Yellow is an element of c.

Yellow ε c.

Green is not element of set c

Green $\notin$ c.

→ ~~Anti Set~~ Empty set.

A set with no elements is called empty set

It is denoted as Φ or $\{\}$.

$|\{\Phi\}| = 1$

$|\Phi\}| = 0$

$|\{\ \}| = 0$.

→. Single tone set

A set with one element is called singleton set.

eg $|\{\Phi\}| = 1$.

→ If 2 sets are equal if and only if they have same elements

$A = \{1, 3, 5\}$ $B = \{3, 5, 1\}$.

$A = B$.

A set is unordered collection of objects.

A set can have positive integers which can be finite or infinite set.

~~If we take~~ repeated elements are listed one.

$A \in \{a, b, c, c, b, a\}$

$A \in \{a, b, c\}$

Common Sets.

Natural numbers.

$A \in \{1, 2, 3, \dots\}$

Set of integers

$A \in \{\dots, -1, 0, 1, \dots\}$.

Rational numbers.

$A \in \{\dots 1, 1/2, 1/3, 2/3, \dots\}$.

Element & Carnality. (Size of set).

Set $C = \{$Yellow, Blue, Red$\}$.

$|C|$

E.G

$|C| = 3$.

Yellow is an element of C.

Yellow $\in C$.

Green is not element of set C

Green $\notin C$.

→ ~~Anti Set~~ Empty set.

A set with no elements is called empty set

It is denoted as ϕ or $\{\}$.

$|\{\phi\}| = 1$

$|\phi| = 0$

$|\{\ \}| = 0$

→ Single tone set

A set with one element is called singleton set.

eg $|\{\phi\}| = 1$.

→ If 2 sets are equal if and only if they have same elements

$A = \{1, 3, 5\}$ $B = \{3, 5, 1\}$.

$A = B$.

Set Notation / Set Builder

1) $A = \{ \ldots 1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \ldots$

$= \{ m/n \mid m, n \in A \text{ where}, n \neq 0 \}$.

2) $A = \{ \ldots -4, -2, 0, 2, 4, \ldots \}$

$= \{ m \mid m \in A \text{ where } m = 2n \}$.

3) $A = \{1, 3, 5, 7\}$

$= \{ \cancel{0 \leq x \leq 8} \mid \cancel{x = n+1} \; x+1,$
$\cancel{x = 2n+1}$

$= \{ \cancel{m+1 \mid m \in W}, \cancel{m \leq 6} \},$
$\{ m = 2n+1, n \in W, n \leq 3 \}$.

4) $A = \{ \ldots, 2, 4, 6, \ldots \}$.

$= \{ 2m \mid m \in N \}$.

Q) $D = \{ x \in Z^+ \mid x < 6 \}$.

$D = \{1, 2, 3, 4, 5\}$

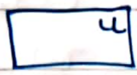
$|D| = 5$.

$A = \{ \phi, \{a, b\} \}$

$|A| = 2$

Venn diagram

It is a pictorial representation of set.

$U \rightarrow$ Universal set. 

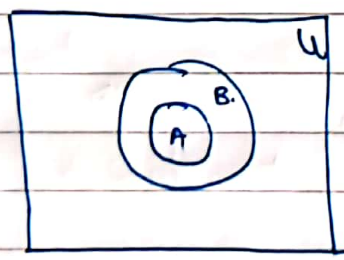
Set $\rightarrow$ 

Subset $(\subseteq)$

$A \subseteq B$ if and only if every element of A is also element of B.

Every

~~$A = \{ x |$~~ $\forall x \{ x \in A \wedge x \in B. \}$.



eg $A = \{1, 2, 3, 4\}$.
$B = \{1, 2, 3, 4, 5, 6\}$.

Proper set:

$A \subseteq B$ and $A \neq B$ then $A \subset B$.

eg $A = \{x, y\}$ $B = \{x, y, z\}$.

$\forall x\ [x \in A \rightarrow x \in B \rightarrow \exists x\ [x \in B \wedge x \notin A]]$

Power set: Given a set S power set is set of all the subsets of set S.

eg. $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

Note: If there are n elements in set then $P(n) = 2^n$.

$|P(A)| = 2^3 = 8$.

Set operation.

→ Union

→ Intersection.

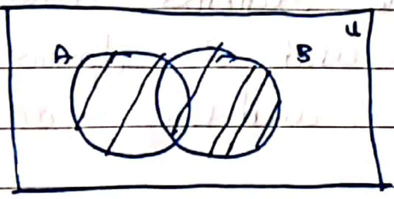
→ Set of compliment

→ Set difference.

→ Cartesian product.

~~Un~~
Union – Let set A and B the union of set all of the elements A or in B. including element which are in both the sets
It is denoted as $[A \cup B]$

$\forall x \{x \in A$ or $x \in B$ or both$\}$



eg :- $A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$|A \cup B| = 6$

Properties of union operation

1> $A \cup B = B \cup A$.

2> $A \cup (B \cup C) = (A \cup B) \cup C$.

3> $A \cup A = A$.

4> $A \cup \phi = A$.

5> $A \cup U = U$.

6> If $A \in B$ then $A \cup B = B$.

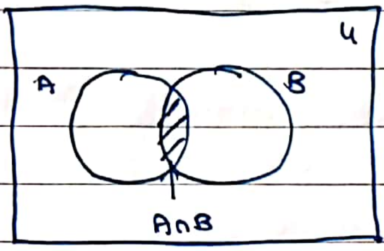
7> $A \subseteq (A \cup B)$ & $B \subseteq (A \cup B)$

8>

2) Intersection :

* Let sets A and B the elements which are common in both A and B forms the intersection of set A and B.

It is denoted as [A ∩ B].



$\forall x \{x \in A \text{ and } x \in B\}$.

eg $A = \{1, 2, 4, 5, 7\}$.

$B = \{1, 3, 4, 5, 6, 8\}$.

$A \cap B = \{1, 4, 5\}$.

Disjoint Sets.

Let set A and B the no elements ~~which are~~ are common in both the set. then they are ~~..~~ disjoint sets.

$A \cap B = \phi$.

eg. $A = \{2, 3\}$
$B = \{4, 2\}$.
$A \cap B = \phi$.

→ Property

1) $A \cap B = B \cap A$.

2) $A \cap A = A$.

3) $A \cap U = A$.

4) $A \cap \overline{A} = \phi$.

5) $A \cap (B \cap C) = (A \cap B) \cap C$.

6). ~~A∩A~~ $A \cap \phi = \phi$.

7). If $A \subseteq B$ then $A \cap B = A$.

8). If. $A \cap B \subseteq A$ & $(A \cap B) \subseteq B$.

→ Compliment of set :

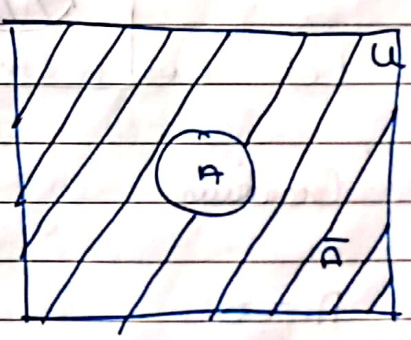
:- Let A is a given set then complement of set A is given by $\overline{A}$.

$\{x \mid x \in U \text{ \& } x \notin A\}$.

$U = \{1, 2, 3, 4, 5, 6\}$.

$A = \{1, 3, 5\}$

$\overline{A} = \{2, 4, 6\}$



Properties.

1) $\overline{\phi} = U$

2) $\overline{U} = \phi$

3) $\overline{\overline{A}} = A$.

4) $A \subseteq B$ then $\overline{B} \subseteq \overline{A}$.

Set Difference

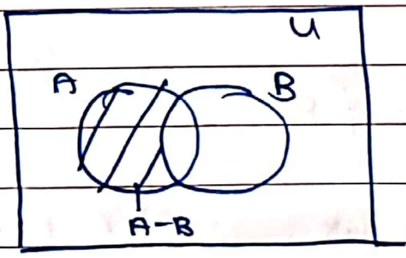
Let A and B are 2 sets then diff is set of elements in A which are not in B is called set difference.

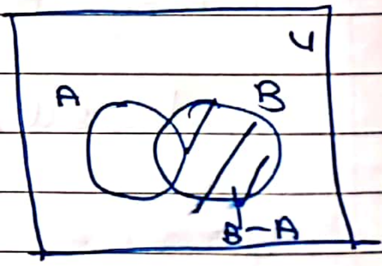
$A - B = \{x \mid x \in A \wedge x \notin B\}$.

$B - A = \{x \mid x \in A \wedge x \notin B\}$.

A − B

B − A





Creative	Compliment
A − B	$A \cap \overline{B}$
B − A	$B \cap \overline{A}$

eg $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

$A = \{1, 2, 3, 6, 7\}$.

$B = \{2, 3, 4, 8\}$.

$A \cap B = \{2, 3\}$

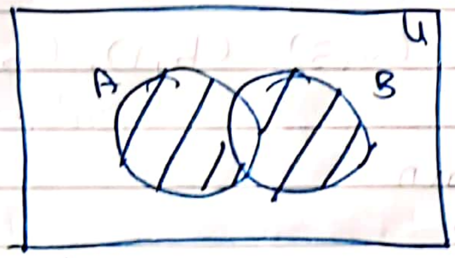
$A - B = \{1, 6, 7\}$

$B - A = \{4, 8\}$.

Symmetric difference.

~~$A - B = B - A$.~~

$A \triangle B = (A - B) \cup (B - A) = A \oplus B$



If there are 2 sets A & B
Union of elements which are not common in both sets.

$\cancel{\text{i.e.}}\ \{x \mid x \in A \text{ or } x \in B \text{ but } x \notin A \cap B\}$

$\{x \mid x \in A - B \text{ or } x \in B - A\}$

eg $A = \{1, 2, 3, 4\}$.

$B = \{3, 4, 5, 6\}$

$A \triangle B = \{1, 2, 5, 6\}$

eg $A = \{a, b, e, g\}$.

$B = \{d, e, f, g\}$

$A - B = \{a, b, \cancel{d, g}\, f, d\}$.

Properties

Cartesian product

Let A & B be 2 sets where cartesian product is $A \times B = \{(x, y)\}$ where $x \in A$, $y \in B$

$A \times B = \{ (x, y) \mid x \in A \;\&\; y \in B \}$.

eg $A = \{a, b\}$ $|A| = m = 2$

$B = \{1, 2, 3\}$ $|B| = n = 3$

$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$|A \times B| = 6 = mn$.

Theorem :-

① Addition principle :-

Let set A & B are finite & disjoint sets

then

$|A \cup B| = |A| + |B|$

$n(A \cap B) = 0$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = n(A) + n(B)$

$|A \cup B| = |A| + |B|$

$|A| = m$ & $|B| = n$.

$$|A \cup B| = |A| + |B|$$

$$= m + n.$$

Hence prove.

eg $A = \{1, 2, 3, 4\}$.

$B = \{5, 6\}$.

$|A| = 4$.

$|B| = 2$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$|A \cup B| = 6$.

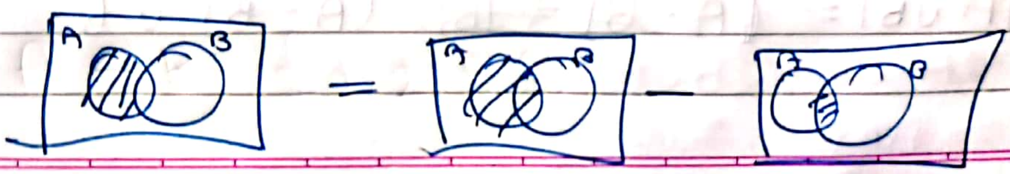
$|A| + |B| = 4 + 2 = 6$.

$|A \cup B| = |A| + |B|$.

Principle of inclusion & exclusion.

Let A ~~& B~~ → finite set & B is not necessary finite set.

$$|A - B| = |A| - |A \cap B|.$$



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From Venn diagram $|A| = |A-B| + |A \cap B|$

eg $A = \{2, 3, 4\}$

$B = \{3, 4\}$

$|A| = 3$

$|B| = 2.$

$A \cap B = \{3, 4\}$

~~nt~~ $|A \cap B| = 2.$

$A - B = \{2\}.$

$|A - B| = 1.$

$|A| = 3.$

$|A-B| \cup |A \cap B| = 2 + 1 = 3.$

$|A| = |A-B| \cup |A \cap B|.$

2) Let A & B be finite set

$|A \cup B| = |A| + |B| - |A \cap B|.$

$A \cup B$ as disjoint union of 2 set.

