Digital Communications 'HW-1

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1) a)

$x_1(t) = 2\,\mathrm{sinc}^{10}(420\pi t)$

It can be written as $2(\mathrm{sinc}(420\pi t))^{20}$

We know that, $2B\,\mathrm{sinc}(2\pi Bt) = \Pi\left(\frac{f}{2B}\right)$

where width $= 2B$

i.e. $2\cancel{\pi}B\cancel{t} = 420\cancel{\pi}\cancel{t}$

$2B = 420$

$B = 210$

If $f_s = 2B$ then we obtain Nyquist rate

$f_s = 2 \times 210$

$= 420$

As the function is raised to 10 power

$10 \times f_s = 10 \times 420$

$= 4200$ samples / sec

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1) b) $x_2(t) = \sin^5 c(6500 \pi t) + \sin c^5(13000 \pi t)$

$(\sin c(6500 \pi t))^5 + (\sin c(13000 \pi t))^5$

Highest Freq Component $= 13000 \pi t$

$2\cancel{\pi} B \cancel{t} = 13000 \cancel{\pi t}$

$B = 6500$

$\therefore f_s = 2B = 2 \times 6500$

$= 13000$ samples / sec

Function raised to power of 5

$\therefore 5 \times f_s = 5 \times 13000$

$= 65000$ samples / sec

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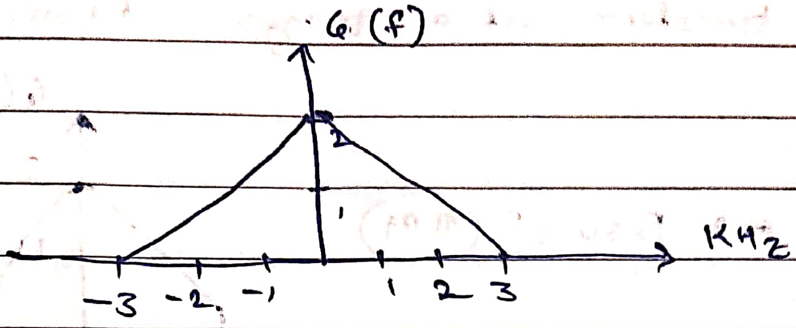
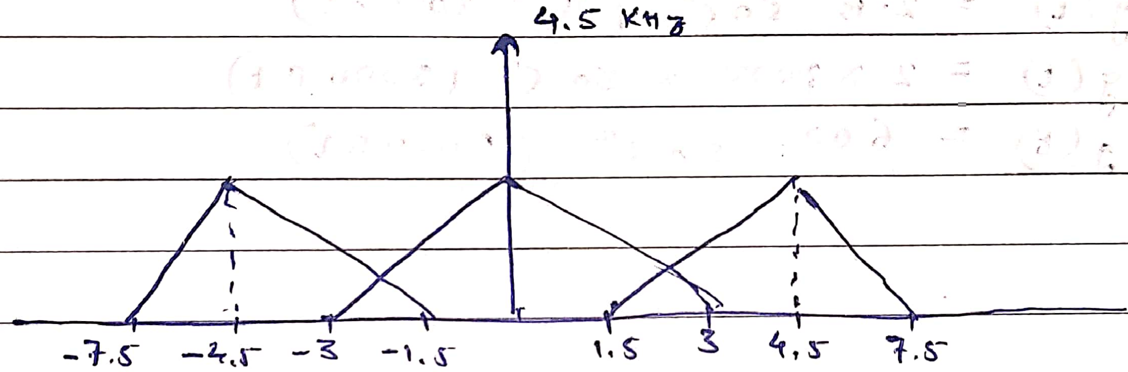
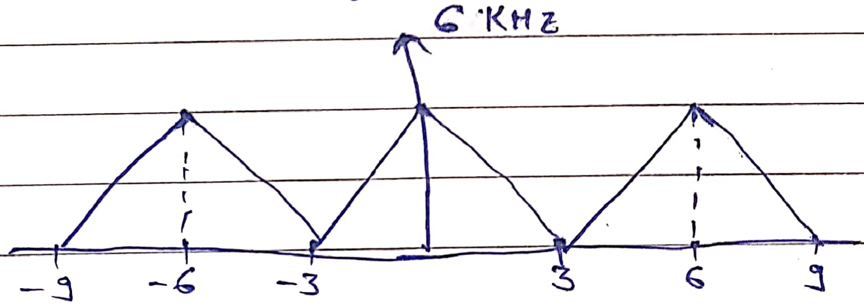
2) Given spectrum



