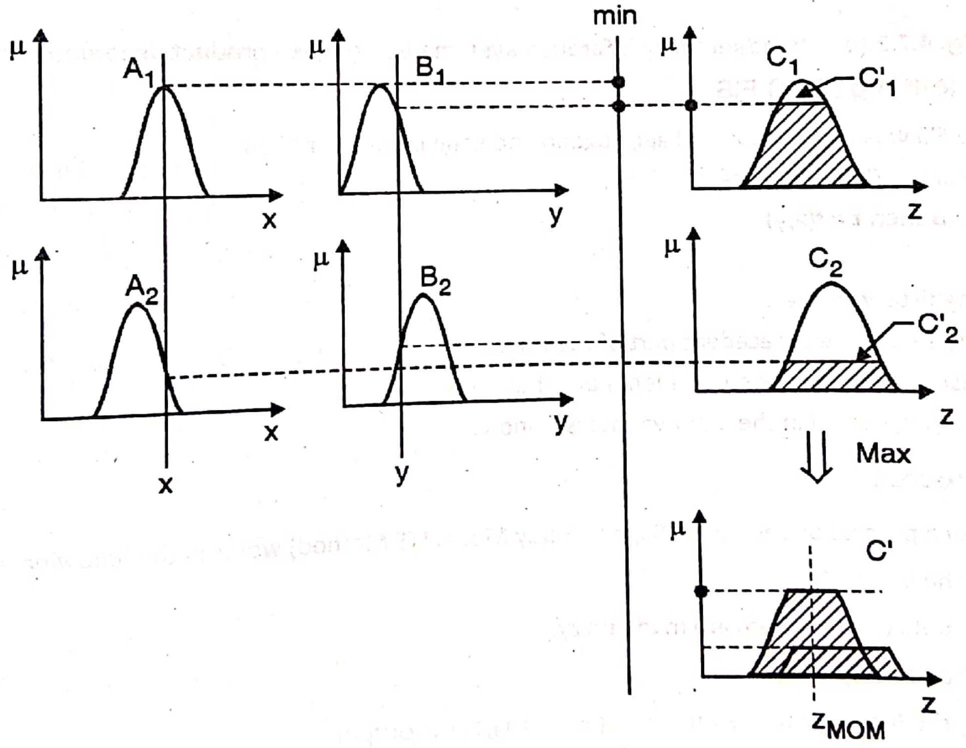

Fig. 4.7.2 (a) : Mamdani fuzzy inference systems using max - min decomposition

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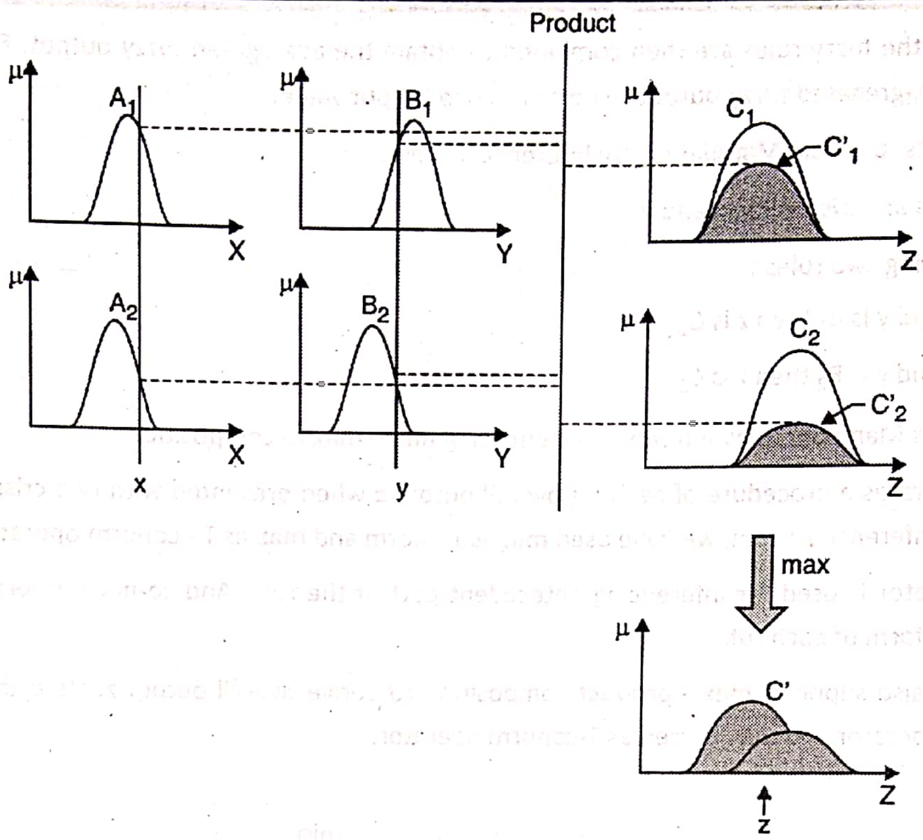

Fig. 4.7.2 (b): Mamdani fuzzy inference systems using max - product decomposition

2. Takagi-Sugeno-Kang (TSK) FIS

- Takagi - Sugeno FIS was proposed by Takagi, Sugeno and Kang in the year 1985.
- A typical fuzzy rule in TSK model has the form ,

  IF x is A and y is B then z = f(x,y)

  Where,

  x, y and z are linguistic variables.

  A and B are fuzzy sets in the antecedent part of the rule.

  Z = f(x, y) is a crisp function in the consequent part of the rule.

  Usually f(x, y) is a polynomial in the input variables x and y.

Fuzzy Inference Process

The fuzzy inference process under Takagi-Sugeno Fuzzy Model (TS Method) works in the following way :

Step 1 : Fuzzifying the inputs

Here, the inputs of the system are made fuzzy.

Step 2 : Applying the fuzzy operator

In this step, the fuzzy operators must be applied to get the output.

First order Sugeno fuzzy model

When f(x,y) is a first order polynomial (e.g. z = ax + by + c) the resulting FIS is called , first order Sugeno fuzzy model.

Zero Order fuzzy model

- In zero order fuzzy model, the output z is a constant (i.e. a = b = 0).
- The typical form of the rule in zero order FIS is

  IF x is A and y is B then z = c

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Where c is a constant

- In this case the output of each fuzzy rules is a constant and hence the overall output is obtained via weighted average method.
- The output level z_i of each rule is weighted by the firing strength w_i of the rule.

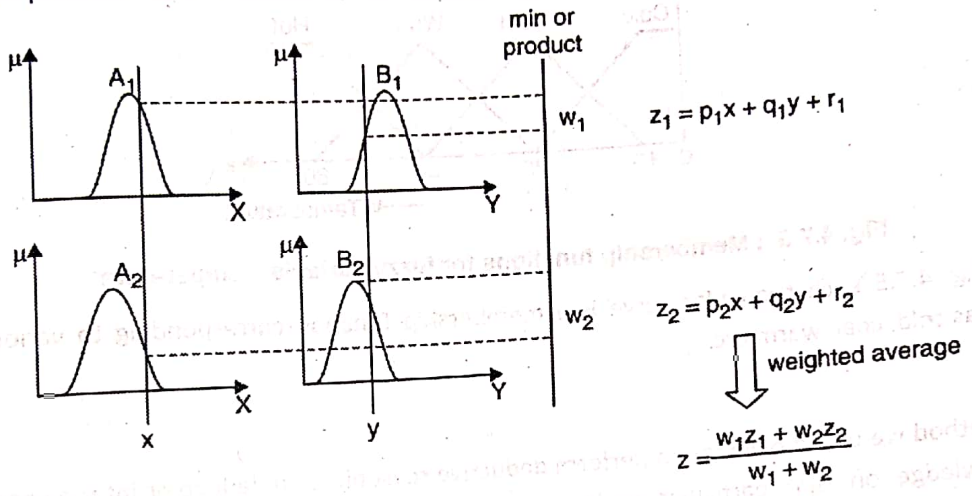

Fig. 4.7.3 : Reasoning in Sugeno FIS

Comparison between the Mamdani System and the Sugeno Model

- **Output Membership Function :** The main difference between them is on the basis of output membership function. The Sugeno output membership functions are either linear or constant.
- **Aggregation and Defuzzification Procedure :** The difference between them also lies in the consequence of fuzzy rules and due to the same their aggregation and defuzzification procedure also differs.
- **Mathematical Rules :** More mathematical rules exist for the Sugeno rule than the Mamdani rule.
- **Adjustable Parameters :** The Sugeno controller has more adjustable parameters than the Mamdani controller.

4.7.2 Fuzzification of Input Variables

- Fuzzification is the process of converting a crisp set into a fuzzy set.
- Here the crisp value is transformed into linguistic variables.
- In real word problems, many a times the input values are not very precise and accurate rather they are uncertain, imprecise and unknown.
- The uncertainty may arise due to the vagueness and incompleteness of data. In such cases, variable may be represented as fuzzy and can be represented as fuzzy membership function.

Methods of membership value of assignment

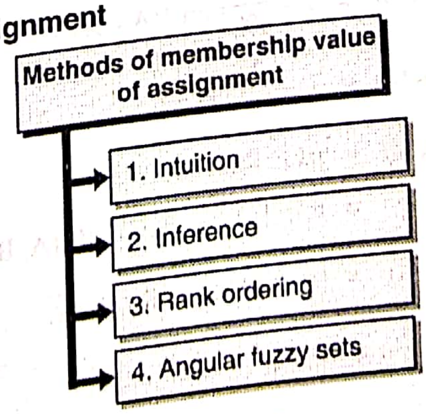

Fig. 4.7.4

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1. Intuition

– As the name suggest, this method is based upon the common intelligence of human. The human develops membership functions based on their own understanding capability.

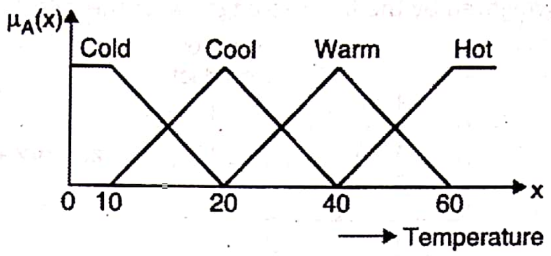

Fig. 4.7.5 : Membership functions for fuzzy variable "temperature"

– As shown in Fig. 4.7.5 each triangular curve is a membership function corresponding to various fuzzy (linguistic) variables such as cold, coal, warm etc.

2. Inference

– In inference method we use knowledge to perform deductive reasoning. To deduce or infer a conclusion, we use the facts and knowledge on that particular problem. Let us consider the example of Geometric shapes for the identification of a triangle.

Let A, B, C be the interior angles of a triangle such that,

$A \geq B \geq C > 0°$ and $A + B + C = 180°$

– For this purpose, we define five types of triangles.

1. R = Approximately Right-angle triangle
2. I = Approximately Isosceles triangle
3. E = Approximately Equilateral triangle
4. I · R = Isosceles Right-angle triangle
5. T = Other type of triangle

– Now, we can infer membership values for all those types of triangles through the method of inference because we possess the knowledge about the geometry of their shapes.

The membership values for five types of triangle can be defined as,

$$\mu R\,(A, B, C) = 1 - \frac{1}{90°} \,|\, A - 90° \,|$$

$$\mu I\,(A, B, C) = 1 - \frac{1}{60°} \min \{(A - B), (B - C)\}$$

$$\mu E\,(A, B, C) = 1 - \frac{1}{80°} \,|\, A - C \,|$$

$$\mu I \cdot R\,(A, B, C) = \mu I \cap R\,(A, B, C)$$

$$= \min \{\mu I\,(A, B, C), \mu R\,(A, B, C)\}$$

$$\mu T\,(A, B, C) = (R \cup I \cup E) = R \cap \bar{I} \cap \bar{E}$$

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Ex :

$$\mu(A, B, C) = \{80, 65, 35\}$$

$$\mu_R(A, B, C) = 1 - \frac{1}{90} \mid 80 - 90 \mid = \frac{8}{9}$$

$$\mu_I(A, B, C) = 1 - \frac{1}{60} \min\{15, 45\} = \frac{3}{4}$$

$$\mu_E(A, B, C) = 1 - \frac{1}{180} \mid 45 \mid = \frac{3}{4}$$

$$\mu_{IR}(A, B, C) = \min\{\mu_I, \mu_R\} \frac{3}{4}$$

$$\mu_T = R^C \cap I^C \cap E^\epsilon$$

$$= \min\left\{\frac{1}{9}, \frac{1}{4}, \frac{1}{4}\right\}$$

$$= \frac{1}{4}$$

3. Rank ordering

- In rank ordering method, preferences are assigned by a single individual, committee, a poll and other opinion methods can be used to assign membership values to fuzzy variables.
- Here the preferences are determined by pair wise comparisons and they are used to determine ordering of the membership.

Example :

Let's suppose 1000 people respond to a questionnaire and their pair wise preferences among the colors red, orange, yellow and blue is given as below.

	Red	Orange	Yellow	Green	Blue
Red	-	517	525	545	661
Orange	483	-	891	477	576
Yellow	474	159	-	534	614
Green	455	523	466	-	643
Blue	339	424	386	357	-

4. Angular fuzzy sets

- Angular fuzzy sets differ from normal fuzzy sets only in their coordinate description.
- Angular fuzzy sets are defined on a universe of angles; hence they are of repeating shapes for every 2π cycles.
- Angular fuzzy sets are used in the quantitative description of the linguistic variables, which are known as "truth values".

Example :

Let's consider that pH values of water samples are taken from a contaminated pond. We know that,

- If P_n value is 7 means it's a neutral solution.

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- Levels of P_n between 14 and 7 are labelled as Absolute Basic (AB), Very Basic (VB), Basic (B), Fairly Basic (FB), Neutral (N) drawn from $\theta = \frac{\pi}{2}$ to $\theta = -\frac{\pi}{2}$
- Levels of P_n between 7 to 0 are called neutral, Fairly Acidic (FA), Acidic (A), Very Acidic (VA), Absolutely Acidic (AA), are drawn from $\theta = 0$ to $\theta = -\frac{\pi}{2}$.
- Linguistic values vary with θ and their membership values are given by equation,

$$E_\theta = t \tan \theta$$

Here 't' is the horizontal projection of the radial vector.

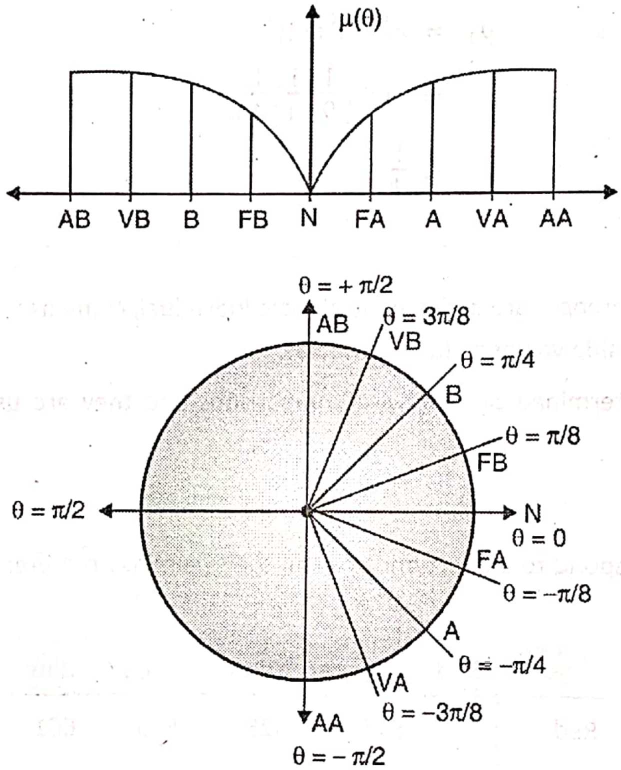

Fig. 4.7.6 : Model of angular fuzzy set

4.7.3 Defuzzification

MU - Dec. 12, Dec. 14

Q.	Explain any four defuzzification methods with suitable example.	**(Dec. 12, 10 Marks)**
Q.	Explain different methods of defuzzification.	**(Dec. 14, 10 Marks)**

- Defuzzification is the process of converting a fuzzy set into a crisp value.
- The output of a fuzzy process may be union of two or more fuzzy membership functions. In that case we need to find crisp value as a representative of the entire fuzzy MF.
- Different methods of defuzzification are listed below :

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Methods of defuzzification

- 1. Max membership principle
- 2. Centre of gravity or centroid
- 3. Weighted average
- 4. Mean of maximum (MoM)
- 5. Centre of sums (CoS)
- 6. Centre of largest area
- 7. First (or last) maxima
- 8. Bisector

Fig. 4.7.7 : Defuzzification methods

1. Max-membership principle / Height method

- This method is limited to peak output functions. It uses the individual clipped or scaled central outputs.

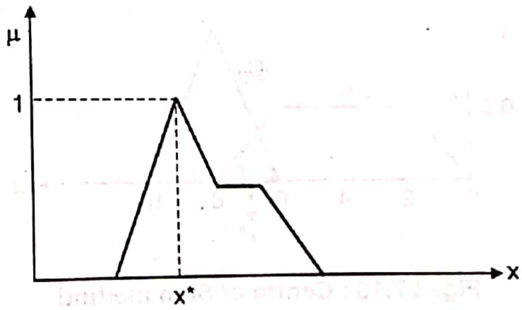

Fig. 4.7.8 : Max-membership

- The algebraic expression is:

$$\mu_{\tilde{c}}(x^*) \geq \mu_{\tilde{c}}(x) \text{ for all } x \in X$$

2. Centre of Area / Gravity (Centroid) Method

- This method is the most preferred and physically appealing of all the defuzzification methods.
- This method determines the centre of the area below the **combined membership function.** (i.e. it takes union of all output fuzzy sets).
- So, if there exist an overlapping area, will be considered only once. Thus overlapping areas are not reflected.
- This operation is computationally complex and therefore results in slow inference cycle.
- Algebraic expression is

For Continuous	For Discrete
$x^* = \dfrac{\int \mu_{\tilde{c}}(x) \cdot x\, dx}{\int \mu_{\tilde{c}}(x)\, dx}$	$x^* = \dfrac{\sum_{i=1}^{n} \mu_{\tilde{c}}(x_i) \cdot x_i}{\sum_{i=1}^{n} \mu_{\tilde{c}}(x_i)}$

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– It is basically used for non-convex membership functions.

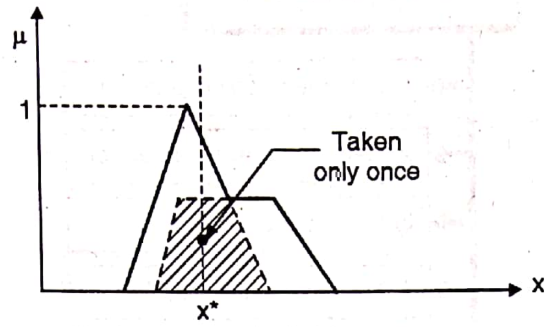

Fig. 4.7.9 : Centroid method

3. **Centre of Sum (COS)**

   – This is faster than many defuzzification methods that are presently in use.
   – This method involves the algebraic sum of individual output fuzzy sets, instead of their union.
   – The idea is to consider the contribution of the area of each output membership curve.
   – In contrast, the centre of area/gravity method considers the union of all output fuzzy sets.
   – In COS method, we take overlapping areas. If such overlapping areas exist, they are reflected more than once.

