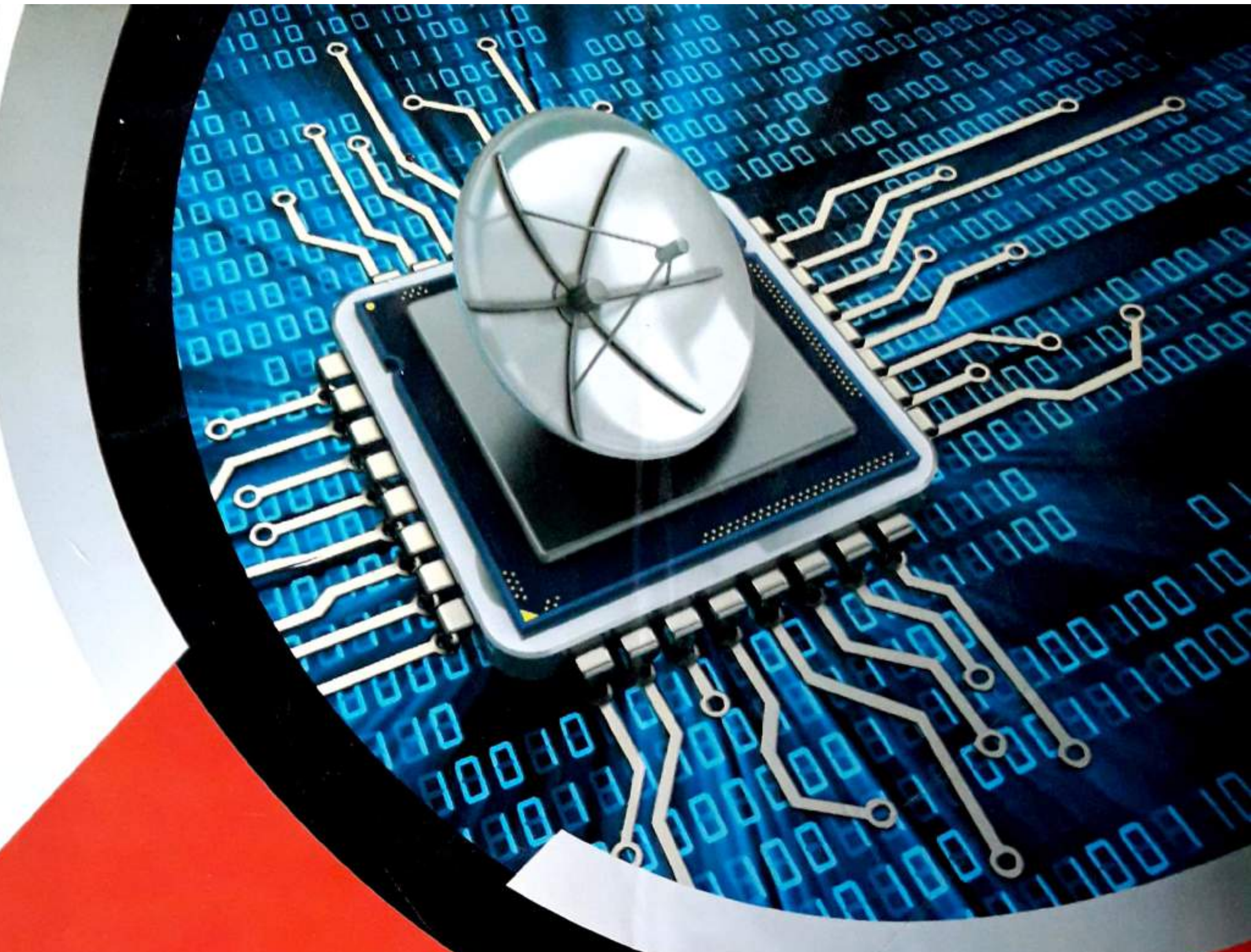

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ELECTRONIC CIRCUITS & COMMUNICATION FUNDAMENTALS

(Code : CSC304)

Semester III – Computer Engineering

J. S. Katre

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(Code : CSC304)

Semester III – Computer Engineering
(Mumbai University)

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J. S. Katre

M.E. (Electronics and Telecommunication)
Formerly, Assistant Professor
Department of Electronics Engineering
Vishwakarma Institute of Technology (V.I.T.), Pune.
Maharashtra, India

Tech-Max Publications, Pune
Innovation Throughout
Engineering Division

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(Semester III, Computer Engineering, Mumbai University)
J. S. Katre

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Preface

Dear students,

I am extremely happy to present the book of **"Electronic Circuits and Communication Fundamentals"** for you. I have divided the subject into small chapters so that the topics can be arranged and understood properly. The topics within the chapters have been arranged in a proper sequence to ensure smooth flow of the subject.

A large number of solved examples have been included. So, I am sure that this book will cater all your needs for this subject.

I am thankful to Shri. Pradeep Lunawat and Shri. Sachin Shah for the encouragement and support that they have extended. I am also thankful to the staff members of Tech-Max Publications and others for their efforts to make this book as good as it is. We have jointly made every possible efforts to eliminate all the errors in this book. However if you find any, please let us know, because that will help me to improve further.

I am also thankful to my family members and friends for their patience and encouragement.

- J. S. Katre

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Syllabus...

Electronic Circuits and Communication Fundamentals :
Sem. III, (Computer Engineering, (MU))

Course Code	Course Name	Credits
CSC304	Electronic Circuits and Communication Fundamentals	4

Course Objectives :

1. To develop the knowledge of semiconductor devices and circuits, and explain their use in communication applications.
2. To inculcate circuit analysis capabilities in students.
3. To gain knowledge in electronic devices and circuits that is useful in real life applications.
4. To understand the fundamental concepts of electronic communication and their use in computer applications.

Course Outcomes : At the end of the course student should be able :

1. To understand the use of semiconductor devices in circuits and analyze them.
2. To understand importance of oscillators and power amplifiers in communication system.
3. To understand basic concepts of operational amplifier and their applications.
4. To understand the fundamental concepts of electronic communication.
5. To apply knowledge of electronic devices and circuits to communication applications.
6. To study basic concepts of information theory.

Prerequisite : Basic Electrical Engineering

Module 1

Electronic Circuits : Bipolar junction transistor :

Input and Output characteristics, Types of Biasing - Fixed bias, self-bias, voltage divider bias, DC load line and significance, CE amplifier using r_e model, (Analysis based Numericals). **(Refer Chapter 1)**

Module 2

Power Amplifiers :

Introduction, Class A and Class C power amplifier, Oscillators : Introduction, Barkhausen criteria, Colpitts oscillator and Crystal oscillator. **(Refer Chapter 2)**

Module 3

Electronic Circuits : Operational Amplifier and its applications :

Op-amp – block diagram, parameters and characteristics, applications - Inverting and Non inverting amplifier, Summing Amplifier (Numerical), Difference amplifier, Basic Integrator and Differentiator, Comparator, Zero Crossing Detector (only theory). **(Refer Chapter 3)**

Module 4

Communication Fundamentals : Analog Communication :

Block diagram and elements of analog communication systems, Theory of amplitude modulation and types of AM (Numerical), Generation of DSB SC using diode based balanced modulator, Generation of SSB using phase shift method, Introduction of FM, and its mathematical representation, Statement of Carson's Rule, Comparison of AM, FM, Block diagram of AM transmitter (HLM and LLM), Block diagram of AM Superheterodyne receiver. **(Refer Chapter 4)**

Module 5

Pulse Modulation and Multiplexing :

Statement of Sampling Theorem, Generation and detection of PAM, PWM, PPM, PCM, DM and ADM, Principle of TDM using PCM and FDM. **(Refer Chapters 5, 6 and 7)**

Module 6

Communication Fundamentals : Information theory :

Amount of information, average information, information rate, Statement of Shannon's theorem, channel capacity (Numericals). **(Refer Chapter 8)**

❑❑❑

Module 1

Chapter 1 : Electronic Circuits : Bipolar Junction Transistor 1-1 to 1-

Module 2

Chapter 2 : Power Amplifiers 2-1 to 2-21

Module 3

Chapter 3 : Operational Amplifier & its Applications 3-1 to 3-72

Module 5

Chapter 6 : Digital Pulse Modulation 6-1 to 6-24

Module 5

Chapter 7 : Multiplexing 7-1 to 7-19

Module 6

□□□

CHAPTER 1

Module 1

Electronic Circuits : Bipolar Junction Transistor

Syllabus :

Input and output characteristics, Types of Biasing – Fixed bias, Self bias, Voltage divider bias, DC load line and significance, CE amplifier using r_e model (Analysis based numericals).

1.1 Introduction :

- The semiconductor device like a diode cannot amplify a signal, therefore its application area is limited.
- A next logical step in the development of semiconductor devices after diode is a Bipolar Junction Transistor (BJT).
- Transistor is a three terminal device. The terminals are collector, emitter and base, out of which base is a control terminal.
- A signal of small amplitude applied to the base is available in the "magnified" form at the collector of the transistor. This is the "amplification" provided by a BJT. Thus a large power signal is obtained from a small power signal.
- The additional power required for this operation is obtained from an external source of dc power.
- BJT is the basic building block of almost all the electronic circuits right from a simple regulator or oscillator circuit, logic gates to a digital computer.
- Before the invention of a transistor, vacuum tubes were being used for amplification applications.

1.1.1 Advantages of a Transistor :

The transistors are more desirable than the vacuum tubes because of their following advantages :

1. Small size light weight and ruggedness.
2. Do not require any filament power.
3. Operate at a low voltage.
4. Higher efficiency.
5. Long life.

1.1.2 Why is it called as a Transistor ?

The term "transistor" was derived from the words TRANSFER and RESISTOR. This term was adopted because it best describes the operation of a transistor, which is the transfer of an input signal current from a low resistance circuit to a high resistance circuit.

1.1.3 Why is it called a "Bipolar" Transistor ?

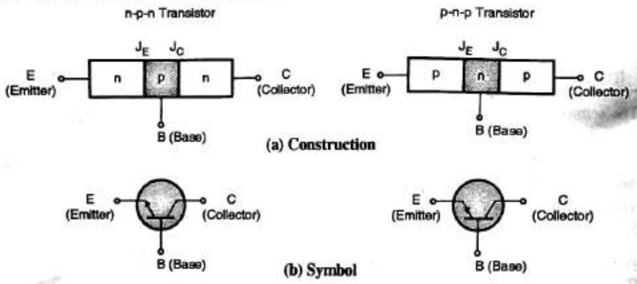
- The conduction in a bipolar junction transistor takes place due to both, electrons and holes. That is why it is called as a "bipolar" transistor.
- If the conduction takes place due to only one type of carriers i.e. majority carriers then the transistor is called as "unipolar" transistor.
- The example of a unipolar device is the Field Effect Transistor (FET).

1.1.4 Types of Transistors :

The bipolar junction transistors are of two types :

1. p-n-p transistors 2. n-p-n transistors.

- The symbols of the p-n-p and n-p-n transistors are as shown in Fig. 1.1.1. The n-p-n transistors are more popular than the p-n-p transistors.
- The arrow is always placed on the emitter terminal and the arrow direction indicates the direction of conventional current flow of emitter current.



(B-163) **Fig. 1.1.1 : Construction and symbols of transistors**

1.2 Construction of a BJT :

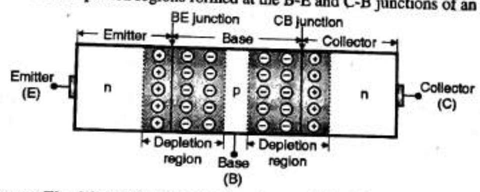
The construction and symbols of the p-n-p and n-p-n transistors is as shown in Fig. 1.2.1(a) and (b) respectively.

The important points about the construction of a transistor are as follows :

- The n-p-n transistor is formed by sandwiching a thin "p" type semiconductor between two "n" type semiconductors whereas a p-n-p transistor is formed by sandwiching a thin "n" type semiconductor between two p type semiconductors.
- In both the types, base comes in between collector and emitter region.
- Base is always a thin and lightly doped layer.
- Emitter and collector layers are much wider than the base and are heavily doped. To be precise, the emitter is the most heavily doped layer because it has to emit or inject electrons and the collector area is slightly larger than the emitter area. The collector area is largest because it is required to dissipate more heat.
- The transistor has two p-n junctions namely the collector base junction and base emitter junction.

1.2.1 The Unbiased Transistor :

- For an unbiased transistor no external power supplies are connected to it.
- We have already seen the formation of a depletion region in an unbiased p-n junction diode.
- As a transistor is formed of two p-n junctions, we can apply the same concept over here as well.
- Fig. 1.2.1 shows the depletion regions formed at the B-E and C-B junctions of an n-p-n transistor.



(B-165) **Fig. 1.2.1 : Depletion regions in an unbiased n-p-n transistor**

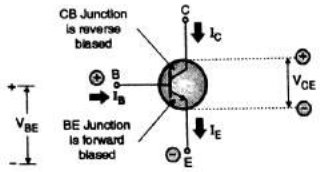
1.3 Transistor Biasing in the Active Region :

- Biasing is the process of applying external voltages to the transistor. The two junctions in a BJT must be biased properly in order to operate it as an amplifier. A transistor cannot work unless external DC power supplies are connected to it.
- A BJT is capable of operating in four different regions, depending on the way in which it is biased. The regions of operation are :
  1. Cutoff region (transistor is OFF)
  2. Saturation region (transistor is fully ON)
  3. Forward active region (in between saturation and cutoff). 4. Inverse active mode
- The biasing conditions for these four regions of operation are as shown in Table 1.3.1.

Table 1.3.1 : Biasing conditions for different regions of operation

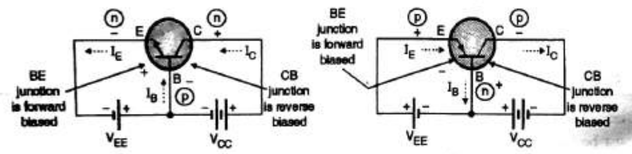
Sr. No.	Region of operation	Base emitter junction	Collector base junction	Application
1.	Cutoff region	Reverse biased	Reverse biased	(B-1619)
2.	Forward active region	Forward biased	Reverse biased	Amplifier
3.	Saturation region	Forward biased	Forward biased	(B-1620)
4.	Inverse active	Reverse biased	Forward biased	

- The biasing of transistor junction for active region are shown in Fig. 1.3.1. A transistor should be biased in the forward active region so as to operate it as an amplifier.
- Fig. 1.3.1 also indicates the conventional directions of the currents I_C, I_B and I_E plus the polarities of voltages V_{BE} and V_{CE}.



(B-166) **Fig. 1.3.1 : Transistor biasing for active region**

- **In order to use the transistor as an amplifier, it must be operated in its active region.** The biasing of the p-n-p and n-p-n transistors for their active region operation and the directions of the currents are as shown in Fig. 1.3.2.



Biasing of n-p-n transistor Biasing of p-n-p transistor

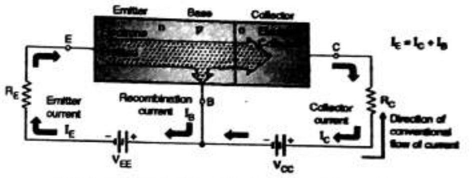
(B-167) **Fig. 1.3.2 : Transistor biasing in the active region**

1.4 The Biased Transistor (Operation of Transistor) :

1.4.1 Operation of n-p-n Transistor :

The operation of n-p-n transistor discussed earlier can be simplified as follows :

- The forward bias at the B-E junction reduces the barrier potential and causes the electrons to flow from n-type emitter to p-type base.
- Holes also will flow from p-type base to n-type emitter. But as the base is more lightly doped than the emitter, almost all the current flowing across the B-E junction consists of electrons entering the base from the emitter. Hence electrons are the majority carriers in an n-p-n transistor.
- Some of the electrons entering into the base region do not reach the collector region. Instead they flow out of the base terminal via the base connection as shown in Fig. 1.4.1, due to recombination. As the base region is very thin and lightly doped, there are very few holes available in the base region for recombination.
- Hence about 2% electrons will flow out of base due to recombination.
- The remaining 98% electrons cross the reverse biased collector junction to constitute the collector current. They cross the collector region and are collected by the supply V_{CC}.



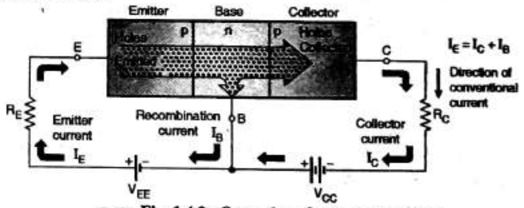
(B-171) **Fig. 1.4.1 : Operation of an n-p-n transistor**

- The emitter current is equal to the sum of collector and base currents.

$$\therefore \quad I_E = I_C + I_B$$

1.4.2 Operation of p-n-p Transistor in the Active Region :

- The p-n-p transistor behaves exactly in the same way as the n-p-n device.
- The only difference is, the majority charge carriers are holes instead of electrons.
- As shown in Fig. 1.4.2 holes are emitted from the p-type emitter across the forward biased EB junction, into the base.



(B-172) **Fig. 1.4.2 : Operation of a p-n-p transistor**

- In the lightly doped base there are very few number of electrons available for recombination.
- Therefore about 2% of total emitted holes will flow out via the base terminal and the remaining are drawn across the collector by the electric field at the reverse biased collector junction.
- As in case of n-p-n transistor, the forward bias at the EB junction controls the collector and emitter currents.

1.5 Transistor Currents :

- As discussed earlier, the electrons injected from emitter into the base constitute the emitter current (I_E).
- Out of these electrons very few will combine with the holes in the thin base region to constitute the base current (I_B).
- The remaining electrons pass through to the collector region and then to the positive end of V_{CC} to constitute the collector current (I_C).

- Therefore we can write that,

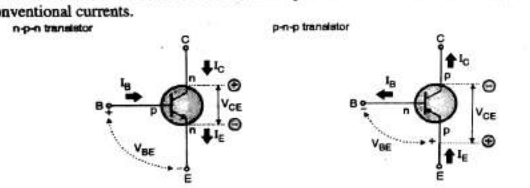
$$I_E = I_C + I_B \qquad ...(1.5.1)$$

- That means, emitter current is always equal to the sum of collector current and base current.
- As I_B is very small compared to I_E we can assume the collector current to be nearly equal to the emitter current.

$$\therefore \; I_C \approx I_E \qquad ...(1.5.2)$$

1.5.1 Circuit Symbols and Conventions :

- The block diagram and conventional circuit symbols of npn and pnp bipolar transistors are as shown in Fig. 1.5.1.
- The arrowhead is always placed on the emitter terminal and it indicates the direction of flow of the conventional emitter current.
- For the npn transistor the emitter current flows out of the emitter terminal whereas for the pnp transistor the emitter current flows into the emitter terminal.
- The current direction for all the other current components have been shown. Note that all these are the conventional currents.



(a) n-p-n Transistor (b) p-n-p Transistor

(B-172(a)) **Fig. 1.5.1 : Circuit symbols and conventions of a bipolar transistor**

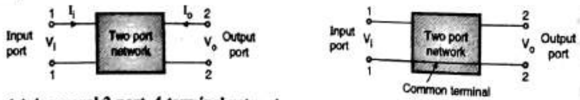
1.6 Transistor Construction :

There are five basic techniques developed to manufacture diodes, transistors and other semiconductor devices. Hence such devices can be classified into one of the following types :

1. Grown type.
2. Alloy type.
3. Electrochemically etched type.
4. Diffusion type.
5. Epitaxial type.

1.7 Transistor Configurations :

- A general two port network is as shown in Fig. 1.7.1(a).
- This network has an input port and an output port. Therefore the total number of terminals are four. But the transistor does not have four terminals, it has only three. Then how to represent it as a two port device ?



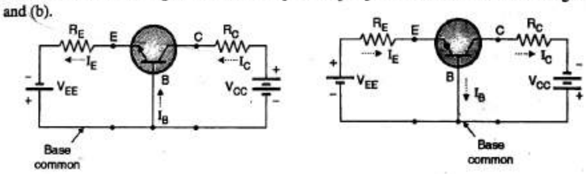
(a) A general 2-port, 4-terminal network (b) A two port 3-terminal device

(B-174) **Fig. 1.7.1**

- It is possible to do so by treating one of the three terminals "common" to input and output ports as shown in Fig. 1.7.1(b).
- The common terminal can be base, emitter or collector.
- Depending on which terminal is made common to input and output port there are three possible configurations of the transistor. They are as follows :
  1. Common Base (CB) configuration
  2. Common Emitter (CE) configuration
  3. Common Collector (CC) configuration
- In the following sections we are going to analyze these three configurations one by one.
- Before we start discussing about the configurations, we must note one important point in our mind. In all the configurations, the emitter base junction is forward biased and collector base junction is reverse biased to operate the transistor in the active region.

1.8 Common Base Configuration :

The common base configuration for the n-p-n and p-n-p transistors is as shown in Fig. 1.8.1(a) and (b).



(a) Common base configuration for n-p-n transistor (b) Common base configuration for p-n-p transistor

(B-175) **Fig. 1.8.1**

The important points about the CB configuration are as follows :

- The input is applied between emitter and base. The base acts as the common terminal between the input and output ports. The input voltage is therefore V_{EB} and the input current is I_E.
- The output is taken between collector and base. Therefore the output voltage is V_{CB} and the output current is I_C.

1.8.1 Current Relations in CB Configuration :

1. The collector current I_C of the common base configuration is given by,

$$I_C = I_{C(INJ)} + I_{CBO} \qquad ...(1.8.1)$$

Let us understand the meaning of each term.

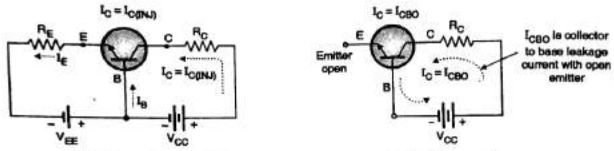
2. $I_{C\,(INJ)}$: It is called as the injected collector current and it is due to the number of electrons crossing the collector base junction, as discussed in the transistor operation earlier.
3. I_{CBO} : This is the reverse saturation current flowing due to the minority carriers between collector and base when the emitter is open. I_{CBO} flows due to the reverse biased collector base junction. As I_{CBO} is negligible as compared to $I_{C\,(INJ)}$ we can neglect it in practice.

$$I_C = I_{C\,(INJ)} \quad .....\text{(practically)} \qquad ...(1.8.2)$$

and

$$I_C = I_{CBO} \quad .....\text{(with emitter open)} \qquad ...(1.8.3)$$

This is shown in Fig. 1.8.2(a) and (b).



(a) Normal operation **(b) With emitter open**

(B-176) **Fig. 1.8.2 : Components of collector current in CB configuration**

Note that the name I_{CBO} can be explained as :

I_{CB} O
→ Emitter is open
→ Collector to base current (B-177)

- Since I_{CBO} flows due to thermally generated minority carriers, it increases with increase in temperature. It doubles its value for every 10°C rise in temperature.

