

# Digital Communications HW-1

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i) (a)

$$x_i(t) = 2 \operatorname{sinc}^{10}(420\pi t)$$

It can be written as  $2(\operatorname{sinc}(420\pi t))^{20}$

We know that,  $2B \operatorname{sinc}(2\pi Bt) = \pi \left( \frac{f}{2B} \right)$

where width =  $2B$

$$\text{i.e. } 2\pi Bt = 420\pi t$$

$$2B = 420$$

$$B = 210$$

If  $f_s = 2B$  then we obtain Nyquist rate

$$f_s = 2 \times 210$$

$$= 420$$

As the function is raised to 10 power

$$10 \times f_s = 10 \times 420$$

$$= 4200 \text{ samples / sec}$$

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i) b)  $x_2(t) = \sin^5(6500\pi t) + \sin^5(13000\pi t)$

$$(\sin^5(6500\pi t))^5 + (\sin^5(13000\pi t))^5$$

Highest Reg Component =  $13000\pi t$

$$2\pi Bt = 13000\pi t$$

$$B = 6500$$

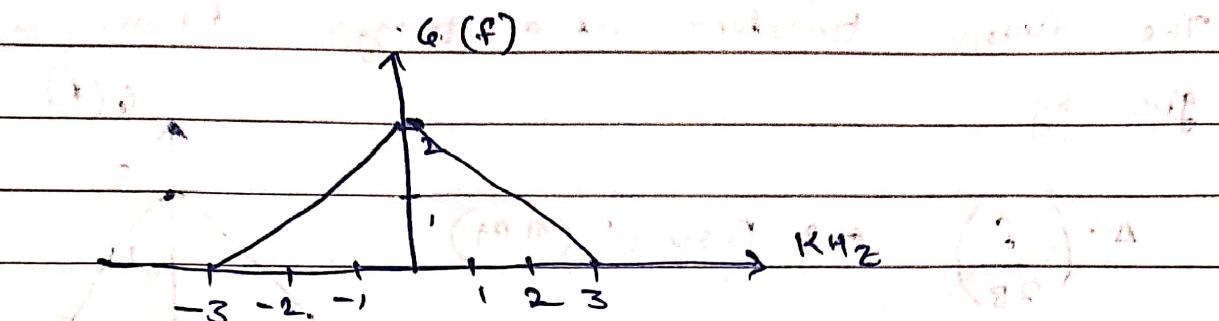
$$\therefore f_s = 2B = 2 \times 6500 \\ = 13000 \text{ samples / sec}$$

Function raised to power of 5

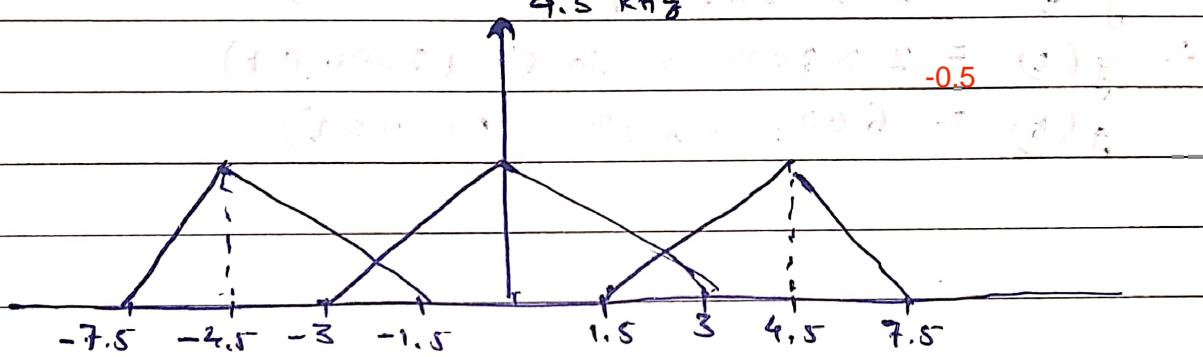
$$\therefore 5 \times f_s = 5 \times 13000^{-2}$$

