

Properties of DFT

① Linearity

$$\text{If } x_1(n) \xleftrightarrow{\text{DFT}} X_1(k) \text{ and } x_2(n) \xleftrightarrow{\text{DFT}} X_2(k) \text{ then}$$
$$a x_1(n) + b x_2(n) \xleftrightarrow{\text{DFT}} a X_1(k) + b X_2(k)$$

② Periodicity

$$\text{If } x(n) \xleftrightarrow{\text{DFT}} X(k) \text{ then } X(k+N) = X(k)$$

i.e. The DFT is periodic with a period N

③ Circular Time Shift

$$\text{If } x(n) \xleftrightarrow{\text{DFT}} X(k) \text{ then } x((n-M))_N \xleftrightarrow{\text{DFT}} e^{-j \frac{2\pi m k}{N}} X(k)$$

$$x(n-M)_N \xleftrightarrow{\text{DFT}} W_N^{mk} X(k)$$

④ Circular Frequency Shift

$$\text{If } x(n) \xleftrightarrow{\text{DFT}} X(k) \text{ then } x(n) e^{+j \frac{2\pi m n}{N}} \xleftrightarrow{\text{DFT}} X((k-m))_N$$

(5) Time Reversal

If $x(n)$ is periodic

$$x((-n))_N = x(N-n)$$

$$\text{If } x(n) \xleftrightarrow[\text{DFT}]{} X(k) \quad 0 \leq n \leq N-1$$

$$x(N-n) \xleftrightarrow{\text{DFT}} X(N-k)$$

$$\text{OR } x((-n))_N \xleftrightarrow{\text{DFT}} X(N-k)$$

(6) Parseval's Energy Theorem

$$\text{If } x(n) \xleftrightarrow[\text{DFT}]{} X(k)$$

Then Energy of the signal is,

$$F = \sum_{n=0}^{N-1} |x(n)|^2$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

(7) Multiplication of Two Sequences

$$x_1(n) \xleftrightarrow{\text{DFT}} X_1(k)$$

$$x_2(n) \xleftrightarrow{\text{DFT}} X_2(k)$$

$$x_1(n) \cdot x_2(n) \xleftrightarrow{\text{DFT}} \frac{1}{N} [X_1(k) \otimes X_2(k)]$$