Solutions Manual
Tutorial 1: Analog-to-Digital Conversion

University of Windsor
Department of Electrical and Computer Engineering
ELEC 4190 - Digital Communications

1. (a) From Fourier transform table,

$$x_1(t) = 4\text{sinc}(420\pi t) \xleftrightarrow{\mathcal{F}} X_1(f) = \frac{1}{105}\Pi\left(\frac{f}{420}\right)$$

The bandwidth of the signal is $B = 210$ Hz and Nyquist rate is $2B = 420$ samples/sec.

(b) From Fourier transform table,

$$x_2(t) = 5\text{sinc}^2(6500\pi t) \xleftrightarrow{\mathcal{F}} X_2(f) = \frac{1}{1300}\Lambda\left(\frac{f}{13000}\right)$$

The bandwidth of the signal is $B = 6500$ Hz and Nyquist rate is $2B = 13000$ samples/sec.

(c) The bandwidth of the first term is 210 Hz and the bandwidth of the second term is 6500. Therefore, the bandwidth of $x_3(t)$ is $B = 6500$ Hz and Nyquist rate is $2B = 13000$ samples/sec.

(d) From the Fourier transform properties,

$$x_1(t) \cdot x_2(t) \xleftrightarrow{\mathcal{F}} X_1(f) * X_2(f)$$

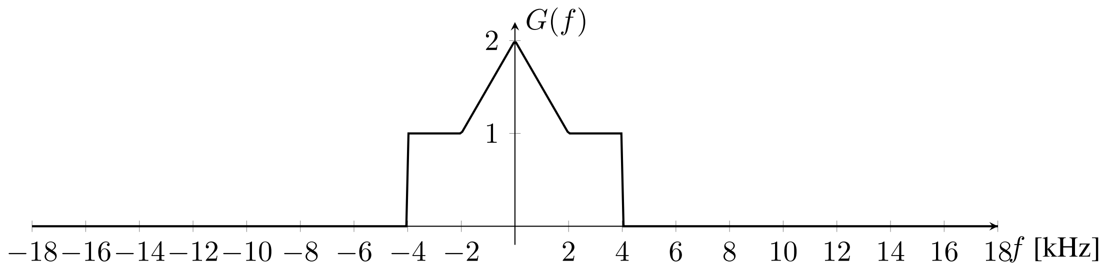
We know that if the width of two functions is A and B, then the width of their convolution is $A + B$. Since the bandwidth of the first term is 210 Hz and the bandwidth of the second term is 6500 Hz, the bandwidth of $x_4(t)$ is $B = 210 + 6500 = 6710$ Hz and Nyquist rate is $2B = 13420$ samples/sec.

2. From the Fourier transform properties,

$$g(t) = x_1(t) * x_2(t) \xleftrightarrow{\mathcal{F}} G(j\omega) = X_1(j\omega) \cdot X_2(j\omega)$$

So, $g(t)$ is bandlimited where $G(j\omega) = 0$ for $|\omega| > \min\{1000\pi, 2000\pi\}$. Therefore, the Nyquist rate of $g(t)$ is 2000π. That means the maximum sampling period T_s can at most be $2\pi/(2000\pi) = 0.001$ sec to be able to recover $g(t)$.

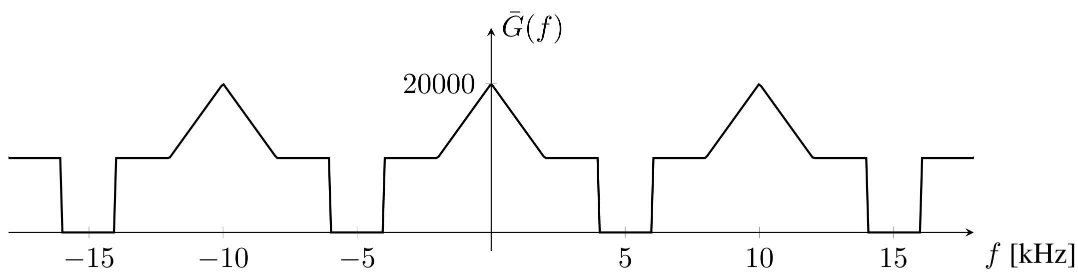
3. First, let's sketch the spectrum of $g(t)$.



