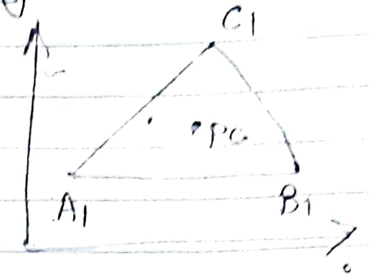
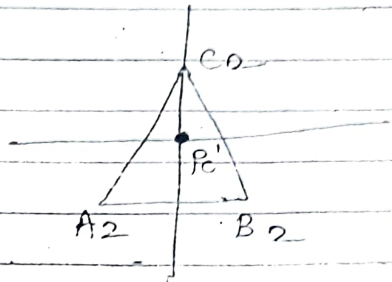
* General Fixed Point Scaling :-

I. Initial position of object $A_1 B_1 C_1$ & fixed point $P_c(x_c, y_c)$.



II. Translate $P_c(x_c, y_c)$ to origin.
The required translation matrix is,

$$T_1 = \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix}$$

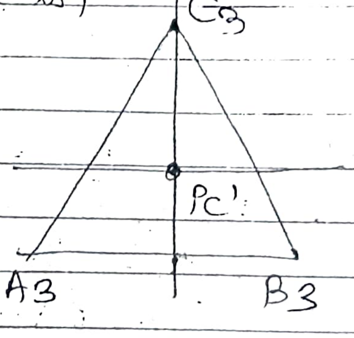


&

$P_c' = T_1 \cdot P_c$
$A_2 = T_1 \cdot A_1$
$B_2 = T_1 \cdot B_1$
$C_2 = T_1 \cdot C_1$

III. Scale the object w.r.t. origin.
Required scaling matrix is,

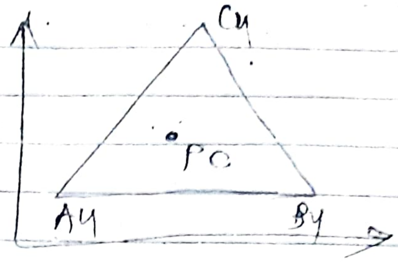
$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$\therefore A_3 = S \cdot A_2$
$B_3 = S \cdot B_2$
$C_3 = S \cdot C_2$

IV. Translate P_c' to original position x_c, y_c
The required translation matrix is,

$$T_2 = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix}$$



$\therefore P_c = T_2 \cdot P_c'$

$A_4 = T_2 \cdot A_3$

$B_4 = T_2 \cdot B_3$

$C_4 = T_2 \cdot C_3$

Composite Transformation Matrix is,

$\therefore T_M = (T_2 \cdot S) \cdot T_1$

Any point P on object will get transformed to P' such that,

$P' = T_M \cdot P$

$\therefore$

$A_4 = T_M \cdot A_1$

$B_4 = T_M \cdot B_1$

$C_4 = T_M \cdot C_1$

* Sums :- (Examples).

1) A point is rotated by 90° about an arbitrary point (3,1). If the original co-ordinate of point is (4,2), what will be the new co-ordinate after rotation.

→ Composite Transformation Matrix is,

$R(\theta) = (T_2 \cdot R) \cdot T_1$

$T_M = T_2 \cdot R \cdot T_1$

(28)

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$$\therefore R(\theta) = \begin{bmatrix} 1 & 0 & xc \\ 0 & 1 & yc \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -xc \\ 0 & 1 & -yc \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, let P' be the new point after rotation.

$$\therefore P' = R(\theta) \cdot P$$

$$= \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

New Co-ordinates are (2,2).

Derive 2D transf^n. which reflects figure in pt (0.5, 0.5)
$T_1 \circ Rfx \circ T_2$

(2b)

2). A triangle A(2,2), B(1,1) & C(3,1) is rotated by 90° about A. Find new co-ordinates of a triangle.

→ The composite transformation matrix is,

$R(\theta) = (T_2 \cdot R) \cdot T_1$

$$TM = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_c \\ 0 & 1 & -y_c \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

let A'B'C' be the new triangle after rotation

$A'B'C' = R(\theta) \cdot ABC$

$$= \begin{bmatrix} 0 & -1 & 4 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

∴ A' (2, 2)
B' (3, 1)
C' (3, 3)

3) Rotate a line AB in counterclockwise direction for 45° angle about origin where A(5,5) & B(20,5).

→ $A = (x, y) = (5, 5)$

$B = (x, y) = (20, 5)$

$\theta = 45°$ & in anticlockwise direction.

To find A' & B'.

$\therefore A'B' = R \cdot AB$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 20 \\ 5 & 5 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 20 \\ 5 & 5 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 20 \\ 5 & 5 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 15/\sqrt{2} \\ 10/\sqrt{2} & 25/\sqrt{2} \\ 1 & 1 \end{bmatrix}$$

$\therefore A'(0, 10/\sqrt{2})$

$B'(15/\sqrt{2}, 25/\sqrt{2})$.

4) Perform 45° rotation of triangle A(0,0) B(1,1) and C(5,2) about arbitrary point (-1,-1).

→ $\theta = 45°$, Point is $(-1, -1)$ $\therefore x_c = -1, y_c = -1$

Composite transformation matrix is,

$TM = T_2 \cdot R \cdot T_1$.

$$Tm = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & (-1) \\ 1/\sqrt{2} & 1/\sqrt{2} & (2/\sqrt{2}-1) \\ 0 & 0 & 1 \end{bmatrix}$$

let $A'B'C'$ be the triangle after rotation.

$$\therefore A'B'C' = Tm \cdot ABC$$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & -1 \\ 1/\sqrt{2} & 1/\sqrt{2} & (2\sqrt{2}-1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & (3/\sqrt{2}-1) \\ (2/\sqrt{2}-1) & (4/\sqrt{2}-1) & (9/\sqrt{2}-1) \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore A'(-1, (2/\sqrt{2}-1))$

$B'(-1, (4/\sqrt{2}-1))$

$C'((3/\sqrt{2}-1), (9/\sqrt{2}-1))$.

5) Scale the triangle ABC with vertices A(0,0) B(1,1) and C(5,2) to twice its size while keeping C(5,2) fixed.

→ $Sx = 2$, $Sy = 2$

It is keeping C(5,2) fixed.

x_c is 5

y_c is 2.

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Composite matrix for Scaling.

$T_m = T_2 . S . T_1$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

~~$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ = final result.~~

~~let A'B'C' be the~~

$$= \begin{bmatrix} 2 & 0 & 5 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

let A'B'C' be the new triangle after ~~so~~ scaling.

$A'B'C' = T_M \cdot ABC$

$$= \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$A' = (-5, -2)$
$B' (-3, 0)$
$C' (5, 2)$.

6) Reflect a line AB with co-ordinates $A(5,5)$, $B(10,10)$ ~~and~~ about line $y=0$.

→ Reflection Matrix for line $y=0$.

$$R_{fx} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

let $A'B'$ be the reflected co-ordinates.

$\therefore A'B' = R_{fx} \cdot AB$.

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 10 \\ 5 & 10 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ -5 & -10 \\ 1 & 1 \end{bmatrix}$$

$A' (5, -5)$
$B' (10, -10)$.

$y = \underset{0}{m}x + \underset{0}{c}$.

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* 3D Transformations :-

i) Translation :-

A point $P(x, y, z)$ is translated to $P'(x', y', z')$ by translation vector such that,

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$

The homogeneous 3D co-ordinate transformation matrix T is given by :

$$T = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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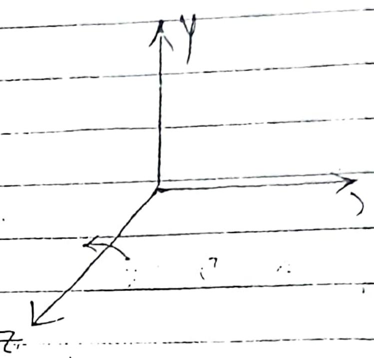
2) Rotation :-

i) Rotation about z-axis :-

Rotation about z-axis by an angle θ in anti-clockwise direction will shift point $P(x, y, z)$ to $P'(x', y', z')$.



All z co-ordinate will remain unchanged.

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$
$z' = z$

The rotation matrix is,

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse rotation i.e. rotation in clockwise direction is obtained by replacing θ by $-\theta$.

