Tutorial 2: Digital Data Transmission

University of Windsor
Department of Electrical and Computer Engineering
ELEC 4190 - Digital Communications

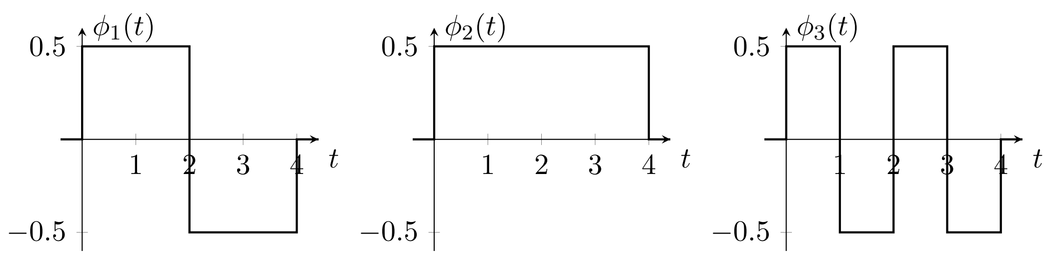
Note: You should try the problems before and during the tutorial session. Your solutions will not be collected or graded. Parts of this tutorial are adapted from the textbooks.

1. Consider the three signal waveforms $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$ shown below:

   (a) Show that these waveforms are orthonormal.

   (b) Express the waveform $s_1(t)$ as a linear combination of $\{\phi_i(t)\}$ and find the coefficients, where $s_1(t)$ is given as

$$s_1(t) = \begin{cases} 2, & 0 \le t < 2, \\ 4, & 2 \le t < 4. \end{cases}$$



2. Consider the two signal waveforms $\{\phi_1(t), \phi_2(t)\}$ given below. Show that these waveforms are orthonormal given that $f_c T_s >> 1$.

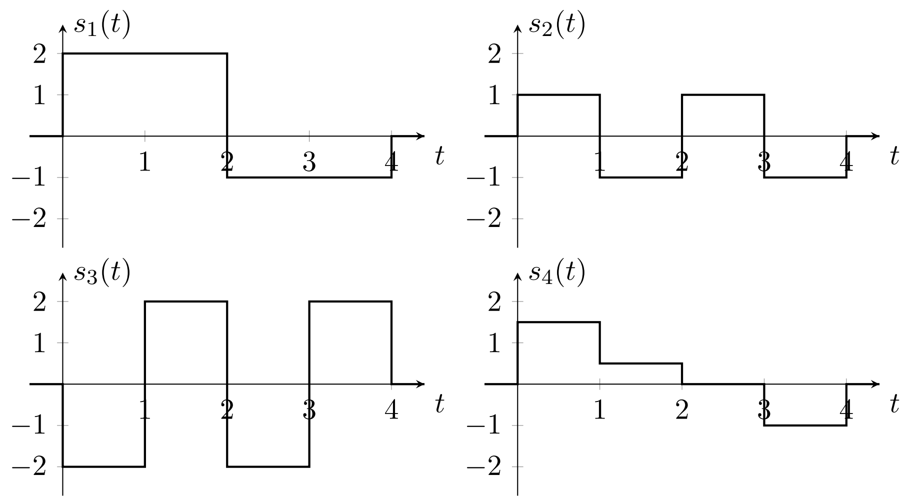
$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t), \quad 0 \le t < T_s$$
$$\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t), \quad 0 \le t < T_s$$

3. Consider the four signal waveforms shown below:

   (a) Determine the dimensionality of the waveforms and a set of basis functions.

(b) Use the basis functions to represent the four waveforms by vectors and plot the signal space representation.

(c) Determine the minimum distance $d_{\min}$ between any pair of vectors.



Self-study: The code below implements the Gram-Schmidt orthonormalization procedure in Octave/MATLAB. The input is the matrix $V = \{s_i(t)\}$ whose rows are the set of given signals $s_1(t), s_2(t), \cdots, s_M(t)$. The output is the matrix $U = \{\phi_j(t)\}$ whose rows are the orthonormal basis functions $\phi_1(t), \phi_2(t), \cdots, \phi_N(t)$ where $N \leq M$. The code assumes that the duration of each data point is 1 sec. Use this function to verify the results in this problem.

