Homework 3

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Q1

a) The corresponding descrambler is

$$S(K) = T(K) \oplus T(K-3) \oplus T(K-5)$$

$$S = T \oplus D^3T \oplus D^5T$$

b)

S	DT	D^2T	D^3T	D^4T	D^5T	T	$T = S \oplus D^3T \oplus D^5T$
1	1	0	1	0	1	1	$T = 1 \oplus 1 \oplus 1 = 1$
1	1	1	0	1	0	1	$T = 1 \oplus 0 \oplus 0 = 1$
1	1	1	1	0	1	1	$T = 1 \oplus 1 \oplus 1 = 1$
1	1	1	1	1	0	0	$T = 1 \oplus 1 \oplus 0 = 0$
1	0	1	1	1	1	1	$T = 1 \oplus 1 \oplus 1 = 1$
0	1	0	1	1	1	0	$T = 0 \oplus 1 \oplus 1 = 0$
0	0	1	0	1	1	1	$T = 0 \oplus 0 \oplus 1 = 1$
0	1	0	1	0	1	0	$T = 0 \oplus 1 \oplus 1 = 0$
0	0	1	0	1	0	0	$T = 0 \oplus 0 \oplus 0 = 0$
0	0	0	1	0	1	0	$T = 0 \oplus 1 \oplus 1 = 0$

T = 1 1 1 0 1 0 1 0 0 0

c)

T	DT	D^2T	D^3T	D^4T	D^5T	S	$S = T \oplus D^3T \oplus D^5T$
1	1	0	1	0	1	1	$S = 1 \oplus 1 \oplus 1 = 1$
1	1	1	0	1	0	1	$S = 1 \oplus 0 \oplus 0 = 1$
1	1	1	1	0	1	1	$S = 1 \oplus 1 \oplus 1 = 1$
0	1	1	1	1	0	1	$S = 0 \oplus 1 \oplus 0 = 1$
1	0	1	1	1	1	1	$S = 1 \oplus 1 \oplus 1 = 1$
0	1	0	1	1	1	0	$S = 0 \oplus 1 \oplus 1 = 0$
1	0	1	0	1	1	0	$S = 1 \oplus 0 \oplus 1 = 0$
0	1	0	1	0	1	0	$S = 0 \oplus 1 \oplus 1 = 0$
0	0	1	0	1	0	0	$S = 0 \oplus 0 \oplus 0 = 0$
0	0	0	1	0	1	0	$S = 0 \oplus 1 \oplus 1 = 0$

$S = 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0$

2) $G(s) = \dfrac{3}{2s+\sqrt{5}}$

a) The closed loop transfer function $H(s)$

$k = 2$

$$H(s) = \frac{0.5\,k\,G(s)}{s + 0.5\,k\,G(s)}$$

$$G(s) = \frac{3}{2s+\sqrt{5}}$$

$$\Rightarrow H(s) = \frac{0.5k\left(\frac{3}{2s+\sqrt{5}}\right)}{s + \left(0.5k\left(\frac{3}{2s+\sqrt{5}}\right)\right)}$$

$$\Rightarrow \frac{0.5k \times 3}{2s+\sqrt{5}} \div s + \frac{0.5k \times 3}{2s+\sqrt{5}}$$

$$\Rightarrow \frac{0.5k \times 3}{2s+\sqrt{5}} \div \frac{s(2s+\sqrt{5}) + 0.5k \times 3}{2s+\sqrt{5}}$$

$$\Rightarrow \frac{0.5k \times 3}{\cancel{2s+\sqrt{5}}} \times \frac{\cancel{2s+\sqrt{5}}}{2s^2+\sqrt{5}s + 0.5k \times 3}$$

$$\Rightarrow \frac{0.5k \times 3}{2s^2+\sqrt{5}s + 0.5k \times 3}$$

Since $k = 2$

$$\Rightarrow \frac{0.5(2) \times 3}{2s^2 + \sqrt{5}\, s + 0.5(2) \times 3}$$

$$\Rightarrow \frac{3}{2s^2 + \sqrt{5}s + 3}$$

$$\Rightarrow \frac{3}{2\left(s^2 + \frac{\sqrt{5}}{2} s + \frac{3}{2}\right)}$$

$$\Rightarrow \frac{3/2}{s^2 + \frac{\sqrt{5}}{2} s + \frac{3}{2}}$$

Find the poles to determine the stability of the system.

$\pm \frac{\sqrt{19}}{4} j - \frac{\sqrt{5}}{4}$ The system is stable since real part is $-ve$

b)

$$s^2 + \frac{\sqrt{5}}{2} s + \frac{3}{2}$$

Damping factor ζ & natural frequency

Compare to $s^2 + 2\zeta\omega_n s + \omega_n^2$

$$\omega_n^2 = \frac{3}{2} \qquad \omega_n = \sqrt{\frac{3}{2}}$$

$$2\zeta\omega_n \cancel{s} = \frac{\sqrt{5}}{2} \cancel{s}$$

$$\cancel{2}\zeta\left(\sqrt{3/2}\right) = \frac{\sqrt{5}}{\cancel{2}} \times \frac{1}{2}$$

$$\zeta\left(\sqrt{3/2}\right) = \frac{\sqrt{5}}{4}$$

$$\zeta = \frac{\sqrt{5}/4}{\sqrt{3}/2}$$

$$= \frac{\sqrt{30}}{12}$$