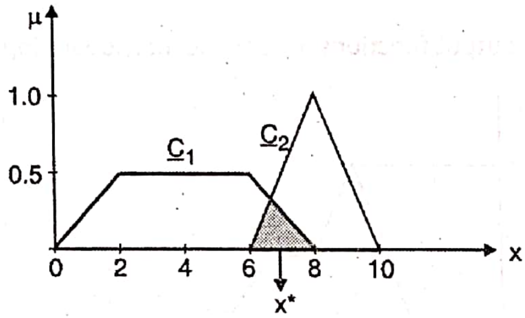

Fig. 4.7.10 : Centre of Sum method

For Continuous	For Discrete
$x^* = \dfrac{\int x \sum_{k=1}^{N} \mu_{\tilde{C}_k}(x)\,dx}{\int \sum_{k=1}^{N} \mu_{\tilde{C}_k}(x)\,dx}$	$x^* = \dfrac{\sum_{i=1}^{N} x_i \sum_{k=1}^{n} \mu_{\tilde{C}_k}(x_i)}{\sum_{i=1}^{N} \sum_{k=1}^{n} \mu_{\tilde{C}_k}(x_i)}$

Advantage : It can be implemented easily and leads to a faster computation.

4. **Weighted average method**

– This method is only valid for **symmetrical** output membership functions.

Algebraic expression is :

$$x^* = \frac{\sum_{i=1}^{n} \mu_{\tilde{c}}(x_i) \cdot xi}{\sum_{i=1}^{n} \mu_{\tilde{c}}(x_i)}$$

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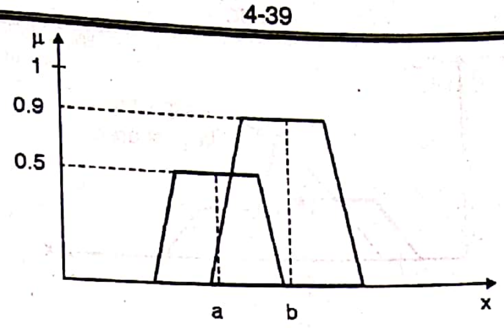

Fig. 4.7.11 : Weighted average method

- The weighted average method is formed by weighting each membership function in the output by its respective maximum membership value.
- The two functions shown in Fig. 4.7.11 would result in the following general form of defuzzification.

$$x^* = \frac{(a \times 0.5) + (b \times 0.9)}{0.5 + 0.9}$$

5. Mean-max membership (Middle of maxima)

- This method is closely related to the max-membership principle (height defuzzification) method; except that the locations of the maximum membership can be non-unique (can be more than one).
- In that case we take the average of the elements having maximum membership value of Maximizing MF.

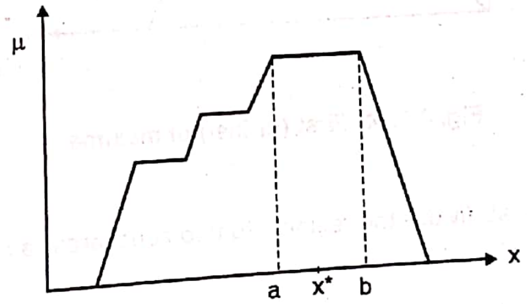

Fig. 4.7.12 : Mean of maximum method

- Algebraic expression is,

$$x^* = \frac{a+b}{2}$$

This method only works for convex.

6. Centre of largest area

- The centre of largest area method is used when the combined output fuzzy set is **non-convex** i.e. it consists of at least two convex fuzzy subsets.
- Then the method determines the convex fuzzy subset with the largest area and defines crisp output value x* to be the centre of area of the largest fuzzy subset.

$$x^* = \frac{\int \mu_{\tilde{c}_m}(x) \cdot x \, dx}{\int \mu_{\tilde{c}_m}(x) \cdot dx}$$

Where, $\tilde{c}_m$ is the convex fuzzy subset that has the largest area.

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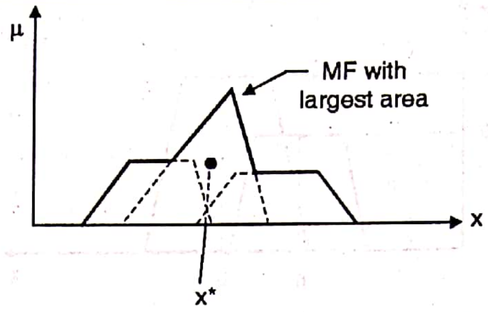

Fig. 4.7.13 : Centre of largest area

7. **First (or last) of maxima**

- This method uses the overall output (i.e. union of all individual output MF).
- First of maxima is determined by taking the smallest value of the domain with maximized membership degree.
- Last of maxima is determined by taking the greatest value of the domain with maximized membership degree.

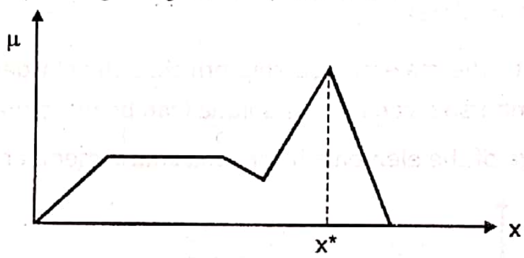

Fig. 4.7.14: First (or last) of maxima

8. **Bisector method**

This method uses the vertical line that divides the region into two equal areas as shown in Fig. 4.7.15 This line is called bisector.

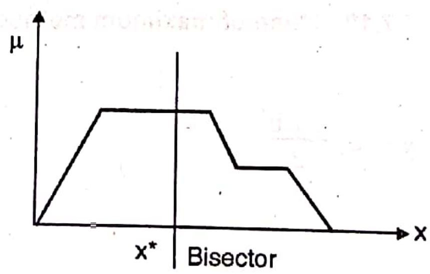

Fig. 4.7.15 : Bisector method of defuzzification

4.8 Fuzzy Controllers

- Most commercial fuzzy products use Fuzzy Knowledge-Based Controllers (FKBC).
- The principal structure of a FKBC is shown in Fig. 4.8.1.

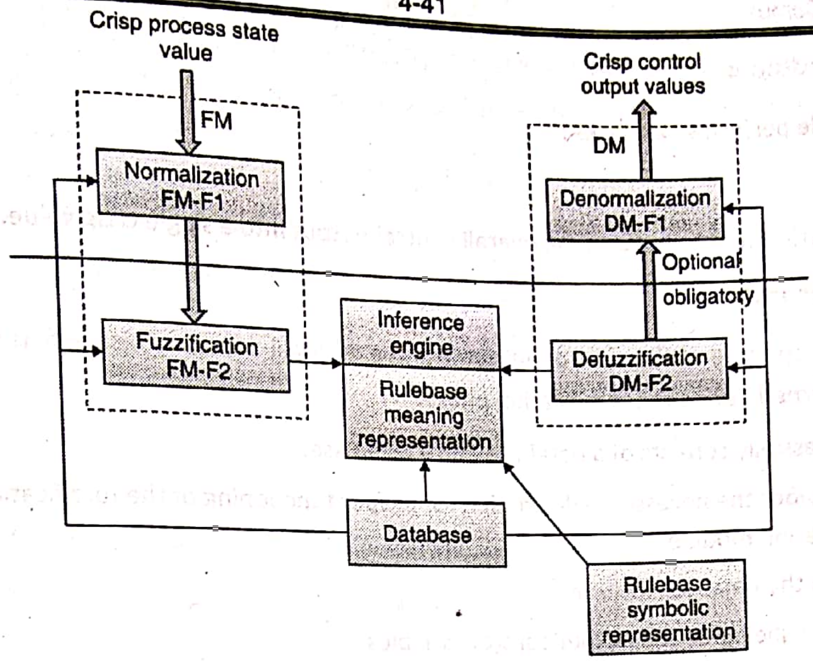

Fig. 4.8.1 : The structure of FKBC

- As shown in Fig.4.8.1, FKBC involves three important modules.

1. Fuzzification module
2. Decision making or Inferencing module
3. Defuzzification module

- In addition to this, it uses two more components

  - Data base and
  - Rule base

  } Knowledge base

1. **Fuzzification module :**

Fuzzification module performs the following two functions.

(a) Normalization

This block performs a scale transformation which maps the physical values of input variables in to a normalized universe of discourse. This block is optional. If a non-normalized domain is used then this block is not required.

(b) Fuzzification

This block performs a fuzzification which converts a crisp input in to a fuzzy set. Here we need to decide the proper fuzzification strategy.

2. **Decision making/Inferencing module :**

- The basic function of the inference engine is to compute the overall value of the control output variable based on the individual contribution of each rule in the rule base.
- The output of the fuzzification module representing the crisp input is matched to each rule-antecedent.
- The degree of match of each rule is established. Based on this degree of match, the value of the control output variable in the rule-consequent is modified. The result is, we get the "clipped" fuzzy set representing the control output variable.
- The set of all clipped control output values of the matched rules represent the overall fuzzy value of control output.

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3. Defuzzification module :

Defuzzification module performs two tasks :

(a) Defuzzification

It performs defuzzification which converts the overall control output into a single crisp value.

(b) Denormalization module

– This block maps the crisp value of the control output into the physical domain. This block is optional. It is used only if normalization is performed during the fuzzification phase.

The knowledge base basically consists of a database and a rule base.

- The **database** provides the necessary information for proper functioning of the fuzzification module, the rule base and the defuzzification module.
- The information in the database includes :
  - Fuzzy MFs for the input and output control variables
  - The physical domains of the actual problems and their normalized values along with the scaling factors.

4.8.1 Steps in Designing FLC

Following are the steps involved in designing FLC

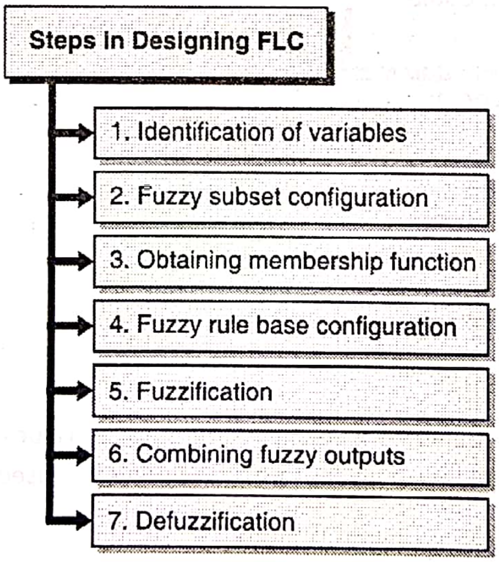

Fig. 4.8.2 Steps in designing FLC

1. **Identification of variables :** Here, the input, output and state variables must be identified of the plant which is under consideration.
2. **Fuzzy subset configuration :** The universe of information is divided into number of fuzzy subsets and each subset is assigned a linguistic label. Always make sure that these fuzzy subsets include all the elements of universe.
3. **Obtaining membership function :** Now obtain the membership function for each fuzzy subset that we get in the above step.
4. **Fuzzy rule base configuration :** Now formulate the fuzzy rule base by assigning relationship between fuzzy input and output.
5. **Fuzzification :**The fuzzification process is initiated in this step.

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6. **Combining fuzzy outputs :** By applying fuzzy approximate reasoning, locate the fuzzy output and merge them.
7. **Defuzzification :** Finally, initiate defuzzification process to form a crisp output

4.8.2 Advantages of FLSs

- It uses very simple Mathematical concepts for reasoning.
- An FLS can be modified by just adding or deleting rules due to flexibility of fuzzy logic.
- Fuzzy logic Systems can take imprecise, distorted, noisy input information.
- FLSs are easy to construct and understand.
- Fuzzy logic is a solution to complex problems in all fields of life, including medicine, as it resembles human reasoning and decision making.

4.8.3 Disadvantages of FLSs

- There is no systematic approach to fuzzy system designing.
- They are understandable only when simple.
- They are suitable for the problems which do not need high accuracy.

4.9 Solved Problems

Ex. 4.9.1 : Model the following as fuzzy set using suitable membership function. "Numbers close to 6". **MU - Dec. 12, Dec. 13, Dec. 14, 6 Marks**

Soln. : Let universe of discourse be the set of all integer numbers.

$$X = \text{Integers}$$

Then fuzzy set "Numbers close to 6" can be defined as

$$\mu_{\bar{A}}(x) = \frac{1}{1+(x-6)^2}$$

Fig. P. 4.9.1 shows the plot of degree of membership of each element.

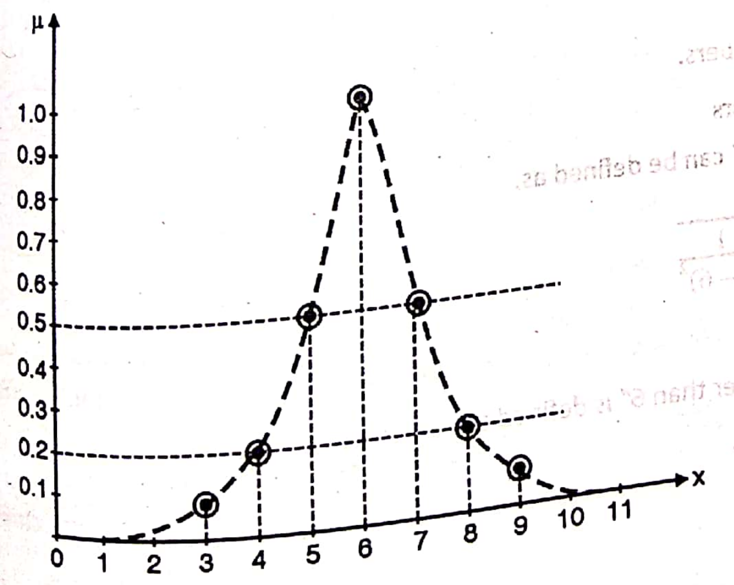

Fig. P. 4.9.1 : Plot of $x \rightarrow \mu_{\bar{A}}(x)$

Table P. 4.9.1: x and corresponding $\mu_{\bar{A}}(x)$

x	$\mu_{\bar{A}}(x)$
2	0.05
3	0.1
4	0.2
5	0.5
6	1
7	0.5
8	0.2
9	0.1
10	0.05

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Ex. 4.9.2 : Model the following fuzzy set using the suitable fuzzy membership function "Number close to 10".

Soln. :

Let X be the universe of discourse.

$$X = \text{Integers}$$

Then fuzzy set "Number close to 10" can be defined as,

$$\mu_{\tilde{A}}(x) = \frac{1}{1+(x-10)^2}$$

Fig. P. 4.9.2 shows the plot of degree of membership for each element.

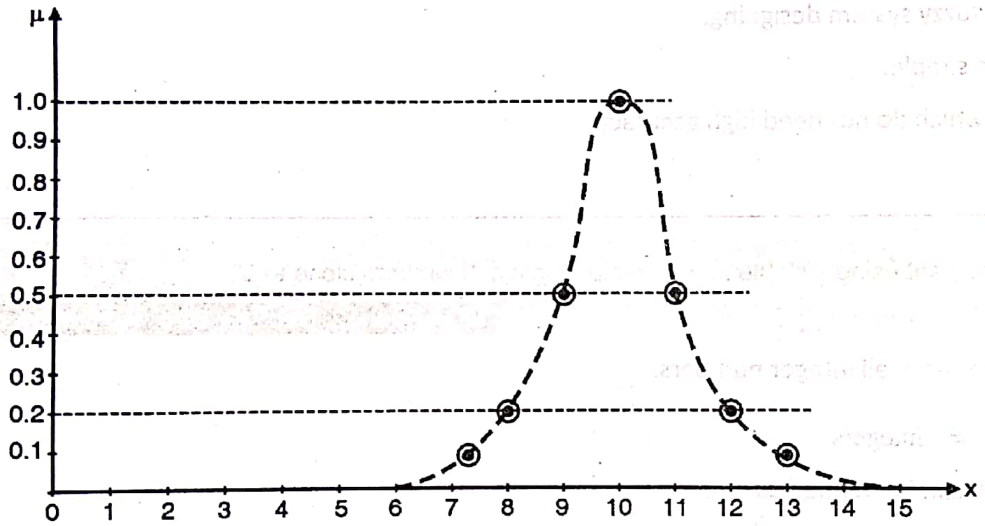

Fig. P. 4.9.2 : Plot of $x \rightarrow \mu_{\tilde{A}}(x)$

Table P. 4.9.2 : x and corresponding $\mu_{\tilde{A}}(x)$

X	$\mu_{\tilde{A}}(x)$
5	0.03
6	0.05
7	0.1
8	0.2
9	0.5
10	1
11	0.5
12	0.2
13	0.1
14	0.05
15	0.03

Ex. 4.9.3 : Model the following fuzzy set using suitable membership function. "Integer number considerably larger than 6".

Soln. :

Here universe of discourse is set of all integer numbers.

$$X = \text{Integers}$$

Then fuzzy set for "Number considerably larger than 6" can be defined as.

$$\mu_{\tilde{A}}(x) = \frac{1}{1+\dfrac{1}{(x-6)^2}}$$

Fig. P.4.9.3 shows the plot of $x \rightarrow \mu_A(x)$

So membership function for "Number considerably larger than 6" is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x \leq 6 \\ \dfrac{1}{1+\dfrac{1}{(x-6)^2}} & , x > 6 \end{cases}$$

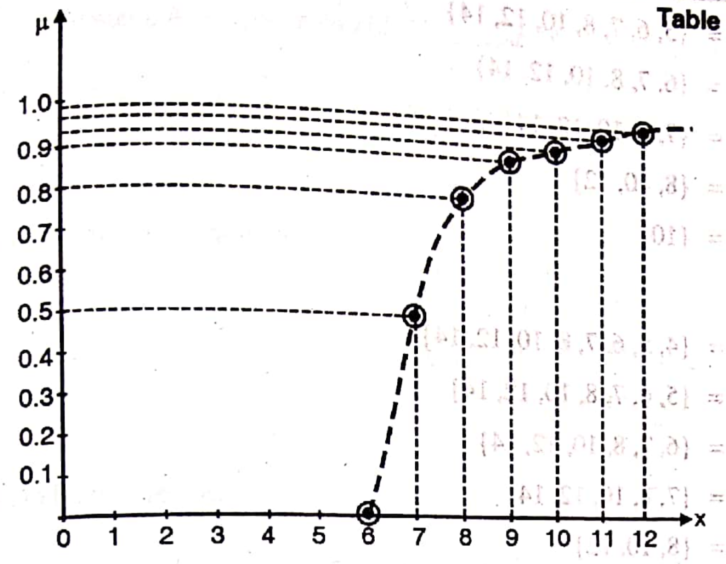

Fig. P. 4.9.3 : Plot of $x \rightarrow \mu_{\tilde{A}}(x)$

Table P. 4.9.3 : x and corresponding $\mu_{\tilde{A}}(x)$

x	$\mu_{\tilde{A}}(x)$
6	0
7	0.5
8	0.8
9	0.90
10	0.94
11	0.96
12	0.97
13	0.98

Ex. 4.9.4 : Determine all α-level sets and strong α-level sets for the following fuzzy set

A = {(1, 0.2), (2, 0.5), (3, 0.8), (4, 1), (5, 0.7), (6, 0.3)}

MU - Dec. 13. Dec. 15. 5/6 Marks

Soln. :

The following are α-level sets

$$A_{0.2} = \{1, 2, 3, 4, 5, 6\}$$
$$A_{0.3} = \{2, 3, 4, 5, 6\}$$
$$A_{0.5} = \{2, 3, 4, 5\}$$
$$A_{0.7} = \{3, 4, 5\}$$
$$A_{0.8} = \{3, 4\}$$
$$A_{1} = \{4\}$$

Following are strong α-level sets.

$$A_{0.2'} = \{2, 3, 4, 5, 6\}$$
$$A_{0.3'} = \{2, 3, 4, 5\}$$
$$A_{0.5'} = \{3, 4, 5\}$$
$$A_{0.7'} = \{3, 4\}$$
$$A_{0.8'} = \{4\}$$
$$A_{1'} = \phi$$

Ex. 4.9.5 : Find out all α-level sets and strong α-level sets for the following fuzzy set

$\tilde{A}$ = {(3, 0.1), (4, 0.2), (5, 0.3), (6, 0.4), (7, 0.6), (8, 0.8), (10, 1), (12, 0.8), (14, 0.6)}

Soln. :

α-level sets

$$A_{0.1} = \{3, 4, 5, 6, 7, 8, 10, 12, 14\}$$
$$A_{0.2} = \{4, 5, 6, 7, 8, 10, 12, 14\}$$

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$$A_{0.3} = \{5, 6, 7, 8, 10, 12, 14\}$$
$$A_{0.4} = \{6, 7, 8, 10, 12, 14\}$$
$$A_{0.6} = \{7, 8, 10, 12, 14\}$$
$$A_{0.8} = \{8, 10, 12\}$$
$$A_{1} = \{10\}$$

Strong α-level sets

$$A_{0.1'} = \{4, 5, 6, 7, 8, 10, 12, 14\}$$
$$A_{0.2'} = \{5, 6, 7, 8, 10, 12, 14\}$$
$$A_{0.3'} = \{6, 7, 8, 10, 12, 14\}$$
$$A_{0.4'} = \{7, 8, 10, 12, 14\}$$
$$A_{0.6'} = \{8, 10, 12\}$$
$$A_{0.8'} = \{10\}$$
$$A_{1'} = \phi$$

Ex. 4.9.6 : A realtor wants to classify the houses he offers to his clients. One indicator of comfort of these houses is the number of bedrooms in them. Let the available types of houses be represented by the following set.