4. Current amplification factor or current gain (α_{dc}) :

- The current amplification factor is the ratio of output current to the total input current. For a CB configuration, we have $I_{C(INJ)}$ as output current and I_E as total input current.

$$\therefore \quad \alpha_{dc} = \frac{I_{C(INJ)}}{I_E} \qquad ...(1.8.4)$$

- The value of α_{dc} for CB configuration will always be less than 1. This is because $I_{C(INJ)} < I_E$.
- Typically the value of α_{dc} ranges between 0.95 to 0.995 depending on the thickness of the base region.
- Larger the thickness of the base is, smaller is the value of α_{dc}.
- From Equation (1.8.4) we write,

$$I_{C(INJ)} = \alpha_{dc} I_E \qquad ...(1.8.5)$$

- Hence the expression for I_C is given by,

$$I_C = \alpha_{dc} I_E + I_{CBO} \qquad ...(1.8.6)$$

- But I_{CBO} is negligibly small.

$$\therefore I_C = \alpha_{dc} I_E \qquad ...(1.8.7)$$

$\therefore$ Current amplification factor or current gain $\alpha_{dc} = \frac{I_C}{I_E}$ ...(1.8.8)

5. Expression for I_B :

We know that , $I_E = I_C + I_B$

Substituting for I_C we get,

$$I_E = (\alpha_{dc} I_E + I_{CBO}) + I_B$$
$$\therefore \quad I_B = (1 - \alpha_{dc}) I_E - I_{CBO}$$

Neglecting I_{CBO} we get,

$$\therefore I_B = (1 - \alpha_{dc}) I_E \qquad ...(1.8.9)$$

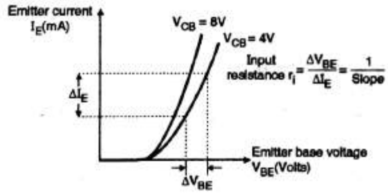
1.8.2 Characteristics of a Transistor in Common Base Configuration :

- The characteristics of a transistor help us to understand its behaviour. The transistor characteristics are of three types :

1. Input characteristics 2. Output characteristics. 3. Transfer characteristics

Input characteristics :

- Input characteristics is always a graph of input current versus input voltage. For the CB configuration, input current is the emitter current (I_E) and input voltage is the emitter to base voltage (V_{BE}).
- The input characteristic is plotted at a constant output voltage V_{CB}. The input characteristics of an npn transistor in CB configuration is as shown in Fig. 1.8.3.
- The emitter base voltage (V_{BE}) is plotted on the X-axis and emitter current I_E is plotted on the Y axis.



(B-179) **Fig. 1.8.3 : Input characteristics of transistor in CB configuration**

- The important observations from the input characteristics of an npn transistor in CB configuration are as follows :

1. The input characteristics is identical to the forward I-V characteristic of a p-n junction diode. This is because there exists a p-n junction between the emitter and base of a transistor.
2. Upto the cut-in voltage, the emitter current is negligible but after the cut-in voltage it increases rapidly with a small increase in the input voltage V_{BE}.
3. The input resistance "r_i" of the transistor in CB configuration is defined as :

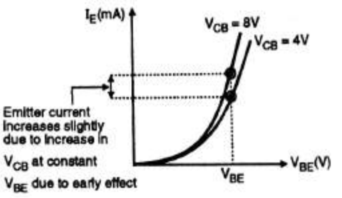
$$r_i = \left.\frac{\Delta V_{BE}}{\Delta I_E}\right|_{constant\ V_{CB}} \qquad ...(1.8.10)$$

Input resistance can be obtained from the input characteristics. It is equal to the reciprocal of the slope of input characteristics in the linear portion of the characteristic.

$$\therefore \quad r_i = \frac{1}{Slope} \qquad ...(1.8.11)$$

As the change in emitter current (ΔI_E) is very large for a small change in input voltage (ΔV_{BE}), the input resistance "r_i" is small.

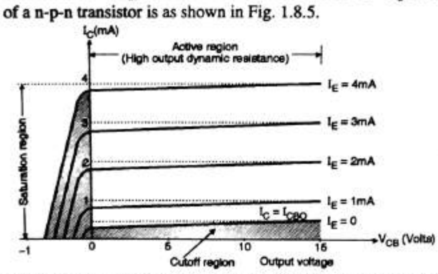
4. **Effect of V_{CB} (output voltage) on the input characteristic :** As shown in Fig. 1.8.4, the emitter current increases slightly with increase in the output voltage V_{CB}. This happens due to a special phenomenon called "Early Effect".



(B-180) Fig. 1.8.4

Output characteristics of transistor in CB configuration :

- Output characteristics is always a graph of output current versus output voltage. For the CB configuration, the output current is collector current (I_C) and the output voltage is collector to base voltage (V_{CB}).
- The output characteristics is plotted for a constant value of input current (I_E). Output characteristics of a n-p-n transistor is as shown in Fig. 1.8.5.



(B-181) **Fig. 1.8.5 : Output characteristics of a npn transistor in CB configuration**

- The important observations from Fig. 1.8.5 are as follows :

1. A transistor can operate in any of three regions of operation.
   1. Cutoff region (transistor off)
   2. Active region and
   3. Saturation region
2. The biasing of the two junctions of a transistor is done as follows :

Region of operation	Emitter base junction	Collector base junction
Cutoff	Reverse biased	Reverse biased
Active	Forward biased	Reverse biased
Saturation	Forward biased	Forward biased

These regions have been shown in Fig. 1.8.5.

3. **Cutoff region :** The region below the curve for $I_E = 0$ in Fig. 1.8.5 is called as cutoff region. The input current $I_E = 0$ and the transistor is in its off state. The output current $I_C = I_{CBO}$ which is very small in magnitude.

4. **Active region :** The collector current I_C is almost equal to the emitter current I_E and it almost remains constant if I_B is held constant. That means if I_E is constant then I_C remains constant irrespective of the variation in the output voltage V_{CB}. Therefore the transistor is said to operate as a "constant current source". (Slight change in I_E does take place due to early effect.)

5. **Dynamic output resistance of the transistor :** The dynamic output resistance of a transistor is defined as :

$$r_o = \left.\frac{\Delta V_{CB}}{\Delta I_C}\right|_{I_E \text{ Constant}} \quad ...(1.8.12)$$

This is nothing but the reciprocal of slope of the output characteristics in the active region. Slope of the output characteristics in the active region is very small.

Therefore the dynamic resistance (r_o) in the active region is large. That is why the voltage drop across a transistor (V_{CE}) is large in the active region.

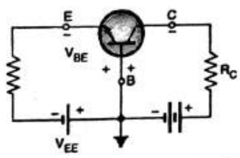
6. **Saturation region :** Both the junctions are forward biased to operate a transistor in the saturation region. Therefore the saturation region corresponds to negative values of V_{CB} as shown in Fig. 1.8.5. The collector current I_C is not constant but increases exponentially with increase in V_{CB}, towards zero. The slope of output characteristics is large in this region. Therefore the dynamic output resistance has a small value. That is why the voltage drop across the transistor (V_{CE}) is small in the saturation region.

7. In the active region, I_C does not depend on V_{CB}. It depends only on the input current I_E. That is why the transistor is called as a "current controlled" or "current operated" device.

Why V_{CB} requires to be negative for operating the transistor in the saturation region in the CB configuration ?

1. To operate a transistor in the saturation region, both the junctions need to be forward biased.
2. Refer Fig. 1.8.6 which shows biasing in CB configuration.

- Since base is common to input and output, it is treated as the reference point or ground point of the circuit.
- All the voltages are measured with respect to base. So base is a "0" potential.
- Then to forward bias the CB junction, the collector must be at – 0.7 Volts. Hence saturation region corresponds to negative values of V_{CB}.



(B-182) **Fig. 1.8.6 : Biasing of transistor in CB configuration**

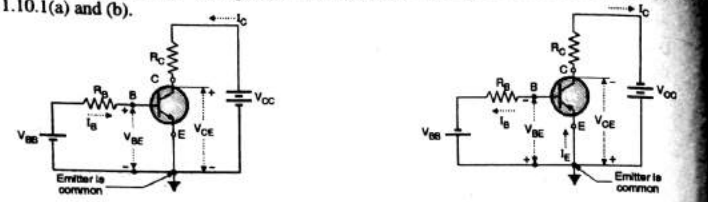
1.9 Features of CB Configuration :

Some important features of CB configuration are :

1.	Common terminal between input and output	:	Base
2.	Input current	:	I_E
3.	Output current	:	I_C
4.	Input voltage	:	V_{BE}
5.	Output voltage	:	V_{CB}
6.	Current gain	:	α_{dc} (less than 1)
7.	Voltage gain	:	Medium
8.	Input resistance	:	Very low (20 Ω)
9.	Output resistance	:	Very high (1 MΩ)
10.	Applications	:	As preamplifier

1.10 Common Emitter (CE) Configuration :

The common emitter configuration for the p-n-p and n-p-n transistors is as shown Figs. 1.10.1(a) and (b).



(a) Common emitter configuration for n-p-n transistor

(b) Common emitter configuration for p-n-p transistor

(B-185) Fig. 1.10.1

The important points about the CE configuration are as follows :

- Now the emitter acts as a common terminal between input and output. The input voltage is appli between base and emitter. Hence V_{BE} is the input voltage and I_B is the input current.
- The output is taken between the collector and emitter. Therefore V_{CE} is the output voltage and I_C the output current. In order to operate the transistor in its active region, the base-emitter (B junction is forward biased and the collector base junction is reverse biased.

1.10.1 Current Relations in CE Configuration :

- For the CB configuration we have defined the relation between the three currents as :

$$I_E = I_C + I_B$$

$$\text{where,} \quad I_C = \alpha_{dc} I_E + I_{CBO}$$

- Rearrange this equation to get,

$$I_C - I_{CBO} = \alpha_{dc} I_E$$

$$\therefore \frac{I_C}{\alpha_{dc}} - \frac{I_{CBO}}{\alpha_{dc}} = I_E = I_C + I_B$$

$$\therefore I_C\left[\frac{1}{\alpha_{dc}} - 1\right] = I_B + \frac{I_{CBO}}{\alpha_{dc}}$$

$$\therefore I_C\left[\frac{1-\alpha_{dc}}{\alpha_{dc}}\right] = I_B + \frac{I_{CBO}}{\alpha_{dc}}$$

$$\therefore I_C = I_B\left[\frac{\alpha_{dc}}{1-\alpha_{dc}}\right] + \frac{I_{CBO}}{(1-\alpha_{dc})} \qquad \text{...(1.10.}$$

- As β_{dc} is the ratio of output current I_C and input current I_B, it is called common emitter curre amplification factor or simply current gain. Thus transistor acts as current amplifier.
- We define a new term at this stage, which is called "beta" and denoted by β_{dc}. The value of β_{dc} much higher than α_{dc}.

$$\text{Let,} \quad \beta_{dc} = \left[\frac{\alpha_{dc}}{1-\alpha_{dc}}\right] \qquad \text{...(1.10.}$$

- Substitute this value in Equation (1.10.1) to get,

$$I_C = \beta_{dc} I_B + \frac{I_{CBO}}{(1-\alpha_{dc})} \qquad \text{...(1.10.3)}$$

$$\text{But,} \quad \beta_{dc} = \frac{\alpha_{dc}}{(1-\alpha_{dc})}$$

$$\therefore 1 + \beta_{dc} = \frac{\alpha_{dc}}{(1-\alpha_{dc})} + 1 = \frac{\cancel{\alpha_{dc}} + 1 - \cancel{\alpha_{dc}}}{1-\alpha_{dc}}$$

$$\therefore 1 + \beta_{dc} = \frac{1}{(1-\alpha_{dc})} \qquad \text{...(1.10.4)}$$

- Substitute this in Equation (1.10.3) to get,

$$I_C = \beta_{dc} I_B + (1+\beta_{dc}) I_{CBO} \qquad \text{...(1.10.5)}$$

- Equation (1.10.5) can be expressed as,

$$I_C = \beta_{dc} I_B + I_{CEO} \qquad \text{...(1.10.6)}$$

where I_{CEO} is the reverse saturation current for the CE configuration which is given by,

$$I_{CEO} = (1+\beta_{dc}) I_{CBO} \qquad \text{...(1.10.7)}$$

- If $\alpha_{dc} = 0.99$ then substituting this value in Equation (1.10.3) we get $\beta_{dc} = \frac{0.99}{1-0.99} = 99$. Thus β_{dc} is much higher than α_{dc}.

1.10.2 Reverse Leakage Current in CE Configuration (I_{CEO}) :

- The reverse leakage current of a transistor operating in the CE configuration is denoted by "I_{CEO}" and is defined as :

Reverse leakage current (CE configuration) :

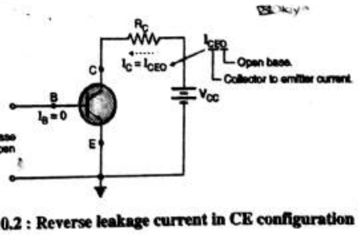
$$I_{CEO} = (1+\beta_{dc}) I_{CBO} \qquad \text{...(1.10.8)}$$

- As the value of β_{dc} is much greater than 1, $I_{CEO} >>> I_{CBO}$. In Equation (1.10.5) if we substitute $I_B = 0$ then we get,

$$I_C = (1+\beta_{dc}) I_{CBO}$$

$$\therefore I_C = I_{CEO} \text{ for } I_B = 0 \qquad \text{...(1.10.9)}$$

- The reverse leakage current (I_{CEO}) increases with increase in the temperature. This current flows in the same direction as that of I_C. Therefore the collector current I_C will increase with increase in temperature even when I_B is constant.
- Refer Fig. 1.10.2 which shows that if base is open then the collector current is equal to I_{CEO}.



(B-186) Fig. 1.10.2 : Reverse leakage current in CE configuration

- This is called as thermal instability. So in CE configuration thermal stabilizing circuit must be included.

1.10.3 Common Emitter Current Gain (β_{dc}) :

We know that, $I_C = \beta_{dc} I_B + I_{CEO}$

Though I_{CEO} is large, it is much smaller as compared to $\beta_{dc} I_B$. Therefore the equation for I_C is modified as :

$$I_C = \beta_{dc} I_B \quad ...(1.10.1$$

$$\text{or} \quad \beta_{dc} = \frac{I_C}{I_B} = \frac{\text{Output current}}{\text{Input current}} \quad ...(1.10.1$$

As β_{dc} is the ratio of output current I_C and input current I_B, it is called common emitter current amplification factor or simply current gain. Thus transistor acts as current amplifier.

1.10.4 Relation between α_{dc} and β_{dc} :

- We know that $\alpha_{dc} = \frac{I_C}{I_E}$

$$\text{But} \quad I_E = I_C + I_B$$

$$\therefore \quad \alpha_{dc} = \frac{I_C}{I_C + I_B}$$

- Divide numerator and denominator by I_B to get,

$$\alpha_{dc} = \frac{(I_C / I_B)}{1 + (I_C / I_B)}$$

$$\text{But} \quad (I_C / I_B) = \beta_{dc}$$

$$\therefore \quad \alpha_{dc} = \frac{\beta_{dc}}{1 + \beta_{dc}} \quad ...(1.10.12)$$

- This is the relation between α_{dc} and β_{dc}. Similarly we can obtain the expression for β_{dc} in terms of α_{dc} as follows :

$$\text{We know that,} \quad \beta_{dc} = \frac{I_C}{I_B}$$

$$\text{But} \quad I_B = I_E - I_C$$

$$\therefore \quad \beta_{dc} = \frac{I_C}{(I_E - I_C)}$$

- Divide numerator and denominator by I_E to get,

$$\beta_{dc} = \frac{(I_C / I_E)}{1 - (I_C - I_E)}$$

$$\text{But} \quad (I_C / I_E) = \alpha_{dc}$$

$$\therefore \quad \beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} \quad ...(1.10.13)$$

- Thus the relations between α_{dc} and β_{dc} are,

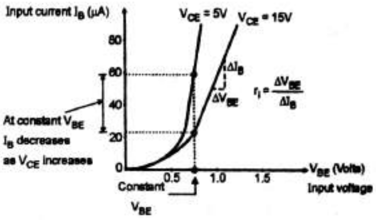
$$\alpha_{dc} = \frac{\beta_{dc}}{1 + \beta_{dc}} \quad \text{and} \quad \beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}}$$

Characteristics of a transistor in common emitter configuration :

Similarly for the common base characteristics, we will plot the input and output characteristics of a transistor.

1.10.5 Input Characteristics (The Base Curve) :

- It is a graph of input current I_B versus input voltage V_{BE} at a constant output voltage (V_{CE}).
- For CE configuration, I_B is the input current and V_{BE} is the input voltage.
- At constant output voltage V_{CE} the input characteristics of a n-p-n transistor is as shown in Fig. 1.10.3. The input characteristics also shows the effect of V_{CE}.



(B-187) **Fig. 1.10.3 : Input characteristics of a transistor in the CE configuration**

The important points about the input characteristics are as follows :

- The input characteristics resembles the forward characteristics of a p-n junction diode. The reason is that B-E junction is a forward biased p-n junction.
- The base current increases rapidly as the base-emitter voltage crosses the cut in voltage of the BE, p-n junction. The dynamic input resistance is defined as :

$$r_i = \left. \frac{\Delta V_{BE}}{\Delta I_B} \right|_{V_{CE\ constant}} \quad ...(1.10.14)$$

- Its value can be obtained from the input characteristics because "r_i" is equal to the reciprocal of slope of the input characteristics.
- The value of dynamic input resistance "r_i" is low (typically 1 kΩ) for the CE configuration but it is not as low as that of CB configuration.

Effect of change in V_{CE} on the input characteristics :

- In CB configuration, the effect of change in V_{CB} on the input characteristics is known as the Early effect.
- Let us see the effect of change in V_{CE} on the input characteristics.
- Fig. 1.10.3 shows that at a constant V_{BB}, if we increase V_{CE} from 5 V to 15 V then the base current decreases from 60 μA to 20 μA.

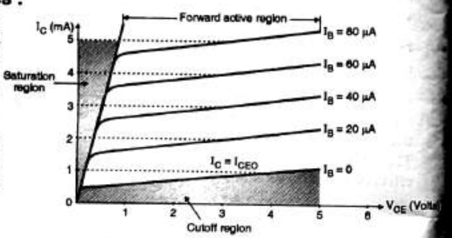
- Thus I_B decreases with increase in V_{CE}. We can explain this as follows.

As V_{CE} increase the CB junction is more reverse biased.
↓
The depletion region at CB junction penetrates more into base.
↓
This reduces the electrical width of the base.
↓
The chances of recombination in the base region reduce.
↓
Hence the base current will reduce.

(B-1109)

1.10.6 Output Characteristics :

- An output characteristics of a CE configuration is the graph of output current (I_C) versus output voltage (V_{CE}) for various fixed values of the input current (I_B).
- The typical output characteristics of a n-p-n transistor operating in the CE configuration are as shown in Fig. 1.10.4.



(B-188) **Fig. 1.10.4 : Output characteristics of a n-p-n transistor in CE configuration**

- As shown in Fig. 1.10.4, there are three regions of operation namely the cutoff region, active region and saturation region.

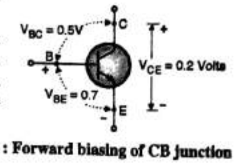
1. **Cutoff region :**

   Both the B-E and C-B junctions are reverse biased to operate the transistor in cutoff region. The base current I_B = 0 and the collector current is equal to the reverse leakage current I_{CEO}. The region below the characteristics for I_B = 0 is cutoff region.

2. **Active region :**
   - The B-E junction is forward biased, and C-B junction is reverse biased to operate the transistor in the active region.
   - At a constant base current I_B, the collector current I_C increases slightly with increase in the voltage V_{CE}. However the collector current is largely dependent on the base current I_B.
   - At a fixed value of V_{CE}, if I_B is increased, then it will cause I_C to increase substantially.
   - This is because $I_C = \beta_{dc} I_B$. This relation is true only for the active region of operation.

3. **Saturation region :**
   - The BE junction as well as the collector junction must be forward biased to operate the transistor in its saturation region.
   - The collector base junction can be forward biased if and only if V_{CE} drops down to about 0.2 Volts. Because then V_{BE} = 0.7 V will forward bias the C-B junction.

   - This is as shown in Fig. 1.10.5. Usually the saturation voltage of a transistor, $V_{CE\,(sat)}$ is between 0.1 to 0.3 Volts.
   - The collector current increases rapidly with increase in V_{CE} as shown in Fig. 1.10.4.
   - Note that in this region I_C is approximately independent of the base current and function of V_{CE}. Therefore in this region the transistor is considered to be a semiconductor resistor of very small value. The transistor is operated as a switch in this region.



(B-189) **Fig. 1.10.5 : Forward biasing of CB junction**

4. **Dynamic output resistance (r_o) :**
   - The dynamic output resistance (r_o) of a transistor in CE configuration is defined as :

$$r_o = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{\text{constant } I_B} \qquad ...(1.10.15)$$

   - The dynamic output resistance can be obtained as reciprocal of slope of output characteristics. Its value is large in the active region because ΔI_C in this region is very small. However value of r_o will be very small in the saturation region. This is because ΔI_C in that region is large for a small value of ΔV_{CE}.
   - In the active region the typical value of r_o for the CE configuration is 10 kΩ.

5. **Definition of β_{ac} :**

   We have already defined β_{dc} as :

$$\beta_{dc} = \frac{I_C}{I_B}$$

   - The value of β_{dc} can be obtained from the output characteristics. At any point on the characteristics we can calculate β_{dc} by taking the ratio of I_C and I_B at that point.
   - Now let us define AC beta of a transistor as :

$$\beta_{ac} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}\text{ constant}} \qquad ...(1.10.16)$$

   - Thus the value of ac beta can be obtained at a constant value of V_{CE} from the output characteristics. The values of β_{dc} and β_{ac} are nearly the same.

6. **Maximum V_{CE} and breakdown :**
   - In the active region the collector junction is reverse biased, so there is a limit on the maximum value of V_{CE}.
   - If V_{CE} exceeds this maximum value, collector junction will breakdown due to the punch through effect.
   - A large current will flow which will generate excessive heat to damage the transistor. Hence for safe operation $V_{CE} < V_{CE\,(max)}$.

1.10.7 Features of CE Configuration :

1.	Common terminal	:	Emitter	6.	Current gain	:	β_{dc} High
2.	Input current	:	I_B	7.	Voltage gain	:	Medium
3.	Output current	:	I_C	8.	Input resistance	:	Moderate (1.1 kΩ)
4.	Input voltage	:	V_{BE}	9.	Output resistance	:	High (40 kΩ)
5.	Output voltage	:	V_{CE}	10.	Applications	:	As audio amplifiers

Ex. 1.10.1 : Calculate the emitter current I_E for a transistor connected in CE configuration. Given $\beta = 50$ and base current $I_B = 20\ \mu A$.

Soln. :

$$I_C = \beta I_B = 50 \times 20 \times 10^{-6} = 1\ mA.$$

$$\text{and} \quad I_E = I_C + I_B = 1mA + 20\ \mu A = 1.02\ mA \quad \text{...Ans.}$$

Ex. 1.10.2 : The collector and base current of n-p-n transistor are measured as $I_C = 5\ mA$, $I_B = 50\ \mu A$ and $I_{CBO} = 1\ \mu A$

1. Determine α, β and I_E.
2. Determine the new level of I_B required to produce $I_C = 10$ mA.

Soln. :

1. $$\beta = \frac{I_C}{I_B} = \frac{5 \times 10^{-3}}{50 \times 10^{-6}} = 100$$

2. $$\alpha = \frac{\beta}{1+\beta} = \frac{100}{1+100} = 0.99$$

3. $$I_E = I_C + I_B = 5.05\ mA.$$

4. $$I_C = \beta I_B + (1+\beta) I_{CBO}$$

$$\therefore 10 \times 10^{-3} = (100 \times I_B) + (101 \times 1 \times 10^{-6})$$

$$\therefore \text{New } I_B = 98.99\ \mu A.$$

Ex. 1.10.3 : In a CE transistor amplifier circuit, V_{CE} is increased from 2 to 12 V, the collector current changes from 3 to 4 mA, determine the output resistance.

