

Q1. MCQs

① c. Quantization

② c. Low Pass Averaging Pass

③ b. 7

④ c. $P_1 - P_3 - P_4 - P_2 - P_5 - P_7 - P_{10}$

⑤ b. Time Reversal

⑥ c. $u(n) - u(n-1)$

⑦ a. Low Pass averaging filter.

⑧ d. 300 Hz

⑨ d. Circular Convolution of sequences

⑩ c. 64 and 32

Q 2 87

Given :

$$x(n) = \begin{cases} n^2 & ; 0 \leq n \leq 3 \\ 10-n & ; 4 \leq n \leq 6 \\ n & ; 7 \leq n \leq 9 \\ 0 & ; \text{otherwise} \end{cases}$$

Formula : i) Energy (E) = $\sum_{n=-\infty}^{\infty} |x(n)|^2$

ii) Power (P) = $\lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$

Solution :

$$\text{Energy (E)} = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^3 |n^2|^2 + \sum_{n=4}^6 |10-n|^2 + \sum_{n=7}^9 |n|^2$$

$$= \sum_{n=0}^3 n^4 + \sum_{n=4}^6 (100 - 20n + n^2) + \sum_{n=7}^9 n^2$$

$$= 1^4 + 2^4 + 3^4 +$$

$$|100 - 20 \times 4 + 4^2| + |100 - 20 \times 5 + 5^2| +$$

$$|100 - 20 \times 6 + 6^2| +$$

$$7^2 + 8^2 + 9^2$$

$$= 98 + |100 - 80 + 16| + |100 - 100 + 25| +$$

$$|100 - 120 + 36| + 194$$

$$= 98 + 36 + 25 + 16 + 194$$

$$\text{Energy (E)} = 369$$

[P.T.O]

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Since, Energy of the signal is a finite value, the given signal $x(n)$ is an energy signal.

Since, the signal is an energy signal
Therefore, the power of signal is zero.

$$\therefore \text{Power (P)} = 0$$

32 []

DFT Properties

① Shifting property

$$\text{If } x(n) \xleftarrow{FT} X(k) \quad \text{OR} \quad x(n) \xleftarrow{FT} X(\omega)$$

$$\text{Then, } x(n-m) \xleftarrow{FT} W_N^{mk} \cdot X(k)$$

$$\text{i.e. } x(n-m) \xleftarrow{FT} e^{-j\omega k} X(\omega)$$

$$\& x(n+m) \xleftarrow{FT} W_N^{-mk} \cdot X(k)$$

Shifting property states that when a signal is shifted by m samples then the magnitude spectrum is unchanged but the phase spectrum is changed by amount $(-\omega k)$.