The matrix for reverse rotation is,

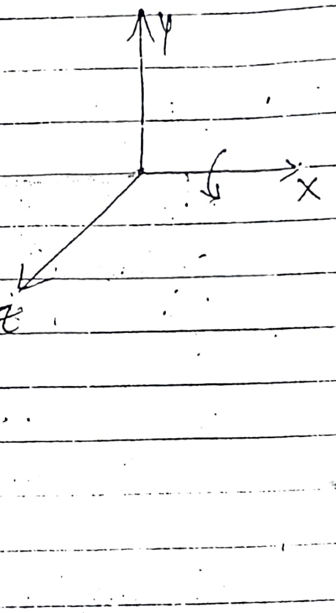
$$R_z^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⟹ Transformation equation for rotation about x-axis and y-axis can be obtained with cyclic permutation of the parameter x, y and z.
Replacing,

in $x \to y \to z \to x$

(ii) Rotation about x-axis :-

Rotation about x-axis by an angle θ in anticlockwise direction will shift point $P(x, y, z)$ to $P'(x', y', z')$.



N: x-co-ordinate will remain unchanged.

New co-ordinates,

$y' = y\cos\theta - z\sin\theta$
$z' = y\sin\theta + z\cos\theta$
$x' = x$

Matrix Representation is,

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Rotation about x-axis i.e. rotation in clockwise direction is obtained by replacing θ by $-\theta$.

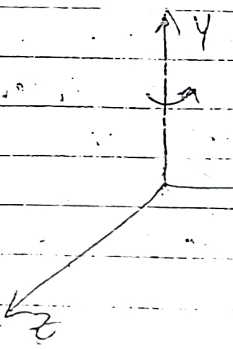
· Matrix for inverse rotation is,

$$R_z^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii) Rotation about Y axis :-

Rotation about Y axis by an angle θ in anticlockwise direction will shift point $P(x, y, z)$ to $P'(x', y', z')$



All the z- co-ordinates will remain unchanged.

The new co-ordinates will then be,

$z' = z \cos\theta - x \sin\theta$

$x' = z \sin\theta + x \cos\theta$

$y' = y$

Matrix representation for Y- axis rotation is,

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$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Rotation ie rotation in clockwise direction is,

$$R_y^{-1} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) ~~General~~ Scaling :-

A pt. $P(x,y,z)$ is scaled to $P'(x',y',z')$ such that,

$$x' = x \cdot S_x$$
$$y' = y \cdot S_y$$
$$z' = z \cdot S_z$$

Homogeneous 3D Co-ordinate transformation matrix S is given by,

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$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

CASE

4) Reflection :-

3D Reflection about x-axis, Y-axis and z-axis is given by,

$$Rf_x = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Rf_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rf_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* General 3D Rotation :-

If the axis of rotation is other than the co-ordinate axis then rotation about any given axis is obtained by using the composite transformation matrix.

Required transformation matrix is obtained by setting-up transformation sequence that

— Moves the selected rotation axis onto ~~the~~ one of the co-ordinate axis.

- Rotates about same co-ordinate axis.
- Returns the selected axis onto its original axis.

CASE 1 :- Rotation about an axis that is parallel to one of the co-ordinate axis :-

Algorithm :-

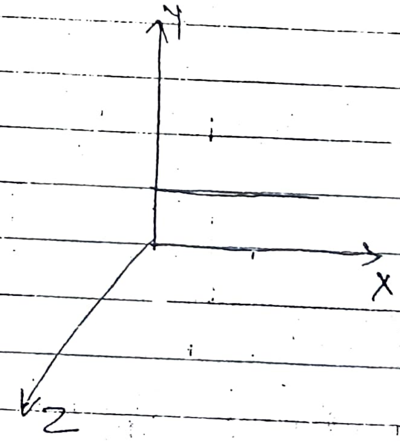
I) Translate the object so that the rotation axis coincides with the parallel co-ordinate axis.

II) Perform the specified rotation about the same co-ordinate axis.

III) Translate the object, so that the rotation axis is moved back to its original position.

Steps

I) Initial position of rotation axis



II) Translate Rotation axis onto X-axis

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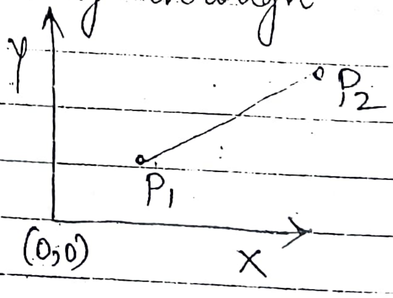
Any point P on the object, we get transformed to P' such that,

$$P' = T_m \cdot P.$$

New transformation object will be the result of rotation about an axis parallel to X-axis.

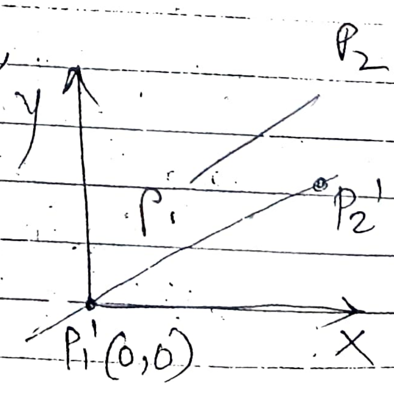
CASE 2: Rotation about an arbitrary axis :-

I. Consider an arbitrary axis passing through two points P_1 & P_2.
Initial position of rotation axis :



II. Translate P_1 to origin.
Required transformation matrix is,

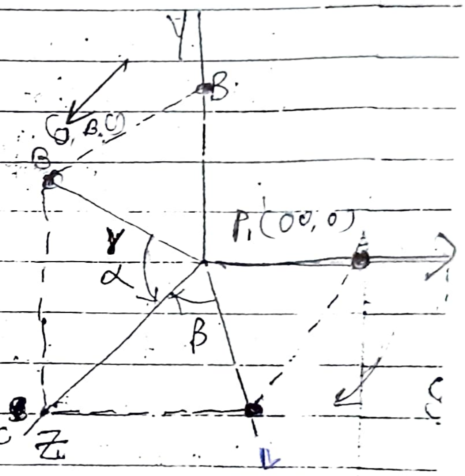
$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -tx \\ 0 & 1 & 0 & -ty \\ 0 & 0 & 1 & -tz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



III. Rotate P_2' onto z-axis. The required transform that will put the rotation axis onto z-axis is obtained in two steps.

Step 1. Rotate the rotation axis about x-axis in anticlockwise direction so that the general rotation axis in the xz plane.

To determine the necessary angle of rotation, consider project of segment $P_1'P_2'$ on YZ plane. The line segment between $P_1'(0,0,0)$ and $P_2'(A,B,C)$ lies on rotation axis. The line segment projection on YZ will extend from (0,0,0) to (0,B,C). Now, if we rotate $P_1'P_2'$ about x-axis until it lies in xz plane, the line segment projection will align along z-axis.



Let the length of projection segment be V.
Then,

$$V = \sqrt{B^2 + C^2} \text{ and}$$

$$\sin\alpha = \frac{B}{V} \text{ and } \cos\alpha = \frac{C}{V}$$

The rotation angle α is the angle between the line segment projection in the yz plane and the positive z-axis.

Required transformation ~~axis~~ matrix to rotate about X-axis is,

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C/V & -B/V & 0 \\ 0 & B/V & C/V & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The above figure shows the general rotation axis, lying on xz plane.

$\therefore$ x co-ordinate value will be unchanged ie A.

y co-ordinate will be 0.

The overall length of segment P_1P_2 is L

$\therefore L = \sqrt{A^2+B^2+C^2}$

z-co-ordinate value will be,

$$\sqrt{L^2 - A^2}$$
$$= \sqrt{B^2+C^2}$$
$$= \underline{\underline{V}}$$

P_2

P_1

$L^2 = A^2 + B^2 + C^2$

$\sqrt{L^2 - A^2} = \sqrt{B^2+C^2}$

Position of point P_2 will be defined as $(A, 0, V)$.

Step 2 : Rotate the rotation axis onto y-axis.

The angle of rotation is β. clockwise

$$\therefore \sin\beta = \frac{A}{L} \qquad \cos\beta = \frac{V}{L}$$

ie. Required rotation matrix is,

$$R_y = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} V/L & 0 & -A/L & 0 \\ 0 & 1 & 0 & 0 \\ A/L & 0 & V/L & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IV. Rotate the object around z-axis.

Specified angle of rotation is θ in anticlockwise direction.

Required transformation matrix is;

$$R_y = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & \\ \sin\beta & 0 & \\ 0 & 0 & \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

V: Rotate the general rotation axis to its original ~~axis~~ orientation.

Step-1: Rotate rotation axis about Y-axis by an angle β in anti-clockwise direction.
Required transformation matrix is,

$$R_y^{-1} = \begin{bmatrix} V/L & 0 & A/L & 0 \\ 0 & 1 & 0 & 0 \\ -A/L & 0 & V/L & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-2: Rotate the rotation axis about X-axis an angle α in clockwise direction.
Reqd. transformation matrix is R_x^{-1}:

$$R_x^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C/V & B/V & 0 \\ 0 & -B/V & C/V & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Important. Randeep Mam.

and core stores No. of bits assigned to each pixels may be minimum 1 & maximum 24. Nowadays, most of the frame buffers are constructed from random access integrated circuit memory with between 1 & 8 bit assigned to each pixel.