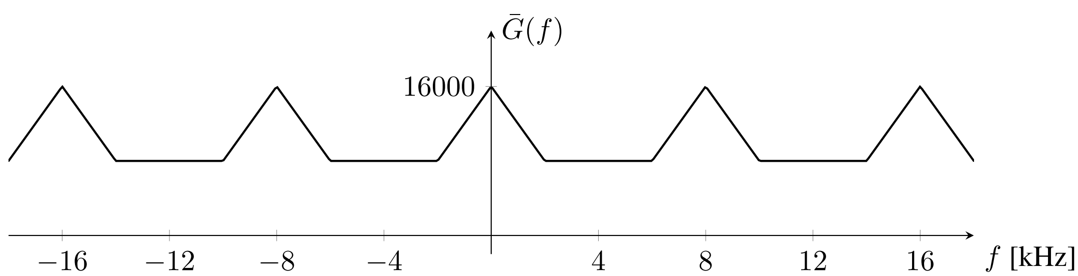
The spectrum of the ideally and uniformly sampled signal $\bar{g}(t)$ is a periodic signal of the spectrum of $g(t)$ repeated every f_s and scaled by $1/T_s$ where $T_s = 1/f_s$ is the sampling time. That is

$$\bar{G}(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} G(f - nf_s)$$

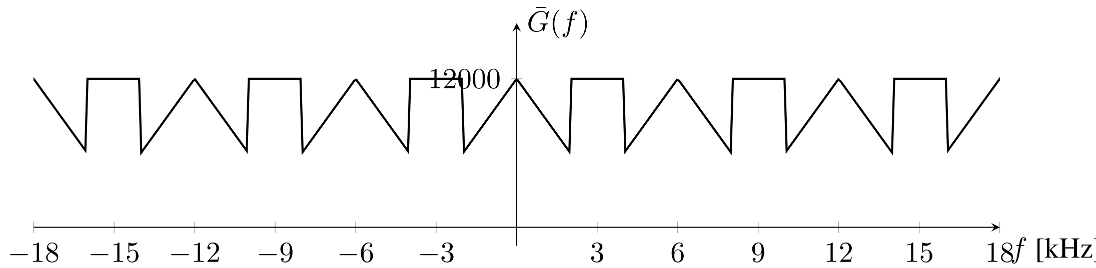
(a) When the sampling frequency is $f_s = 10000$ Hz, the spectrum of $\bar{g}(t)$ is



(b) When the sampling frequency is $f_s = 8000$ Hz, the spectrum of $\bar{g}(t)$ is



(c) When the sampling frequency is $f_s = 6000$ Hz, the spectrum of $\bar{g}(t)$ is



(d) It is clear that $g(t)$ can be perfectly reconstructed using an ideal LPF only in cases (a) and (b) because the sampling frequency is higher than or equal to the Nyquist rate $2B = 8000$ Hz.

4. (a) The number of binary digits per sample $n = \log_2 L = 16$ bits because $2^6 = 65536$.

   (b) Given that $P_m = 0.1$ W and $m_p = 1$ V,

$$\text{SNR} = 3L^2 \frac{P_m}{m_p^2} = 91.1 \text{ dB}$$

   (c) The bandwidth of the signal is 15 kHz, hence the Nyquist rate is 30 kHz. So, the bit rate is $R_b = 30000 \times 16 = 480000$ bit/s.

   (d) In this case, the bit rate is $R_b = 44100 \times 16 = 705600$ bit/s. The bandwidth required to transmit the encoded signal is $R_b/2 = 352800$ Hz.

5. Given that $m_p = 1$ V and $P_m = 120$ mW,

$$\text{SNR} = 3L^2 \frac{P_m}{m_p^2} = 0.36L^2 \geq 10^{3.6} \quad \rightarrow \quad L \geq 105.16$$

Since, binary PCM is used, $L = 128$ which is the nearest power of 2. Then, the minimum number of bits to achieve the desired SNR is 7 bits.

