ELEC 4190 - Digital Communications

Homework 2

Amey Mahendra Thakur 110107589

Q1

a) Dimensionality & Basis function

For s_1

$d_1 = s_1$

$$\therefore \phi_1 = \frac{d_1}{\sqrt{E_{s_1}}} = \frac{s_1}{\sqrt{E_{s_1}}}$$

$$E_{s_1} = \int_0^T s_1^2$$

$$= \int_0^1 2^2 \, dt + \int_1^2 2^2 \, dt + 0$$

$$= 8$$

Remember

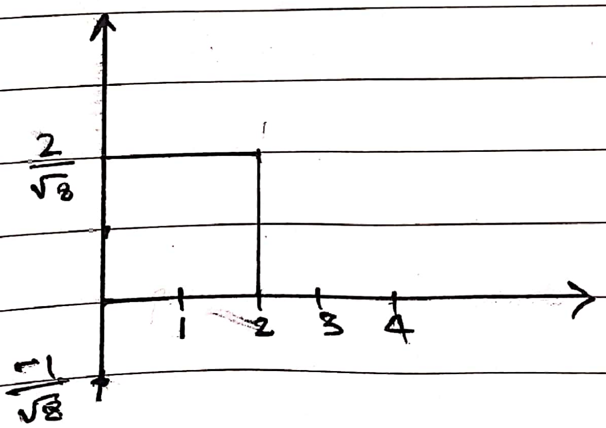
$$d_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij} \phi_j(t)$$

$$\phi_1 = \frac{1}{\sqrt{8}} = s_1(t)$$

$\phi_1(t)$

$$\phi(t) = \begin{cases} \frac{2}{\sqrt{8}} & ; 0 \le t < 2 \\ 0 & ; 2 \le t \le 4 \end{cases}$$

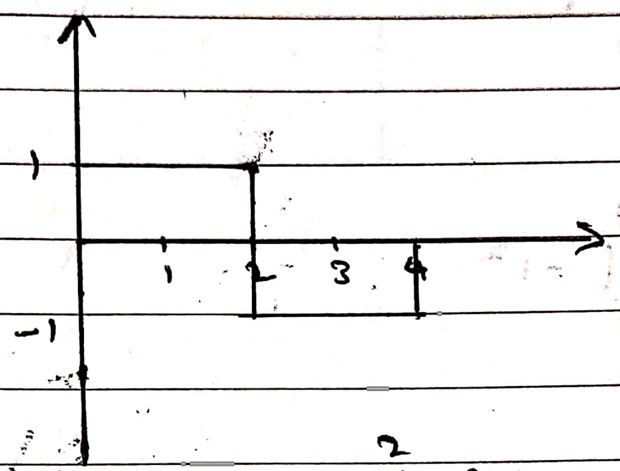
$\therefore$



b) For S_2

$d_2(t) = S_2(t) - [S_2 \phi_1(t)]$

But $S_{21} = \int_0^T S_2 \phi_1$

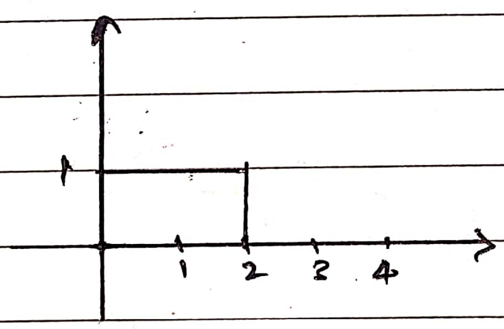


$\int_0^1 (1 \times \frac{2}{\sqrt{8}}) + \int_1^2 (\frac{2}{\sqrt{8}}) + \int_2^3 (-1 \times 0) + \int_3^4 (-1 \times 0)$

$= \sqrt{2}$

$\therefore d_2(t) = S_2(t) - \sqrt{2}\, \phi_1(t)$

$\sqrt{2}\, \phi_1(t) \Rightarrow \sqrt{2} \times \frac{2}{\sqrt{8}}$



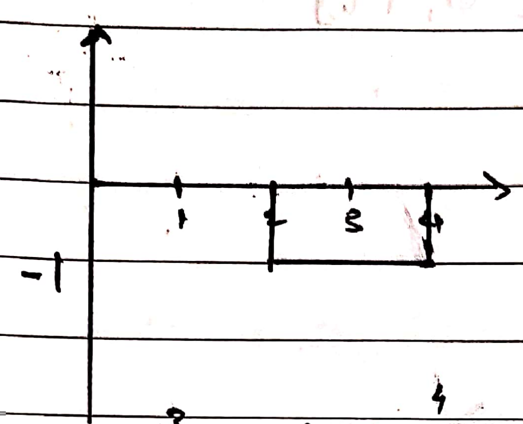
$\therefore 1 - 1 = 0$

$1 - 1 = 0$

$-1 - 0 = -1$

$-1 - 0 = -1$

$\therefore d_2(t)$

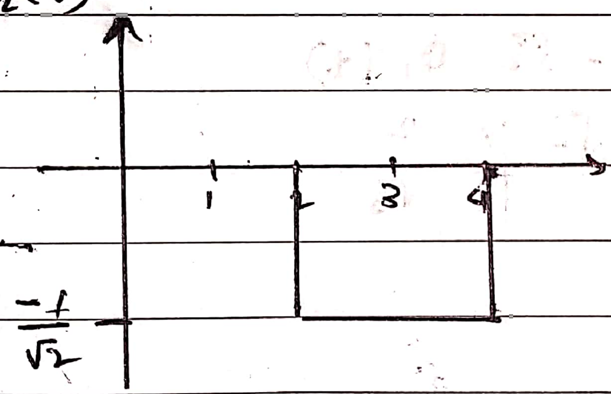


$$E\,d_2 = \int_2^3 -1^2 + \int_3^4 -1^2$$

$$= 2$$

$$\phi_2(t) = \frac{d_2(t)}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot d_2(t)$$

