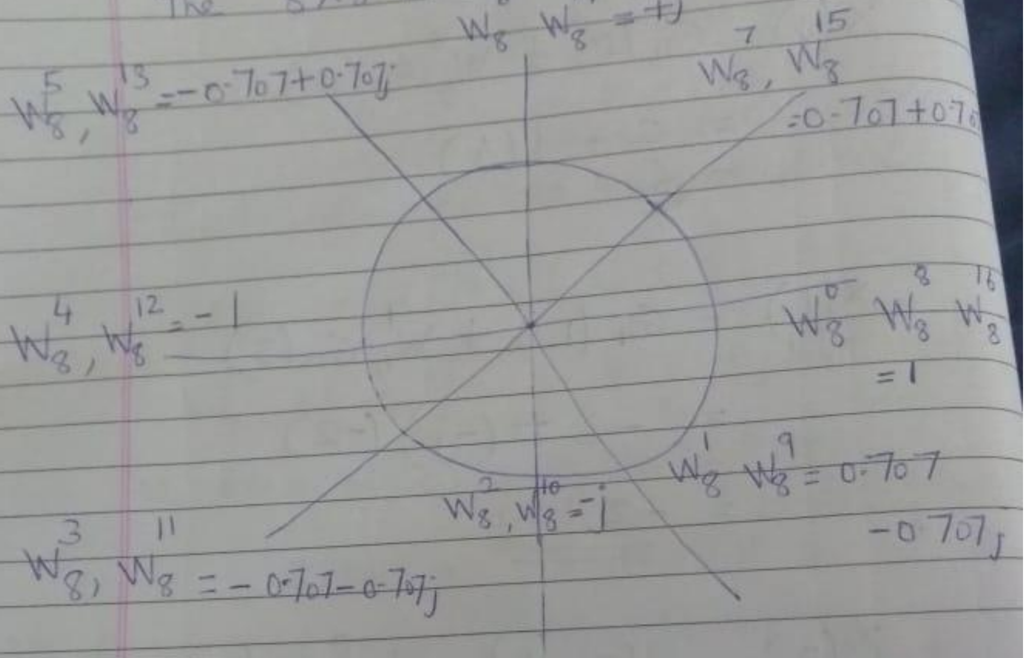
For W_8 we can draw the values of the 8×8 matrix using a circle



$$W_8^{nk} = \begin{array}{c|c} & n \rightarrow \\ k\downarrow & \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} & \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \end{array}$$

$$
W_8 = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & +0.707-j0.707 & -j & -0.707-j0.707 & -1 & -0.707+j0.707 & j & +0.707+j0.707 \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & -0.707-j0.707 & j & +0.707-j0.707 & -1 & +0.707+j0.707 & -j & -0.707+j0.707 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -0.707+j0.707 & -j & +0.707+j0.707 & -1 & +0.707-j0.707 & j & -0.707-j0.707 \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & +0.707+j0.707 & j & -0.707+j0.707 & -1 & -0.707-j0.707 & -j & +0.707-j0.707
\end{bmatrix}
\end{array}
$$

Compute the 8-point DFT of the sequence $x(n) = \{0, 1, 2, 3\}$

Ans: Since we require a 8-point DFT, we append four zeros to the original input sequence

$$\therefore x(n) = \{0, 1, 2, 3, 0, 0, 0, 0\}$$

As $N = 8$; we generate a 8×8 DFT matrix

$$X(k) = [W_8]\, x(n)$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707-j0.707 & -j & -0.707-j0.707 & -1 & -0.707+j0.707 & j & 0.707+j0.707 \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -0.707-j0.707 & j & 0.707-j0.707 & -1 & 0.707+j0.707 & -j & -0.707+j0.707 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.707+j0.707 & -j & 0.707+j0.707 & -1 & 0.707-j0.707 & j & -0.707-j0.707 \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & 0.707+j0.707 & j & -0.707+j0.707 & -1 & -0.707-j0.707 & -j & 0.707-j0.707 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving the above matrix operation, we obtain

$$X(k) = \begin{bmatrix} 6 \\ -1.414-j4.83 \\ -2+j2 \\ 1.414-j0.83 \\ -2 \\ 1.414+j0.83 \\ -2-j2 \\ -1.414+j4.83 \end{bmatrix}$$

$\cancel{X(k)}$

Calculate the 8 point DFT of

$$x(n) = \{1, 2, 1, 2\}$$

First we will make length of given sequence '8' by appending zeros to $x(n)$

$$\therefore \quad x(n) = \{1, 2, 1, 2, 0, 0, 0, 0\}$$

Now, $\quad X(k) = [W_8]\, x(n)$

We have already learnt how to compute W_8, which is an 8X8 DFT matrix

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0.707-j0.707 & -j & -0.707-j0.707 & -1 & -0.707+j0.707 & j & 0.707+j0.707 \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & -0.707-j0.707 & j & 0.707-j0.707 & -1 & 0.707+j0.707 & -j & -0.707+j0.707 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -0.707+j0.707 & -j & 0.707+j0.707 & -1 & 0.707-j0.707 & j & -0.707-j0.707 \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & 0.707+j0.707 & j & -0.707+j0.707 & -1 & -0.707-j0.707 & -j & 0.707-j0.707 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore X(k) = \begin{bmatrix} 1+2+1+2+0+0+0+0 \\ 1+1.414-j1.414-j-1.414-j1.414+0+0+0+0 \\ 1-2j-1+2j+0+0+0+0 \\ 1-1.414-j1.414+j+1.414-j1.414+0+0+0+0 \\ 1-2+1-2+0+0+0+0 \\ 1-1.414+j1.414-j+1.414+j1.414+0+0+0+0 \\ 1+2j-1-2j+0+0+0+0 \\ 1+1.414+j1.414+j-1.414+j1.414+0+0+0+0 \end{bmatrix}$$

$$\therefore X(k) = \begin{bmatrix} 6 \\ 1-j3.828 \\ 0 \\ 1-j1.828 \\ -2 \\ 1+j1.828 \\ 0 \\ 1+j3.828 \end{bmatrix}$$

This is the required DFT

$\therefore$ The 8-point DFT of the sequence is

$\therefore X(k) = \{6, 1-j3.828, 0, 1-j1.828, -2, 1+j8.28, 0, 1+j3.828\}$