$|A \cup B| = \cancel{|A-B| + |B|}. \; (A-B) \cup B$

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By principle of addition.

$$|A \cup B| = |A - B| + |B|.$$

By principle of inclusion exclusion.

$$= |A| - |A \cap B| + |B|$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

eg 1) In a survey of 260 college students the foll data be obtained 64 had taken Maths. 94 had take CS. , 58 had taken business course, 28 had taken both ~~CS~~ bus & maths, ~~24~~ 26 had taken both Maths & CS, 22 CS+Bus & 14 had taken all 3 types of course.

1) How many students ~~h~~ have taken none of 3 types of cours

2) Of the students survey how many had taken only CS cours

n ~~(A ∪)~~

$n(A) = 64$

$n(B) = 94$

$n(C) = 58.$

$n(A \cap C) = 28$

$n(A \cap B) = 26$

$n(B \cap C) = 22$

$n(A \cap B \cap C) = 14$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C).$$

$= 64 + 94 + 58 - 28 - 26 - 22 + 14$

$= 154$

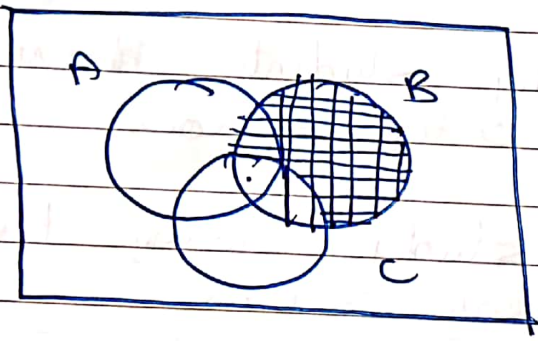
Neither $= 260 - 154 = 106$

$n(B) = n(B-A) + n(B-C) + n(A \cap B \cap C)$

$= n(B) - n(B \cap A) + n(B) - n(B \cap C) + n(A \cap B \cap C)$.

$= 94 - 26 + 94 - 22 + 14$

$= 154$.



$n(B) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$= 94 - 26 - 22 + 14$

$= 60$.

Law of Set Theory

1) Commutative Law

1) $A \cup B = B \cup A$
2) $A \cap B = B \cap A$

2> Associative law.

1> $A \cup (B \cup C) = (A \cup B) \cup C$.

2> $A \cap (B \cap C) = (A \cap B) \cap C$.

3> Distributive law.

1> $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2> $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

4> Idempotent law

$A \cup A = A$

$A \cap A = A$.

5> De Morgan's law

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$.

6> Absorption law.

$A \cup (A \cap B) = A$.

$A \cap (A \cup B) = A$.

7> Double complement.

$\overline{\overline{A}} = A$.

a) In a survey of 60 people we first find that 25 ~~reed~~ read a New Week magzine, 26 read Time & 26 read Fortune. Also 9 read both both New Week & Fortune. 11 read both New Week & time, 8 read both time & fortune : 8 read no magzine at all.

1) Find number of people who read all 3 magzine.

2) Fill the correct number of people in each of region.

Here

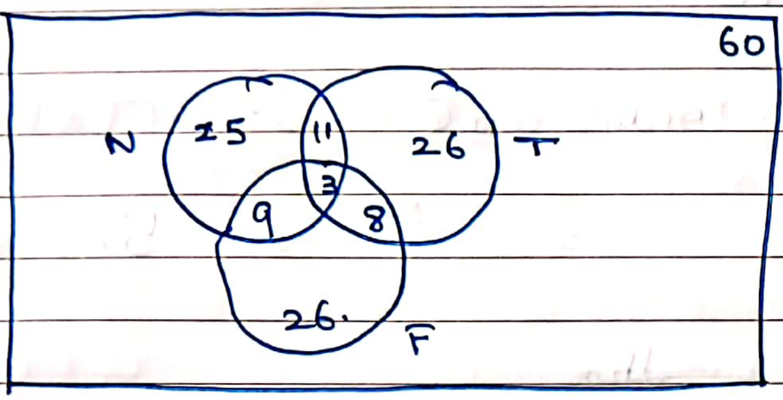
N, T, F be set of people who read news week, time & fortune respectively

3) Determine number of people who read exactly 1 magzine.

1) $n(N \cup T \cup F) = n(N) + n(T) + n(F) - n(N \cap T) - n(N \cap F) - n(T \cap F) + n(N \cap F \cap T)$.

$60 - 8 = 25 + 26 + 26 - 9 - 11 - 8 + n(N \cap F \cap T)$

$n(N \cap F \cap T) = 3$



3) $n(N)$ only $= n(N) - n(N \cap T) - n(N \cap F) + n(N \cap T \cap F)$
$= 25 - 11 - 9 + 3 = 8$

$n(T)$ only $= n(T) - n(T \cap N) - n(T \cap F) + n(N \cap T \cap F)$
$= 26 - 11 - 8 + 3 = 10$

$n(F)$ only $= n(F) - n(F \cap N) - n(F \cap T) + n(N \cap T \cap F)$
$= 26 - 9 - 8 + 3 = 12.$

Number of students who read exactly 1 magazine $= 8 + 10 + 12 = \underline{\underline{30}}.$

- Calculate number of people who N & T but not F.

$n(\cancel{F} N \cap T)$ only $= n(N \cap T) - n(N \cap T \cap F)$

$= \cancel{9 - 3 = 6}.\ 11 - 3 = 8.$

Calculate number of people who read N & F but not T.

$n(N \& F)$ only $= n(N \cap F) - n(N \cap F \cap T \cap F)$
$= 9 - 3 = 6.$

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Calculate people who read T & F but not N

$n(T \cap F)$ only $= n(T \cap F) - n(T \cap F \cap N)$

$= 8 - 3 = 5.$

~~Calculate those~~

Q Find how many integers 1 & 60 are ~~not div~~ divisible neither by 3 nor by 5.

$A = \{3, 6, 9, \cdots, 60\}.$

$n(A) = \cancel{12}\ 20$

$B = \{5, 10, \cdots, 60\}$

$n(B) = 12$

~~$n(A \cup B) = 60$~~

$n(A \cap B) = \{15, 30, 45, 60\}$

$n(A \cap B) = 4.$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 20 + 12 - 4$

$= 28.$

A Numbers not divisible by 3 or 5

$= 60 - 28$

$= 32.$

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Q) Number of integers 2, 3, 5 are A_1, A_2, A_3

$n(A_1) = \frac{60}{2} = \cancel{15}\ 30 \quad n(A_2) = \frac{60}{3} = 20 \quad n(A_3) = \frac{60}{5} = 12$

$n(A_1 \cap A_2) = \frac{60}{2 \times 3} = \frac{60}{6} = 10$

$n(A_1 \cap A_3) = \frac{60}{2 \times 5} = \frac{60}{10} = 6$

$n(A_2 \cap A_3) = \frac{60}{3 \times 5} = \frac{60}{15} = 4$

$n(A_1 \cap A_2 \cap A_3) = \frac{60}{2 \times 3 \times 5} = \frac{60}{30} = 2$

$n(A_1 \cup A_2 \cup A_3) = 30 + 20 + 12 - 10 - 6 - 4 + 2$

$= 44$

$n(\overline{A_1 \cup A_2 \cup A_3}) = 60 - 44 = 16.$

How many integers 1- 2000 are divisible 2, 3, 5 or 7.

$\cancel{A_1}$ $n(A_1) = \frac{2000}{2} = 1000$

$n(A_2) = \frac{2000}{3} = 666.\cancel{67}$

$n(A_3) = \frac{2000}{5} = 400$

$n(A_4) = \frac{2000}{7} = 285$

$n(A_1 \cap A_2) = \frac{2000}{2 \times 3} = 333$

$n(A_1 \cap A_3) = \frac{2000}{2 \times 5} = 200$

$n(A_1 \cap A_4) = \frac{2000}{2 \times 7} = 142$

$n(A_2 \cap A_3) = \frac{2000}{3 \times 5} = 133$

$n(A_2 \cap A_4) = \frac{2000}{3 \times 7} = 95$

$n(A_3 \cap A_4) = \frac{2000}{5 \times 7} = 57$

$n(A_1 \cap A_2 \cap A_3) = \frac{2000}{2 \times 3 \times 5} = 66$

$n(A_1 \cap A_2 \cap A_4) = \frac{2000}{2 \times 3 \times 7} = 47$

$n(A_1 \cap A_3 \cap A_4) = \frac{2000}{2 \times 5 \times 7} = 28$

$n(A_2 \cap A_3 \cap A_4) = \frac{2000}{3 \times 5 \times 7} = 19$

$n(A_1 \cap A_2 \cap A_3 \cap A_4) = \frac{2000}{2 \times 3 \times 5 \times 7} = 9$

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$n(A_1 \cup A_2 \cup A_3 \cup A_4) = n(A_1) + n(A_2) + n(A_3) + n(A_4)$
$- n(A_1 \cap A_2) - n(A_1 \cap A_3) - n(A_2 \cap A_3)$
$- n(A_2 \cap A_4) - n(A_1 \cap A_4) -$
$n(A_3 \cap A_4) + n(A_1 \cap A_2 \cap A_3)$
$+ n(A_1 \cap A_2 \cap A_4) + n(A_1 \cap A_3 \cap A_4)$
$+ n(A_2 \cap A_3 \cap A_4)$
$- n(A_1 \cap A_2 \cap A_3 \cap A_4)$

$= 1000 + 666 + 400 + 285 -$
$333 - 200 - 142 - 133 - 95$
$- 57 + 66 + 47 + 28 + 19 \div 9$

$= 1542.$

3) Determine no. of integers 1 & 250. is divisible by any of 2, 3, 5, 7.

$n(A) = \cancel{120}\ \frac{250}{2} = 125$

$n(B) = \frac{250}{3} = 83$

$n(C) = \frac{250}{5} = 50$

$n(D) = \frac{250}{7} = 35$

$n(A \cap B) = \frac{250}{2 \times 3} = 41$

$n(A \cap C) = \frac{250}{2 \times 5} = 25$

$n(A \cap D) = \frac{250}{2 \times 7} = 17$

$n(B \cap C) = \frac{250}{3 \times 5} = 16.$

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$$n(B\cap D) = \frac{250}{3\times 7} = 11$$

$$n(C\cap D) = \frac{250}{5\times 7} = 7$$

$$n(A\cap B\cap C) = \frac{250}{2\times 3\times 5} = 8$$

$$n(A\cap B\cap D) = \frac{250}{2\times 3\times 7} = 5$$

$$n(A\cap C\cap D) = \frac{250}{2\times 5\times 7} = 3$$

$$n(B\cap C\cap D) = \frac{250}{3\times 5\times 7} = 2$$

$$n(A\cap B\cap C\cap D) = \frac{250}{2\times 3\times 5\times 7} = 1.$$

$$n(A\cup B\cup C\cup D) = 125+83+50+35-41-25-17-16-11-7+8+5+3+2-1$$

$$= 193$$

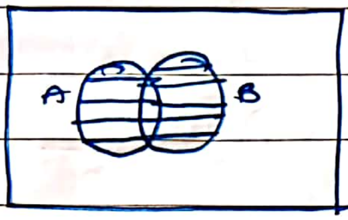
→ $A\cup B = B\cup A.$

LHS $= \cancel{\{x} \{x \mid x\in A \text{ or } x\in B\}.$

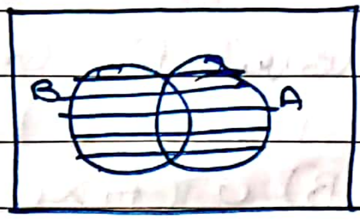
$= \{x\in B \text{ or } x\in A\}.$

$= x\in \{B\cup A\}$

$= B\cup A.$



$A \cup B$



$B \cup A$.

