

ELEC 4190 - Digital Communications

Homework 2

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Q1

a) Dimensionality & Basis function

For s_1 ,

$$d_1 = s_1$$

$$\therefore \Phi_1 = d_1 = s_1$$

$$\sqrt{E_{S_1}} \quad \sqrt{E_{S_1}}$$

$$E_{S_1} = \int_0^T s_1^2 dt$$

$$= \int_0^1 2^2 dt + \int_1^2 2^2 dt + 0$$

$$= 8$$

Remember

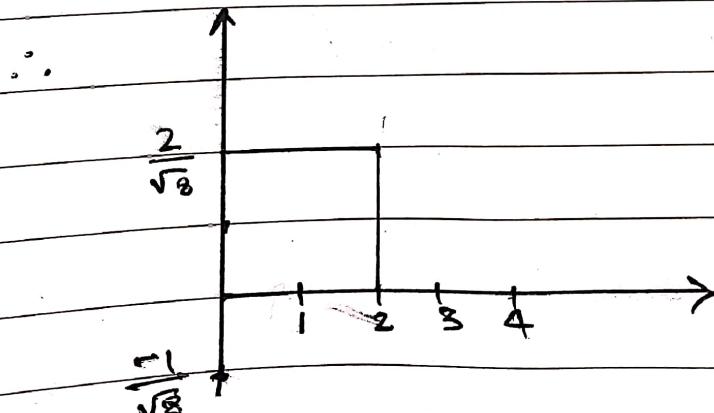
$$d_i(t) = s_i(t) - \sum_{j=1}^i s_{ij} \Phi_j(t)$$

$$\Phi_1 = \frac{1}{\sqrt{8}} = s_1(t)$$

$$\sqrt{8}$$

$$\Phi_1(t)$$

$$\Phi_1(t) = \begin{cases} \frac{2}{\sqrt{8}} & ; 0 \leq t < 2 \\ 0 & ; 2 \leq t \leq 4 \end{cases}$$

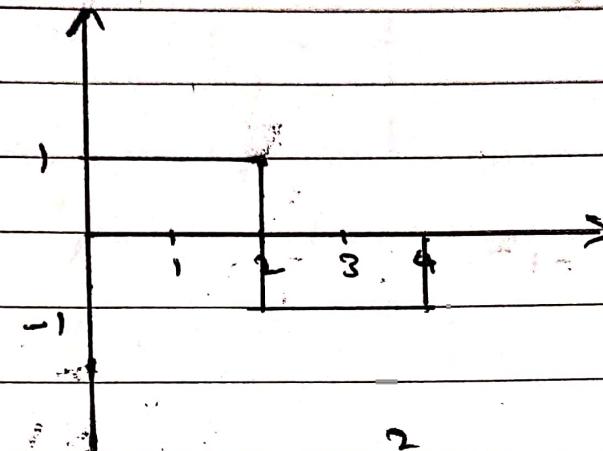


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b) For s_2

$$d_2(t) = s_2(t) - [s_2 \Phi_1(t)]$$

$$\text{But } s_{21} = \int_0^t s_2 \Phi_1$$

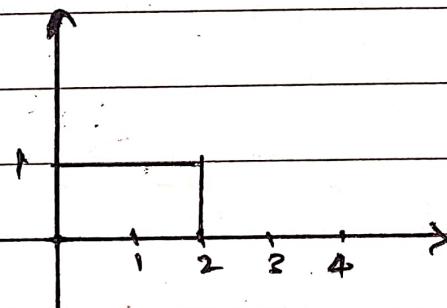


$$\int_0^1 (1 \times \frac{2}{\sqrt{8}}) + \int_1^2 (1 \times \frac{2}{\sqrt{8}}) + \int_2^3 (-1 \times 0) + \int_3^4 (-1 \times 0)$$

$$= \sqrt{2}$$

$$\therefore d_2(t) = s_2(t) - \sqrt{2} \Phi_1(t)$$

$$\sqrt{2} \Phi_1(t) \Rightarrow \sqrt{2} \times \frac{2}{\sqrt{8}}$$



$$\therefore 1 - 1 (= 0) \times 2 + (1) \times 0 \times 1 = 0 \times 2 + 0 \times 1$$