$$= 65000 \text{ samples / sec}$$

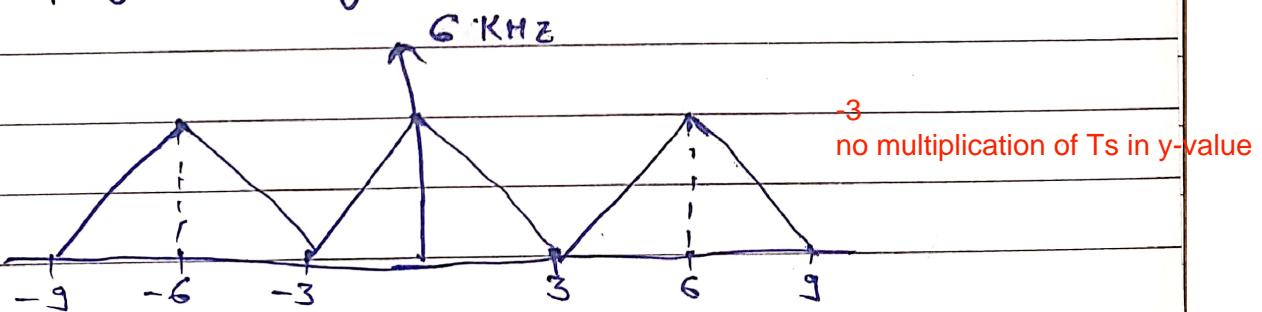
2) Given spectrum ( $G(f)$ ) of message signal to be sampled



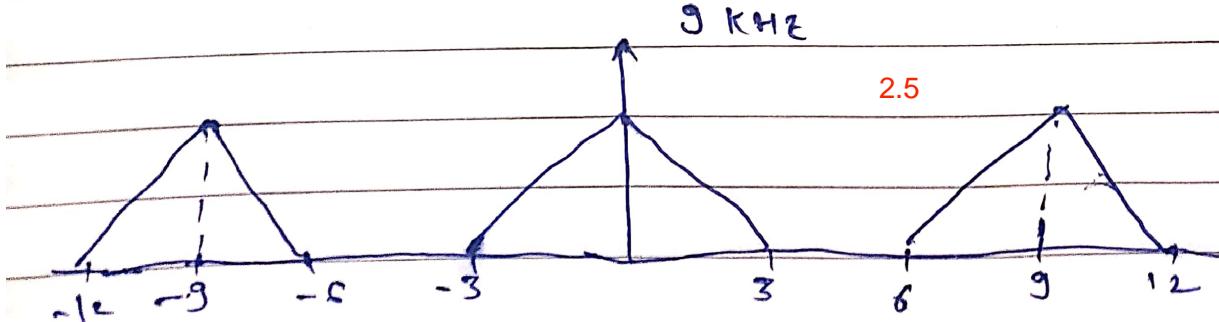
a) Sampling frequency  $f_s = 4500 \text{ Hz}$  or  $4.5 \text{ kHz}$