Distributive:

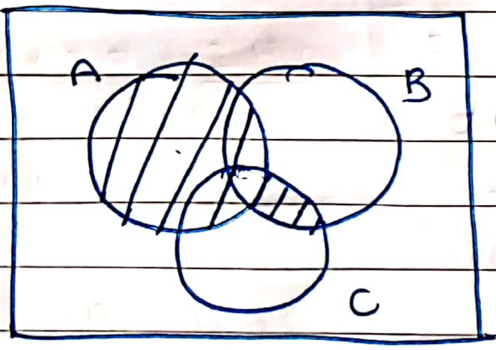
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

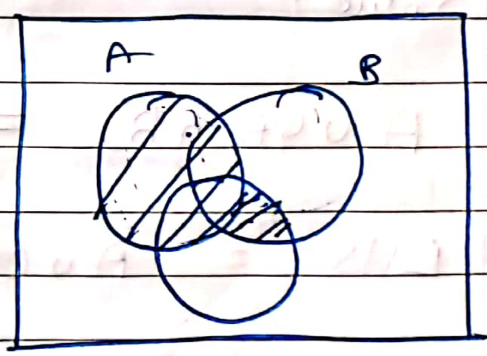
$$= \{x : (x \in A \text{ or } x \in B) \,\&\, (x \in A \text{ or } x \in C)\}$$

$$= \{x : x \in A \text{ or } (x \in B \,\&\, x \in C)\}$$

$$= \{x : x \in A \cup (B \cap C)\}.$$

$$= A \cup (B \cap C).$$





$A \times (B \cup C) =$

By distributive law

$(A \times B) \cup (A \times C) =$

$(x, y) \in \{ \cancel{x \in A}$ $(A \times (B \cup C))$.

$x \in A$ and $y \in (B \cup C)$

$x \in A$ and $(y \in B$ or $y \in C)$

$(x \in A$ & $y \in B)$ or $(x \in A$ & $y \in C)$

$(x, y) \in (A \times B)$ $\cup$ $(x, y) \in (A \times C)\}$

$(x, y) \in [(A \times B) \cup (A \times C)]$

$(A \times B) \cup (A \times C)$

Solve.

1) $A \cup (\bar{A} \cap B) = A \cup B$.

LHS $= A \cup (\bar{A} \cap B)$.

Using distributive law

$= (A \cup \bar{A}) \cap (A \cup B)$

$= U \cap (A \cup B)$ – Complement law

~~Idempotent law~~

$= A \cup B$ – Intersection process

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2) $A \cap (\bar{A} \cup B)$

LHS = $A \cap (\bar{A} \cup B)$.

$= (A \cap \bar{A}) \cup (A \cap B)$ — Distributive law

$= \phi \cup (A \cap B)$ — Complement law

$= A \cap B$ — ~~Indoo~~ Union property

3) $\overline{\overline{(\bar{A} \cup B)} \cap C \cup \bar{B}} = \overline{(\bar{A} \cap \bar{B})} \cup \overline{(C \cup \bar{B})}$

$= (\overline{\bar{A} \cap \bar{B}}) \cup (\bar{C} \cap B)$

$= (A \cup B) \cup (\bar{C} \cap B)$

=

$\overline{\overline{A \cup B} \cap C} \cap B$.

$= [(A \cup B) \cup \bar{C}] \cap B$

$= [$ ~~$B \cup (A \cup \bar{C}) \cap B$~~.

$= [B \cap (A \cup B)] \cup [B \cap \bar{C}]$ Distributive

$= B \cup [B \cap \bar{C}]$ Absorption

$= B$. Absorption

$[A \cap B] \cup [B \cap [(C \cap D) \cup (C \cap \bar{D})]]$
$= B \cap (A \cup C)$

$[A \cap B] \cup [B \cap [C \cap (D \cup \bar{D})]]$ Distributive

$[A \cap B] \cup [B \cap [C \cap U]]$ ~~Union Complement~~ Complement

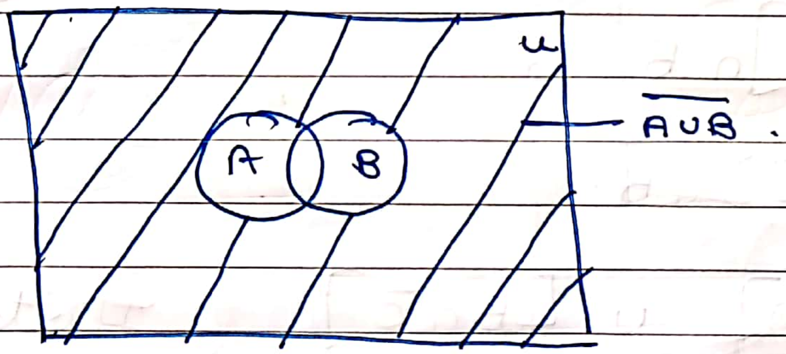
$(A \cap B) \cup (B \cap C)$ Absorption

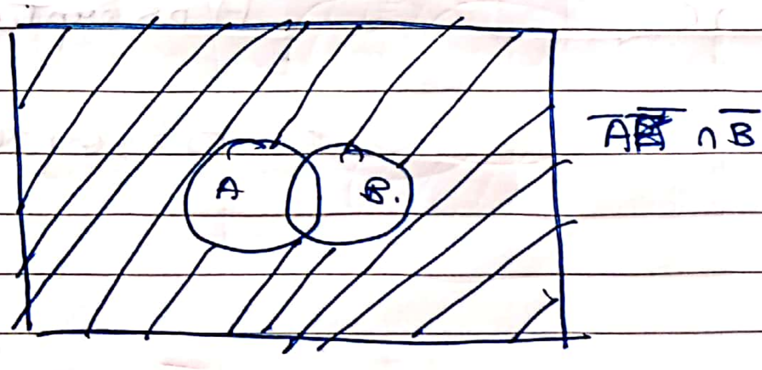
$B \cap (A \cup C)$. Distributive

$\sim(A \cup B) = \sim A \cap \sim B$, $A \cup (B \cap C)$

Proof based on Venn diagram.

$\overline{A \cup B} = \bar{A} \cap \bar{B}$.





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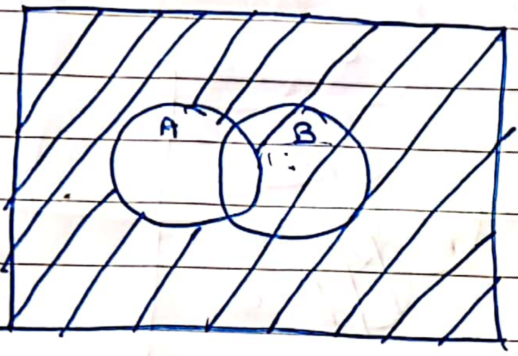
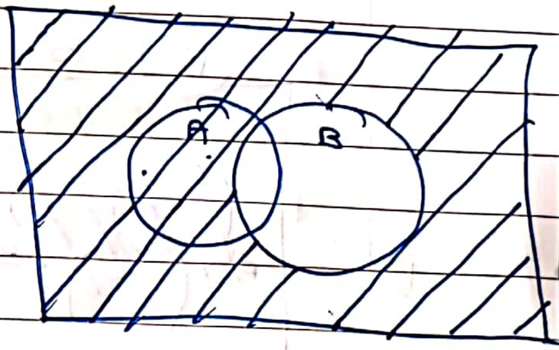
$\overline{A}$

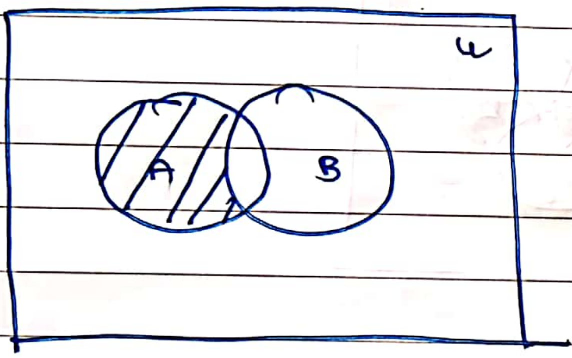


$\overline{B}$

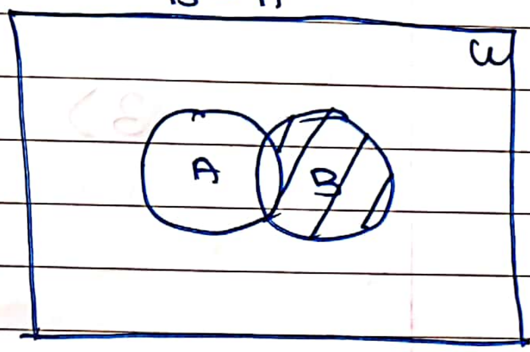


2) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

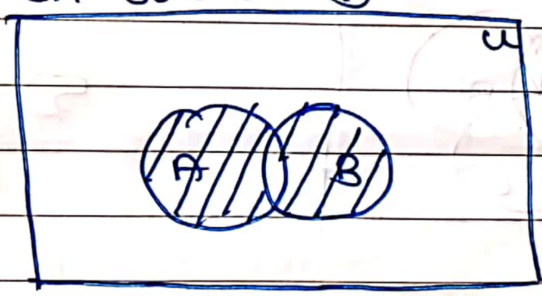
$A - B$



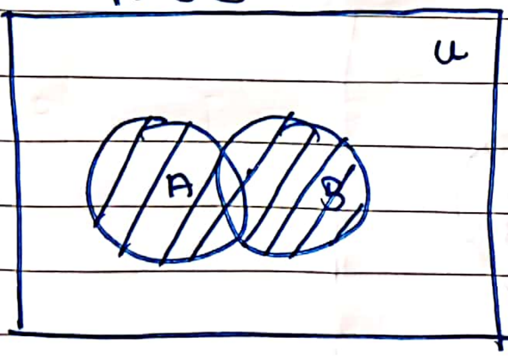
$B - A$



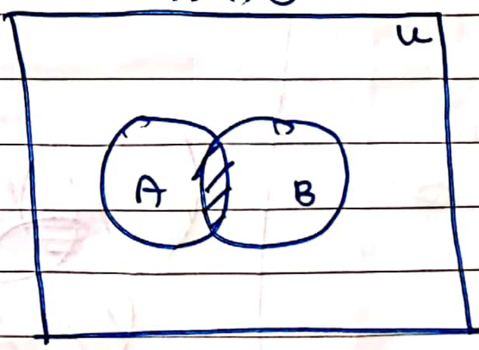
$(A-B) \cup (B-A)$



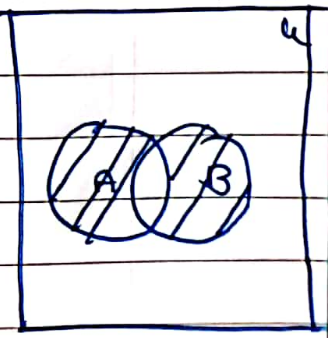
$A \cup B$



$A \cap B$



$(A \cup B) - (A \cap B)$



LHS = RHS

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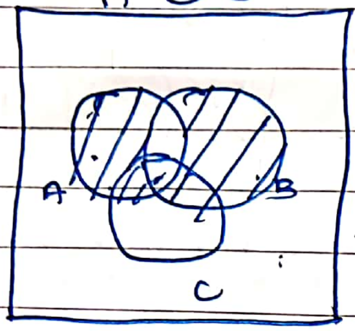
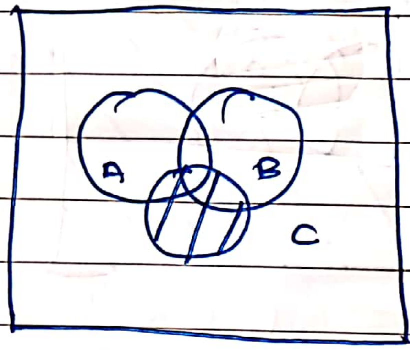
3) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$