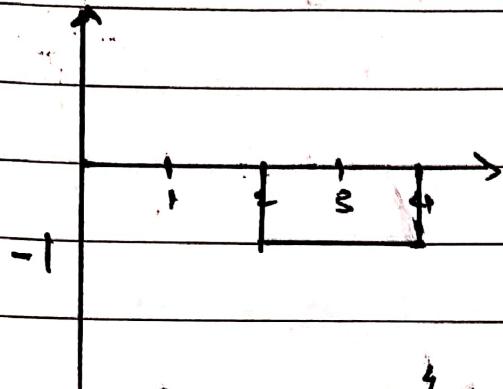
$$1 - 1 = 0$$

$$-1 - 0 (= -1) \times 2 + (0) \times 0 \times 1 = 0 \times 2 + 0 \times 1$$

$$-1 - 0 = -1$$

$\therefore d_2(t)$

$$[\phi_1(t), \phi_2(t)] + \alpha_1 \phi_1 + \alpha_2 \phi_2$$

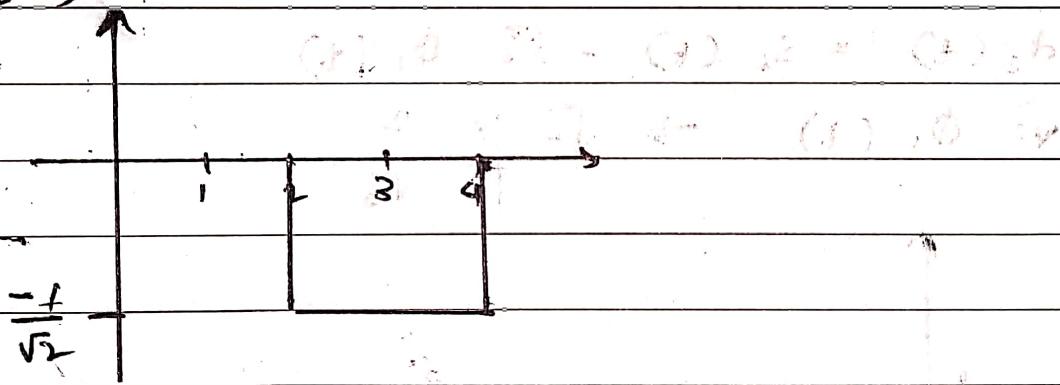


$$\Sigma d_2 = \int_{-1}^2 -1^2 + \int_{-1}^4 -1^2$$

$\therefore 2$

$$\Phi_2(t) = \frac{d_2(t)}{\sqrt{2}} + \frac{d_2(t+1)}{\sqrt{2}}$$

$\Phi_2(t)$

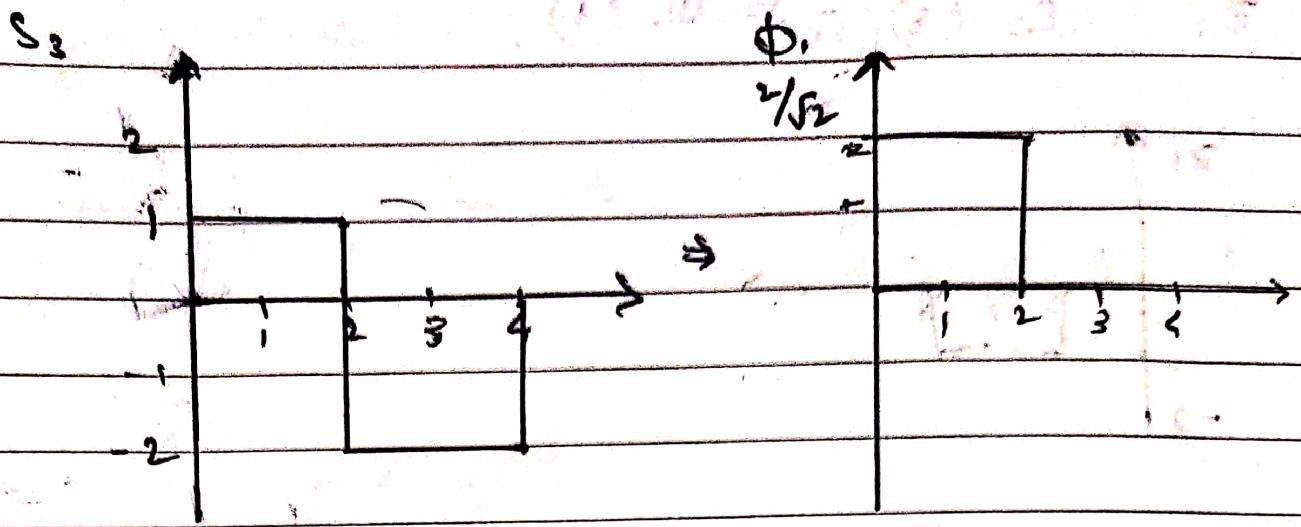


For s_3

$$d_3 = s_3(t) = [s_2, \Phi_1(t) + s_{22} \Phi_2(t)]$$

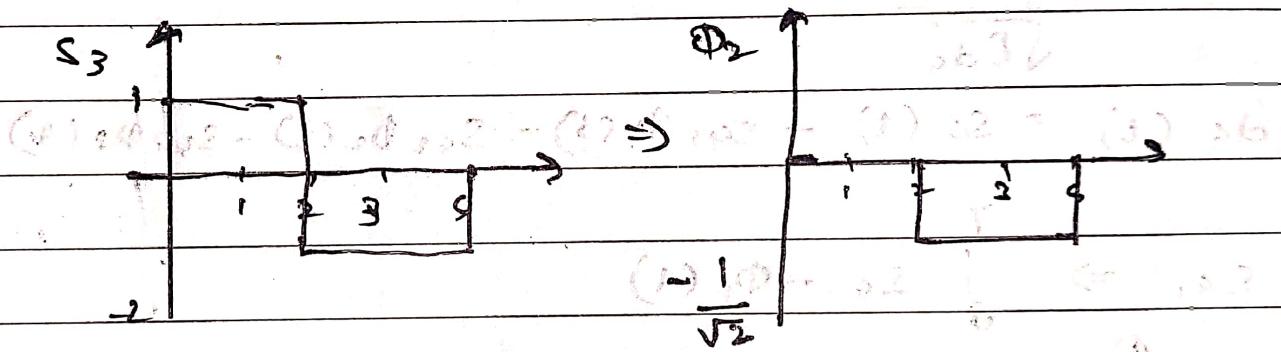
$$= s_3(t) - s_2 \Phi_1(t) - s_{22} \Phi_2(t)$$

