Q. $x(t) = 5\cos 2\pi(1000t) + 10\cos 2\pi(5000t)$

(i) What is Nyquist rate for this Signal

(ii) If this analog signal is sampled at 4kHz; will the signal be recovered from its samples.

Sol^n:- (i) Nyquist Rate

Given :- $x(t) = 5\cos 2\pi(1000)t + 10\cos 2\pi(5000)t$

The given signal consists of following frequencies,

$$F_1 = 1000\ Hz \text{ and}$$
$$F_2 = 5000\ Hz$$

Thus $F_{max} = 5000\ Hz = 5kHz$

$\therefore$ Nyquist Rate $= 2F_{max}$
$= 10\ KHz$

(ii) Analog Signal is Sampled at 4kHZ

A discrete signal is obtained by substituting

$$t = \frac{n}{F_s} = \frac{n}{4000} \text{ in Eq (1)}$$

$$\therefore x(n) = 5\cos 2\pi\left(\frac{1000n}{4000}\right) + 10\cos 2\pi\left(\frac{5000n}{4000}\right)$$

$$\therefore x(n) = 5\cos\left(\frac{\pi}{2}\right)n + 10\cos\left(\frac{5\pi}{2}\right)n$$

Now, in the second term

$\left(\frac{5\pi}{2}\right)$ can be written as,

$$\frac{5\pi}{2} = \left(2\pi + \frac{\pi}{2}\right)$$

$$\therefore 10\cos\left(\frac{5\pi}{2}\right)n = 10\cos\left(2\pi + \frac{\pi}{2}\right)n$$

$$= 10\cos\left(\frac{\pi}{2}\right)n$$

Hence we observe that $10\cos\left(\frac{5\pi}{2}\right)n$ is identical to $10\cos\left(\frac{\pi}{2}\right)n$. This is called <u>aliasing</u>.

$$\therefore x(n) = 15\cos\left(\frac{\pi}{2}\right)n$$

The continuos time signal had 2 different frequencies while the discrete time signal has only one.

(iii) The reconstructed signal is obtained by substituting $n = tF_s$ in the discrete time signal

$$\text{Since } F_s = 4000$$

$$\therefore n = 4000t$$

$$\therefore x(t) = 15\cos\left(\frac{\pi}{2}\right)4000t$$

$$\therefore x(t) = 15\cos 2\pi\, 1000t$$

We observe that the reconstructed signal has only 1 frequency of 1000 Hz.