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

The houses in this set are specified by the number of bedrooms in a house. Describe comfortable house for 4-person family" using a fuzzy set.

Soln. :

The fuzzy set for "comfortable type of house for a 4-person family" may be described as,

$$\tilde{A} = \{(1, 0.2), (2, 0.5), (3, 0.8), (4, 1), (5, 0.7), (6, 0.3)\}$$

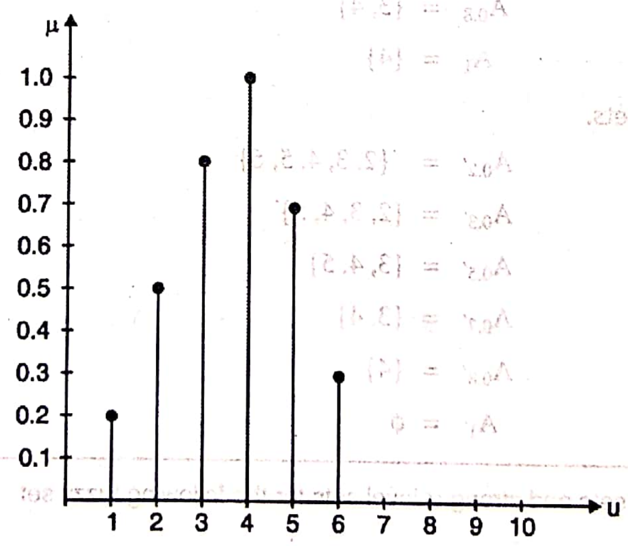

Fig. P. 4.9.6: Plot of u $\rightarrow \mu_A(u)$

Ex. 4.9.7 : Assume $\tilde{A}$ = "x considerably larger than 10" and $\tilde{B}$ = "x approximately 11" characterized by

$\tilde{A} = \{x, \mu_{\tilde{A}}(x) \mid x \in X\}$ Draw the plot for both the sets and show $\tilde{A} \cup \tilde{B}$ and $\tilde{A} \cap \tilde{B}$ in a plot.

Soln. :

Fuzzy set $\tilde{A}$ can be defined as,

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x \le 10 \\ \dfrac{1}{1+\dfrac{1}{(x-10)^2}} & , x > 10 \end{cases}$$

Set $\tilde{B}$ can be defined as,

$$\mu_{\tilde{B}}(x) = \frac{1}{1+(x-11)^2}$$

Then,

$$\mu_{\tilde{A} \cap \tilde{B}}(x) = \begin{cases} \min[(1+(x-10)^{-2})^{-1}, (1+(x-11)^2)^{-1}] & , x > 10 \\ 0 & , x \le 10 \end{cases}$$

That is, intersection operation on fuzzy set $\tilde{A}$ and $\tilde{B}$ represents a new fuzzy set "x considerably larger than 10 and approximately 11".

and

$$\mu_{\tilde{A} \cup \tilde{B}}(x) = \{\max[(1+(x-10)^{-2})^{-1}, (1+(x-11)^2)^{-1}], x \in X$$

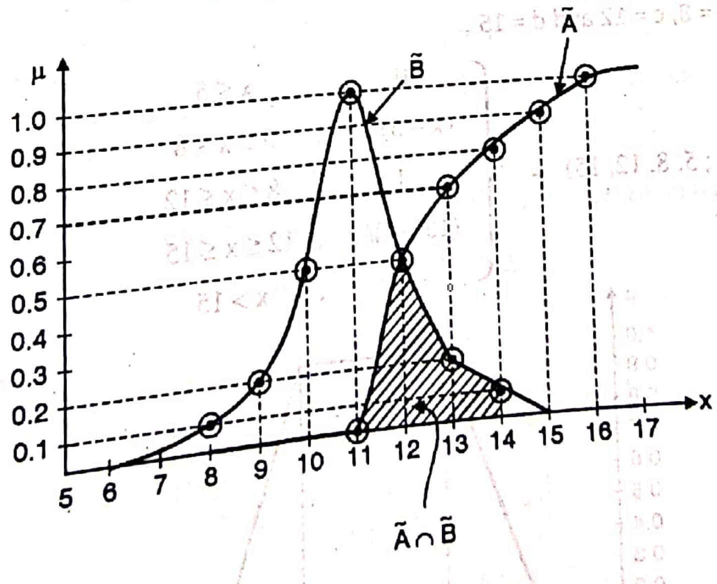

Fig. P. 4.9.7 : Plot of $\tilde{A} \cap \tilde{B}$

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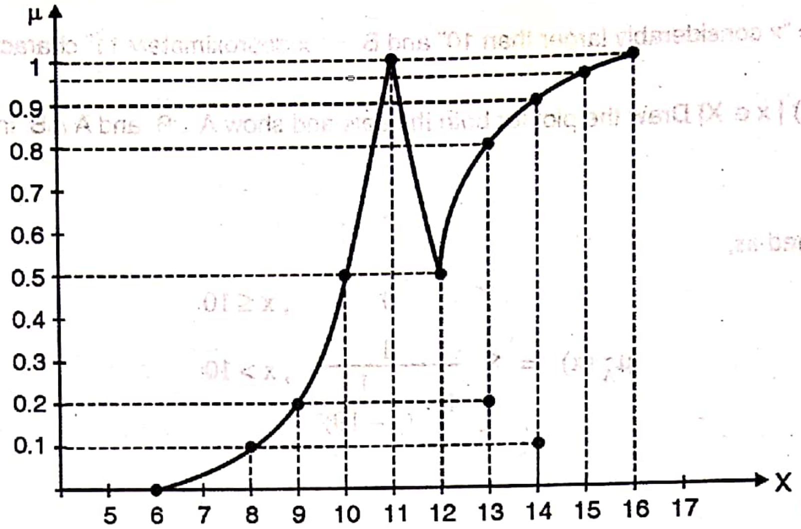

Fig. P. 4.9.7(a) : Plot of $\tilde{A} \cup \tilde{B}$

Ex. 4.9.8 : Model the following as fuzzy set using trapezoidal membership function "Number close to 10".

Soln. :

"Number close to 10" can be represented by,

$$\text{Trapezoid}(x\,;a,b,c,d) = \begin{cases} 0 & , \quad x \le a \\ (x-a)/(b-a) & , \quad a \le x \le b \\ 1 & , \quad b \le x \le c \\ (d-x)/(d-c) & , \quad c \le x \le d \\ 0 & , \quad x > 15 \end{cases}$$

We have selected a = 5, b = 8, c = 12 and d = 15

$$\text{Trapezoid}(x\,;5,8,12,15) = \begin{cases} 0 & , \quad x \le 5 \\ (x-5)/3 & , \quad 5 \le x \le 8 \\ 1 & , \quad 8 \le x \le 12 \\ (15-x)/3 & , \quad 12 \le x \le 15 \\ 0 & , \quad x > 15 \end{cases}$$

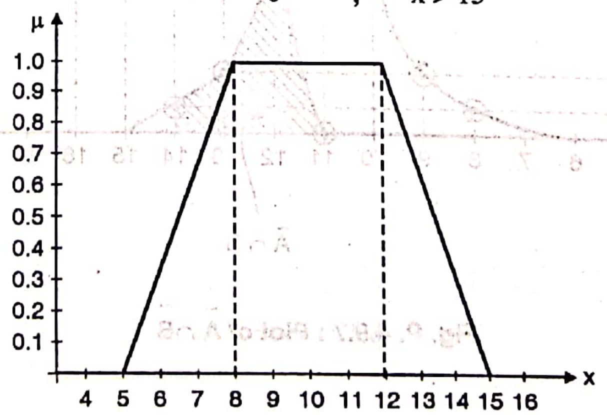

Fig. P. 4.9.8 : Trapezoidal MF for "Number close to 10"

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Ex. 4.9.9 : Let A = {a_1, a_2} , B = {b_1, b_2, b_3}, C = {c_1, c_2}. Let R be a relation from A to B defined by matrix.

	b_1	b_2	b_3
a_1	0.4	0.5	0
a_2	0.2	0.8	0.2

Let S be a relation from B to C defined by matrix.

	c_1	c_2
b_1	0.2	0.7
b_2	0.3	0.8
b_3	1	0

Find : (1) Max-min composition of R and S. (2) Max-product composition of R and S.

Soln. :

(1) Max-min composition

T = ROS =

T	c_1	c_2
a_1	0.3	0.5
a_2	0.3	0.8

$$T(a_1, c_1) = \max(\min(0.4, 0.2), \min(0.5, 0.3), \min(0, 1))$$
$$= \max(0.2, 0.3, 0) = 0.3$$
$$T(a_1, c_2) = \max(\min(0.4, 0.7), \min(0.5, 0.8), \min(0, 0))$$
$$= \max(0.4, 0.5, 0) = 0.5$$
$$T(a_2, c_1) = \max(\min(0.2, 0.2), \min(0.8, 0.3), \min(0.2, 1))$$
$$= \max(0.2, 0.3, 0.2) = 0.3$$
$$T(a_2, c_2) = \max(\min(0.2, 0.7), \min(0.8, 0.8), \min(0.2, 0))$$
$$= \max(0.2, 0.8, 0) = 0.8$$

(2) Max-product composition

T = ROS =

T	c_1	c_2
a_1	0.15	0.4
a_2	0.24	0.64

$$T(a_1, c_1) = \max(0.4 \times 0.2, 0.5 \times 0.3, 0 \times 1)$$
$$= \max(0.08, 0.15, 0) = 0.15$$
$$T(a_1, c_2) = \max(0.4 \times 0.7, 0.5 \times 0.8, 0 \times 0)$$
$$= \max(0.28, 0.40, 0) = 0.4$$
$$T(a_2, c_1) = \max(0.2 \times 0.2, 0.8 \times 0.3, 0.2 \times 1)$$

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$$= \max(0.04, 0.24, 0.2) = 0.24$$

$$T(a_2, c_2) = \max(0.2 \times 0.7, 0.8 \times 0.8, 0.2 \times 0)$$

$$= \max(0.14, 0.64, 0) = 0.64$$

Ex. 4.9.10 : High speed rail monitoring devices sometimes make use of sensitive sensors to measure the deflection of the earth when a rail car passes. These deflections are measured with respect to some distance from the rail car and, hence are actually very small angles measured in micro-radians. Let a universe of deflection be A = [1, 2, 3, 4] where A is the angle in micro-radians, and let a universe of distance be D = [1, 2, 5, 7] where D is distance in feet, suppose a relation between these two parameters has been determined as follows :

R =

	D_1	D_2	D_3	D_4
A_1	1	0.3	0.1	0
A_2	0.2	1	0.3	0.1
A_3	0	0.7	1	0.2
A_4	0	0.1	0.4	1

Now let a universe of rail car weights be W = [1, 2], where W is the weight in units of 100,000 pounds. Suppose the fuzzy relation of W to A is given by,

S =

	W_1	W_2
A_1	1	0.4
A_2	0.5	1
A_3	0.3	0.1
A_4	0	0

Using these two relations, find the relation $R^T oS = T$.

(a) Using max-min composition.

(b) Using max-product composition.

Soln. :

First find R^T.

R^T =

	A_1	A_2	A_3	A_4
D_1	1	0.2	0	0
D_2	0.3	1	0.7	0.1
D_3	0.1	0.3	1	0.4
D_4	0	0.1	0.2	1

and S =

	W_1	W_2
A_1	1	0.4
A_2	0.5	1
A_3	0.3	0.1
A_4	0	0

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(a) Using max-min composition

T =

	W_1	W_2
D_1	1	0.4
D_2	0.5	1
D_3	0.3	0.3
D_4	0.2	0.1

$$T(D_1, W1) = \max(1, 0.2, 0, 0) = 1$$
$$T(D_1, W_2) = \max(0.4, 0.2, 0, 0) = 0.4$$
$$T(D_2, W_1) = \max(0.3, 0.5, 0.3, 0) = 0.5$$
$$T(D_2, W_2) = \max(0.3, 1, 0.1, 0) = 1$$
$$T(D_3, W_1) = \max(0.1, 0.3, 0.3, 0) = 0.3$$
$$T(D_3, W_2) = \max(0.1, 0.3, 0.1, 0) = 0.3$$
$$T(D_4, W_1) = \max(0, 0.1, 0.2, 0) = 0.2$$
$$T(D_4, W_2) = \max(0, 0.1, 0.1, 0) = 0.1$$

(a) Using max product composition

T =

	W_1	W_2
D_1	1	0.4
D_2	0.5	1
D_3	0.3	0.3
D_4	0.06	0.1

$$T(D_1, W_1) = \max(1 \times 1, 0.2 \times 0.5, 0 \times 0.3, 0 \times 0) = \max(1, 0.10, 0, 0) = 1$$
$$T(D1, W2) = \max(1 \times 0.4, 0.2 \times 1, 0 \times 0.1, 0 \times 0)$$
$$= \max(0.4, 0.2, 0, 0) = 0.4$$
$$T(D2, W1) = \max(0.3 \times 1, 1 \times 0.5, 0.7 \times 0.3, 0.1 \times 0)$$
$$= \max(0.3, 0.5, 0.21, 0) = 0.5$$
$$T(D2, W2) = \max(0.3 \times 0.4, 1 \times 1, 0.7 \times 0.1, 0.1 \times 0)$$
$$= \max(0.12, 1, 0.07, 0) = 1$$
$$T(D3, W1) = \max(0.1 \times 1, 0.3 \times 0.5, 1 \times 0.3, 0.4 \times 0)$$
$$= \max(0.1, 0.15, 0.3, 0) = 0.3$$
$$T(D3, W2) = \max(0.1 \times 0.4, 0.3 \times 1, 1 \times 0.1, 0.4 \times 0)$$
$$= \max(0.04, 0.3, 0.1, 0) = 0.3$$
$$T(D4, W1) = \max(0 \times 1, 0.1 \times 0.5, 0.2 \times 0.3, 1 \times 0)$$
$$= \max(0, 0.05, 0.06, 0) = 0.06$$
$$T(D4, W2) = \max(0 \times 0.4, 0.1 \times 1, 0.2 \times 0.1, 1 \times 0)$$
$$= \max(0, 0.1, 0.02, 0) = 0.1$$

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Ex. 4.9.11 : Model the following fuzzy set using trapezoidal membership function. "Middle age". MU - May 13. 5 Marks

Soln. :

Let X be a reasonable age interval of human being.

$$X = \{0, 1, 2, 3, \ldots, 100\}$$

Then a fuzzy set "Middle age" can be represented using Trapezoidal MF as follows.

$$\text{Trapezoid}(x; 30, 40, 60, 70) = \begin{cases} 0, & x \leq 30 \\ (x-30)/10, & 30 \leq x \leq 40 \\ 1, & 40 \leq x \leq 60 \\ (70-x)/10, & 60 \leq x \leq 70 \\ 0, & x > 70 \end{cases}$$

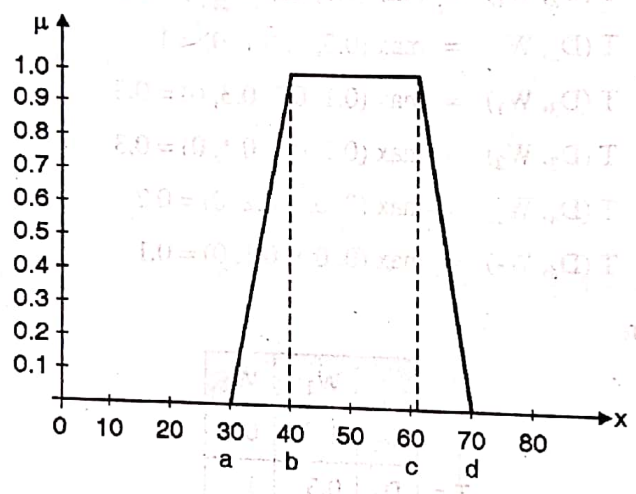

Fig. P. 4.9.11 : Trapezoidal MF for "Middle age"

Ex. 4.9.12 : Represent the set of old people as a fuzzy set using appropriate membership function.

Soln. :

Let X = (0,120) set of all possible ages.

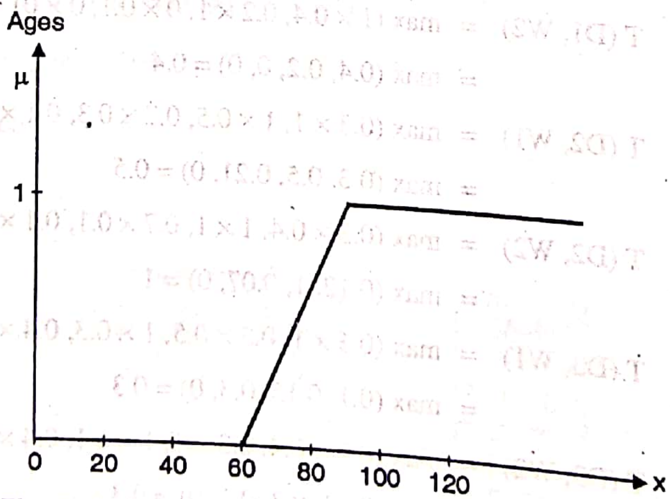

Fig. P. 4.9.12 : Membership function for "old people"

$$\mu_{old}(x) = \begin{cases} 0, & 0 \leq x \leq 60 \\ (x-60)/20, & 60 \leq x \leq 80 \\ 1, & x \geq 80 \end{cases}$$

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Ex. 4.9.13 : Develop graphical representation of membership function to describe linguistic variables "cold", "warm" and "hot". The temperature ranges from 0°c to 100°c. Also show plot for "cold and warm" and "warm or hot" temperature.