② Frequency shifting

$$W_N^{mn} \cdot X(k) \xleftarrow{FT} x(k+m)$$

$$W_N^{-mn} \cdot X(k) \xleftarrow{FT} x(k-m)$$

③ Conjugate property

$$x(n) \xleftarrow{FT} X(k)$$

$$x^*(n) \xleftarrow{FT} X^*(-k)$$

④ Symmetric Property

$$x(n) \xleftarrow{FT} X(k)$$

$$\text{If } x(n) = x^*(-n)$$

$$\text{Then } X(k) = X^*(N-k)$$

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⑤ Convolution Property

If $x_1(n) \xleftrightarrow{FT} X_1(k)$ & $x_2(n) \xleftrightarrow{FT} X_2(k)$

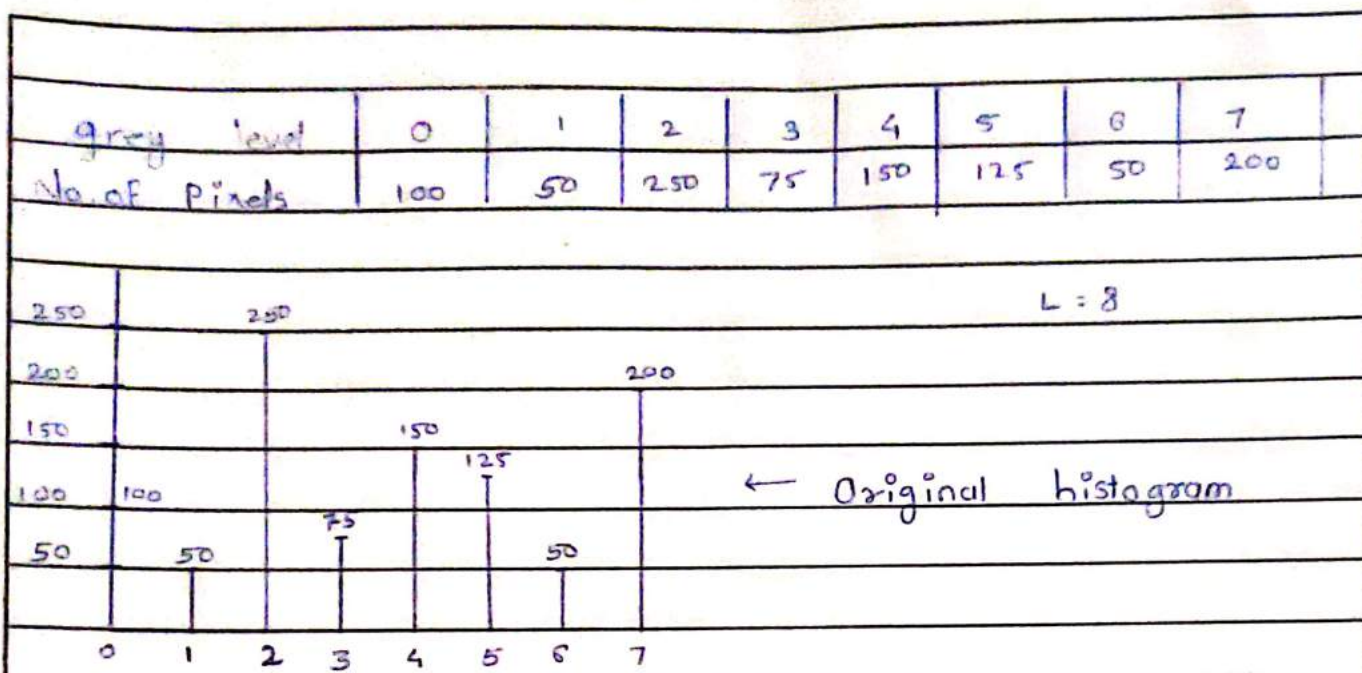
Then $x_1(n) * x_2(n) \xleftrightarrow{FT} X_1(k) \cdot X_2(k)$

Convolution of two signals in time domain is equivalent to multiplication in frequency domain.

Q 3 A)

Histogram

- It is a graphical representation of the intensity distribution of an image
- In simple terms, it represents the no. of pixels for each intensity value considered.
- Histogram is not a unique representation of an image.
- It is a graph of gray values vs frequency occurrences of gray value.
- It depends on the probability or frequency of gray value.
- So no matter how the grey values are distributed over the image, if the frequency of occurrences of gray values is not changed, the histogram will not change.
- Therefore, histogram is not unique representation of images.
- It means that it is possible that two or more different images have same histograms.



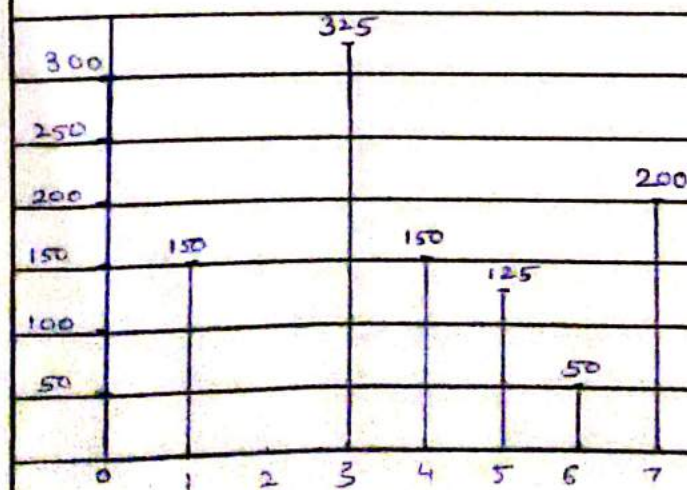
grey level	n_k	PDF	SK	$L-1 \times SK$	Rounding off
0	100	0.1	0.1	0.7	1
1	50	0.05	0.15	1.05	1
2	250	0.025	0.4	2.8	3
3	75	0.075	0.475	3.325	3
4	150	0.15	0.625	4.375	4
5	125	0.125	0.75	5.25	5
6	50	0.05	0.80	5.6	6
7	200	0.2	1	7	7

N = 1000

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old grey level	Equalized grey level	New grey level
0	100 }	1
1	50 }	1
2	250 }	3
3	75 }	3
4	150	4
5	125	5
6	50	6
7	200	7

Equalized grey level	No of Pixels
0	0
1	$100 + 50 = 150$
2	0
3	$250 + 75 = 325$
4	150
5	125
6	50
7	200



Q3 a]

Image segmentation based on discontinuity.

