ELEC 4190 – Digital Communications

Review: Signals and Systems

Outline

- Basics of signals and systems
- Signal representation in the frequency domain
- Analysis of LTI systems

- Recommended reading: Proakis and Salehi – Chapter 2
- Extra reading: Lathi and Ding – Chapters 2 and 3

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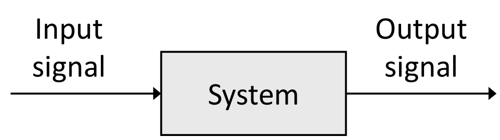
Signals and Systems

Signals*: functions of one or more independent variables that contain information about some phenomenon*

Systems*: entities that respond to particular signals by producing other signals or some desired behavior*



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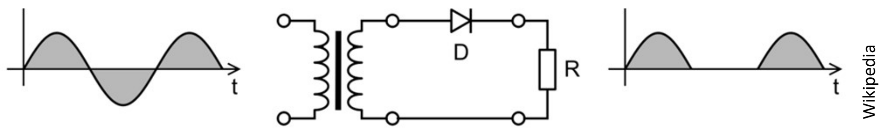
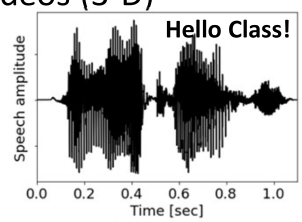
Examples

- Electrical circuits: voltages and currents



- Cameras: audio (1-D), images (2-D), videos (3-D)

Monochromatic brightness (x,y) or R(x,y), G(x,y), and B(x,y)



- Cellphones: speech, text messages, internet data
- Automobiles: applied force, speed

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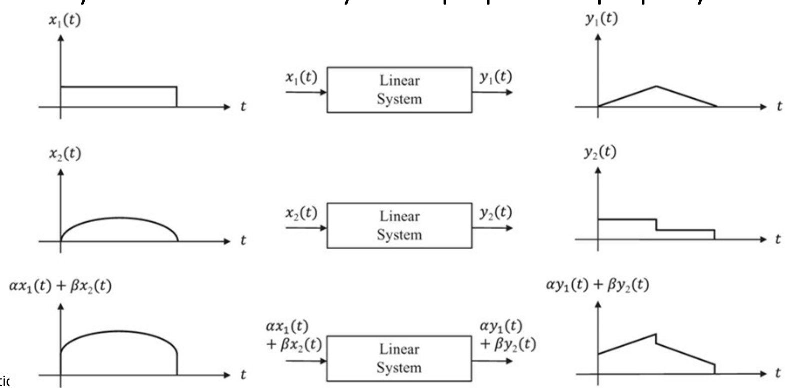
Classification of Signals

- Continuous-time vs. Discrete-time signals:
  - Continuous-time (CT) signals: time is continuous (i.e., $t \in \mathbb{R}$)
  - Discrete-time (DT) signals: time is only a discrete set of values (i.e., $t \in \mathbb{Z}$)
- Analog vs. Digital signals:
  - Analog signals: amplitude varies over a continuous range
  - Digital signals: amplitude is limited to a finite set of symbols
- Note that continuous-time ≠ analog and discrete-time ≠ digital

Classification of Systems

- Linear vs. Nonlinear systems:
  - Linear systems: satisfy the superposition (scaling and additivity) property
    - That is, response of the system to linear combination of the inputs is the linear combination of the response to the corresponding inputs
  - Nonlinear systems: do not satisfy the superposition property



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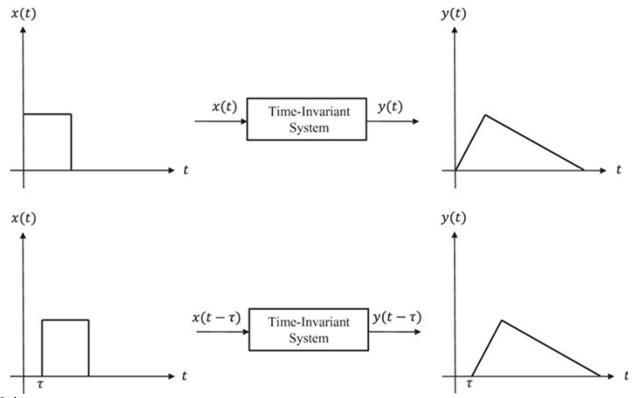
Classification of Systems (cont.)

- Time-invariant vs. Time-varying systems:
  - Time-invariant systems: input-output relationship does not change with time
    - That is, a delayed version of an input results in a delayed version of the output
  - Time-varying systems: input-output relationship changes with time



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Example

Classify this system:

$$y(t) = \frac{d}{dt}x(t)$$

- Linear or nonlinear?
- Time-invariant or time-varying?
- Linear Time-Invariant (LTI)?

