Properties of DFT

① **Linearity**

If $x_1(n) \overset{DFT}{\longleftrightarrow} X_1(k)$ and $x_2(n) \overset{DFT}{\longleftrightarrow} X_2(k)$ then

$$a\,x_1(n) + b\,x_2(n) \overset{DFT}{\longleftrightarrow} a\,X_1(k) + b\,X_2(k)$$

② **Periodicity**

If $x(n) \overset{DFT}{\longleftrightarrow} X(k)$

then $X(k+N) = X(k)$

i.e The DFT is periodic with a period N

③ **Circular Time Shift**

If $x(n) \overset{DFT}{\longleftrightarrow} X(k)$

then $x((n-M))_N \overset{DFT}{\longleftrightarrow} e^{-j\frac{2\pi mk}{N}} \cdot X(k)$

$$x(n-M)_N \overset{DFT}{\longleftrightarrow} W_N^{mk} \cdot X(k)$$

④ **Circular Frequency shift**

If, $x(n) \overset{DFT}{\longleftrightarrow} X(k)$

then, $x(n)\,e^{+j\frac{2\pi mn}{N}} \overset{DFT}{\longleftrightarrow} X((k-m))_N$

(5) Time Reversal

If $x(n)$ is periodic

$$x((-n))_N = x(N-n)$$

$0 \le n \le N-1$

If $x(n) \overset{DFT}{\longleftrightarrow} X(k)$

$$x(N-n) \overset{DFT}{\longleftrightarrow} X(N-k)$$

OR

$$x((-n))_N \overset{DFT}{\longleftrightarrow} X(N-k)$$

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(6) Parseval's Energy Theorem

If $x(n) \overset{DFT}{\longleftrightarrow} X(k)$

Then Energy of the signal is,

$$E = \sum_{n=0}^{N-1} |x(n)|^2$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

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(7) Multiplication of Two sequences

$$x_1(n) \overset{DFT}{\longleftrightarrow} X_1(k)$$

$$x_2(n) \overset{DFT}{\longleftrightarrow} X_2(k)$$

$$x_1(n) \cdot x_2(n) \overset{DFT}{\longleftrightarrow} \frac{1}{N} \left[X_1(k) \circledast X_2(k) \right]$$

Property	Time Domain	Frequency Domain
1. Linearity	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$
2. Time-shifting	$x[n - m]$	$e^{-j2\pi km}X(k)$
3. Frequency-shifting (modulation)	$e^{-j2\pi k_0 n/N}x[n]$	$X(k - k_0)$
4. Time reversal	$x[-n]$	$X(-k)$
5. Conjugation	$x^*[n]$	$X^*(-k)$
6. Time-convolution	$x_1[n] \otimes x_2[n]$	$X_1[k]X_2[k]$
7. Frequency-convolution	$x_1[n]x_2[n]$	$\frac{1}{N} X_1[k] \otimes X_2[k]$
8. Parseval's relation	$E_x = \sum_{n=0}^{N-1} \lvert x[n] \rvert^2$	$E_x = \frac{1}{N} \sum_{k=0}^{N-1} \lvert X[k] \rvert^2$