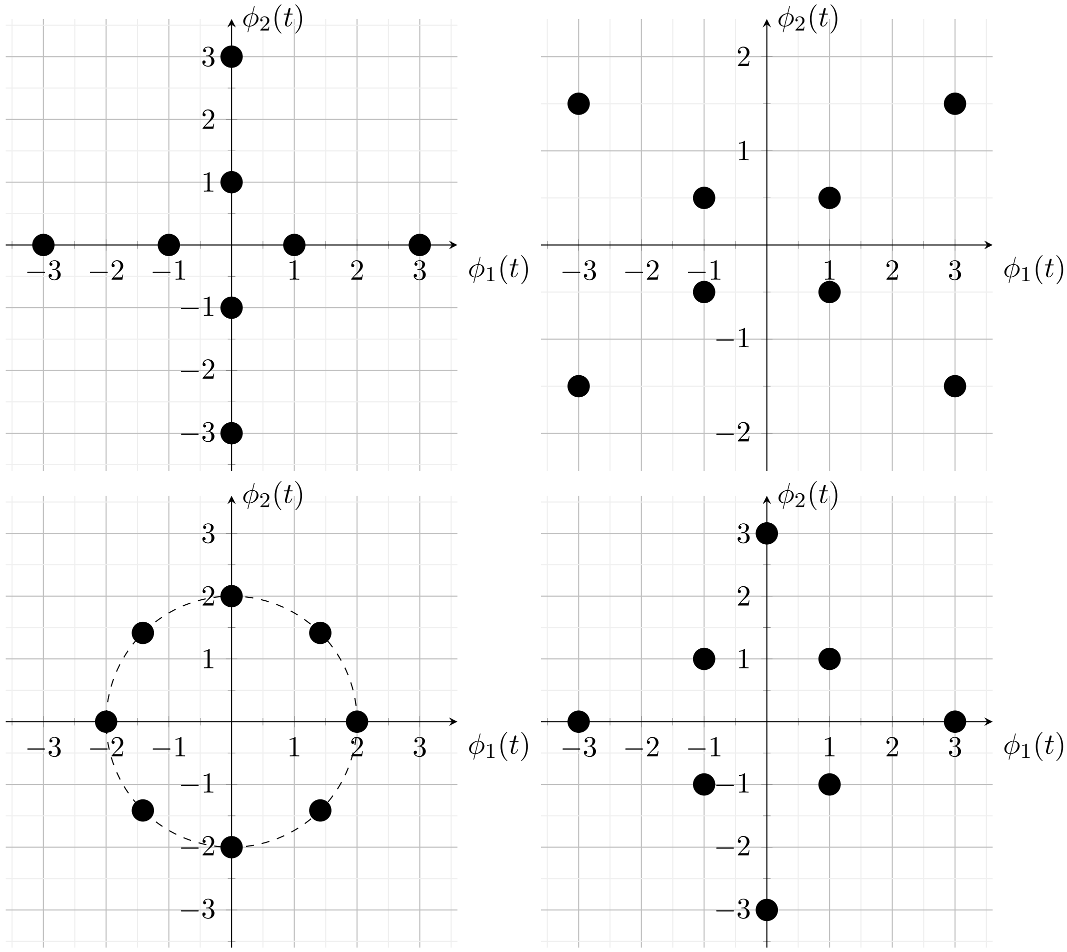
```
function U = gramschmidt(V)
    [n, k] = size(V);
    U = V(1,:) / norm(V(1,:));
    for i = 2:n
        d_i = V(i,:);
        for j = 1:size(U, 1)
            d_i = d_i - (U(j,:)*V(i,:)') * U(j,:);
        end
        if abs(norm(d_i))<1e-6
            continue
        else
            U = [U; d_i / norm(d_i)];
        end
    end
end
```

4. For each of the signal point constellation shown below:

(a) Find the minimum distance between the constellation points $d_{\min}$.

(b) Determine the average symbol energy.

(c) Sketch the decision regions Z_i that minimize error probability assuming AWGN.



5. For 8-ASK with $d_{\min} = 2$ and pulse shaping $g(t) = \sqrt{2/T_s}, 0 \leq t < T_s$,

   (a) Sketch the signal space representation.

   (b) Find a Gray code mapping such that adjacent symbols differ by a single bit.

   (c) Sketch the decision regions Z_i that minimize error probability assuming AWGN.

   (d) What is the additional energy to double the number of bits per symbol (i.e., 64-ASK) while keeping the same bit error probability?

6. Consider a 4-PSK constellation with $d_{\min} = \sqrt{3}$ and a unit-energy NRZ pulse shaping,

(a) Sketch the signal space representation.

(b) Find a Gray code mapping such that adjacent symbols differ by a single bit.

(c) Sketch the decision regions Z_i that minimize error probability assuming AWGN.