$$s_{31} = \int_0^1 s_3 \Phi_1$$



$$\int_0^1 (1 \times 2/\sqrt{2}) + \int_1^2 (1 \times 0) = \sqrt{2}$$

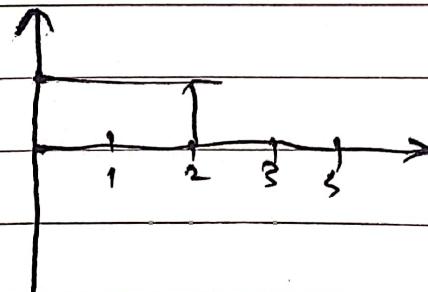
$$S_{32} = \int S_3 \Phi_2$$



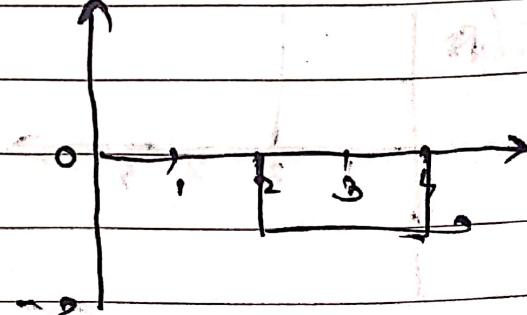
$$\int_2^3 (-2 \times -1/\sqrt{2}) + \int_3^4 (-2 \times -1/\sqrt{2}) = 2\sqrt{2}$$

$$d_2 = S_3(+)-\sqrt{2}\Phi_1(+)-2\sqrt{2}\Phi(0)$$

$$2\Phi_1(0)$$

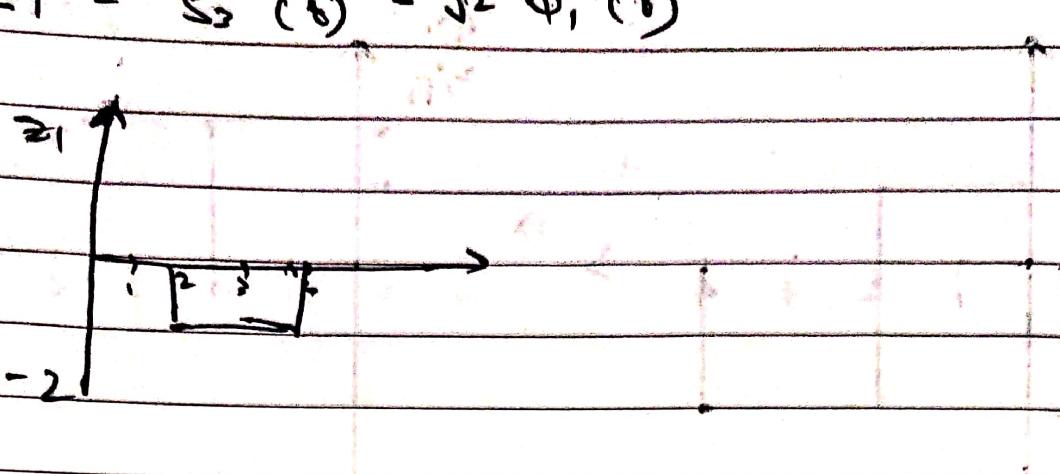


$$2\sqrt{2}\Phi(0)$$



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$$z_1 = s_3(t) - \sqrt{2} \phi_1(t)$$



$$z_1 - 2\sqrt{2} \phi_2(t) = 0$$

$$z_1 = (s_3(t) + 2\sqrt{2} \phi_2(t)) / \sqrt{2}$$

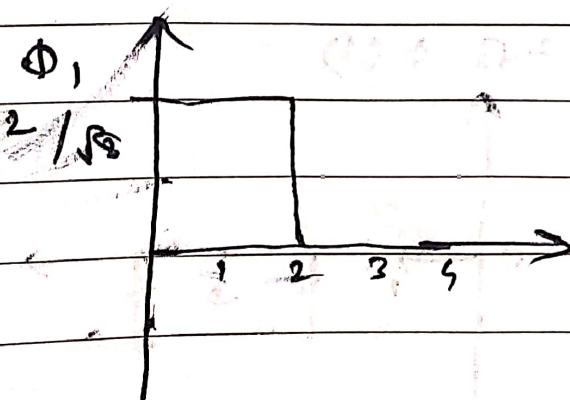
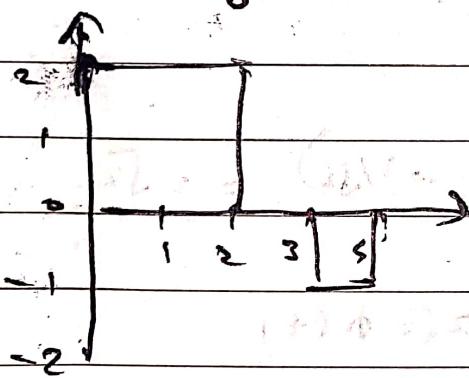
\therefore There is no ϕ_3

$$\phi_4 = \frac{d s_4}{\sqrt{E d s}}$$

$$\sqrt{E d s}$$

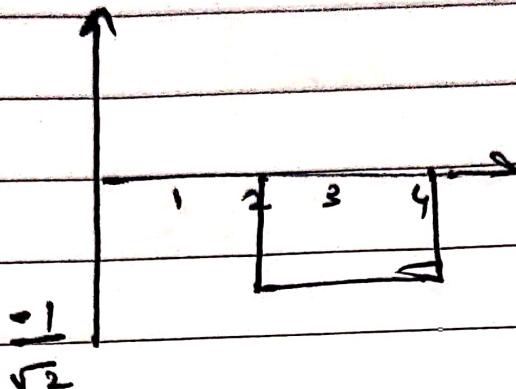
$$d s_4(t) = s_4(t) - s_{41} \phi_1(t) - s_{42} \phi_2(t) - s_{43} \phi_3(t)$$

$$s_{41} \Rightarrow \int_0^T s_4 - \phi_1(t)$$



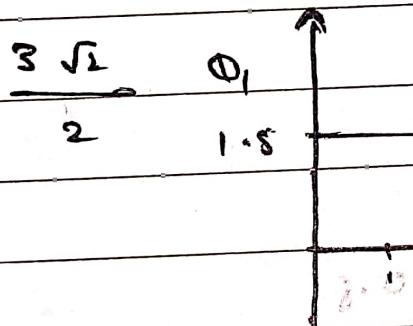
$$\int_0^T (1.5 \times e^{-\sqrt{2}t}) + \int_0^T (1.5 \times t^2 / \sqrt{2}) = \frac{3\sqrt{2}}{2}$$