Soln. :

Given : $\Delta V_{CE} = 12 - 2 = 10$ V and $\Delta I_C = 4 - 3 = 1$ mA.

$$\therefore \text{Output resistance } R_o = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{10}{1 \times 10^{-3}} = 10\ k\Omega \quad \text{...Ans.}$$

Ex. 1.10.4 : In an n-p-n transistor $\alpha = 0.98$, $I_E = 10$ mA, leakage current $I_{CBO} = 1\ \mu A$. Determine I_C, I_B, β, I_{CEO}.

Soln. :

Given : $\alpha = 0.98$, $I_E = 10$ mA, $I_{CBO} = 1\ \mu A$.

To find : I_C, I_B, β and I_{CEO}.

1. **Find β :**

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.98}{1-0.98} = 49 \quad \text{...Ans.}$$

2. **Find I_C :**

$$\therefore I_C = \alpha I_E + I_{CBO} = (0.98 \times 10 \times 10^{-3}) + 1 \times 10^{-6} = 9.801\ mA \quad \text{...Ans.}$$

3. **Find I_B :**

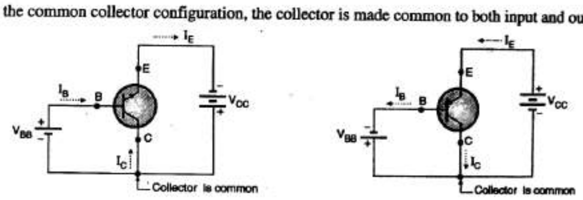
$$\therefore I_B = I_E - I_C = 10 - 9.801 = 0.199\ mA \quad \text{...Ans.}$$

4. **Find I_{CEO} :**

$$I_{CEO} = (1+\beta) I_{CBO} = (1+49) \times 1 \times 10^{-6} = 50\ \mu A \quad \text{...Ans.}$$

1.11 Common Collector (CC) Configuration :

- The Common Collector (CC) configuration for p-n-p and n-p-n transistors is as shown in Figs. 1.11.1 (a) and (b).
- In the common collector configuration, the collector is made common to both input and output.



(a) Common collector configuration for n-p-n transistor

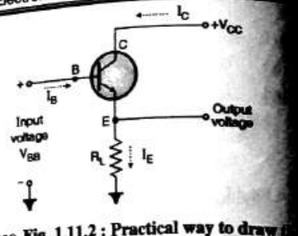
(b) Common collector configuration for p-n-p transistor

(B-195) Fig. 1.11.1

- The V_{BC} is input voltage and I_B is the input current whereas V_{EC} is the output voltage and I_E is the output current. This configuration is also known as "emitter follower" configuration.
- The common collector configuration is used primarily for impedance matching purpose because it has a high input impedance and low output impedance as compared to the C-B and C-E configurations.

1.11.1 Practical Way to Draw Common Collector Configuration :

- Fig. 1.11.1(a) does not show the practical way of representing the CC configuration. Practically it is drawn as shown in Fig. 1.11.2.



(B-196) **Fig. 1.11.2 : Practical way to draw the common collector configuration**

- As seen from Fig. 1.11.2, the common collector configuration is basically same as the Common Emitter (CE) configuration.
- The only difference between them is that the load is now connected in the emitter lead rather than in the collector terminal.
- Then how do we justify that the output is taken with respect to collector ?
- The answer is that even though the collector is connected to + V_{CC} it is a dc voltage. As far as analysis is concerned, + V_{CC} will be treated as "ac ground".
- Thus collector is connected to ground, and output is measured with respect to ground i.e. colle for all the ac analysis.

Current relations in CC configurations :

- The basic relation between the transistor currents is,

$$I_E = I_C + I_B$$

- Substitute $I_C = \alpha_{dc} I_E + I_{CBO}$ to get,

$$I_E = \alpha_{dc} I_E + I_B + I_{CBO}$$

$$\therefore \quad (1 - \alpha_{dc})\, I_E = I_B + I_{CBO}$$

$$\therefore \quad I_E = \frac{I_B}{(1 - \alpha_{dc})} = \frac{I_{CBO}}{(1 - \alpha_{dc})} \qquad ...(1.11$$

- But, $\beta_{dc} = \dfrac{\alpha_{dc}}{1 - \alpha_{dc}}$

$$\therefore \quad (1 - \alpha_{dc}) = \frac{\alpha_{dc}}{\beta_{dc}}$$

- But, $\alpha_{dc} = \dfrac{\beta_{dc}}{1 + \beta_{dc}}$

$$\therefore \quad (1 - \alpha_{dc}) = \frac{\beta_{dc}}{(1 + \beta_{dc})\, \beta_{dc}} = \frac{1}{(1 + \beta_{dc})}$$

$$\therefore \quad \frac{1}{(1 - \alpha_{dc})} = 1 + \beta_{dc}$$

- Substitute this value into Equation (1.11.1) to get,

$$I_E = (1 + \beta_{dc})\, I_B + (1 + \beta_{dc})\, I_{CBO} \qquad ...(1.1$$

- Neglecting I_{CBO} we get, $I_E = (1 + \beta_{dc})\, I_B$

$$\therefore \quad \frac{I_E}{I_B} = (1 + \beta_{dc}) \qquad ...(1.1$$

- The ratio I_E / I_B is the current gain for CC configuration.

Current gain of CC configuration : The current gain of a transistor in common collector configuration is denoted by γ (gamma) and is defined as,

$$\text{Current gain} \quad \gamma = \frac{I_E}{I_B} = \frac{I_E + I_B}{I_B}$$

$$\therefore \quad \gamma = (1 + \beta_{dc}) \qquad ...(1.11.4)$$

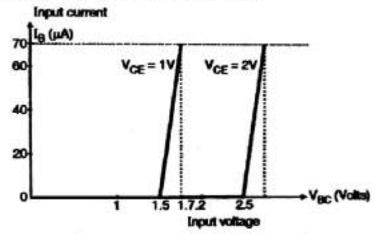
Input and output resistance of CC configuration : The CC configuration is used primarily for the impedance matching purpose, because it has a high input impedance and low output impedance which is opposite to that of CB and CE configurations.

Voltage gain of CC configuration :

In order to forward bias the base emitter junction, it is necessary that the input voltage V_{BC} must be higher than the output voltage V_{EC}. Therefore the voltage gain which is the ratio of output voltage to input voltage is always less than 1.

1.11.2 Input Characteristics of n-p-n Transistor in CC Configuration :

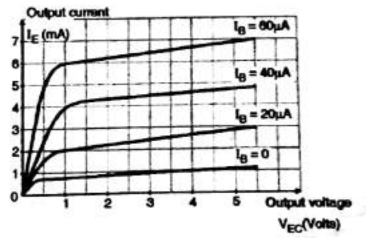
- Input characteristics of a transistor in CC configuration is the plot of input voltage V_{BC} versus the input current I_B at a constant output voltage V_{BC} = 1.5 V, as shown in Fig. 1.11.3.
- Consider the characteristic for V_{EC} = 1 V. The base-emitter junction is not forward biased upto V_{BC} = 1.5 V. Therefore the base current is zero upto V_{BC} = 1.5 V.
- Then it increases rapidly as the V_{BC} is increased beyond 1.5 V. This is because B-E junction is more and more forward biased.



(B-197) **Fig. 1.11.3 : Input characteristics of a transistor in CC configuration**

1.11.3 Output Characteristics of a n-p-n Transistor in CC Configuration :

- Output characteristics of a transistor in the CC configuration is a plot of output voltage V_{BC} versus the output current I_E for constant value of input current I_B. The set of output characteristics is as shown in Fig. 1.11.4.
- From Fig. 1.11.4, it is clear that the output characteristics of CC configuration are similar to those for the CE configuration. This is because I_C is approximately equal to I_E.



(B-198) **Fig. 1.11.4 : Output characteristics of a transistor in CC configuration**

1.11.4 Features of CC Configuration :

1. Common terminal : Collector
2. Input current : I_B
3. Output current : I_E
4. Input voltage : V_{BC}
5. Output voltage : V_{EC}
6. Current gain : γ High
7. Voltage gain : Less than 1
8. Input resistance : High (500 kΩ)
9. Output resistance : Low (50 Ω)
10. Application : As output stage, for impedance matching

1.12 Comparison of Configurations :

Table 1.12.1

Sr. No.	Parameter	CB	CE	CC
1.	Common terminal between input and output	Base	Emitter	Collector
2.	Input current	I_E	I_B	I_B
3.	Output current	I_C	I_C	I_E
4.	Current gain	$\alpha_{dc} = \frac{I_C}{I_E}$	$\beta_{dc} = \frac{I_C}{I_B}$	$\gamma = \frac{I_E}{I_B} = (1 + \beta_{dc})$
5.	Input voltage	V_{EB}	V_{BE}	V_{BC}
6.	Output voltage	V_{CB}	V_{CE}	V_{EC}
7.	Voltage gain	Medium	Medium	Less than 1
8.	Input resistance	Very low (20 Ω)	Low (1 kΩ)	High (500 kΩ)
9.	Output resistance	Very high (1 MΩ)	High (40 kΩ)	Low (50 Ω)
10.	Applications	As preamplifier	Audio amplifiers	For impedance matching.

Why is CE configuration most preferred configuration ?

Out of the three configurations discussed, the CE configuration is the most popular and wid used configuration. The reasons are as follows :

1. It has a high voltage gain as well as a high current gain.
2. As voltage gain as well as current gain are high, it has a very high power gain. This is beca power gain is the product of voltage gain and current gain.
3. The CE configuration has moderate values of R_i and R_o. Therefore many such stages ca coupled to each other without using any additional impedance matching circuits. Due to automatic impedance matching, maximum power transfer will take place from one stage t other.

1.13 Transistor as a Current Amplifier :

- When used in the CE configuration, the relation between the output current (I_C) and the i current (I_B) of a transistor is given by,

$$I_C = \beta I_B \qquad ...(1.1$$

- And in the CC configuration, the relation between the output current (I_E) and the input current (I_B) is given by,

$$I_E = (1 + \beta) I_B \qquad ...(1.13.2)$$

- The current gain is defined as,

$$\text{Current gain } A_I = \frac{\text{Output Current}}{\text{Input Current}}$$

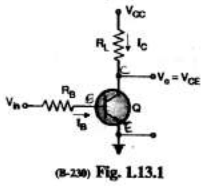
- Hence current gain of CE configuration $= \frac{I_C}{I_B} = \beta$
- And current gain of CC configuration $= \frac{I_E}{I_B} = (1 + \beta)$

As the value of β is much higher than 1, we can say that the current gain of CE and CC configuration is large. Hence for a small change in the input current we get a large change in output current. Thus current amplification takes place and the transistor acts as a current amplifier.

Note that the transistor does not act as current amplifier when used in the CB configuration.

1.13.1 BJT as a Voltage Amplifier :

- If for a small change in input voltage, a proportional large change in output voltage is obtained, then we say that voltage amplification has taken place. Let us see how BJT provides the voltage amplification.
- Consider the circuit shown in Fig. 1.13.1. Note that this is a simplified circuit used just to introduce the concept of voltage amplification.



(B-230) Fig. 1.13.1

- The transistor is operated in the CE configuration. The output voltage V_o is taken at the collector with respect to ground.

$$\therefore \quad V_o = V_{CE}$$

- Due to a small change ΔV_{in}, there will be a small change in I_B.

$$\therefore \quad \Delta I_B = \frac{\Delta V_{in}}{R_B} \qquad ...(1.13.3)$$

- Hence the corresponding change in collector current is given by,

$$\Delta I_C = \beta \Delta I_B = \beta \frac{\Delta V_{in}}{R_B}$$

- Hence the corresponding change in output voltage is given by,

$$\Delta V_o = \Delta I_C R_L \qquad ...(1.13.4)$$

Substituting the value of ΔI_C we get,

$$\Delta V_o = \beta \frac{\Delta V_{in}}{R_B} \times R_L$$

$$\therefore \quad \Delta V_o = \frac{\beta R_L}{R_B} \times \Delta V_{in} \qquad ...(1.13.5)$$

Thus for a small change in V_{in} we get a large change in V_o and the voltage amplification has taken place. Hence the BJT acts as a voltage amplifier.

1.14 Transistor Applications :

Some of the important applications of a transistor are as follows :

1. Amplifiers
2. Switching circuits
3. Oscillators
4. Waveshaping circuits
5. Logic circuits
6. Timers and multivibrator
7. Delay circuits.

1.15 Need of Biasing :

- You have already learnt the basics of a bipolar junction transistor. In this chapter we will go o step ahead to learn the techniques to connect the dc power supplies to the transistor.
- We will discuss various such techniques and analyze each one of them.

What is meant by dc biasing of a transistor ?

- We know that transistor can operate in any of the three regions of operations namely cuto active region and saturation.
- To operate the transistor in these regions the two junctions of a transistor should be forward reverse biased as shown in Table 1.15.1.

Table 1.15.1 : Regions of operation and applications

Region of operation	Base emitter junction	Collector base junction	Application
Cutoff	Reverse biased	Reverse biased	As a switch
Active	Forward biased	Reverse biased	Amplifier
Saturation	Forward biased	Forward biased	As a switch

- In order to do so, we need to connect external dc power supplies with correct polarities a magnitudes. This process is called as dc biasing of a transistor.

1.15.1 Requirements of Transistor Biasing :

A transistor biasing circuit is expected to satisfy the following requirements :

1. The transistor should be biased in the active region if it is to be used for amplification and saturation and cut off if it is to be used as a switch.
2. The Q point should be adjusted approximately at the center of the load line for voltage amplifi application.
3. The value of stability factor (S) should be as small as possible. Ideal value of stability factor is
4. Q point should be stabilized by introducing a negative feedback in the biasing circuit.
5. The Q-point should not be affected (its position should not change) due to temperature changes device to device variation.
6. Bypass capacitor should be included to avoid reduction in voltage gain due to negative feedbac
7. Transistor should be biased in the linear region of the transfer characteristics.

1.15.2 Factors to be Considered while Design a Biasing Circuit :

Following are some of the important factors to be considered while designing a biasing circuit :

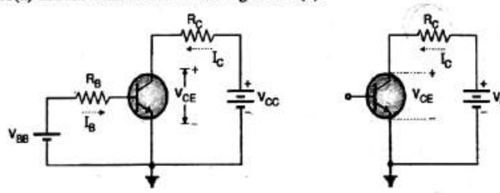
1. Position of a Q point.
2. Value of I_C (collector current) at quiescent point i.e. I_{CQ}.

3. Value of every stability factor should be as low as possible. This is essential for ensuring higher stability of the Q-point.
4. Transistor should be biased in the linear portion of its transfer characteristics.
5. Forward bias the BE junction and reverse bias the CB junction to bias the transistor in its forward active region.
6. Maximum output swing without producing any distortion.

1.16 Transistor Biasing :

1.16.1 DC Load Line :

- To understand the concept of dc load line consider the common emitter configuration of Fig. 1.16.1(a) and the collector circuit of Fig. 1.16.1(b).



(a) Common emitter configuration (b) Collector circuit

(F-282) **Fig. 1.16.1**

Procedure to plot the DC load line :

- Refer to the collector circuit of the CE configuration drawn in Fig. 1.16.1(b) and apply KVL to this circuit to write,

$$V_{CC} - V_{CE} - I_C R_C = 0 \quad ...(1.16.1)$$

- Rearranging the Equation (1.16.1) we get,

$$I_C = \left[-\frac{1}{R_C}\right] V_{CE} + \frac{V_{CC}}{R_C} \quad ...(1.16.2)$$

- Compare Equation (1.16.2) equation with the general equation of a straight line.

$$\text{i.e.} \quad y = mx + C \quad ...(1.16.3)$$

- The comparison yields the following results,

$$y = I_C \qquad x = V_{CE}$$

$$m = -1/R_C \qquad C = V_{CC}/R_C$$

- This comparison shows that Equation (1.16.3) represents a straight line. This straight line is called as the **dc load line.**

$$\underbrace{I_C}_{y} = \underbrace{[-\frac{1}{R_C}]}_{m} \underbrace{V_{CE}}_{x} + \underbrace{\frac{V_{CC}}{R_C}}_{C}$$

(B-2544)

- Now substitute $V_{CE} = 0$ in Equation (1.16.1) to get $I_C = V_{CC} / R_C$ which is $I_{C\,max}$ or point "A" in Fig. 1.16.2 and substituting $I_C = 0$ we get $V_{CE} = V_{CC}$, which represents point "B" in Fig. 1.16.2.

Why the name dc load line ?

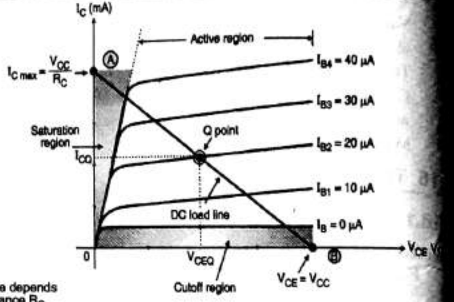
The "DC" word indicates that this line is drawn under the dc operating conditions without any ac signal at the input. And the word load line is used because the slope of this line is $-1/R_C$ where R_C is the load resistance.

(F-283)

DC — This line is drawn under DC operating conditions

Load line — The slope of this line depends on the load resistance R_C, slope $= -1/R_C$

The dc load line is drawn on the output characteristics as shown in Fig. 1.16.2.



(F-284) **Fig. 1.16.2 : DC load line showing the Q point on output characteristics of the transistor**

1.16.2 The Operating Point or Quiescent Point (Q Point) :

- The term quiescent means quiet, still or inactive. Therefore the Q point is also called as "operatin point" or "bias point". Q point is the point on the load line which represents the dc current throug a transistor (I_{CQ}) and the voltage across it (V_{CEQ}), when no ac signal is applied at the input. In sho it represents the dc bias condition. Co-ordinates of Q point are (V_{CEQ}, I_{CQ}).
- The dc load line is a set of infinite number of such operating points and the user or designer c choose any point on the dc load line as the operating point.
- The position of operating point on the load line is dependent on the application of the transist We need to design a biasing circuit so as to locate the Q point on the dc load line as per requirement of an application. If the transistor is being used for "amplification" purpose, then Q point should be exactly at the center of load line (see Q point in Fig. 1.16.2 to avoid a distortions getting introduced in the amplified ac waveform).
- The co-ordinates of the Q-point are :

$$Q = (V_{CEQ}, I_{CQ})$$

- The DC load line is actually a set of infinite Q points. Any point on the dc load line can be used the Q point by the designer of the circuit.

1.16.3 Selection of Q Point :

- Depending on the application, we can select the position of the Q point on the load line. This shown in Table 1.16.1.
- The shape of amplifier output signal depends on the position of Q point.

Table 1.16.1 : Position of Q point and application

Application	Position of Q point
Open switch	In the cut off region
Closed switch	In the saturation region
Amplifier	In the active region

ZAKIYA ZAKIYA ZAKIYA

1.16.4 Factors Affecting the Stability of Q Point :

- Ideally the Q point is expected to be "stable" i.e. should not shift up or down on the load line. But practically it is not so. In fact the Q point is quite unstable and keeps changing its position on the dc load line.
- The factors affecting the stability of Q point are :
  1. Changes in temperature.
  2. Changes in the value of β_{dc}
  3. Variations of parameters from one transistor to the other. However the Q point instability due to any reason is not desirable because it will introduce distortion in the amplified signal.

1.17 Bias Stabilization :

- Bias stabilization is a process of stabilizing the Q point (bias point) of the circuit.
- Hence we need to design a biasing circuit which will keep the position of Q point stable on the load line.
- First let us understand the effects of various parameters on the stability of Q point and then discuss various biasing circuits to counter these effects.

1.17.1 Q Point Instability due to Temperature :

- The junction temperature of a transistor is dependent on the amount of current flowing through the transistor. Due to increase in temperature the following parameters of a transistor will change : 1. V_{BE} 2. β_{dc} 3. I_{CBO}

1. **Change in V_{BE} :** The base to emitter voltage decreases at a rate of 2.5 mV/°C with increase in temperature. The base current I_B will therefore increase and it will force I_C to change, and hence the Q point will change its position.
2. **Change in current gain β_{dc} :** The current gain β_{dc} of a transistor is temperature dependent. As $I_C = \beta_{dc} I_B$, changes in β_{dc} will change the collector current I_C. This will change the position of Q point.
3. **Reverse saturation current I_{CBO} :**
   - The reverse saturation current of the reverse biased CB junction flows due to the minority carriers hence it is dependent on temperature. We know that,

$$I_C = \beta_{dc} I_B + (1 + \beta_{dc}) I_{CBO}$$

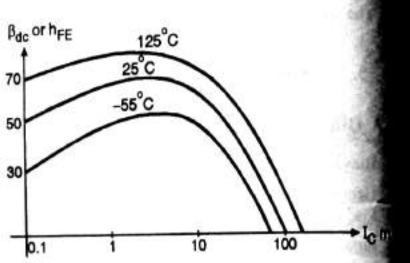
   - Therefore change in I_{CBO} due to temperature will force the collector current I_C and hence the Q point to change.
   - To overcome this problem the biasing circuit must include some kind of "temperature compensation", or "temperature stability" so that the changes in the values of these parameters can be kept under control.

1.17.2 Q Point Instability due to Changes in β_{dc} :

The current gain β_{dc} for a given type of transistor normally has a very wide range. The value of β_{dc} may typically range from 50 to 150 or more depending on the value of I_C. So if we replace a faulty transistor by a new one having the same number (say BC147) then its β_{dc} can be largely different than that of the old one. This will change Q point.

1.17.3 Variations in Current Gain (β_{dc}) :

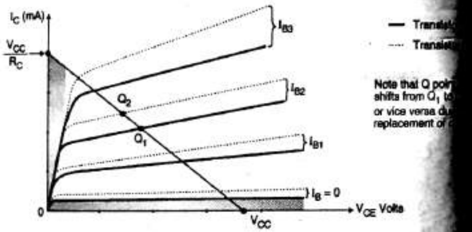
- The current gain β_{dc} of a transistor depends on three factors :
  1. The transistor 2. The collector current 3. Temperature.
- The value of β_{dc} depends on which transistor is being used. The value of β_{dc} also depends on the value of collector current and temperature as illustrated in Fig. 1.17.1.
- From Fig. 1.17.1 we conclude that at fixed value of I_C, the current gain increases with increase in temperature. And at constant temperature, the current first increases with increase in I_C, reaches maximum value and then decreases with increase in the value of I_C.



(F-3810) **Fig. 1.17.1 : Variation of current gain**

1.17.4 Q Point Instability due to Variation in Parameters from One Device to Other :

- Two transistors of identical number (e.g. BC 147) do not have exactly the same characteristics. Important parameters such as β_{dc} will differ in value from one transistor to the other.
- Hence if we replace one transistor by the other of same number, the Q point is going to get shifted.



(F-286) **Fig. 1.17.2 : Q point instability due to variation in parameters from one device to other**

- The characteristics shown in Fig. 1.17.2 explains why the Q point position varies due to dev variation. These are the output characteristics of two transistors having the same number. T dashed characteristics are for the transistor having a β_{dc} much larger than the other one.
- Initially when the first transistor with small value of β_{dc} is being used, its Q point is denoted by But if this transistor becomes faulty and hence replaced by the other transistor, then Q point gets shifted to Q_2 as shown in Fig. 1.17.2.