Soln. :

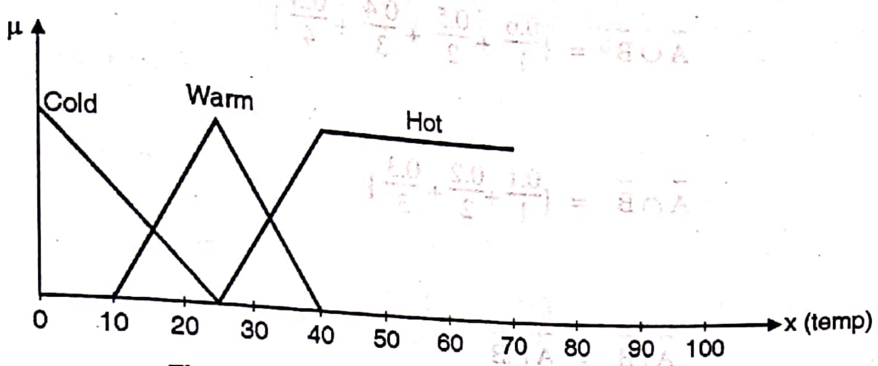

Fig. P. 4.9.13 : MF for cold, warm and hot temp.

a. Plot for "cold and warm"

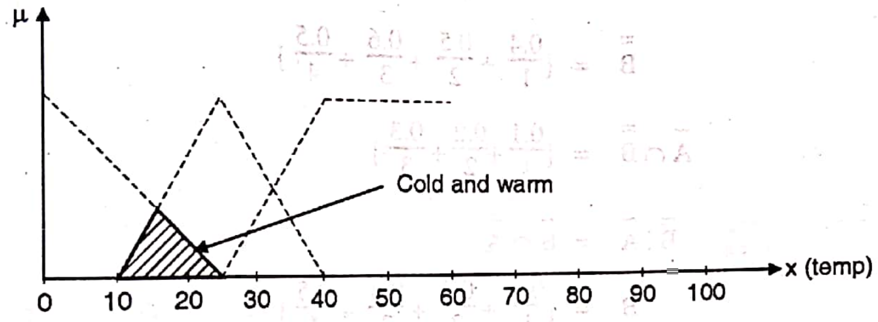

Fig. P. 4.9.13 (a) : MF for "cold and warm"

b. Plot for "warm or hot"

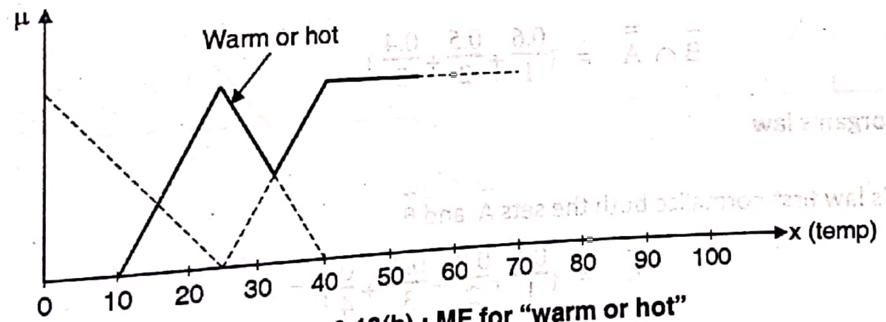

Fig. P. 4.9.13(b) : MF for "warm or hot"

Ex. 4.9.14 : Given two fuzzy sets.

$$\tilde{A} = \left\{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3}\right\}$$

$$\tilde{B} = \left\{\frac{0.6}{1} + \frac{0.5}{2} + \frac{0.4}{3} + \frac{0.5}{4}\right\}$$

Perform following operations on $\tilde{A}$ and $\tilde{B}$.

(1) Union

(2) Intersection

(3) Set difference

(4) Verify Demorgan's law.

Soln. :

1. **Union**

$$\tilde{A} \cup \tilde{B} = \{\frac{0.6}{1} + \frac{0.5}{2} + \frac{0.4}{3} + \frac{0.5}{4}\}$$

2. **Intersection**

$$\tilde{A} \cap \tilde{B} = \{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3}\}$$

3. **Set difference**

$$\tilde{A} \mid \tilde{B} = \tilde{A} \cap \bar{\tilde{B}}$$

$$\tilde{A} = \{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3}\}$$

$$\bar{\tilde{B}} = \{\frac{0.4}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\}$$

$$\tilde{A} \cap \bar{\tilde{B}} = \{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3}\}$$

$$\tilde{B} \mid \tilde{A} = \tilde{B} \cap \bar{\tilde{A}}$$

$$\tilde{B} = \{\frac{0.6}{1} + \frac{0.5}{2} + \frac{0.4}{3} + \frac{0.5}{4}\}$$

$$\bar{\tilde{A}} = \{\frac{0.9}{1} + \frac{0.8}{2} + \frac{0.7}{3}\}$$

$$\tilde{B} \cap \bar{\tilde{A}} = \{\frac{0.6}{1} + \frac{0.5}{2} + \frac{0.4}{3}\}$$

4. **Verification of Demorgan's law**

To verify Demorgan's law first normalize both the sets $\tilde{A}$ and $\tilde{B}$.

$$\tilde{A} = \{\frac{0.1}{1} + \frac{0.2}{2} + \frac{0.3}{3} + \frac{0}{4}\}$$

$$\tilde{B} = \{\frac{0.6}{1} + \frac{0.5}{2} + \frac{0.4}{3} + \frac{0.5}{4}\}$$

a. $\overline{\tilde{A} \cup \tilde{B}} = \bar{\tilde{A}} \cap \bar{\tilde{B}}$

L.H.S : $\overline{\tilde{A} \cup \tilde{B}}$

$$\tilde{A} \cup \tilde{B} = \{\frac{0.6}{1} + \frac{0.5}{2} + \frac{0.4}{3} + \frac{0.5}{4}\}$$

$$\overline{\tilde{A} \cup \tilde{B}} = \{\frac{0.4}{1} + \frac{0.5}{2} + \frac{0.6}{3} + \frac{0.5}{4}\} \quad ...(1)$$

R.H.S : $\bar{\tilde{A}} \cap \bar{\tilde{B}}$

$$\bar{\tilde{A}} = \left\{\frac{0.9}{1} + \frac{0.8}{2} + \frac{0.7}{3} + \frac{1}{4}\right\}$$

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$$\overline{\overline{B}} = \left\{\frac{0.4}{1}+\frac{0.5}{2}+\frac{0.6}{3}+\frac{0.5}{4}\right\}$$

$$\overline{\overline{A}} \cap \overline{\overline{B}} = \left\{\frac{0.4}{1}+\frac{0.5}{2}+\frac{0.6}{3}+\frac{0.5}{4}\right\} \quad ...(2)$$

Since L.H.S. = R.H.S. hence proved.

b. $\overline{\tilde{A}\cap\tilde{B}} = \overline{\overline{A}} \cup \overline{\overline{B}}$

L.H.S. : $\overline{\tilde{A}\cap\tilde{B}}$

$$\tilde{A}\cap\tilde{B} = \left\{\frac{0.1}{1}+\frac{0.2}{2}+\frac{0.3}{3}+\frac{0}{4}\right\}$$

$$\overline{\tilde{A}\cap\tilde{B}} = \left\{\frac{0.9}{1}+\frac{0.8}{2}+\frac{0.7}{3}+\frac{1}{4}\right\} \quad ...(1)$$

R.H.S. : $\overline{\overline{A}} \cup \overline{\overline{B}}$

$$\overline{\overline{A}} \cup \overline{\overline{B}} = \left\{\frac{0.9}{1}+\frac{0.8}{2}+\frac{0.7}{3}+\frac{1}{4}\right\} \quad ...(2)$$

Since L.H.S. = R.H.S. hence proved.

Ex. 4.9.15 : For the given membership functionas shown in Fig. P.4.9.15, determine the defuzzified output value by any two methods. **MU - May 13, 10 Marks**

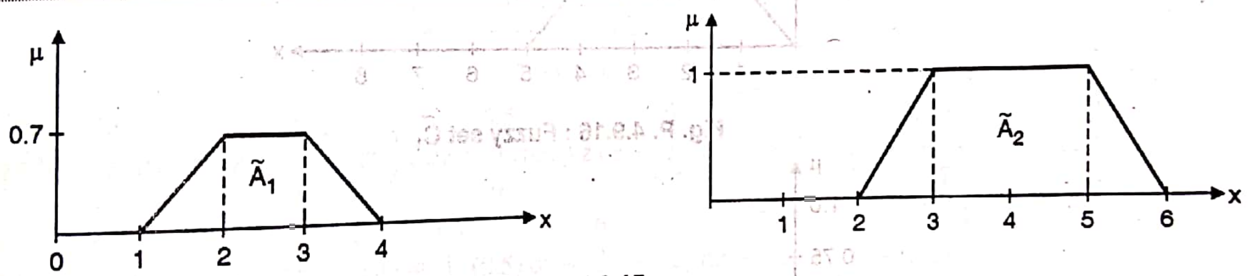

Fig. P. 4.9.15

Soln. :

1. **Weighted average method**

In weighted average method we first find the centre of each individual furzy set

Centre of $\tilde{A}_1$ = 2.5 Centre of $\tilde{A}_2$ = 4.0

Next we find the membership value at the centre

Membership value of centre of $\tilde{A}_1$ = 0.7

Membership value of centre of $\tilde{A}_2$ = 1

$$x^* = \frac{(0.7 * 2.5) + (1 \times 4.0)}{0.7+1} = \frac{1.75+4}{1.7} = 3.38$$

2. **Centre of sum method**

First find the area of each individual curve.

We know that, Area of Trapezoid = $\frac{[\text{(Sum of length of parallel lines)} \times \text{(distance between parallel lines)}]}{2}$

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$$\text{Therefore, Area of } \tilde{A}_1 = \frac{(1+3)*0.7}{2} = 1.4$$

$$\text{Similarly, Area of } \tilde{A}_2 = \frac{(2+4)*1}{2} = \frac{6}{2} = 3$$

Next, find center of each curve.

$$\text{Centre of } \tilde{A}_1 = 2.5$$

$$\text{Centre of } \tilde{A}_2 = 4$$

$$X^* = \frac{(2.5 \times 1.4) + (4 \times 3)}{1.4 + 3} = \frac{15.5}{4.4} = 3.52$$

Ex. 4.9.16 : Consider three fuzzy sets $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$ given below. Find defuzzified value using :

(1) mean of max (2) centroid

(3) centre of sum and (4) weighted avg. method.

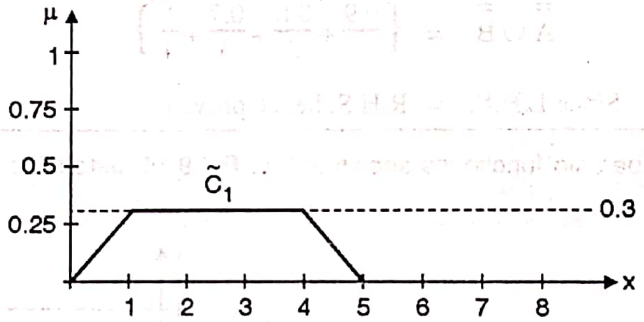

Fig. P. 4.9.16 : Fuzzy set $\tilde{C}_1$

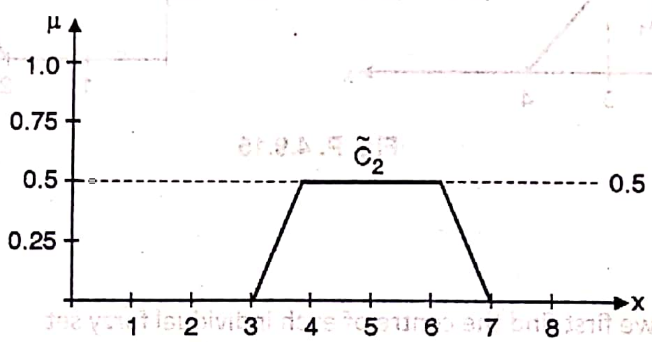

Fig. P. 4.9.16 (a) : Fuzzy set $\tilde{C}_2$

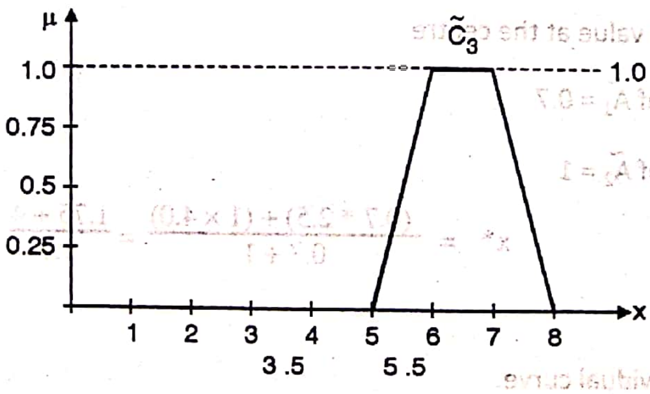

Fig. P. 4.9.16(b) : Fuzzy set $\tilde{C}_3$

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Soln. :

First find aggregation of all MFs (union).

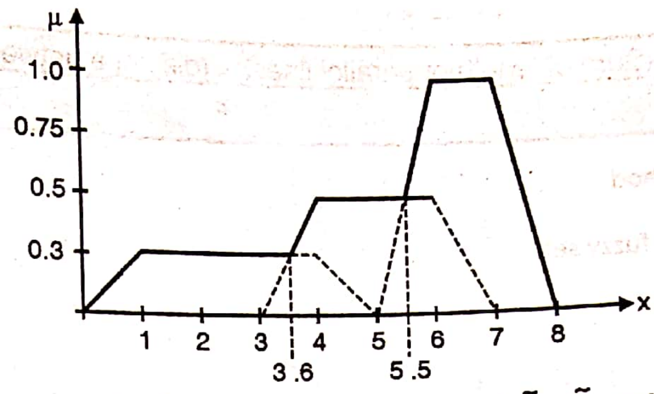

Fig. P. 4.9.16(c) : Aggregated fuzzy set of $\tilde{C}_1$, $\tilde{C}_2$ and $\tilde{C}_3$

1) **Using Mean of max**

Since $\tilde{C}_3$ is the maximizing MF, we take the mean (average) of all the elements having maximum membership value in $\tilde{C}_3$

$$x^* = \frac{6+7}{2} = \frac{13}{2} = 6.5$$

2) **Using Centroid method**

$$z^* = \frac{\int \mu_\beta (z) \bullet z dz}{\int \mu_\beta (z) dz}$$

$$= \left[\int_0^1 (0.3z)\, z dz + \int_1^{3.6} (0.3z)\, dz + \int_{3.6}^{4} \left(\frac{z-3}{2}\right) z\, dz + \int_4^{5.5} (0.5)\, z\, dz + \int_{5.5}^{6} (z-5)\, z\, dz + \int_6^7 z\, dz + \int_7^8 (8-z)\, z\, dz\right]$$

$$+ \left[\int_0^1 (0.3z)\, dz + \int_1^{3.6} (0.3)\, dz + \int_{3.6}^{4} \left(\frac{z-3}{2}\right) dz + \int_4^{5.5} (0.5)\, dz + \int_{5.5}^{6} (z-5)\, dz + \int_6^7 dz + \int_7^8 (8-z)\, dz\right]$$

$= 4.9$

3) **Using centre of sum method**

First find area of each individual fuzzy set

Area of $\tilde{C}_1$ = 1.2

Area of $\tilde{C}_2$ = 1.5

Area of $\tilde{C}_3$ = 2

Then find centre of each individual fuzzy set.

Centre of $\tilde{C}_1$ = 2.5

Centre of $\tilde{C}_2$ = 5

Centre of $\tilde{C}_3$ = 6.5

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$$x^* = \frac{(2.5 \times 1.2) + (1.5 \times 5) + (2 \times 6.5)}{1.2 + 1.5 + 2}$$

$$\therefore \quad x^* = 5$$

Note: Area of Trapezoid = $\frac{[\text{(Sum of length of parallel lines)} \times \text{(distance between parallel lines)}]}{2}$

4) Using weighted average method

Find centre of each individual fuzzy set.

Centre of $\tilde{C}_1$ = 2.5

Centre of $\tilde{C}_2$ = 5

Centre of $\tilde{C}_3$ = 6.5

Find membership values of these centres.

Membership value of centre of $\tilde{C}_1$ = 0.3

Membership value of centre of $\tilde{C}_2$ = 0.5

Membership value of centre of $\tilde{C}_3$ = 1

$$x^* = \frac{(2.5 \times 0.3) + (5 \times 0.5) + (6.5 \times 1)}{0.3 + 0.5 + 1} = \frac{9.75}{1.8}$$

$$\therefore \quad x^* = 5.146$$

Ex. 4.9.17 : Given fuzzy set.

$$\tilde{A} = \left\{ \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.8}{3} + \frac{1}{4} + \frac{1}{5} + \frac{0.8}{6} \right\}$$

Find core and support of fuzzy set $\tilde{A}$

Soln. :

Core of $\tilde{A}$ = { 4, 5 } $\rightarrow$ Membership value equal to 1

Support of $\tilde{A}$ = {1, 2, 3, 4, 5, 6} $\rightarrow$ Membership value > 0

Ex. 4.9.18 : Consider following two fuzzy sets

$$\tilde{A} = \left\{ \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} + \frac{0.5}{4} \right\} \qquad \tilde{B} = \left\{ \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.2}{3} + \frac{1}{4} \right\}$$

Find : 1) Algebraic sum 2) Algebraic product

3) Bounded sum 4) Bounded difference

Soln. :

1) Algebraic sum

$$\mu_{\tilde{A} + \tilde{B}}(x) = [\mu_A(x) + \mu_B(x)] - [\mu_A(x) \cdot \mu_B(x)]$$

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$$= \left\{\frac{0.3}{1}+\frac{0.5}{2}+\frac{0.6}{3}+\frac{1.5}{4}\right\} - \left\{\frac{0.02}{1}+\frac{0.06}{2}+\frac{0.08}{3}+\frac{0.5}{4}\right\}$$

$$= \left\{\frac{0.28}{1}+\frac{0.44}{2}+\frac{0.52}{3}+\frac{1}{4}\right\}$$

2) Algebraic product

$$\mu_{\tilde{A}\cdot\tilde{B}}(x) = \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(x)]$$

$$= \left[\frac{0.02}{1}+\frac{0.06}{2}+\frac{0.08}{3}+\frac{0.5}{4}\right]$$

3) Bounded sum

$$\mu_{\tilde{A}\oplus\tilde{B}}(x) = \min[1, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x)]$$

$$= \min\left\{1, \left\{\frac{0.3}{1}+\frac{0.5}{2}+\frac{0.6}{3}+\frac{1.5}{4}\right\}\right\} = \left\{\frac{0.3}{1}+\frac{0.5}{2}+\frac{0.6}{3}+\frac{1}{4}\right\}$$

4) Bounded difference

$$\mu_{\tilde{A}\odot\tilde{B}}(x) = \max[0, \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)]$$

$$= \max\left\{0, \left\{\frac{0.1}{1}+\frac{0.1}{2}+\frac{0.2}{3}+\frac{-0.5}{4}\right\}\right\} = \left\{\frac{0.1}{1}+\frac{0.1}{2}+\frac{0.2}{3}+\frac{0}{4}\right\}$$

Ex. 4.9.19 : For given two fuzzy sets

$$\tilde{A} = \left\{\frac{0.3}{x_1}+\frac{0.7}{x_2}+\frac{1}{x_3}\right\} \text{ and } \tilde{B} = \left\{\frac{0.4}{y_1}+\frac{0.9}{y_2}\right\}$$

Perform Cartesian product.