$$S_{42} \Rightarrow \int_0^T S_4 - \phi_2(t)$$

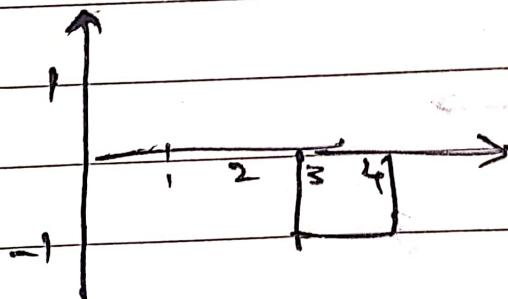


$$\int_0^T (-1 \times -1/\sqrt{2}) = \frac{\sqrt{2}}{2}$$

$$d_4(t) = S_4(t) - \frac{3\sqrt{2}}{2} \phi_1(t) - \frac{\sqrt{2}}{2} \phi_2(t)$$

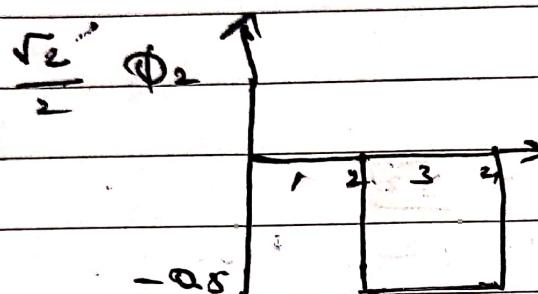
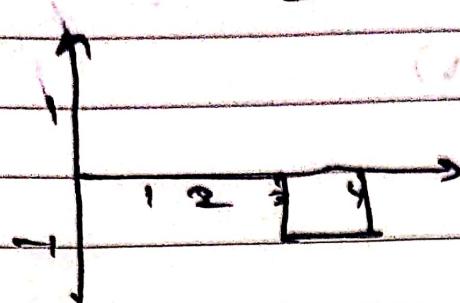


$$S_4(t) - \frac{3\sqrt{2}}{2} \phi_1 = z_1$$



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$$S_4(t) = \frac{3\sqrt{2}}{2} \left(\cos(\omega t + \pi/2) + \sin(\omega t + 3\pi/2) \right)$$



$$(1)_{z=0} \Rightarrow 1 = (\cos \phi_2 - i \sin \phi_2) + (\cos \phi_2 + i \sin \phi_2)$$

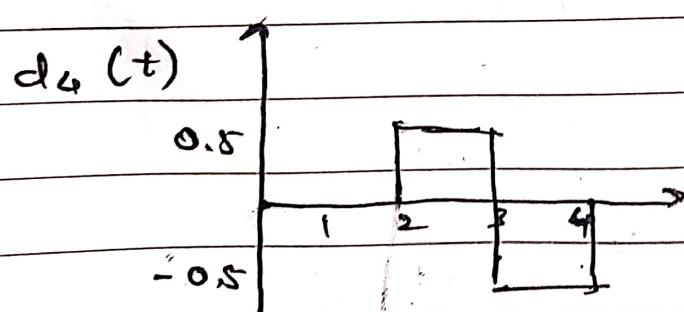
$$z_1 = \frac{\sqrt{2}}{2} \phi_2$$

$$0 \rightarrow 1 = 0$$

$$1 \rightarrow 2 = 0$$

$$2 \rightarrow 3 \Rightarrow 0 - (-0.5) = 0.5$$

$$3 \rightarrow 4 \Rightarrow -1 - (-0.5) = -1 + 0.5 = 0.5$$

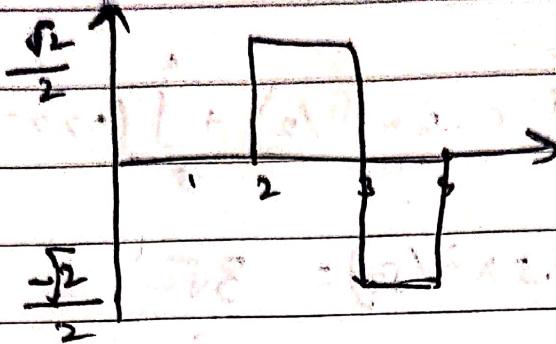


$$\phi_4 = \frac{d_4(t)}{\sqrt{E_{d4}}}$$

$$E_{d4} = \frac{1}{2} \int_0^3 (0.15)^2 + \int_3^4 (0.5)^2 = 0.5$$

$$\phi_4 = \frac{1}{\sqrt{2}} (\text{even } \Phi_4 (+))$$

i. Φ_4 :



$$b) S_1 = S_{11} \Phi_1 + S_{12} \Phi_2 + S_{13} \Phi_3 + S_{14} \Phi_4$$

$$S_2 = S_{21} \Phi_1 + S_{22} \Phi_2 + S_{23} \Phi_3 + S_{24} \Phi_4$$

$$S_3 = S_{31} \Phi_1 + S_{32} \Phi_2 + S_{33} \Phi_3 + S_{34} \Phi_4$$

$$S_4 = S_{41} \Phi_1 + S_{42} \Phi_2 + S_{43} \Phi_3 + S_{44} \Phi_4$$

$$S_{11} = \int S_1 \Phi_1 = \int_0^2 (2 \times 2/\sqrt{2}) = 2\sqrt{2}$$

$$S_{12} = \int S_1 \Phi_2 = 0$$

$$S_{13} = \int S_1 \Phi_3 = 0$$

$$S_{21} = \int S_2 \Phi_1 = \int_0^2 (2\sqrt{2} \times 1) = 2\sqrt{2}$$

$$S_{22} = \int S_2 \Phi_2 = \int_0^2 (-1 \times -1/\sqrt{2}) = \sqrt{2}$$

$$S_{24} = \int S_2 \Phi_4 = \int_0^2 (-1 \times \sqrt{2}/2) + \left(-1 \times -\frac{\sqrt{2}}{2} \right) = 0$$