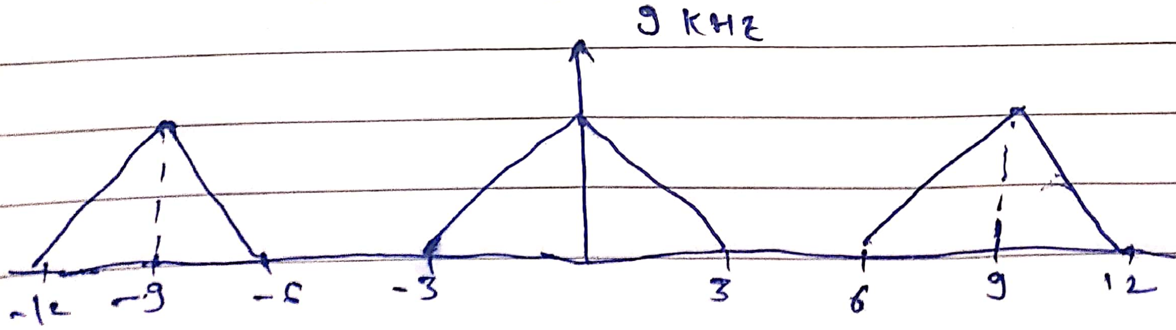
a) Sampling frequency f_s = 4500 Hz or 4.5 KHz



b) Sampling frequency f_s = 6000 Hz or 6 KHz



c) Sampling frequency f_s = 9000 Hz or 9 KHz



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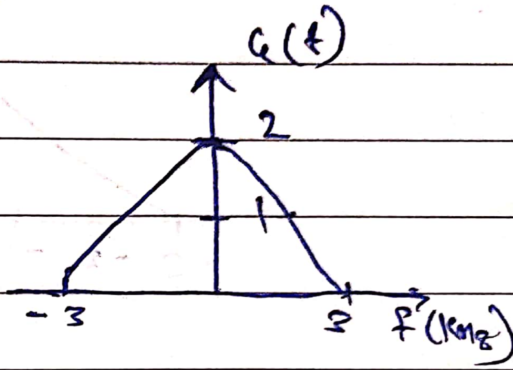
3) a) Expression for $g(t)$

The fourier transform of a triangular function is given by



$$\Delta\left(\frac{f}{2B}\right) \longleftrightarrow B \operatorname{sinc}^2(\pi B t)$$

Given - Amplitude = 2

Bandwidth = 3 KHz

$\therefore\ g(t) = 2 \cdot B \cdot \operatorname{sinc}^2(\pi \times 3000 \times t)$

$\therefore\ g(t) = 2 \times 3000 \times \operatorname{sinc}^2(3000 \pi t)$

$g(t) = 6000 \operatorname{sinc}^2(3000 \pi t)$

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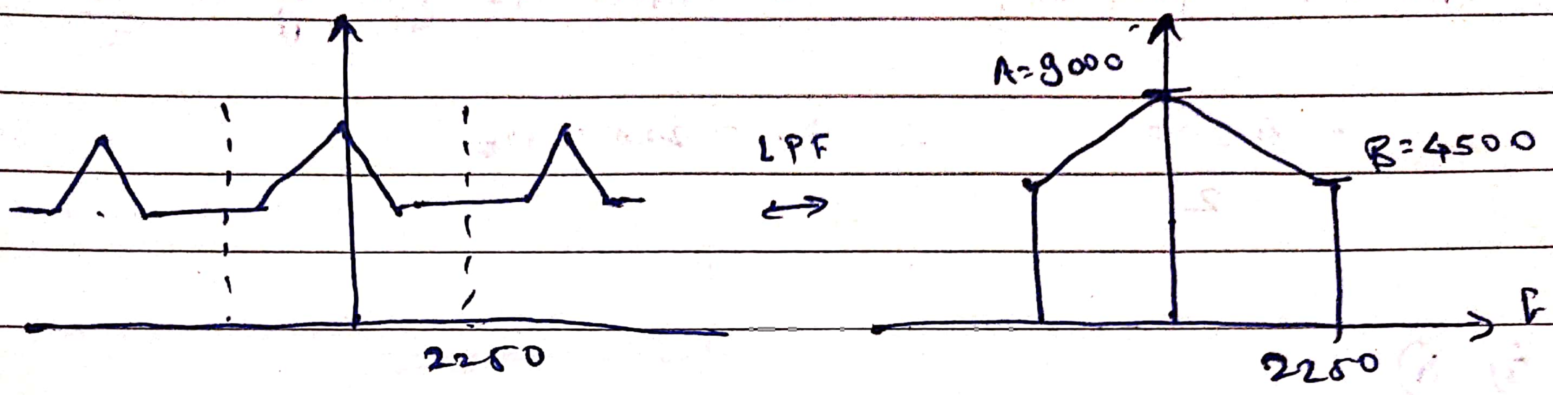
3) b) Expression for output signal $\tilde{g}(t)$