Rotating Memory Frame Buffer in ...ster Display 13

Earliest frame buffer made use of disks and drums for image storage. The rotational frequency of this device can be made to coincide with the frequency selected for the TV refresh. Therefore, it is possible to read the intensity values from the drum or disk, convert them into analog voltage and thus, construct a video signal. Each track of the memory device provides a single stream of bits, therefore one bit of memory represents each pixel. For more intensity precisions, several tracks must be used in parallel. Fig. shows use of four tracks in parallel to provide the frame buffer with 16 different intensity levels.

$2^4 = 16$

Important

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and core stores. No. of bits assigned to each pixels may be minimum 1 & maximum 24. Nowadays, most of the frame buffers are constructed from random access integrated circuit memory with between 1 & 8 bit assigned each pixel.

Rotating Memory Frame Buffer on Raster Scan Display

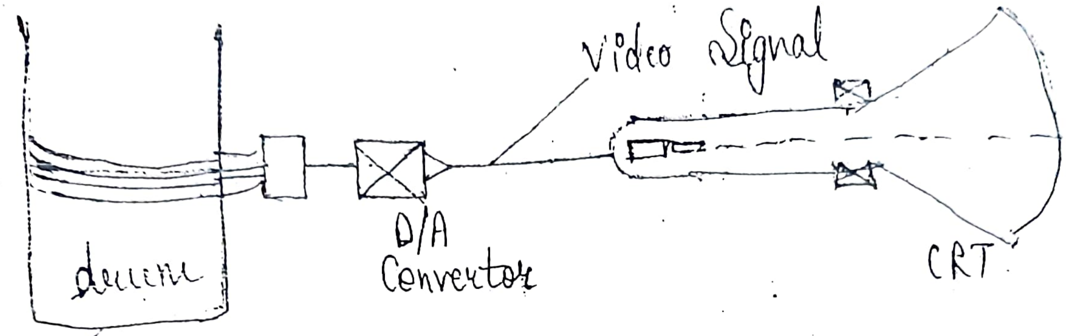
Earliest frame buffer made use disks and drums for image storage. The rotational frequency of this device can be made to coincide with the frequency selected for the TV refresh. Therefore, it is possible to read the intensity values from the drum or disk, convert them into analog voltage and thus, construct a video signal. Each track of the memory device provides a single stream of bits, therefore one bit of memory represents each pixel. For more intensity precisions, several tracks must be used in parallel. Fig. shows use of four tracks in parallel to provide the frame buffer with 16 different intensity levels.

$2^4 = 16$

Disadvantages :-

1) High cost of the memory

2) Requires more time due to latency problem



* Shift register frame buffer :-

To Generate a video signal from a digital memory, we must read the contents of the memory at a constant high frequency. Refresh intensity value is required each time, the electron beam moves to a new pixel. On a standard TV monitor this occurs every 90 nano sec. or less and high resolution display, time drops to 25 ns or less. Hence, integrated circuits are better able to achieve high memory bandwidth needed.

Shift register is a particularly appropriate circuit for use in a frame buffer, everytime the pulse applied to the shift register contents are shifted by one place placing one bit from one end register and allowing one bit to added at the other end. As each emerges from the shift registers can be used as an intensity value and then inserted into the other end the register to keep the contents circulating. Several shift registers in parallel can be used where more than one bit of intensity per pixel is required. eg 256 scan lines each of 340 pixels, we use 340 shift reg. of 256 bits each.

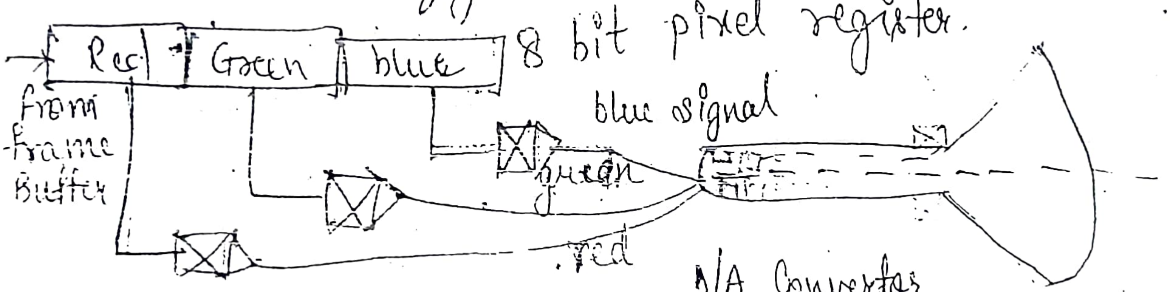
Random Access Frame Buffer

Modern frame-buffer uses random access integrated memory circuits. Each pixels intensity is represented by 1, 2, 4, 8 or more bits of memory. 1 bit is sufficient for text and simple graphics. 2 & 4 bit are useful in application that requires the display of solid areas of grey or colour and 8 or more bits are needed for high quality shaded pictures.

encoding to store in FB

The simplest method can be used to encode the colour pictures for storage in a frame buffer is to define the color component of each pixel. The bit representing the pixel can be divided into three groups of bits, each indicating the intensity of one of the three primary colour components.

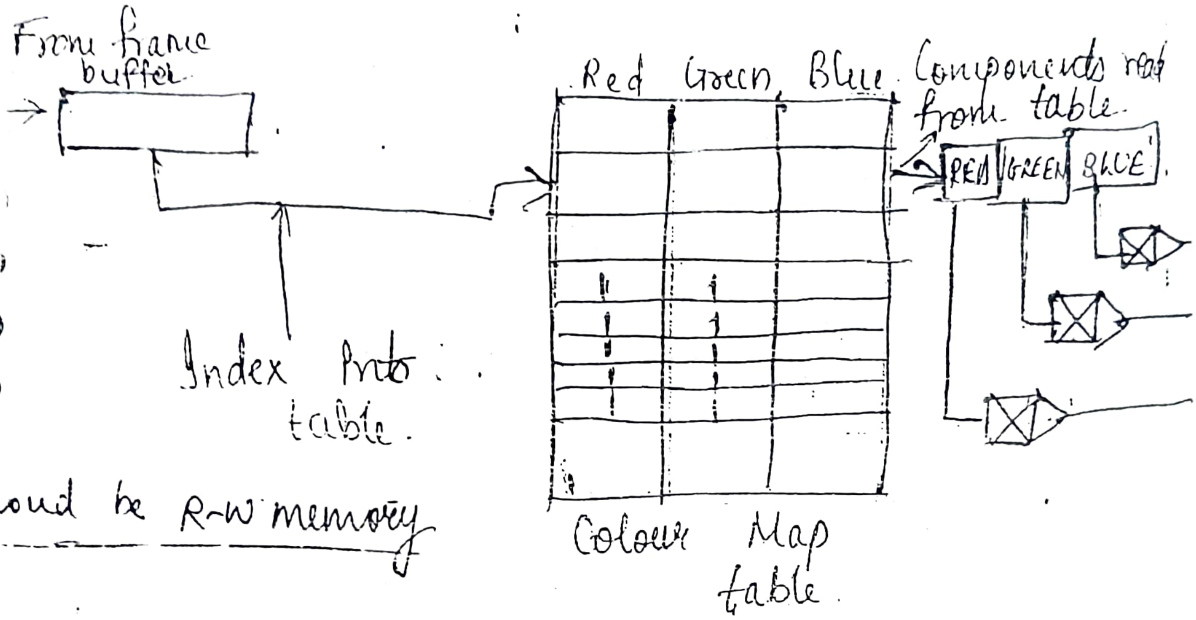
In 8-bit byte, 3 bits normally allocated to red, three bits to green & 2 bits to blue. The three components are then fed to the three guns of the color TV monitor. This arrangement is shown in figure.



Color Mapping

The simplest colour component encoding scheme has the disadvantages of limiting the range of colours. A more flexible screen involves the use of colour map. The values stored in the frame buffer are treated as addresses into a table of colours defined by their red, green & blue components. Thus, 8 bit per point frame buffer could address a 256 color table. Each of the color component can be defined to high precision, thus, providing very accurate control

...er the colour display. Figure shows the organisation of colour map.



...ould be R-W memory

* **Multiple Plane Frame Buffer** :- Provision of multiple bits per pixel is not only useful in representing the intensity and colour but allows the frame buffer to be treated as several planes each containing a seperate image. Division in two planes can be made in several diff. ways such as e.g. 8 bit per pixel frame buffer can represent a single image to 8 bits of intensity precision, 2 images to 4 bit of precision, 4 images to 2 bit of precision or 8 seperate B/W

images. Other assignments of bits can be are such as 4-2-2.