- In this approach, the image is partition based on abrupt changes in grey level.
- The principle area of interest within this category is detection of isolated points and detection of lines and edges in an image.

① Point detection

- The detection of points is done by using following mask

-1	-1	-1
-1	8	-1
-1	-1	-1

- If $|R| > T$ then isolated point is detected.
- Where T is non-negative point threshold.
- $R = w_1z_1 + w_2z_2 + w_3z_3 + \dots + w_9z_9$
- The idea is that gray level of an isolated image will be quite different from gray level of its neighbours.

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② Line Detection

- Masks for line detection

-1	-1	-1		-1	-1	2
2	2	2		-1	2	-1
-1	-1	-1		2	-1	-1

① Horizontal

② $+45^\circ$

-1	2	-1		2	-1	-1
-1	2	-1		-1	2	-1
-1	2	-1		-1	-1	2

③ Vertical

④ -45°

- The first mask respond more strongly to line oriented horizontally.
- While third mask respond more strongly to line oriented vertically and 2nd and 4th mask respond more strongly to line oriented $+45^\circ$ & -45° .
- With constant background the maximum response would result when the line is passing through the middle row of the mask.

③ Edge Detection

- It is most common approach for detection of discontinuities in grey level
- Edge detection characterizes the object boundaries
- Edge point can be thought of as pixel location of abrupt gray levels.
- It is the boundary between two regions with relatively distinct gray level properties.
- Types of edges
 - ① Step edge
 - ② Ramp edge
- Step edges are detected using first order derivatives like sobel, Robert, Prewitt.
- Ramp edges are detected using second order derivatives like Laplacian Filter.

Q4 A]

Zero Memory Point Operations

- It is also known as point processing
- In zero memory point operations, single pixels are used. i.e. T is 1×1 operator.
- It means that the new value $f(x, y)$
- For every input image pixel value, Transformation function gives corresponding output image pixel value. No memory location is required to store intermediate results.
- Let x denotes input image pixel value and S denotes output image pixel value.
- Then $S = T(x)$, where T is any zero memory point operation transformation function.

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Dynamic Range Compression

- Most of the times the dynamic range of the image exceeds the capability of the display device.
- What happens is that some pixel values are so large that the other low value pixels get obscured.
- A simple example of such is that during daytime we cannot see the stars.
- The reason behind this is that the intensity of sun is so large and that of the stars is so low that the eye cannot adjust to such a large dynamic range.
- In image processing, a classic example of such large difference is the Fourier spectrum.
- In Fourier spectrum, only some of the values are very large while most of the values are too small.
- The dynamic range of pixels is of the order of 10^6 .
- Hence when we plot Fourier spectrum, we see only small dots, which represent the large values.

Bit Plane Slicing

- In this technique, we find out the contribution made by each bit to the final image.
- Image is defined as a $256 \times 256 \times 8$ image.
- In this 256×256 is the number of pixels present in the image and 8 is the number of bits required to represent each pixel.
- 8 bits simply means 28 or 256 grey levels.
- Now each pixel will be represented by 8 bits.
- Example
Black is represented as 00000000 and white is represented as 11111111 and between them 254 grey levels are accommodated.
- In bit plane slicing, we see the importance of each bit in the final image.
- This can be followed as: —
 - ① Consider the LSB value of each pixel and plot image using only LSB.
 - ② Continue doing this for each bit till we come to the MSB.
 - ③ Note that we will get 8 different images and all the 8 images will be binary.

Q4. B

$$x(n) = \{1, -2, 3, 2\}$$

① Since $x(n)$ is of length 4.

