* Octal Number System

— The number System with base eight is known as the octal number System.

* Octal - to - Decimal Conversion

— Any octal number can be converted into its equivalent decimal number using the weights assigned to each octal digit position.

- **Prob:** Convert $(6327.4051)_8$ into its equivalent decimal number.

Sol: $(6327.4051)_8 = 6\times8^3+3\times8^2+2\times8^1+7\times8^0+$
$4\times8^{-1}+0\times8^{-2}+5\times8^{-3}+1\times8^{-4}$

$= 3072+192+16+7+\frac{4}{8}+0+\frac{5}{512}+\frac{1}{4096}$

$\therefore (6327.4051)_8 = (3287.5100098)_{10}$

- **Prob:** Convert the following octal numbers to decimal numbers.

a) $(416)_8$ b) $(360.15)_8$

Sol: $(270)_{10}$ $(240.2031)_{10}$

* Decimal - to - Octal Conversion

— The conversion from decimal to octal is similar to the conversion procedure for base-10 to base-2 Conversion. The only difference is that number 8 is used in place of 2 for division in the case of integers and for multiplication in the case of fractional numbers.

Prob: Convert the following decimal numbers to [octal] numbers:

a) $(234)_{10}$

$$\begin{array}{r|l} 8 & 234 \\ \hline 8 & 29 - 2 \\ \hline & 3 - 5 \end{array}$$

$(234)_{10} = (352)_8$

b) $(2988.6875)_{10}$

Integer part:

$$\begin{array}{r|l} 8 & 2988 \\ \hline 8 & 373 - 4 \\ \hline 8 & 46 - 5 \\ \hline & 5 - 6 \end{array}$$

$(2988)_{10} = (5654)_8$

Fractional part:

Fractional No	prod	Fractional part	Int part
.6875	$.6875 \times 8 = 5.5$	.5	5 MSB
.5	$.5 \times 8 = 4.0$	.0	4

$\therefore (2988.6875)_{10} = (5654.54)_8$

c) $(3287.5100098)_{10} = (6327.4051)_8$

Int

$$\begin{array}{r|l} 8 & 3287 \\ \hline 8 & 410 - 7 \\ \hline 8 & 51 - 2 \\ \hline & 6 - 3 \end{array}$$

MSB

Frac

$.5100098 \times 8$	$= 4.0800784$	4	.0800784
$.0800784 \times 8$	$= 0.6406272$	0	.6406272
$.6406272 \times 8$	$= 5.1250176$	5	.1250176
$.1250176 \times 8$	$= 1.0001408$	1	.0001408

Note: The conversion for fractional no.s may not be exact. In general, an approximate equivalent can be determined by terminating the process at a desired p[osition]

Octal-to-Binary Conversion

- Octal numbers can be converted into equivalent binary numbers by replacing each octal digit by its 3-bit equivalent binary.

Octal	Decimal	Binary
0	0	000
1	1	001
2	2	010
3	3	011
4	4	100
5	5	101
6	6	110
7	7	111
10	8	001 000
11	9	001 001
12	10	001 010
13	11	001 011
14	12	001 100
15	13	001 101
16	14	001 110
17	15	001 111

Prob: Convert $(736)_8$ into an equivalent binary.

Sol: $(736)_8 = (111\,011\,110)_2$

Prob: $(725.63)_8$ to binary

$= (111\ 010\ 101 . 110\ 011)_2$

Prob: $(364.25)_8$ to binary

$= (011\ 110\ 100 . 010\,101)_2$

Binary-to-Octal Conversion

— Binary numbers can be converted into equivalent octal numbers by making groups of 3-bits starting from LSB and moving towards MSB for integer part of the number and then replacing each group of 3-bits by its octal representation. For fractional part, the groupings of 3-bits are made starting from the binary point.

