DFT Computation Using DFT properties.

(1) Linearity

$$x_1(n) = \{1, 2, 3, 4\}; \quad x_2(n) = \{5, 6, 7, 8\}$$

Compute the DFT of the sequence

$$x_3(n) = 2x_1(n) + 3x_2(n)$$

From the linearity property, we know

If $x_1(n) \xleftrightarrow{DFT} X_1(k)$

$x_2(n) \xleftrightarrow{DFT} X_2(k)$

then, $ax_1(n) + bx_2(n) \xleftrightarrow{DFT} aX_1(k) + bX_2(k)$

Now; $x_3(n) = 2x_1(n) + 3x_2(n)$

$$\therefore DFT\{x_3(n)\} = DFT\{ax_1(n)\} + DFT\{bx_2(n)\}$$

$$x_3(n) \xleftrightarrow{DFT} aX_1(k) + bX_2(k)$$

We calculate DFT of $x_1(n)$ & $x_2(n)$

$$X_1(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\therefore X_1(k) = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Similarly,

$$X_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$$

$$\therefore X_2(k) = \begin{bmatrix} 26 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$\therefore DFT\{x_3(n)\} = X_3(k) = 2X_1(k) + 3X_2(k)$$

$$= 2\begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} + 3\begin{bmatrix} 26 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ -4+4j \\ -4 \\ -4-4j \end{bmatrix} + \begin{bmatrix} 78 \\ -6+6j \\ -6 \\ -6-6j \end{bmatrix}$$

$$\therefore X_3(k) = \begin{bmatrix} 98 \\ -10+10j \\ -10 \\ -10-10j \end{bmatrix}$$

$$\therefore X_3(k) = \{98, -10+10j, -10, -10-10j\}$$

② Circular Time shift

$$x(n) \xleftrightarrow{DFT} X(k)$$

$$x(n-m) \xleftrightarrow{DFT} e^{-j\frac{2\pi mk}{N}} X(k)$$

Given $x(n) = \{1, 2, 3, 4\}$

(i) Find the DFT of $x(n)$

(ii) Using the results obtained in (i) obtain the DFT of the following sequences

$$x_1(n) = \{4, 1, 2, 3\}$$

$$x_2(n) = \{3, 4, 1, 2\}$$

(i) We first find the DFT of $x(n)$ using the matrix notation.

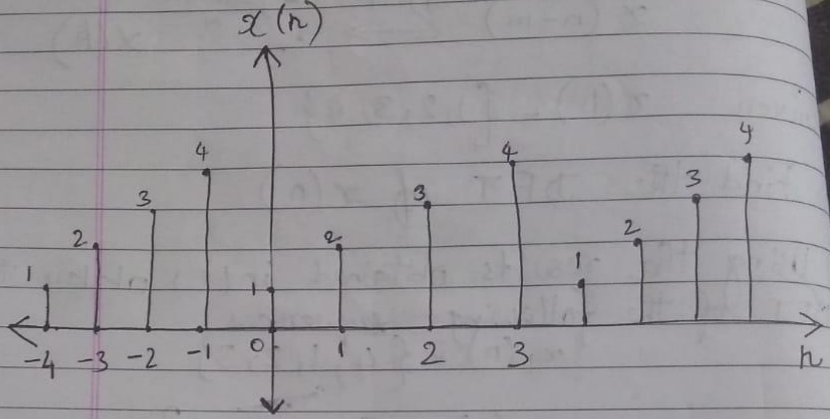
$$X(k) = W_N^{nk} \cdot x(n)$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\therefore X(k) = \{10, -2+2j, 2, -2-2j\}$$