$$\tilde{g}(t) = A\,\Delta\left(\frac{f}{2B}\right) + B\,\Pi\left(\frac{f}{2B}\right)$$

$$= 9000 \times \operatorname{sinc}^2(\pi \times 4500 \times t) + 4500 \times \operatorname{sinc}(2 \times \pi \times 2250 \times t)$$

$$= 9000\,\operatorname{sinc}^2(1500\pi t) + 4500\,\operatorname{sinc}(4500\pi t)$$

$g(t)$ & $\tilde{g}(t)$ are not same as sampling frequency f_s is less than nyquist rate. This leads to aliasing effect. There by it generates a rectangular signal in combination to triangular signal.

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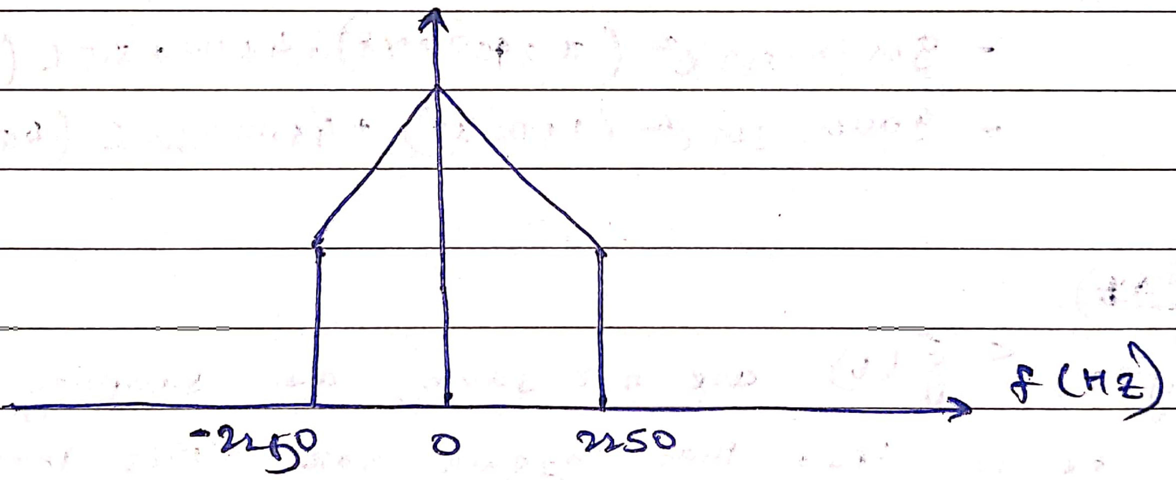
3) c)

Cut off frequency of the filter is given by $f_c = \frac{f_s}{2}$

$= \frac{4500}{2}$ $\therefore$ $f_c = 2250$ Hz

3) d)

The spectrum of the output is



4) Given normalized peak voltage = 1V = m_p

Average message power = 150 mW = 150×10^{-3} W

SNR $\geq$ 40 dB

$$10 \log (x) = 40$$

$$\log x = \frac{40}{10} = 4$$

$$x = 10^4$$

$$\frac{S_o}{N_o} \leq 3 \cdot L^2 \frac{\overline{m^2(t)}}{m_p^2}$$

$$10^4 \leq 3 \times L^2 \times \frac{150 \times 10^{-3}}{1^2}$$

$$L^2 \geq \frac{10^4}{3 \times 150 \times 10^{-3}}$$

$$L^2 > \frac{10^6}{45}$$

$$L \geq \sqrt{\frac{10^6}{45}}$$

$$L \geq 149.07$$

L should be power of 2 > 149

$\therefore$ $L = 2^7$

i.e. $2^8 = 256$ $n = 8$ bit

$$L = 256$$

$$\frac{S}{N} = 3 \times (256)^2 \times \frac{150 \times 10^{-3}}{1^2}$$

$$= 3 \times 65536 \times 150 \times 10^{-3}$$

$$= 28491.2 \text{ Hz}$$

$$\frac{S_o}{N_o} = 44.70 \text{ dB}$$

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5) a) Given amplitudes are $[-8, 8]$
and samples $\{2.1, -0.9, 2.5, 1.2, -7.8\}$