By dividing the frame buffer into planes we can apply a variety of diff. kinds of video mixing. One plane can be used to show a static picture and another to show a symbol or picture part that the users wishes to drag around the screen. In animation system, several moving objects can be displayed as a seperate planes. ~~One~~ 1 bit plane can be used as a mask to select certain regions of another plane for display.

* Strength and weakness of the frame buffer

Frame buffer is one of the most versatile display device. Given 8 or more bits of intensity precision, it can produce the colour & monochrome images whose quality are limited only by the performance and complexity of the monitor on which they are displayed.

For application that involves shading solid areas of colour, high quality text or any type of image processing, the frame buffer offers the only satisfactory form of display.

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PROGRESSIVE THEORY TEST I / II / 20.....

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vmware-config.pl
path
cd ddos
ns trust.tcl
nam out.nam

cd ns-allin
cd ns2
make cl
make

Ch 4 WINDOWING AND CLIPPING :

* The Co-ordinate System :-

If a counter-clockwise rotation of 90° about the origin aligns positive x-axis with the positive Y-axis, the co-ordinate system is called right-handed co-ordinate system, otherwise it is left handed co-ordinate system.

1) World Co-ordinate System :- (WCS)

- World Co-ordinate system is right-handed co-ordinate system. The object model that resides in object space is described in WCS. This model represents the object in the physical units of length.

- WCS is theoretically infinite in extent.

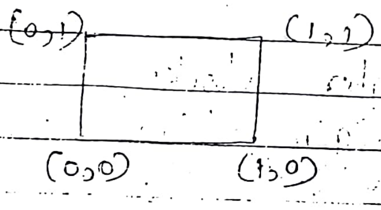
2) Physical Device Co-ordinate System :- (DCS).

:- Co-ordinate system that corresponds to device or workstation where the image or picture ~~of~~ object model is to be displayed.

- DCS can be left handed or Right-hand co-ordinate system depending on workstation.

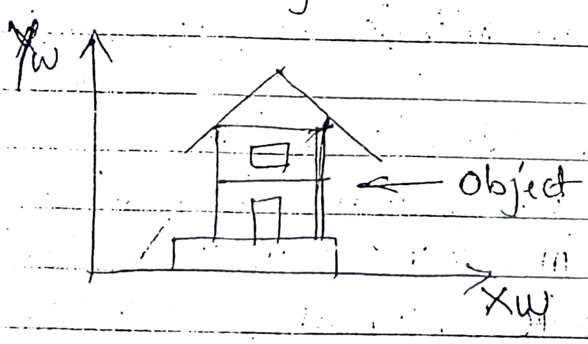
3) Normalised device co-ordinate system: ND

- It is right handed co-ordinate system in which the display area of the virtual display device corresponds to the unit square.

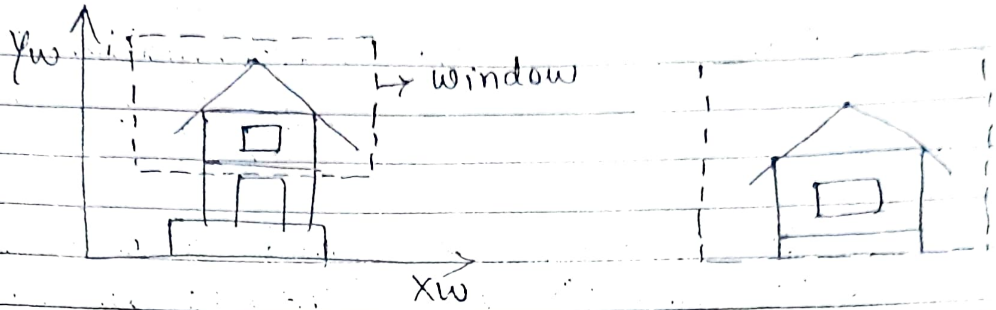


* Concept of Window and Viewport :-

Consider building described in WCS.

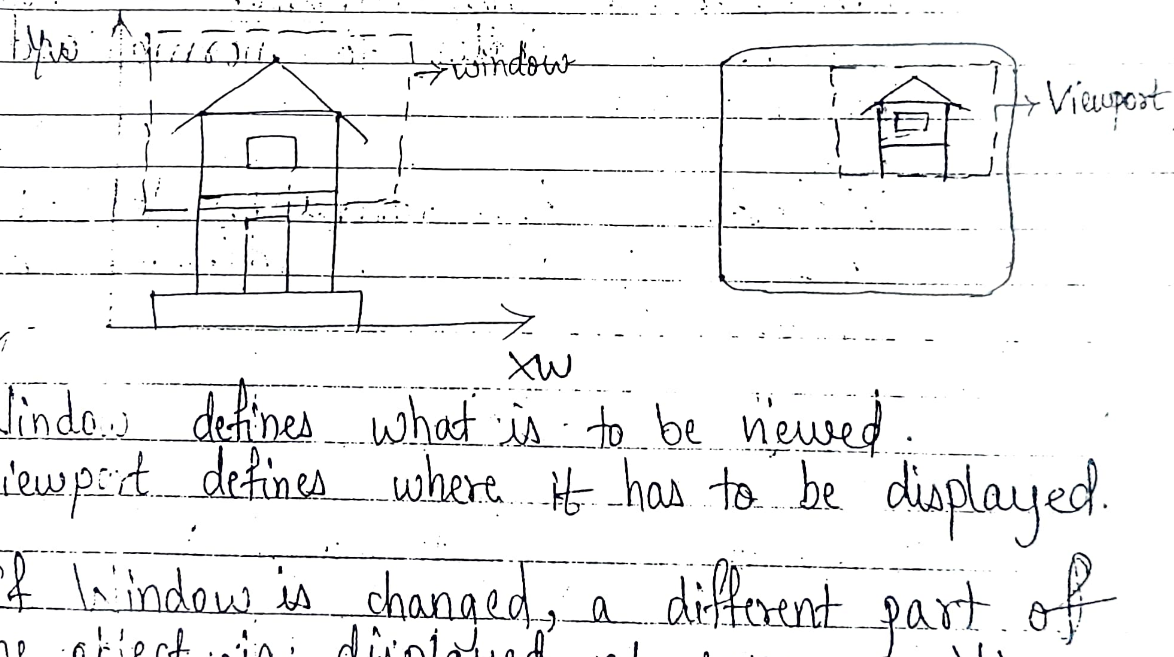


A World Co-ordinate area selected for display is called as window.



The window can be directly mapped onto the display area or onto a subregion of the display device.

An area on display device to which a window is mapped is called a viewport.



Window defines what is to be viewed.
Viewport defines where it has to be displayed.

If window is changed, a different part of the object is displayed at same position on the screen.

If viewport is changed, the same portion of the object is displayed at different position on the display device.

* Window to Viewport transformation :-

Also called as Viewing transformation OR, Windowing transformation.

The mapping of world co-ordinates to device co-ordinates is referred to as viewing transformation. This is the transformation from window to viewport.

Viewing transformation is formed by the following transformations :-

1) Normalization transformation N that maps world co-ordinates to normalized device co-ordinates.

2) Workstation transformation W, that maps normalized device co-ordinates to device co-ordinates.

$$\therefore V = N \cdot W$$

Normalization transformation :-

As different display devices have different screen sizes measured in pixels, we need to make our program display device independent. Therefore, device independent co-ordinate system is used to display picture on screen. This device independent units system is called as "Normalized Device Co-ordinate System".

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VI. Translate the rotation axis to its original position. Required transformation matrix is,

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The composite transformation matrix is,

$$TM = T^{-1} \cdot Rx^{-1} \cdot Ry^{-1} \cdot \cancel{Rz} \cdot Ry \cdot Rz \cdot T.$$

Any point P on object will get transformed to P' such that,

$$P' = TM \cdot P.$$

(44)

* Sums :-

1) A triangle ABC defined by 3 vertices are A (0,2,1) B(2,3,0) and C(1,2,1). Find the final co-ordinates after it is rotated by 45° around a line joining the points (1,1,1) and (0,0,0).

→ Here, the axis of rotation passes through origin. (as one of the point is (0,0,0))

∴ Translation matrices are not required.

$T_{RM} = R_{xy}^{-1} \cdot R_{z} \cdot R_{xy}$

We know, $a=1, b=1, c=1$.