$\therefore N=4$ we generate a DFT matrix of size 4×4

$$② \quad X(k) = [W_4]_{4 \times 4} x(n)$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2 + 3 + 2 \\ 1 + 2j - 3 + 2j \\ 1 + 2 + 3 - 2 \\ 1 - 2j - 3 - 2j \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 + 4j \\ 4 \\ -2 - 4j \end{bmatrix}$$

$$\therefore X(k) = \{4, -2 + 4j, 4, -2 - 4j\}$$

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③ For magnitude and Phase spectrum

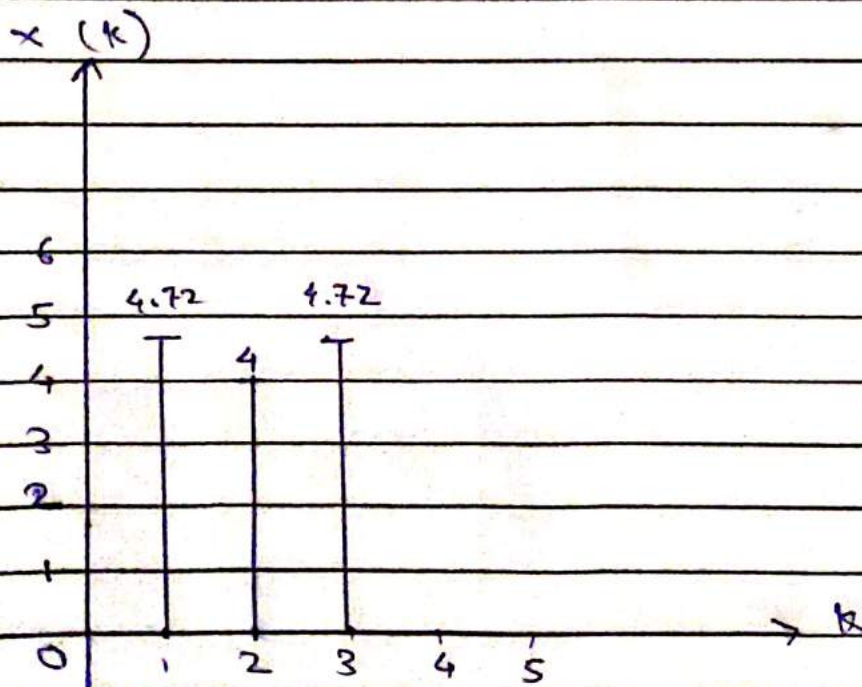
$$|x(k)| = \sqrt{(\text{Real})^2 + (\text{Imaginary})^2}$$

$$\therefore |x(k)| = \{4, 4.472, 4, 4.472\}$$

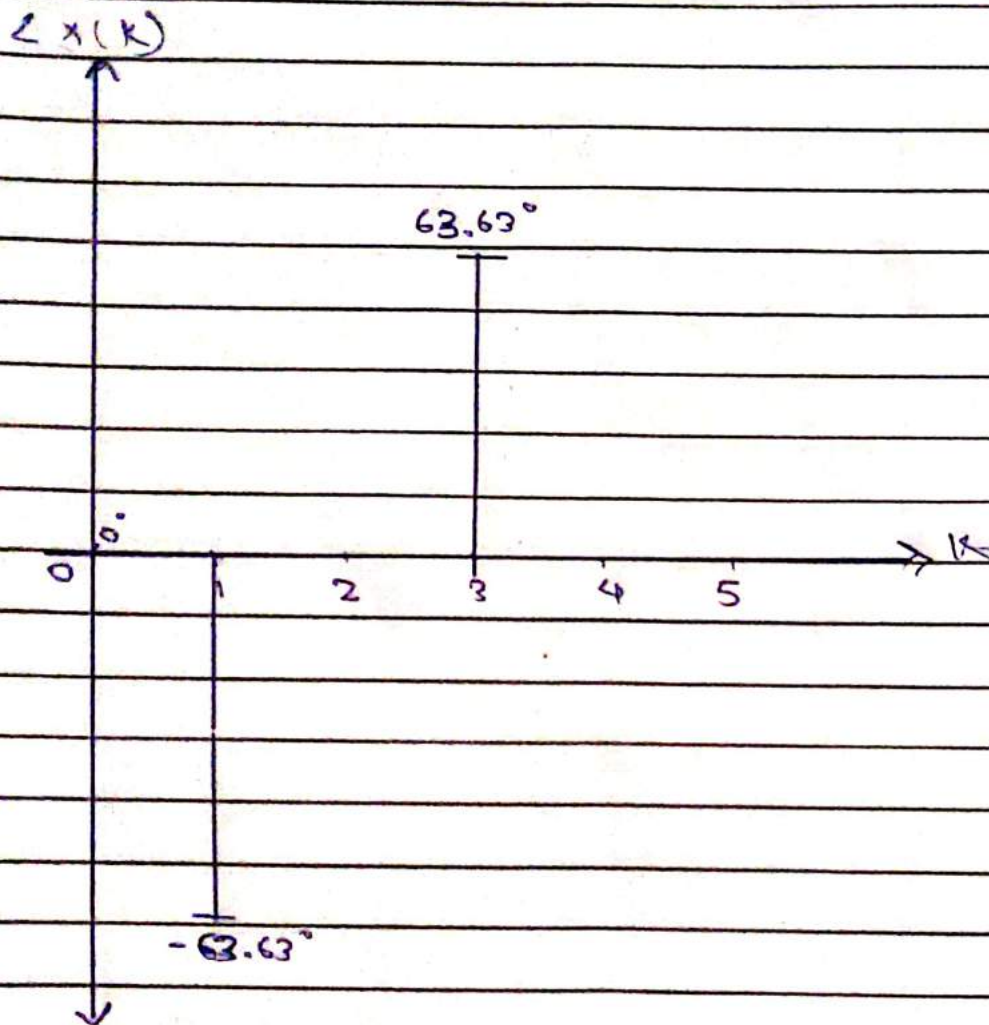
$$\textcircled{4} \therefore \angle x(k) = \tan^{-1} \left(\frac{\text{Imaginary}}{\text{Real}} \right)$$