$\phi_2(t)$

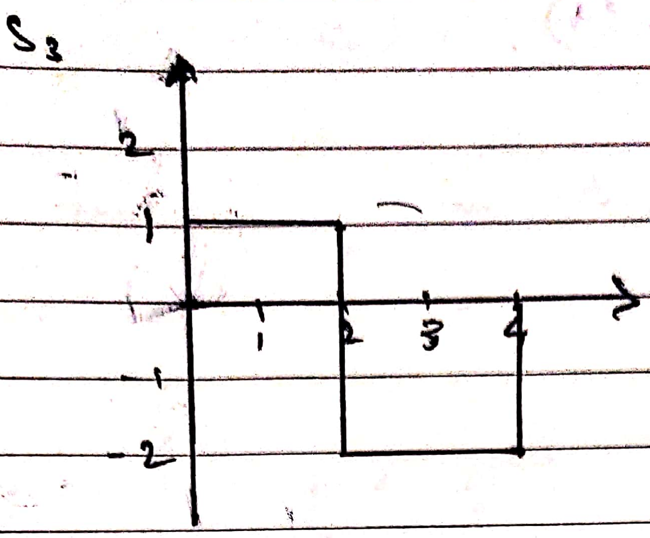


For S_3

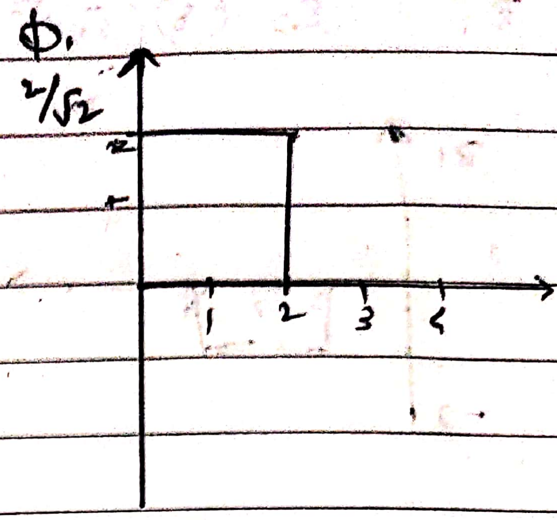
$$d_3 = s_3(t) - [S_{31}\,\phi(t) + S_{32}\,\phi_2(t)]$$

$$= S_3(t) - S_{31}\,\phi_1(t) - S_{32}\,\phi_2(t)$$

$$S_{31} = \int_0^1 S_3\,\phi_1$$

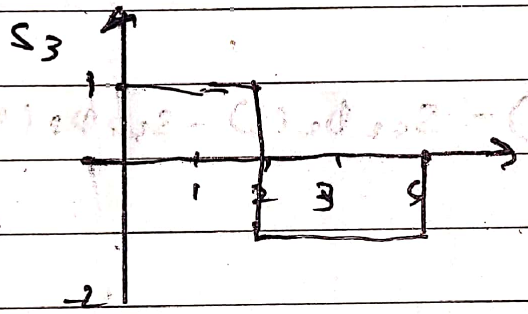


$\Rightarrow$

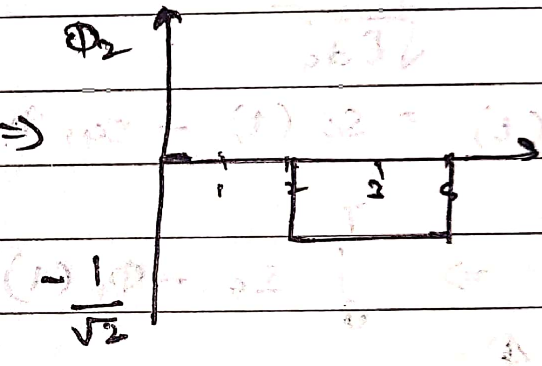


$$\int_0^1 (1 \times 2/\sqrt{8}) + \int_1^2 (1 \times 2/\sqrt{8}) = \sqrt{2}$$

$$S_{32} = \int S_3 \Phi_2$$



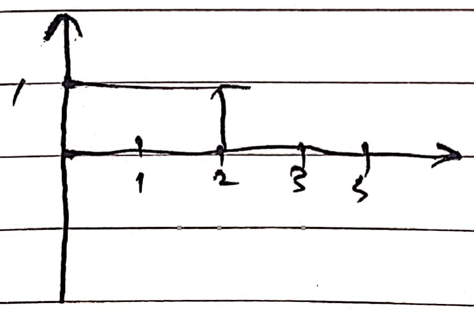
$\Rightarrow$



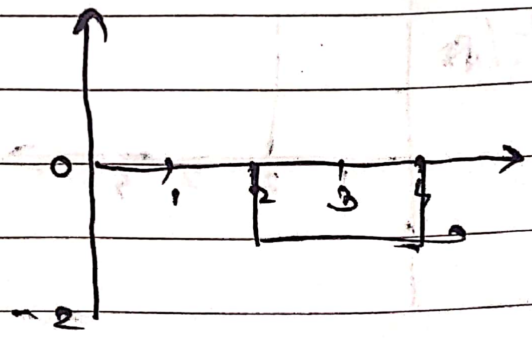
$$\int_2^3 (-2 \cdot -1/\sqrt{2}) + \int_3^4 (-2 \times -1/\sqrt{2}) = 2\sqrt{2}$$