$L = \sqrt{a^2+b^2+c^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

$V = \sqrt{b^2+c^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$

$R_{xy} = R_x \cdot R_y$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C/V & -B/V & 0 \\ 0 & B/V & C/V & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V/L & 0 & -A/L & 0 \\ 0 & 1 & 0 & 0 \\ A/L & 0 & V/L & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} V/L & 0 & -A/L & 0 \\ -AB/LV & C/V & -B/L & 0 \\ AC/VL & B/V & C/L & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/\sqrt{3} & 0 & -1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(113)

$$R_{xy}^{-1} = \begin{bmatrix} V/L & -AB/LV & AC/VL & 0 \\ 0 & C/V & B/V & 0 \\ -A/L & -B/L & C/L & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/\sqrt{3} & -1/\sqrt{6} & 1/\sqrt{6} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos 45 & \sin 45 & 0 & 0 \\ -\sin 45 & \cos 45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\therefore TM = \begin{bmatrix} \sqrt{2}/\sqrt{3} & 0 & -1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} & 0 \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/\sqrt{3} & -1/\sqrt{6} & 1/\sqrt{6} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} \sqrt{4}/\sqrt{9} & \\ & \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix}}$$

$$\begin{bmatrix} 0.816 & 0 & -0.577 & 0 \\ -0.408 & 0.707 & -0.577 & 0 \\ 0.408 & 0.707 & 0.577 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.816 & -0.408 & 0.408 & 0 \\ 0 & 0.707 & 0.707 & 0 \\ -0.577 & -0.577 & 0.577 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

4/6

$$\begin{bmatrix} 0.707 & 0.707 & 0 & 0 \\ -0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 0.998 & 0 & 0 & 0 \\ -0.499 & 0 & 0 & 0 \\ 0 & 0 & 0.999 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 & 0 & 0 \\ -0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.705 & 0.705 & 0 & 0 \\ -0.352 & -0.352 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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2) Find new co-ordinates of line joinging two points (0,0,0) and (5,5,5) after shifting it right by 2 units and up by 3 units.

→ Shifting means moving i.e. translation.
Right means x-direction & up means Y direction
∴ $t_x = 2$ & $t_y = 3$, $t_z = 0$.

∴ let A'B' be the new co-ordinates of line after translation.

∴ $A'B' = T \cdot AB$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 \\ 0 & 5 \\ 0 & 5 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 7 \\ 3 & 8 \\ 0 & 5 \\ 1 & 1 \end{bmatrix}$$

∴ $A'(2,3,0)$
$B'(7,8,5)$

Scale square ABCD with co-ordinates A(0,0) B(5,0) C(5,5) & D(0,5) by 2 units for x-direction & 3 units for y direction.

Translate square ABCD where A(0,0) B(3,0) C(3,3) D(0,3) by 2 units in both directions & then scale it by 1.5 units in x direction & 0.5 units in y-direction.

Rotate triangle ABC where A(0,0) B(6,0) C(3,3) by 90° about origin.

Find transformation matrix that transforms square ABCD whose center is at (2,2) is reduced to half of its size with center still remaining at (2,2). The co-ordinates of ABCD are A(0,0), B(0,4) C(4,4) & D(4,0). Find co-ordinates of new square.

Perform x-shear & y-shear on triangle having A(2,1) B(4,3) & C(2,3). Consider Shx = 2 & Shy = 2.

X-Shear A'(4,1) B'(10,3) C'(8,3).

Y Shear A'(2,5) B'(4,11) C'(2,7).

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PROGRESSIVE THEORY TEST I / II / 20..... 2

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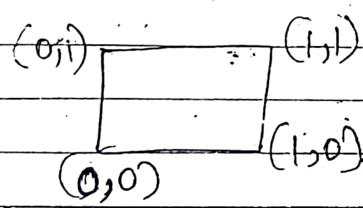
In this Normalized Device Co-ordinate System, the screen is measured as 1 unit in width and 1 unit in height ie. 1X1 units. Lower left corner of screen is at the origin of co-ordinate system and upper right corner is the point (1,1). The point (0.5,0.5) is the center of the screen. It is shown in fig.



Here some interpreter is used to convert these normalized Device Co-ordinates to appropriate pixel values for respective display device.

Interpreter converts Normalised Device Co-ord. by using foll. eq^n

$$X = X_n + X_w$$
$$Y = Y_n + Y_H$$

X,Y → Actual device x & y co-ord.

X_n, Y_n — Normalised device x & y.

X_w — width of Actual screen in pixels

Y_H — Height of Actual screen in pixels.

Workstation transformation :-

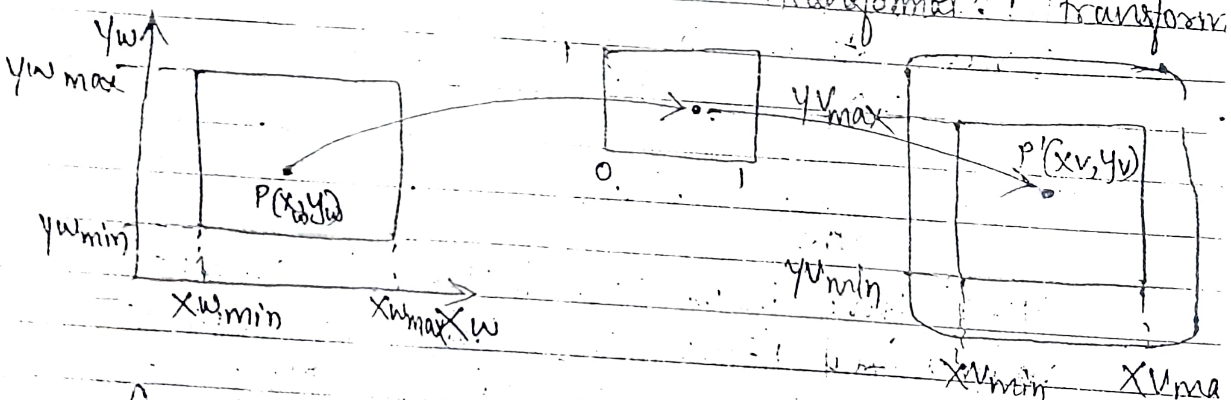
The process that converts normalised device co-ordinates to physical device coordi is called as workstation transformation.

Window :- A world co-ordinate area selected fo display is called as window.
Window defines what is to be viewed.

Viewport :- A area on display device to whic window is mapped is called viewpor
Viewport defines where it is to be viewed.

$V = W \cdot N$.

ie: Viewing transformation = Normalisation transformat? * Workstat transform



Consider a point P(xw,yw) in world co-ordinate system mapped to P'(xv,yv) in device co-ordinate system.

then,

$$\frac{Xv - Xvmin}{XVmax - Xvmin} = \frac{Xw - Xwmin}{Xwmax - Xwmin} \quad (1)$$

$$\frac{y_v - y_{vmin}}{y_{vmax} - y_{vmin}} = \frac{y_w - y_{wmin}}{y_{wmax} - y_{wmin}} \quad \text{--- (2)}$$

From eqn. (1) & (2),

$$x_v = x_{vmin} + (x_w - x_{wmin}) \cdot S_x$$

$$y_v = y_{vmin} + (y_w - y_{wmin}) \cdot S_y$$

where,

$$S_x = \frac{x_{vmax} - x_{vmin}}{x_{wmax} - x_{wmin}} = \frac{\text{Viewport x extent}}{\text{window x extent}}$$

$$S_y = \frac{y_{vmax} - y_{vmin}}{y_{wmax} - y_{wmin}} = \frac{\text{Viewport y extent}}{\text{window y extent}}$$

Window To Viewport Transformation Algorithm :-

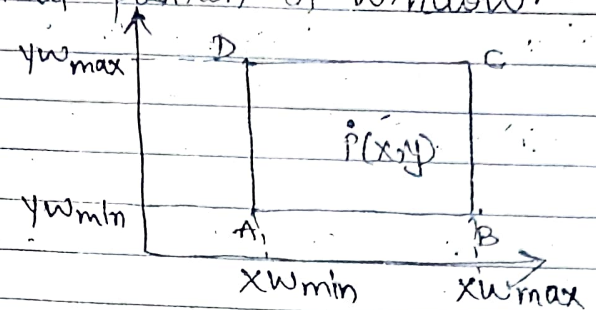
— Translate the object so that lower left corner of the window is moved to the origin.

— Scale the object so that the window ~~to~~ has the dimensions of the viewport.

— Translate the object so that the scaled window area is positioned to viewport.