1.17.5 Stability Factors :

- The stability of Q point of a transistor amplifier depends on the following three parameters :
  1. Leakage current I_{CO} 2. β_{dc} 3. Base to emitter voltage
- The effect of these parameters can be expressed mathematically by defining the stability factor the three parameters individually as follows :

1. Stability factor, $$S = \left.\frac{\Delta I_C}{\Delta I_{CO}}\right|_{\text{Constant } V_{BE} \text{ and } \beta_{dc}} \text{ or } \frac{\partial I_C}{\partial I_{CO}} \quad ...(1.17.1)$$

- **This represents the change in collector current due to change in reverse saturation current I_{CO}. The other two parameters that means V_{BE} and β_{dc} are assumed to be constant.**

2. Stability factor, $$S' = \left.\frac{\Delta I_C}{\Delta V_{BE}}\right|_{\text{Constant } I_{CO} \text{ and } \beta_{dc}} \text{ or } \frac{\partial I_C}{\partial V_{BE}} \quad ...(1.17.2)$$

S' represents the change in I_C due to change in V_{BE} at constant I_{CO} and β_{dc}.

3. Stability factor, $$S'' = \left.\frac{\Delta I_C}{\beta_{dc}}\right|_{\text{Constant } I_{CO} \text{ and } V_{BE}} \text{ or } \frac{\partial I_C}{\partial \beta_{dc}} \quad ...(1.17.3)$$

- **These expressions indicate that I_C is collectively dependent on ΔI_{CO}, ΔV_{BE} and $\Delta \beta_{dc}$. The combined effect of these parameters on I_C is mathematically given by,**

Total change in collector current, $$\Delta I_C = S \, \Delta I_{CO} + S' \, \Delta V_{BE} + S'' \, \Delta \beta_{dc} \quad ...(1.17.4)$$

- **Ideally the values of all the stability factors should be zero and practically they should be as small as possible.**
- **Practically the value of S is significantly higher than the other two stability factors. Hence while comparing the biasing circuits, we should focus our attention more on the value of S.**

1.18 Stabilization (Biasing) Techniques :

- Bias stabilization means techniques used to stabilize the Q point. Some of them are as follows :
  1. Fixed bias circuit (Single base resistor biasing)
  2. Collector to base bias circuit
  3. Voltage divider bias circuit
  4. Emitter bias.

1.18.1 General Procedure for the DC Analysis :

- If a biasing circuit is given and the co-ordinates of Q point i.e. V_{CEQ} and I_{CQ} are to be obtained, then the following procedure is to be followed.

Procedure :

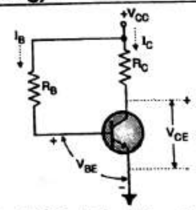
Step 1 : Calculate the base current I_B by applying KVL to the base loop of the given circuit.

Step 2 : Calculate $I_{CQ} = \beta_{dc} I_B$.

Step 3 : Calculate V_{CEQ} by applying KVL to the collector loop of the given circuit.

1.19 Fixed Bias Circuit (Single Base Resistor Biasing) :

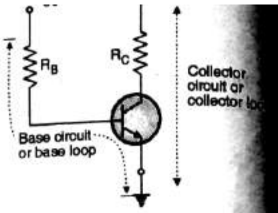
- The simplest biasing circuit used to bias a BJT is called as the fixed bias circuit which is as shown in Fig. 1.19.1.
- Till now we have always shown two separate power supplies V_{CC} and V_{BB} to bias a transistor. But in this circuit only one power supply (V_{CC}) has been used to supply power to collector as well as base.
- R_B is the single base biasing resistor, hence this circuit is also called as single base resistor biasing.



(F-287) **Fig. 1.19.1 : A fixed bias circuit for n-p-n transistor**

1.19.1 Analysis of Fixed Bias Circuit :

For the analysis of fixed bias circuit let us redraw this circuit as shown in Fig. 1.19.2. As seen from the Fig. 1.19.2 there are two loops or two circuits, namely base circuit and the collector circuit.



(F-288) **Fig. 1.19.2 : Fixed bias circuit redr to show base and collector circuit**

Analysis :

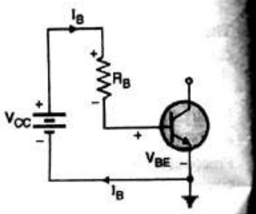
Step 1 : Expression for base current I_{BQ} or I_B :

- Consider the base circuit shown in Fig. 1.19.3. Here deliberately the collector resistance R_C is assumed to be open circuited. Apply the Kirchhoff's voltage law to the base circuit to get,

$$V_{CC} - I_B R_B - V_{BE} = 0 \qquad ...(1.19.1)$$

- Rearranging the Equation (1.19.1) we get,

$$\therefore \quad I_B = \frac{V_{CC} - V_{BE}}{R_B}$$



(F-289) **Fig. 1.19.3 : Base circuit or base loo**

- For silicon transistors V_{BE} = 0.7 V and for germanium transistors it is 0.3 V. There $V_{BE} < V_{CC}$, hence neglecting V_{BE} we can write :

$$I_B = \frac{V_{CC}}{R_B} \qquad ...(1.19$$

This is the approximate expression for the base current corresponding to Q point i.e. I_{BQ}.

- In this equation supply voltage V_{CC} and R_B both are of fixed value. Therefore the base current also remains constant. **Therefore the name of this biasing circuit is "fixed bias circuit".**

Step 2 : Expression for I_{CQ} or I_C :

- The fixed bias circuit is designed to operate in the active region
- Hence the collector current is given by the following expression :

$$I_C = \beta_{dc} I_B + I_{CEO} \qquad ...(1.1$$

- But as, $I_{CEO} << \beta_{dc} I_B$ we can neglect it to get,

$$I_{CQ} = \beta_{dc} I_{BQ}$$

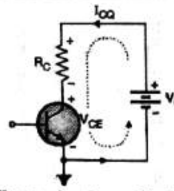
Step 3 : Expression for V_{CEQ} or V_{CE} :

- Now consider the collector circuit shown in Fig. 1.19.4. Here the base resistance is assumed t open circuited.

- Apply KVL to the collector circuit to write :

$$V_{CC} - I_{CQ} R_C - V_{CEQ} = 0$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C \qquad ...(1.19.4)$$



(F-290) **Fig. 1.19.4 : Collector circuit or collector loop**

Co-ordinate of Q point :

- The co-ordinates of Q point of any biasing circuits are (V_{CEQ}, I_{CQ}).
- Hence the expression for V_{CEQ} and I_{CQ} will give us the co-ordinates of Q point.

Stabilization of Q point :

- Let us see how far the fixed bias circuit is able to stabilize the Q point.
- As temperature increases, I_{CBO} increases. So I_C will increase because

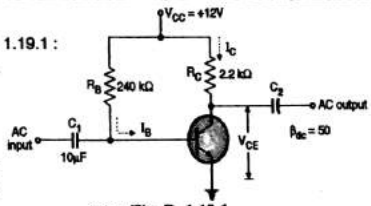
$$I_C = \beta I_B + (1 + \beta) I_{CBO}$$

- In the fixed bias circuit I_B is constant. So I_C will keep varying with change in temperature.
- The fixed bias circuit cannot automatically keep I_C constant and stabilize the Q point.
- Thus no stabilization is provided by the fixed bias circuit.

Ex. 1.19.1 : Determine the following for the fixed bias configuration of Fig. P. 1.19.1 :

(a) I_{BQ} and I_{CQ} (b) V_{CEQ}

(c) V_B and V_C (d) V_{BC}.



(F-291) **Fig. P. 1.19.1**

Soln. :

- In this example we have been asked to find the voltages and currents corresponding to the Q point.
- At Q point we assume that the ac input applied to the circuit is zero and the ac output produced is also zero.
- For DC analysis therefore assume that the capacitors C_1 and C_2 are open circuited and hence do not exist in the circuit at all.

Step 1 : Obtain I_{BQ} and I_{CQ} :

$$I_{BQ} = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{240 \times 10^3} = 47.08\ \mu A \qquad \text{...Ans.}$$

$$I_{CQ} = \beta_{dc} \times I_{BQ} = 50 \times 47 \times 10^{-6} = 2.35\ mA \qquad \text{...Ans.}$$

Step 2 : Obtain V_{CEQ} :

Consider the collector circuit of Fig. P. 1.19.1. Applying the Kirchhoff's voltage law to it we get,

$$V_{CC} = V_{CEQ} - I_{CQ} R_C \qquad ...(1)$$

Substituting the values,

$$V_{CEQ} = 12 - (2.35 \times 10^{-3} \times 2.2 \times 10^3) = 6.83\ \text{Volts.} \qquad \text{...Ans.}$$

Step 3 : Obtain V_B and V_C :

V_B and V_C are the voltages measured at base and collector with respect to ground.

$$\therefore \; V_B = V_{BE} = 0.7 \text{ Volts and } V_C = V_{CE} = 6.83 \text{ Volts.}$$

Obtain V_{BC} :

$$V_{BC} = V_{BE} - V_{CE}$$

Substituting the values we get, $\quad V_{BC} = 0.7 - 6.83 = -6.13$ Volts.

Ex. 1.19.2 : Derive the expression for the stability factor "S" of a fixed bias circuit. Comment o result.

Soln. :

- We have defined the stability factor "S" as follows :

$$S = \left.\frac{\Delta I_C}{\Delta I_{CO}}\right|_{\text{Constant } V_{BE} \text{ and } \beta_{dc}}$$

S gives us the change in I_C due to change in the reverse saturation current I_{CBO}. As I_{CBO} chan by ΔI_{CBO}, the base current I_B will change by ΔI_B and the collector current I_C changes by ΔI_C

- For a CE configuration we know that,

$$I_C = \beta_{dc} I_B + I_{CEO} = \beta_{dc} I_B + (1 + \beta_{dc}) I_{CBO}$$

- Therefore change in I_C is given by, $\quad \Delta I_C = \beta_{dc} \Delta I_B + (1 + \beta_{dc}) \Delta I_{CBO}$
- Dividing both the sides by ΔI_C we get,

$$1 = \beta_{dc}\left[\frac{\Delta I_B}{\Delta I_C}\right] + (1 + \beta_{dc})\left[\frac{\Delta I_{CBO}}{\Delta I_C}\right]$$

$$\therefore \; 1 - \beta_{dc}\left[\frac{\Delta I_B}{\Delta I_C}\right] = (1 + \beta_{dc})\left[\frac{\Delta I_{CBO}}{\Delta I_C}\right] \qquad \therefore \; \frac{\Delta I_{CBO}}{\Delta I_C} = \frac{1 - \beta_{dc}[\Delta I_B / \Delta I_C]}{(1 + \beta_{dc})}$$

- But, $S = \dfrac{\Delta I_C}{\Delta I_{CBO}} \qquad \therefore \quad S = \dfrac{(1 + \beta_{dc})}{1 - \beta_{dc}[\Delta I_B / \Delta I_C]}$ ...(1.1

- But for the fixed bias circuit, $\quad I_B = \dfrac{V_{CC} - V_{BE}}{R_B}$

In this equation V_{CC}, V_{BE} and R_B all are fixed. Therefore I_B cannot change. $\therefore \; \Delta I_B$ Substituting this in Equation (1.19.5) we get,

$$S = (1 + \beta_{dc}) \qquad \text{...(1.1}$$

Comment on the expression for S :

Substitute $\beta_{dc} = 49$ in Equation (1.19.5). The value of S = 50. i.e. collector current change times as large as change in the reverse saturation current I_{CBO}. Fixed bias circuit thus gives a very stability of the Q point. It is the worst configuration as far as the stability of Q point is concerned.

Advantages of fixed bias circuit :

1. The fixed bias circuit is simple and has less number of components.

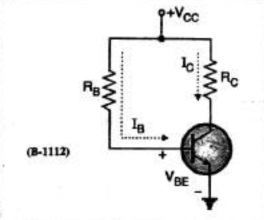
2. It give very good flexibility as the Q point can be set at any point in the active region by just adjusting the value of R_B.

Disadvantages of fixed bias circuit :

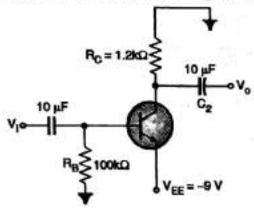
1. Very poor thermal stability as $S = (1 + \beta_{dc})$.
2. With changes in β_{dc} due to changes in temperature, the operating point keeps on shifting its position.

To overcome these disadvantages the second bias stabilization circuit i.e. collector to base bias circuit is used.

Table 1.19.1 : Summary of results of fixed bias circuit

Circuit diagram	Expressions
 (B-1112)	**Analysis equations** $I_B = \dfrac{V_{CC} - V_{BE}}{R_B}$, $I_C = \beta_{dc} I_B$, $V_{CE} = V_{CC} - I_C R_C$ **Stability factors** $S = 1 + \beta_{dc}$, $S' = \beta_{dc}/R_B$, $S'' = -\beta_{dc}/R_B$ **Design equations** $R_B = \dfrac{V_{CC} - V_{BE}}{I_B}$, $R_C = \dfrac{V_{CC} - V_{CE}}{I_C}$

Ex. 1.19.3 : Determine V_C and V_B for the network shown in Fig. P. 1.19.3 with $\beta = 45$ and $V_{BE} = 0.7$ V.



(B-1113) **Fig. P. 1.19.3**

Soln. :

Given : $\beta = 45$, $V_{BE} = 0.7$ V

Step 1 : Calculate V_B :

V_B is the base voltage with respect to ground. Referring Fig. P. 1.19.3(a) we get,

$$V_B = V_{BE} - V_{EE}$$

$$\therefore V_B = 0.7 - 9 = -8.3 \text{ Volts} \qquad \text{...Ans.}$$

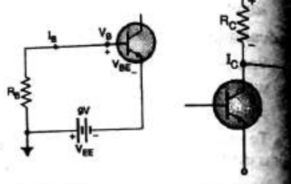
Step 2 : Calculate I_B and I_C :

Apply KVL to the base loop of Fig. P. 1.19.3(a) to write,

$$I_B R_B + V_{BE} = V_{EE}$$

$$\therefore \quad I_B = \frac{V_{EE} - V_{BE}}{R_B} = \frac{9 - 0.7}{100\ k\Omega} = 83\ \mu A$$

$$\therefore \quad I_C = \beta I_B = 45 \times 83 \times 10^{-6} = 3.735\ mA.$$



(B-1114) (a) (B-1115) (b)

Fig. P. 1.19.3

Step 3 : Calculate V_C :

V_C is the collector voltage with respect to ground. Refer Fig. P. 1.19.3(b) to write,

$$V_C = -I_C R_C = -3.735 \times 1.2 = -4.482 \text{ Volts}$$

Ex. 1.19.4 : **Draw the dc load line and locate the operating point for the fixed biasing transistor ci[rcuit] shown in Fig. P. 1.19.4(a). What will be its stability factor ? V_{BE} = 0.7 V.**

Soln. :

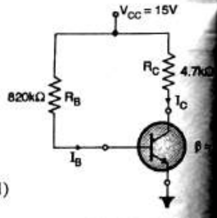
Step 1 : DC load line :

Apply KVL to the collector circuit to write,

$$V_{CC} = I_C R_C + V_{CE}$$

$$\therefore \quad I_C = -\frac{1}{R_C} V_{CE} + \frac{V_{CC}}{R_C} = -\frac{1}{4700} V_{CE} + \frac{15}{4700}$$

$$\therefore \quad I_C = -\frac{1}{4700} V_{CE} + 3.19\ mA \qquad ...(1)$$



(B-1116) **Fig. P. 1.19.4(a)**

This is the equation for DC load line.

- Substitute $I_C = 0$ in Equation (1) to get,

$$V_{CE} = V_{CC} = 15\ V$$

$\therefore$ Point "A" on the load line will have coordinates (15 V, 0 mA)

$$A = (15\ V, 0\ mA)$$

- Substitute $V_{CE} = 0$ in Equation (1) to get,

$$I_C = 3.19\ mA.$$

$\therefore$ Point "B" on the load line will have coordinates (0, 3.19 mA)

- Fig. P. 1.19.4(b) shows the dc load line.

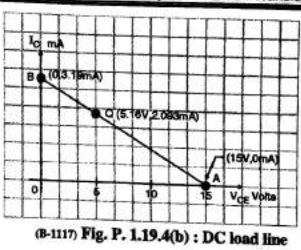
Step 2 : Q point :

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{15 - 0.7}{820 \times 10^3} = 17.44\ \mu A$$

$$I_C = \beta I_B = 120 \times 17.44 \times 10^{-6} = 2.093\ mA$$

$$V_{CE} = V_{CC} - I_C R_C$$

$$= 15 - (2.093 \times 4.7) = 5.16 \text{ Volts}$$



(B-1117) **Fig. P. 1.19.4(b) : DC load line**

$\therefore$ Q point coordinates are : (5.16 V, 2.093 mA) **...Ans.**

1.20 Collector to Base (Collector Feedback) Bias Circuit :

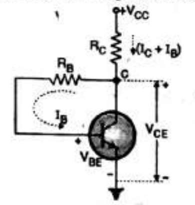
- The collector to base bias circuit which is an improvement over the fixed bias circuit is as shown in Fig. 1.20.1.
- The base resistance R_B is now connected to the collector and not to the supply voltage V_{CC}. Actually R_B is connected between the collector and base terminals of the transistor.
- The current flowing through R_C is the sum of I_C and I_B as shown in Fig. 1.20.1.

1.20.1 Analysis of Collector to Base Bias Circuit :

We will analyze this circuit by breaking it into two circuits or loops i.e. base circuit and collector circuit.

Step 1 : Expression for the base current :

- To obtain the expression for base current I_{BQ}, we have to apply KVL to the base loop.
- The base circuit is as shown in Fig. 1.20.2. The collector terminal is assumed to be open circuited. Applying the Kirchhoff's voltage law to the base circuit of Fig. 1.20.2, we can write,



(F-304) **Fig. 1.20.1 : Collector to base bias circuit**

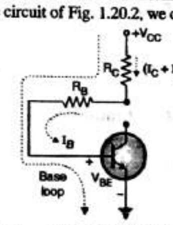


Fig. 1.20.2 : Base circuit

$$V_{CC} - (I_C + I_B) R_C - I_B R_B - V_{BE} = 0$$

$$\text{But } I_C = \beta_{dc} I_B$$

$$\therefore \quad V_{CC} = (R_B + R_C) I_B + \beta_{dc} I_B R_C + V_{BE}$$

$$\therefore \quad V_{CC} = [R_B + R_C + \beta_{dc} R_C] I_B + V_{BE}$$

$$\therefore \quad I_{BQ} = \frac{V_{CC} - V_{BE}}{[R_B + (1+\beta_{dc}) R_C]} \quad ...(1.20...$$

$$\therefore \quad V_{CC} = (R_B + R_C) I_B + I_C R_C + V_{BE}$$

Step 2 : Expression for the collector current :

$$\text{Collector current } I_{CQ} = \beta_{dc} I_{BQ}$$

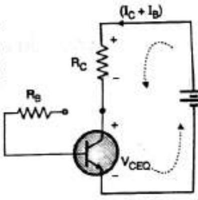
Now substitute the expression for I_{BQ} to get,

$$I_{CQ} = \frac{\beta_{dc} [V_{CC} - V_{BE}]}{[R_B + (1+\beta_{dc}) R_C]} \quad ...(1.20...$$

This is the expression for collector current I_{CQ} at the Q point of the circuit.

Step 3 : Expression for V_{CEQ} :

- Now consider the collector circuit shown in Fig. 1.20.3 where the base is assumed to be open.
- Apply KVL to the collector circuit of Fig. 1.20.3 to get,



(F-305) **Fig. 1.20.3 : Collector loop**

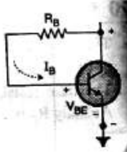


Fig. 1.20.4

$$V_{CC} = (I_C + I_B) R_C + V_{CEQ}$$

$$\therefore \quad V_{CEQ} = V_{CC} - (I_C + I_B) R_C \quad ...(1.20...$$

- This is the expression for collector to emitter voltage V_{CEQ} at the Q point of the circuit.

Alternate expression for I_B :

Refer Fig. 1.20.4 to write,

$$V_{CE} = I_B R_B + V_{BE}$$

$$\therefore \quad I_B = \frac{V_{CE} - V_{BE}}{R_B} \quad ...(1.2...$$

1.20.2 Q Point Stabilization in Collector to Base Bias Circuit :

- We have seen the factors that affect the stability of Q point. They are change in β_{dc} or I_{CO} d... change in temperature or due to piece to piece variation in characteristics.

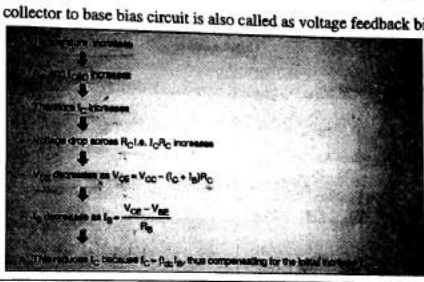
$$\text{But} \quad I_C = \beta_{dc} I_B + I_{CEO}$$

- Therefore due to changes in β_{dc} or I_{CEO}, the collector current changes. Suppose that β_{dc} and... increase then the sequence of events takes place as follows :

Stabilization of Q point by collector to base bias circuit :

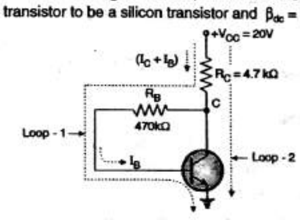
- Thus the value of I_C is maintained constant irrespective of changes in β_{dc} or I_{CEO}, to stabilize t... point.
- As R_B is connected between the collector (output) and base (input), a part of the output is bein... back to the input. This is a negative feedback.

- Therefore the collector to base bias circuit is also called as voltage feedback bias circuit.

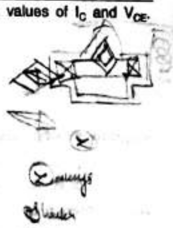


(F-306)

Ex. 1.20.1 : For the circuit shown in Fig. P. 1.20.1, determine the Q point values of I_C and V_{CE}. Assume the transistor to be a silicon transistor and $\beta_{dc} = 100$.



(B-855) **Fig. P. 1.20.1**



Soln. :

Step 1 : Calculate I_{BQ} :

Apply KVL to loop-1 to get,

$$V_{CC} - (I_C + I_B) R_C - I_B R_B - V_{BE} = 0$$

Substitute $I_C = \beta_{dc} I_B$

$$\therefore \quad V_{CC} - (1+\beta_{dc}) I_B R_C - I_B R_B - V_{BE} = 0$$

$$\therefore \quad I_{BQ} = \frac{V_{CC} - V_{BE}}{R_B + (1+\beta_{dc}) R_C} = \frac{20 - 0.7}{470 \times 10^3 + (1+100) \times 4.7 \times 10^3} = 20.43\ \mu A.$$

Step 2 : Calculate I_{CQ} and V_{CEQ} :

1. $I_{CQ} = \beta_{dc} \times I_{BQ} = 100 \times 20.43\ \mu A = 2.043\ mA$ **...Ans.**
2. Apply KVL to loop 2 in Fig. P. 1.20.1 to get,

$$\therefore \quad V_{CC} - (I_C + I_B) R_C - V_{CE} = 0$$

$$\therefore \quad V_{CEQ} = V_{CC} - (I_C + I_B) R_C$$

$$= 20 - (2.043\ mA + 20.43\ \mu A)\ 4.7 \times 10^3$$

$$= 10.30\ V \quad \text{...Ans.}$$

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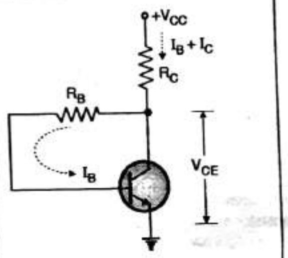
Summary of results for collector to base bias :

Table 1.20.1

Circuit diagram	Important equations
 (B-1132) **Collector to base bias circuit**	$I_B = \frac{(V_{CC} - I_C R_C) - V_{BE}}{R_C + R_B}$ $I_C = \frac{\beta_{dc} (V_{CC} - I_C R_C - V_{BE})}{R_C + R_B}$ $S = \frac{1 + \beta_{dc}}{1 + \beta_{dc}\left[\frac{R_C}{R_B + R_C}\right]}$ $S' = \frac{-\beta_{dc}}{R_B + (1 + \beta_{dc}) R_C}$ $S'' = \frac{-I_C (R_B + R_C)}{\beta_{dc} [R_B + (1 + \beta_{dc}) R_C]}$

Advantages of collector to base bias :

1. Improvement in stability.
2. Needs only one dc power supply.
3. All the advantages of negative feedback are obtained.

Disadvantages of collector to base bias :

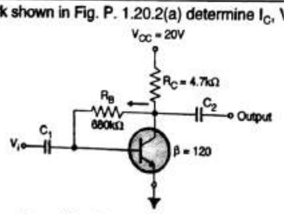
1. If the value of R_C is very small as in a transformer coupled circuit then the equation for stability factor "S" which is originally given as :

$$S = \frac{1 + \beta_{dc}}{1 + \beta_{dc}\left[\frac{R_C}{R_B + R_C}\right]} \qquad \text{Reduces to, } S = (1 + \beta_{dc})$$

This is same as the expression for the stability factor of fixed bias circuit. Thus for small values of R_C there is hardly any improvement in the stability factor S.