$$\therefore \angle x(k) = \{0^\circ, -63.43^\circ, 0^\circ, 63.43^\circ\}$$

⑤ Magnitude spectrum



⑥ Phase Spectrum



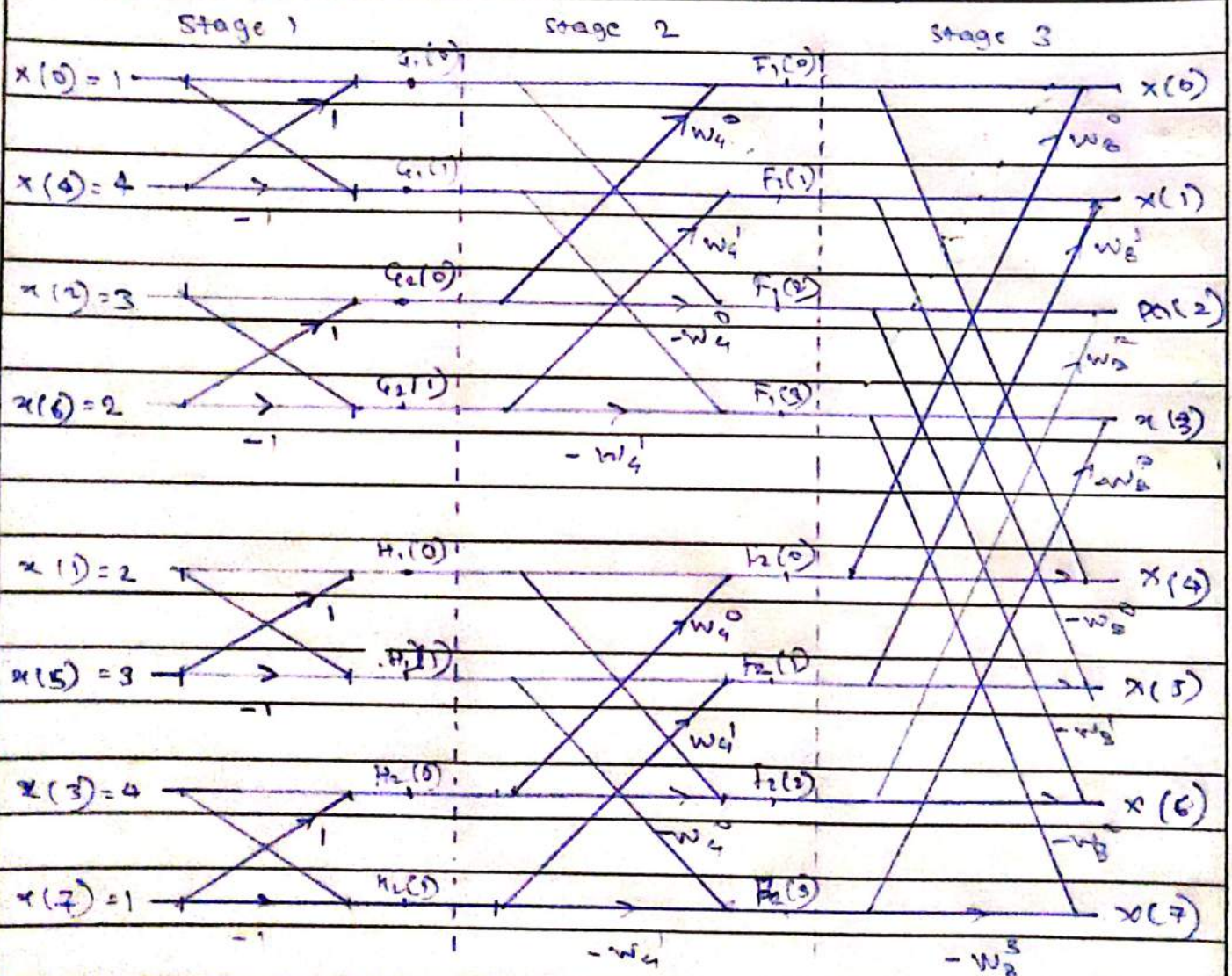
Q4. c]

