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A sinusoidal carrier has an amplitude of 10V and freq 30KHz. It is amplitude modulated by a sinusoidal voltage of amplitude 3V and frequency 1KHz. Modulated voltage is developed across 50Ω

[12 Marks]

- ① Write eqn of modulated wave
- ② Plot modulated wave showing maxima & minima
- ③ Determine modulation Index
- ④ Draw the spectrum of modulated wave
- ⑤ Calculate total power
- ⑥ Calculate Power carried by sidebands

Solution

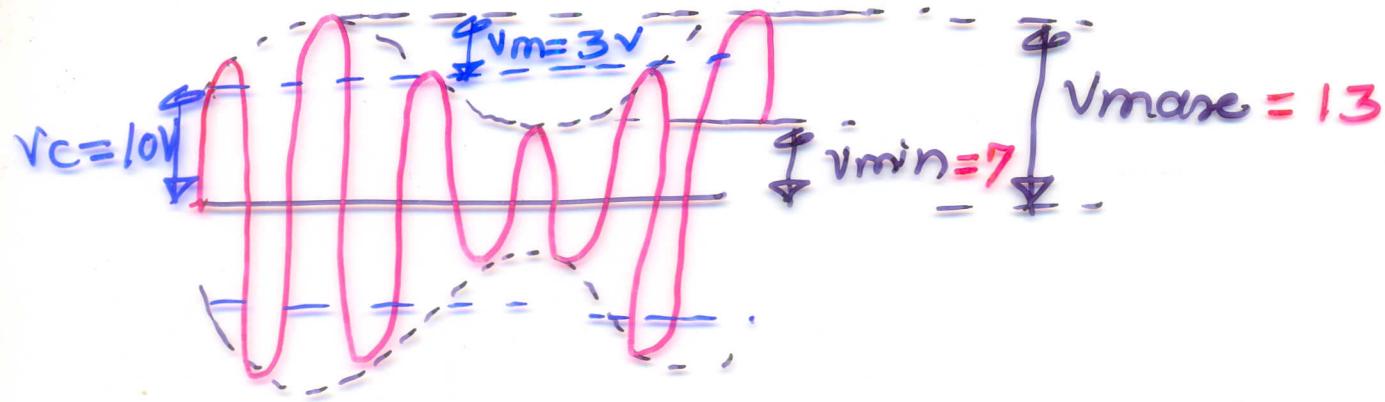
$$V_c = 10V, V_m = 3V$$

$$m = \frac{V_m}{V_c} = \frac{3}{10} = 0.3, \boxed{m = 0.3}$$

$$f_m = 1\text{KHz}, f_c = 30\text{KHz}$$

$$\theta = V_c \sin \omega_c t + \frac{m V_c}{2} \cos (\omega_c - \omega_m) t - \frac{m V_c}{2} \cos (\omega_c + \omega_m) t$$

$$\theta = 10 \sin(60\pi \times 10^3 t) + 1.5 \cos(58\pi \times 10^3 t) - 1.5 \cos(62\pi \times 10^3 t)$$