Soln. :

For Cartesian product we consider min operator (denoted by $\wedge$)

$$\tilde{A}\times\tilde{B} = \left\{\frac{0.3\wedge 0.4}{(x_1, y_1)}+\frac{0.3\wedge 0.9}{(x_1, y_2)}+\frac{0.7\wedge 0.4}{(x_2, y_1)}+\frac{0.7\wedge 0.9}{(x_2, y_2)}+\frac{1\wedge 0.4}{(x_3, y_1)}+\frac{1\wedge 0.9}{(x_3, y_2)}\right\}$$

$$= \left\{\frac{0.3}{(x_1, y_1)}+\frac{0.3}{(x_1, y_2)}+\frac{0.4}{(x_2, y_1)}+\frac{0.7}{(x_2, y_2)}+\frac{0.4}{(x_3, y_1)}+\frac{0.9}{(x_3, y_2)}\right\}$$

It can be represented in a matrix form

$\tilde{A}\times\tilde{B}$ =

	y_1	y_2
x_1	0.3	0.3
x_2	0.4	0.7
x_3	0.4	0.9

Ex. 4.9.20 : Consider the following fuzzy sets

$$\text{Low temperature} = \left\{\frac{1}{131}+\frac{0.8}{132}+\frac{0.6}{133}+\frac{0.4}{134}+\frac{0.2}{135}+\frac{0}{136}\right\}$$

$$\text{High temperature} = \left\{\frac{0}{134}+\frac{0.2}{135}+\frac{0.4}{136}+\frac{0.6}{137}+\frac{0.8}{138}+\frac{1}{139}\right\}$$

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$$\text{High pressure} = \left\{ \frac{0.1}{400} + \frac{0.2}{600} + \frac{0.4}{700} + \frac{0.6}{800} + \frac{0.8}{900} + \frac{1}{1000} \right\}$$

Temperature ranges are 130° F to 140° F and pressure limit is 400 psi to 1000 psi. Find the following membership functions :

1) Temperature not very low.
2) Temperature not very high
3) Pressure slightly high.
4) Pressure very very high.

Soln. :

1. Temperature not very low

$$\text{Very low} = \text{low2}$$

$$= \left\{ \frac{1}{131} + \frac{0.64}{132} + \frac{0.36}{133} + \frac{0.16}{134} + \frac{0.04}{135} + \frac{0}{136} \right\}$$

$$\text{Not very low} = 1 - \text{very low}$$

$$= \left\{ \frac{0}{131} + \frac{0.36}{132} + \frac{0.64}{133} + \frac{0.84}{134} + \frac{0.96}{135} + \frac{1}{136} \right\}$$

2. Temperature not very high

$$\text{SVery high} = \text{high}^2$$

$$\therefore \text{Temp very high} = \left\{ \frac{0}{134} + \frac{0.04}{135} + \frac{0.16}{136} + \frac{0.36}{137} + \frac{0.64}{138} + \frac{1}{139} \right\}$$

$$\text{Temp not very high} = 1 - \text{very high}$$

$$= \left\{ \frac{1}{134} + \frac{0.96}{135} + \frac{0.84}{136} + \frac{0.64}{137} + \frac{0.36}{138} + \frac{0}{139} \right\}$$

3. Pressure slightly high

$$\text{Slightly high} = \text{dilation (high)} = \sqrt{\text{high}}$$

$$\therefore \text{Pressure slightly high} = (\text{high pressure})^{1/2}$$

$$= \left\{ \frac{\sqrt{0.1}}{400} + \frac{\sqrt{0.2}}{600} + \frac{\sqrt{0.4}}{700} + \frac{\sqrt{0.6}}{800} + \frac{\sqrt{0.8}}{900} + \frac{\sqrt{1}}{1000} \right\}$$

$$= \left\{ \frac{0.31}{400} + \frac{0.44}{600} + \frac{0.63}{700} + \frac{0.77}{800} + \frac{0.89}{900} + \frac{1}{1000} \right\}$$

4. Pressure very very high

$$\text{Very very high} = (\text{high})^4$$

$$\therefore \text{Pressure very very high} = (\text{high pressure})^4$$

$$= \left\{ \frac{0.0001}{400} + \frac{0.0016}{600} + \frac{0.025}{700} + \frac{0.12}{800} + \frac{0.40}{900} + \frac{1}{1000} \right\}$$

Ex. 4.9.21 : Two fuzzy relations are given by

$$R = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.6 & 0.3 \\ x_2 & 0.2 & 0.9 \end{array}$$

$$S = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline y_1 & 1 & 0.5 & 0.3 \\ y_2 & 0.8 & 0.4 & 0.7 \end{array}$$

Obtain fuzzy relation T as a max-min composition and max-product composition between the fuzzy relations.

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Soln. :

1. **Using max-min composition**

$$T(x1, z1) = \max(\min(0.6, 1), \min(0.3, 0.8))$$
$$= \max(0.6, 0.3)$$
$$= 0.6$$
$$T(x1, z2) = \max(\min(0.6, 0.5), \min(0.3, 0.4))$$
$$= \max(0.5, 0.3)$$
$$= 0.5$$
$$T(x2, z3) = \max(\min(0.6, 0.3), \min(0.3, 0.7))$$
$$= \max(0.3, 0.3)$$
$$= 0.3$$
$$T(x2, z1) = \max(\min(0.2, 1), \min(0.9, 0.8))$$
$$= \max(0.2, 0.8)$$
$$= 0.8$$
$$T(x2, z2) = \max(\min(0.2, 0.5), \min(0.9, 0.4))$$
$$= \max(0.2, 0.4)$$
$$= 0.4$$
$$T(x2, z3) = \max(\min(0.2, 0.3), \min(0.9, 0.7))$$
$$= \max(0.2, 0.7)$$
$$= 0.7$$
$$T = R \circ S \quad \begin{array}{c} X1 \\ X2 \end{array} \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix}$$

2. **Using max-product**

$$T(X1, Z1) = \max(0.6 \times 1, 0.3 \times 0.8)$$
$$= \max(0.6, 0.24)$$
$$= 0.6$$
$$T(X1, Z2) = \max(0.6 \times 0.5, 0.3 \times 0.4)$$
$$= \max(0.30, 0.12)$$
$$= 0.3$$
$$T(X1, Z3) = \max(0.6 \times 0.3, 0.3 \times 0.7)$$
$$= \max(0.18, 0.21)$$
$$= 0.21$$
$$T(X2, Z1) = \max(0.2 \times 1, 0.9 \times 0.8)$$
$$= \max(0.2, 0.72)$$
$$= 0.72$$
$$T(X2, Z2) = \max(0.2 \times 0.5, 0.9 \times 0.4)$$

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$$= \max\ (0.1, 0.36)$$

$$= 0.36$$

$$T(X2, Z3) = \max\ (0.2 \times 0.3\ ,\ 0.9 \times 0.7)$$

$$= \max\ (0.06\ ,\ 0.63)$$

$$= 0.63$$

$$T = R\,O\,S = \begin{array}{c} \\ X1 \\ X2 \end{array} \begin{array}{c} \begin{array}{ccc} Z1 & Z2 & Z3 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & 0.21 \\ 0.72 & 0.36 & 0.63 \end{bmatrix} \end{array}$$

Ex. 4.9.22 : Given two fuzzy relations R_1 and R_2 defined on $X \times Y$ and $Y \times Z$ respectively, where $X = \{1, 2, 3\}$, $Y = \{\alpha, \beta, \gamma, \delta\}$ and $z = \{a, b\}$. Find :

1. Max - min composition.
2. Max - product composition.

$$R_1 = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.5 \\ 0.4 & 0.3 & 0.7 & 0.9 \\ 0.6 & 0.1 & 0.8 & 0.2 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0.1 & 0.9 \\ 0.2 & 0.3 \\ 0.5 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}$$

Soln. :

1. Max - min composition

Here R_1 is defined on $X \times Y$ where $X = \{1, 2, 3\}$ and $Y = (\alpha, \beta, \gamma, \delta)$

$$R_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} \alpha & \beta & \gamma & \delta \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.5 \\ 0.4 & 0.3 & 0.7 & 0.9 \\ 0.6 & 0.1 & 0.8 & 0.2 \end{bmatrix} \end{array}$$

Similarly R_2 is defined on $Y \times Z$ where $Y = \{\alpha, \beta, \gamma, \delta\}$ and $Z = \{a, b\}$

So,

$$R_2 = \begin{array}{c} \\ \alpha \\ \beta \\ \gamma \\ \delta \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0.1 & 0.9 \\ 0.2 & 0.3 \\ 0.5 & 0.6 \\ 0.7 & 0.3 \end{bmatrix} \end{array}$$

So composition of R_1 and R_2 will be defined on $X \times Z$ where $X = \{1, 2, 3\}$ and $Z = \{a, b\}$.

Here $R_1 o\ R_2$ is 3×2 matrix

$$R_1 o\ R_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0.5 & 0.3 \\ 0.7 & 0.6 \\ 0.5 & 0.6 \end{bmatrix} \end{array}$$

We compute each element of $R_1 o\ R_2$ as follows :

$$\mu_{R1\ o\ R2}(1, a) = \max\ (\min\ (0.1, 0.1),\ \min\ (0.2, 0.2),\ \min(0.3, 0.5),\ \min\ (0.5, 0.7))$$

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$= \max(0.1, 0.2, 0.3, 0.5)$

$= 0.5$

$\mu_{R1 \circ R2}(1, b) = \max(\min(0.1, 0.9), \min(0.2, 0.3), \min(0.3, 0.6), \min(0.5, 0.3))$

$= \max(0.1, 0.2, 0.3, 0.3)$

$= 0.3$

$\mu_{R1 \circ R2}(2, a) = \max(\min(0.4, 0.1), \min(0.3, 0.2), \min(0.7, 0.5), \min(0.9, 0.7))$

$= \max(0.1, 0.2, 0.5, 0.7)$

$= 0.7$

$\mu_{R1 \circ R2}(2, b) = \max(\min(0.4, 0.9), \min(0.3, 0.3), \min(0.7, 0.6), \min(0.9, 0.3))$

$= \max(0.4, 0.3, 0.6, 0.3)$

$= 0.6$

$\mu_{R1 \circ R2}(3, a) = \max(\min(0.6, 0.1), \min(0.1, 0.2), \min(0.8, 0.5), \min(0.2, 0.7))$

$= \max(0.1, 0.1, 0.5, 0.2)$

$= 0.5$

$\mu_{R1 \circ R2}(3, b) = \max(\min(0.6, 0.9), \min(0.1, 0.3), \min(0.8, 0.6), \min(0.2, 0.3))$

$= \max(0.6, 0.1, 0.6, 0.2)$

$= 0.6$

2. Max - product composition

$$R_1 \circ R_2 = \begin{array}{c c} & \begin{array}{cc} a & b \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.35 & 0.18 \\ 0.63 & 0.42 \\ 0.40 & 0.54 \end{bmatrix} \end{array}$$

$\mu_{R1 \circ R2}(1, a) = \max(0.1 \times 0.1, 0.2 \times 0.2, 0.3 \times 0.5, 0.5 \times 0.7)$

$= \max(0.01, 0.04, 0.15, 0.35)$

$= 0.35$

$\mu_{R1 \circ R2}(1, b) = \max(0.1 \times 0.9, 0.2 \times 0.3, 0.3 \times 0.6, 0.5 \times 0.3)$

$= \max(0.09, 0.06, 0.18, 0.15)$

$= 0.18$

$\mu_{R1 \circ R2}(2, a) = \max(0.4 \times 0.1, 0.3 \times 0.2, 0.7 \times 0.5, 0.9 \times 0.7)$

$= \max(0.04, 0.06, 0.35, 0.63)$

$= 0.63$

$\mu_{R1 \circ R2}(2, b) = \max(0.4 \times 0.9, 0.3 \times 0.3, 0.7 \times 0.6, 0.9 \times 0.3)$

$= \max(0.36, 0.09, 0.42, 0.27)$

$= 0.42$

$\mu_{R1 \circ R2}(3, a) = \max(0.6 \times 0.1, 0.1 \times 0.2, 0.8 \times 0.5, 0.2 \times 0.7)$

$= \max(0.06, 0.02, 0.40, 0.14)$

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$$= 0.40$$

$$\mu_{R1 \circ R2}(3, b) = \max(0.6 \times 0.9, 0.1 \times 0.3, 0.8 \times 0.6 \times 0.2 \times 0.3)$$

$$= \max(0.54, 0.03, 0.48, 0.06) = 0.54$$

Ex. 4.9.23 : Let R be the relation that specifies the relationship between the 'color of a fruit' and 'grade of maturity'. Relation S specifies the relationship between 'grade of maturity' and 'taste of a fruit', where color, grade and taste of a fruit are characterized by crisp sets x, y, z respectively as follows.

X = {green, yellow, red}

Y = {verdant, half mature, mature}

Z = {sour, tasteless, sweet}

Consider following relations R and S and find the relationship between 'color and taste' of a fruit using

1. Max - min composition
2. Max - product composition

R	Verdant	Half mature	Mature
Green	1	0.5	0
Yellow	0.3	1	0.4
Red	0	0.2	1

S	Sour	Tasteless	Sweet
Verdant	1	0.2	0
Half mature	0.7	1	0.3
Mature	0	0.7	1

Soln. :

1. Max - min composition

T	Sour	Tasteless	Sweet
Green	1	0.5	0.3
Yellow	0.7	1	0.4
Red	0.2	0.7	1

$$T(\text{green, sour}) = \max(\min(1, 1), \min(0.5, 0.7), \min(0, 0))$$

$$= \max(1, 0.5, 0)$$

$$= 1$$

$$T(\text{green, tasteless}) = \max(\min(1, 0.2), \min(0.5, 1), \min(0, 0.7))$$

$$= \max(0.2, 0.5, 0)$$

$$= 0.5$$

$$T(\text{green, sweet}) = \max(\min(1, 0), \min(0.5, 0.3), \min(0, 1))$$

$$= \max(0, 0.3, 0)$$

$$= 0.3$$

$$T(\text{yellow, sour}) = \max(\min(0.3, 1), \min(1, 0.7), \min(0.4, 0.7))$$

$$= \max(0.3, 0.7, 0.4)$$

$$= 0.7$$

$$T(\text{yellow, tasteless}) = \max(\min(0.3, 0.2), \min(1, 1), \min(0.4, 0.7))$$

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$= \max(0.2, 1, 0.4)$

$= 1$

T (yellow, sweet) $= \max(\min(0.3, 0), \min(1, 0.3), \min(0.4, 1))$

$= \max(0, 0.3, 0.4)$

$= 0.4$

T (red, sour) $= \max(\min(0, 1), \min(0.2, 0.7), \min(1, 0))$

$= \max(0, 0.2, 0) = 0.2$

T (red, tasteless) $= \max(\min(0, 0.2), \min(0.2, 1), \min(1, 0.7))$

$= \max(0, 0.2, 0.7)$

$= 0.7$

T (red, sweet) $= \max(\min(0, 0), \min(0.2, 0.3), \min(1, 1))$

$= \max(0, 0.2, 1)$

$= 1$

2. Max - product composition

T	Sour	Tasteless	Sweet
Green	1	0.5	0.15
Yellow	0.7	1	0.4
Red	0.14	0.7	1

T (green, sour) $= \max(1 \times 1, 0.5 \times 0.7, 0 \times 0)$

$= \max(1, 0.35, 0)$

$= 1$

T (green, tasteless) $= \max(1 \times 0.2, 0.5 \times 1, 0 \times 0.7)$

$= \max(0.2, 0.5, 0)$

$= 0.5$

T (green, sweet) $= \max(1 \times 0, 0.5 \times 0.3, 0 \times 1)$

$= \max(0, 0.15, 0)$

$= 0.15$

T (yellow, sour) $= \max(0.3 \times 1, 1 \times 0.7, 0.4 \times 0.7)$

$= \max(0.3, 0.7, 0.28)$

$= 0.7$

T (yellow, tasteless) $= \max(0.3 \times 0.2, 1 \times 1, 0.4 \times 0.7)$

$= \max(0.06, 1, 0.28)$

$= 1$

T (yellow, sweet) $= \max(0.3 \times 0, 1 \times 0.3, 0.4 \times 1)$

$= \max(0, 0.3, 0.4)$

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$$= 0.4$$

$$T\text{ (red, sour)} = \max(0 \times 1, 0.2 \times 0.7, 1 \times 0)$$
$$= \max(0, 0.14, 0)$$
$$= 0.14$$

$$T\text{ (red, tasteless)} = \max(0 \times 0.2, 0.2 \times 1, 1 \times 0.7)$$
$$= \max(0, 0.2, 0.7)$$
$$= 0.7$$

$$T\text{ (red, sweet)} = \max(0 \times 0, 0.2 \times 0.3, 1 \times 1)$$
$$= \max(0, 0.06, 1)$$
$$= 1$$

4.10 Design of Controllers (Solved Problems)

Note : All the problems based on controller design have been solved using Mamdani Fuzzy Inference model and mean of max defuzzification method.

1. Domestic Shower Controller

Ex. 4.10.1 : Design a fuzzy controller to regulate the temperature of a domestic shower. Assume that:

(a) The temperature is adjusted by single mixer tap.

(b) The flow of water is constant.

(c) Control variable is the ratio of the hot to the cold water input.

The design should clearly mention the descriptors used for fuzzy sets and control variables, set of rules to generate control action and defuzzification. The design should be supported by figures where ever possible.

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

– Here **input** is the position of mixer tap. Assume that position of mixer tap is measured in degrees (0° to 180°). It represents opening of the mixer tap in degrees. 0° indicates tap is closed and 180°indicates tap is fully opened.

– **Output** is temperature of water according to the position of mixer tap. It is measured in°C. We take **five descriptors** for each input and output variables.

– Descriptors for input variable (position of mixer tap) are given below.

EL - Extreme Left

L - Left

C - Centre

R - Right

ER - Extreme Right

{ EL, L, C, R, ER }

– Descriptors for output variable (Temperature of water) are given below :

VCT - Very Cold Temperature

CT - Cold Temperature

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WT - Warm Temperature

HT - Hot Temperature

VHT - Very Hot Temperature

{ VCT, CT, WT, HT, VHT }

Step 2 : Define membership functions for input and output variables.

We use triangular MFs because of its simplicity.