$$d_3 = S_3(t) - \sqrt{2}\,\Phi_1(t) - 2\sqrt{2}\,\Phi(t)$$

$2\,\Phi_1(t)$

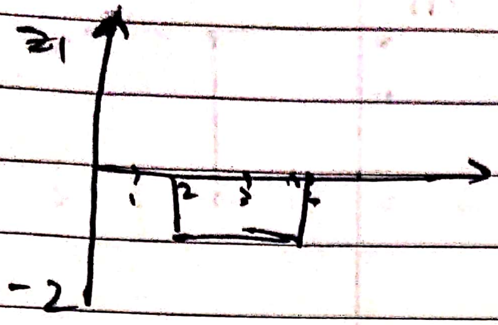


$2\sqrt{2}\,\Phi(t)$



5

$$z_1 = S_3(t) - \sqrt{2}\,\Phi_1(t)$$



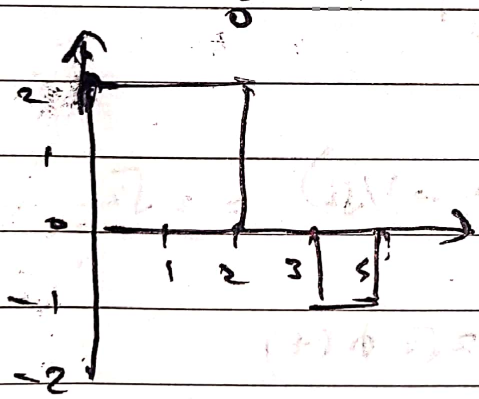
$$z_1 - 2\sqrt{2}\,\Phi_2(t) = 0$$

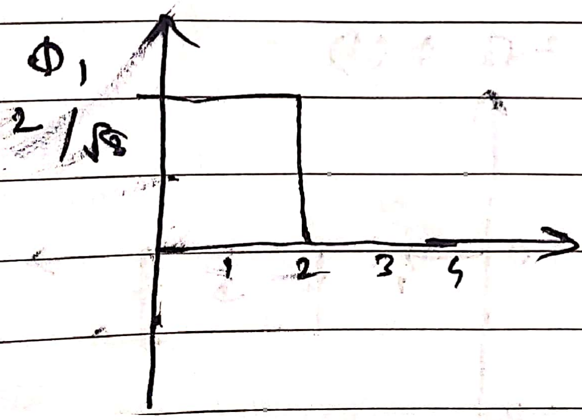
$\therefore$ There is no Φ_3

$$\Phi_4 = \frac{d_{s4}}{\sqrt{E_{d4}}}$$

$$d_4(t) = S_4(t) - S_{41}\Phi_1(t) - S_{42}\Phi_2(t) - S_{43}\Phi_3(t)$$

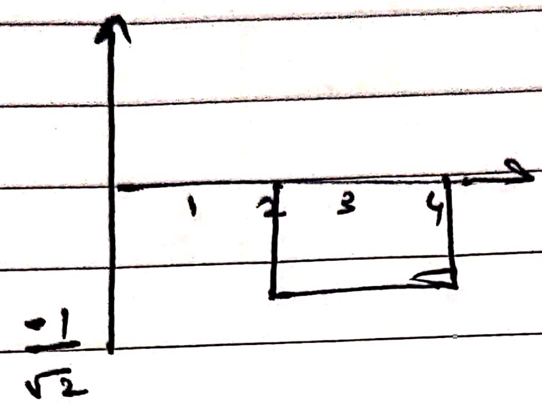
$$S_{41} \Rightarrow \int_0^T S_4 - \Phi_1(t)$$





$$\int_0^1 \left(1.5 \times {}^{2}/\sqrt{8}\right) + \int_1^2 \left(1.5 \times {}^{2}/\sqrt{8}\right) = \frac{3\sqrt{2}}{2}$$

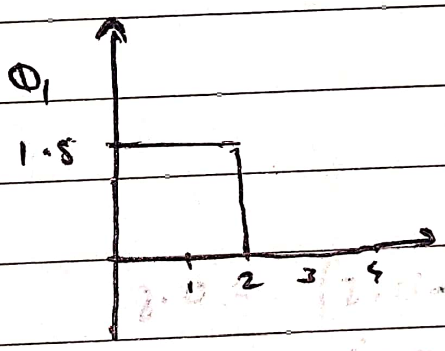
$$S_{42} \Rightarrow \int_0^T S_4 - \phi_2(t)$$



$$\int_3^4 \left(-1 \times {}^{-1}/\sqrt{2}\right) = \frac{\sqrt{2}}{2}.$$

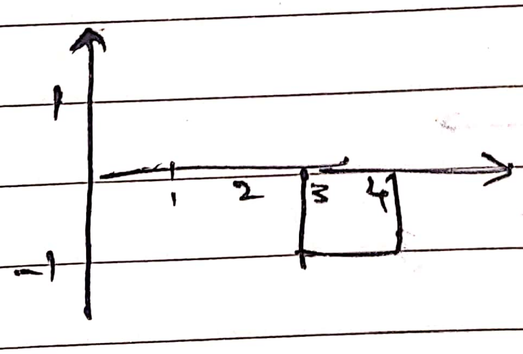
$$d_4(t) = S_4(t) - \frac{3\sqrt{2}}{2}\,\phi_1(t) - \frac{\sqrt{2}}{2}\,\phi_2(t)$$