2. Circuit design becomes complicated.
3. Voltage gain is reduced due to negative feedback.
4. Q-point stability is not as good as expected.

Ex. 1.20.2 : For the network shown in Fig. P. 1.20.2(a) determine I_C, V_{CE}, V_B and V_C.



(B-1133) **Fig. P. 1.20.2(a) : Given circuit**

Soln. :

Given : $V_{CC} = 20$ V, $R_C = 4.7$ kΩ, $R_B = 680$ kΩ, $\beta = 120$

To find : I_C, V_{CE}, V_B and V_C.

Step 1 : Find I_B :

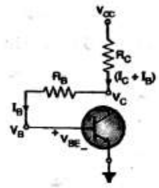
Apply KVL to the base loop shown in Fig. P. 1.20.2(b), to get,

$$V_{CC} = (I_C + I_B) R_C + I_B R_B + V_{BE}$$

$$\therefore V_{CC} - V_{BE} = (\beta I_B + I_B) R_C + I_B R_B = (1 + \beta) I_B R_C + I_B R_B$$

$$\therefore I_B = \frac{V_{CC} - V_{BE}}{R_B + (1 + \beta) R_C} = \frac{20 - 0.7}{680 \times 10^3 + [(1 + 120) \times 4.7 \times 10^3]}$$

$$\therefore I_B = 15.46\ \mu A \qquad ...(1)$$



(B-1134) **Fig. P. 1.20.2(b) : Base loop**

Step 2 : Find I_C :

$$I_C = \beta I_B = 120 \times 15.46 \times 10^{-6} = 1.854 \text{ mA} \quad \text{...Ans.}$$

Step 3 : Find V_B :

$$V_B = V_{BE} = 0.7 \text{ Volts} \quad \text{...Ans.}$$

Step 4 : Find V_C :

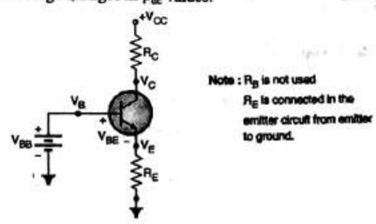
$$V_C = V_B + I_B R_B = 0.7 + (15.46 \times 10^{-6} \times 680 \times 10^3) = 10.51 \text{ Volts} \quad \text{...Ans.}$$

Step 5 : Find V_{CE} :

$$V_{CE} = V_C = 10.51 \text{ Volts} \quad \text{...Ans.}$$

1.21 Emitter Bias :

- When we use transistor in an amplifier circuit the dc biasing circuit should be such that the Q point should not change its position (location) on the DC load line despite the changes in β_{dc}.
- A circuit which can practically achieve this is called as the **emitter bias** circuit and it is as shown in Fig. 1.21.1. It has got a resistor connected in its emitter (R_E) and base resistor R_B is not being used.
- Due to this small change, the Q point of this circuit becomes very stable. It does not change its position even due to large changes in β_{dc} values.



(F-3821) **Fig. 1.21.1 : Emitter bias**

1.21.1 Analysis :

- From Fig. 1.21.1 we get,

$$V_B = V_{BB} \text{ Volts}$$

- Emitter voltage $V_E = V_B - V_{BE} = V_{BB} - V_{BE}$

1.21.2 Q Point Co-ordinates :

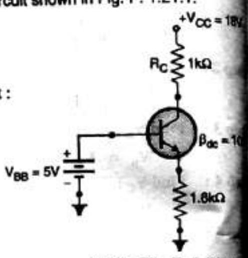
Let us solve the following example to understand the procedure for finding the Q point co-ordinates of an emitter bias circuit.

Ex. 1.21.1 : Find the co-ordinates of Q point for the emitter bias circuit shown in Fig. P. 1.21.1.

Soln. :

- The co-ordinates of Q point are (V_{CE}, I_C).
- We will follow the procedure given below to find the Q point :

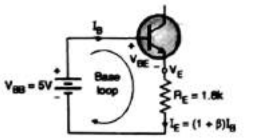
1. Find I_B
2. Find I_C and I_E
3. Find V_{CE}



(F-3822) **Fig. P. 1.21.1**

Step 1 : Find I_B :

Let $V_{BE} = 0.7$ V. Apply KVL to the base loop of Fig. P. 1.21.1(a) to write,



(F-3823) **Fig. P. 1.21.1(a) : Base loop**

$$V_{BB} = V_{BE} + V_E$$

$$= V_{BE} + I_E R_E$$

$$\text{But } I_E = I_C + I_B = \beta_{dc} I_B + I_B$$

$$\therefore I_E = (1 + \beta_{dc}) I_B$$

$$\therefore V_{BB} = V_{BE} + (1 + \beta_{dc}) R_E I_B$$

$$\therefore I_B = \frac{V_{BB} - V_{BE}}{(1 + \beta_{dc})R_E} = \frac{5 - 0.7}{(1 + 100) \times 1.8 \times 10^3}$$

$$= 23.65\ \mu A \quad ...(1)$$

Step 2 : Find I_C and I_E :

$$I_C = \beta_{dc} \times I_B = 100 \times 23.65 \times 10^{-6} = 2.365 \text{ mA}$$

$$I_E = (1 + \beta_{dc}) I_B = 101 \times 23.65 \times 10^{-6} = 2.389 \text{ mA}$$

Step 3 : Find V_{CE} :

Apply KVL to the collector loop of Fig. P. 1.21.1(b) to write,

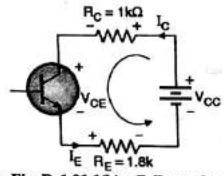
$$V_{CC} = I_C R_C + V_{CE} + I_E R_E$$

$$\therefore V_{CE} = V_{CC} - I_C R_C - I_E R_E$$

$$= 18 - (2.365 \times 1) - (2.389 \times 1.8)$$

$$= 11.33 \text{ Volts}$$

$$\therefore \text{Q point} = (11.33 \text{ V}, 2.365 \text{ mA}) \quad \text{...Ans.}$$



(F-3824) **Fig. P. 1.21.1(b) : Collector loop**

1.21.3 The Emitter Bias Circuit is Immune to Changes in β_{dc} :

1. If the collector current (I_C) tends to increase due to either rise in temperature or change in β_{dc} due to replacement of transistor then the emitter current I_E also will increase.
2. Due to increase in I_E, the voltage drop across R_E i.e. V_E will increase.
3. Therefore V_{BE} will decrease. This will reduce the value of I_B and therefore the increased collector current will be reduced. Thus the stabilization of Q point takes place. In other words, the emitter bias circuit is immune to changes in the value of β_{dc}.

1.21.4 Minor Effect of Current Gain :

- The β_{dc} has a minor effect on the value of I_C. This can be shown as follows,

$$I_E = I_C + I_B$$

$$\therefore I_E = I_C + \frac{I_C}{\beta_{dc}} = \frac{\beta_{dc} I_C + I_C}{\beta_{dc}}$$

$$\therefore I_E = \frac{(1 + \beta_{dc})}{\beta_{dc}} \times I_C \qquad \therefore I_C = \frac{\beta_{dc}}{(1 + \beta_{dc})} I_E \quad ...(1.21.1)$$

- In this expression, I_E is called as the **correlation factor.**
- Since $\beta_{dc}/(1 + \beta_{dc}) \approx 1$ we can safely assume $I_C = I_E$ without introducing much error.

Note :

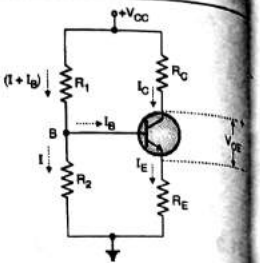
- The base bias circuit produces a constant base current while the emitter bias circuit produces a constant value of emitter current I_E.
- Due to the problems with current gain variation the base bias circuit is used for biasing the transistor as a switch, i.e in either saturation or cut off.
- Due to stability of Q point the emitter bias circuit is used for biasing the transistor as an amplifier i.e in the active region.

1.22 Voltage Divider Bias (VDB) or Self Bias :

- This is the third bias stabilizing circuit. The circuit diagram of voltage divider bias is as shown in Fig. 1.22.1.

Features of the circuit :

- The resistors R_1 and R_2 form a potential divider to apply a fixed voltage V_B to the base.
- A resistance R_E has been connected in the emitter circuit. This resistor is not present in the fixed bias or collector to base bias circuits.



(F-316) Fig. 1.22.1 : Voltage divider bias

Bias stabilization using voltage divider bias circuit :

(F-317)

- If I_C increases due to change in temperature or β_{dc}
- Then I_E increases
- Hence drop across R_E increases ($V_E = I_E R_E$)
- But V_B is constant. Hence V_{BE} decreases.
- Hence I_B decreases.
- Hence I_C also decreases. Thus the compensation for increase in I_C is achieved.

1.22.1 Approximate Analysis of the Voltage Divider Bias :

- The approximate analysis of voltage divider bias circuit will be done by considering the base and collector circuits separately.
- The approximate analysis can be applied if and only if $I > I_B$.

Base circuit :

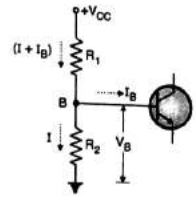
Consider the base circuit shown in Fig. 1.22.2. Here we assume that the collector and emitter terminals are open circuited. The base voltage V_B is nothing but the voltage across resistor R_2.

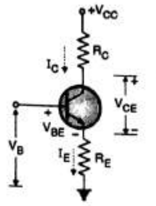
$$\therefore \quad V_B = V_{R2} = \frac{R_2}{(R_1 + R_2)} \times V_{CC} \qquad ...(1.22.1)$$

This is because, current through R_1 and R_2 is approximately same and is equal to I.

Collector circuit :

Consider the collector circuit shown in Fig. 1.22.3. The voltage across emitter resistance R_E can be obtained as follows :





(F-318) Fig. 1.22.2 : Base circuit Fig. 1.22.3 : Collector circuit

$$V_E = I_E R_E = V_B - V_{BE}$$

$$\therefore \quad I_E = \frac{V_B - V_{BE}}{R_E} \qquad ...(1.22.2)$$

Applying the KVL to the collector circuit we get,

$$V_{CC} = I_C R_C + V_{CE} + R_E I_E$$

$$\therefore \quad V_{CE} = V_{CC} - I_C R_C - I_E R_E \qquad ...(1.22.3)$$

1.22.2 Accurate VDB Analysis using Thevenin's Equivalent Circuit :

- If the condition $I_B <<< I$ is not satisfied by the self bias circuit then the exact analysis should be performed.
- The procedure to be followed for the exact analysis is as follows :

Procedure for exact analysis :

Step 1 : Draw the Thevenin's equivalent of self bias circuit.

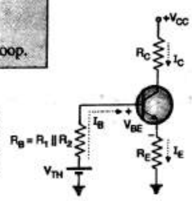
Step 2 : Obtain I_B by applying KVL to the base loop.

Step 3 : Obtain $I_C = \beta I_B$

Step 4 : Obtain the expression for V_{CE} by applying KVL to collector loop.

Step 1 : Thevenin's equivalent circuit :

The Thevenin's equivalent circuit of the voltage divider bias circuit is as shown in Fig. 1.22.4. The resistances R_1 and R_2 are replaced by R_B and V_{TH}, where V_{TH} is the Thevenin's voltage and R_B is the Thevenin's equivalent resistance which is equal to the parallel combination of R_1 and R_2.



(F-319) Fig. 1.22.4 : Thevenin's equivalent circuit for voltage divider bias

$$\therefore \quad R_B = (R_1 \| R_2) = \frac{R_1 R_2}{(R_1 + R_2)} \quad ...(1.21.4) \quad \text{and} \quad V_{TH} = \frac{R_2 V_{CC}}{(R_1 + R_2)} \quad ...(1.22.5)$$

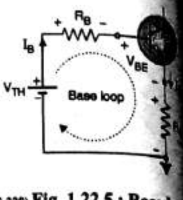
Step 2 : Obtain the expression for I_B :

Applying KVL to the base loop of Fig. 1.22.5 we get,

$$V_{TH} - I_B R_B - V_{BE} - I_E R_E = 0$$

$$\therefore \quad V_{TH} - I_B R_B - V_{BE} - (1 + \beta_{dc}) I_B R_E = 0$$

$$\therefore \quad I_B = \frac{V_{TH} - V_{BE}}{R_B + (1 + \beta_{dc}) R_E} \quad ...(1.22.6)$$



(F-320) Fig. 1.22.5 : Base loop

This is the required expression for I_B.

Step 3 : Obtain expression for I_C :

$$I_C = \beta_{dc} \times I_B = \frac{\beta_{dc}(V_{TH} - V_{BE})}{R_B + (1 + \beta_{dc}) R_E} \quad ...(1.22.7)$$

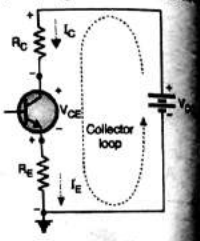
Step 4 : Obtain the expression for V_{CE} :

Apply KVL to the collector loop shown in Fig. 1.22.6 to get,

$$V_{CC} = I_C R_C + V_{CE} + I_E R_E$$

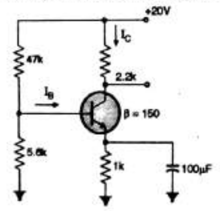
$$\therefore \quad V_{CE} = V_{CC} - I_C R_C - I_E R_E \quad ...(1.22.8)$$

This is the required expression for V_{CE}.



(F-321) Fig. 1.22.6 : Collector loop

Ex. 1.22.1 : For a voltage divider biasing circuit shown in Fig. P. 1.22.1, find I_C, V_{CE}, I_B, V_E and V_B.



(B-1173) Fig. P. 1.22.1

Soln. :

Step 1 : Draw Thevenin's equivalent circuit :

The Thevenin's equivalent circuit is shown in Fig. P. 1.22.1(a).

$$R_B = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{47 \times 5.6}{47 + 5.6} = 5\ k\Omega$$

$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{5.6}{47 + 5.6} \times 20 = 2.13\ V$$

Step 2 : Find I_B and I_C :

$$I_B = \frac{V_{TH} - V_{BE}}{R_B + (1 + \beta) R_E}$$

$$= \frac{2.13 - 0.7}{5k + (151 \times 1k)}$$

$$= 9.167\ \mu A \quad ...Ans.$$

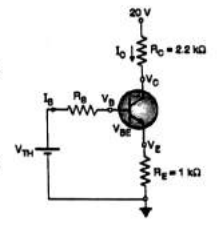
$$I_C = \beta I_B$$

$$= 150 \times 9.167 \times 10^{-6}$$

$$= 1.375\ mA \quad ...Ans.$$

$$I_E = (1 + \beta) I_B = 151 \times 9.167 \times 10^{-6}$$

$$= 1.384\ mA$$



(B-1174) Fig. P. 1.22.1(a) : Thevenin's equivalent circuit

Step 3 : Find V_C, V_E and V_{CE} :

$$V_C = V_{CC} - I_C R_C = 20 - (1.375 \times 2.2)$$

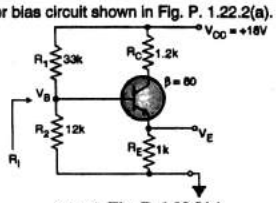
$$= 16.975\ V \quad ...Ans.$$

$$V_E = I_E R_E = 1.384 \times 1 = 1.384\ V \quad ...Ans.$$

$$V_{CE} = V_C - V_E = 16.975 - 1.384 = 15.591\ V \quad ...Ans.$$

Ex. 1.22.2 : Determine the following for the voltage divider bias circuit shown in Fig. P. 1.22.2(a).

1. I_C
2. V_E
3. V_B
4. V_{CE} and
5. R_i



(B-1175) Fig. P. 1.22.2(a)

Soln. :

Step 1 : Find I_B and I_C :

$$R_B = R_1 \| R_2$$

$$= \frac{33k \times 12\ k}{33k + 12k} = 8.8\ k\Omega$$

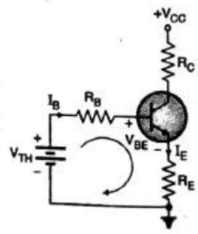
$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{12}{33 + 12} \times 18 = 4.8\ V$$

Applying KVL to base loop of Fig. P. 1.22.2(b),

$$V_{TH} = I_B R_B + V_{BE} + I_E R_E$$

$$= I_B R_B + V_{BE} + (1 + \beta) I_B R_E$$

$$\therefore \quad I_B = \frac{V_{TH} - V_{BE}}{R_B + (1 + \beta) R_E} = \frac{4.8 - 0.7}{8.8 + (81 \times 1)}$$



(B-1176) Fig. P. 1.22.2(b) : Base loop

$$\therefore \quad I_B = 45.65\ \mu A$$

$$\therefore \quad I_C = \beta I_B = 80 \times 45.65 \times 10^{-6} = 3.65\ mA \quad ...Ans.$$

$$I_E = (1 + \beta) I_B = 81 \times 45.65 \times 10^{-6} = 3.69\ mA \quad ...Ans.$$

Step 2 : Find V_E and V_B :

$$V_E = I_E R_E = 3.69 \times 1 = 3.69 \text{ V}$$

$$V_B = V_E + V_{BE} = 3.69 + 0.7 = 4.39 \text{ V}$$

Step 3 : Find V_{CE} :

$$V_{CE} = V_{CC} - I_C R_C - V_E = 18 - (3.65 \times 1.2) - 3.69 = 9.93 \text{ V}$$

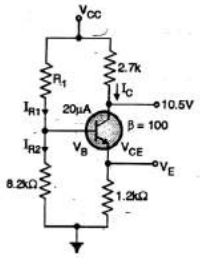
Step 4 : Find R_i :

$$R_i = [(1 + h_{fe}) R_E + h_{ie}] \parallel R_B$$

Assume $h_{ie} = 1 \text{ k}\Omega$, $h_{fe} = \beta = 80$

$$\therefore R_i = [(81 \times 1k) + 1k] \parallel 8.8 k = 7.947 \text{ k}\Omega$$

Ex. 1.22.3 : Determine the following for the voltage divider configuration shown in Fig. P. 1...
1. I_C 2. V_E 3. V_{CC} 4. V_{CE} 5. V_B 6. R_1



(B-2249) **Fig. P. 1.22.3**

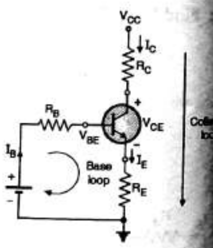
Soln. :

Given : $R_C = 2.7 \text{ k}\Omega$, $R_E = 1.2 \text{ k}\Omega$, $V_C = 10.5 \text{ V}$, $\beta = 100$, $R_2 = 8.2 \text{ k}\Omega$, $I_B = 20 \ \mu\text{A}$.

To find : 1. I_C 2. V_E 3. V_{CC} 4. V_{CE} 5. V_B 6. R_1

Step 1 : Draw Thevenin's equivalent :

Fig. P. 1.22.3(a) shows Thevenin's equivalent circuit.



(B-2250) **Fig. P. 1.22.3(a) Thevenin's equivalent ci...**

Step 2 : Calculate I_C : From Fig. P. 1.22.3(a).

$$I_C = \beta I_B = 100 \times 20 \times 10^{-6} = 2 \text{ mA} \quad \text{...Ans.}$$

Step 3 : Calculate V_E :

$$V_E = I_E R_E = (1 + \beta) I_B R_E$$

$$= (101) \times 20 \times 10^{-6} \times 1.2 \times 10^3 = 2.424 \text{ V}$$

Step 4 : Calculate V_{CC} :

$$V_{CC} = V_C + I_C R_C$$

$$= 10.5 + (2 \times 10^{-3} \times 2.7 \times 10^3) = 15.9 \text{ V} \quad \text{...Ans.}$$

Step 5 : Calculate V_{CE} :

$$V_{CE} = V_C - V_E = 10.5 - 2.424 = 8.076 \text{ V} \quad \text{...Ans.}$$

Step 6 : Calculate V_B :

$$V_B = V_E + V_{BE} = 2.424 + 0.7 = 3.124 \text{ V} \quad \text{...Ans.}$$

Step 7 : Calculate R_1 : From Fig. P. 1.22.3,

$$R_1 = \frac{V_{CC} - V_B}{I_{R1}} \quad \text{...(2)}$$

But, $$I_{R2} = \frac{V_B}{R_2} = \frac{3.124}{8.2 \text{ k}} = 380.97 \ \mu\text{A}$$

$$\therefore I_{R1} = I_B + I_{R2} = 20 \ \mu\text{A} + 380.97 \ \mu\text{A} = 400.97 \ \mu\text{A}$$

Substitute value of I_{R1} in Equation (2), we get

$$R_1 = \frac{15.9 - 3.124}{400.97 \ \mu\text{A}} = 31.86 \text{ k}\Omega \quad \text{...Ans.}$$

Ex. 1.22.4 : Derive the expression for the stability factor S of the voltage divider bias circuit. Comment on the result.

Soln. : To derive the expression for S we are going to use the same equation which we had used to obtain "S" for the collector to base bias which is,

$$S = \frac{1 + \beta_{dc}}{1 - \beta_{dc} [\Delta I_B / \Delta I_C]} \quad \text{...(1)}$$

and substitute the value of $\frac{\Delta I_B}{\Delta I_C}$ for the self bias circuit to obtain the required expression for S.

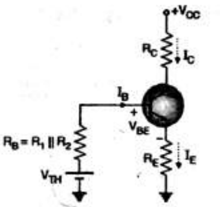
To obtain the value of $\Delta I_B / \Delta I_C$:

Consider the Thevenin's equivalent circuit which we have discussed in section 1.21.4. The same circuit has been repeated in Fig. P. 1.22.4.