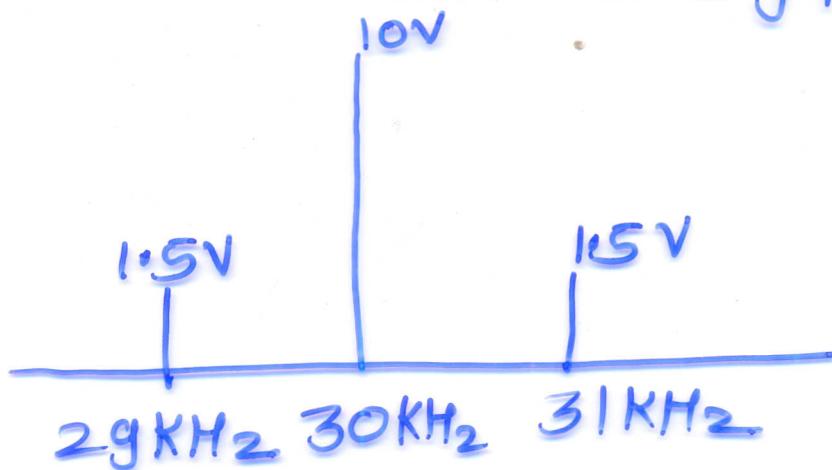
$$V_{max} = V_c + V_m = 10 + 3 = 13$$

$$V_{min} = V_c - V_m = 10 - 3 = 7$$

Spectrum of modulated wave

$$f_c + f_m = 30 + 1 = 31 \text{ kHz}$$

$$f_c - f_m = 30 - 1 = 29 \text{ kHz}$$



$$P_T = P_C (1 + m^2/2)$$

$$P_C = \frac{V_c^2}{2R} = \frac{10 \times 10}{2 \times 50} = 1W$$

$$P_T = 1 \left(1 + \frac{(0.3)^2}{2}\right) = 1.045 \text{ watt}$$

$$\boxed{P_T = 1.045 \text{ watt}}$$

$$\boxed{P_{PSB} = P_T - P_C = 0.045 \text{ W}}$$

$$\boxed{P_{LSB} = P_{USB} = \frac{0.045}{2} = 0.0225 \text{ W}}$$

Antenna current of an AM Tx is 3
 8 amp when only the carrier is sent, but increases to 8.93 Amp when carrier is modulated by a single sine wave. Find the percentage modulation. Determine the antenna current when the depth of modulation changes to 0.8

Solution -

$$i) \frac{I_t}{I_c} = \sqrt{1 + \frac{m^2}{2}}, \quad I_c = 8 \text{ amp}$$

$$I_t = 8.93 \text{ amp}$$

$$\left(\frac{I_t}{I_c} \right)^2 - 1 = \frac{m^2}{2}, \quad m = \sqrt{2 \left[\left(\frac{I_t}{I_c} \right)^2 - 1 \right]}$$

$$m = \sqrt{2 \left[\left(\frac{8.93}{8} \right)^2 - 1 \right]}$$

$$= 0.701447$$

$$m = 70.1\%$$

$$ii) m = 0.8, I_c = 8 \text{ amp}, I_t = ?$$

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

$$= 8 \sqrt{1 + (0.8)^2}$$

$$I_t = 9.19 \text{ Amp}$$

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A certain Tx⁷ radiates 9 kW with the carrier unmodulated and 10.125 kW when the carrier is sinusoidally modulated. Calculate modulation index.

If another sine wave corresponding to 40°, modulation is transmitted simultaneously, determine total radiated power.

i) $P_c = 9 \text{ kW}, P_t = 10.125 \text{ kW}$

$$\frac{P_t}{P_c} = 1 + \frac{m^2}{2}$$

$$\frac{10.125}{9} = 1 + \frac{m^2}{2}$$

$$\frac{10.125 - 9}{9} = \frac{m^2}{2}$$

$$m = \sqrt{\frac{1.125}{9} \times 2}$$

$$m = 0.5$$

$$2) m_1 = 0.5, m_2 = 0.4$$

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$$m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{(0.5)^2 + (0.4)^2}$$

$$m_t = \sqrt{0.25 + 0.16} = 0.64$$

$$\boxed{m_t = 0.64}$$

$$\frac{P_t}{P_c} = 1 + \frac{m_t^2}{2}$$

$$P_t = g \left(1 + \frac{(0.64)^2}{2} \right)$$

$$= g (1 + 0.205)$$

$$\boxed{P_t = 10.845 \text{ kW}}$$

A 360 W carrier is simultaneously modulated by two audio waves with modulation percentage of 55 and 65 respectively. What is total sideband power radiated.