Prob: Convert the following binary numbers to octal

a) $(10\,111\,001\,110 \cdot 001\,100)_2 = (5716.14)_8$

b) $(\cdot 111\,001\,111)_2 = (\cdot 717)_8$

c) $(001\,011\,011\,110 \cdot 110\,010\,100\,110)_2 = (1336 \cdot 6246)_8$

* Octal Arithmetic :

Addition

- The sum of two octal digits is the same as their decimal sum, provided the decimal sum is less than 8. If the decimal sum is 8 or greater Subtract 8 to obtain the octal digit.
- A carry of 1 is produced when the decimal sum is corrected this way.

Prob: a) $4_8 + 2_8$ b) $6_8 + 7_8$ c) $1_8 + 7_8$

Sol:

a) $4_8 + 2_8 = 6_8$

b) $6_8 + 7_8 = (13 - 8)$

$6_8 + 7_8 = 5_8$ & carry 1

c) $1_8 + 7_8 = (8 - 8)$

$= 0_8$ carry

d) Add 167_8 and 325_8

Sol:

$$\begin{array}{r} {\scriptstyle 1\;1\;\;} \\ 1\,6\,7 \\ 3\,2\,5 \\ \hline (5\,1\,4)_8 \end{array}$$

e) Add 341_8, 125_8, 472_8 & 577_8

Sol:

$$\begin{array}{r} {\scriptstyle 2\;1\;\;} \\ 3\,4\,1 \\ 1\,2\,5 \\ 4\,7\,2 \\ 5\,7\,7 \\ \hline (1\,7\,5\,7)_8 \end{array}$$

Note: Octal addition and Subtraction can also be performed by converting the numbers to binary perform addition / Subtraction and convert the result back to octal.

Eg: Add $(23)_8$ and $(67)_8$

Sol:

$$\begin{array}{r} 23 \\ +67 \\ \hline (112)_8 \end{array} \qquad \begin{array}{r} = 010\ 011 \\ 110\ 111 \\ \hline 001\ 001\ 010 \end{array} = (112)_8$$

Eg: Subtract $(37)_8$ from $(53)_8$ using 8-bit representation

Sol:

$$\begin{array}{r} 53 \\ -37 \\ \hline +14 \end{array} \qquad \begin{array}{r} 00101011 \\ +11100001 \\ \hline \boxed{1}00001100 \end{array} \text{ (2's comp of 37)}$$

discard ↑ 014

* Subtraction with 7's Complement:

— The 7's complement of an octal number is found by subtracting each digit from 7.

Prob: Find 7's Complement of $(612)_8$

Sol:

$$\begin{array}{r} 7\ 7\ 7 \\ -\ 6\ 1\ 2 \\ \hline (1\ 6\ 5)_8 \end{array}$$

* Steps for Octal Subtraction using 7's Comp. method : (A − B)

Step-1 : Find 7's complement of Subtrahend (7's c of B)

Step-2 : Add two octal numbers (first number (A) & 7's complement of the second number (B))

Step-3 : If carry is produced in the addition, add carry in the LSB of the sum ;

Step-4 : ~~Otherwise~~ If no carry, find 7's complement of the sum as a result with negative sign.

Prob: Use the 7's complement method of subtraction to compute $176_8 - 157_8$

Sol: 7's comp of 157 →

$$\begin{array}{r} 7\,7\,7 \\ -\,1\,5\,7 \\ \hline 6\,2\,0 \end{array}$$

Add 176 & 620 →

$$\begin{array}{r} 1\,7\,6 \\ +\,6\,2\,0 \\ \hline \boxed{1}\,0\,1\,6 \\ +\,1 \\ \hline (0\,1\,7)_8 \end{array}$$

Use the 7's complement method of subtraction (4)
to compute $153_8 - 243_8$

7's compl. of 243 →

$$\begin{array}{r} 7\,7\,7 \\ -\,2\,4\,3 \\ \hline 5\,3\,4 \\ \hline \end{array}$$

Add 153 & 534 →

$$\begin{array}{r} 1\,5\,3 \\ 5\,3\,4 \\ \hline \boxed{0}\,7\,0\,7 \end{array}$$

7's comp of 707 →

$$\begin{array}{r} 7\,7\,7 \\ -\,7\,0\,7 \\ \hline (-\,7\,0)_8 \end{array}$$

Subtraction using 8's Complement

The 8's complement of an octal number is found by adding a 1 to the LSB of the 7's comple of an octal number.