b) Sampling frequency  $f_s = 6000 \text{ Hz}$  or  $6 \text{ kHz}$



c) Sampling frequency  $f_s = 9000 \text{ Hz}$  or  $9 \text{ kHz}$



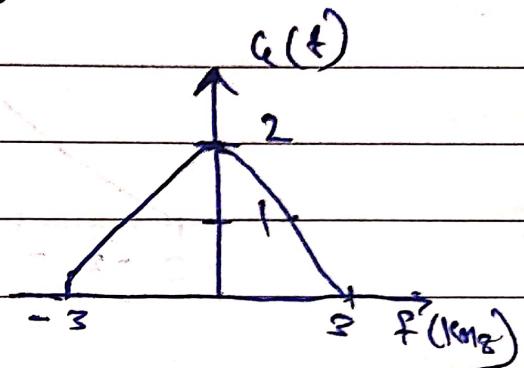
3) a) Expression for  $g(t)$

The Fourier transform of a triangular function is given by

$$\Delta \cdot \left( \frac{f}{2B} \right) \leftrightarrow B \sin C^2 (\pi Bt)$$

Given - Amplitude = 2

Bandwidth = 3 kHz



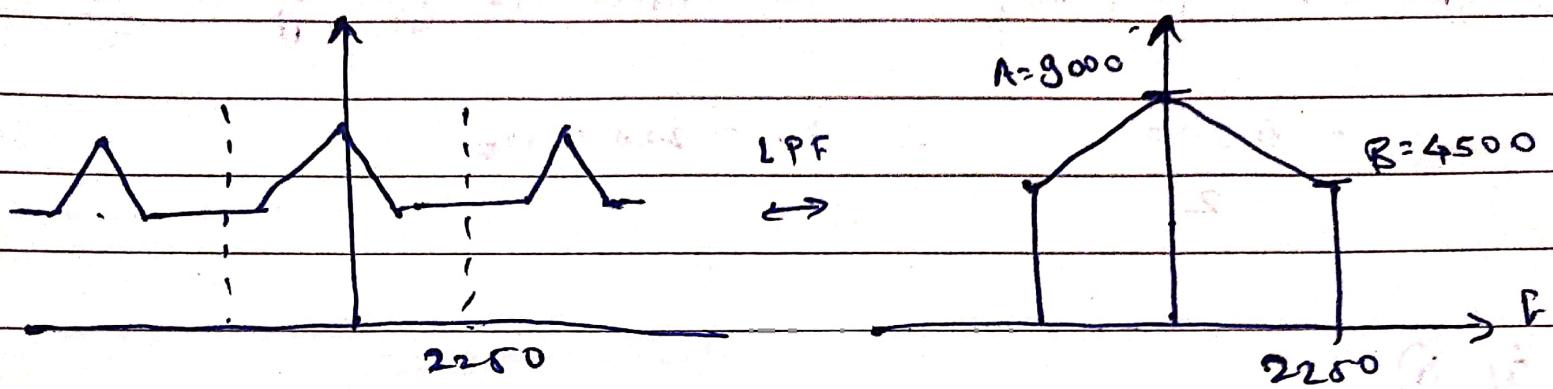
$$\therefore g(t) = 2 \cdot B \cdot \sin C^2 (\pi \times 3000 \times t)$$

$$\therefore g(t) = 2 \times 3000 \times \sin C^2 (3000 \pi t)$$

$$g(t) = 6000 \sin C^2 (3000 \pi t)$$

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3) b) Expression for output signal  $\tilde{g}(t)$



$$\begin{aligned}\tilde{g}(t) &= A \Delta \left( \frac{f}{2B} \right) \sin(\omega_B t + \phi_B) \left( \frac{f}{2B} \right) \\ &= 9000 \sin^2(\pi \times 4500 \times t) + 4500 \sin C (2 \times \pi \times 2250 \times t) \\ &= 9000 \sin^2(1500\pi t) + 4500 \sin C (4500\pi t)\end{aligned}$$

~~Sampling~~

$g(t) < \tilde{g}(t)$  are not same as sampling frequency  $f_s$  is less than nyquist rate. This leads to aliasing effect. Then by it generates a rectangular signal in combination to triangular signal.