6 marks

Soln $P_c = 360 \text{ W}$

$$m_1 = 0.55, m_2 = 0.65$$

$$m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{(0.55)^2 + (0.65)^2}$$

$$\boxed{m_t = 0.85}$$

$$P_{SBT} = \frac{P_c(m_t^2)}{2}$$

$$\boxed{P_{SBT} = 130 \text{ W}}$$

(6)

The amplitude modulated waveform has the form

$$x(t) = 10(1 + 0.5 \cos 2000\pi t + 0.5 \cos 4000\pi t) \times \cos 20,000\pi t$$

- a) Sketch the amplitude spectrum of $x(t)$
- b) Find average power content of each spectral component including carrier
- c) Find the total power and sideband power
- d) What is the modulation index.

08marks



$$x(t) = 10(1 + 0.5 \cos 2000\pi t + 0.5 \cos 4000\pi t) \times \cos 20,000\pi t$$

$$\times \cos 20,000\pi t$$

This is the case of Amplitude modulation by several waves

$$m_1 = 0.5$$

$$m_2 = 0.5$$

$$2\pi f m_1 = 2000\pi$$

$$f m_1 = 1 \text{ kHz}$$

$$2\pi f m_2 = 4000\pi$$

$$f m_2 = 2 \text{ kHz}$$

$$USB_1 = f_c + f m_1$$

$$USB_1 = (10 + 1) \text{ kHz}$$

$$USB_1 = 11 \text{ kHz}$$

$$2\pi f_c = 20,000\pi$$

$$f_c = 10 \text{ kHz}$$

(17)

$$USB_2 = f_c + f_{m2} = (10 + 2) \text{ kHz}$$

$$\boxed{USB_2 = 12 \text{ kHz}}$$

$$LSB_1 = f_c - f_{m1} = (10 - 1) \text{ kHz}$$

$$\boxed{LSB_1 = 9 \text{ kHz}}$$

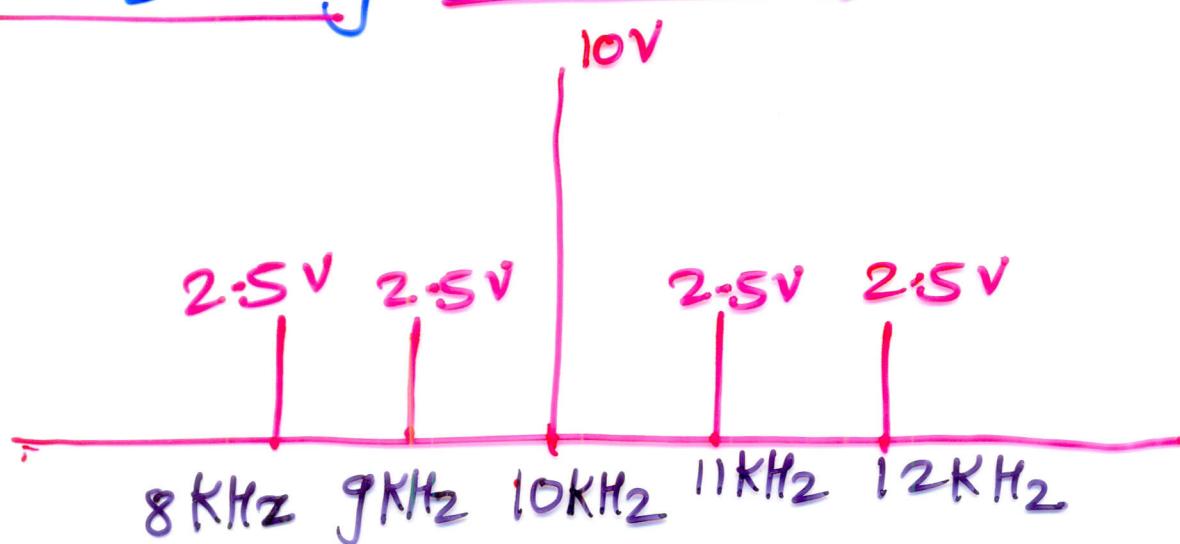
$$LSB_2 = f_c - f_{m2} = (10 - 2) \text{ kHz}$$