Apply KVL to the base circuit of Fig. P. 1.22.4 we can write,

$$V_{TH} = I_B R_B + V_{BE} + (I_C + I_B) R_E \quad \text{...(2)}$$

If we consider V_{BE} to be independent of I_C, we can differentiate Equation (2) with respect to I_C to obtain,



(B-861) **Fig. P. 1.22.4 : Thevenin's equivalent circuit for voltage divider bias circuit**

$$0 = R_B \frac{\partial I_B}{\partial I_C} + 0 + R_E + R_E \frac{\partial I_B}{\partial I_C}$$

$$\therefore 0 = \frac{\partial I_B}{\partial I_C} (R_B + R_E) + R_E$$

$$\therefore \frac{\partial I_B}{\partial I_C} = \frac{-R_E}{(R_B + R_E)}$$

$$\therefore \frac{\Delta I_B}{\Delta I_C} = -\frac{R_E}{(R_B + R_E)}$$

Substitute this in Equation (1) to obtain,

$$S = \frac{1 + \beta_{dc}}{1 - \beta_{dc}\left[\frac{-R_E}{R_B + R_E}\right]} = \frac{1 + \beta_{dc}}{1 + \beta_{dc}\left[\frac{R_E}{R_B + R_E}\right]}$$

$$S = \frac{(1 + \beta_{dc})(R_B + R_E)}{R_B + R_E + \beta_{dc} R_E} = \frac{(1 + \beta_{dc})(R_B + R_E)}{R_B + (1 + \beta_{dc}) R_E}$$

Divide numerator and denominator by R_E to get,

$$S = (1 + \beta_{dc}) \frac{1 + (R_B/R_E)}{(1 + \beta_{dc}) + (R_B/R_E)}$$

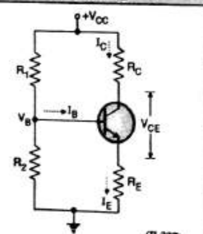
This is the desired result.

Comments on the result :

- The value of S depends on the ratio (R_B/R_E). If R_B/R_E is small then the value of S = 1 and [if the] ratio $(R_B/R_E) \rightarrow \infty$ then $S \approx (1 + \beta_{dc})$. Thus the self bias circuit is more stable for smaller [values] of the ratio (R_B/R_E).
- If the ratio (R_B/R_E) is fixed then S increases with increase in the value of β_{dc}. Thus st[ability] decreases with increase in β_{dc}.
- S is independent of β_{dc} for small values of β_{dc}.
- Smaller values of R_B give better stabilization.

Summary of analysis for the voltage divider bias :

Table 1.22.1 : Summary of results for the voltage divider bias

Circuit diagram	Important equations
 (F-333)	$V_{TH} = \frac{R_2}{(R_1 + R_2)} \times V_{CC}$
	$R_B = R_1 \parallel R_2 = \frac{R_1 R_2}{(R_1 + R_2)}$, $I_E = \frac{V_B - V_{BE}}{R_E}$
	$S = (1 + \beta_{dc}) \frac{1 + (R_B / R_E)}{(1 + \beta_{dc}) + (R_B / R_E)}$
	$S' = \frac{-\beta_{dc}}{R_B + (1 + \beta_{dc}) R_E}$, $S'' = \frac{S I_C}{\beta_{dc}(1 + \beta_{dc})}$

Advantages of voltage divider bias circuit :

The advantages of voltage divider bias circuit are as follows :

1. It has the smallest value of S among all the biasing circuits. This shows that the bias point stability is highest for the self bias circuit.
2. It is possible to avoid the loss of signal gain by connecting an emitter bypass capacitor across R_E. This does not have any adverse effect on the other advantages of self bias circuit.
3. R_E introduces a negative feedback. This will make the self bias circuit more stable. So all the other advantages of negative feedback get attached to this circuit.

Voltage divider bias circuit is therefore the most widely used biasing circuit.

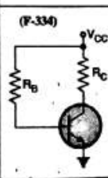
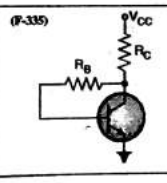
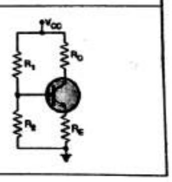
Disadvantages of voltage divider bias circuit :

1. The ratio R_B / R_E needs to be low for better Q-point stabilization. So R_B should be small and R_E high. But this reduces the input resistance.
2. Reduction in gain due to negative feedback if R_E is unbypassed.

Note : Due to very high Q-point stability and other advantages, the voltage divider bias circuit is the most widely used biasing circuit.

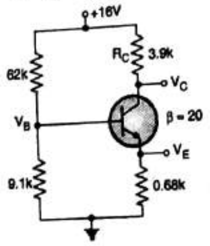
1.22.3 Comparison of Different Biasing Circuits :

Table 1.22.2 : Comparison of different biasing circuit

Sr. No.	Parameter	Fixed bias	Collector to base bias	Voltage divider bias
1.	Emitter resistance	Not used	Not used	Used
2.	Negative feedback	Not used	Included	Included
3.	Stability factor	$S = (1 + \beta_{dc})$	$S = \frac{1 + \beta_{dc}}{1 + \beta_{dc}\left[\frac{R_C}{R_C + R_B}\right]}$	$S = (1 + \beta_{dc}) \frac{1 + (R_B / R_E)}{1 + \beta_{dc} + (R_B / R_E)}$
4.	Q point stability	Poor	Moderate	Good
5.	Configuration	(F-334) 	(F-335) 	(F-336) 

Ex. 1.22.5 : Determine the following for the voltage divider configuration :

1. I_C 2. V_E 3. V_B 4. V_C 5. R_i. Refer Fig. P. 1.22.5(a).



(B-1149) **Fig. P. 1.22.5(a)**

Soln. :

Step 1 : Calculate I_B, I_C :

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{62 \times 9.1}{62 + 9.1} = 7.94\ k\Omega$$

$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{9.1}{62 + 9.1} \times 16 = 2\ V$$

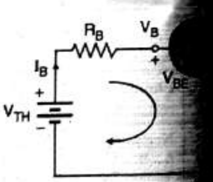
Applying KVL to the base loop and simplifying we get,

$$I_B = \frac{V_{TH} - V_{BE}}{R_B + (1 + \beta) R_E} = \frac{2 - 0.7}{7.94\ k + (21 \times 0.68\ k)}$$

$$\therefore I_B = 58.5\ \mu A \qquad ...(1)$$

$$\therefore I_C = \beta I_B = 20 \times 58.5 \times 10^{-6} = 1.17\ mA \qquad \textbf{...Ans.}$$

$$I_E = (1 + \beta) I_B = 21 \times 58.5 \times 10^{-6} = 1.23\ mA$$



(B-1150) **Fig. P. 1.22.5(b) : Bas**

Step 2 : Find V_B and V_E :

$$V_E = I_E R_E = 1.23 \times 0.68 = 0.8364\ V$$

$$V_B = V_E + V_{BE} = 0.8364 + 0.7 = 1.5364\ V$$

Step 3 : Find V_C : $\quad V_C = V_{CC} - I_C R_C = 16 - (1.17 \times 3.9) = 11.437\ V$

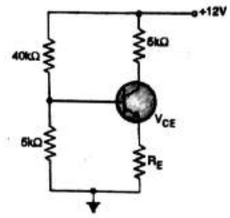
Step 4 : Find R_i :

For voltage divider bias circuit, $R_i = [h_{ie} + (1 + h_{fe}) R_E] \parallel R_B$

1.22.4 Load Line and Q Point of VDB Circuit :

- The procedure to draw the load line and locate Q point on it for a VDB circuit is **exactly** that discussed for the fixed bias circuit.
- The following examples will make this procedure clear.

Ex. 1.22.6 : Calculate the characteristics of the circuit shown in Fig. P. 1.22.6(a). Assume β = 60 and V_{BE} = 0.3 V.



(B-864) **Fig. P. 1.22.6(a)**

Soln. :

The two important characteristics are :

1. Q - point 2. DC - load line of the circuit

Part I : To obtain the Q point

We are expected to calculate the dc bias voltage and currents. That means we are supposed to obtain the Q point.

1. **To calculate the value of I_B :**

The Thevenin's equivalent circuit is as shown in Fig. P. 1.22.6(b).

Where $V_{TH} = \frac{5\ k}{(5 + 40)\ k} \times V_{CC} = \frac{12}{9} = 1.33$ Volts.

And $R_{TH} = (5\ k\Omega \parallel 40\ k\Omega) = \frac{5 \times 40}{(5 + 40)} = 4.44\ k\Omega$

Applying KVL to the base emitter loop we get,

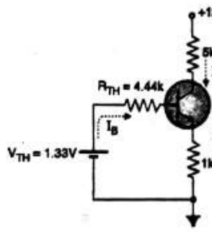
$$V_{TH} = (I_B \times 4.44\ k) + V_{BE} + (I_E \times 1\ k)$$

But $\quad I_E = (1 + \beta) I_B = 61\ I_B$

Since the transistor is in active region

$$\therefore 1.33 = 4.44\ k\ I_B + 0.3 + 61\ k\ I_B$$

$$\therefore I_B = 15.73\ \mu A$$



(B-1157) **Fig. P. 1.22.6(b) : Thevenin's equivalent circuit**

2. The collector current $I_C = \beta I_B = 60 \times 15.73 \times 10^{-6} = 0.944$ mA.
3. The emitter current $I_E = (1 + \beta) I_B = 61 \times 15.73 \times 10^{-6} = 0.96$ mA.
4. **To calculate V_{CE} :**

Applying KVL to the collector circuit of Fig. P. 1.22.6(b) we can write that,

$$V_{CC} = (I_C \times 5\ k) + V_{CE} + (I_E \times 1\ k)$$

$$\therefore V_{CE} = 12 - (0.944 \times 5) - (0.96 \times 1) = 6.32\ \text{Volts.}$$

$$\therefore V_{CE} = 6.32\ V \text{ and } I_C = 0.944\ mA$$

These are the co-ordinates of Q point.

Part II : To draw the load line

The expression for the collector loop is modified as,

$$V_{CC} = I_C R_C + V_{CE} + I_E R_E$$

$$V_{CE} = V_{CC} - I_C \left[R_C + \frac{R_E}{\alpha} \right] \qquad \text{... Since } I_E = \frac{I_C}{\alpha}$$

$$\therefore V_{CE} = V_{CC} - I_C\left[R_C + \frac{(1+\beta)}{\beta} R_E\right] \quad \text{... Since } \alpha = \frac{\beta}{1+\beta}$$

Substituting the values we get,

$$V_{CE} = 12 - I_C\left[5 + \left(\frac{61}{60} \times 1\right)\right] = 12 - I_C\,(6.01)$$

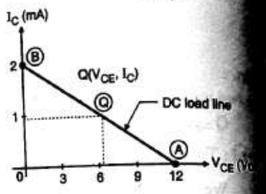
$$\therefore V_{CE} = 12 - 6\, I_C \quad \text{...(1)}$$

Substitute $I_C = 0$ to get point A as,

Point A : $V_{CE} = 12 \quad ...(I_C = 0)$

Substitute $V_{CE} = 0$ to get point B as,

Point B : $I_C = 2\text{ mA} \quad ...(V_{CE} = 0)$



(B-1158) **Fig. P. 1.22.6(c) : DC load li...**

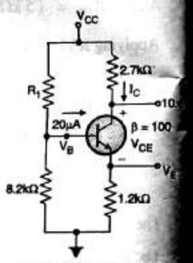
The DC load line is drawn by joining points A and B as shown in Fig. P. 1.22.6(c).

Thus the two most important characteristics in the DC analysis of a bipolar transistor are Q-point and the DC load line.

Ex. 1.22.7 : For the voltage-divider bias configuration of Fig. P. 1.22.7 determine :

1. I_C 2. V_E 3. V_{CC}

4. V_{CE} 5. V_B 6. R_1



(B-2686) **Fig. P. 1.22.7**

Soln. :

Given : $R_C = 2.7\text{ k}\Omega$, $R_E = 1.2\text{ k}\Omega$, $\beta = 100$, $V_C = 10.6$ V

$I_B = 20\ \mu A$ $R_2 = 8.2\text{ k}\Omega$

To find : 1. I_C 2. V_E 3. V_{CC} 4. V_{CE} 5. V_B 6. R_1

1. To find I_C :

$$\beta = \frac{I_C}{I_B} \quad \therefore 100 = \frac{I_C}{20 \times 10^{-6}}$$

$$\therefore I_C = 2\text{ mA}$$

2. To find V_E :

$$V_E = I_E R_E \text{ and } I_E = (1+\beta) I_B$$

$$= (1 + 100)\, 20 \times 10^{-6} \times 1.2 \times 10^3 = 2.424\text{ V}$$

3. To find V_{CC} :

$$V_{CC} = I_C R_C + V_C$$

$$= 2 \times 10^{-3} \times 2.7 \times 10^3 + 10.6$$

$$= 16\text{ V}$$

4. To find V_{CE} :

$$V_{CE} = V_C - V_E = 10.6 - 2.424$$

$$= 8.176\text{ V}$$

5. To find V_B :

$$V_B = V_E + V_{BE}$$

$$= 2.424 + 0.7 \quad (\because V_{BE} = 0.7\text{ V})$$

$$V_B = 3.124\text{ V} \quad \text{...Ans.}$$

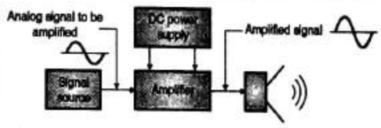
6. To find R_1 :

$$R_1 = \frac{V_{CC} - V_B}{I_B}$$

$$= \frac{16 - 3.124}{20 \times 10^{-6}} = 643.8\text{ k}\Omega \quad \text{...Ans.}$$

1.23 Amplifier :

- The **linear amplifiers** are designed to amplify the analog signals. Analog signals are the signals the magnitude can take any value and the signal can vary continuously with time.

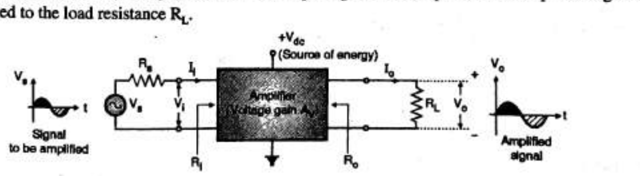


(F-389) **Fig. 1.23.1 : Block diagram of an amplifier**

- A **linear amplifier** is then suppose to multiply the input signal by a constant to produce the output. This multiplying factor is greater than 1 and called as "gain" of the amplifier.

1.23.1 Block Diagram of an Amplifier :

- The block diagram of an amplifier is as shown in Fig. 1.23.2.
- In order to magnify the input signal V_s, all the amplifiers need a source of energy. This may be provided by a battery or a dc power supply. The dc power supply is also essential for biasing the BJT used in the amplifier circuit.
- The amplifier should contain atleast one active device such as a transistor or Field Effect Transistor (FET) or an operational amplifier (OP-AMP). If a transistor is used then it should be biased in the active region.
- In Fig. 1.23.2, the source V_s represents the ac input signal to be amplified. The amplified signal is applied to the load resistance R_L.



(B-225) **Fig. 1.23.2 : Block diagram of an amplifier showing important parameters**

1.23.2 Amplifier Characteristics :

For comparing the amplifier performance we have to define certain important characteristics. Some of them are :

1. Voltage gain A_V and current gain A_I
2. Input resistance R_i
3. Output resistance R_o
4. Power gain A_p
5. The current gain A_I

1. Gain :

- The gain of an amplifier is defined as the ratio of output quantity to the input quantity.
- So the ratio of output voltage to input voltage will be called as "voltage gain A_V" of the amplifier.
- Similarly the ratio of output current to input current is called as "current gain A_I" of the amplifier.

$$\therefore \text{ Current gain, } A_I = \frac{I_o}{I_i} \quad \text{...(1.23.1)}$$

$$\text{And voltage gain, } A_V = \frac{V_o}{V_i} \quad \text{...(1.23.2)}$$

- The voltage and current gains of an amplifier should be as large as possible. Gain is unitless quantity.

2. Input resistance (R_i) :

- It is the resistance seen looking into the input terminals of an amplifier.
- More generally an input impedance Z_i is defined, however Z_i is resistive for the transistorised amplifiers hence it is replaced by R_i.
- Ideally R_i should be infinite and practically it should be as large as possible.

3. Output resistance (R_o) :

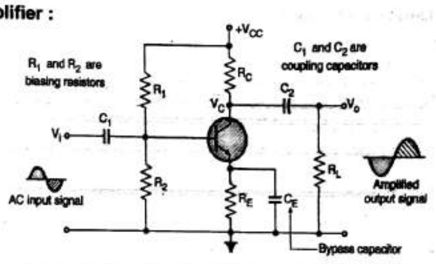
- It is the resistance seen looking into the output terminals of an amplifier when the input signal source V_s is short circuited ($V_i = 0$) and output terminals are open circuited.
- R_o should be ideally equal to zero and practically it should be as small as possible.
- Thus current gain, voltage gain, input resistance and output resistance are the four important parameters of an amplifier.

1.24 Types of BJT Amplifiers :

- The transistor amplifiers are classified into three categories :
  1. Common Emitter (CE) amplifier.
  2. Common Base (CB) amplifier.
  3. Common Collector (CC) amplifier.
- Let us discuss them one by one.

1.24.1 Base Biased C.E. Amplifier :

- Let us consider a practical transistor amplifier circuit, discuss its operation and then analyze the behaviour of this circuit using the dc load line.
- A single stage RC coupled amplifier using transistor as an active device is as shown in Fig. 1.24.1.



(F-390) **Fig. 1.24.1 : Single stage RC coupled CE amplifier**

Description :

- The capacitors C_1 and C_2 are called as the coupling capacitors. As the load resistor R_L is coupled to the amplifier through the coupling capacitor, this amplifier is called as RC coupled amplifier.
- The transistor is connected in the Common Emitter (CE) configuration. Therefore this amplifier is called CE amplifier.

Circuit components and their functions :

- Let us now know about the function of each component in the RC coupled amplifier.
- Resistors R_1, R_2 and R_E are used for biasing the transistor in the active region, because for operating the transistor as an amplifier it is necessary to bias it in the active region. The type of biasing circuit used here is voltage divider bias or self bias.
- R_C is the collector resistor used for controlling the collector current.

Input coupling capacitor C_1 :

- The input coupling capacitor C_1 is used for coupling the ac input voltage V_i to the base of the transistor.
- As capacitors block dc, this capacitor helps to block any dc component present in V_i and couples only the ac component of the input signal. This capacitor also ensures that the dc biasing conditions of the transistor remain unchanged even after application of the input signal.
- For the coupling capacitor C_1 to work properly its value should be large enough so that its reactance is less than 10 % of the input resistance of the amplifier at the lowest operating frequency of the amplifier.

Output coupling capacitor C_2 :

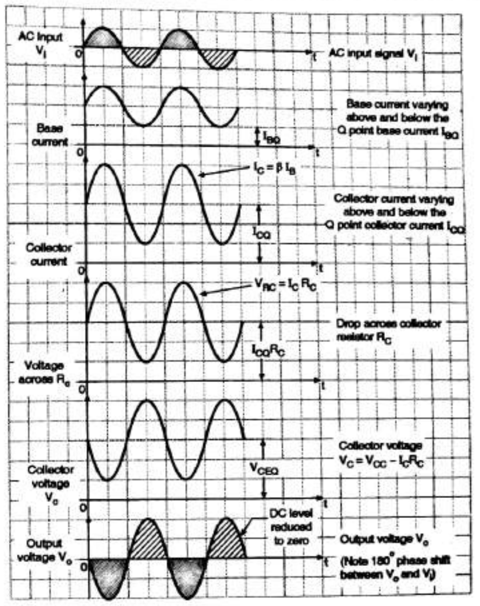
- This capacitor couples the amplifier output to the load resistance or to the next stage of the amplifier.
- It is used for blocking the dc part and passing only the ac part of the amplified signal to the load.

Operation of the amplifier :

- When the ac input signal is absent, the value of dc base current of the transistor is I_{BQ} wh called as the quiescent point base current.
- Corresponding to I_{BQ}, a quiescent dc collector current I_{CQ} also flows through the tran ($I_{CQ} = \beta_{dc} I_{BQ}$) and the collector emitter voltage is V_{CEQ}. All this happens because we have bi the transistor in its active region using the biasing components.
- When small ac sinusoidal signal is applied at the input of the amplifier, an alternating base cur starts flowing in the circuit.
- This base current varies above and below the Q point value of the base current (I_{BQ}) as shown Fig. 1.24.2. Thus we can say that the ac signal is superimposed on the DC current I_{BQ}.
- Due to these variations in the base current, proportional variations take place in the colle current, because $I_C = \beta I_B$. As I_B increases I_C also increases and with decrease in I_B, I_C will decrease as shown in Fig. 1.24.2.
- Thus I_B and I_C are in phase but I_C is a magnified version of I_B. **I_C varies above and below** th **point value I_{CQ}.**
- This varying I_C passes through the collector resistor R_C to produce a varying voltage drop I_C across it. This voltage drop $I_C R_C$ is in phase with the collector and base currents as shown Fig. 1.24.2.
- The collector voltage is given by,

$$V_C = V_{CC} - I_C R_C \quad ...(1.24$$

- Therefore with changes in the voltage drop $I_C R_C$, the collector voltage also will vary as shown Fig. 1.24.2.
- However V_C and $I_C R_C$ will vary in exactly opposite manner with respect to each other, because $I_C R_C$ increases V_C has to decrease according to Equation (1.24.1).
- This collector voltage is then coupled to the load through the coupling capacitor C_2. It will b the dc part of V_C and allow only ac part to pass through as shown in Fig. 1.24.2.
- See the ac signal amplitude obtained after C_2. This is the output voltage. Its magnitude is higher than that of the input signal and its shape is exactly same as that of the input signal. the input ac signal has been successfully amplified.



(F-391) **Fig. 1.24.2 : Waveforms showing the process of amplification**

Phase relationship between input and output :

As seen from Fig. 1.24.2, there is a 180° phase shift between the output and input or the output is said to be an "inverted" version of input. Thus for a CE amplifier there is a phase shift of between V_i and V_o.

Role of coupling capacitor C_2 :

The coupling capacitor C_2 couples (allows to pass without any change) the AC part of the signal to the output and blocks the DC part as shown in Fig. 1.24.2. Therefore the signal obtained after the coupling capacitor C_2 has a zero DC value.

Does the frequency change in the amplification process ?

Comparing the waveforms of V_i and V_o we conclude that the frequency of the amplified output signal V_o is same as that of the input voltage. Thus in the amplification process, the frequency of the signal remains unchanged.

1.24.2 Voltage Gain A_V and Output Voltage :

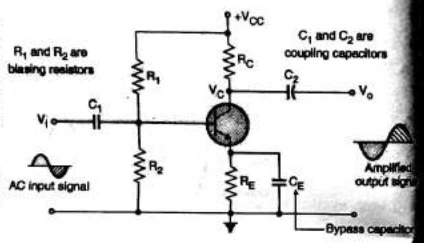
- Amplifier voltage gain $A_V = \frac{V_o}{V_i}$
- Hence the output voltage $V_o = A_V \times V_i$.

1.24.3 Features of CE Amplifier :

1. Input signal is applied at the base and amplified output is obtained at the collector.
2. Voltage gain is high.
3. Current gain is high.
4. Input impedance is moderately high.
5. Output impedance is moderately high.
6. There is a phase shift of 180° between the input and output.

1.24.4 Emitter Biased Amplifier :

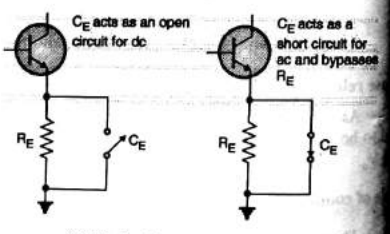
- The Q point of base biased amplifier is unstable. Therefore this amplifier is not much used. Instead the emitter biased amplifier (either VDB or TSEB) is preferred due to its stable Q point.
- The circuit of a CE amplifier with voltage divider bias is as shown in Fig. 1.24.3.
- Note that R_1, R_2 and R_E provide the voltage divider biasing. C_E is known as the bypass capacitor whereas C_1 and C_2 are the coupling capacitors.



(B-231) **Fig. 1.24.3 : CE amplifier with VD bias**

1.24.5 Bypass Capacitor C_E :

- The capacitor connected in parallel with the emitter resistor R_E is called as the emitter bypass capacitor.
- Resistance R_E should be as high as possible for very good, Q point stability under dc conditions but R_E should be as small as possible for the ac voltage gain to be high.
- These are contradicting requirements. But use of C_E fulfills both of them satisfactorily. The function of C_E is as explained below. Refer Fig. 1.24.4.



