Solutions Manual
Tutorial 3: Carrier and Symbol Synchronization

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ELEC 4190 - Digital Communications

1. Using the delay operator D, we can rewrite the input-output relationship as:

$$T = S \oplus DT \oplus D^3T$$

   (a) To design a descrambler to extract S, we need to apply $\oplus DT \oplus D^3T$ on both sides. This way we get:

$$T \oplus DT \oplus D^3T = S$$

   (b) Given that $T = S \oplus DT \oplus D^3T$:

S	DT	D^2T	D^3T	T
1	0	1	1	0
1	0	0	1	0
1	0	0	0	1
1	1	0	0	0
1	0	1	0	1
0	1	0	1	0
0	0	1	0	0
0	0	0	1	1
0	1	0	0	1
0	1	1	0	1

   (c) Given that $S = T \oplus DT \oplus D^3T$:

T	DT	D^2T	D^3T	$\hat{S}$
0	0	1	1	1
0	0	0	1	1
1	0	0	0	1
0	1	0	0	1
1	0	1	0	1
0	1	0	1	0
0	0	1	0	0
1	0	0	1	0
1	1	0	0	0
1	1	1	0	0

This output $\hat{S} = S$.

2.

(a)

$$H(s) = \frac{0.5KG(s)}{s + 0.5KG(s)} = \frac{1}{s^2 + \sqrt{2}s + 1}$$

The poles of the system are at the roots of the denominator. That is solution of the equation:

$$s^2 + \sqrt{2}s + 1 = 0 \rightarrow -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$$

Hence, the system is stable since the real part of the poles is negative (i.e., in the left half of the s-plane).

(b) To find the damping factor and natural frequency of the loop, we compare the denominator to the form $s^2 + 2\zeta\omega_n s + \omega_n^2$. Hence, $\omega_n = 1$ and $\zeta = 1/\sqrt{2}$.

3.

(a)

$$H(s) = \frac{0.5KG(s)}{s + 0.5KG(s)} = \frac{\frac{K_1}{\tau_1}}{s^2 + \frac{1}{\tau_1}s + \frac{K_1}{\tau_1}}$$

The gain at $f = 0$ is $|H(0)| = 1$.

(b) The poles of the system are at the roots of the denominator. That is solution of the equation:

$$s^2 + \frac{1}{\tau_1}s + \frac{K_1}{\tau_1} = 0 \rightarrow -\frac{1}{2\tau_1} \pm \frac{\sqrt{1 - 4K_1\tau_1}}{2\tau_1}$$

In order for the system to be stable, the real part of the poles must be negative (i.e., in the left half of the s-plane). Knowing that $K > 0$, that means as long as τ_1 is positive, both roots will be in the left half of the s-plane.
Furthermore, since $\zeta = \frac{1}{2\sqrt{K_1\tau_1}}$ needs to be less than 1, we need $K_1\tau_1 > 0.25$.

4. Using KVL, we get

$$G(s) = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} = \frac{1 + R_2Cs}{1 + (R_1 + R_2)Cs} = \frac{1 + \tau_2 s}{1 + \tau_1 s}$$

By comparison, we get $\tau_1 = (R_1 + R_2)C$ and $\tau_2 = R_2C$.