1. Membership functions for input variable – position of mixer tap.

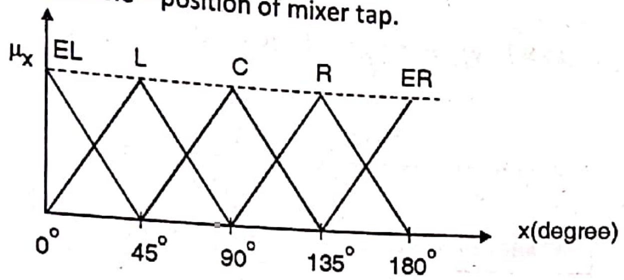

Fig. P. 4.10.1 : Membership function for position of mixer tap

$$\mu_{EL}(x) = \frac{45-x}{45}, 0 \le x \le 45$$

$$\mu_L(x) = \begin{cases} \frac{x}{45}, & 0 \le x \le 45 \\ \frac{90-x}{45}, & 45 < x \le 90 \end{cases}$$

$$\mu_c(x) = \begin{cases} \frac{x-45}{45}, & 45 \le x \le 90 \\ \frac{135-x}{45}, & 90 < x \le 135 \end{cases}$$

$$\mu_R(x) = \begin{cases} \frac{x-90}{45}, & 90 \le x \le 135 \\ \frac{180-x}{45}, & 135 < x \le 180 \end{cases}$$

$$\mu_{ER}(x) = \frac{x-135}{45}, 135 \le x \le 180$$

2. Membership functions for output variable - temperature of water.

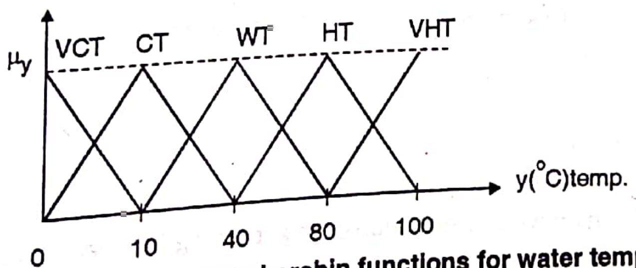

Fig. P. 4.10.1(a) : Membership functions for water temperature

$$\mu VCT(y) = \frac{10-y}{10}, 0 \le y \le 10$$

$$\mu CT(y) = \begin{cases} \frac{y}{10}, & 0 \le y \le 10 \\ \frac{40-y}{30}, & 10 < y \le 40 \end{cases}$$

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$$\mu WT(y) = \begin{cases} \dfrac{y-10}{30}, & 10 \le y \le 40 \\ \dfrac{80-y}{40}, & 40 < y \le 80 \end{cases}$$

$$\mu HT(y) = \begin{cases} \dfrac{y-40}{40}, & 40 \le y \le 80 \\ \dfrac{100-y}{20}, & 80 < y \le 100 \end{cases}$$

$$\mu VHT(y) = \frac{y-80}{20}, 80 \le y \le 100$$

Step 3 : Form a rule base.

Table P. 4.10.1

Input (Mixer tap position)	Output (Temperature of water)
EL	VCT
L	CT
C	WT
R	HT
ER	VHT

We can read the rule base shown in Table P. 4.10.1 in terms of If-then rules.

Rule 1 : If mixer tap position is EL (Extreme Left) then temperature of water is VCT (Very Cold Temperature).

Rule 2 : If mixer tap position is L (Left) then temperature of water is CT (Cold).

Rule 3 : If mixer tap position is C (Centre) then temperature of water is WT (Warm)

Rule 4 : If mixer tap position is R (Right) then temperature of water is HT (Hot)

Rule 5 : If mixer tap position is ER (Extreme Right) then temperature of water is VHT (Very Hot).

Thus, we have five rules.

Step 4 : Rule Evaluation.

Assume that mixer tap position is 75°. This value x = 75° maps to following two MFs of Rule 2 and Rule 3 respectively.

Rule 2 : $\mu_L(x) = \dfrac{90-x}{45}$

Rule 3 : $\mu_c(x) = \dfrac{x-45}{45}$

Now, substitute the value of x = 75 in above two equations, we get strength of each rule

Strength of Rule 2 $\Rightarrow \mu_L(75) = \dfrac{90-75}{45} = \dfrac{1}{3}$

Strength of Rule 3 $\Rightarrow \mu_c(75) = \dfrac{75-45}{45} = \dfrac{2}{3}$

Step 5 : Defuzzification

We apply **mean of maximum** defuzzification technique.

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We find the rule with the maximum strength

$$= \max(\text{Strength of Rule 1, strength of Rule 2})$$
$$= \max(\mu_L(x), \mu_c(x))$$
$$= \max\left(\frac{1}{3}, \frac{2}{3}\right) = \frac{2}{3}$$

Thus, Rule 3 has the maximum strength.

According to Rule 3, If mixer tap position is C (center) then water temperature is **Warm.** So, we use Output MFs of **warm water temperature** for defuzification. We have following two equations for warm water temperature.

$$\mu_{WT}(y) = \frac{y-10}{30} \quad \text{and}$$
$$\mu_{WT}(y) = \frac{80-y}{40}$$

Since, the strength of rule 3 is $\frac{2}{3}$, substitute $\mu_{WT}(y) = \frac{2}{3}$ in the above two equations.

$$\frac{y-10}{30} = \frac{2}{3} \Rightarrow y = 30$$
$$\frac{80-y}{40} = \frac{2}{3} \Rightarrow y = 53$$

Now take the average (mean) of these values.

$$y^* = \frac{30+53}{2} = \mathbf{41.5°C}$$

2. Washing Machine Controller

Ex. 4.10.2 : Design a controller to determine the wash time of a domestic washing machine. Assume that input is dirt and grease on cloths. Use three descriptors for input variables and five descriptors for output variables. Derive set of rules for controller action and defuzzification. The design should be supported by figures wherever possible. Show that if the cloths are soiled to a larger degree the wash time will be more and vice-versa.

MU - May 12, Dec. 12, Dec. 13, Dec. 14, 20 Marks

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

- Here inputs are 'dirt' and 'grease'. Assume that they are measured in percentage (%). That is amount of dirt and grease is measured in percentage.
- Output is 'wash time' measured in minutes.
- We use three descriptors for each of the input variables.

Descriptors for dirt are as follows :

SD - Small Dirt

MD - Medium Dirt

LD - Large Dirt

{SD, MD, LD}

Descriptors for grease are {NG, MG, LG}

NG - No Grease

MG - Medium Grease

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LG - Large Grease

We use five descriptors for output variable.

So, descriptors for wash time are {VS, S, M, L, VL}

VS - Very Short

S - Short

M - Medium

L - Large

VL - Very Large

Step 2 : Define membership functions for each of the input and output variables.

We use triangular MFs because of their simplicity.

(1) Membership functions for dirt

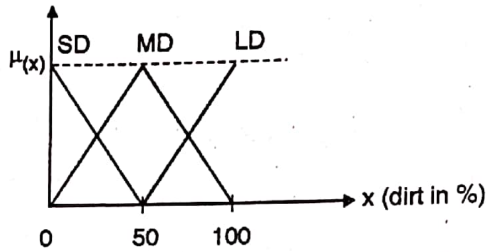

Fig. P. 4.10.2 : Membership functions for dirt

$$\mu_{SD}(x) = \frac{50-x}{50}, \quad 0 \leq x \leq 50$$

$$\mu_{MD}(x) = \begin{cases} \frac{x}{50}, & 0 \leq x \leq 50 \\ \frac{100-x}{50}, & 50 < x \leq 100 \end{cases}$$

$$\mu_{LD}(x) = \frac{x-50}{50}, \quad 50 \leq x \leq 100$$

(2) Membership functions for grease

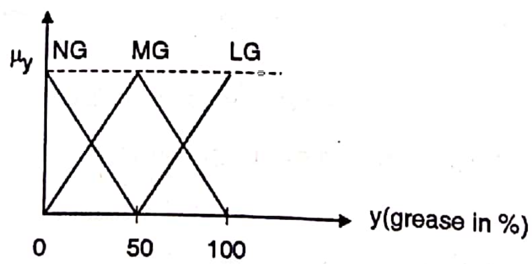

Fig. P. 4.10.2(a) : Membership functions of grease

$$\mu_{NG}(y) = \frac{50-y}{50}, \quad 0 \leq y \leq 50$$

$$\mu_{MG}(y) = \begin{cases} \frac{y}{50}, & 0 \leq y \leq 50 \\ \frac{100-y}{50}, & 50 < y \leq 100 \end{cases}$$

$$\mu_{LG}(y) = \frac{y-50}{50}, \quad 50 \leq y \leq 100$$

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Membership functions for wash time

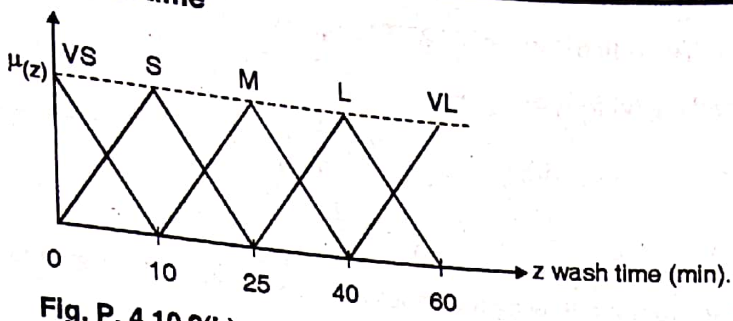

Fig. P. 4.10.2(b) : Membership functions for wash time

$$\mu VS(z) = \frac{10-z}{10}, \quad 0 \le z \le 10$$

$$\mu S(z) = \begin{cases} \frac{z}{10}, & 0 \le z \le 10 \\ \frac{25-z}{15}, & 10 < z \le 25 \end{cases}$$

$$\mu M(z) = \begin{cases} \frac{z-10}{15}, & 10 \le z \le 25 \\ \frac{40-z}{15}, & 25 < z \le 40 \end{cases}$$

$$\mu L(z) = \begin{cases} \frac{z-25}{15}, & 25 \le z \le 40 \\ \frac{60-z}{20}, & 40 < z \le 60 \end{cases}$$

$$\mu VL(z) = \frac{z-40}{20}, \quad 40 \le z \le 60$$

Step 3 : Form a Rule base

x \ y	NG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

The above matrix represents in all nine rules. For example, first rule can be **"If dirt is small and no grease then wash time is very short"** similarly all nine rules can be defined using if --- then.

Step 4 : Rule Evaluation

Assume that dirt = 60 % and grease = 70%

dirt = 60 % maps to the following two MFs of "dirt" variable

$$\mu_{MD}(x) = \frac{100-x}{50} \text{ and } \mu_{LD}(x) = \frac{x-50}{50}$$

Similarly grease = 70 % maps to the following two MFs of "grease" variable.

$$\mu_{MG}(y) = \frac{100-y}{50} \text{ and } \mu_{LG}(y) = \frac{y-50}{50}$$

Evaluate $\mu_{MD}(x)$ and $\mu_{LD}(x)$ for x = 60, we get

$$\mu_{MD}(60) = \frac{100-60}{50} = \frac{4}{5} \qquad \dots (1)$$

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$$\mu_{LD}(60) = \frac{60-50}{50} = \frac{1}{5} \quad \text{...(2)}$$

Similarly evaluate $\mu_{MG}(y)$ and $\mu_{LG}(y)$ for y = 70, we get

$$\mu_{MG}(70) = \frac{100-70}{50} = \frac{3}{5} \quad \text{...(3)}$$

$$\mu_{LG}(70) = \frac{70-50}{50} = \frac{2}{5} \quad \text{... (4)}$$

The above four equation leads to the following four rules that we are suppose to evaluate.

(1) Dirt is medium and grease is medium.

(2) Dirt is medium and grease is large.

(3) Dirt is large and grease is medium.

(4) Dirt is large and grease is **large.**

Since the antecedent part of each of the above rule is connected by **and** operator we use **min** operator to evaluate strength of each rule.

$$\text{Strength of rule 1: } S_1 = \min(\mu_{MD}(60), \mu_{MG}(70)) = \min(4/5, 3/5) = 3/5$$

$$\text{Strength of rule 2: } S_2 = \min(\mu_{MD}(60), \mu_{LG}(70)) = \min(4/5, 2/5) = 2/5$$

$$\text{Strength of rule 3: } S_3 = \min(\mu_{LD}(60), \mu_{MG}(70)) = \min(1/5, 3/5) = 1/5$$

$$\text{Strength of rule 4: } S_4 = \min(\mu_{LD}(60), \mu_{LG}(70)) = \min(1/5, 2/5) = 1/5$$

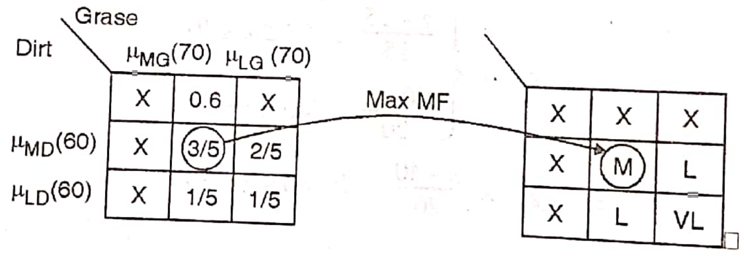

Fig. P.4.10.2(c): Rule strength and its mapping to corresponding output MF

Step 5 : Defuzzification

– Since, we use **"Mean of max"** defuzzification technique, we first find the rule with the maximum strength.

$$= \text{Max}(S1, S2, S3, S4)$$

$$= \text{Max}(3/5, 2/5, 1/5, 1/5)$$

$$= 3/5$$

– This corresponds to rule 1.

– This rule 1 – "dirt is **medium** and grease is **medium**" has maximum strength 3/5.

– The above rule corresponds to the output MF $\mu_M(z)$. This mapping is shown in Fig. P. 4.10.2(c).

– To find out the final defuzzified value, we now take average (mean) of $\mu_M(z)$.

$$\mu_M(z) = \frac{z-10}{15} \quad \text{and} \quad \mu_M(z) = \frac{40-z}{15}$$

$$\therefore\ 3/5 = \frac{z-10}{15} \qquad 3/5 = \frac{40-z}{15}$$

$$\therefore\ Z = 19 \qquad z = 31$$

$$\therefore\ z^* = \frac{19+31}{2}$$

$$= 25 \text{ min}$$

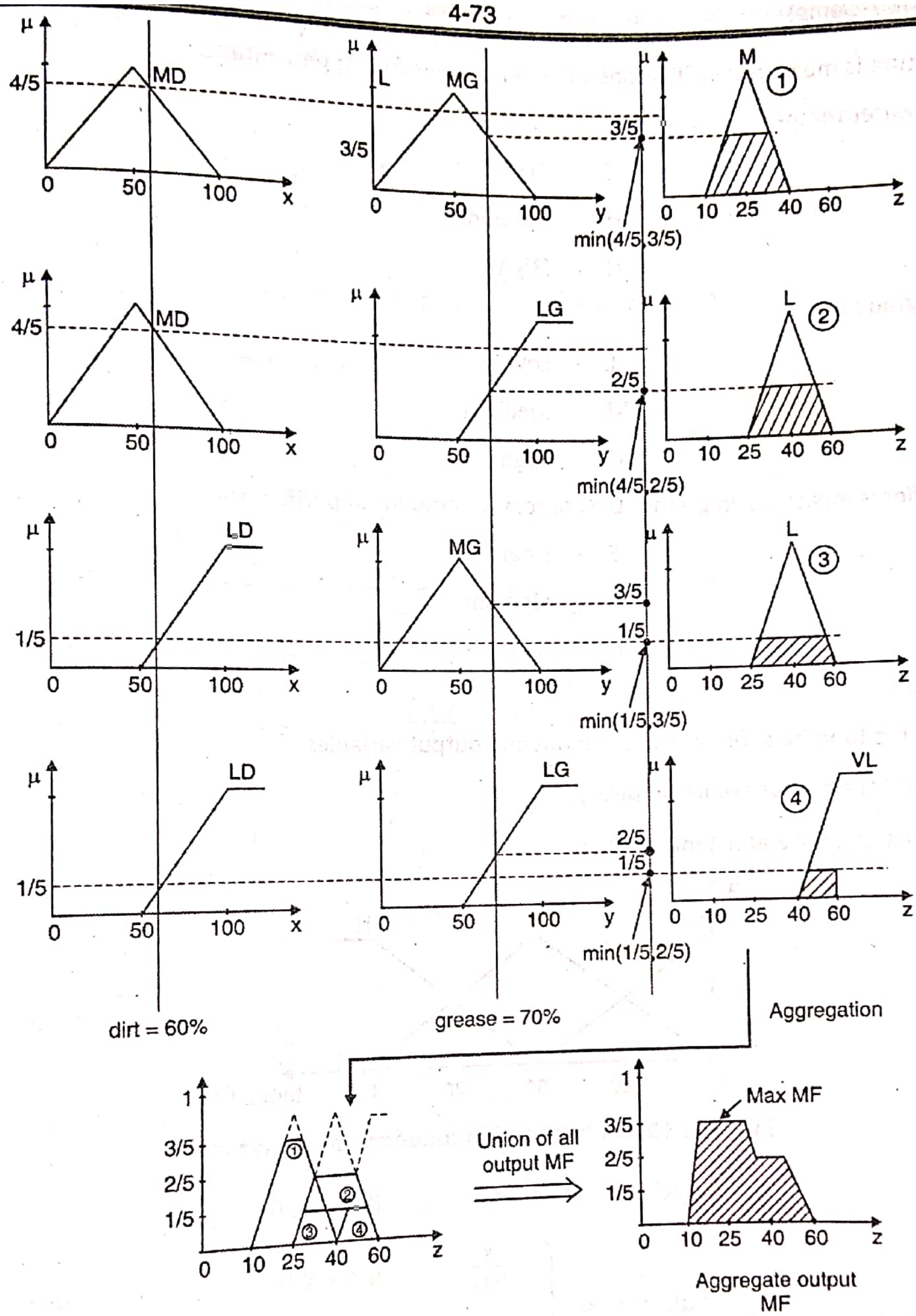

Fig. P. 4.10.2(d) : Process of rule evaluation and defuzzification

3. Water Purifier Controller

Ex. 4.10.3 : Design a fuzzy controller to control the feed amount of purifier for the water purification plant. Raw water is purified by injecting chemicals. Assume input as water temperature and grade of water. Output as amount of purifier. Use three descriptors for input and output variables. Design rules to control action and defuzzification. Design should be supported by figures whenever necessary. Clearly indicate that when temperature is low, grade is low then chemical used is in large amount. **MU - May 13, 20 Marks**

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

Here input variables are water temperature and grade of water.

- Water temperature is measured in °C. grade of water is measured in percentage.
- Descriptors for water temperature are

{C, M, H}
C - Cold
M - Medium
H - High

- Descriptors for grade are

{L, M, H}
L - Low
M - Medium
H - High

- Amount of purifier is measured in grams. Descriptors for amount of purifier are

{S, M, L}.
S - Small
M - Medium
L - Large

Step 2 : Fuzzification

Define membership functions for each of the input and output variables.

We use triangular MFs because of its simplicity.