(a) Under dc conditions **(b) Under ac conditions**

(B-926) **Fig. 1.24.4 : Function of bypass capacitor C_E**

- This capacitor offers a low reactance to the amplified ac signal. Therefore the emitter resistor R_E gets bypassed through C_E for only the ac signals. This will increase the voltage gain of the amplifier.

- Moreover as C_E acts as an open circuit for dc voltages, it does not bypass R_E for dc conditions. So R_E is present in the circuit only for dc conditions and high Q point stability is obtained.
- Thus presence of C_E does not alter the dc biasing conditions.

Selection criteria for the emitter bypass capacitor C_E :

- In order to successfully bypass (short circuit) the emitter resistance R_E, the reactance X_{CE} of the bypass capacitor C_E should be very small as compared to R_E even at the lowest input frequency $f = f_{min}$.
- So the selection criteria for C_E is as follows :

$$X_{CE} \leq \frac{R_E}{10} \quad \text{... at } f = f_{min}$$

$$\text{or } X_{CE} \leq 0.1\, R_E \quad \text{... at } f = f_{min} \qquad \text{...(1.24.2)}$$

Operation of the Amplifier :

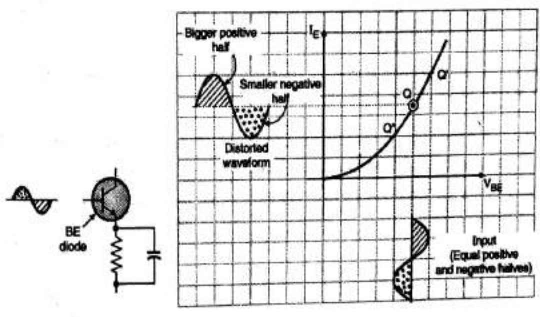
- The operation of the CE amplifier with emitter bias is same as that of the CE amplifier with base bias (discussed earlier).
- The waveforms of this amplifier are same as those of the base biased CE amplifier.

1.25 Small Signal Operation :

- Fig. 1.25.1(a) shows the VI characteristics of a forward biased BE junction diode of a transistor. The sinusoidal variation in V_{BE} corresponds to the ac signal applied at the base terminal of a transistor.

Q point :

- With the variation in input voltage and V_{BE}, the operating point Q moves between the two extremes Q′ and Q″ as shown in Fig. 1.25.1(b).
- The size of ac input voltage decides the amount of variation in the position of Q point.



(a) Partially drawn amplifier **(b) Distortion when the input single is too large**

(F-4406) **Fig. 1.25.1**

1.25.1 Distortion :

- The variation in V_{BE} results in corresponding sinusoidal variation in the emitter current as show in Fig. 1.25.1(b). The frequency of the ac emitter current is same as that of the ac V_{BE} voltage. Th shape of I_E waveform also is approximately same as that of the V_{BE} waveform.
- If the input voltage is too large then the emitter current waveform is **distorted** as shown Fig. 1.25.1(b). That means shape of I_E is not exactly same as that of the V_{BE} waveform.
- This happens due to the **nonlinear** nature of the characteristics of BE diode. Such a distortion **undesirable** because it degrades the quality of output.

Reducing the distortion :

- One way of reducing the distortion in Fig. 1.25.1(b) is to keep the ac base voltage **small.** Th smaller base voltage would result in a **smaller** swing of Q point.
- The smaller swing or variation in the Q point position means less curvature of the graph. If th input signal is small then the graph appears to be **linear** and the distortion will be either **reduce** or **eliminated.**

1.25.2 Definition of Small Signal Operation :

The 10 % rule :

- In Fig. 1.25.1(b) the total emitter current consists of a dc component and an ac component. Henc the total emitter current is given by,

$$\underbrace{I_E}_{\text{Total emitter current}} = \underbrace{I_{EQ}}_{\text{DC component}} + \underbrace{i_e}_{\text{AC component}}$$

(F-4407)

- If the ac component i_e is smaller than the dc component I_{EQ} then the distortion can be reduced This operation is called as small **signal operation.**
- We can define the small signal operation as follows. The condition for small signal operation is a follows :

$$\text{Small signal} \;:\; i_{e\,(\text{peak to peak})} < 0.1\, I_{EQ}$$

- That means the peak to peak ac emitter current is less than **10 percent** of the dc emitter curren This is known as the **10 percent** and the amplifiers which satisfy this rule are called as **sma signal amplifiers.**
- The small signal amplifiers are used at the **front end** (input blocks) of the radio and TV receivers

1.26 AC Resistance of the Emitter Diode :

- Fig. 1.26.1 shows the V – I characteristics of the forward biased emitter diode (V_{BE} versus I_E).
- The total emitter current is given by,

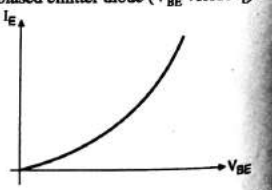
$$I_E = I_{EQ} + i_e \qquad \text{...(1.26.1)}$$

- Similarly the total base emitter voltage is given by,

$$V_{BE} = \underbrace{V_{BEQ}}_{\text{DC part}} + \underbrace{V_{be}}_{\text{AC part}} \qquad \text{...(1.26.2)}$$

(F-4406(a))

- The ac emitter resistance r_e' is defined as :

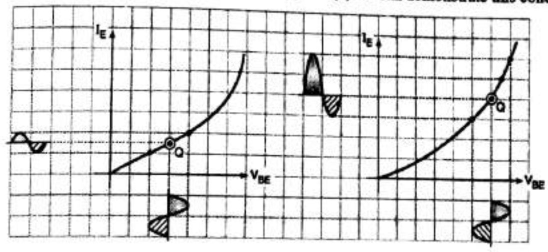


(F-4409) Fig. 1.26.1 : AC resistance of an emitter diod

$$r_e' = \frac{v_{be}}{i_e} \qquad \text{...(1.26.3)}$$

Where both v_{be} and i_e are ac quantities.

- The ac emitter resistance (r_e') is inside the transistor. Its vale is dependent not only on v_{be} and i_e but also on the location of Q point. From Fig. 1.26.2(a) and (b) we can demonstrate this concept.



(a) Small emitter resistance (b) Large emitter resistance

(F-4410) Fig. 1.26.2 : Calculation of r_e'

- A simple formula for ac resistance r_e' can be derived as follows :

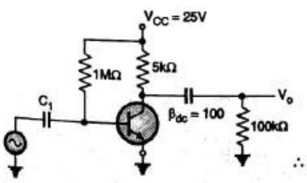
$$r_e' = \frac{25\text{ mA}}{I_E} \qquad \text{...(1.26.4)}$$

Where I_E = dc emitter current (at Q point)

- The importance of r_e' is that it is used to calculate the value of amplifier gain.

Ex. 1.26.1 : For base biased amplifier of Fig. P. 1.26.1 find the value of r_e'.

Soln. :



$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{25 - 0.7}{1 \times 10^6}$$

$$= 24.3\ \mu A \qquad \text{...(1)}$$

$$I_E = (1 + \beta_{dc}) \times I_B = 101 \times 24.3 \times 10^{-6}$$

$$I_E = 2.45\text{ mA}$$

$$\therefore\; r_e' = \frac{25\text{ mV}}{I_E} = 10.20\ \Omega \qquad \text{...Ans.}$$

(F-4411) Fig. P. 1.26.1 : Base biased amplifier

1.27 Analysis of Transistor Amplifiers :

- In this chapter we are going to analyse various transistor amplifier configurations.
- The procedure to be followed for such an analysis is as follows :

Procedure for analysis :

Step 1 : Draw the ac equivalent circuit of the given amplifier.
Step 2 : Replace the transistor by its h-parameter or r_e equivalent circuit.
Step 3 : Calculate voltage gain, current gain, input impedance and output impedance.

- Hence before proceeding further, we have to learn about the ac equivalent circuit and h-parameter and r parameter equivalent circuits.

1.27.1 AC Equivalent Circuit :

In order to carry out only the ac analysis of an amplifier the first step is to draw its ac equivalent circuit. The rules to be followed for drawing the ac equivalent circuit are as follows :

Rules to be followed to draw the ac equivalent circuit :

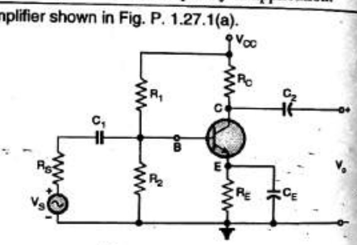
1. Replace all the dc sources by a short circuit. The dc voltages are important only in deciding the Q point of the amplifier.
2. The coupling capacitors C_1 and C_2 and the bypass capacitor C_E are to be replaced by short circuit. This is because the reactance of these capacitors is very small at the frequency of application.

Ex. 1.27.1 : Draw the ac equivalent circuit of the amplifier shown in Fig. P. 1.27.1(a).

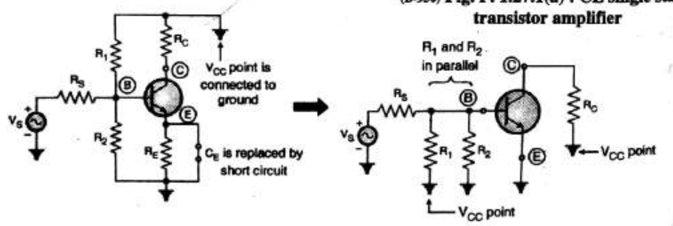
Soln. :

- The ac equivalent circuit shown in Fig. P. 1.27.1(b) is drawn by following the procedure given below :

1. As the bypass capacitor C_E is assumed to be short circuited, the resistor R_E is bypassed i.e. shorted out, thus emitter is directly connected to ground point.



(B-930) **Fig. P. 1.27.1(a) : CE single stage transistor amplifier**

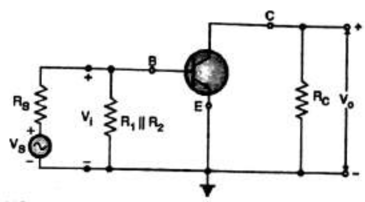


(b) (c)

(B-931) **Fig. P. 1.27.1**

2. As the dc supplies are to be replaced by short circuits, the + V_{CC} point is connected to ground. Therefore R_C appears between collector and ground and R_1 appears in parallel with R_2 as shown in Fig. P. 1.27.1(c). The ac equivalent circuit is as shown in Fig. P. 1.27.1(b).

- The final ac equivalent circuit is shown in Fig. P. 1.27.1(d).



(B-932) **Fig. P. 1.27.1(d) : AC equivalent circuit of the amplifier shown in Fig. P. 1.27.1(a)**

1.27.2 Various Models used for AC Analysis :

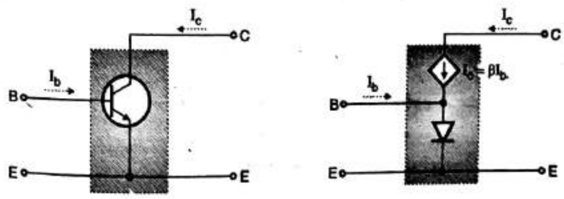
- After drawing the AC equivalent circuit of an amplifier, the next step is to replace the transistor by one of its models. The commonly used models for the small signal ac analysis are :
  1. Small signal r_e model.
  2. Small signal hybrid model. (using h-parameters)
  3. Small signal hybrid - π model.
- The word model has a meaning of equivalent circuit. Let us discuss these equivalent circuits one by one.

1.27.3 The "r_e" Transistor Model :

The r_e model makes use of a diode and controlled current source to represent the behaviour of a transistor in the region of interest.

r_e Model for common emitter configuration :

- The common emitter configuration is as shown in Fig. 1.27.1(a). The input is connected between base and emitter terminals and BE junction is forward biased.
- The output is obtained between the collector and emitter and the CB junction is reverse biased. The BE junction is replaced by a diode and a controlled current source appears between the collector and base terminals as shown in Fig. 1.27.1(b).



(a) A common emitter configuration (b) Approximate model for the CE configuration

(F-1425) **Fig. 1.27.1**

- The controlled current source between collector and base terminals has a value of I_c and it is given by,

$$I_c = \beta I_b \quad \text{...(1.27.1)}$$

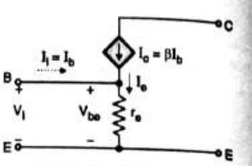
- The current through the diode in Fig. 1.27.1 (b) is given by :

$$I_e = I_c + I_b = (1 + \beta) I_b \qquad \ldots(1.27.2)$$

- But as the value of β is much higher than 1 we can write that,

$$I_e \approx \beta I_b \qquad \ldots(1.27.3)$$

- We can replace the diode in the approximate model of Fig. 1.27.1 (b) by the ac resistance r_e and draw the r_e model for the common emitter configuration as shown in Fig. 1.27.2.



(F-1426) **Fig. 1.27.2 : r_e model for the CE configuration**

Merits of r_e model :

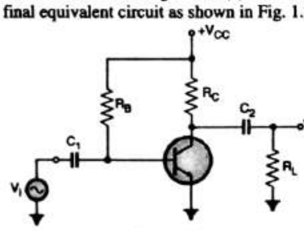
1. The parameters of r_e model can be determined for any region of operation within the active region.
2. It is a simple and less elaborate model of a transistor.
3. These parameters can be obtained easily from the "h" parameters which are specified by the manufacturer.

Demerits of r_e model :

1. r_e model does not take in to consideration the output impedance level of a device and feedback effect from the output to input.
2. This model is sensitive to the dc level of operation of the amplifier. Therefore the input resistance will vary with the dc operating point.

1.28 Analysis of Base Biased CE Amplifier :

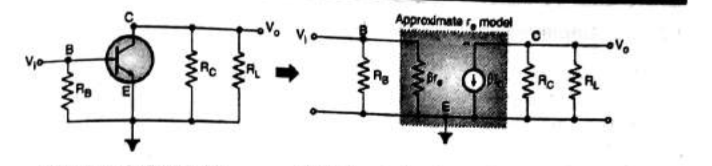
Draw the ac equivalent circuit as shown in Fig. 1.28.1(a) and then replace the transistor by the approximate r_e model to get the final equivalent circuit as shown in Fig. 1.28.1(b).



(F-1436) **Fig. 1.28.1**

Voltage gain :

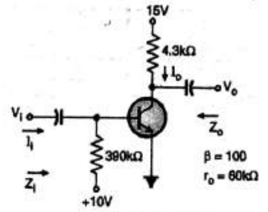
$$\text{Voltage gain} \quad A_V = \frac{V_o}{V_i} = \frac{-\beta I_b (R_C \| R_L)}{I_b \times \beta_{re}} = \frac{-(R_C \| R_L)}{r_e} \qquad \ldots(1.28.1)$$



(a) AC equivalent circuit **(b) AC equivalent circuit with approximate r_e model**

(F-1437) **Fig. 1.28.1**

Ex. 1.28.1 : For the common emitter or fixed bias configuration in Fig. P. 1.28.1(a), determine r_e, Z_i, Z_o, A_V.



(B-2710) **Fig. P. 1.28.1(a)**

Soln. :

Step 1 : Draw the equivalent circuit :

The equivalent circuit is as follows :

Step 2 : Find r_e' :

Apply KVL to the base loop of Fig. P. 1.28.1(a) to write,

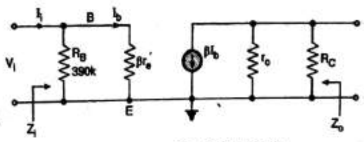
$$+10 = I_B R_B + V_{BE}$$

$$\therefore \quad I_B = \frac{10 - 0.7}{390 \times 10^3} = 23.85\ \mu A$$

$$\therefore \quad I_E = (1 + \beta) I_B$$

$$= 101 \times 23.85 \times 10^{-6} = 2.4\ mA$$

$$\therefore \quad r_e' = \frac{25\ mV}{I_E} = \frac{25}{2.4} = 10.42\ \Omega \qquad \ldots\text{Ans.}$$



(B-2761) **Fig. P. 1.28.1(b)**

Step 3 : Find Z_i, Z_o, A_V :

$$Z_i = R_B \| \beta r_e' = 390\ k \| (100 \times 10.42) = 1.04\ k\Omega \qquad \ldots\text{Ans.}$$

$$Z_o = R_C \| r_o = 60\ k \| 4.3\ k = 4.01\ k\Omega \qquad \ldots\text{Ans.}$$

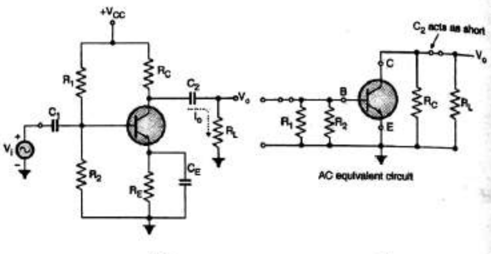
$$A_V = \frac{-\beta I_b (R_C \| r_o)}{\beta r_e' I_b} = \frac{-R_C \| r_o}{r_e'} = \frac{-4.01 \times 10^3}{10.42} = -385 \qquad \ldots\text{Ans.}$$

1.29 CE Amplifier with Bypassed R_E :

- The common emitter amplifier with bypassed emitter resistance is shown in Fig. 1.29.1(a) and its ac equivalent circuit is shown in Fig. 1.29.1(b).

Approximate analysis :

- In the approximate analysis we will neglect "r_o" in the r_e model of the transistor.

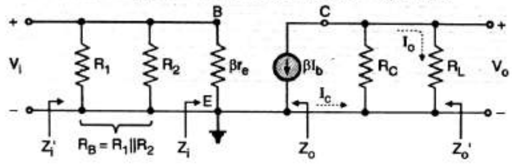


(a) (b)

(F-1438) **Fig. 1.29.1 : CE amplifier with bypassed R_E**

Step 1 : Draw the ac equivalent circuit using the r_e model :

- The ac equivalent circuit with approximate r_e model is shown in Fig. 1.29.2.



(F-1439) **Fig. 1.29.2 : Approximate r_e model**

Step 2 : Calculate the input impedance :

- Looking at Fig. 1.29.2 we get,

$$\text{Input impedance } Z_i = \beta r_e \text{ ohms} \qquad \ldots(1.29.1)$$

- And the input impedance, taking the biasing resistances into consideration is given by,

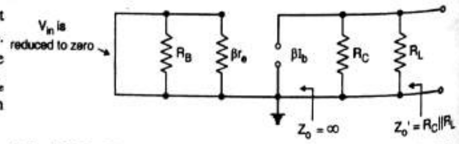
$$Z_i' = (R_1 \| R_2 \| Z_i)$$

$$\therefore \; Z_i' = R_B \| \beta r_e \qquad \ldots(1.29.2)$$

$$\text{where } R_B = R_1 \| R_2 \quad \therefore \quad Z_i' = \frac{R_B \, \beta r_e}{R_B + \beta r_e} \qquad \ldots(1.29.3)$$

Step 3 : Output impedance (Z_o and Z_o') :

- To calculate Z_o and Z_o', the input voltage V_i should be set to zero. If $V_i = 0$, then $I_b = 0$ and the source $\beta I_b = 0$. Hence the r_e model is modified as shown in Fig. 1.29.3.



(F-1440) **Fig. 1.29.3 : Equivalent circuit to calculate output resistance**

- Hence the output impedance $Z_o = \infty$.
- And the output impedance taking the load resistance into consideration is given by,

$$Z_o' = R_C \| R_L = \frac{R_C R_L}{(R_C + R_L)} \qquad \ldots(1.29.4)$$

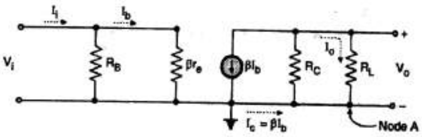
Step 4 : Voltage gain A_V :

- The voltage gain is given by,

$$A_V = \frac{V_o}{V_i} = \frac{I_o R_L}{I_b \times \beta r_e} \qquad \ldots(1.29.5)$$

For further calculations refer Fig. 1.29.4.

- At node A of Fig. 1.29.4 the collector current gets divided.



(F-1441) **Fig. 1.29.4 : Equivalent circuit for voltage gain**

$$\therefore \; I_o = -\frac{R_C}{(R_C + R_L)} \times I_c \qquad \ldots(1.29.6)$$

- The negative sign is due to the opposite directions of I_o and I_c.
- Substituting expression for I_o into Equation (1.29.5) we get,

$$A_V = \frac{-R_C R_L I_c}{(R_C + R_L) \beta r_e I_b}$$

- But $I_c = \beta I_b$

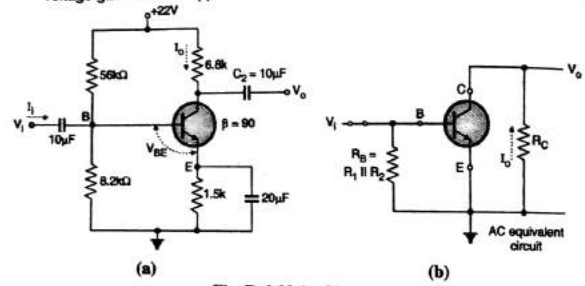
$$\therefore \; A_V = \frac{-R_C R_L \beta I_b}{(R_C + R_L) \beta I_b r_e} = \frac{-R_C R_L}{(R_C + R_L) r_e}$$

$$\text{But } \frac{R_C R_L}{(R_C + R_L)} = (R_C \| R_L)$$

$$\therefore \; A_V = \frac{-(R_C \| R_L)}{r_e} \qquad \ldots(1.29.7)$$

- The negative sign indicates that there is a 180° phase shift between the input and output voltages.

Ex. 1.29.1 : For the circuit shown in Fig. P. 1.29.1(a) and (b) determine the input impedance and voltage gain. Use the approximate analysis.



(F-1443) **Fig. P. 1.29.1 : Given Circuit**

Soln. :

Step 1 : Calculate r_e :

- For the DC conditions test $\beta R_E > 10\ R_2$. Substituting the values we get,

$$\beta R_E = 90 \times 1.5\ k = 135\ k\Omega \quad \text{and} \quad 10\ R_2 = 10 \times 8.2\ k = 82\ k\Omega$$

$\therefore$ **$\beta R_E > 10\ R_2$ is satisfied. So we will use the approximate analysis.**

$$\therefore\ V_B = \frac{R_2}{(R_1 + R_2)} \times V_{CC} = \frac{8.2}{56 + 8.2} \times 22$$

$$\therefore\ V_B = 2.81 \text{ Volts}$$

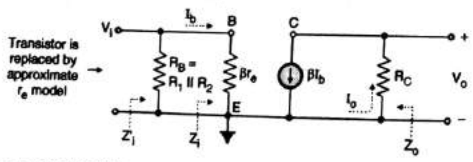
$$\therefore\ V_E = V_B - V_{BE} = 2.81 - 0.7 = 2.11 \text{ Volts}$$

$$\therefore\ \text{Emitter current } I_E = \frac{V_E}{R_E} = \frac{2.11}{1.5 \times 10^3} = 1.41 \text{ mA}$$

$$\therefore\ r_e = \frac{26 \text{ mV}}{I_E} = \frac{26}{1.41} = 18.44\ \Omega$$

Step 2 : Draw the AC equivalent circuit using the approximate r_e model :

The ac equivalent circuit is shown in Fig. P. 1.29.1(b) and the ac equivalent circuit using approximate r_e model is shown in Fig. P. 1.29.1(c).