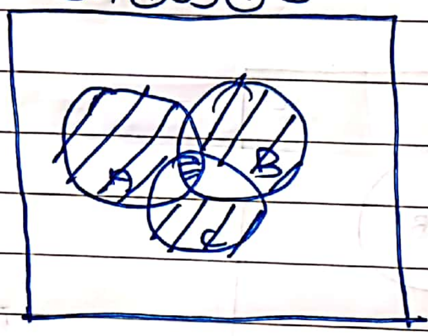
1) $A \oplus B$



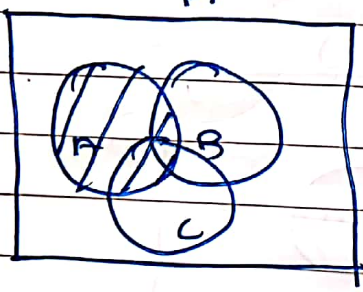
2) C



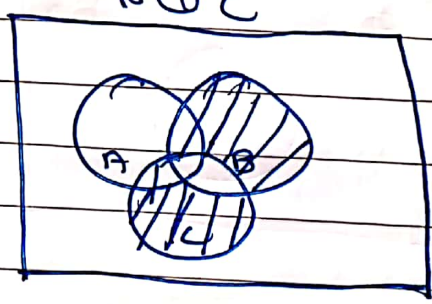
3) $(A \oplus B) \oplus C$



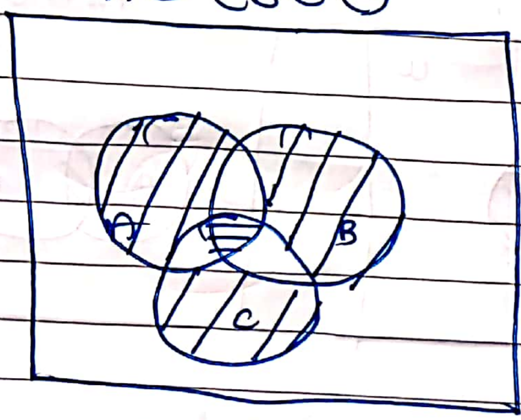
4) A



5) $B \oplus C$



6) $A \oplus (B \oplus C)$

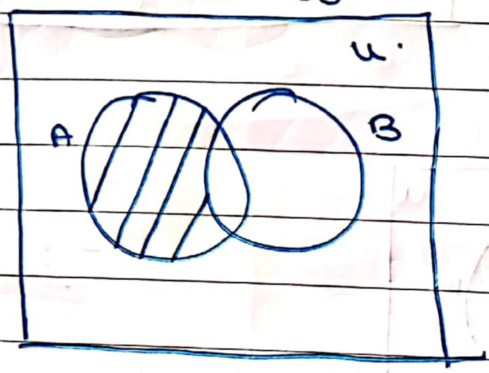


LHS = RHS

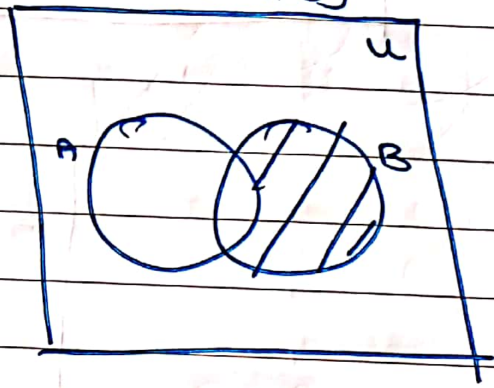
1) $A \oplus B = B \oplus A$.

2) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$,

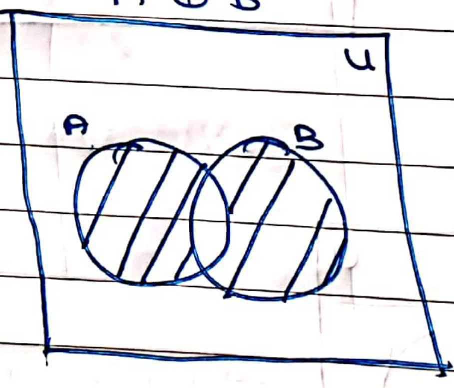
$A - (A \cap B)$

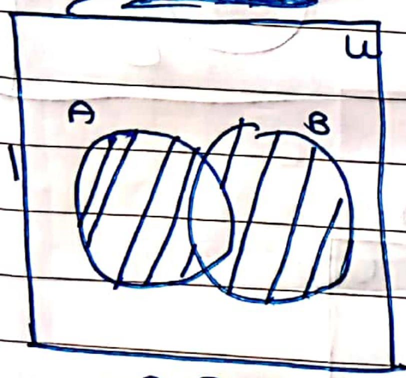


$B - (A \cap B)$

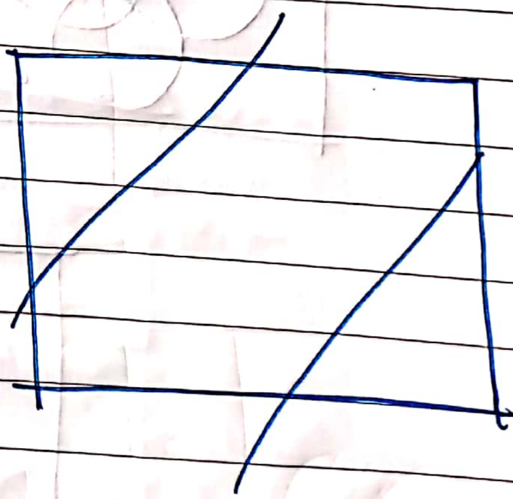


$A \oplus B$.





$B \oplus A$.



LHS = RHS.

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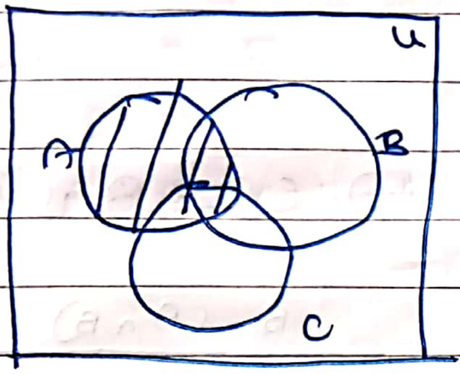
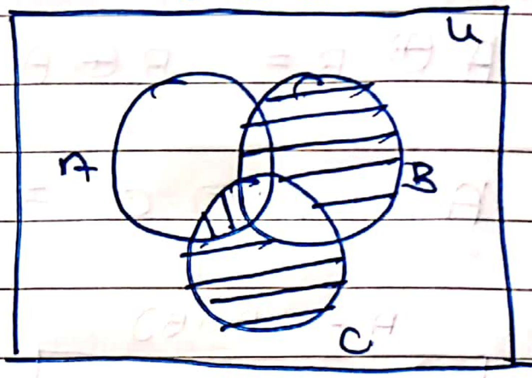
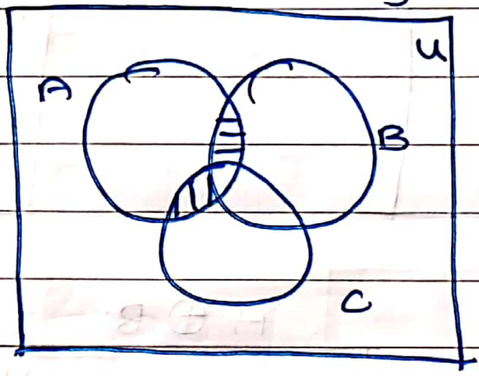
A



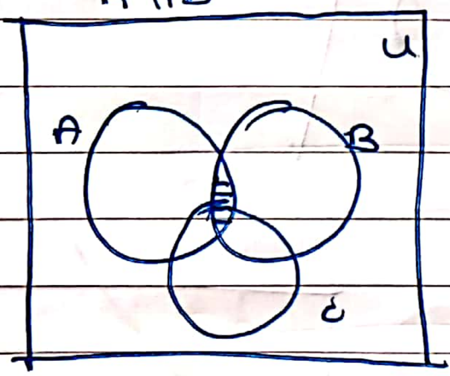
$B \oplus C$



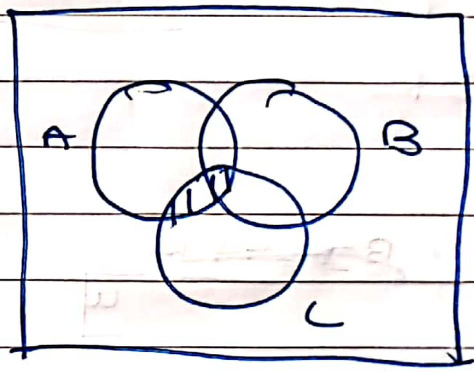
$A \cap (B \oplus C)$



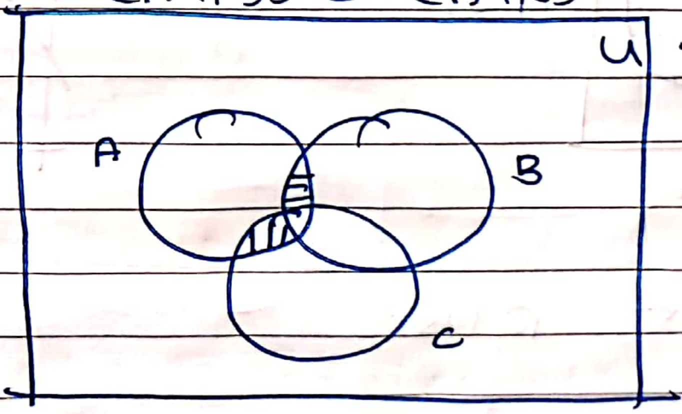
$A \cap B$



$A \cap C$



$(A \cap B) \oplus (A \cap C)$.



Partition of set:

Collection of $\{A_i\}$ of non empty subsets. is called partition of sets.

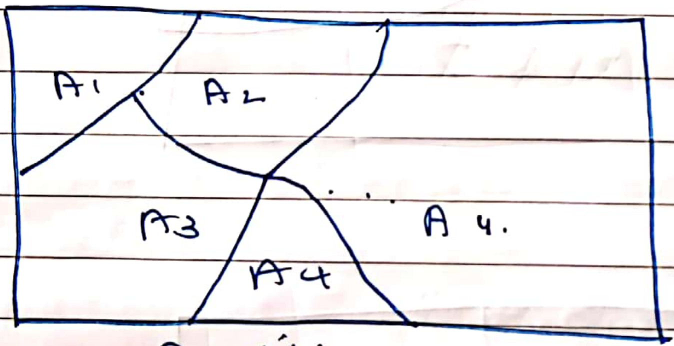
Each element of S belongs to one subset of A_i

$A_1 \cup A_2 \cup A_3 \ldots \ldots \cup A_i = S$.

Subset of A_i are mutually disjoint

$A_i \cap A_j = \phi$

Subsets in a partitions are called cells set.



Partition.

eg $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Consider the foll collection of subset of S.

1) $A_1 = \{\{1, 2, 3\}, \{3, 4, 5\}, \{6, 7\}\}$

2) $A_2 = \{\{1, 2, 3\}, \{4, 5\}, \{6, 7\}, \{8\}\}$

3) $A_3 = \{\{1, 2\}, \{3, 4, 5\}, \{6, 7\}\}$.

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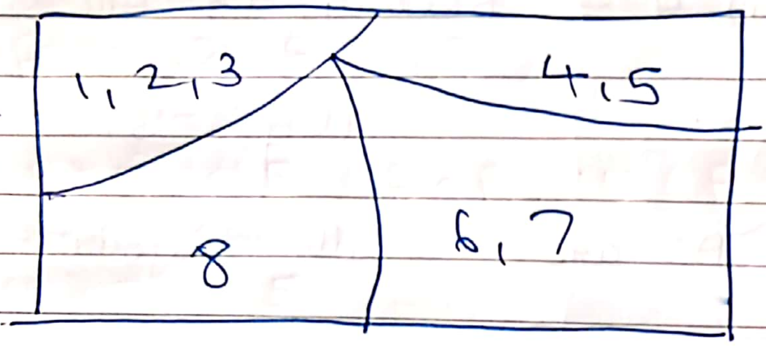
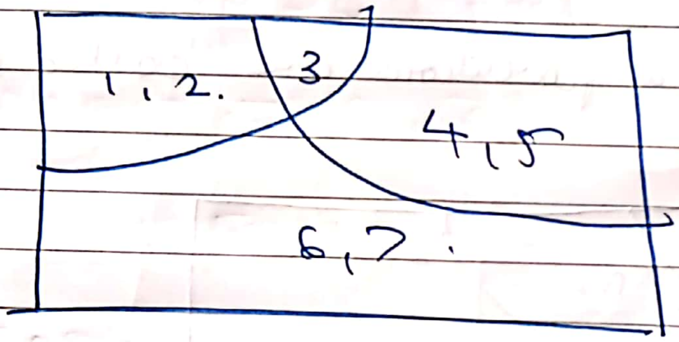
which of them is a partition of S & why also draw a diagram for this

A_2 is a partition of S

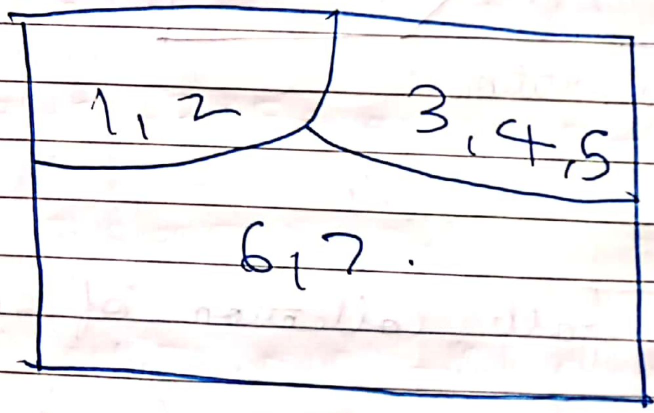
A2



A_1



A_3



A_1 is not a partition of S. There is intersection of 3.