Solution:

Since $N=8$, we draw a 8-point DIT-FFT diagram.

$$x(n) = \{ 1, 2, 3, 4, 4, 3, 2, 1 \}$$

$$x(0) \quad x(4) \quad x(2) \quad x(6) \quad x(1) \quad x(5) \quad x(3) \quad x(7)$$



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Stage 1

$$G_1(0) = x(0) + x(4) = 1 + 4 = 5$$

$$G_1(1) = x(0) - x(4) = 1 - 4 = -3$$

$$G_2(0) = x(2) + x(6) = 3 + 2 = 5$$

$$G_2(1) = x(2) - x(6) = 3 - 2 = 1$$

$$H_1(0) = x(1) + x(5) = 2 + 3 = 5$$

$$H_1(1) = x(1) - x(5) = 2 - 3 = -1$$

$$H_2(0) = x(3) + x(7) = 4 + 1 = 5$$

$$H_2(1) = x(3) - x(7) = 4 - 1 = 3$$

Stage 2

$$F_1(0) = 5 + 5 = 10$$

$$F_2(0) = 10$$

$$F_1(1) = -3 - j$$

$$F_2(1) = -1 - 3j$$

$$F_1(2) = 5 - 5 = 0$$

$$F_2(2) = 5 - 5 = 0$$

$$F_1(3) = -3 + j$$

$$F_2(3) = -1 + 3j$$

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Stage 3

$$x(0) = 10 + 10 = 20$$

$$\begin{aligned} x(1) &= -3 - j + (0.707 - j 0.707) (-1 - 3j) \\ &= -3 - j + (-0.707 - j 2.121 + j 0.707 - 2.121j) \\ &= -3 - j + (-2.828 - j 1.414j) \\ &= -5.828 - j 2.414 \end{aligned}$$

$$x(2) = 0$$

$$\begin{aligned} x(3) &= -3 + j + (-0.707 - 0.707j) (-1 + 3j) \\ &= -3 + j + (0.707 - 2.121j + j 0.707) \\ &= -3 + j + (2.828 - j 1.414) \\ &= -0.172 - j 0.414 \end{aligned}$$

$$x(4) = 10 - 10 = 0$$

$$\begin{aligned} x(5) &= -3 - j - (0.707 - j 0.707) (-1 - j 3) \\ &= -0.172 + j 0.414 \end{aligned}$$

$$x(6) = 0 - (-j)(0) = 0$$

$$\begin{aligned} x(7) &= -3 - j - (-0.707 - j 0.707) (-1 + j 3) \\ &= -5.828 + j 2.414 \end{aligned}$$

$$\begin{aligned} x(8) &= \{ 20, -5.828 - j 2.414, 0, -0.172 - j 0.414, \\ &\quad -0.172 + j 0.414, 0, -5.828 + j 2.414 \} \end{aligned}$$

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