(d) What is the additional energy to double the number of bits per symbol (i.e., 16-PSK) while keeping the same bit error probability?

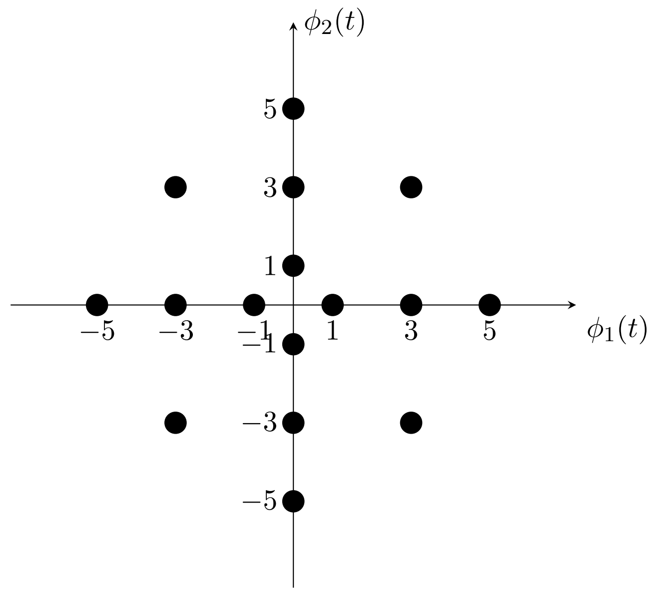
7. Use the symbol error probability of M-ASK systems to prove that the symbol error probability for M-QAM systems is

$$P_s = 1 - \left[1 - 2\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)\right]^2$$

8. Prove that the minimum frequency separation for FSK such that the $\cos(2\pi f_i t)$ and $\cos(2\pi f_j t)$ are orthogonal is $\Delta f = \frac{1}{2T_s}$. Note that in practical systems, the carrier frequency is much higher than 1.

9. Assume a 8-PAM communication system with a unit-energy NRZ pulse shaping filter. If the desired average bit error probability is 10^{-6}, determine the SNR per bit, E_b/N_0.

10. The 16-QAM signal constellation shown below is an international standard for telephone line modems (called V.29). Determine the optimum decision boundaries for the detector. Assume that the SNR is sufficiently high so that errors only occur between adjacent points.



11. An ideal voice-band telephone line channel has a bandpass frequency-response characteristic spanning the frequency range 600-3000 Hz. Design a QPSK system for transmitting data at a rate of 2400 bits/sec and a carrier frequency $f_c = 1800$. For spectral shaping, use a raised cosine frequency-response characteristic. Sketch a block diagram of the system and describe its functional operation. Repeat if the bit rate is R = 4800 bits/sec.

12. A voice-band telephone channel passes the frequencies in the band from 300-3300 Hz. We want to design a modem that transmits at a symbol rate of 2400 symbols/sec, and our objective is to achieve 9600 bits/sec. Select an appropriate QAM signal constellation and the roll-off factor of a pulse with a raised cosine spectrum that utilizes the entire frequency band.

13. Consider a digital communication system that transmits information via QAM over a voice-band telephone channel at a rate 2400 symbols per second. The additive noise is assumed to be white and Gaussian.

    (a) Determine E_b/N_0 required to achieve an error probability of 10^{-5} at 4800 bps.

    (b) Determine E_b/N_0 required to achieve an error probability of 10^{-5} at 9600 bps.

    (c) Determine E_b/N_0 required to achieve an error probability of 10^{-5} at 19200 bps.

    (d) What conclusions do you reach from these results?

14. **Self-study**: Write a program to plot symbol error probability (i.e., P_s) vs. SNR per bit (i.e., γ_b) in an AWGN channel for 4-ASK, 8-ASK, 16-ASK, BPSK, QPSK, 8-PSK, 8-QAM, 16-QAM, and 64-QAM. In your program, compare the results obtained from simulation to that from theoretical expressions. **Hint**: Follow the steps below to simulate each case.

    (a) Assume $E_b = 1$ to generate a random sequence of modulated symbols. That is, use E_b to find $+A$'s and $-A$'s for BPSK, $\pm d$'s and $\pm 3d$'s for 4-ASK, etc.

    (b) To simulate the channel, generate a zero-mean Gaussian random variable with a variance that achieves the desired SNR per bit, then add the noise to the data sequence.

    (c) Demodulate the received sequence based on the decision regions of each constellation.

    (d) Find the percentage of symbol errors.

    (e) Repeat for different values of γ_b.