A_2 is a partition of S as all elements of sets are present and no elements is common in any 2 partition.

A_3 is not a partition of sets as $8 \in S$ is not present in the partition.

~~Practic~~ Principle of Duality.

In any law ~~to~~ we exchange $\cup$ by $\cap$ & vice versa.

If necessary swipe is required.

It is dual of each other.

$\overline{A\cup B} = \overline{A} \cap \overline{B}$

$\overline{A\cap B} = \overline{A} \cup \overline{B}$

Let A, B, C are subsets of universal set U. given that

$A\cap B = A\cap C$

& $\cancel{\overline{A\cap B}}$ = $\overline{A}\cap B = \overline{A}\cap C$

It is necessary that $B=C$. Justify

$A\cap B = A\cap C$.

$\cancel{\overline{A\cap B}} = \cancel{\overline{A\cap C}}$

$B = B \cap U$

$A \cup \overline{B} = U \rightarrow$ By intersection prope

$B = B \cap (A \cup \overline{A})$

$B = \cancel{B \cap} [(B \cap A) \cup (B \cap \overline{A})$,
Distributive law

$B = (A \cap C) \cup (\overline{A} \cap C)$

$B = C \cap (A \cup \overline{A})$

$B = C \cap U$

$B = C$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

$A = \{1, \cancel{2}, 3, 4, 5, 7\}$,

$B = \{3, 5, 7, 8, 9\}$.

$C = \{2, 3, 5, 6, 7, \cancel{8}\}$.

$A \cap B = \{3, 5, 7\}$.

$A \cap C = \{3, 5, 7\}$

$\overline{A} = \{2, 6, 8, 9, 10\}$

$\overline{A} \cap B = \{8, 9\}$

$\overline{A} \cap C = \{2, \cancel{8}\}$

$A \cap B = A \cap C$ — i

$\overline{A} \cap B \neq \overline{A} \cap C$ — ii

It is not satisfied.

modified set c such that $A \cap B = A \cap C$ is also true

Set C should have elements 8, 9 to satisfy 2^{nd} condition.

$c = \{3, 5, 7, 8, 9\}$.

eg1) A survey of 500 television ~~shoppers~~ watchers produce foll inform:
285 watch football, 195 watch hockey, 115 watch basketball, 45 watch (F & B) 70 watch (F & H), 50 watch (H & B) 50 donot watch neither

~~U = 500~~

$n(U) =$ 1) How many people watch all the 3 kinds of games.

2) How many people watch exactly one of sports.

3)

$n(U) = 500$

$n(A \cup B \cup C) = 500 - 50$
$= 450$

$n(A) = 285$

$n(B) = 195$

$n(C) = 115$

$n(A \cap C) = 45$

$n(A \cap B) = 70$

$n(B \cap C) = 50$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$450 = 285 + 195 + 115 - 45 - 70 - 50 + x$

$x = \cancel{450} = 20$

$n(A)$ only $= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 285 - 70 - 45 + 20 = 190$

$n(B)$ only $= n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

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$= 195 - \cancel{4}70 - 50 + 20$

$= 95$

$n(C)$ only $= n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 115 - 45 - 50 + 20$

$= 40$

n(People who watch 1 channel) $= \cancel{19}$

$190 + 95 + 40 = 325$

Logic.

Proposition:

It is a statement is a declarative sentence wh[ich] is either true or false.

a) The Earth is flat $\rightarrow$ F.

b) $2+2=4 \rightarrow T$.

c) $3^3 \geq 21 \rightarrow T$.

d) $2^{8^2} \leq 3 \rightarrow F$.

$x+3=5$ If $x=2$. (T)
If $x=3$. (F).

It is true when $x=2$ and false when $x \neq 2$.

Notation :

$p, q, r, \dots, x$.

p = The sky is blue.

q = Earth is flat.

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Logic operations.

1. While using NOT or while performing statements with NOT it is denoted as $\sim p$.

If $p = F$ $\therefore \sim p = \sim F = T$.
" $p = T$ $\therefore \sim p = \sim T = F$.

eg p: I am going to buy a ~~fruit~~ book.

$\sim p$: I am not going to buy a book.

2. Conjunction.

If p & q are compound statements then conjunction of p & q is $p \wedge q$.

p: The sun is shining.
q: The day is bright

$p \wedge q$ = The sun is shining & the day is bright.

eg: and, but

Disjunction.

If p & q are compound statements then disjunction of p & q is p or q = $p \vee q$.

p: There is an error in the programme.
q: Anita is intelligent girl.

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$p \vee q$: There is an error in ~~pro~~ the programme ~~when~~ or Anita is intelligent girl.

I will watch Star news . = p

~~p~~ I will watch Discovery . = q.

~~I will watch~~

~~I will~~ Exclusive OR
~~Either~~ I will watch Star news or Discovery.

In EX-OR one or other will happen but not both

If - then .

If p & q are compound statements ~~it~~ then If p, then q
i.e $p \rightarrow q$. ($\rightarrow$ is implication)

p is Hypothesis or antecedant.

q is Consequence

p: Hari works ~~card~~ hard
q: Hari will pass the exam.

$p \rightarrow q$ = If Hari works hard then he will pass the exam.

$p \rightarrow q = \sim q \rightarrow p$

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Contrapositive

$\sim(p \rightarrow q) = \sim[(\sim p \vee q)]$

$= p \wedge \sim q.$

$\sim(q \rightarrow p).$

∴ If Hari ~~work~~ will not work hard ~~if~~ then ~~a~~ he will not pass the exam. — ~~A~~ Inverse

∴ If Hari will not pass the exam then he will not work hard.

p: It is raining
q: I carry an umbrella.

$q \rightarrow p$ = If I carry an umbrella then it is raining.

$(\sim q \rightarrow \sim p)$ = If I don't carry an umbrella then it will not rain.

For $p \rightarrow q$: Both of the statements must be sufficient.

p is sufficient for q & q is sufficient for p

Biconditional:

$(p \rightleftarrows q).$

If p & q are compound statements then biconditional statement $p \rightleftarrows q$ is

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$(\sim p \vee q) \wedge (\sim q \vee p)$.

p if & only if q.

If a integer is even number if it is divisible by 2.

~~Ist~~

It is raining if & only if I carry an umbrella.

p: Mohan is rich

q: Mohan is happy.

Mohan is happy if and only if he is rich. $(q \Leftrightarrow p)$

Proposition & statement forms.

eg: ~~For~~ $(p \vee q) \rightarrow q$.

p = Mohan is rich

~~p~~q = Mohan is happy.

(~~Mohan~~ If Mohan is rich or happy then he is happy.

3) $q \to p$

2) $p \vee \sim q$

1) $\sim p \wedge q$

$\sim p \vee (p \wedge q)$

1) ~~Mohan is not rich but he is !~~

Mohan is poor or he is both rich & happy

2) Mohan is not rich but he is happy.

3) Mohan is rich ~~but~~ or he is unhappy.

4) If Mohan is happy then he is rich.

2. Write the foll statements in symbolic form

1) Whenever whether is nice then only we will have a picnic.

p : Weather is nice
q : We will have a picnic.

$q \to p$

2) If either Anil takes science or Aparna takes Maths then Deepa will take Arts.
p = Anil takes science.
q : Aparona takes ~~po~~ maths
r : Deepa ~~to~~ will take Arts.

The symbolic form of is $(p \vee q) \rightarrow r$

Programme is readable ~~if~~ only if it is well structured.

p: Programme is readable

q: It is well structured.

$q \longrightarrow p$.

Unless he studies, he will fail in exam.

p: Unless He studies.

q: He will fail in exam.

$\sim p \rightarrow q$.

Truth tables

- If the statements is

$2^2 = 4$ or $2^3 = 8$. then $2^n =$:

Draw the truth of negation.

p	$\sim p$
T	F
F	T

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Conjuction ($p \wedge q$)

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

Disjunction ($p \vee q$)

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

Condition ($p \rightarrow q$) $\equiv [\sim p \vee q]$.

p	q	$p \rightarrow q$	$q \rightarrow p$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

Biconditional ($p \rightleftharpoons q$) $\equiv [(\sim p \vee q) \wedge (\sim q \vee p)]$

p	q	$p \rightleftharpoons q$
F	F	T
F	T	F
T	F	F
T	T	T

X-OR

p	q	$p \bar{\vee} q$
F	F	F
F	T	T
T	F	T
T	T	F

Q) Draw TT for foll statements.

1) $\sim[p \wedge (p \vee \sim q)]$

(i)	(ii)	(iii)	(iv) A	(v) B	(vi)
p	q	$\sim q$	$p \vee \sim q$	$p \wedge A$	$\sim B$
F	F	T	T	F	T
F	T	F	F	F	T
T	F	T	T	T	F
T	T	F	T	T	F

No. of rows $= 2^n = 2^2 = 4$

No of columns $= m+n = 2+4 = 6$.

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$(p \to q) \vee (p \leftrightarrow q)$.

(i)	(ii)	(iii) A	(iv) B	(v)
p	q	$p \to q$	$p \leftrightarrow q$	$A \vee B$
F	F	T	T	T
F	T	T	F	T
T	F	F	F	F
T	T	T	T	T

No. of statments $(m) = 2$

No of operands $(n) = 3$.

No of rows $(2^n) = 2^2 = 4$

No of columns $(\geq m+n) = 2+3 = 5$

If $p \to q$ is F determine truth value of $(\sim(p \wedge q)) \to q$.

$p = T$
$q = F$

p	q	$p \wedge q$	$\sim(p \wedge q)$ A	$A \to q$
T	F	F	T	F.

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Tautology

If a statement has truth value (T) irrespective of T assigning to its variables

Contradiction:

If a statement has truth value (F) irrespective of T F assigning to its variables.

If T F value is neither T nor F then it is contingency

Verify foll proposition.

1) $p \vee \sim(p \wedge q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
F	F	F	T	T
F	T	F	T	T
T	F	F	T	T
T	T	T	F	T

It is tautology.

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From last statement column we can conclude that it is Tautology

$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.

p	q	$p \xrightarrow{A} q$	$\sim q$	$\sim p$	$\sim q \xrightarrow{B} \sim p$	$A \Leftrightarrow$
F	F	T	T	T	T	T
F	T	T	F	T	T	T
T	F	F	T	F	F	T
T	T	T	F	F	T	T

From last ~~st~~ column we can conclude that it is tautology.

$(q \wedge p) \vee (q \wedge \sim p)$

$(p \vee \sim q) \wedge F$.

p	q	$\sim p$	$\overset{A}{q \wedge p}$	$\overset{B}{q \wedge \sim p}$	$A \vee B$
F	F	T	F	F	F
F	T	T	F	T	T
T	F	F	F	F	F
T	T	F	T	F	T

Contingency

p	q	$\sim q$	$p \overset{A}{\vee} \sim q$	$A \wedge p$
F	F	T	T	$\cancel{F}$ F
F	T	F	F	$\cancel{F}$ F
T	F	T	T	$\cancel{F}$ T
T	T	F	$\cancel{F}$ T	$\cancel{F}$ $\cancel{F}$ T.

