Compute 4-point DFT of the sequence $x(n) = \{1, 2, 3, 1\}$

using DIT - FFT Radix-2 algorithm

★ Pre-check using DFT

$$X(k) = [W_4]\, x(n)$$

$$[W_4] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\therefore X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ -2-j \\ 1 \\ -2+j \end{bmatrix}$$

$$\therefore X(k) = \{7, -2-j, 1, -2+j\}$$

For FFT

Here $N = 4$

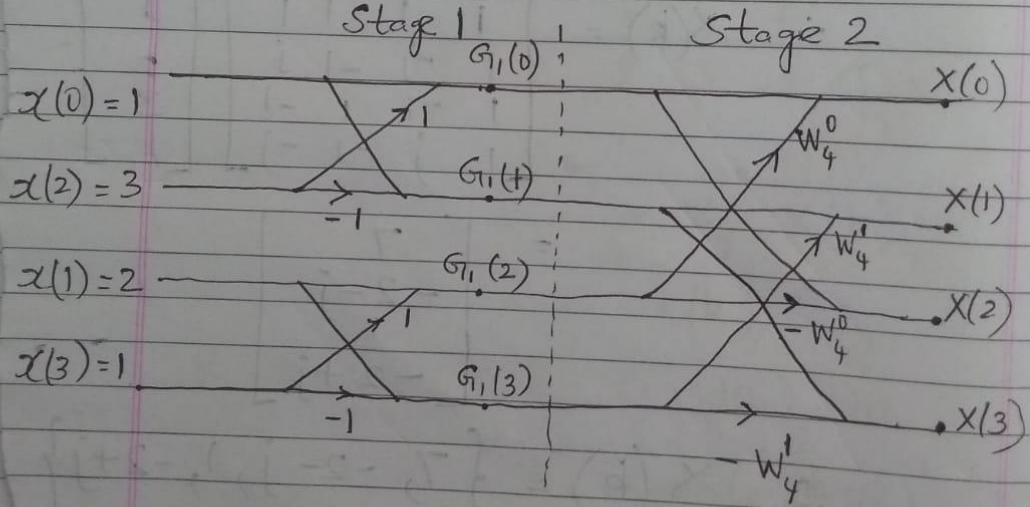
$$x(n) = \{1, 2, 3, 1\}$$

$x(0)\quad x(1)\quad x(2)\quad x(3)$

The twiddle factors required are:

$$W_4^0 = e^{-j\frac{2\pi}{4}(0)} = 1$$

$$W_4^1 = e^{-j\frac{2\pi}{4}(1)} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = -j$$



Stage 1	Stage 2
$G_1(0) = x(0) + x(2) = 1+3 = 4$	$X(0) = G_1(0) + W_4^0 G_1(2)$
$G_1(1) = x(0) - x(2) = 1-3 = -2$	$X(1) = G_1(1) + W_4^1 G_1(3)$
$G_1(2) = x(1) + x(3) = 2+1 = 3$	$X(2) = G_1(0) - W_4^0 G_1(2)$
$G_1(3) = x(1) - x(3) = 2-1 = 1$	$X(3) = G_1(1) - W_4^1 G_1(3)$

Stage 2

$$X(0) = G_1(0) + W_4^0 G_1(2) = 4 + (1)(3) = 7$$
$$X(1) = G_1(1) + W_4^1 G_1(3) = -2 + (-j)(1) = -2 - j$$
$$X(2) = G_1(0) - W_4^0 G_1(2) = 4 - (1)(3) = 1$$
$$X(3) = G_1(1) - W_4^1 G_1(3) = -2 - (-j)(1) = -2 + j$$

$$\therefore X(k) = \{7, -2-j, 1, -2+j\}$$

Given $x(n) = [0, 1, 2, 3]$

Compute $X(k)$ using DIT-FFT

* Pre-check using DFT

$$X(k) = [W_4] \cdot x[n]$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & +1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore X(k) = \begin{bmatrix} 0+1+2+3 \\ 0-j-2+3j \\ 0-1+2-3 \\ 0+j-2-3j \end{bmatrix} = \begin{bmatrix} 6 \\ +2j-2 \\ -2 \\ -2-2j \end{bmatrix}$$

$$\therefore X(k) = \{6, -2+2j, -2, -2-2j\}$$

For FFT

Here $N = 4$

$$x(n) = \{ \underset{x(0)}{1}, \underset{x(1)}{2}, \underset{x(2)}{3}, \underset{x(3)}{1} \}$$

The twiddle factors required are:

$$W_4^0 = e^{-j\frac{2\pi}{4}(0)} = 1$$

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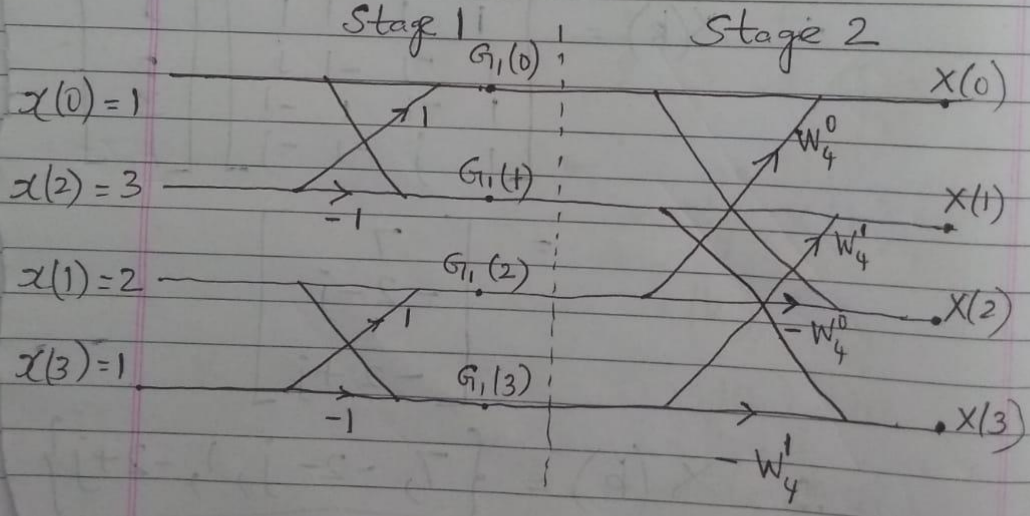

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Stage 2:

$X(0) = G_1(0) + W_4^0 G_1(2)$

$= 2 + 1(4)$

$= \underline{\underline{6}}$

$X(1) = G_1(1) + W_4^1 G_1(3)$

$= -2 + (-j)(-2)$

$= \underline{\underline{-2+2j}}$

$X(2) = G_1(0) - W_4^0 G_1(2)$

$= 2 - (1)(4)$

$= 2 - 4$

$= \underline{\underline{-2}}$

$X(3) = G_1(1) - W_4^1 G_1(3)$

$= -2 - (-j)(-2)$

$= \underline{\underline{-2-2j}}$

$\therefore X(k) = \{6, -2+2j, -2, -2-2j\}$