$L = 16 \quad m_p = 8$

$$\Delta V = \frac{2m_p}{L} = \frac{2 \times 8}{16} = 1$$

$$\therefore \text{Levels} = \frac{\Delta V}{2} = \{\pm 7.5, \pm 6.5, \pm 5.5, \pm 4.5, \pm 3.5, \pm 2.5, \pm 1.5, \pm 0.5\}$$

Quantized samples $m_q[k]$
$= \{2.5, -0.5, 2.5, 1.5, -7.5\}$

5) b) For non-uniform quantization with 16 levels
$\mu = 20$.

Step 1: Normalise the levels $y = \frac{m}{m_p}$

$y = \{\pm 0.93, \pm 0.81, \pm 0.68, \pm 0.56, \pm 0.43, \pm 0.31, \pm 0.18, \pm 0.06\}$

Step 2: Use $\frac{m}{m_p} = \frac{(1+\mu)^y - 1}{\mu}, \quad 0 \leq y \leq 1$

(i) $\frac{m}{8} = \frac{21^{7.5/8} - 1}{20} = \pm 6.544$

(ii) $\frac{m}{8} = \frac{21^{6.5/8} - 1}{20} = \pm 4.34$

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(iii) $\frac{M}{8} = \frac{21^{5.5/8} - 1}{20} = \pm 2.84$

(iv) $\frac{M}{8} = \frac{21^{4.5/8} - 1}{20} = \pm 1.81$

(v) $\frac{M}{8} = \frac{21^{3.5/8} - 1}{20} = \pm 1.11$

(vi) $\frac{M}{8} = \frac{21^{2.5/8} - 1}{20} = \pm 0.63$

(vii) $\frac{M}{8} = \frac{21^{1.5/8} - 1}{20} = \pm 0.30$

(viii) $\frac{M}{8} = \frac{21^{0.5/8} - 1}{20} = \pm 0.083$

$\therefore$ Quantized samples are $\{1.81, -1.11, 2.84, 1.11, -6.544\}$

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6) Encoder:

K	0	1	2	3	4	5
m [K]	2.2	1.8	0.75	1.25	2.9	2.25
mq [K-1]	1	2.25	1.5	0.75	1.5	2.75
d [K]	1.2	-0.75	-0.75	0.5	1.4	-0.5
dq [K]	1.25	-0.75	-0.75	0.75	1.25	-0.25
mq [K]	2.25	1.5	0.75	1.5	2.75	2.50

Quantization Error:

d [K] - dq [K]   -0.05   0   0   -0.25   0.15   -0.25

Decoder:

dq [K]	1.25	-0.75	-0.75	0.75	1.25	-0.25
mq [K-1]	1	2.25	1.5	0.75	1.5	2.75
mq [K]	2.25	1.5	0.75	1.5	2.75	2.5

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7) Given input signal $m(t) = 2\cos^2(890\pi t) \cancel{\;}$
$-5\sin^2(1000\sqrt{3}\,\pi t)$

We know that $\frac{d}{dt}(\cos^2 x) = -\sin 2x$ & $\frac{d}{dx}(\sin^2 x) = \sin 2x$

$$m(t) = \frac{d}{dt}\left[\left(2\cos^2(890\pi t)\right) - 5\sin^2(1000\sqrt{3}\,\pi t)\right]$$

$$= 2 \times 890\pi \times (-\sin(1780\pi t)) - 5 \times 1000\sqrt{3}\pi \times (\sin(2000\sqrt{3}\pi t))$$

$$= -1780\pi \sin(1780\pi t) - 8660\pi \sin(2000\sqrt{3}\pi t)$$

$$\approx -1780\pi - 8600\pi$$

$$\simeq = -10440\pi$$

$$|m(t)|_{max} = 32798.22 \leq \Delta f_s$$

$$f_s = 2000\sqrt{3}$$

$$\Delta v = \frac{32798.22}{2000\sqrt{3}}$$

$$= \frac{32798.22}{3464.10} = 9.468$$

$$\Delta v \geq 9.468$$