$$S_{21} = \int S_2 \Phi_1 = \int_0^2 (1 \times 2/\sqrt{2}) = \sqrt{2}$$

$$S_{22} = \int S_2 \Phi_2 = \int_2^4 (-1/\sqrt{2} \times -2) = 2\sqrt{2}$$

$$S_{23} = \int S_2 \Phi_3 = \int_2^3 (1 - 2 \times \sqrt{2}/2) + \int_3^4 (1 - 2 \times -2\sqrt{2}/2) = 0$$

$$S_{41} = \int S_4 \Phi_1 = \int_0^2 (1.5 \times 2/\sqrt{3}) = \frac{3\sqrt{2}}{2}$$

$$S_{42} = \int S_4 \Phi_2 = \int_2^3 (-1 \times -1/\sqrt{2}) = \sqrt{2}$$

$$S_{43} = \int S_4 \Phi_3 = \int_3^4 (-\sqrt{2}/2 \times -1) = \frac{\sqrt{2}}{2}$$

$$S_1 = (2\sqrt{2}, 0, 0, 0)$$

$$S_2 = (\sqrt{2}, \sqrt{2}, 0, 0)$$

$$S_3 = (\sqrt{2}, 2\sqrt{2}, 0, 0)$$

$$S_4 = \left(\frac{3\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \right)$$

$$\|S_1 - S_2\| = \sqrt{(2\sqrt{2} - \sqrt{2})^2 + (0 - \sqrt{2})^2 + 0 + 0} = \sqrt{10}$$

$$\|S_1 - S_3\| = \sqrt{(2\sqrt{2} - \sqrt{2})^2 + (0 - 2\sqrt{2})^2 + 0 + 0} = 2$$

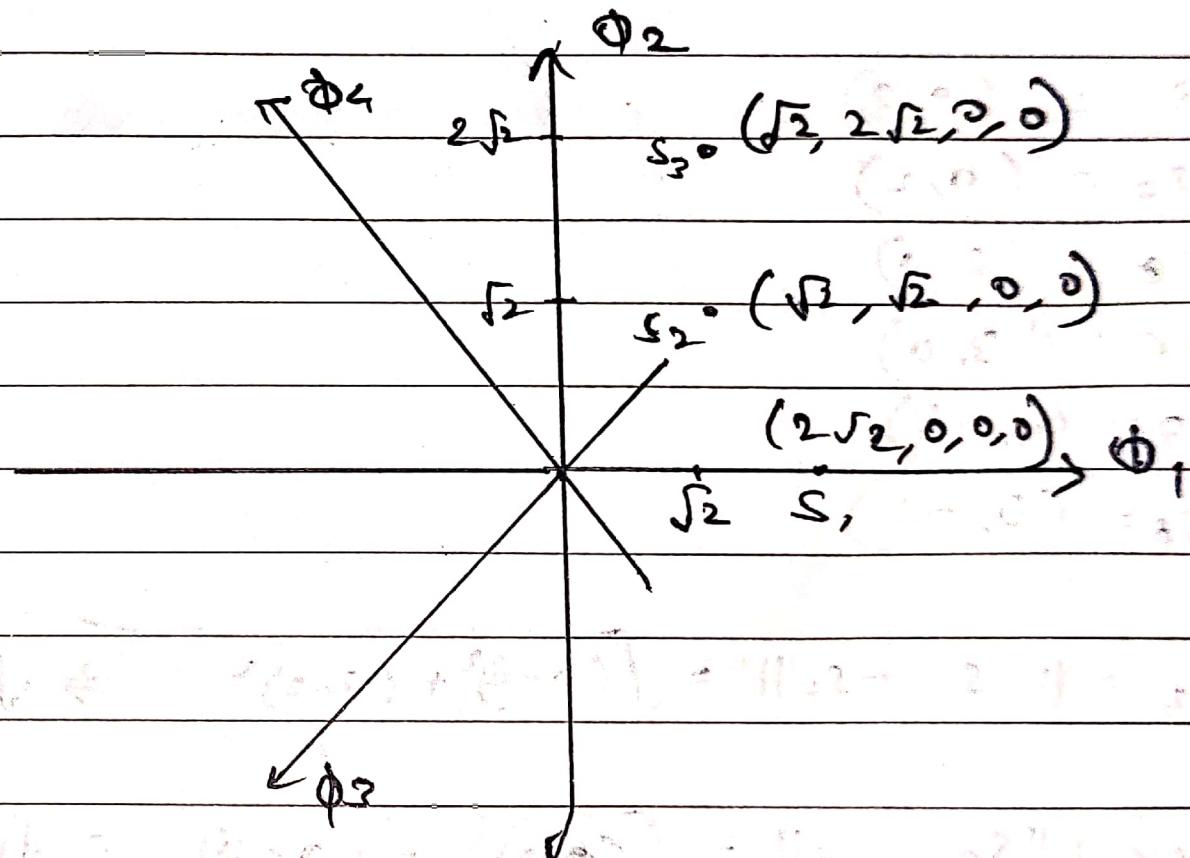
$$\|S_1 - S_4\| = \sqrt{\left(2\sqrt{2} - \frac{3\sqrt{2}}{2}\right)^2 + \left(0 - \frac{\sqrt{2}}{2}\right)^2 + 0 + \left(0 - \frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{6}}{2}$$