Find the 8's complement of $(346)_8$

$$\begin{array}{r} 7\,7\,7 \\ -\,3\,4\,6 \\ \hline 4\,3\,1 \\ +\,1 \\ \hline (4\,3\,2)_8 \end{array}$$

* Steps for octal Subtraction using 8's comp Use

Step-1: Find 8's complement of Subtrahend

Step-2: Add two octal numbers (first number and 8's complement of the sec. number)

Step-3: If carry is produced in the addition it is discarded;

Step-4: If no carry, find 8's complement of the Sum as a result with negative sign.

Prob: Use the 8's complement method of subtraction to compute $516_8 - 413_8$

Sol: 8's comple of 413 →

$$\begin{array}{r} 7\;7\;7 \\ -\,4\;1\;3 \\ \hline 3\;6\;4 \\ +\,1 \\ \hline (3\;6\;5)_8 \end{array}$$

Add 516 & 365 →

$$\begin{array}{r} 5\;1\;6 \\ +\,3\;6\;5 \\ \hline \boxed{1}\;1\;0\;3 \end{array}$$

Discard the carry →

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Use the 8's complement method of subtraction to compute $316_8 - 451_8$

8's comple of 451 ⟶

$$\begin{array}{r} 777 \\ -\,451 \\ \hline 326 \\ +1 \\ \hline (327)_8 \end{array}$$

- Add 316 & 328 ⟶

$$\begin{array}{r} 316 \\ +\,327 \\ \hline \boxed{0}645 \end{array}$$

- 7's compl of 645 ⟶

$$\begin{array}{r} 777 \\ -\,645 \\ \hline (132)_8 \\ +1 \\ \hline (-133)_8 \end{array}$$

Applications of octal Number System

— It is highly inconvenient to handle long strings of binary numbers. It may cause errors also.

Eg: The binary number 01111110 can easily be rembered as 376 and can be entered as 376 using keys. Since digital circuits can process only 0's & 1's, the octal numbers have to be converted into binary form using special circuits known as oct-to-bin converters before being processed by the digital circuits.

* Hexadecimal Number System

— Hexadecimal number system is very popular in computer uses. This consists of 16-distinct symbols. These are numerals 0 through 9 and alphabets A through F. Since numeric digits and alphabets both are used to represent the digits in the hexadecimal system, therefore, this is an alphanumeric number system.

Hexa	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Table: Binary & Decimal equivalent of Hexadecimal Numbers.

* Hexadecimal-to-Decimal Conversion :

Prob: obtain decimal equivalent of $(3A.2F)_{16}$

Sol: $(3A.2F)_{16} = 3\times16^1 + 10\times16^0 + 2\times16^{-1} + 15\times16^{-2}$

$= 48 + 10 + \frac{2}{16} + \frac{15}{16^2}$

$(3A.2F)_{16} = (58.1836)_{10}$

Prob: $(1A82)_{16}$

$(1A82)_{16} = 1\times16^3 + 10\times16^2 + 8\times16^1 + 2\times16^0$

$= 4096 + 2560 + 128 + 2$

$(1A82)_{16} = (6786)_{10}$

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Decimal - to - Hexadecimal Conversion:

Convert the following decimal numbers to hexadecimal numbers:

a) $(0\cdot625)_{10}$ b) $(2824\cdot725)_{10}$

$0\cdot625 \times 16 = 10$ A

$(0\cdot625)_{10} = (\cdot A)_{16}$

$$
\begin{array}{r|l}
16 & 2824 \\
\hline
16 & 176-8 \\
\hline
 & 11-0
\end{array}
$$

$(B08)_{16}$

$\cdot725 \times 16 = 11\cdot6$ $\cdot6$ B

$\cdot6 \times 16 = 9\cdot6$ $\cdot6$ 9

$\cdot6 \times 16 = 9\cdot6$ $\cdot6$ 9

$(B08\cdot B99)_{16}$

Hexadecimal - to - Binary Conversion:

- Hexadecimal numbers can be converted into equivalent binary numbers by replacing each hex digit by its equivalent 4-bit binary number.