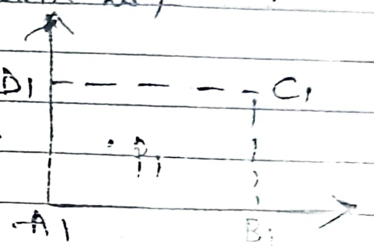
8

I. Initial position of window.



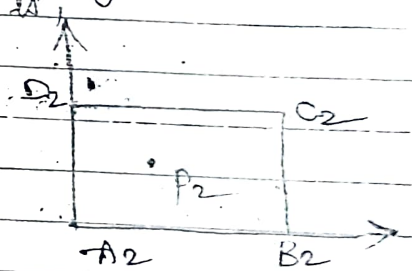
II. Translate (xw_{min}, yw_{min}) to the origin.
The Required translation matrix is,

$$T_1 = \begin{bmatrix} 1 & 0 & -xw_{min} \\ 0 & 1 & -yw_{min} \\ 0 & 0 & 1 \end{bmatrix}$$



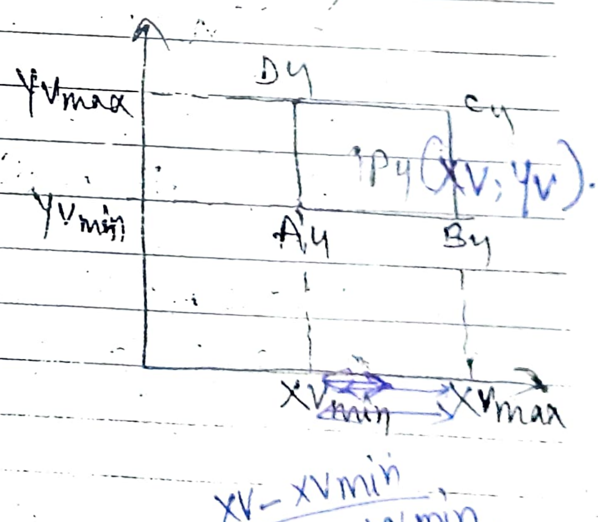
III. Scale with respect to origin.
Required scaling matrix is,

$$S = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



IV. Translate to (xv_{min}, xv_{max}).
The required translation matrix is,

$$T_2 = \begin{bmatrix} 1 & 0 & xv_{min} \\ 0 & 1 & yv_{min} \\ 0 & 0 & 1 \end{bmatrix}$$



$$\frac{xv - xv_{min}}{xv_{max} - xv_{min}}$$

(9)

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The composite transformation matrix is,

$$V = T_2 \cdot S \cdot T_1$$

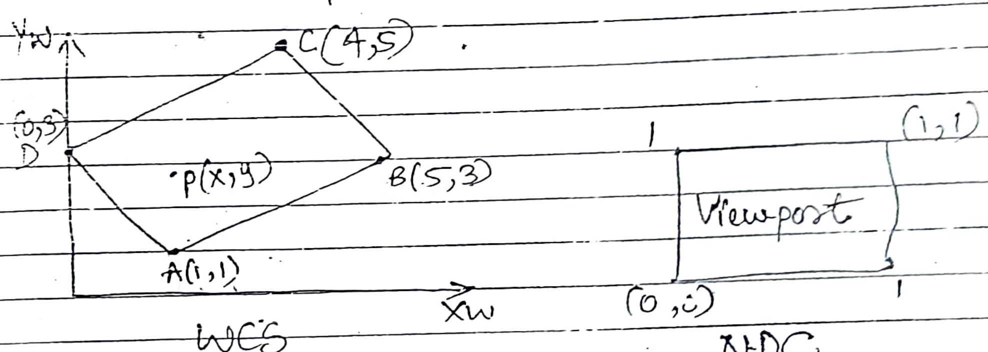
Any point $P(x,y)$ on the object will get transformed to $P'(x',y')$ such that,

$$P' = V \cdot P.$$

Examples :-

(1) Find a normalisation transformation which uses the rectangle ABCD $A(1,1)$ $B(5,3)$ $C(4,5)$ and $D(0,3)$ as window & NDC as a viewport.

⟹

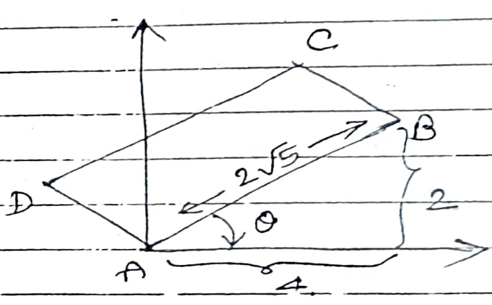


Step I :-

Translate point A(1,1) to the origin.

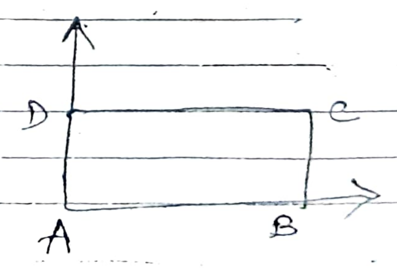
The reqd. translation matrix is,

$$T_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$



ep II :-

Rotate the rectangle about rigin by an angle θ in clockwise irection so that it is aligned with -ordinate axis.



The required rotation matrix is,

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

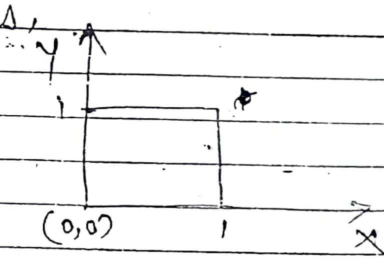
$$\cos\theta = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}, \quad \sin\theta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\therefore R = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step III :- Perform scaling transformation that transforms window to viewport.

The reqd. scaling matrix is,

$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



where,

$$S_x = \frac{\text{Viewport x extent}}{\text{windows x extent}} = \frac{1}{d(AB)}$$

$$S_y = \frac{\text{Viewport Y extent}}{\text{window Y extent}} = \frac{1}{d(AD)}$$

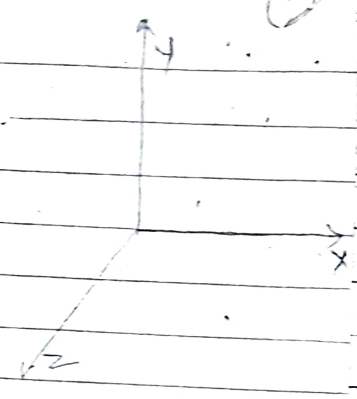
$$\therefore d(AB) = \sqrt{(3-1)^2 + (5-1)^2} = \sqrt{20} = 2\sqrt{5}.$$

$$d(AD) = \sqrt{(0-1)^2 + (3-1)^2} = \sqrt{5}.$$

$$\therefore S_x = \frac{1}{2\sqrt{5}}, \quad S_y = \frac{1}{\sqrt{5}}$$

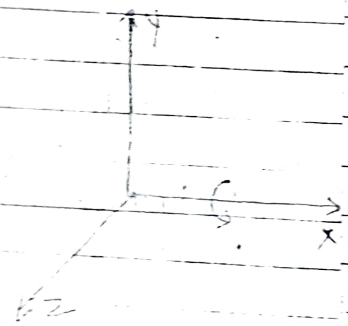
$$\therefore S = \begin{bmatrix} 1/2\sqrt{5} & 0 & 0 \\ 0 & 1/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The Reqd Transformation matrix is



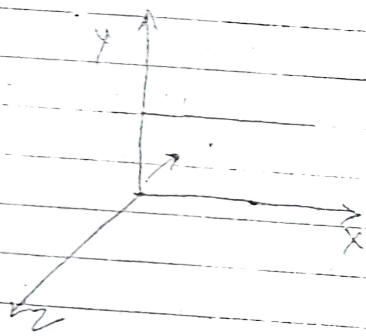
$$T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

III] Rotate about x-axis
The Reqd Transformation matrix is



$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

IV] Translate the Rotation axis to its original position
The Reqd Transformation matrix is



$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The composite Transformation matrix is

$$T_m = T_2 \cdot R_x \cdot T_1 = (T_2 \cdot R_x) \cdot T_1$$

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The composite transformation (N_T) matrix is :-

$$N_T = S \cdot R \cdot T$$

$$= \begin{bmatrix} 1/2\sqrt{5} & 0 & 0 \\ 0 & 1/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore N_T = \begin{bmatrix} 1/5 & 1/10 & -1/10 \\ -1/5 & 2/5 & -1/5 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Find normalization transformation matrix that maps a window whose lower left corner is at (1,1) and upper right corner is as (3,5) on to

a) a viewport which is entire normalized device screen.

b) A viewport that has lower left corner is at (0,0) and upper right corner $(\frac{1}{2}, \frac{1}{2})$

$\Rightarrow$ $N_T = T_2 \cdot S \cdot T_1$

$$= \begin{bmatrix} 1 & 0 & X_{vmin} \\ 0 & 1 & Y_{vmin} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -X_{wmin} \\ 0 & 1 & -Y_{wmin} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & X_{vmin} \\ 0 & S_y & Y_{vmin} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -X_{wmin} \\ 0 & 1 & -Y_{wmin} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & -S_x \cdot x_{wmin} + x_{vmin} \\ 0 & S_y & -S_y \cdot y_{wmin} + y_{vmin} \\ 0 & 0 & 1 \end{bmatrix}$$

where $S_x = \dfrac{X_{vmax} - X_{vmin}}{x_{wmax} - x_{wmin}}$ & $S_y = \dfrac{Y_{vmax} - Y_{vmin}}{y_{wmax} - y_{wmin}}$

$= \dfrac{1-0}{3-1} = \dfrac{1}{2}$ $\qquad = \dfrac{1-0}{5-1} = \dfrac{1}{4}$

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a) Window parameters,

$xw_{min} = 1$ $xw_{max} = 3$

$yw_{min} = 1$ $yw_{max} = 5$.