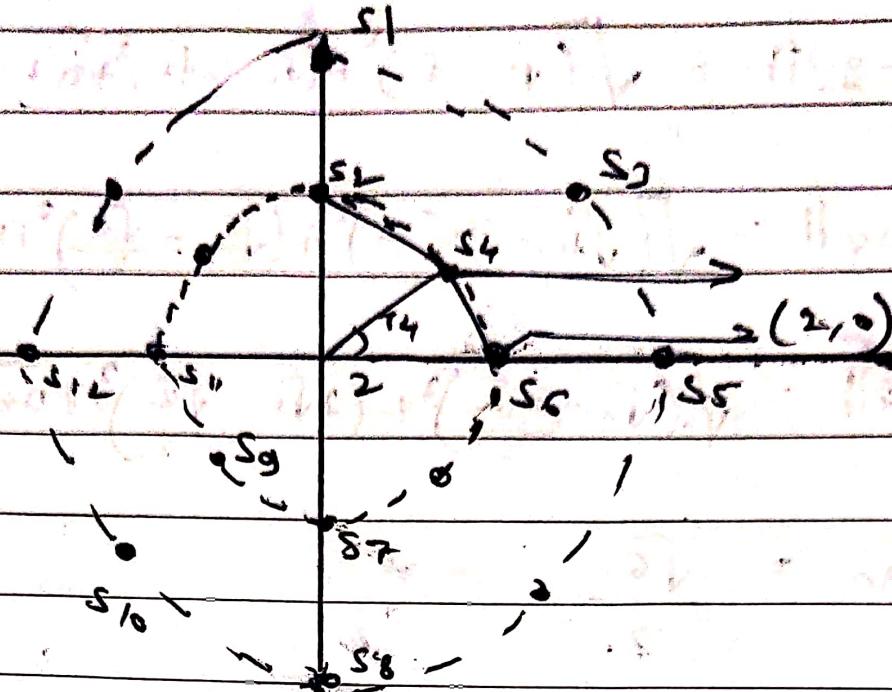
$$\|\mathbf{s}_2 - \mathbf{s}_3\| = \sqrt{(\sqrt{2} - \sqrt{2})^2 + (\sqrt{2} - 2\sqrt{2})^2 + 0 + 0} = \sqrt{2}$$

$$\|\mathbf{s}_2 - \mathbf{s}_4\| = \sqrt{\left(\sqrt{2} - \frac{3\sqrt{2}}{2}\right)^2 + \left(\sqrt{2} - \frac{\sqrt{2}}{2}\right)^2 + 0 + \left(0 - \frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{6}}{2}$$

$$\|\mathbf{s}_3 - \mathbf{s}_4\| = \sqrt{\left(\sqrt{2} - \frac{3\sqrt{2}}{2}\right)^2 + \left(2\sqrt{2} - \frac{\sqrt{2}}{2}\right)^2 + 0 + \left(0 - \frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{10}}{2}$$

$$d_{\min} = \frac{\sqrt{6}}{2}$$





$$s_1 = (0, 3)$$

$$s_2 = (0, 2)$$

$$s_3 = (2, 0)$$

$$s_4 = (1, 1)$$

$$s_5 = (3, 0)$$

$$s_6 = (2, -2)$$

$$s_7 = (0, -2)$$

$$s_8 = (0, -3)$$

a) $d_{12} = \|s_1 - s_2\| = \sqrt{(0-0)^2 + (3-2)^2} \Rightarrow \sqrt{1^2} = 1$

$$d_{56} = \|s_5 - s_6\| = \sqrt{(3-2)^2 + (0-0)^2} = \sqrt{1^2} = 1$$

closest distance is $s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}$

$$d_{\min} = 1$$

b) Average symbol energy

$$S_1 = S_5 = 3^2$$

$$S_2 = S_6 = 2^2$$

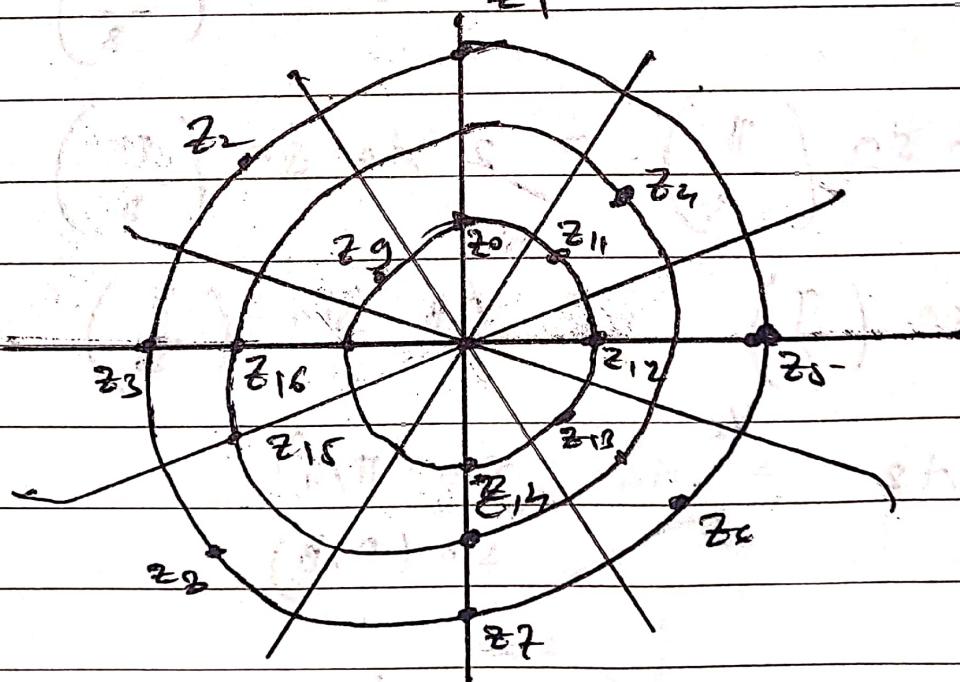
$$S_3 = S_{12} = (-3)^2$$

$$S_{11} = S_7 = (-2)^2$$

$E_s = A^2$ [The energy of Psk for all signal is same]