$\frac{3\sqrt{2}}{2}$ ϕ_1

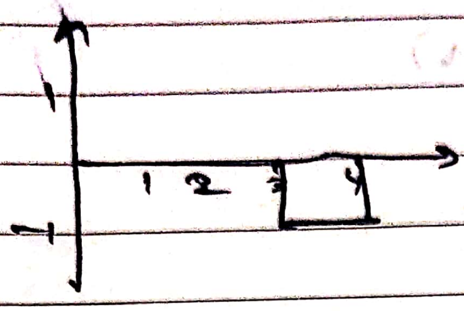


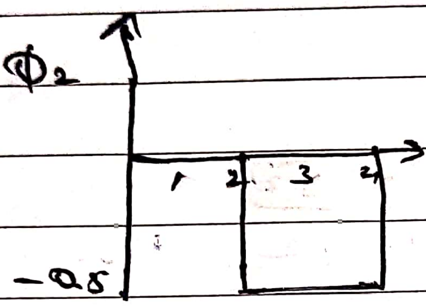
3 to 4 $\Rightarrow$ $-1 - 0$

$$S_4(t) - \frac{3\sqrt{2}}{2}\,\phi_1 = Z_1$$



$S_4(t) - \frac{3\sqrt{2}}{2}\,\phi_1 = z_1$





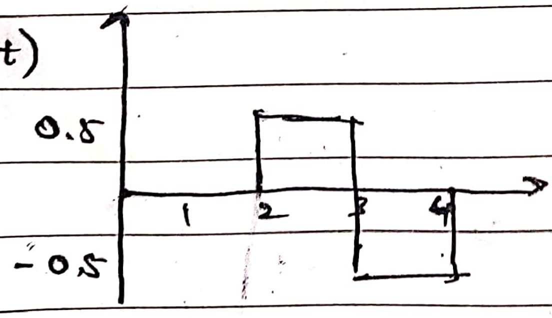
$z_1 - \frac{\sqrt{2}}{2}\,\phi_2$

0 to 1 = 0

1 to 2 = 0

2 to 3 $\Rightarrow 0 - (-0.5) = 0.5$

3 to 4 $\Rightarrow -1 - (-0.5) = -1 + 0.5 = -0.5$

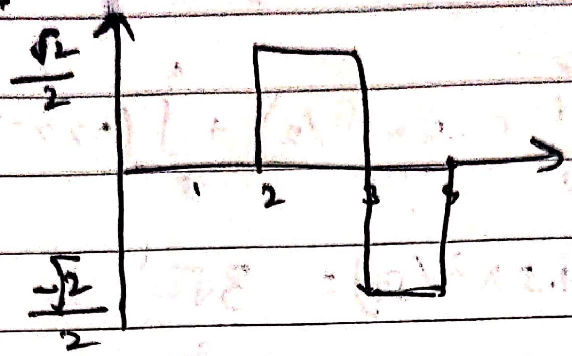


$\phi_4 = \frac{d_4(t)}{\sqrt{E_{d4}}}$

$E_{d4} = \int_2^3 (0.5)^2 + \int_3^4 (0.5)^2 = 0.5$

$$\phi_4 = \frac{1}{\sqrt{0.5}} \times d_4(t)$$

$\therefore \phi_4$:



b) $S_1 = S_{11}\phi_1 + S_{12}\phi_2 + S_{13}\phi_3 + S_{14}\phi_4$

$S_2 = S_{21}\phi_1 + S_{22}\phi_2 + S_{23}\phi_3 + S_{24}\phi_4$

$S_3 = S_{31}\phi_1 + S_{32}\phi_2 + S_{33}\phi_3 + S_{34}\phi_4$

$S_4 = S_{41}\phi_1 + S_{42}\phi_2 + S_{43}\phi_3 + S_{44}\phi_4$

$$S_{11} = \int S_1\phi_1 = \int_0^2 (2 \times 2/\sqrt{8}) = 2\sqrt{2}$$

$$S_{12} = \int S_1\phi_2 = 0$$

$$S_{14} = \int S_1\phi_4 = 0$$

$$S_{21} = \int S_2 \phi_1 = \int_0^2 (2\sqrt{8} \times 1) = \sqrt{2}$$

$$S_{22} = \int S_2\phi_2 = \int_2^4 (-1 \times -1/\sqrt{2}) = \sqrt{2}$$

$$S_{24} = \int S_2\phi_4 = \int_2^3 (-1 \times \sqrt{2}/2) + \int_3^4 \left(-1 \times -\frac{\sqrt{2}}{2}\right) = 0$$

$$S_{31} = \int S_3 \phi_1 = \int_0^2 (1 \times 2/\sqrt{2}) = \sqrt{2}$$

$$S_{32} = \int S_3 \Phi_2 = \int_2^4 (-1/\sqrt{2} \times -2) = 2\sqrt{2}$$

$$S_{34} = \int S_3 \Phi_4 = \int_2^3 (-2 \times \sqrt{2}/2) + \int_3^4 \left(+2 \times \frac{-2\sqrt{2}}{2}\right) = 0$$

$$S_{41} = \int S_4 \Phi_1 = \int_0^2 (1.5 \times 2/\sqrt{8}) = \frac{3\sqrt{2}}{2}$$

$$S_{42} = \int S_4 \Phi_2 = \int_3^4 (-1 \times {}^{-1}/\sqrt{2}) = \frac{-\sqrt{2}}{2}$$

$$S_{44} = \int S_4 \Phi_4 = \int_3^4 (-\sqrt{2}/2 \times -1) = \frac{\sqrt{2}}{2}$$

$$s_1 = (2\sqrt{2}, 0, 0, 0)$$

$$S_2 = (\sqrt{2}, \sqrt{2}, 0, 0)$$

$$S_3 = (\sqrt{2}, 2\sqrt{2}, 0, 0)$$

$$S_4 = \left(\frac{3\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right)$$

$$\| s_1 - S_2 \| = \sqrt{(2\sqrt{2} - \sqrt{2})^2 + (0 - 2\sqrt{2})^2 + 0 + 0} = \sqrt{10}$$

$$\| s_1 - S_2 \| = \sqrt{(2\sqrt{2} - \sqrt{2})^2 + (0 - \sqrt{2})^2 + 0 + 0} = 2$$

$$\| S_1 - S_4 \| = \sqrt{\left(2\sqrt{2} - \frac{3\sqrt{2}}{2}\right)^2 + \left(0 - \frac{\sqrt{2}}{2}\right)^2 + 0 + \left(0 - \frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{6}}{2}$$

