

Compute the DFT of $x(n) =$

$$\{1, 2, 3, 4\}$$

Since $x(n)$ is of length 4, $N=4$,
and we generate a DFT matrix
of size 4×4

$$\therefore X(k) = [W_4]_{4 \times 4} x(n)$$

$$\therefore X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$X(k) = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$$\therefore X(k) = \{10, -2+2j, -2, -2-2j\}$$

Compute the DFT of the sequence
 $x(n) = \{1, 1, 1, 1\}$

The DFT is given by the equation

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

where,

$$W_N^{nk} = e^{-j \frac{2\pi nk}{N}}$$

Since the length of $x(n) = 4$, the DFT equation reduces to,

$$X(k) = \sum_{n=0}^3 x(n) W_4^{kn}; k=0, 1, 2, 3$$

In matrix form it is written as,

$$X(k) = [W_4] x(n)$$

where,

$$x(n) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore X(k) = \begin{matrix} W_4 \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \end{matrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore X(k) = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \text{ Hence } X(k) = [4, 0, 0, 0]$$