Compute the DFT of $x(n) = \{1, 1, 0, 0\}$

Since $x(n)$ is of length 4, $N = 4$ and we generate a DFT matrix of size 4×4

$$\therefore X(k) = [W_4]_{4\times 4}\, x(n)$$

$$\therefore X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore X(k) = \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix}$$

$$\therefore X(k) = \{2, 1-j, 0, 1+j\}$$

Compute the IDFT of

$$X(k) = \{2, 1-j, 0, 1+j\}$$

The IDFT is given by the equation

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, W_N^{-nk}, \quad n = 0, 1, 2, \ldots, N-1$$

In matrix form, it is written as,

$$x(n) = \frac{1}{N}\left[W_N^*\right] X(k)$$

Since, $X(k)$ is of length 4, $N=4$ and we generate a IDFT matrix of size 4×4

$$\therefore x(n) = \frac{1}{4} \begin{array}{cc} W_4^* & X(k) \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & +1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} & \begin{bmatrix} 2 \\ 1-j \\ 0 \\ 1+j \end{bmatrix} \end{array}$$

$$\therefore x(n) = \frac{1}{4} \begin{bmatrix} 4 \\ 4 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore x(n) = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore x(n) = \{1, 1, 0, 0\}$$