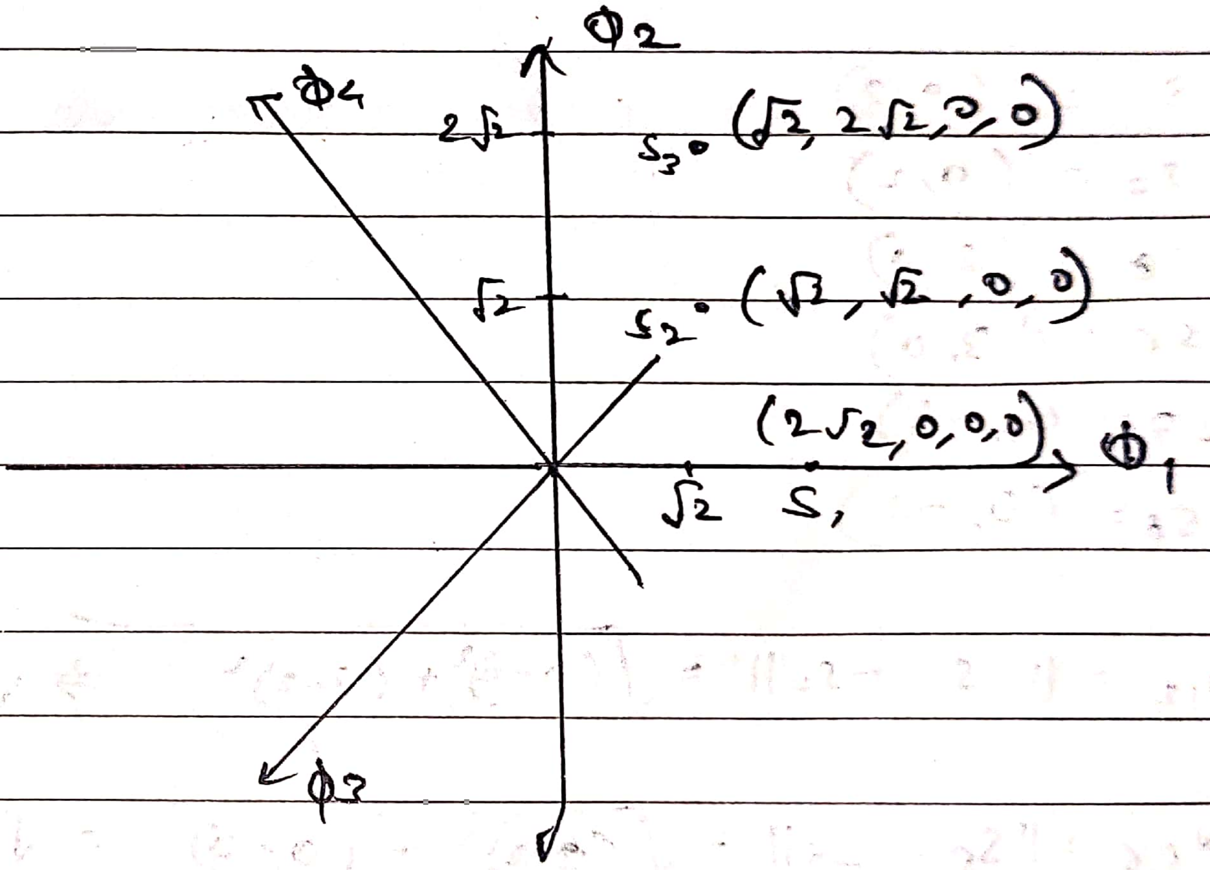
$$\|S_2 - S_3\| = \sqrt{(\sqrt{2}-\sqrt{2})^2 + (\sqrt{2}-2\sqrt{2})^2 + 0 + 0} = \sqrt{2}$$

$$\|S_2 - S_4\| = \sqrt{\left(\sqrt{2} - \frac{3\sqrt{2}}{2}\right)^2 + \left(\sqrt{2} - \frac{\sqrt{2}}{2}\right)^2 + 0 + \left(0 - \frac{\sqrt{2}}{2}\right)} = \frac{\sqrt{6}}{2}$$

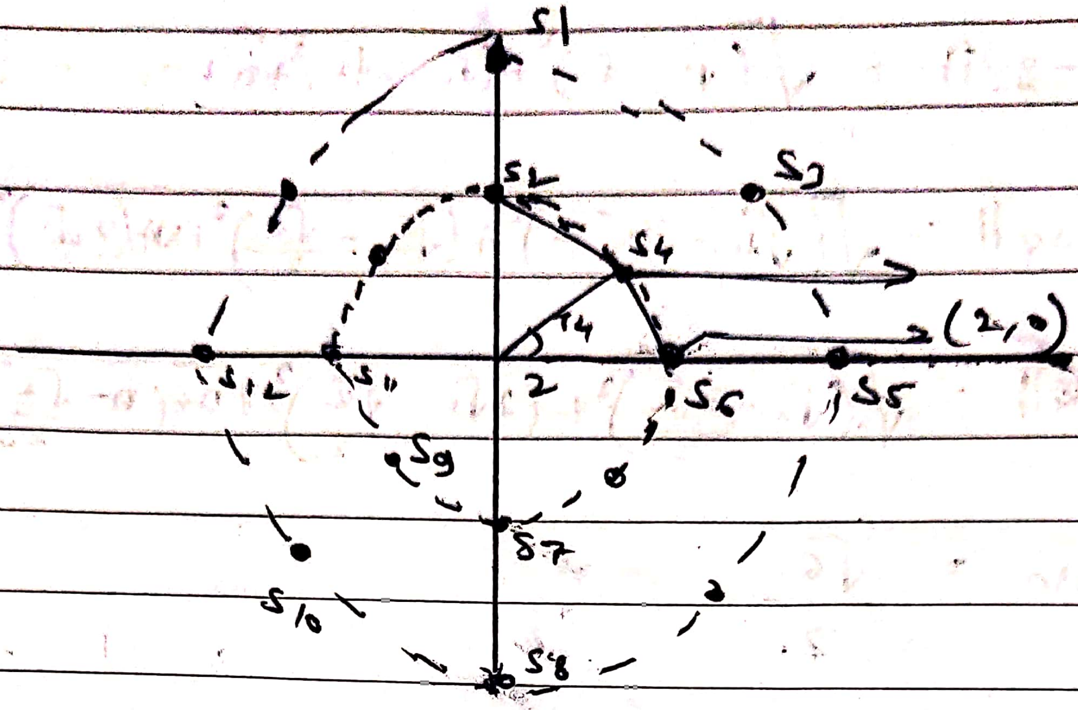
$$\|S_3 - S_4\| = \sqrt{\left(\sqrt{2} - \frac{3\sqrt{2}}{2}\right)^2 + \left(2\sqrt{2} - \frac{\sqrt{2}}{2}\right)^2 + 0 + \left(0 - \frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{22}}{2}$$