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Linear Time-invariant (LTI) Systems

- The class of linear time-invariant (LTI) systems plays an important role both in communication and system theory systems
- The system can be completely characterized by its impulse response
  - That is, the response to inputs may simply be derived in the time domain using only its impulse response

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

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Signal Representation in the Frequency Domain

- Spectral representation of a general (aperiodic) signal → Fourier Transform
- Given a continuous-time signal $x(t)$, the Fourier transform (CTFT) of the signal is given by

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt$$ ← Analysis equation

and the Inverse Fourier Transform of the signal is given by

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi f t} df$$ ← Synthesis equation

- FT is the spectrum of the signal

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(f)$$

- There are Fourier transform versions for discrete-time signals (DTFT)

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt$$

Example

Determine the Fourier transform of the signal below and plot its spectra.

$$x(t) = e^{-at}u(t), \quad a > 0$$

$$= \frac{1}{a + j2\pi f}$$

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$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt$$

Example

Determine the Fourier transform of the signal below and plot its spectra.

$$x(t) = \Pi\left(\frac{t}{\tau}\right)$$

$$= \tau \text{sinc}(\pi f \tau)$$

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Notes on Fourier Transform

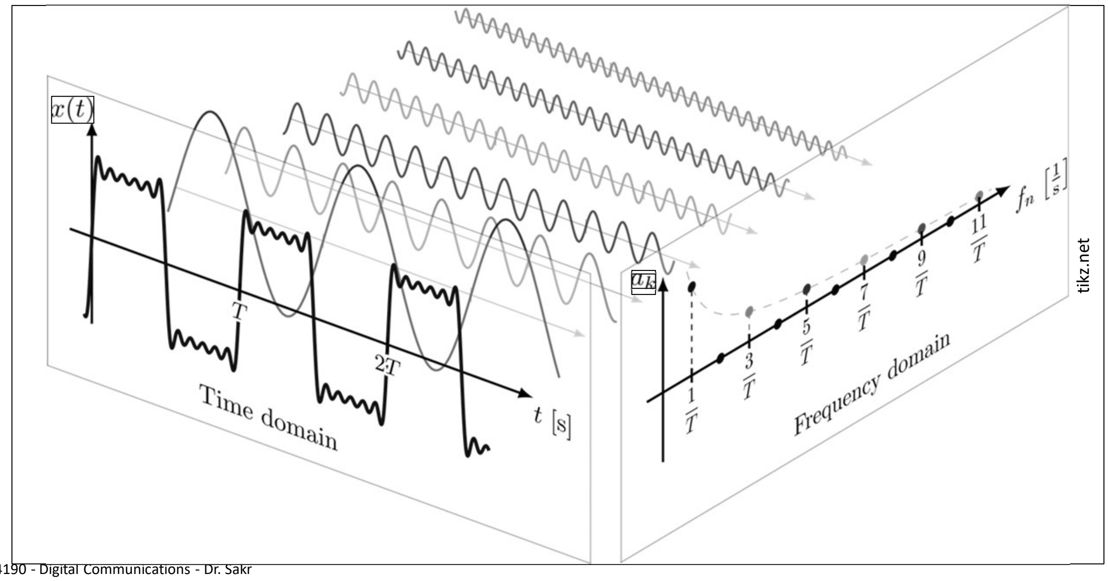

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Table of Fourier Transforms

- Luckily, there are tables for basic Fourier transform pairs
- Example:

	$g(t)$	$G(f)$	Condition
1	$e^{-at}u(t)$	$\frac{1}{a+j2\pi f}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j2\pi f}$	$a>0$
3	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+(2\pi f)^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j2\pi f)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j2\pi f)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$\delta(f)$	
8	$e^{j2\pi f_0 t}$	$\delta(f-f_0)$	
9	$\cos 2\pi f_0 t$	$0.5[\delta(f+f_0)+\delta(f-f_0)]$	
10	$\sin 2\pi f_0 t$	$j0.5[\delta(f+f_0)-\delta(f-f_0)]$	

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Properties of Fourier Transform

- Properties of Fourier transform can be used as follows

Operation	**$g(t)$**	**$G(f)$**
Linearity	$\alpha_1 g_1(t) + \alpha_2 g_2(t)$	$\alpha_1 G_1(f) + \alpha_2 G_2(f)$
Duality	$G(t)$	$g(-f)$
Time scaling	$g(at)$	$\frac{1}{\lvert a \rvert} G\left(\frac{f}{a}\right)$
Time shifting	$g(t - t_0)$	$G(f) e^{-j2\pi f t_0}$
Frequency shifting	$g(t) e^{j2\pi f_0 t}$	$G(f - f_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(f) G_2(f)$
Frequency convolution	$g_1(t) g_2(t)$	$G_1(f) * G_2(f)$
Time differentiation	$\frac{d^n g(t)}{dt^n}$	$(j2\pi f)^n G(f)$
Time integration	$\int_{-\infty}^{t} g(x)\, dx$	$\frac{G(f)}{j2\pi f} + \frac{1}{2} G(0) \delta(f)$