Viewport parameters,

$xv_{min} = 0$ $xv_{max} = 1$

$yv_{min} = 0$ $yv_{max} = 1$.

and $S_x = \frac{1}{2}$

$S_y = \frac{1}{4}$

$$\therefore N_T = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/4 & -1/4 \\ 0 & 0 & 1 \end{bmatrix}$$

b) Window parameters :-

$xw_{min} = 1$ $xw_{max} = 3$

$yw_{min} = 1$ $yw_{max} = 5$

Viewport parameters :-

$xv_{min} = 0$ $xv_{max} = 1/2$

$yv_{min} = 0$ $yv_{max} = 1/2$

and $S_x = \frac{1/2}{2} = \frac{1}{4}$

$S_y = \frac{1/2}{4} = \frac{1}{8}$

$$\therefore N_T = \begin{bmatrix} 1/4 & 0 & -1/4 \\ 0 & 1/8 & -1/8 \\ 0 & 0 & 1 \end{bmatrix}$$

③ Find normalization transformation matrix from window whose lower left corner is at (0,0) and upper right corner is at (4,3) onto normalised device screen so that aspect ratios are preserved.

11

3

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The window aspect ~~are~~ ratio is $a_w = 4/3$.
we should choose a viewport that is as large as possible w.r.t. normalized device screen.

Therefore, we choose x extent from 0 to 1 and y extent from 0 to 3/4.

$\therefore a_v = \frac{4}{3}$

$\therefore a_v = \frac{1}{3/4}$

Window Parameters,

$xw_{min} = 0$ $yw_{min} = 0$
$xw_{max} = 4$ $yw_{max} = 3$.

Viewport Parameters,

$xv_{min} = 0$ $xv_{max} = 1$
$yv_{min} = 0$. $yv_{max} = 3/4$.

$$N_T = \begin{bmatrix} S_x & 0 & -S_x \cdot xw_{min} + xv_{min} \\ 0 & S_y & -S_y \cdot yw_{min} + yv_{min} \\ 0 & 0 & 1 \end{bmatrix}$$

where, $S_x = \frac{xv_{max} - xv_{min}}{xw_{max} - xw_{min}} = \frac{1}{4}$

and $S_y = \frac{yv_{max} - yv_{min}}{yw_{max} - yw_{min}} = \frac{3/4}{3} = \frac{1}{4}$

$$\therefore N_T = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ind NTM that maps window (1,1) & (6,6) to NCS.

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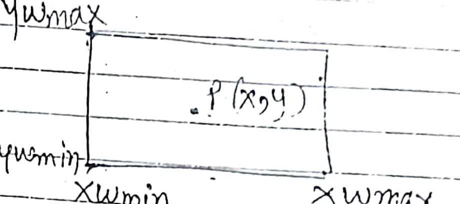
* Clipping :-

The process which divides each element of the picture into its visible and invisible positions, allowing invisible portion to be discarded is called as Clipping.

(1) Point Clipping :-

In point clipping, if a specified point is inside the clipping window then it is accepted and displayed on the screen. And if it is outside the clipping window then it is rejected and is not displayed on the screen.

Any point P (x, y) is inside the window if all of the following inequalities are satisfied.

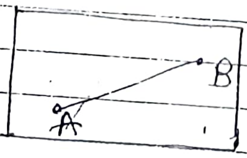


$xw_{min} \le x \le xw_{max}$

$yw_{min} \le y \le yw_{max}$

(2) Line Clipping :-

Consider a line segment with end points A (x_1, y_1) and B (x_2, y_2).



Category 1 : If both endpoints of line segment are within the window, the line segment is VISIBLE.

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Category 2 :- If the line segment from (x_1, y_1) to (x_2, y_2) satisfies any one of the following, the line segment is NOT - VISIBLE.

$x_1\ x_2 < xw_{min}$ $\quad$ $y_1\ y_2 < yw_{min}$

$x_1\ x_2 > xw_{max}$ $\quad$ $y_1\ y_2 > yw_{max}$.

Category 3 :- If the line segment is neither of category 1 nor of category 2 then it is a clipping candidate.

Cohen - Sutherland Line Clipping Algorithm :-

STEP I :-
Accept the end points of line segments and window boundaries.
i.e. (x_1, y_1) (x_2, y_2)
(xw_{min}, yw_{min}) (xw_{max}, yw_{max})

STEP II :-
Assign a four bit code to each end point of the line segment i.e. $B_4\ B_3\ B_2\ B_1$.

if $x_1 < xw_{min}$, then $B_1 = 1$ else $B_1 = 0$
if $x_2 > xw_{max}$, then $B_2 = 1$ else $B_2 = 0$
if $y_1 < yw_{min}$, then $B_3 = 1$ else $B_3 = 0$
if $y_2 > yw_{max}$, then $B_4 = 1$ else $B_4 = 0$.

1001	1000	1010
0001	0000	0010
0101	0100	0110

TBRL

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STEP III :-

a) If both the end point codes are 0000, the line is VISIBLE.
Display the line segment.
Stop.

b) If logical AND of the endpoint code is not 0000, then line segment is not visible (INVISIBLE).
Discard the line segment
Stop.

c) If logical AND of the end point code is 0000, then the line segment is clipping candidate.

STEP IV :-

Determine the intersecting boundary.

If $B_1 = 1$, intersect with $x = xwmin$
If $B_2 = 1$, intersect with $x = xwmax$
If $B_3 = 1$, intersect with $y = ywmin$
If $B_4 = 1$, intersect with $y = ywmax$.

STEP V :-

Determine the intersecting point co-ordinates (x', y')

The equations are :-

$$x' = x_1 + \frac{1}{m}(y' - y_1) \quad \text{where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

and here, $y' = ywmin$ OR
$y' = ywmax$.

$y' = y_1 + m(x' - x_1)$

Here, $x' = xwmin$ OR

$x' = xwmax$.

STEP VI :-

Goto Step II until you get totally visible line segment.

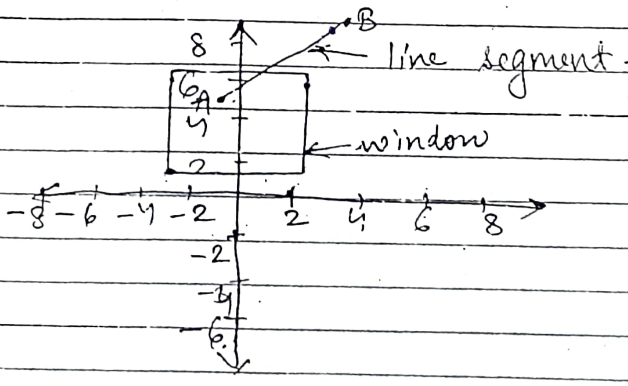
STEP VII :- STOP

Ex. Find the clipping co-ordinates of line joining A(-1,5) and B(3,8). A(-2,3) B(7,2) W(0,0) (5,5)

→ Step I :- A($x_1=-1$, $y_1=5$) B($x_2=3$, $y_2=8$)

xwmin = -3   xwmax = 2

ywmin = 1   ywmax = 6



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1001	1000	1010
0001	0000	0010
0101	0100	0110

Step 2 :- For A(-1,5).

Code = 0000

Point A is visible

For B(3,8).

Code = 1010

Point B is not visible.

0000
⊙ 1010
0000   ∴ It is a clipping candidate

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Step 3 :- Intersecting boundary,

$B_2 = 1$, intersecting with $x = xwmax$.
$B_4 = 1$, intersecting with $y = ywmax$.

Select $x = xwmax$ as a clipping ~~candidate~~ boundary.

Step 4 :- Intersecting Point Co-ordinates.

$$x' = xwmax = 2$$

$$y' = y_1 + m(x' - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8-5}{3-(-1)} = \frac{3}{4}.$$

$$\therefore y' = 5 + \frac{3}{4}(2-(-1))$$

$$= \frac{29}{4}.$$

$$\therefore I_1 = \left(2, \frac{29}{4}\right)$$ ~~AI_1~~ is Clipped line segment.