$$d_{min} = \frac{\sqrt{6}}{2}$$

c)



Q



$S_1 = (0,3)$

$S_2 = (0,2)$

$S_5 = (2,0)$

$S_6 = (3,0)$

$S_7 = (0,-2)$

$S_8 = (0,-3)$

a) $d_{12} = \|S_1 - S_2\| = \sqrt{(0-0)^2 + (3-2)^2} \Rightarrow \sqrt{1^2} = 1$

$d_{56} = \|S_5 - S_6\| = \sqrt{(3-2)^2 + (0-0)} = \sqrt{1^2} = 1$

closest distance is $S_1, S_2, \& S_3, S_4 \& S_5, S_6 \& S_7, S_8 \& S_{11}, S_{12}$

$d_{min} = 1$

b) Average symbol energy

$S_1 = S_5 = 3^2$

$S_2 = S_6 = 2^2$

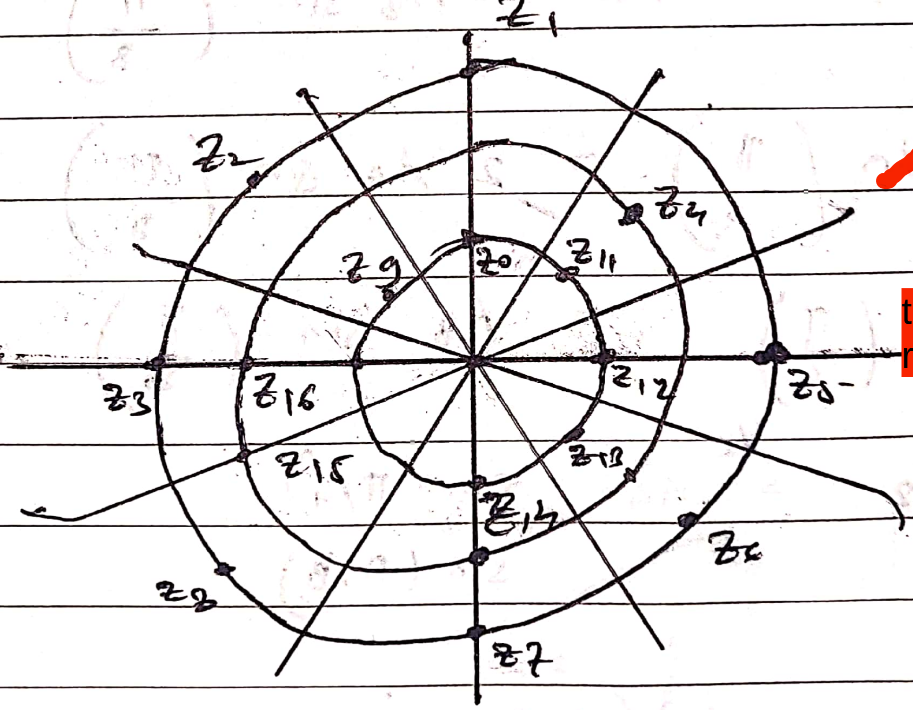
$S_8 = S_{12} = (-3)^2$

$S_{11} = S_7 = (-2)^2$

$E_s = A^2$ [The energy of PSK for all signal is same]

[Amplitude doesn't change]