Prob: Convert the following hexadecimal numbers to binary numbers:

a) $(A4C)_{16}$

$(1010\ 0100\ 1100)_2$

b) $(3E7\cdot DA)_{16}$

$(0011\ 1110\ 0111\cdot 1101\ 1010)_2$

Binary - to - Hexadecimal Conversion:

- Binary numbers can be converted into the equivalent hexadecimal numbers by making groups of 4-bits starting from LSB and moving towards MSB for integer part and then replacing each group of 4-bits by its hexadecimal representation.
- For the fractional part, the above procedure is repeated starting from the bit next to the binary point & moving toward the right

Prob: Convert the following binary numbers to hexadecimal numbers.

a) 1011 1100 1011 $= (BCB)_{16}$

b) 0010 1111 0011 . 0011 0010 $= (2F3.32)_{16}$

c) 0010 1101 1110 . 1100 1010 0110 $= (2DE.CA6)_{16}$

d) 0001 1111 0001 . 1001 1001 1010 $= (1F1.99A)_{16}$

* Hexadecimal-to-octal Conversion:

— Hexadecimal numbers can be converted to equivalent octal numbers by converting hexa number to equivalent binary and then to octal.

Prob: Convert the following hexadecimal number to octal number:

a) $(B7A4)_{16}$

001 011 011 110 100 100

$(133644)_8$

b) D43E.5A

001 101 010 000 111 110 . 010 110 100

$(152076.264)_8$

c) $(0.BF85)_{16}$

. 101 111 111 000 010 100

$(0.577024)_8$

Octal-to-Hexadecimal Conversion:

- Octal numbers can be converted to equivalent hex numbers by converting octal to equivalent binary and then to hex.

Prob: Convert the following octal number to hex no.

a) $(744)_8$

000111100100

$(1E4)_{16}$

b) $(3472.56)_8$

011100111010 . 10111000

$(73A.B8)_{16}$

c) $(247.36)_8$

000010100111 . 01111000

$(A7.78)_{16}$

Hexadecimal Arithmetic

Addition

- The sum of two hexadecimal digits is the same as their equivalent decimal sum, provided the decimal equivalent is less than 16.
- If the decimal sum is 16 or greater, subtract 16 to obtain the hexadecimal digit.
- A carry of 1 is produced when the decimal sum is corrected this way.

Prob: a) $3_{16} + 9_{16}$ b) $9_{16} + 7_{16}$ c) $A_{16} + 8_{16}$

Sol: $3_{16} + 9_{16} = C_{16}$

$9_{16} + 7_{16} = 16 - 16$

$= 0$ Carry 1

$A_{16} + 8_{16} = 18 - 16$

$= 2$

Carry 1

Prob: Add $3F8_{16}$ and $5B3_{16}$

Sol:

$$\begin{array}{ccc} 3 & F & 8 \\ 5 & B & 3 \\ \hline (9 & A & B)_{16} \end{array}$$

* Subtraction

— Hex subtraction is best accomplished using the complement method. The 15's & 16's complements for hex numbers are used like 1's & 2's Complements to perform subtraction.

* Subtraction with 15's complement

— The 15's complement of a hexadecimal number is found by subtracting each digit from 15.

Prob: Find 15's complement of $A9B_{16}$

Sol:

$$\begin{array}{cccc} & 15 & 15 & 15 \\ - & A & 9 & B \\ \hline & 5 & 6 & 4_{16} \end{array}$$

Steps for hex Subtraction using 15's complement method:

Step-1 : Find 15's complement of subtrahend

Step-2 : Add two hex numbers (first no. & 15's comple of the second no.)