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3) c)

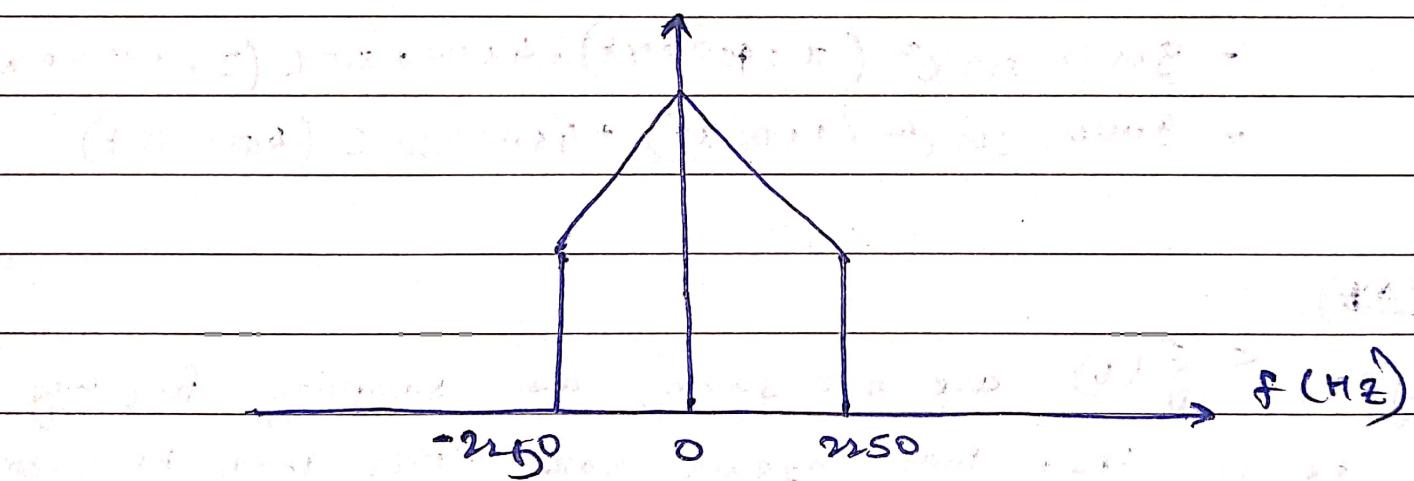
Cut off frequency of the filter is given by  $f_c = \frac{f_s}{2}$

$$= \frac{4500}{2}$$

$$\therefore f_c = 2250 \text{ Hz}$$

3) d)

The spectrum of the output is



4) Given normalized peak voltage  $= 1 \text{ V} = \text{mV}$

Average message power  $= 150 \text{ mW} = 150 \times 10^{-3} \text{ W}$

$$\text{SNR} \geq 40 \text{ dB}$$

$$10 \log(x) \geq 40$$

$$\log x \geq \frac{40}{10} = 4$$

$$x \geq 10^4$$

$$\frac{s_0}{n_0} \leq 3 \frac{L^2}{m_p^2(1)}$$

$$10^4 \leq 3 \times L^2 \times 150 \times 10^{-3}$$

$$L^2 \geq \frac{10^4}{3 \times 150 \times 10^{-3}}$$

$$L^2 \geq \frac{10^6}{45}$$

$$L \geq \sqrt{\frac{10^6}{45}}$$

$L$  should be power of 2  $\geq 149$

$$\therefore L = 2^7$$

i.e.  $2^7 = 128 \Rightarrow n = 8 \text{ bits}$

$$L = 128$$

$$\frac{s_0}{n_0} = 3 \times (128)^2 \times \frac{150 \times 10^{-3}}{128}$$

$$= 3 \times 65536 \times 150 \times 10^{-3}$$

$$= 28481.2 \text{ Hz}$$

$$\frac{s_0}{n_0} = 44.70 \text{ dB}$$

5) a) Given amplitudes are  $[-8, 8]$   
and samples  $\{2.1, -0.9, 2.5, 1.2, \pm 7.8\}$

$$L = 16 \quad m_p = 8$$

$$\Delta V = \frac{2m_p}{L} = \frac{2 \times 8}{16} = 1$$

$$\therefore \text{Levels} = \frac{\Delta V}{2} = \{ \pm 7.8, \pm 6.8, \pm 5.8, \pm 4.8, \\ \pm 3.8, \pm 2.8, \pm 1.8, \pm 0.8 \}$$