$\Rightarrow \frac{1}{16} \Sigma (8 \times 4 + 8 \times 9) = \frac{11}{2}$

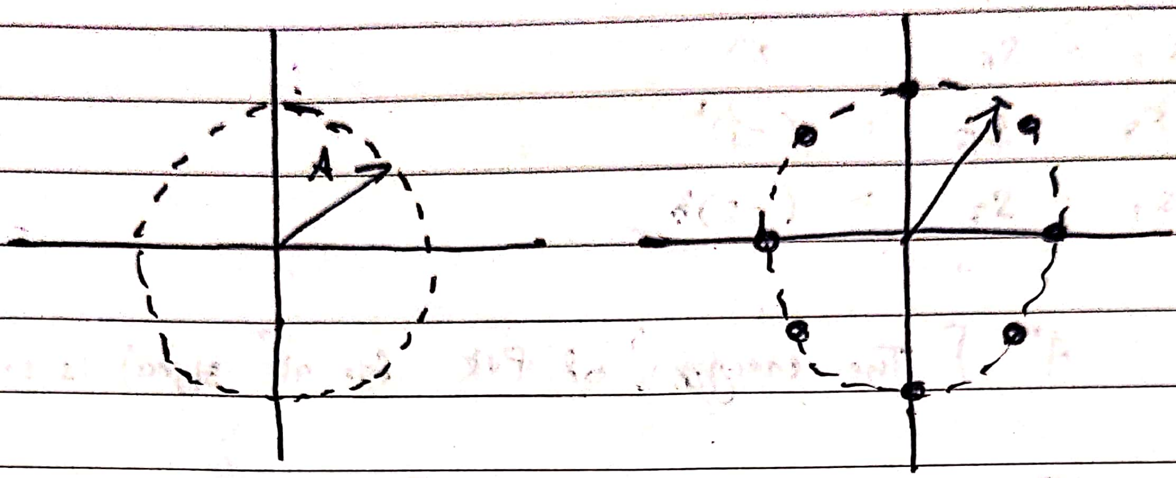


-2

there is no distinguishable region

Q3

For QPSK & 8-PSK



To have the same minimum distance, The radii are

$d_{min} \rightarrow$ For QPSK $= 2A_4 \sin\left(\frac{\pi}{4}\right)$

$d_{min} \rightarrow$ For 8 PSK $= 2A_8 \sin\left(\frac{\pi}{8}\right)$

$$\Rightarrow \cancel{2} A_4 \sin\left(\frac{\pi}{4}\right) = \cancel{2} A_8 \sin\left(\frac{\pi}{8}\right)$$

$$A_4 \sin\left(\frac{\pi}{4}\right) = A_8 \sin\left(\frac{\pi}{8}\right)$$

$$A_8 = A_4 \frac{\sin(\pi/4)}{\sin(\pi/8)}$$

$$A_8 = A_4 (1.8477)$$

$$\frac{A_8}{A_4} = 1.8477.$$

⇒ From looking at the amplitude of 8 Psk we can see the amplitude to larger & this will require more energy.
We can also do it in this manner.