~~Conta~~ Contradiction.

Laws of Logic.

1) Idempotent law

$p \vee p = p$ eg $T \vee T = T$

$p \wedge p = p$ eg $T \wedge T = T$

2) Commutative.

$p \vee q = q \vee p$

$p \wedge q = q \wedge p$

3) Associative

$(p \vee q) \vee r \cong p \vee (q \vee r)$

$(p \wedge q) \wedge r \cong p \wedge (q \wedge r)$

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4) Distributive law.

$p \vee (q \wedge r) = (p \vee q) \wedge (p \wedge r)$

$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

Identity law.

$p \vee F = p$

$p \vee T = T$

$p \wedge T = p$

$p \wedge F = F$

Absorption law.

$p \wedge (p \vee q) = p$

$p \vee (p \wedge q) = p$

Implication.

$p \to q = \sim p \vee q$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	T	F	T

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Complement law

$p \vee \sim p = T$.

$p \wedge \sim p = F$.

p.

De Morgan law.

$\sim(p \vee q) = \sim p \wedge \sim q$

$\sim(p \wedge q) = \sim p \wedge \sim q$.

1) The given truth value of p & q as T & those of r & s as F. Find truth value of following

(i) $[p \wedge (q \wedge r)] \vee \sim[(p \vee q) \vee (r \vee s)]$

$[T \wedge (T \wedge F)] \vee \sim[(T \vee T) \vee (F \vee F)]$

$= [T \wedge F] \vee \sim[T \vee F]$

Idempotent law.

$= F \wedge \sim T$

Complement law

$= F \wedge F$

Complement law

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$= F.$

2) Show that $\cancel{\ell}\, a \vee (\bar{a} \wedge b) = (\cancel{\ell}\, a \vee b)$

$LHS = a \vee (\bar{a}\ \cancel{\wedge} b).$

$= (a \vee \bar{a}) \cancel{\vee} \wedge \cancel{(a \vee b)}\,(a \vee b)$ ~~Associative~~. Distributive

$= T\ \cancel{\vee} \wedge (a \vee b)$ Complement law

$= \cancel{a \vee}\ a \vee b$ Identity law

3) $\cancel{\ell}\, a \wedge (\bar{a} \vee b) = a \wedge b.$

$(a \wedge \bar{a}) \vee (a\ \cancel{\vee} \wedge b)$ Distributive law

$F \vee (a \wedge b)$ Complement Law

$a \wedge b$ Identity law.

4) Show that foll equivalence without considering ~~truth~~ table.

1) $\sim(p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) = r.$

$LHS = \cancel{\sim}(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$

$= \cancel{\sim}(\sim p \wedge (\sim q \wedge r)) \vee (r \wedge (p \vee q))$ Distributive

$= \cancel{\sim}[r \wedge (\sim p \wedge \sim q)] \vee [r \wedge (p \vee q)]$ Associative

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$\sim \wedge [(\sim p \wedge \sim q) \vee (p \wedge q)]$

De Morgan

$\sim \wedge (\sim (p \vee q) \vee (p \vee q)]$

$\sim \wedge F.$ Complement

$\sim.$ Identity

Q) $(p \wedge \sim q) \vee q \vee (\sim p \wedge q) = (p \vee q)$

LHS $(p \wedge \sim q) \vee [q \vee (\sim p \wedge q)]$

$(p \wedge \sim q) \vee [\cancel{(q \vee \sim p) \wedge (q \vee q)}]$

$(p \wedge \sim q) \vee q. \rightarrow$ Absorption law

$(p \vee q) \wedge (q \vee \sim q)$ Distributive law

$(p \vee q) \wedge T$ ~~Identity~~ complement law

$p \vee q$ Identity law

Q) $[\sim p \wedge (p \vee q)] \rightarrow q$

LHS : $[\sim p \wedge (p \vee q)] \rightarrow q$

: $[(\sim p \wedge p) \vee (\sim p \vee q)] \rightarrow q$
Distributive law

$[F \vee (\sim p \vee q)] \rightarrow q$

Complement law

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$(\sim p \vee q) \rightarrow q$ Identity law

$\sim(\sim p \vee q) \vee q$

$\sim(q \vee \sim p) \vee q$

$(p \wedge \sim q) \vee q$ DeMorgan law

$\sim(\sim q \vee p)$

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Show that a given statement is tautology or not

1) $(p \vee q) \wedge \sim(\sim p \wedge (\sim \theta \vee \sim R)) \vee (\sim p \wedge \sim \theta) \vee (\sim p \wedge \sim R)$

$(p \vee \theta) \wedge \sim(\sim p \wedge (\sim \theta \vee \sim R)) \vee (\sim p \wedge (\sim \theta \vee \sim R))$

$(p \vee \theta) \wedge 1$

$\cancel{p \vee \theta}$

$(p \vee q) \wedge [\sim(\sim p \wedge \sim(\theta \wedge R))] \vee (\sim p \wedge \sim \theta) \vee (\sim p \wedge \sim R)$

$(p \vee q) \wedge [p \vee (q \wedge r)] \vee [\sim(p \vee q) \vee \sim(p \vee r)]$

$((p \vee q) \wedge (p \vee q) \wedge (p \vee r)) \vee [\sim(p \vee q) \vee \sim(p \vee r)]$

$[(p \vee q) \wedge (p \vee r)] \vee \sim [(p \vee q) \wedge (p \vee r)]$

$\cancel{A}$ T : tautology.

2). $[p \wedge (p \rightarrow q)] \rightarrow q$.

$[p \wedge (\sim p \vee q)] \rightarrow q$

$[(p \wedge \sim p) \vee (p \wedge q)] \rightarrow q$

$[F \vee (p \wedge q)] \rightarrow q$.

$[p \wedge q] \rightarrow q$.

$\sim(p \wedge q) \vee q$.

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$(\sim p \vee \sim q) \vee q$.

$\sim p \vee (\sim q \vee q)$

$\sim p \vee T$

T

Logical implication:

If r and s are the prepositions then r logically implies s. If $r \rightarrow s$ is tautology.

Show that p & q is logically implies $(p \rightarrow q)$.

If $(p \wedge q)$ is $\rightarrow (p \rightarrow q)$ is tautology.

p	q	$p \wedge q$ (A)	$p \rightarrow q$ (B)	$A \rightarrow B$
F	F	F	T	T
F	T	F	T	T
T	F	F	F	T
T	T	T	T	T

From last column we prove that it is a tautology. p & q are logically implies.

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Show that $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

Simplify:

$(p \vee \sim q) \vee q \vee (\sim p \wedge q)$.

$[(p \wedge \sim q) \vee q] \vee [\sim p \wedge q]$

Associative law.

$[\cancel{(p \vee q) \vee (\sim q}$

$\cancel{[p \vee (q \vee \sim q)] \vee [\sim p \wedge q]}$

$[(p \vee q) \wedge (q \vee \sim q)] \vee [\sim p \wedge q]$

Distributive law.

$[\cancel{p \vee q}]$

$[(p \vee q) \wedge T] \vee [\sim p \wedge q]$ Idempotent

$[(p \vee q)] \vee [\sim p \wedge q]$. Identity.

$[p \vee q] \vee \cancel{\sim (p \vee \sim q)} (\sim p \wedge q)$.

$p \vee [q \vee (\sim p \wedge q)]$.

$\cancel{p \vee [(q \vee \sim p) \vee (q}$

$q \vee [p \vee (\sim p \wedge q)]$

$q \vee [(p \vee \sim p) \wedge (p \vee q)]$ Distributive

$q \vee [p \vee q]$ Identity.

$(q \vee q) \vee p$

$q \vee p$

$p \vee q$ — Identity law.

Equivalence of Statement form.

→ The statements are logically ~~ke~~ equivalent if both have same truth value irrespective of value assigned the statement value.

C⟺A eg $(p \vee q) \wedge (p \vee \sim q) \vee q \longleftrightarrow p \vee q$.

	p	q	$\overset{A}{p \vee q}$	$\sim pq$	$\overset{B}{p \vee \sim q}$	$\overset{C}{A \wedge B}$	$\overset{D}{C \vee q}$
~~F~~T	F	F	F	T	T	F	F
T	F	T	T	F	F	F	T
T	T	F	T	T	T	T	T
T	T	T	T	F	T	T	T

$(p \vee q) \wedge (p \vee \sim q) \vee q \longleftrightarrow p \vee q$.

From column A & D given statements are equivalent to each other

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Predicates.

1) x is tall & handsome.

2) $x + 3 = 5$

3) $x + y \geq 10$.

These are not propositions because they don't have any values. If value assigned to variable then each become true or false.

eg He is tall & handsome ~~False~~ True.

eg $3 + 2 = 5$ False.

~~eg~~ Assertion

Sentence that contains one or more ~~pos~~ variable is predicate. It is truth value is predicated after assigning ~~to truth~~ value. is true or false.

Predicate contains n variables such that $x_1, x_2, \ldots, x_n$ is called as n place predicate.

$P(x_1, x_2, \ldots, x_n)$.

It is an argument.

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eg $x + y \geq z$.

$P(x, y, z)$

eg x is the father of y.

$P(x, y)$.

→ A set of values based on based on which values of predicate is decided is called as universe of discourse.

When we specify a value of a variable appearing in the predicate is called ~~define~~ bind that variable.

A predicate becomes a proposition only when all the variables are bind.

$x + 3 = 5$.

$-1 + 3 = 5 \longrightarrow$ False.

$2 + 3 = 5 \longrightarrow$ true.

U = The set of integers = Universe of discourse.

2). For 2^{nd} method, for binding predicates by using quantification of variables.

→ Universal quantifier.
→ Essential quantifier.

If $P(x)$ is a predicate with one variable x then for all x $P(x) \rightarrow T$. Universal quantifier ($\forall$)

$\forall x : P(x)$

$\forall x \Rightarrow$ all, every "b"

eg $P(x) = x \geq 0.$ $x = 0, 1, 2, \ldots$

$x \rightarrow$ +ve integers.

$\forall x : P(x) \rightarrow$ true.

$x \rightarrow$ Any real no.

$\forall x \; P(x) \rightarrow$ False.

Essential quantifier.

Suppose ~~for~~ for Predicate $P(x)$ all x $P(x)$ is false. but there exists at least one value of x for which $P(x)$ is true then x is said to be counted by essential quantifier.

$\exists x \; P(x) \Rightarrow T.$

$\rightarrow$. Some eg a eg $P(x)$ — 1 place. variable.

$x + 3 = 5. \longrightarrow T$

$\exists x \; P(x)$ is true.

$\forall x \; P(x)$ is false.

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$\exists x \forall y \; P(x,y)$.

There exists a ~~variably~~ x ~~&y~~ for all values of y for which $P(x,y)$ is true.

$\forall x \; \exists y \; P(x,y)$.

For all values of x there exists a value of y for which $P(x,y)$ is true.

$\forall y \; \exists x \; (P(x,y))$.

For all values of y there exists a value of x for which $P(x,y)$ is true.

$\exists y \; \exists x \; (P(x,y))$.

There exists a value any ~~of~~ & x for which $P(x,y)$ is true.

$\forall x \; \forall y \; P(x,y)$.

~~It~~ For all values of x & y the condition $P(x,y)$ is true.

Negation of Predicate / Quantifying statement.

1. ~~Ax~~ $\forall x \; P(x)$ is a predicate.

   $\neg[\forall x \; P(x)]$

   There exists a value x for which $P(x)$ is false.

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a) What is a universal and essential sentence.

~~Whites~~ given an English sentence

1) $\forall x \, P(x)$.

2) $\exists x \, Q(x)$

3) $P(x)$: x is alive ~~or~~ than it is mortal.

4) ~~R~~ $Q(x)$ is ~~too~~ live in water.

1) $\exists x \, [\sim(P(x))]$

2) $\forall x \, [\sim Q(x)]$

3) $P(x)$: x is not ~~alive~~ than

1) For all values of x ; x is alive than it is mortal.

Neg There exists value of x where if x is not alive than it is mortal.

2) There exists a value ~~o~~ x where x is living water.

Neg For all value of x, x is not living in water.

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$\forall x\, [\sim P(x)]$.