The actual SNR obtained is

$$\text{SNR} = 3L^2 \frac{P_m}{m_p^2} = 0.36 \times 128^2 = 37.7 \text{ dB}$$

6. (a) Given $m_p = 4$ and $L = 8$, the uniform quantization step size is $2m_p/L = 1$. This means the 8 quantization levels are at $\{3.5, 2.5, 1.5, 0.5, -0.5, -1.5, -2.5, -3.5\}$. Hence, the output sequence of the uniform quantizer is

   Input: $\{2.1, -0.9, 2.5, 1.2, 1.7, -3.8\}$ $\rightarrow$ Outuput: $\{2.5, -0.5, 2.5, 1.5, 1.5, -3.5\}$

   Note that the output is the closest value in $\{3.5, 2.5, 1.5, 0.5, -0.5, -1.5, -2.5, -3.5\}$ for each value in the input sequence.

(b) We first find the nonuniform quantization levels using the inverse formula. For a μ-law compressor, we know that

$$y = \frac{1}{\ln(1+\mu)} \ln\left(1 + \frac{\mu m}{m_p}\right), \quad 0 \le m \le m_p \to \frac{m}{m_p} = \frac{(1+\mu)^y - 1}{\mu}, \quad 0 \le y \le 1$$

Now, we use the inverse formula to obtain the nonuniform quantization levels corresponding to the uniform quantization levels. Note that, the uniform levels has to be normalized first, i.e., $y \leftarrow l/m_p$. Given that $m_p = 4$ and $l = \{\pm 3.5, \pm 2.5, \pm 1.5, \pm 0.5\}$, we get

$$y = \{\pm 0.875, \pm 0.625, \pm 0.375, \pm 0.125\}$$

Hence, the nonuniform quantization levels are

$$\frac{m}{4} = \frac{10^y - 1}{9} \quad \to \quad m = \{\pm 2.88, \pm 1.43, \pm 0.61, \pm 0.15\}$$

Note that the formula above is only valid for positive amplitudes. For negative amplitudes, we plug in the absolute value and multiply the result by -1.
Now, the output sequence of the nonuniform quantizer is

Input: $\{2.1, -0.9, 2.5, 1.2, 1.7, -3.8\} \to$ Outuput: $\{1.43, -0.61, 2.88, 1.43, 1.43, -2.88\}$

Note that the output is the closest value in $\{\pm 2.88, \pm 1.43, \pm 0.61, \pm 0.15\}$ for each value in the input sequence.

7. (a) Let $x = k\omega_m T_s + \theta_m$, then we get

$$\begin{aligned} f(x) &= A_m[\cos(x) - \cos(x + 0.5\theta_m)] \\ &= -2A_m \sin(\theta_m)\sin(x - \theta_m) \end{aligned}$$

The maximum can be obtained by using the first derivative of $f(x)$ as follows.

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = -2A_m \sin(\theta_m)\cos(x - \theta_m) = 0 \quad \to \quad x - \theta_m = \pi/2 + n\pi$$

and the peak value is $2A_m \sin(\theta_m)$.

(b) The SNR improvement factor is

$$\left(\frac{m_p}{d_p}\right)^2 = \left(\frac{A_m}{2A_m \sin(\theta_m)}\right)^2 = \frac{1}{4\sin^2(\theta_m)}$$

8. The condition to avoid slope overload is

$$|\dot{m}(t)|_{\max} < \Delta v f_s$$

We can find $\dot{m}(t)$ as follows.

$$\frac{\mathrm{d}}{\mathrm{d}t} m(t) = 3 \times 890\pi \sin(890\pi t) - 0.7 \times 1000\sqrt{3}\pi \cos(1000\sqrt{3}\pi t)$$

Since $\sqrt{3}$ is not a rational number, $|\dot{m}(t)|_{\max} = 3 \times 890\pi + 0.7 \times 1000\sqrt{3}\pi = 12197$ and $\Delta v_{\min} = 12197/f_s$ where $f_s = 1000\sqrt{3}$ samples/sec.