$$\boxed{LSB_2 = 8 \text{ kHz}}$$

amplitude of sidebands = $\frac{mV_c}{2}$

$$= \frac{0.5 \times 10}{2} = 2.5 \text{ V}$$

frequency spectrum



$$\text{Carrier Power } P_c = \frac{V_c^2}{2R} = \frac{100}{2 \times 1}$$

$$\boxed{P_c = 50 \text{ W}}$$

$$P_{LSB_1} = P_{LSB_2} = P_{USB_1} = P_{USB_2} = \frac{m^2}{4} P_c$$

$$= \frac{(0.5)^2}{4} \times 50 = 3.125 \text{ W}$$

(8)

Total Power

$$P_T = P_C \left(1 + \frac{m_t^2}{2} \right)$$

$$m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{(0.5)^2 + (0.5)^2} \\ = \sqrt{0.5}$$

$$\boxed{m_t = 0.707}$$

$$\therefore P_T = 50 \left(1 + \frac{(0.707)^2}{2} \right) = 50 \left(1 + \frac{0.5}{2} \right)$$

$$\boxed{P_T = 62.5W}$$

Total sideband power = $P_T - P_C$

$$= 62.5 - 50$$

$$\boxed{P_{SBT} = 12.5W}$$

or

$$P_{SBT} = P_C \frac{m_t^2}{2}$$

$$= 50 \times \frac{0.5}{2} = 12.5W$$

$$P_{SBT} \stackrel{\text{or}}{=} P_{LSB1} + P_{USB1} + P_{LSB2} + P_{USB2} \\ = 12.5W$$

Draw the spectrum of amplitude modulated waveform, if the modulating signal is

$$m(t) = \cos 2000\pi t + 0.5 \cos 4000\pi t$$

and carrier is

$$c(t) = 1.5 \cos 10,000\pi t$$

Also calculate the total and sideband power.

Sol^N →

$$V_c = 1.5 \text{ Volt}$$

$$V_{m_1} = 1 \text{ Volt}$$

$$V_{m_2} = 0.5 \text{ Volt}$$

$$\textcircled{1} m_1 = \frac{V_{m_1}}{V_c} = \frac{1}{1.5} = 0.67 \quad m_1 = 0.67$$

$$\textcircled{2} m_2 = \frac{V_{m_2}}{V_c} = \frac{0.5}{1.5} = 0.33 \quad m_2 = 0.33$$

$$\textcircled{3} m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{(0.67)^2 + (0.33)^2}$$

$$m_t = 0.745355992 \approx 0.75$$

$$m_t = 0.75$$

④ $P_{LSB,1} = P_{USB,1}$ for modulating signal 1

$$= \frac{P_c m_1^2}{4}$$

$$P_c = \frac{V_c^2}{2R} = \frac{(1.5)^2}{2 \times 1} = 1.125 \text{ watt}$$

$$P_c = 1.125 \text{ watt}$$

$$P_{LSB1} + P_{USB1} = \frac{P_c m_1^2}{2} = 1.125 \times (0.667)^2 \\ = 0.24999$$
(10)

$P_{LSB1} + P_{USB1} = 0.25$

$P_{LSB1} = P_{USB1} = \frac{0.25}{2} = 0.125$

$$P_{USB2} = P_{LSB2} = \frac{P_c m_2^2}{4} = \frac{1.125 \times (0.33)^2}{2}$$

$P_{USB2} = P_{LSB2} = 0.03125 \text{ W}$

$P_{LSB2} + P_{USB2} = 0.0625 \text{ watt}$

Total sideband band power

$$= 0.25 + 0.0625$$

$P_{SBT} = 0.3125 \text{ watt}$

$$P_T = P_c + P_{SBT} = 1.125 + 0.3125$$

$P_T = 1.4375 \text{ W}$