$\equiv \exists x\, [P(x)]$

For all values of ~~x~~ if x is ~~not~~ ~~alive then it~~ is mortal.

There exists ~~of~~ x where ~~x is alive then it~~ is mortal.

For all values of x P(x) is false.

There exists a value of x where P(x) is true

P(x) : x is man.

C(x) : x is clever.

1) $\exists x\, (m(x) \rightarrow c(x))$ $\sim p \vee q$

There exist a value x where if x is man & then x is clever.

For all values of x where ~~if~~ x is man & x is not clever.

2) $\forall x\, (m(x) \wedge c(x))$

For all values of x, x is man & clever.

There exists a value x where x is not man or x is not clever.

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Condition	Negation
$\forall x . P(x)$	$\exists x\ [\sim P(x)]$.
$\exists x\ P(x)$	$\forall x\ [\sim P(x)]$
$\forall x\ [\sim P(x)]$	$\exists x\ [P(x)]$
$\exists x\ [\sim (P(x))]$	$\forall x\ [P(x)]$.

- Translate universal disclosure . be a real number

a) For any value of $x, x^2 \rightarrow$ non negative

$$\forall x\ [x^2 \geq 0]$$

b) For every value of x, there is some value of y ; $x \cdot y = 1$.

$$\forall x\ \exists y\ [xy = 1]$$

c) There are positive values of x, y where $x, y > 0$.

$$\exists x\ \exists y\ [x, y > 0].$$

$$\exists x\ \exists y\ [(x > 0) \wedge (y > 0) \wedge (xy > 0)$$

d) There is a value of x & for all value of y such that if y is +ve then $x + y$ is −ve.

$$\exists x\ \forall y\ [(y > 0) \wedge (x < 0) \wedge (|x| > |y|)$$

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Write down the foll propositions in symbols.

1) For every number x there is a number y such that $y = x+1$

2) There is a number y such that for every number x, $y = x+1$.

1) $\forall x \exists y (y = x+1)$

2) ~~$\exists x \forall$~~ $\exists y \forall x (y = x+1)$

Negate the foll sentence such that the symbol negation does not appear outside the square bracket.

1) $\forall x [x^2 \geq 0]$.

2) $\exists x [x \cdot 2 = 1]$.

3) $\forall x \exists y [x + y = 1]$.

Ans) 1) $\exists x [x^2 < 0]$

2) $\forall x [x \cdot 2 \neq 1]$

3) $\exists x \forall y [x + y \neq 1]$

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Mathematical induction.

Let $P(n)$ is a statement is involved in natural numbers

① If $P(n)=$ true for $n=n_0$.
$\uparrow \rightarrow$ Basis of induction

② Assume $P(k)$ is true $(k \geq n_0)$

Induction Start We have to prove $P(k+1)$ is a true.

$P(n)=1$ all numbers $n > n_0$

$n=k \Rightarrow$ Induction ~~by~~ hypothesis.

③ ~~$P(n)$~~

$$P(n)=1+2+3+\ldots = \frac{n(n+1)}{2}$$

$P(1)=$ ~~FC~~ $\frac{1(1+1)}{2}=1$

LHS $=1$.

LHS $=$ RHS.

It is true for $n=k$ terms.

$P(k)=\frac{k(k+1)}{2}$

$P(k+1)=\frac{(k+1)(k+2)}{2}$

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$P(k) + k+1 = \frac{k(k+1)}{2} + k+1$

$= \frac{k(k+1) + 2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2}$

LHS = RHS.

Hence it is true for each & every term

Q2) Prove that.

~~$5^n - 1$~~

$5^n - 1$ is divisible by 4 for $n \geq 1$

$P(n) = 5^n - 1$

Let us check for $n = 1$

$5^1 - 1 = 5 - 1 = 4$

$4/4 = 1$

4 is divisible by 4.

It is assumed that it is true for $n = k$ terms

$P(k) = 5^k - 1$ is divisible by 4.

$5^k - 1 = 4m$. $\therefore 5^k = 4m+1$.

Let us check for $n = k+1$. where m is an integer.

$5^{k+1} - 1 = 5(5^k) - 1 = 5(4m+1) - 1$

$= 20m + 5 - 1 = 20m + 4$

$= 4(5m+1)$

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It is true for $n = k+1$ terms

Hence it is true for all terms

3) Prove that $7^n - 1$ is divisible by 6 for $n \geq 1$.

Let us check for $n = 1$.

$LHS = 7^1 - 1 = 7 - 1 = 6$

6 is divisible by 6.

It is assumed that it is true for $n = k$ terms.

$7^k(7) - 7 + 6$

$7(7^k - 1) + 6$

Let us check for $n = k+1$

$\cancel{6}\,7^{k} - 1 = 6\cancel{k}m$ where m is an integer

$7^k = 6m + 1$

$7^{k+1} - 1 = 7^k(6m+1) - 1$

$= \cancel{7^k(6m+}$

$= 42m + 7 - \cancel{6}1 =$

$= 42m + 6$

$= 6(7m+1)$

Hence it is true for $k+1$ terms.

Hence it is true for all terms.

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4) Prove that $n! \geq 2^{n-1}$ for $n \geq 1$.

Let us check for $n = 1$

$LHS = n! = 1! = 1$

$RHS = 2^{n-1} = 2^{1-1} = 2^0 = 1.$

$LHS = RHS$

Let us assume for $n =$ ~~p~~ k terms:

$k! \geq 2^{k-1} \rightarrow$ Hypothesis Induction.

Let us prove for $n = k+1$ terms

$(k+1)!$ $\quad\quad\quad$ $\cancel{2^{k}}$ $1 \quad 2^{k+1-1}$

$\cancel{k(k+1)}$

$(k+1)\, k! \geq 2^{k}$ $\quad\quad\quad$ $2^{\cancel{k}\, k}$.

$\cancel{2^{k-1}(2)}$

$\cancel{(k+1) \geq 2^{k-1}}$

While giving normal form we can determine whether the statement is contradiction or tautology by TT.

If variable in statement is more than one then it is difficult to construct a TT so that we develop this into normal form.

$p \wedge \sim p$ = Contradiction

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A statement from which is disjunctive of fundamental ~~or~~ conjunction gives us disjunctive ~~power~~ normal form

$(p \wedge q) \vee \sim q$.

$(p \vee \sim q) \wedge (q \vee \sim q)$ Distributive law

$(p \vee \sim q) \wedge T$. Idempotent law

$(p \vee \sim q)$. Identity law

The replacement negation ~~to~~ before conjunction or disjunction using De Morgan's law & property.

Apply distributive law & simply the expression. (CNF) law.

2) $p \wedge (p \Rightarrow q)$

$p \wedge (\sim p \vee q)$

$(p \wedge \sim p) \vee (p \wedge q)$ Distributive

$F \vee (p \wedge q)$ Idempotent

$p \wedge q$ Identity

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⇒ $(p \to q) \wedge (\sim p \wedge q)$.

$(\sim p \vee q) \wedge (\sim p \wedge q)$

$[(\sim p \vee q) \wedge \sim p] \wedge q$.

$[(\sim p \wedge \sim p) \vee (\sim p \wedge q)] \wedge q$.

~~$\sim [p \vee (\sim p \vee q)] \wedge q]$~~

$[\sim p \vee (\sim p \wedge q)] \wedge q$.

$[(\sim p \wedge q) \vee [(\sim p \wedge q) \wedge q)]$

$[(\sim p \wedge q) \vee [\sim p \wedge q]]$

$\sim p \wedge q$.

⑥ $\sim [p \to (q \wedge r)]$

$\sim [\sim p \vee (q \wedge r)]$ Implication

$p \wedge \sim (q \wedge r)$ De morgan law

$p \wedge [\sim q \vee \sim r]$.

$(p \wedge \sim q) \vee (p \wedge \sim r)$ Distributive

$(a \times b)$

Disjunction of statement (AND, OR, ~) are called as fundamental disjunction.

eg p

$\sim p$ $\qquad (\wedge) \vee (\wedge)$

$\sim p \vee q$

$p \vee q$.

Conjunction of fundamental disjunction is called CNF (Conjunction normal form.

eg $p \wedge r$.

$\sim p \wedge (p \vee q)$ $\qquad (\vee) \wedge (\vee)$

$(p \vee q) \wedge (q \vee r)$

2) $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$.

$[p \vee r] \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$.

$[(p \vee r) \wedge (p \vee \sim q)] \wedge [\sim p \vee q]$

$[p \vee (r \wedge \sim q)] \wedge [\sim p \vee q]$.

It is in CNF form.

Q) ~~$(\sim p \wedge q)$~~ $(\sim p \wedge q \wedge r) \vee (p \wedge q)$.

$= q \wedge [(\sim p \wedge r) \vee p]$ $\qquad$ Distributive

$= q \wedge [(\sim p \vee p) \wedge (p \wedge r)]$ Distributive Idempotent

$= q \wedge [p \wedge r] = p \wedge q \wedge r$

Relations :

→ A and B are the non empty sets.

A relation R from A to B is a subset of $A \times B$.

$A \, R \, B$.

If A relation R from A to B is not a subset of $A \times B$

$A \not R B$.

eg $A = \{1, 2, 3\}$.

$B = \{r, s\}$

$R = \{(1, r), (2, r), (2, s), (3, r), (3, s)\}$.

$A \times B = \{(1, r), (1, s), (2, r), (2, s), (3, r), (3, s)\}$

$|A \times B| = 6$.

$a \, R \, b$.

$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$B \times B = \{(r, r), (r, s), (s, r), (s, s)\}$.

$R_1 = \{(1, 2), (3, 1), (3, 3)\}$.

$a \, R \, a$

eg $A = \{1, 2, 3, 4, 5\}$.

Define R < on A. ie $a \; R \; b$. if

$a < b$.

$R = \{ \cancel{(1,1)}, (1,2), (1,3), (1,4), (1,5),$
$(2,3), (2,4), (2,5), (3,4), (3,5),$
$(4,5) \}$

$R \subseteq A \times B \Rightarrow a R b$.

Domain.

It is a set of all elements in A that are related to some element in B.

ie. it is a set of first elements in the pair that take a $\underline{R}$.

$A = \{1, 2, 3\}$.

$B = \{r, s\}$

$R = \{(1, r), (2, r), (3, r)\}$

$D = \{1, 2, 3\}$

$Dom(R) = \{1, 2, 3\}$

Range.

It is set of elements in B that are second elements of pair in the relation R. i.e all the elements in B that are related to some elements in A.

Range.

RAN(R) = { r }.

Representation of function.

A. Matrix form

R = {(1,2), (2,3), (3,2), (1,3)}.

A = {1,2,3}.
B = {2,3}.

$$
M_R = \begin{array}{c|c}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}
\end{array}
$$

Q) A = {1, 2, 3, 4, 8}.

B = {e·1, 4, 6, 9}

A R b = if a divides b. (a/b).

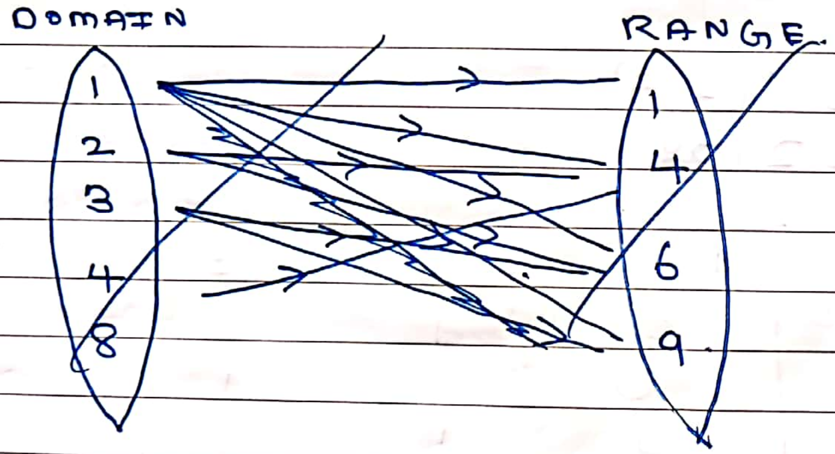
R = {(1,1), (2,1), (4,1), (3,1), (4,4), (8,1), (8,4)}.

R = {(1,1), (4,1), (4,2), (4,4), (6,1), (6,2), (6,3), (9,1), (9,3)}.