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Essential Bandwidth of a Signal

- The spectra of many signals extend to infinity
- In practice, most of signal energy is contained within *B* Hz and beyond that may be ignored
- Hence, bandwidth *B* is called the essential bandwidth of the signal
- The criteria to select *B* depends on the application

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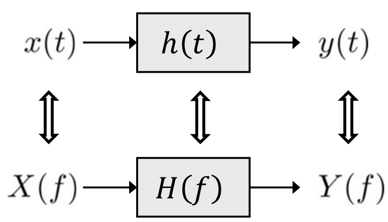
Signal Transmission over an LTI System

- Recall: LTI system can be completely characterized by its impulse response
- Input and output signals: $x(t)$ and $y(t)$
- Input and output signals spectra: $X(f)$ and $Y(f)$
- Impulse response: $h(t)$
- Frequency response (or transfer function): $H(f) = |H(f)|e^{j\theta_h(f)}$
- Communication channel:

Time domain:

Frequency domain:



Analysis of LTI Systems

Time-domain analysis: $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t)$

Convolution Integral

$$\Updownarrow \qquad \Updownarrow \qquad \Updownarrow$$

Convolution Property

Frequency-domain analysis: $X(f) \longrightarrow \boxed{H(f)} \longrightarrow Y(f) = H(f) \cdot X(f)$

$$Y(f) = \underbrace{|X(f)||H(f)|}_{\text{amplitude of output signal after transmission}} e^{j(\underbrace{\theta_x(f)+\theta_h(f)}_{\text{phase of output signal after transmission}})}$$

$2B\mathrm{sinc}(2\pi Bt) \leftrightarrow \Pi\left(\frac{f}{2B}\right)$

Example

Determine the output signal $y(t)$ *of an LTI system if* $x(t) = \mathrm{sinc}(\pi\tau_1 t)$ *and* $h(t) = \mathrm{sinc}(\pi\tau_2 t)$**.**

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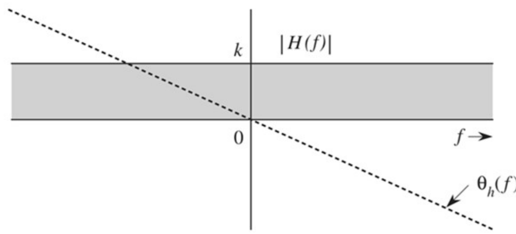
Distortionless Transmission

- Distortionless means the output waveform is a replica of the input waveform (i.e., same shape even if delayed or multiplied by a constant)
- That is, a distortionless transmission of $x(t)$ satifies

$$y(t) = k \cdot x(t - t_d) \Leftrightarrow Y(f) = kX(f)e^{-j2\pi f t_d} \rightarrow H(f) = \underbrace{k}_{\text{constant gain}}\underbrace{e^{-j2\pi f t_d}}_{\text{linear phase, or constant delay}}$$

- This is the amplitude and phase response of an LTI system for a distortionless transmission:



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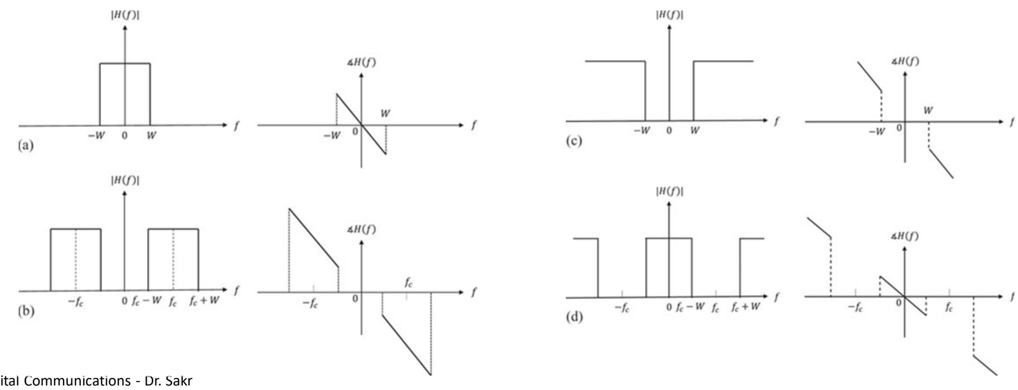
Example: Ideal LPF, HPF, BPF, and BSF Filters

- An ideal filter exactly passes signals at certain sets of frequencies and completely rejects the rest
- Distortionless: flat magnitude characteristic and a linear phase characteristic over the passband of the filter



Summary

- By now you should know:
  - Definition of signals and systems
  - Different classes of signals and systems
  - How signals are represented in the frequency domain
  - Fourier transform of some useful signals
  - Properties of Fourier transform
  - Essential bandwidth of a signal
  - Analysis of LTI systems in time and frequency domains