Step 2 :- Code for $A(-1,5)$ = 0000
Code for $I_1\left(2, \frac{29}{4}\right)$ = 1000

$\therefore$ Point A is visible
Point I_1 is invisible.

A AND I_1 = 0000
1000
————
0000

$\therefore AI_1$ is Clipping Candidate,

Step 3 :- Determine ~~clip~~ intersecting boundary
$B_4 = 1$, intersect with $y = ywmax$.

Step 4 :- Determine intersecting ~~boundary~~ point co-ordinates.

~~$B_4 = 1$, int~~

(6.)

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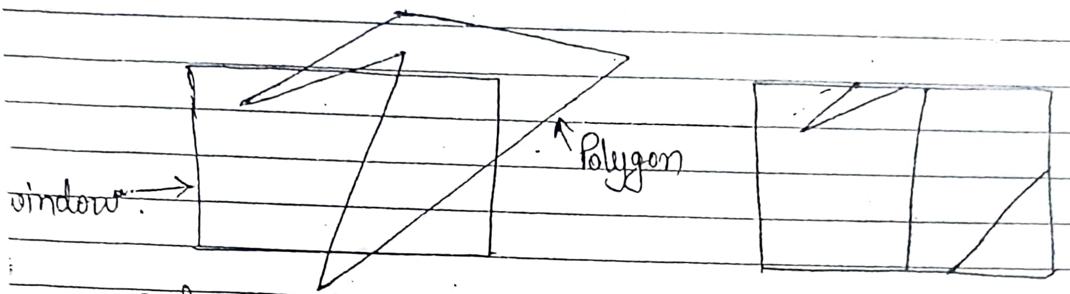
* Polygon Clipping :-



Before clipping. After Clipping

* Sutherland - Hodgeman Polygon Clipping Algo. :-

A polygon is clipped ~~as a~~ by processing all its edges as a single whole unit against each edges of clipping window. It can be done by processing all polygon vertices against each edge of clipping window as follows :-

i) Original set of polygon vertices are first clipped against left edge of window to produce new set of vertices (left clipper).

) New set of vertices is now clipped against right edge to produce new set of vertices (right clipper).

) Again this new set is clipped against top and then bottom edge of clip window to produce final sequence of polygon vertices.

r producing new set of vertices, all vertices are processed in foll. way against boundary nd this processing is repeated against rest l boundaries till we get final set of vertices.

Processing vertices :-

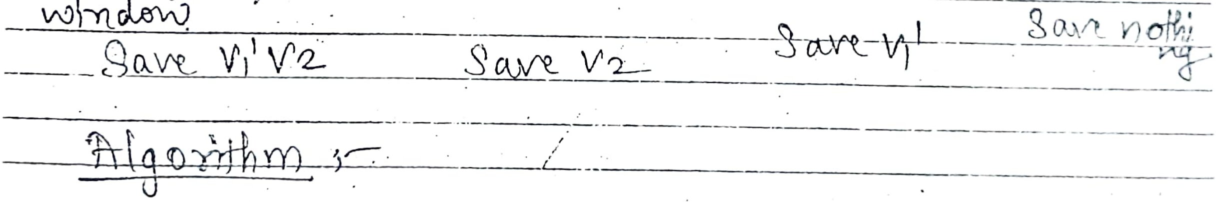
1. If first vertex is OUTSIDE & second vertex is INSIDE the window boundary, then intersecting point of polygon edge with window boundary & second vertex are added to output vertices set.

2. If both the vertices of edge are inside window boundary, then add only second vertex to output set.

3. If first vertex of edge is INSIDE & second is OUTSIDE the window boundary, then point of intersection of edge with window boundary is stored in output set.

4. If both vertices of edge are OUTSIDE of window boundary then, nothing is added to output set.

Processing of vertices is shown below :-



Save V1' V2   Save V2   Save V1'   Save nothing

Algorithm :-

Step 1 :- Start

Step 2 :- Read co-ordinates of all vertices of the polygon and the window

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$y' = yw_{max} = 6$ $\quad m = \frac{3}{4}$

$x' = x_1 + \frac{1}{m}(y' - y_1)$

$= -1 + \frac{4}{3}(6-5)$

$= \frac{1}{3}$

$I_2(x', y') = I_2\left(\frac{1}{3}, 6\right)$

Step 2 :- Code for $A(-1,5)$ is 0000
Code for $I_2\left(\frac{1}{3}, 6\right)$ is 0000

∴ The Both points are visible
Display the line segment AI_2
Stop.

2) Midpoint Subdivision line Clipping Algorithm

Step I :-
Accept the two end points of the line (x_1, y_1) & (x_2, y_2) and the window parameters (xw_{min}, yw_{min}) & (xw_{max}, yw_{max})

Step II :-
Assign 4 bit region codes to both end points.

if $x_1 < xw_{min}$, $B_1 = 1$ else $B_1 = 0$
if $x_2 > xw_{max}$, $B_2 = 1$ else $B_2 = 0$
if $y_1 < yw_{min}$, $B_3 = 1$ else $B_3 = 0$
if $y_2 > yw_{max}$, $B_4 = 1$ else $B_4 = 0$

Step III :

a) If both the end point codes are 0000, the line is visible.

Display the line segment.
Stop

b) If logical AND of the endpoint code is not 0000, then the line segment is INVISIBLE.
Discard the line segment.
Stop.

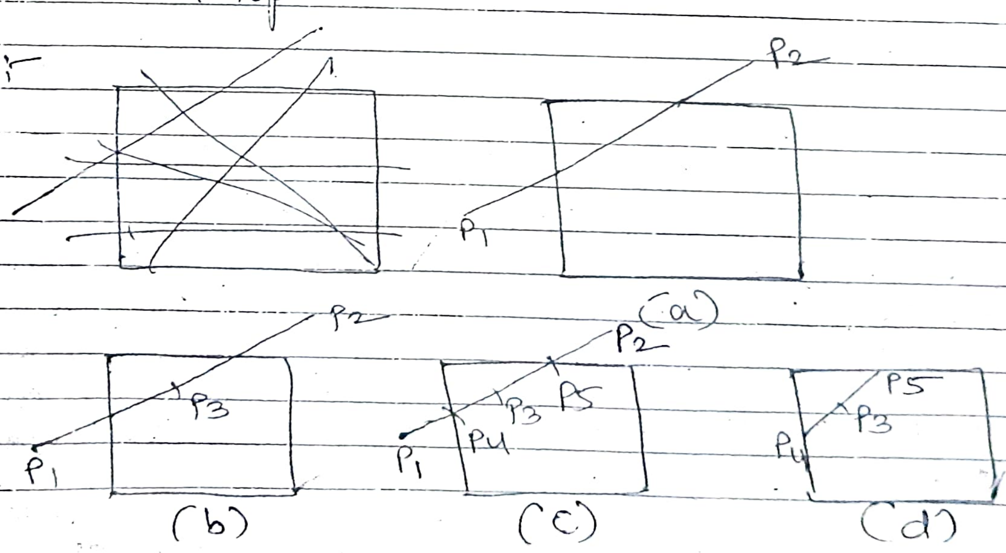
c) If logical AND of end point code is 0000, then the line segment is clipping candidate.

Step IV :-
For the clipping candidate line segment, find out its midpoint and divide it into two equal line segments.

Step V :-
For both divided line segments, repeat the steps 2 to 4 until you get completely visible and completely invisible line segments i.e. category 1 & 2

Step VI :-
Stop.

Ex :-



lower left & upper right corner co-ordinate.

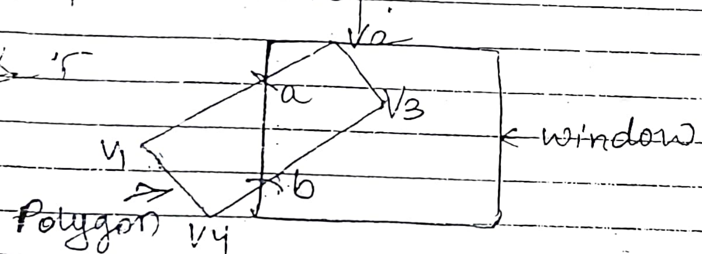
Step 3 :- Process the vertices against left edge of window to get output vertices set.

Step 4 :- Repeat Step 3 for right, top & bottom edges of window to get final output vertices set.

Step 5 :- Display polygon connecting all vertices from output vertices set.

Step 6 :- Stop.

Ex :-



1. V_1 - outside V_2 - inside
   save a V_2
2. V_2 V_3 - inside save V_3
3. V_3 - inside V_4 - outside
   save b.
4. V_4 V_1 - outside save nothing

∴ Output Set = a V_2 V_3 b.

Draw the polygon from output set.