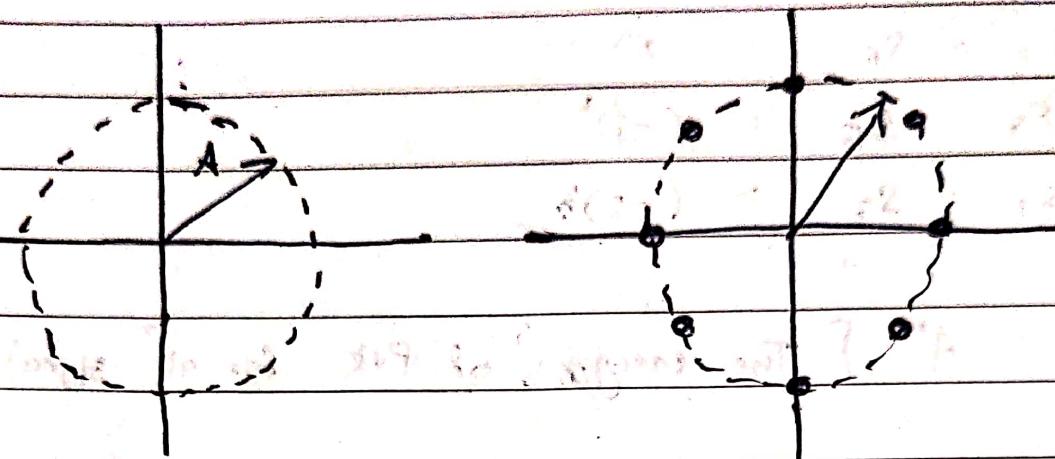
[Amplitude doesn't change]

$$\Rightarrow \frac{1}{16} \sum (8 \times 4 + 8 \times 9) = \frac{71}{2}$$



Q3

For QPSK & 8-PSK



To have the same minimum distance, The radii are

$$d_{\min} \rightarrow \text{For QPSK} = 2A_4 \sin\left(\frac{\pi}{4}\right)$$

$$d_{\min} \rightarrow \text{For 8-PSK} = 2A_8 \sin\left(\frac{\pi}{8}\right)$$

~~$$\Rightarrow 2A_4 \sin\left(\frac{\pi}{4}\right) = 2A_8 \sin\left(\frac{\pi}{8}\right)$$~~

$$A_4 \sin\left(\frac{\pi}{4}\right) = A_8 \sin\left(\frac{\pi}{8}\right)$$

$$A_8 = A_4 \frac{\sin(\pi/8)}{\sin(\pi/4)}$$

$$A_8 = A_4 (1.8477)$$

$$\frac{A_8}{A_4} = 1.8477$$

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From looking at the amplitude of 8 ps⁻¹
 we can see the amplitude to larger of λ_8
 will require more energy.
 We can also do it in this manner.

$$E_8 = \frac{1}{8} \sum_8 (A_8)^2 = A_8^2$$

$$E_4 = \frac{1}{4} \sum_4 (A_4)^2 = A_4^2$$

$$\frac{(A_8)^2}{(A_4)^2} = (.18477)^2 = 3.4128$$

$$BS(8 \text{ ps}^{-1}) = 3.4128 BS(4 \text{ ps}^{-1})$$

Q4 Tell & draw a net for regular prism.