$$E_8 = \frac{1}{8} \Sigma_8 (A_8)^2 = A_8^2$$

$$E_4 = \frac{1}{4} \Sigma_4 (A_4)^2 = A_4^2$$

$$\frac{(A_8)^2}{(A_4)^2} = (1.8477)^2 = 3.4128$$

$$E_S\ (8\ Psk) = 3.4128\ E_S\ (4\ Psk)$$

Q4

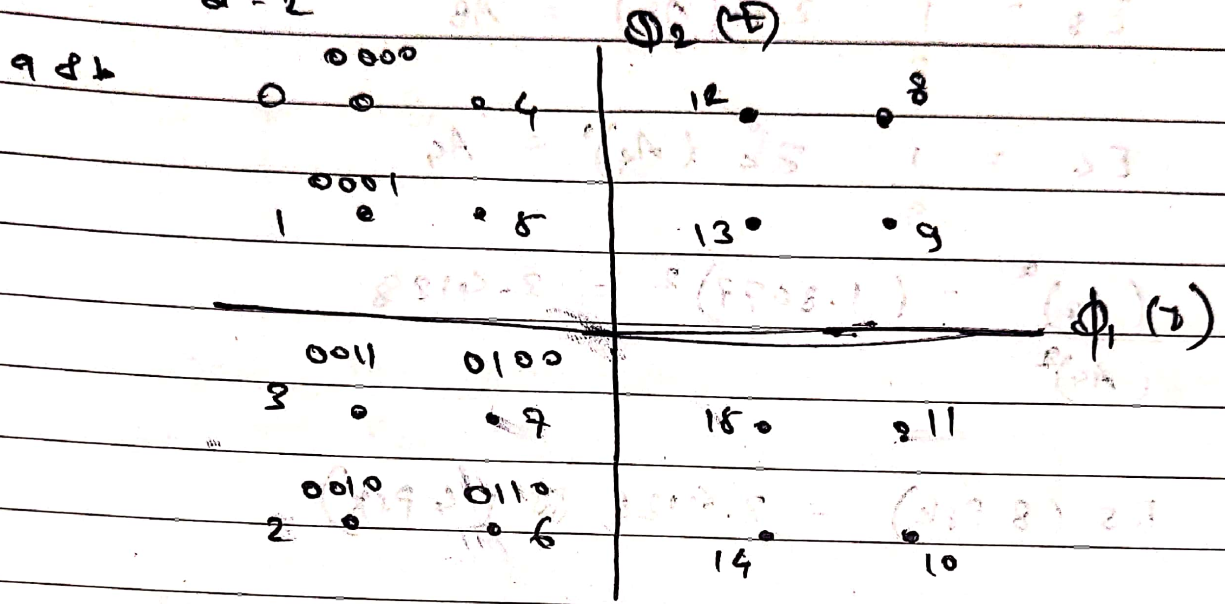
16 - QAM

$d_{min} = 4$

$d_{min} = 2d$

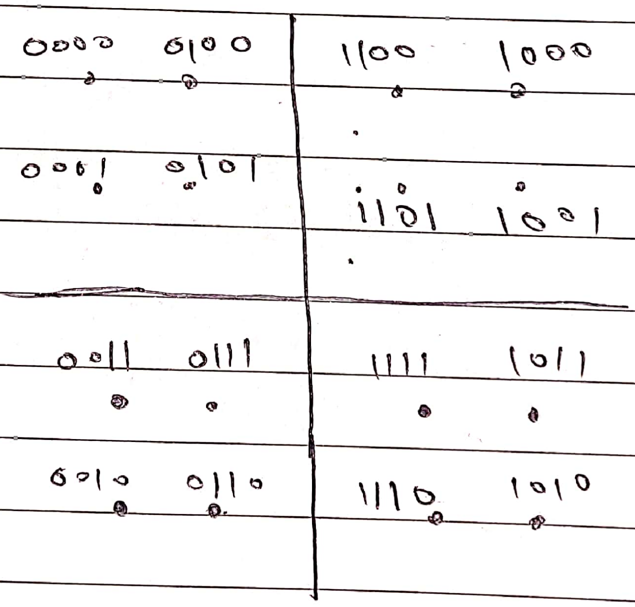
$4 = 2d$

$d = 2$

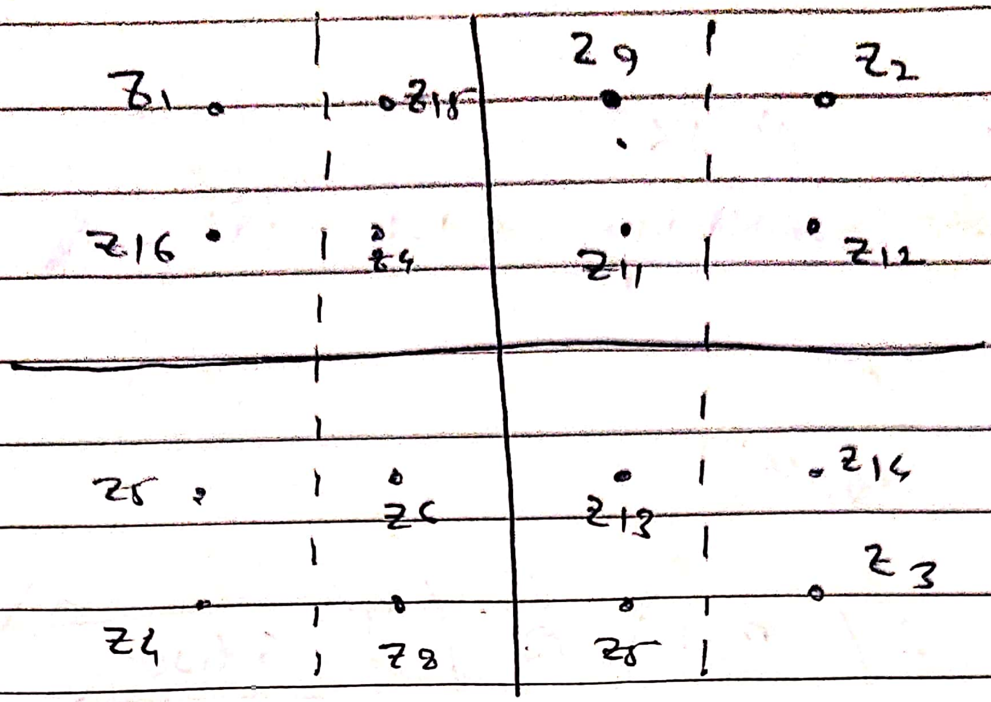


M = 16 K = 4

Binary	Graycode
0000	0000
0001	0001
0010	0011
0011	0010
0100	0110
0101	0111
0110	0101
0111	0100
1000	1100
1001	1101
1010	1111
1011	1110
1100	1010



1101	1011
1110	1001
1111	1000



$d_{min} = 4$

$d = 2$

$d_{min} = 2d$

d) For 16 QAM $k = 4$ || $E_s = \frac{2}{3}(M-1)(d)^2$

$$E_s = \frac{2}{3}(16-1)(2)^2 = 40$$

For 32 QAM $k = 5$

$$E_s = \frac{2}{3}(32-1)(2)^2$$

$$= 82.66$$

Q5

16 PAM $\quad M = 16$

Bit error probability $= 1.1 \times 10^{-5} = P_b$

$$P_b = \frac{P_s}{\log_2 M}$$

$$P_s = 2\left(\frac{M-1}{M}\right) Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_o}}\right)$$

$$P_b = \frac{2}{\log_2 M}\left(\frac{M-1}{M}\right) Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right) = 1.1 \times 10^{-5}$$

$$\Rightarrow \frac{15}{32} Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_o}}\right) = 1.1 \times 10^{-5}$$

$$\Rightarrow \frac{15}{32} Q(x) = 1.1 \times 10^{-5}$$

Let $x = \sqrt{\frac{6E_s}{(M^2-1)N_0}}$

$Q(x) = 2.3 \times 10^{-5}$

$x = 4.07$

$Q(4.07) = 2.3 \times 10^{-5}$

$$\therefore \sqrt{\frac{6\,E_s}{(M^2-1)\,N_o}} = 4.07$$

$$\boxed{\frac{E_s}{N_o} = y}$$

$$\sqrt{\frac{6}{(16^2-1)}\,y} = 4.07$$

$$0.023529\,y = (4.07)^2$$

$$y = \frac{(4.07)^2}{0.023529}$$

$$\Rightarrow \frac{E_s}{N_o} = 704.02$$

$$E_b = \frac{E_s}{\log_2 M} = \frac{704.02}{\log_2(16)} = \frac{704.02}{4} = \underline{176 \text{ per bit}}$$

$$dB = 10 \log (SNR) = 10 \log (17.6) = \underline{22.45\,dB}$$

As

Q6

B = 25 KHz

Symbol rate = 4800 symbol/sec

bit rate = 19200 bit/sec

For FSK

$$W = (1+2)\frac{M R_b}{2\log_2 M} \text{ Hz}$$

No. of bit/symbol $= \frac{19200}{4800} = 4$ bits/symbol

$K = 4$ bits, $M = 2^4 = 16$

⇒ 16-FSK will be used

2

To find α = roll off

$$25000 = (1+\alpha) \cdot \frac{16 \times 19200}{2\log_2(16)}$$

$$25000 = (1+\alpha)\, 38,400$$

$$\frac{25000}{38400} = (1+\alpha)$$

$$0.65 = 1+\alpha$$

$\alpha = 0.65 - 1$

$$\boxed{\alpha = -0.35}$$