Step-3 : If carry is produced in the addition, add carry to the LSB of the sum

Step-4 : If no carry, find 15's complement of the sum as a result with a negative sign.

Prob: Use the 15's complement method of subtraction to compute $B02_{16} - 98F_{16}$

Sol: 15's compl of 98F $\longrightarrow$

$$\begin{array}{ccc} 15 & 15 & 15 \\ 9 & 8 & F \\ \hline 6 & 7 & 0 \end{array}$$

Add B02 & 670 $\longrightarrow$

$$\begin{array}{rcccc} & & 6 & 7 & 0 \\ + & & B & 0 & 2 \\ \hline & \boxed{1} & 1 & 7 & 2 \\ & & & + & 1 \\ \hline & & (1 & 7 & 3)_{16} \end{array}$$

Prob: Use the 15's complement method to comp[ute]

$69B_{16} - C14_{16}$

Sol: 15's compl of C14 →

$$\begin{array}{ccc} 15 & 15 & 15 \\ C & 1 & 4 \\ \hline 3 & E & B \end{array}$$

Add 69B & 3EB

$$\begin{array}{cccc} & 3 & E & B \\ & 6 & 9 & B \\ \hline \boxed{0} & A & 8 & 6 \end{array}$$

15's Comple of A86 →

$$\begin{array}{ccc} 15 & 15 & 15 \\ A & 8 & 6 \\ \hline \end{array}$$

$(-5\ 7\ 9)_{16}$

* Subtraction with 16's complement:

- The 16's comple of a hex no. is found by adding a 1 to the LSB of the 15's Comp of a hex number.

Prob: Find the 16's complement of $A8C_{16}$

Sol:

$$\begin{array}{ccc} 15 & 15 & 15 \\ -\ A & 8 & C \\ \hline 5 & 7 & 3 \\ + & & 1 \\ \hline \end{array}$$

$(5\ 7\ 4)_{16}$

Steps for hex Subtraction using 16's complement method:

Step-1: Find 16's complement of subtrahend

Step-2: Add two hex numbers (first no & 16's comple of the 2nd no.)

Step-3: If carry is produced in the addition it is discarded

Step-4: If no carry, find 16's complement of the sum as a result with negative sign.

Prob: Use the 16's complement method of subtraction to compute $CB2_{16} - 972_{16}$

Sol: 16's comp 972 ⟶

$$\begin{array}{ccc} 15 & 15 & 15 \\ 9 & 7 & 2 \\ \hline 6 & 8 & D \\ & + & 1 \\ \hline 6 & 8 & E \end{array}$$

Add CB2 to 68E

$$\begin{array}{cccc} & 6 & 8 & E \\ + & C & B & 2 \\ \hline \boxed{1} & 3 & 4 & 0 \\ \hline & 3 & 4 & 0 \end{array}$$

carry is ignored

Prob: Use the 16's complement method of subtraction to compute $3B7_{16} - 854_{16}$

Sol: 16's compl of 854 →

$$
\begin{array}{ccc}
15 & 15 & 15 \\
8 & 5 & 4 \\
\hline
7 & A & B \\
 & + & 1 \\
\hline
7 & A & C \\
\end{array}
$$

Add 3B7 to 7AC →

$$
\begin{array}{cccc}
 & 7 & A & C \\
\hline
+ & 3 & B & 7 \\
 & 1 & 1 & \\
\hline
\boxed{0} & B & 6 & 3 \\
\hline
\end{array}
$$

16's comp of B63 →

$$
\begin{array}{cccc}
 & 15 & 15 & 15 \\
- & B & 6 & 3 \\
\hline
 & 4 & 9 & C \\
 & & + & 1 \\
\hline
\end{array}
$$

$$(-49D)_{16}$$