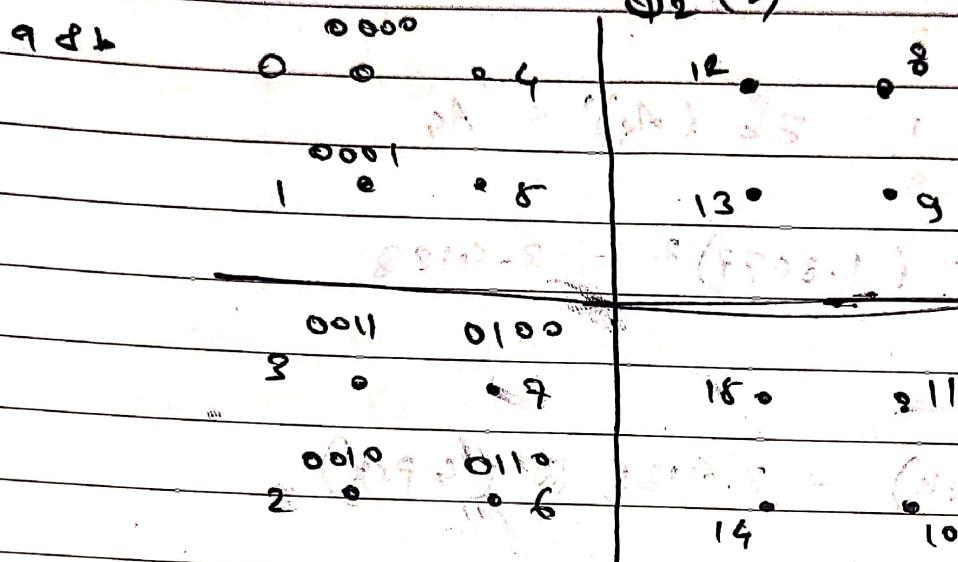
A- 16 - QAM of 4-ary constellations

$$d_{\min} = 4$$

$$d_{\min} = 2d$$

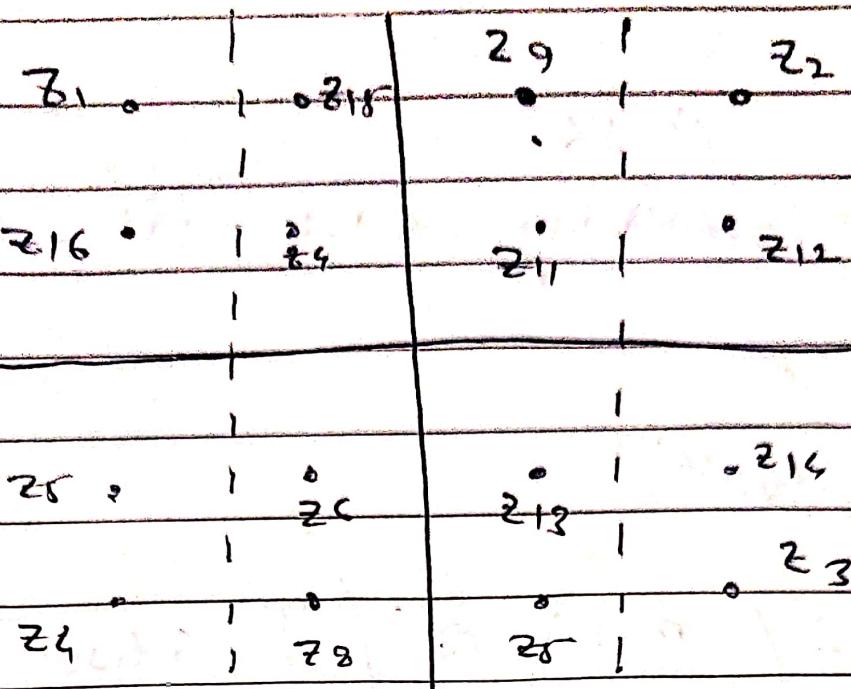
$$4 = 2d$$

$$d = 2$$



$$M = 16 \quad K = 4$$

Binary	Graycode	0000 0100	1100 1000
0000	0000	0000 0100	1100 1000
0001	0001	0001 0101	1101 1001
0010	0011	0011 0111	1111 1011
0011	0010	0011 0111	1111 1011
0100	0110	0110 0110	1110 1010
0101	0111	0110 0110	1110 1010
0110	0101		
0111	0100		
1000	1100	1101	1011
1001	1101	1110	1001
1010	1111	1111	1000
1011	1110		
1100	1010		



$$d_{\min} = 4$$

$$d = 2$$

$$d_{\min} = 2d$$

d) For 16-QAM $k=4$ || $E_s = \frac{2}{3} (m-1) (d)^2$

$$E_s = \frac{2}{3} (16-1) (2)^2 = 40$$

For 32-QAM $k=5$

$$E_s = \frac{2}{3} (32-1) (2)^2$$

$$= 82.66$$

Q 516 PAM $M = 16$

$$\text{Bit error probability} \approx 1.1 \times 10^{-5} = P_b$$

$$P_b = \underline{P_s}$$

$$\log_2 M$$

$$P_s = \frac{2}{\log_2 M} \left(\frac{m-1}{m} \right) Q \left(\sqrt{\frac{6 E_s}{(M^2-1) N_0}} \right)$$

$$P_b = \frac{2}{\log_2 M} \left(\frac{m-1}{m} \right) Q \left(\sqrt{\frac{6 E_s}{(M^2-1) N_0}} \right) = 1.1 \times 10^{-5}$$

$$\Rightarrow \frac{15}{32} Q \left(\sqrt{\frac{6 E_s}{(M^2-1) N_0}} \right) = 1.1 \times 10^{-5}$$

$$\Rightarrow \frac{15}{32} Q(x) = 1.1 \times 10^{-5}$$

$$\text{Let } x = \sqrt{\frac{6 E_s}{(M^2-1) N_0}}$$

$$Q(x) = 2.3 \times 10^{-5}$$

$$x = 4.07$$

$$Q(4.07) = 2.3 \times 10^{-5}$$

$$\therefore \sqrt{\frac{6 E_s}{(m^2-1) N_0}} = 4.07$$

$$\frac{E_s}{N_0} = y$$

$$\sqrt{\frac{6}{(16^2-1)}} y = 4.07$$

$$0.023829 y = (4.07)$$

$$y = \frac{(4.07)^2}{0.023829}$$

$$\Rightarrow \frac{E_s}{N_0} = 704.02$$

$$E_b = \frac{E_s}{\log_2 M} = \frac{704.02}{\log_2(16)} = 704.02 = 176 \text{ per bit}$$

$$dB = 10 \log(SNR) = 10 \log(176) = 22.45 dB$$

A_s

Q6

$$B = 25 \text{ kHz}$$

$$\text{Symbol rate} = 4800 \text{ symbol/sec}$$

$$\text{bit rate} = 19200 \text{ bit/sec}$$

For FSK

$$W = (1+2) \frac{M R_b}{H_B}$$

$$2 \log_2 M$$

$$\text{No. of bit/symbol} = \frac{19200}{4800} = 4 \text{ bits/symbol}$$

$$K = 4 \text{ bits}, M = 2^4 = 16$$

$\Rightarrow 16 - \text{FSK}$ will be used

To find $\alpha = \text{roll off}$

$$25000 = (1 + \alpha) \cdot \frac{16 \times 19200}{2 \log_2 (16)}$$

$$25000 = (1 + \alpha) 38400$$

$$\frac{25000}{38400} = (1 + \alpha)$$

$$0.65 = 1 + \alpha$$

$$\alpha = 0.65 - 1$$

$$\boxed{\alpha = -0.35}$$