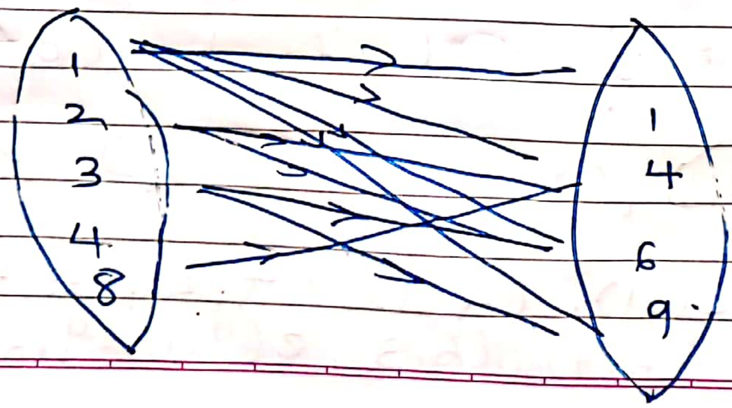
R = { (1,1), (1,4), (1,6), (1,9), (2,4), (4,4), ~~(1,6)~~, (2,6), (3,6), (3,9) }

	1	4	6	9
1	1	1	1	1
2	0	1	1	0
3	0	0	1	1
4	0	1	0	0
8	0	0	0	0

Digraph :



R is set of relation of set A which is finite elements of A . R node operation of A R I

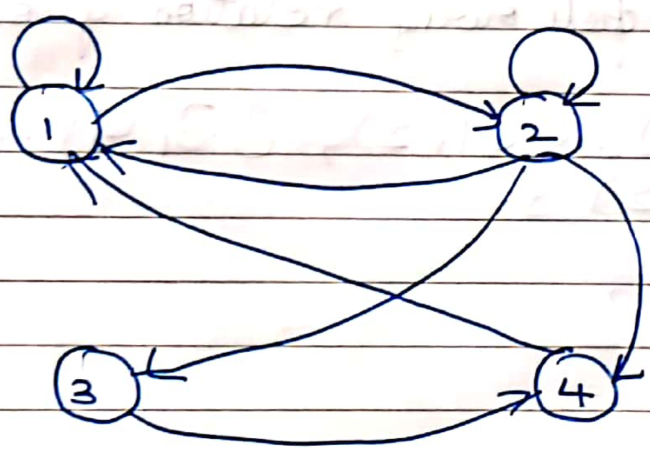


$A = \{1, 2, 3, 4\}$.

$R = \{(1,1), (1,2), (2,1), (2,2),$
$(2,3), (2,4), (3,4), (4,1)\}$

$DOM(R) = \{1, 2, 3, 4\}$

$RAN(R) = \{1, 2, 3, 4\}$.



Degree of vertices

Indegree ~~Self~~ 3

Outdegree 1

Logic.

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

~~A = {~~

$A = \{1, 2, 3, 4\}$.

$B = \{X, Y, Z\}$

Let R be the following relation from A to B.

$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$.

Matrices ?
Diagraph
Domain & Range.

	X	Y	Z
1	0	1	1
2	0	0	0
3	0	1	0
4	1	0	1

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1, 2, 3

1 x y

2 . 2 w

3 . 1 z

Diagraphs

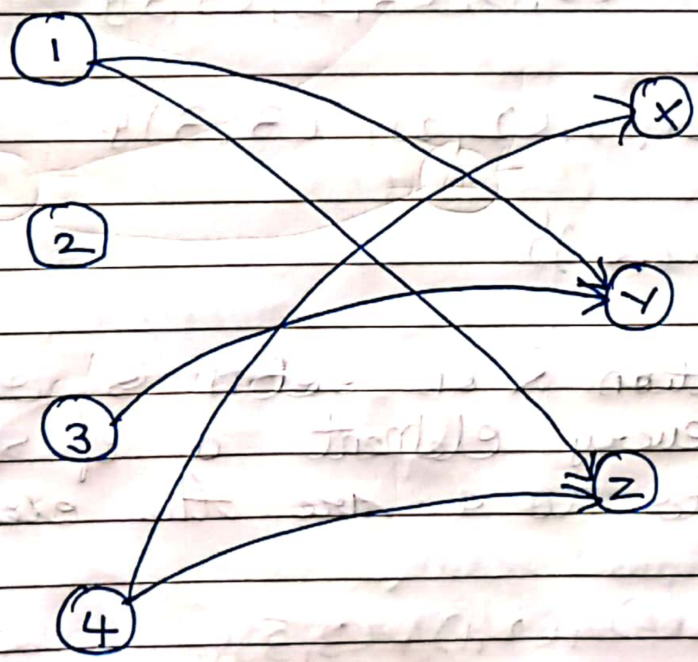
	1	2
2	x	w
3	2	

3

Domain = {1, 3, 4}.

Range = {x, y, z}.

Diagraphs



The diagraph is not possible when 2 different sets on which the relation has been defined.

Find inverse of a given relation R.

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(a, b)

(b, a)

$R^{-1} = \{(4,1), (2,1), (4,3), (X,4), (2,4)\}$

Properties of relation:

1) Reflexive property:

$A = \{1, 2, 3\}$.

$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$.

$R = \{(1,1), (2,2), (3,3)\}$.

$R = \{(a,a)\}$

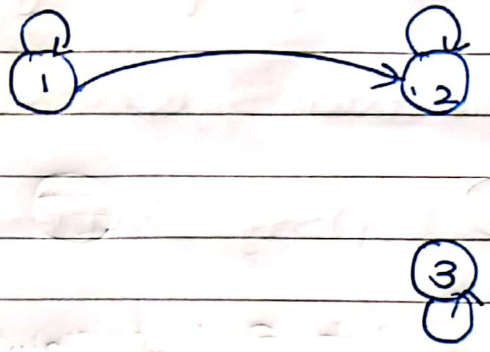
A relation R on set A is reflexive if for every element a of set A, a relates to a. are the elements of relation

$R = \{(1,2), (2,1), (3,3)\}$.

$(1,2) \notin R$.
$(2,1) \notin R$.

$R = \{(1,1), (2,2), (3,3), (1,2)\}$.

As all elements of reflexive are present it is a reflexive function.



$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}$$

Irreflexive function

A relation R on set A is irreflexive of A if (a,a) is not an element of R for every $a \in A$.

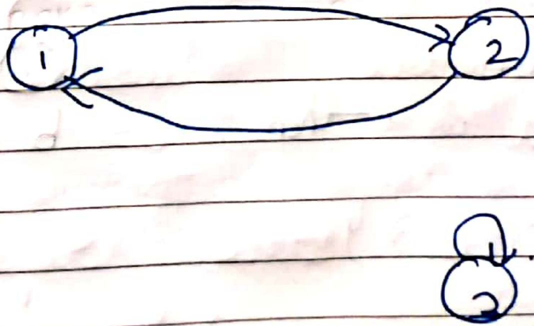
R is irreflexive if no element is related to itself.

$R = \{(1,2), (2,1), (3,3)\}$.

It is an irreflexive.

$\Rightarrow (a,a) \notin R$.

Here R is an irreflexive relation because (1,2) & (2,1) are the elements of R as $(a,a) \notin R$).



$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 0 & 1 \end{array}$$

A irreflexive all diagonal elements should be 0.

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→ In irreflexive relation has no cycle of length 1.

Symm · aRb. bRa

Assy. aRb b$\not R$a

A relation R of set A is assymetric relation if whenever A relates to B, B not relates to A.

~~A = {1,2,3}.~~

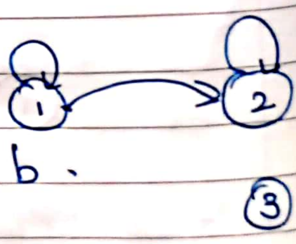
A = {1,2,3}.

R = {(1,2), (1,1)}.

$$R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

If the given relation is assymetric then the relation is always antisymmetric.

Antisymmetric



aRb, bRa as · then a = b.

eg : a = {1, 2, 3}

R = { (1,2), (2,2), (1,1)}.

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 \end{array}$$

Contrapositive

If R is antisymmetric if a R b in a & $a \neq b$ & b R̸ a. then it is contrapositive

If $A = \{1, 2, 3, 4\}$.

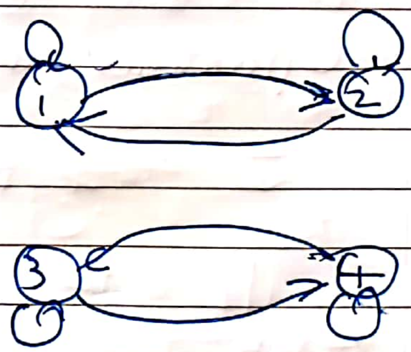
determine the given relation is reflexive, irreflexive, symmetric, antisymmetric.

~~R=~~

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,3), (3,4), (4,4)\}$

Reflexive

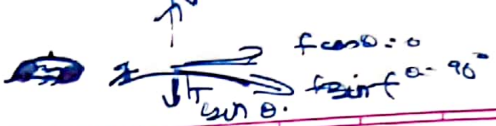
Symmetric.



It is not irreflexive as (1,1), (2,2), (3,3), (4,4) elements are in R.
$R = \{(1,1), (2,2), (3,3), (4,4)\}$

The given relation is symmetric because $\{(1,2), (2,1), (3,4), (4,3)\}$.

It is not antisymmetric.



Transitive relation.

aRb . bRc then aRc.

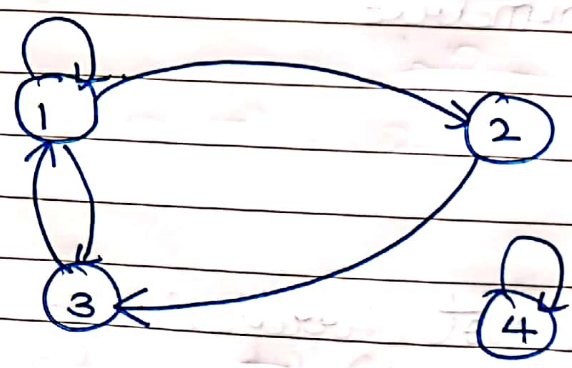
eg. ~~R = {(2,1),(1,2),(2,3)}~~

R = {(2,1),(1,2),(2,2)}

R ~~ft~~ is a transitive relation.

~~R = {~~ The Relation R is previous equation is transitive relation.

Q). R = {(1,1), (1,3), (3,1), (1,2), (2,3), (4,4)}



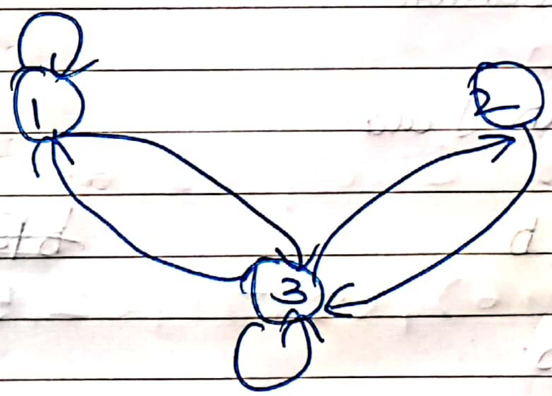
Irreflexive ~~Reflexive~~, Symmetric, Transitive.

Determine symmetric or assymetric antisymmetric

$$R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \hline \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

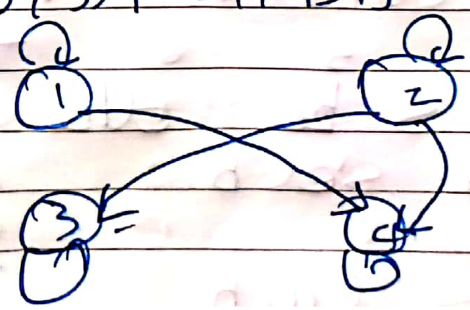
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

1) $R = \{(1,1), (1,3), (\cancel{3}2,3), (3,1), (3,2), (3,3)\}$



Symmetric; Irreflexive; Transitive

2). $R = \{(1,1), (1,4), (2,2), (2,3)\ (2,4), (3,3), (4,4)\}$



Reflexive
Antisymmetric.
Assymetric.