(ii) We now assume that $x(n)$ is periodic, we plot $x(n)$

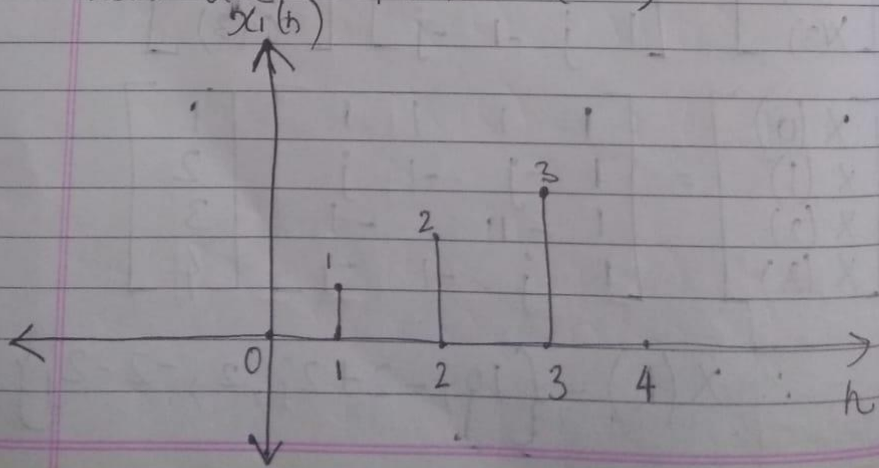


Now $x_1(n)$ is $x(n)$ shifted to the right by one.

Note $x_1(n)$ can also be $x(n)$ shifted to the left by 3.

i.e $x_1(n) = x(n-1)$ OR $x_1(n) = x(n+3)$

Let's take $x_1(n) = x(n-1)$



We know,

If $x(n) \xleftrightarrow{DFT} X(k)$

$$x(n-p) \xleftrightarrow{DFT} e^{\frac{-j2\pi pk}{N}} \cdot X(k)$$

i.e $x(n-p) \xleftrightarrow{DFT} W_N^{pk} X(k)$

$\therefore x(n-1) \xleftrightarrow{DFT} W_N^{k} \cdot X(k)$

$\therefore x_1(n) = x(n-1) \xleftrightarrow{DFT} W_N^{k} \cdot X(k)$

We already know, $X(k) = \{10, -2+2j, -2, -2-2j\}$

i.e. $X_1(k) = W_4^k X(k)$

$X_1(0) = W_4^0 X(0) = (1)(10) = 10$

$X_1(1) = W_4^1 X(1) = (-j)(-2+2j) = 2+j2$

$X_1(2) = W_4^2 X(2) = (-1)(-2) = 2$

$X_1(3) = W_4^3 X(3) = j(-2-2j) = 2-j2$

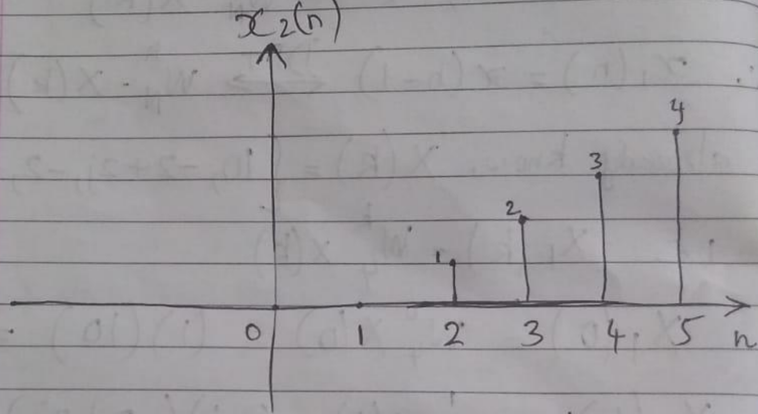
$\therefore DFT\{4, 1, 2, 3\} = \{10, 2+j2, 2, 2-j2\}$

$\therefore X_1(k) = \{10, 2+j2, 2-j2\}$

Similarly for $x_2(n)$, we observe that $x_2(n)$ is simply $x(n)$ shifted to the left by 2 or to the right by 2.

$$\therefore \; x_2(n) = x(n+2) = x(n-2)$$

Let us take $x_2(n) = x(n-2)$



$$\therefore \; x(n-2) \xleftrightarrow{DFT} W_4^{2k} \cdot X(k)$$

$$\therefore \; X_2(k) = W_4^{2k} \cdot X(k)$$

$$X_2(0) = W_4^{0} X(k) = (1)(10) = 10$$

$$X_2(1) = W_4^{2} X(k) = (-1)(-2+2j)$$
$$= 2-2j$$

$$X_2(2) = W_4^{4} X(k) = (1)(-2) = -2$$

$$X_2(3) = W_4^{6} X(k) = (-1)(-2-2j) = 2+2j$$

$$\therefore \; DFT\{3, 4, 1, 2\} = \{10, 2-j2, -2, 2+2j\}$$

$$\therefore \; X_2(k) = \{10, 2-2j, -2, 2+2j\}$$