(1) Membership functions for water temperature

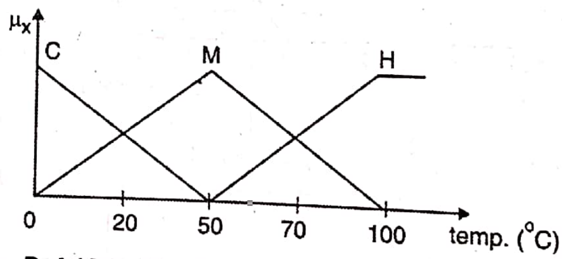

Fig. P. 4.10.3 : Membership functions for water temp.

$$\mu C(x) = \frac{50-x}{50}, \quad 0 \le x \le 50$$

$$\mu M(x) = \begin{cases} \dfrac{x}{50}, & 0 \le x \le 50 \\ \dfrac{100-x}{50}, & 50 < x \le 100 \end{cases}$$

$$\mu H(x) = \frac{x-50}{50}, \quad 50 \le x \le 100$$

(2) Membership functions for grade of water

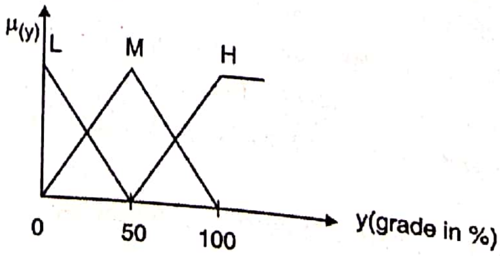

Fig. P. 4.10.3(a) : Membership functions for grade of wat...

$$\mu L(y) = \frac{50-y}{50}, \quad 0 \le y \le 50$$

$$\mu M(y) = \begin{cases} \frac{y}{50}, & 0 \le y \le 50 \\ \frac{100-y}{50}, & 50 < y \le 100 \end{cases}$$

$$\mu H(y) = \frac{y-50}{50}, \quad 50 \le y \le 100$$

(3) Membership functions for amount of purifier

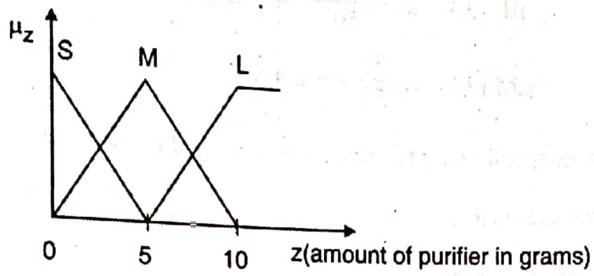

Fig. P. 4.10.3(b) : Membership functions for purifier

$$\mu S(z) = \frac{5-z}{5}, \quad 0 \le z \le 5$$

$$\mu M(z) = \begin{cases} \frac{z}{5}, & 0 \le z \le 5, \\ \frac{10-z}{5}, & 5 < z \le 10 \end{cases}$$

$$\mu L(z) = \frac{z-5}{5}, \quad 5 < z \le 10$$

Step 3 : Form a rule base

Temp \ grade	L	M	H
C	L	M	S
M	L	M	M
H	M	S	S

- The above matrix represents in all nine rules. For example,
- First rule can be, "If temperature is cold and grade is low then amount of purifier required is large."
- Similarly all nine rules can be defined using if-then rules.

Step 4 : Rule Evaluation

- Assume water temperature = 5° and grade = 30
- Water temperature = 5° maps to the following two MFs of "temperature" variable.

$$\mu_c(x) = \frac{50-x}{50} \text{ and } \mu_M(x) = \frac{x}{50}$$

- Similarly, grade = 30 maps to the following two MFs of "grade" variable.

$$\mu_L(y) = \frac{50-y}{50} \text{ and } \mu_M(y) = \frac{y}{50}$$

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– Evaluate $\mu_c(x)$ and $\mu_M(x)$ for x = 5°

We get,

$$\mu_c(5) = \frac{50-5}{50} = \frac{9}{10} = 0.9 \quad ...(1)$$

$$\mu M(5) = \frac{5}{50} = \frac{1}{5} = 0.1 \quad ...(2)$$

– Evaluate $\mu_L(y)$ and $\mu_M(y)$ for y = 30

$$\mu L(30) = \frac{50-30}{50} = \frac{2}{5} = 0.4 \quad ...(3)$$

$$\mu M(30) = \frac{30}{50} = \frac{3}{5} = 0.6 \quad ...(4)$$

– The above four equation represents following for rules that we need to evaluate.

1. If temperature is cold and grade is low.
2. If temperature is cold and grade is medium.
3. If temperature is medium and grade is low.
4. If temperature is medium and grade is **medium**.

– Since the antecedent part of each rule is connected by ***and*** operator we use ***min*** operator to evaluate strength of each rule

Strength of rule1 : S_1 = min (μ_C (5), μ_L (30)) = min (0.9, 0.4) = 0.4

Strength of rule2 : S_2 = min (μ_c(5), μ_M(30)) = min (0.9, 0.6) = 0.6

Strength of rule3 : S_3 = min (μ_M(5), μ_L(30)) = min (0.1, 0.4) = 0.1

Strength of rule4 : S_4 = min (μ_M(5), μ_M(30)) = min (0.1, 0.6) = 0.1

Temp \ Grade	$\mu_L(30)$	$\mu_M(30)$	
$\mu_C(5)$	0.4	0.6	X
$\mu_M(5)$	0.1	0.1	X
	X	X	X

(a) Rule strength table

Max MF

Temp \ Grade			
	L	M	S
	L	M	M
	M	S	S

(b) Rule base table

Fig. P. 4.10.3(c) :Rule strength and its mapping to corresponding output MF

Step 5 : Defuzzification

Since, we use "mean of max" defuzzification technique, we first find the rule with maximum strength.

= max (S_1, S_2, S_3, S_4) = max (0.4, 0.6, 0.1, 0.1) = 0.6

– This corresponds to rule 2.

Thus rule 2 : "Temperature is cold and grade is medium" has maximum strength 0.6.

– The above rule corresponds to the output MF $\mu_M(z)$. This is show is shown in Fig. P.4.10.3(c).

– To find out final defuzzified value, we now take average (i.e. mean) of $\mu_M(z)$.

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$$\mu_M(z) = \frac{10 - z}{5} \quad \text{and} \quad \mu_M(z) = \frac{z}{5}$$

$$0.6 = \frac{10 - z}{5} \qquad \therefore 0.6 = \frac{z}{5}$$

$$\therefore z = 13 \qquad \therefore z = 3$$

$$\therefore z^* = \frac{13 + 3}{2} = 8 \text{ gms}$$

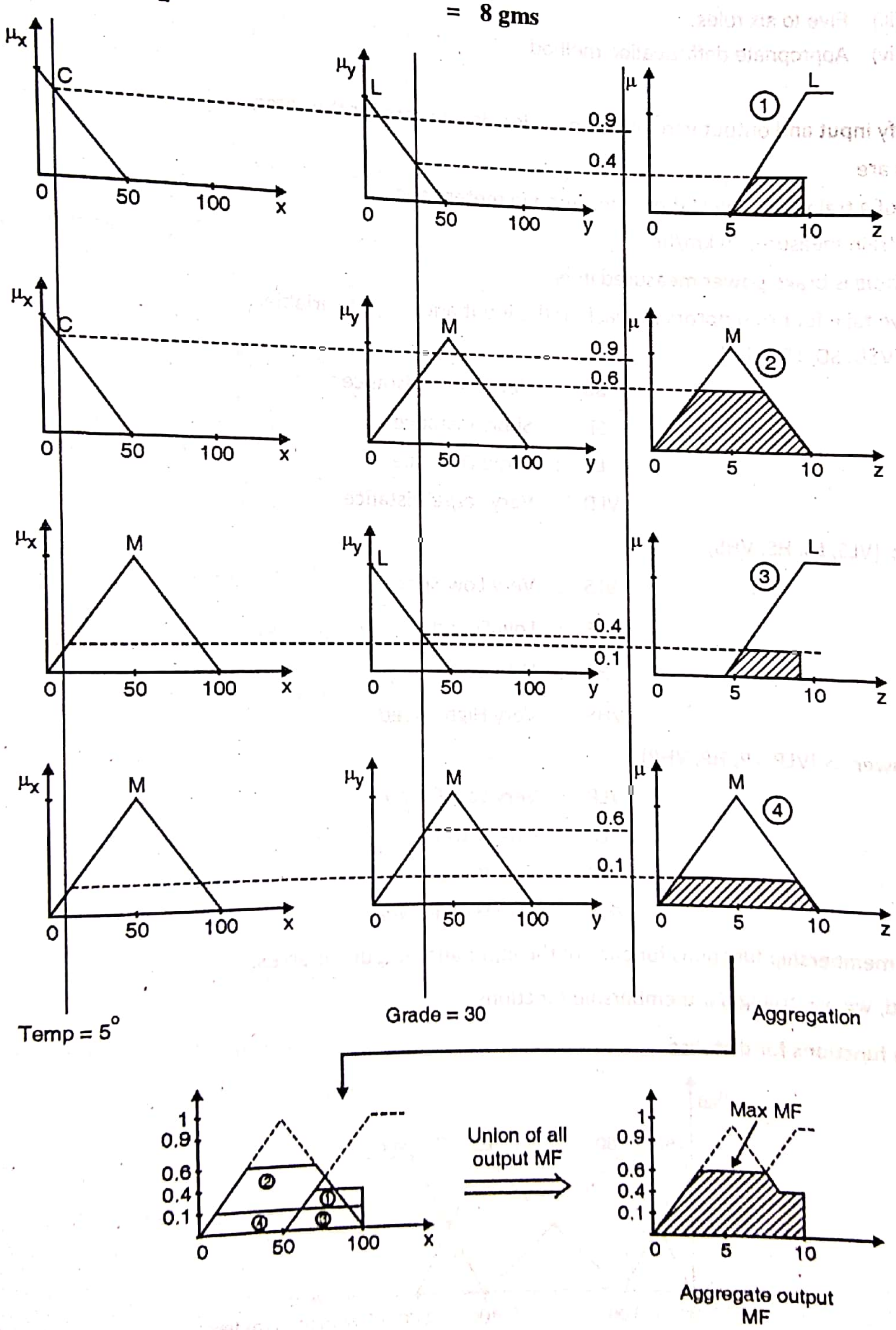


Fig. P. 4.10.3(d) : Process of rule evaluation and defuzzification

4. Train Brake Power Controller

Ex. 4.10.4 : Design a fuzzy controller for a train approaching or leaving a station. The inputs are distance from a station and speed of the train. The output is brake power used. Use,

(i) Triangular membership functions

(ii) Four descriptors for each variables

(iii) Five to six rules.

(iv) Appropriate deffuzication method.

Soln. :

Step 1 : Identify input and output variables and decide descriptors for the same.

– Here inputs are

Distance of a train from the station, measured in meters and

Speed of train measured in km/hr.

– Output variable is brake power measured in %.

As mentioned, we take four descriptors for each of the input and output variables.

For distance ⇒ {VSD, SD, LD, VLD}

VSD : Very Short Distance

SD : Short Distance

LD : Large Distance

VLD : Very Large Distance

– For speed ⇒ {VLS, LS, HS, VHS}

VLS : Very Low Speed

LS : Low Speed

HS : High Speed

VHS : Very High Speed

– For brake power ⇒ {VLP, LP, HP, VHP}

VLP : Very Low Power

LP : Low Power

HP : High Power

VHP : Very High Power

Step 2 : Define membership functions for each of the input and output variables.

– As mentioned, we use triangular membership functions.

1. Membership functions for distance

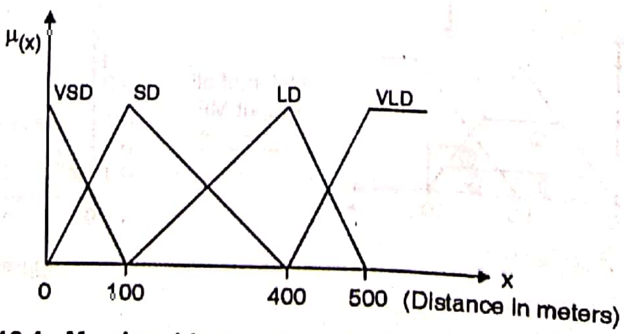


Fig. P. 4.10.4 : Membership functions for distance (distance in meters)

$$\mu VSD(x) = \frac{100-x}{100}, \quad 0 \le x \le 100$$

$$\mu SD(x) = \begin{cases} \frac{x}{100}, & 0 \le x \le 100 \\ \frac{400-x}{300}, & 100 < x \le 400 \end{cases}$$

$$\mu LD(x) = \begin{cases} \frac{x-100}{300}, & 100 \le x \le 400 \\ \frac{500-x}{100}, & 400 < x \le 500 \end{cases}$$

$$\mu VLD(x) = \frac{x-400}{100}, \quad 400 \le x \le 500$$

2. **Membership functions for speed**

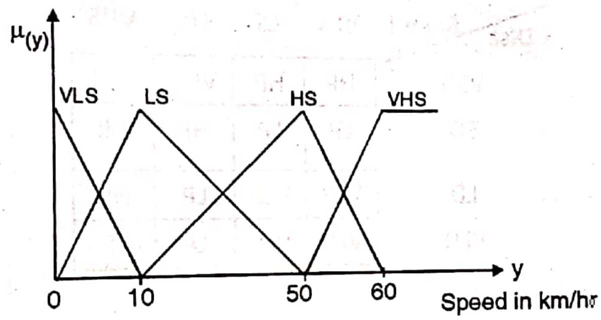


Fig. P. 4.10.4(a) : Membership functions for speed

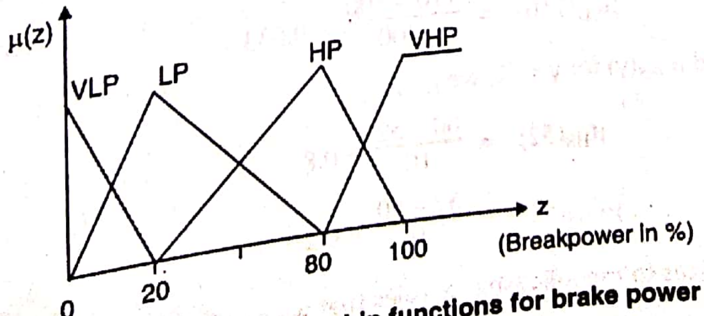
$$\mu VLS(y) = \frac{10-y}{10}, \quad 0 \le y \le 10$$

$$\mu LS = \begin{cases} \frac{y}{10}, & 0 \le y \le 10 \\ \frac{50-y}{40}, & 10 < y \le 50 \end{cases}$$

$$\mu HS(y) = \begin{cases} \frac{y-10}{40}, & 10 \le y \le 50 \\ \frac{60-y}{10}, & 50 < y \le 60 \end{cases}$$

$$\mu VHS(y) = \frac{y-50}{10}, \quad 50 \le y \le 60$$

3. **Membership functions for brake power**



P. 4.10.4(b) : Membership functions for brake power

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$$\mu VLP(z) = \frac{20-z}{20}, \quad 0 \le z \le 20$$

$$\mu LP(z) = \begin{cases} \frac{z}{20}, & 0 \le z \le 20 \\ \frac{80-z}{60}, & 20 < z \le 80 \end{cases}$$

$$\mu HP(z) = \begin{cases} \frac{z-20}{60}, & 20 \le z \le 80 \\ \frac{100-z}{20}, & 80 < z \le 100 \end{cases}$$

$$\mu VHP(z) = \frac{z-80}{20}, \quad 80 \le z \le 100$$

Step 3 : Form a rule base.

Dist \ Speed	VLS	LS	HS	VHS
VSD	HP	HP	VHP	VHP
SD	LP	LP	HP	VHP
LD	VLP	VLP	LP	HP
VLD	VLP	VLP	LP	LP

The above matrix represents in all 16 rules.

For example, First rule can be "If distance of a train is Very Short (VSD) and speed is Very Low (VLS) then required brake power is High (HP)".

Similarly all 16 rules can be defined using If then rules.

Step 4 : Rule Evaluation

Assume distance = 110 meters and speed = 52 km/hr

Distance = 110 maps to the following two MFs of "distance" variable.

$$\mu_{SD}(x) = \frac{400-x}{300} \text{ and } \mu_{LD}(x) = \frac{x-100}{300}$$

– Similarly speed = 52 maps to the following two MFs of "speed" variable.

$$\mu_{HS}(y) = \frac{60-y}{10} \text{ and } \mu_{VHS}(y) = \frac{y-50}{10}$$

– Evaluate $\mu_{SD}(x)$ and $\mu_{LD}(x)$ for x = 110, we get,

$$\mu_{SD}(110) = \frac{400-110}{300} = 0.96 \quad ...(1)$$

$$\mu_{LD}(110) = \frac{110-100}{300} = 0.033 \quad ...(2)$$

– Similarly evaluate $\mu_{HS}(y)$ and $\mu_{VHS}(y)$ for y = 52, we get,

$$\mu_{HS}(52) = \frac{60-52}{10} = 0.8 \quad ...(3)$$

$$\mu_{VHS}(52) = \frac{52-50}{10} = 0.2 \quad ...(4)$$

– The above four equations leads to the following for rules that we needs to evaluate.