Quantized samples  $m_q[k]$

$$= \{ 2.5, -0.5, 2.5, 1.5, -7.5 \}$$

5) b) For non-uniform quantization with 16 levels

$$\mu = 20.$$

Step 1: Normalize the levels  $y = \frac{x}{m_p}$

$$y = \{ \pm 0.93, \pm 0.61, \pm 0.68, \pm 0.56, \pm 0.42, \pm 0.31, \pm 0.18, \pm 0.06 \}$$

Step 2: Use  $\frac{m}{8} = \frac{(1+y)^{1/8}-1}{\mu}, \quad 0 \leq y \leq 1$

$$\textcircled{i} \quad \frac{m}{8} = \frac{2^{1^{7.5/8}-1}}{20} = \pm 6.044$$

$$\textcircled{ii} \quad \frac{m}{8} = \frac{2^{1^{6.5/8}-1}}{20} = \pm 4.34$$

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(iii)  $\frac{m}{8} = \frac{21^{5.5/8} - 1}{20} = \pm 2.84$

(iv)  $\frac{m}{8} = \frac{21^{4.5/8} - 1}{20} = \pm 1.81$

(v)  $\frac{m}{8} = \frac{21^{3.5/8} - 1}{20} = \pm 1.11$

(vi)  $\frac{m}{8} = \frac{21^{2.5/8} - 1}{20} = \pm 0.63$

(vii)  $\frac{m}{8} = \frac{21^{1.5/8} - 1}{20} = \pm 0.30$

(viii)  $\frac{m}{8} = \frac{21^{0.5/8} - 1}{20} = \pm 0.083$

$\therefore$  Quantized samples are  $\{1.81, -1.11, 2.84, 1.11, -0.63\}$

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### 6) Encoder:

$K$	0	1	2	3	4	5
$m[k]$	2.2	1.8	0.75	1.25	2.9	2.25
$m_q[k-1]$	1	2.25	1.5	0.75	1.5	2.75
$d[k]$	1.2	-0.75	-0.75	0.5	1.4	-0.5
$d_q[k]$	1.25	-0.75	-0.75	0.75	1.25	-0.25
$m_q[k]$	2.25	1.5	0.75	1.5	2.75	2.50

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### Quantization Error

$$d[k] - d_q[k] = -0.05 \quad 0 \quad 0 \quad -0.25 \quad 0.18 \quad -0.75$$

### Decoder:

$d_q[k]$	1.25	-0.75	-0.75	0.75	1.25	-0.25
$m_q[k-1]$	1	2.25	1.5	0.75	1.5	2.75
$m_q[k]$	2.25	1.5	0.75	1.5	2.75	2.50

7) Given input signal  $m(t) = 2 \cos^2(890\pi t) - 5 \sin^2(1000\sqrt{3}\pi t)$

We know that  $\frac{d}{dt} (\cos^2 x) = -\sin 2x \quad \text{and} \quad \frac{d}{dx} (\sin^2 x) \sin 2x$

$$m(t) = \frac{d}{dt} \left[ \left( 2 \cos^2(890\pi t) \right) - 5 \sin^2(1000\sqrt{3}\pi t) \right]$$

$$= 2 \times 890\pi \times (-\sin(1780\pi t)) - 5 \times 1000\sqrt{3}\pi \times (\sin(2000\sqrt{3}\pi t))$$

$$= -1780\pi \sin(1780\pi t) - 8600\pi \sin(2000\sqrt{3}\pi t)$$

$$\approx -1780\pi - 8600\pi$$

$$\approx -10440\pi$$

$$|m(t)|_{\max} = 32798.22 \leq \Delta V$$

$$f_s = 2000\sqrt{3}$$

$$\Delta V = \frac{32798.22}{2000\sqrt{3}}$$

$$= \frac{32798.22}{3464.10} = 9.468$$

$$\Delta V \Rightarrow 9.468 \quad 2$$