Tutorial 1: Analog-to-Digital Conversion

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ELEC 4190 - Digital Communications

Note: You should try the problems before and during the tutorial session. Your solutions will not be collected or graded. Parts of this tutorial are adapted from the textbooks.

1. Determine the Nyquist rate of the following signals:
   (a) $x_1(t) = 4\text{sinc}(420\pi t)$
   (b) $x_2(t) = 5\text{sinc}^2(6500\pi t)$
   (c) $x_3(t) = x_1(t) + 2x_2(t)$
   (d) $x_4(t) = x_1(t) \cdot x_2(t)$

2. Consider two signals $x_1(t)$ is bandlimited to $\omega_1 = 1000\pi$, and $x_2(t)$ is bandlimited to $\omega_2 = 2000\pi$. Determine the maximum sampling interval T_s such that the signal $g(t) = x_1(t) * x_2(t)$ is recoverable after sampling using an ideal lowpass filter. Assume impulse-train sampling.

3. For the bandlimited signal $g(t)$ whose Fourier transform is $G(f) = \Pi(f/8000)+\Lambda(f/4000)$, sketch the spectrum of its ideally and uniformly sampled signal $\bar{g}(t)$ at
   (a) Sampling frequency $f_s = 10000$ Hz.
   (b) Sampling frequency $f_s = 8000$ Hz.
   (c) Sampling frequency $f_s = 6000$ Hz.
   (d) If we attempt to reconstruct $g(t)$ from $\bar{g}(t)$ using an ideal LPF with a cutoff frequency $f_s/2$ Hz, which of these sampling frequencies will result in the same signal $g(t)$?

4. A compact disc (CD) records audio signals digitally by using PCM. Let the audio signal bandwidth be 15 kHz.
   (a) If the Nyquist samples are uniformly quantized into $L = 65,536$ levels and then binary-coded, determine the number of binary digits required to encode a sample.
   (b) If the audio signal has average power of 0.1 W and peak voltage of 1 V, find the resulting ratio of signal to quantization noise (SNR) of the uniform quantizer output in part (a).

(c) Determine the number of binary digits per second (bit/s) required to encode the audio signal.

(d) For practical reasons, signals are sampled at a rate well above the Nyquist rate. Practical CDs use $44,100$ samples per second. If $L = 65,536$, determine the number of bits per second required to encode the signal and the minimum bandwidth required to transmit the encoded signal.

5. A message signal $m(t)$ is normalized to peak voltages of ± 1 V. The average message power equals 120 mW. To transmit this signal by binary PCM without compression, uniform quantization is adopted. To achieve a required SNR of at least 36 dB, determine the minimum number of bits required to code the uniform quantizer. Determine the actual SNR obtained with this newly designed uniform quantizer.

6. Given a signal with amplitudes in $[-4, 4]$ and samples $\{2.1, -0.9, 2.5, 1.2, 1.7, -3.8\}$,

   (a) Find the quantized sequence if quantization is uniform with 8 levels.

   (b) Find the quantized sequence if quantization is nonuniform with 8 levels. Assume μ-law quantizer with $\mu = 9$.

7. Consider a simple DPCM encoder in which $N = 1$ is used for $m(t) = A_m \cos(\omega_m t + \theta_m)$. The sampling interval is T_s such that $m[k] = m(kTs)$ with $\theta_m = 0.5\omega_m T_s$. The first-order estimator is formed by

$$\hat{m}_q[k] = m[k-1]$$

   with prediction error

$$\begin{aligned} d[k] &= m[k] - \hat{m}_q[k] \\ &= m[k] - m[k-1] \\ &= A_m[\cos(k\omega_m T_s + \theta_m) - \cos(k\omega_m T_s - 0.5\theta_m)] \end{aligned}$$

   (a) Determine the peak value of $|d[k]|$.

   (b) Evaluate the amount of SNR improvement in dB that can be achieved by this DPCM over a standard PCM.

8. Consider a message signal as input to the DM system:

$$m(t) = 3\cos(890\pi t) - 0.7\sin(1000\sqrt{3}\pi t)$$

   Determine the minimum step size Δv necessary to avoid DM slope overload.