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1. Distance is short and speed is high
2. Distance is short and speed is very high
3. Distance is large and speed is high
4. Distance is large and speed is very high

Since the antecedent part of each rule is connected by ***and*** operator we use ***min***. Operator to evaluate strength of each rule

$$\text{Strength of rule1: } S_1 = \min(\mu_{SD}(110), \mu_{HS}(52)) = \min(0.96, 0.8) = 0.8$$

$$\text{Strength of rule2 : } S_2 = \min(\mu_{SD}(110), \mu_{VHS}(52))$$

$$= \min(0.96, 0.2) = 0.2$$

$$\text{Strength of rule3 : } S_3 = \min(\mu_{LD}(110), \mu_{HS}(52))$$

$$= \min(0.033, 0.8) = 0.033$$

$$\text{Strength of rule: } S_4 = \min(\mu_{LD}(110), \mu_{VHS}(52))$$

$$= \min(0.033, 0.2) = 0.033$$

Distance \ Speed	VLS	LS	HS	VHS
VSD	X	X	X	X
SD	X	X	0.8	0.2
LD	X	X	0.03	0.03
VLD	X	X	X	X

(a) Rule strength table

	VLS	LS	HS	VHS
VSD				
SD			HP	VHP
LD			LP	HP
VLD				

(b) Rule base table

Fig. P. 4.10.4 (c) : Rule strength table and its mapping to corresponding output MF

Step 5 : Defuzzification

- We use "mean of max" defuzzification technique.
- We first find the rule with maximum strength

$$= \max(S_1, S_2, S_3, S_4)$$

$$= \max(0.8, 0.2, 0.33, 0.2) = 0.8$$

- This corresponds to rule 1.
- Thus rule 1 – "If dist is short and speed is high" has maximum strength 0.8.
- The above rule corresponds to the output MF $\mu_{HP}(z)$. This mapping is shown in Fig. P. 4.10.4(c).
- To compute the final defuzzified value, we take average of $\mu_{HP}(z)$.

$$\mu_{HP}(z) = \frac{z-20}{60} \qquad \mu_{HP}(z) = \frac{100-z}{20}$$

$$\therefore 0.8 = \frac{z-20}{60} \qquad 0.8 = \frac{100-z}{20}$$

$$\therefore z = 68 \qquad \therefore z = 84$$

$$\therefore z^* = \frac{68+84}{2} = 76\%$$

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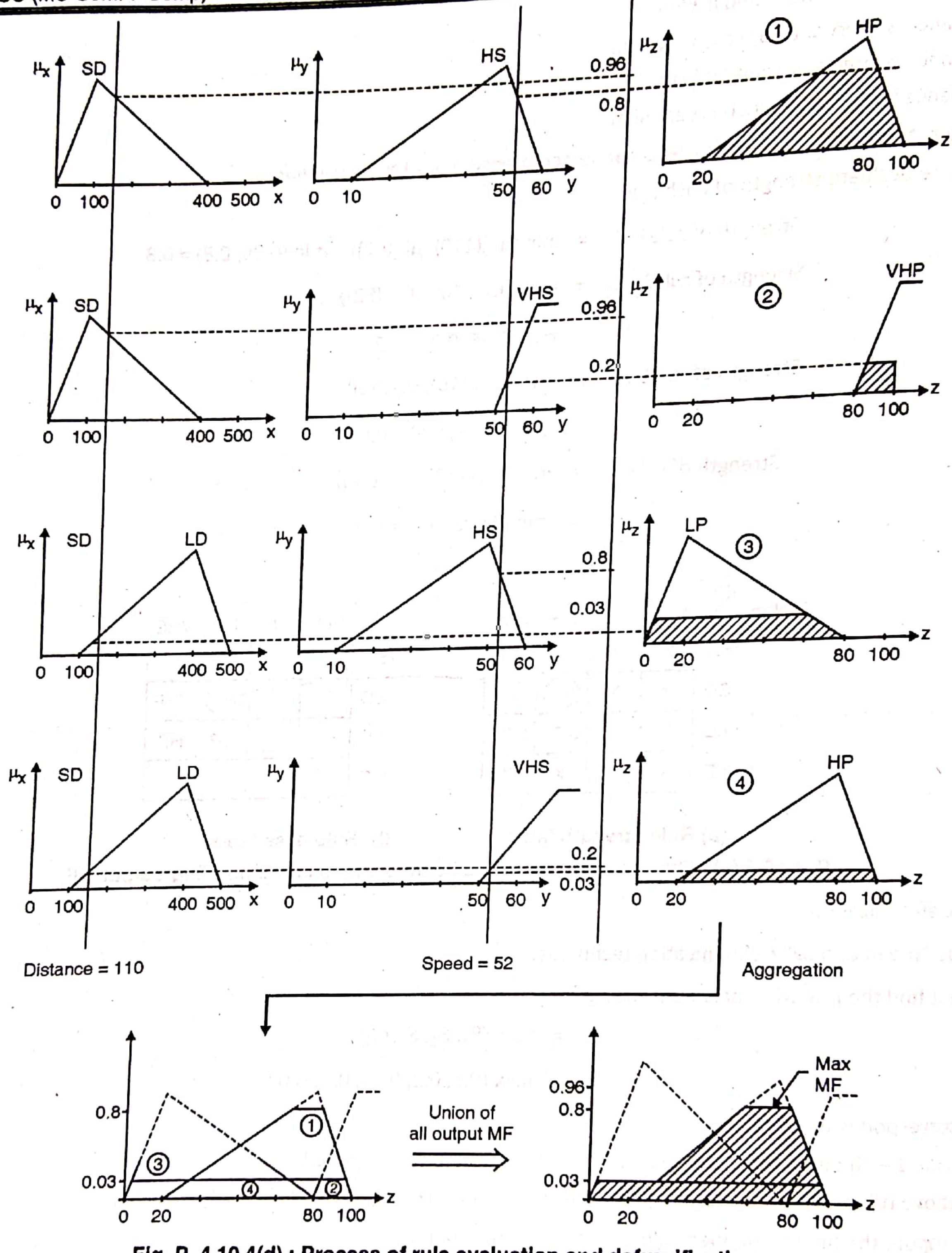

Fig. P. 4.10.4(d) : Process of rule evaluation and defuzzification

5. Water Tank Temperature Controller

Ex. 4.10.5 : Design a fuzzy controller for maintaining the temperature of water in the tank at a fixed level. Input variables are the cold water flow into the tank and steam flow into the tank. For cooling, cold water flow is regulated and for raising the temperature steam flow is regulated. Define the fuzzification scheme for input variables. Device a set of rules for control action and defuzzification. Formulate the control problem in terms of fuzzy inference rules incorporating the degree of relevance for each rule. Design a scheme which shall regulate the water and steam flows properly.

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Soln. :

Step 1 : Identify input and out variables and decide descriptors for each variables.

Here inputs are,

1. Amount of valve opening for cold water.
2. Amount of valve opening for steam.

- Valve opening is measured in degree from 0° to 180°. we take five descriptors for each of the input variables.
- Descriptors for value opening for cold water flow are

{ELCV, LCV, CCV, RCV, ERCV}

ELCV : Extreme Left Cold Valve

LCV : Left Cold Valve

CCV : Centre Cold Valve

RCV : Right Cold Valve

ERCV : Extreme Right Cold Valve

Descriptors for valve opening of steam flow are {ELSV, LSV, CSV, RSV, ERSV}

ELSV : Extreme Left Steam Valve

LSV : Left Steam Value

CSV : Centre Steam Value

RSV : Right Steam Value

ERSV : Extreme Right Steam Value

- Output is temperature of water in the tank measured in °C.

We use seven descriptors for temperature of water {VVCT, VCT, CT, WT, HT, VHT, VVHT}

VVCT : Very Very Cold Temperature

VCT : Very Cold Temperature

CT : Cold Temperature

WT : Warm Temperature

HT : Hot Temperature

VHT : Very Hot Temperature

VVHT : Very Very Hot Temperature

Step 2 : Define membership functions for each of the input and output variables.

1. Membership functions for valve opening for cold water flow

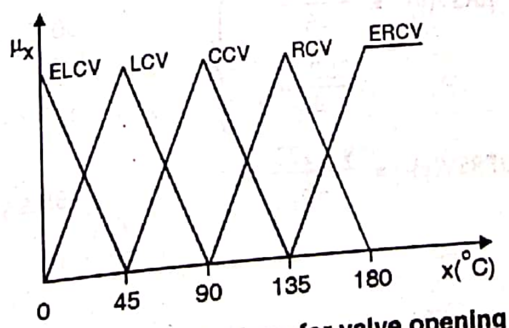

Fig. P. 4.10.5 : Membership functions for valve opening of cold water

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$$\mu ELCV(x) = \frac{45-x}{45}, \quad 0 \le x \le 45$$

$$\mu LCV(x) = \left.\begin{array}{ll} \frac{x}{45}, & 0 \le x \le 45 \\ \frac{90-x}{45}, & 45 < x \le 90 \end{array}\right\}$$

$$\mu CCV(x) = \left.\begin{array}{ll} \frac{x-45}{45}, & 45 \le x \le 90 \\ \frac{135-x}{45}, & 90 < x \le 135 \end{array}\right\}$$

$$\mu RCV(x) = \left.\begin{array}{ll} \frac{x-90}{45}, & 90 \le x \le 135 \\ \frac{180-x}{45}, & 135 < x \le 180 \end{array}\right\}$$

$$\mu ERCV(x) = \frac{x-135}{45}, \quad 135 \le x \le 180$$

2. **Membership functions for valve opening for steam flow**

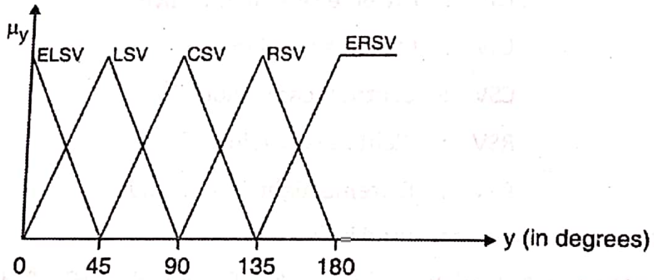

Fig. P.4.10.5(a) : Membership functions for value opening for steam flow

$$\mu ELSV(y) = \frac{45-y}{45}, \quad 0 \le y \le 45$$

$$\mu LSV(y) = \left.\begin{array}{ll} \frac{y}{45}, & 0 \le y \le 45 \\ \frac{90-y}{45}, & 45 < y \le 90 \end{array}\right\}$$

$$\mu ESV(y) = \left.\begin{array}{ll} \frac{y-45}{45}, & 45 \le y \le 90 \\ \frac{135-y}{45}, & 90 < y \le 135 \end{array}\right\}$$

$$\mu RSV(y) = \left.\begin{array}{ll} \frac{y-90}{45}, & 90 \le y \le 135 \\ \frac{180-y}{45}, & 135 < y \le 180 \end{array}\right\}$$

$$\mu ERSV(y) = \frac{y-135}{45}, \quad 135 \le y \le 180$$

3. **Membership functions for temperature of a water in tank**

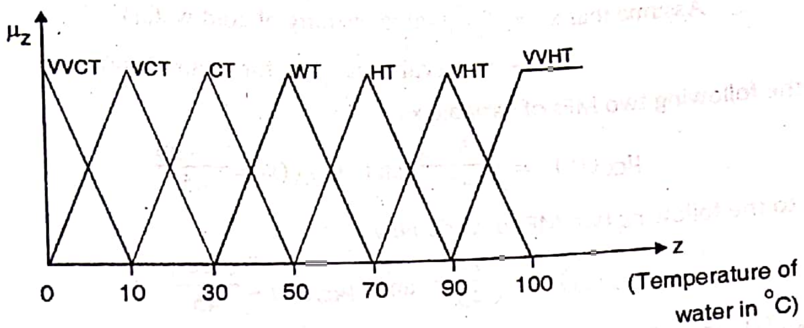

Fig. P.4.10.5(b): Membership functions for temperature of water in tank

$$\mu VVCT(z) = \frac{10-z}{10}, \quad 0 \le z \le 10$$

$$\mu VCT(z) = \begin{cases} \frac{z}{10}, & 0 \le z \le 10 \\ \frac{30-z}{20}, & 10 < z \le 30 \end{cases}$$

$$\mu CT(z) = \begin{cases} \frac{z-10}{20}, & 10 \le z \le 30 \\ \frac{50-z}{20}, & 30 < z \le 50 \end{cases}$$

$$\mu WT(z) = \begin{cases} \frac{z-30}{20}, & 30 \le z \le 50 \\ \frac{70-z}{20}, & 50 < z \le 70 \end{cases}$$

$$\mu HT(z) = \begin{cases} \frac{z-50}{20}, & 50 \le z \le 70 \\ \frac{90-z}{20}, & 70 < z \le 90 \end{cases}$$

$$\mu VHT(z) = \begin{cases} \frac{z-70}{20}, & 70 \le z \le 90 \\ \frac{100-z}{10}, & 90 < z \le 100 \end{cases}$$

$$\mu VVHT(z) = \frac{z-90}{10}, \quad 90 \le z \le 100$$

Step 3 : Form a rule base.

X \ Y	ELSV	LSV	CSV	RSV	ERSV
ELCV	WT	WT	HT	VHT	VVHT
LCV	CT	WT	HT	VHT	VVHT
CCV	VCT	CT	WT	HT	VHT
RCV	VCT	VCT	CT	WT	HT
ERCV	VVCT	VVCT	VCT	CT	WT

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Step 4 : Rule evaluation

$$\text{Assume that } x = 95^\circ \text{ (valve opening of cold water)}$$

$$y = 50^\circ \text{ (valve opening for steam flow)}$$

- Here x = 95° maps to the following two MFs of variable x :

$$\mu_{CCV}(x) = \frac{135 - x}{45} \text{ and } \mu_{RCV}(x) = \frac{x - 90}{45}$$

- Similarly y = 50° maps to the following two MFs of variable y :

$$\mu_{LSV}(y) = \frac{90 - y}{45} \text{ and } \mu_{CSV}(y) = \frac{y - 45}{45}$$

- Evaluate $\mu_{CCV}(x)$ and $\mu_{RCV}(x)$ for $x = 95^\circ$ we get,

$$\mu_{CCV}(95) = \frac{135 - 95}{45} = 0.88 \quad \text{...(1)}$$

$$\mu_{RCV}(95) = \frac{95 - 90}{45} = 0.11 \quad \text{...(2)}$$

- Evaluate $\mu_{LSV}(y)$ and $\mu_{CSV}(y)$ for $y = 50^\circ$, we get

$$\mu_{LSV}(50) = \frac{90 - 50}{45} = 0.88 \quad \text{...(3)}$$

$$\mu_{CSV}(50) = \frac{50 - 45}{45} = 0.11 \quad \text{..(4)}$$

Above four equations lead to the following four rules that we need to evaluate.

1. Cold water valve is in **center** and steam flow valve is in **left.**
2. Cold water valve is in **center** and steam flow valve is in **center.**
3. Cold water valve is in **right** and steam flow valve is in **left.**
4. Cold water valve is in **right** and steam flow valve is in **center.**

Table P.4.10.5 shows rule strength table.

Table P. 4.10.5 : Rule strength table

	μ_{ELSV}	μ_{LSV}	μ_{CSV}	μ_{RSV}	μ_{ERSV}
μ_{ELSV}	X	X	X	X	X
μ_{LSV}	X	X	X	X	X
μ_{CSV}	X	0.88	0.11	X	X
μ_{RSV}	X	0.11	0.11	X	X
μ_{ERSV}	X	X	X	X	X

Step 5 : Defuzzification

- We find the rule with maximum strength

$$= \max(0.88, 0.11, 0.11, 0.11)$$
$$= 0.88$$

Which corresponds to rule 1.

- The rule 1 corresponds to the output MF $\mu_{CT}(z)$. To compute final defuzzified value we take average of $\mu_{CT}(z)$.

$$\mu CT(z) = \frac{z-10}{20} \Rightarrow 0.88$$

$$= \frac{z-10}{20} \Rightarrow z = 27.7$$

$$\mu CT(z) = \frac{50-z}{20} \Rightarrow 0.88$$

$$= \frac{50-z}{20} \Rightarrow z = 32.3$$

$$\therefore\ z^* = \frac{27.7 + 32.3}{2}$$

$$= 30^\circ C$$

Review Questions

Q. 1 Explain support and core of a fuzzy set with examples.

Q. 2 Model the following as fuzzy set using trapezoidal membership function : "Numbers close to 10".

Q. 3 Using Mamdani fuzzy model, Design a fuzzy logic controller to determine the wash time of a domestic washing machine. Assume that the inputs are dirt and grease on cloths. Use three descriptors for each input variables and five descriptor for the output variable. Derive a set of rules for control action and defuzzification. The design should be supported by figures wherever possible.

Q. 4 Let A = $\{a_1, a_2\}$, B = (b_1, b_2, b_3), C = (c_1, c_2)

Let R be a relation from A to B defined by matrix :

	b_1	b_2	b_3
a_1	0.4	0.5	0
a_2	0.2	0.8	0.2

Let S be a relation from B to C defined by matrix :

	c_1	c_2
b_1	0.2	0.7
b_2	0.3	0.8
b_3	1	0

Find (i) Max-min composition of R and S

(ii) Max-products composition of R and S.

Q. 5 Define Supports, Core, Normality, Crossover paints and α – cut for fuzzy set.

Q. 6 High speed rail monitoring devices sometimes make use of sensitive sensors to measure the deflection of the earth when a rail car passes. These deflections are measured with respect to some distance from the rail car and, hence are actually very small angles measured in micro-radians. Let a universe of deflection be A = [1, 2, 3, 4] where A is the angle in micro-radians, and let a universe of distance be D = [1, 2, 5, 7] where D is distance in feet, suppose a relation between these two parameters has been determined as follows :

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R =

	D_1	D_2	D_3	D_4
A_1	1	0.3	0.1	0
A_2	0.2	1	0.3	0.1
A_3	0	0.7	1	0.2
A_4	0	0.1	0.4	1

Now let a universe of rail car weights be W = [1, 2], where W is the weight in units of 100,000 pounds. Suppose the fuzzy relation of W to A is given by,

S =

	W_1	W_2
A_1	1	0.4
A_2	0.5	1
A_3	0.3	0.1
A_4	0	0

Using these two relations, find the relation $R^T oS = T$.

(a) Using max-min composition. (b) Using max-product composition.

Q. 7 Design a fuzzy logic controller for a train approaching or leaving a station. The inputs are the distance from the station and speed of the train. The output is the amount of brake power used. Use four descriptors for each variable use Mamdani Fuzzy model.

Q. 8 Explain the Fuzzy Inference System in detail.

Q. 9 Explain the different Fuzzy membership function.

Q. 10 Explain any four defuzzification methods with suitable Example.

Q. 11 State the different properties of Fuzzy set.

Q. 12 Determine all α - level sets and strong α - level sets for the following fuzzy set.

A = {(1, 0.2) (2, 0.5), (3, 0.8), (4, 1) (5, 0.7), (6, 0.3)}.

Q. 13 Design a fuzzy controller to determine the wash time of a domestic washing machine. Assume that the inputs are dirt and grease on clothes. Use three descriptors for each I/P variable. Device a set of rules for control action and defuzzification. The design should be supported by figures wherever possible. Clearly indicate that if the clothes are soiled to a larger degree the wash time required will be more.

Q. 14 Explain different methods of defuzzification.

Q. 15 Explain cylindrical extension and projection operations on fuzzy relation with example.

Q. 16 Model the following as fuzzy set using trapezoidal membership function "Middle age".

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Q. 17 For the given membership function as shown in Fig. Q. 17, determine the defuzzified output value by any 2 methods.

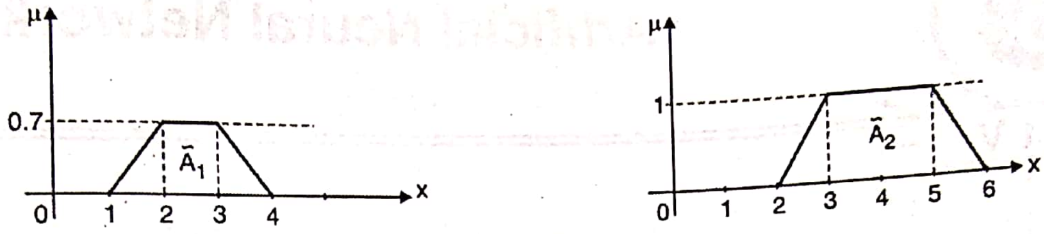

Fig. Q. 17

Q. 18 Compare Mamdani and Sugeno fuzzy models.

Q. 19 Design fuzzy logic controller for water purification plant. Assume the grade of water and temperature of water as the inputs and the required amount of purifier as the output. Use three descriptors for input and output variables. Derive set of rules for control the action and deffuzzification. The design should be supported by figures. Clearly indicate that if water temperature is low and grade of water is low, then amount of purifier required is large.

Q. 20 Discuss fuzzy composition techniques with suitable example.

Q. 21 Two fuzzy relations are given by

$$R = \begin{array}{c} \\ x1 \\ x2 \end{array}\begin{array}{c} \begin{array}{cc} y1 & y2 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 \\ 0.2 & 0.9 \end{bmatrix} \end{array}$$

$$S = \begin{array}{c} \\ y1 \\ y2 \end{array}\begin{array}{c} \begin{array}{ccc} z1 & z2 & z3 \end{array} \\ \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.8 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain fuzzy relation T as a max-min composition and max-product composition between the fuzzy relations.

Q. 22 Describe in detail the formation of inference rules in a Mamdani Fuzzy inference system.

□□□

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