(F-1444) **Fig. P. 1.29.1(c) : Approximate ac equivalent circuit using r_e model**

Step 3 : Calculate input impedance :

- Referring to Fig. P. 1.29.1(c) we can write,

$$Z_i = \beta r_e = 90 \times 18.44 = 1.66\ k\Omega \quad \text{...Ans.}$$

- Considering the effect of biasing resistors R_1 and R_2.

$$Z_i' = R_B \parallel Z_i$$

- But $R_B = R_1 \parallel R_2 = 56\ k \parallel 8.2\ k = \frac{56 \times 8.2}{56 + 8.2}$

$$\therefore\ R_B = 7.15\ k\Omega$$

$$\therefore\ Z_i' = 7.15\ k \parallel 1.66\ k\Omega = \frac{7.15 \times 1.66}{(7.15 + 1.66)} = 1.35\ k\Omega \quad \text{...Ans.}$$

Step 4 : Voltage gain A_V :

$$\text{Voltage gain } A_V = \frac{V_o}{V_i} = \frac{-I_o R_C}{I_b Z_i}$$

- But $I_o = I_c = \beta I_b$ and $Z_i = \beta r_e$

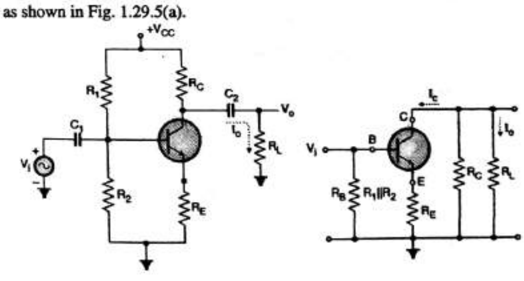
$$\therefore\ \text{Voltage gain } A_V = \frac{-\beta I_b R_C}{I_b \times \beta r_e} = \frac{-R_C}{r_e} = \frac{-6.8\ k\Omega}{18.44\ \Omega}$$

$$= -368.76 \quad \text{...Ans.}$$

- The negative sign indicates that there is a phase shift of 180° between the input and output.

1.29.1 CE Amplifier with Unbypassed R_E :

The amplifier configuration discussed in this section consists of an emitter resistance R_E which is unbypassed as shown in Fig. 1.29.5(a).



(a) CE amplifier with unbypassed R_E (b) AC equivalent circuit

(F-1447) **Fig. 1.29.5**

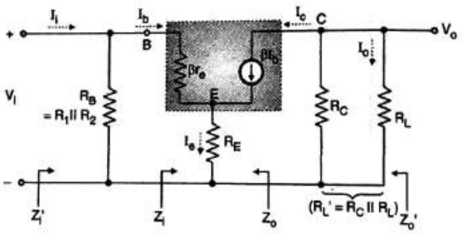
Approximate analysis :

Step 1 : Draw ac equivalent circuit using r_e model :

- The ac equivalent by replacing all capacitors and V_{CC} by short circuit is shown in Fig. 1.29.5(b).
- The ac equivalent circuit with r_e model is shown in Fig. 1.29.6. Note that the approximate r_e model is being used which does not include r_o.

Step 2 : Calculate the input impedances :

- Applying KVL to the input side of Fig. 1.29.6 we get,



(F-1448) **Fig. 1.29.6 : Approximate r_e model**

$$V_i = \beta r_e I_b + I_e R_E$$
$$= \beta r_e I_b + (1+\beta) I_b R_E$$
$$\therefore V_i = I_b [\beta r_e + (1+\beta) R_E]$$

$$\therefore \text{Input impedance } Z_i = \frac{V_i}{I_b} = \beta r_e + (1+\beta) R_E \quad \ldots(1.29.8)$$

$$\text{As } \beta \approx (1+\beta)$$
$$Z_i = \beta r_e + \beta R_E = \beta (r_e + R_E) \quad \ldots(1.29.9)$$

- Input impedance by taking the biasing resistors R_1 and R_2 into consideration is given by,

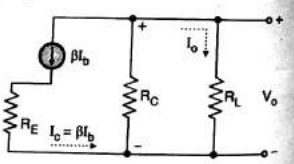
$$Z_i' = R_B \| Z_i = R_B \| \beta (r_e + R_E)$$
$$Z_i' = \frac{\beta R_B (r_e + R_E)}{R_B + \beta (r_e + R_E)} \quad \ldots(1.29.10)$$

Step 3 : To obtain the voltage gain A_V :

- To obtain the voltage gain refer Fig. 1.29.7.

$$A_V = \frac{V_o}{V_i} = \frac{I_o R_L}{I_b Z_i}$$

But $I_o = -I_c \cdot \dfrac{R_C}{R_C + R_L}$



(F-1450) **Fig. 1.29.7 : Output side (for voltage gain)**

$$\therefore A_V = \frac{-I_c R_C \times R_L}{I_b (R_C + R_L) Z_i}$$

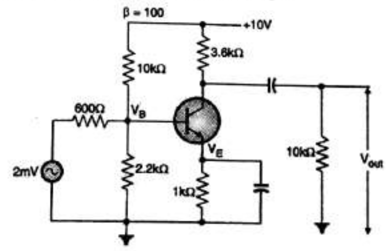
But $I_c / I_b = \beta$ and $Z_i = \beta (r_e + R_E)$

$$\therefore A_V = \frac{-\beta R_C R_L}{(R_C + R_L) \times \beta (r_e + R_E)}$$

But $\dfrac{R_C R_L}{(R_C + R_L)} = R_C \| R_L = R_L'$

$$\therefore A_V = \frac{-R_L'}{(r_e + R_E)} \quad \ldots(1.29.11)$$

Ex. 1.29.2 : In transistor amplifier circuit as shown in Fig. P. 1.29.2, the ac generator has an internal resistance of 600 Ω. Determine the output voltage.



(B-2716) **Fig. P. 1.29.2**

Soln. :

Given : $R_E = 1$ k, $R_1 = 10$ k, $R_2 = 2.2$ k, $R_C = 3.6$ k, $V_i = 2$ mV

Step 1 : Calculate r_e :

For the DC conditions test $\beta R_E > 10 R_2$.

- Assume $\beta = 10$, $\beta R_E = 100 \times 1000 = 100000$

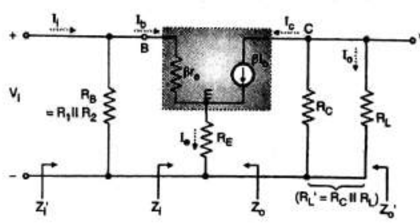
$$10 R_2 = 10 \times 2200 = 22000$$

$\therefore \beta R_E > 10 R_2$ is satisfied. So we will use approximate analysis.

$$\therefore V_B = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{2.2\text{ k}}{10\text{ k} + 2.2\text{ k}} \times 10 = 1.8032 \text{ Volts}$$
$$\therefore V_E = V_B - V_{BE} = 1.8032 - 0.7 = 1.1032 \text{ V}$$
$$\therefore \text{Emitter current } I_E = \frac{V_E}{R_E} = 1.1 \text{ mA}$$
$$\therefore r_e = \frac{26 \text{ mV}}{I_E} = \frac{26}{1.1} = 23.56\ \Omega$$

Step 2 : Draw AC equivalent circuit using r_e model :

$$A_V = \frac{R_L'}{r_e} = \frac{R_C \| R_L}{r_e}$$
$$= \frac{\left(\frac{R_C \times R_L}{R_C + R_L}\right)}{r_e}$$
$$= \frac{\left(\frac{3.6 \times 10}{3.6 + 10}\right)}{23.56}$$
$$= 112.35$$



(F-1448) **Fig. P. 1.29.2(a) : Approximate r_e model**

$$\therefore \text{ Output voltage } V_o = V_i \times A_V$$
$$= 2 \times 10^{-3} \times 112.35$$
$$= 0.22467 \text{ V}$$ ...Ans.

Review Questions

Q. 1 Draw the equivalent circuit of n-p-n transistor and explain the operation of an unbiased transistor.

Q. 2 State the regions of operation of a transistor and explain the biasing conditions for the three regions.

Q. 3 With the help of neat figures, explain the operation of an n-p-n transistor.

Q. 4 With the help of neat diagrams, explain the operation of a p-n-p transistor.

Q. 5 What is reverse saturation current ?

Q. 6 With the help of neat circuit diagram, explain the common emitter configuration of a transistor.

Q. 7 Define and explain the significance of common emitter amplification factor βdc.

Q. 8 What are the applications of CE configuration ?

Q. 9 Define the reverse leakage current of a CE configuration.

Q. 10 List specification for transistors.

Q. 11 Derive $\alpha = \frac{\beta}{\beta + 1}$

Q. 12 Why the E-B junction is forward biased and the C-B junction is reverse biased ?

Q. 13 Define alpha of the transistor.

Q. 14 Define beta of transistor.

Q. 15 Name the three operating regions of the transistor.

Q. 16 Justify that current gain α in common base transistor configuration is less than and nearly equal to 1.

Q. 17 Why transistor is called a bipolar device ?

Q. 18 Define α and β for a transistor and establish the relationship between them.

Q. 19 With the help of experimental setup, explain the procedure to plot the input and output characteristics of a transistor in CE configuration.

Q. 20 List biasing methods for transistor. State necessity for biasing.

Q. 21 State the requirements of biasing circuit.

Q. 22 State the function of biasing circuit.

Q. 23 Define biasing of a transistor and state four methods of transistor biasing.

Q. 24 Define stability factor of biasing circuit.

Q. 25 Explain the need of biasing in transistor amplifier. Draw circuit of fixed bias.

Q. 26 Draw circuit diagram of potential divider biasing circuit.

Q. 27 What are the advantages of voltage divider bias circuit ?

Q. 28 List the three sources of instability of collector current and define three stability factors.

Q. 29 Why is it preferred to locate the Q point at the centre of the active region for amplification purpose ?

Q. 30 State the importance of stability of operating point. List techniques used for the same.

Q. 31 List various biasing methods. Compare them.

Q. 32 Draw voltage divider bias circuit. How it stabilizes operating point ?

Q. 33 How transistor is used as an amplifier ?

Q. 34 What is amplification ? What type of devices do we need to amplify a signal ?

Q. 35 Define R_i, R_o, A_I, A_V and A_P for a voltage amplifier.

Q. 36 What should be the values of R_i and R_o for an ideal and practical voltage amplifier and explain why ?

Q. 37 What do you understand by small signal operation ?

Q. 38 Derive the expressions for A_i, A_V, R_i, R_o for a CE amplifier with bypassed R_E.

□□□

CHAPTER 2

Power Amplifiers

Module 2

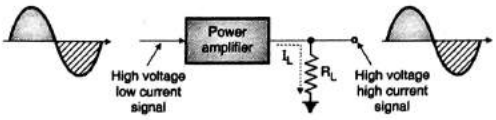
Syllabus :

Introduction, Class A and Class C power amplifiers.

Oscillators : Introduction, Barkhausen criteria, Colpitt's oscillator and crystal oscillator.

2.1 Concept of Power Amplifier :

- The small signal amplifiers are designed to amplify signals of very small magnitude.
- They operate in the linear portion of the transfer characteristics which is very close to the Q point of the amplifier.
- Small signal amplifiers can be analyzed with the help of small signal h-parameters.
- **Small signal amplifiers** are also known as **"voltage amplifiers".** This is because these amplifiers are used primarily for voltage amplification but they are not capable of supplying a large amount of power to the loads such as loud speakers.
- Whenever the load demands a large power, we have to use specially designed amplifiers called "power amplifiers".
- As shown in Fig. 2.1.1, these amplifiers convert low power signal at their input to a high power signal.

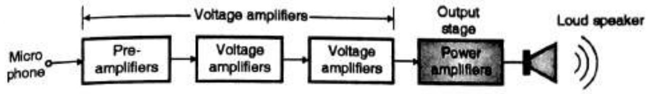


(F-871) **Fig. 2.1.1 : Concept of power amplifiers**

- The input signal to the large signal amplifier is a high voltage low current signal obtained from a chain of voltage amplifiers.
- Power is equal to the product of voltage and current. Hence high voltage, low current signal at the input corresponds to a low power signal.
- The large signal amplifier (power amplifier) will increase the current sourcing and sinking capability. Therefore at its output we get a high voltage, high current signal that means a high power signal. Thus the power amplifier is basically a **current amplifier.**
- Power amplifier is an amplifier which amplifies power or current. It is also called as large signal amplifier.

- The simplest example of power amplifier is the "emitter follower" circuit which has a unity voltage gain but high current gain.
- Power amplifier is usually the last (final) stage in most of the high power circuits. Its output is connected directly to the load. Fig. 2.1.2 shows the block schematic of an AF amplifier which demonstrates this concept.
- The applications of power amplifiers are :

1.	Radio receivers	2.	Public address (P.A.) systems.
3.	CD/casette players	4.	TV receivers.



(F-872) **Fig. 2.1.2 : Simplified block diagram of an AF amplifier**

2.2 Important Features of a Power Amplifier :

Some of the important features of a large signal amplifiers are as follows :

1. Output impedance of the power amplifier should be matched with the load.
2. Power transistors are required to be used.
3. Power amplifiers are bulky.
4. Harmonic distortion is present in their output.
5. They are capable of handling a large power.

Impedance matching :

- As the power amplifiers are handling a large amount of power it is important to transfer maximum power to the load. To do so, impedance matching between the output impedance of the power amplifier and load has to be ensured.
- As the loads like loud speakers have low impedance, the output impedance of a power amplifier also must be low. Therefore the common collector or emitter follower circuit is normally used as the power amplifiers, because it has a low output impedance.
- A transformer may also be used for impedance matching, on the output side.

Use of power transistors :

- As the power amplifiers are designed to handle large powers, the transistors used must be capable of withstanding to large voltages and currents.
- A large power gets dissipated in these transistors in the form of heat. Therefore we cannot use the ordinary logic level transistors.
- Instead specially manufactured power transistors are used.

Power amplifiers are bulky :

- The power transistors are bigger in size than the low power transistors. In addition to this, they are mounted on " heat sinks " which are of large surface area.
- The " heat sinks " help to reduce the temperature of the power transistors by dissipating the heat to the surroundings.
- Due to the use of heat sinks and large size power transistors, the power amplifiers become bulky.

Harmonic distortion :

- Due to the non-linear characteristics of transistors, harmonic distortion will be present on the output side of the amplifier.
- That means, those frequency components which are not present on the input side, will be present in the output.
- This will distort the shape of the output waveform. To measure the percentage of distortion, analysis of the output waveform is carried out.

h-parameters cannot be used for the analysis :

- The h-parameters are called as small signal h-parameters. They are valid if and only if the input signal is small enough to operate the amplifier close to Q point, on the linear portion of the transfer characteristics of the transistor.
- Power amplifiers can not satisfy this condition. Therefore we can not use h-parameters for the analysis of power amplifiers.
- The analysis of power amplifiers is carried out with the help of a load line drawn on the characteristics of the transistor.

Efficiency :

- Efficiency of a power amplifier is defined as the ratio of output power to the total input power.

$$\therefore \quad \% \text{ efficiency } (\eta) = \frac{P_{out}}{P_{in}} \times 100 \qquad ...(2.2.1)$$

- But $P_{in} = P_{out} + P_{losses}$

$$\therefore \quad \% (\eta) = \frac{P_{out}}{P_{out} + P_{losses}} \times 100 \qquad ...(2.2.2)$$

Where P_{losses} is the power lost in the power transistor.

- If the transistor is biased to operate in the active region, then power loss taking place in it will be high and efficiency will be low.
- But if the transistor is biased in the cut off region or below cut off and in the saturation region then the power loss taking place in it is low and the efficiency will be increased.

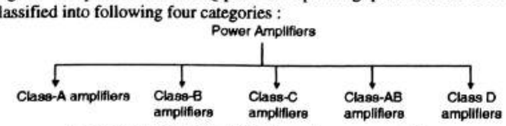
2.2.1 Conversion Efficiency :

- The conversion efficiency or theoretical efficiency is a measure of the ability of an active device to convert the dc power of the supply into an ac (signal) power delivered to the load.
- We can define it as percentage efficiency as,

$$\eta = \frac{\text{Signal power delivered to the load}}{\text{DC power supplied to the output circuit}} \times 100$$

2.3 Classification of Power Amplifiers :

Depending on the position of the Q-point or operating point on the load line, the power amplifiers are classified into following four categories :



(F-873) **Fig. 2.3.1 : Classification of power amplifiers**

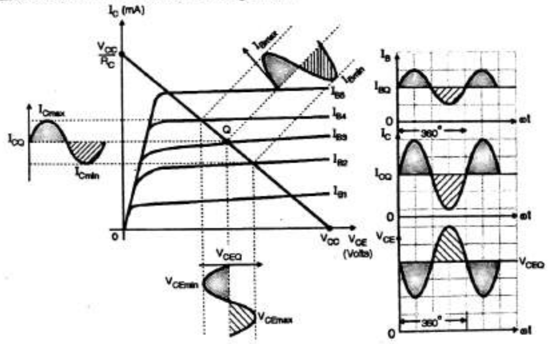
- This classification has been done on the basis of position of Q point on the load line.
- The type of amplifier and the position of Q point are listed in Table 2.3.1.

Table 2.3.1 : Position of Q point for various power amplifiers

Sr. No.	Type of power amplifier	Position of Q-point
1.	Class - A	At the center of load line
2.	Class - B	In the cut off region
3.	Class - AB	Just above the cut off
4.	Class - C	Below the cut off

2.3.1 Class A Power Amplifier :

- A power amplifier is referred to as a class A power amplifier if the transistor used for amplification conducts for the full cycle duration of the input ac signal.
- The Q point is adjusted exactly at the center of the load line as shown in Fig. 2.3.2. Due to this the output signal is obtained for the full cycle of the ac input i.e. for 360° as shown in Fig. 2.3.2.
- The power transistor is biased such that the operating point (Q point) is approximately at the center of the load line as shown in Fig. 2.3.2.



(F-874) **Fig. 2.3.2 : Graphical representation of class A operation**

- Now as we apply the ac signal to the base of the transistor, the base current changes sinusoidally above and below the quiescent base current I_{BQ} as shown in Fig. 2.3.2.
- In response to the changes in I_B, the collector current changes sinusoidally above and below its quiescent current value I_{CQ}. The collector current and base current are in phase with each other.
- Due to changes in I_C, the voltage V_{CE} will also fluctuate sinusoidally as shown in Fig. 2.3.2. Note that V_{CE} and I_C are 180° out of phase.
- The transistor remains in the "active region" for all the values of input signal and never enters into the saturation or cutoff regions.

- As shown in Fig. 2.3.2, the input signal is amplified faithfully, without introducing any distortions. Thus harmonic contents in the output will be low.
- As the transistor continuously operates in its active region, the voltage V_{CE} across it and current I_C through it, both are simultaneously high.
- Therefore a large power will be dissipated in the transistor in the form of heat. Therefore the efficiency of class A power amplifiers is low. In fact it is the lowest of all the power amplifiers.
- Typically the efficiency (η) of a class A power amplifier lies between 25% to 50%.

Advantages of class A amplifier :

1. Simple construction.
2. Distortionless output voltage.

Disadvantages of class A amplifier :

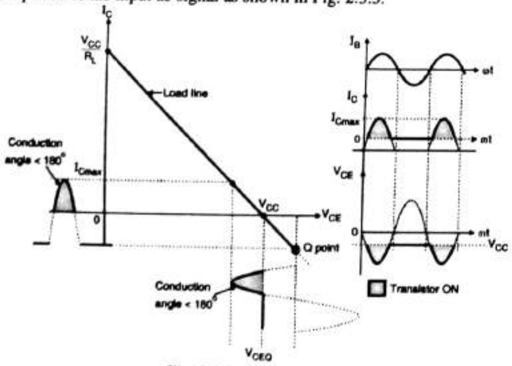
1. Very low efficiency (25% or 50%)
2. Large power dissipation in the power transistors.

Applications of Class A Amplifier :

1. Linear amplifiers.
2. High gain voltage amplifiers.
3. RF and IF amplifiers in radio and TV.

2.3.2 Class C Amplifiers :

- A power amplifier is referred to as a class C power amplifier if its output is obtained for less than a half cycle period of the input ac signal as shown in Fig. 2.3.3.



(F-877) **Fig. 2.3.3 : Class C operation**

- Thus the power transistor in a class C configuration will conduct for a duration which is less than the period of a half cycle of the ac input signal.
- For this the operating point is adjusted to be below the X-axis as shown in Fig. 2.3.3. Thus the transistor is biased below cut-off.

- Due to reduced conduction angle, the output waveform is heavily distorted. The percent distortion is higher than that for a class B amplifier. Therefore class C amplifiers are not used as A.F. power amplifiers.
- The efficiency of class C amplifiers is very high. In fact it is the highest of all the power amplifiers. Typically the efficiency is above 95%.

Advantages of class C amplifier :

1. Very high efficiency (higher than 95%).
2. Low power loss in the power transistors.

Disadvantage of class C amplifier :

1. The output waveform can be distorted.

Applications of class C power amplifier :

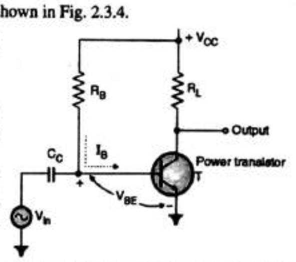
1. The class C amplifiers generally use a tuned circuit as load. Such amplifiers are called as class - C tuned amplifiers.
2. These amplifiers are used as the collector modulator to produce the amplitude modulated signal.

2.3.3 BJT as a Class A Power Amplifier : MU : May 14

University Questions

Q. 1 Explain class C BJT power amplifier in detail. Compare it with class A BJT power amplifier. **(May 14, 10 Marks)**

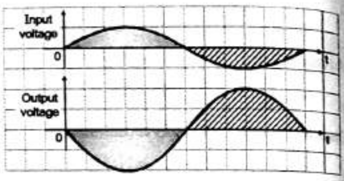
- A series fed directly coupled class A amplifier is as shown in Fig. 2.3.4.
- Fixed biasing is being used for the circuit and resistance R_B is adjusted in such a way that the "Q" point lies exactly at the center of the load line.
- Transistor " T " is a power transistor and the level of voltage that can be applied at the input is of the order of a few volts.
- The circuit shown in Fig. 2.3.4 is capable of handling a large amount of power however the voltage gain of the circuit is not very high, because this is a power amplifier.



(F-888) **Fig. 2.3.4 : Series fed directly coupled class A amplifier**

- This amplifier is called as series fed directly coupled amplifier because, the load R_L is connected directly (coupled directly) to the collector circuit without using any coupling technique.
- The graphical representation of series fed directly coupled amplifier is as shown in Fig. 2.3.4 which shows that the input and output voltage waveforms are pure sinewaves, and there is no harmonic distortion present in the waveform.
- The transistor remains in the "active region" for all the values of input signal and never enters into the saturation or cutoff regions.
- The transistor conducts for the complete cycle of ac input i.e. for 360°. Thus the angle of collector current flow is 360° or full cycle.

- The input signal is amplified faithfully, without introducing any distortions. Thus harmonic contents in the output will be low.
- As the transistor continuously operates in its active region, the voltage V_{CE} across it and current I_C through it, both are simultaneously high.
- Therefore a large power will be dissipated in the transistor in the form of heat. Therefore the efficiency of class A power amplifiers is low. In fact it is the lowest of all the power amplifiers.
- Typically the efficiency (η) of a class A power amplifier lies between 25% to 50%.
- The input and output voltage waveforms are as shown in Fig. 2.3.5.



(B-1666) **Fig. 2.3.5 : Input output voltage waveforms of a class A amplifier**

Advantages of Directly Coupled Class A Amplifier :

1. The circuit design is simple.
2. Less number of components are required. Hence it is cheaper and less bulky.
3. Output transformer is not used as the load is directly coupled.

Disadvantages of Directly Coupled Class A Amplifier :

1. Large power dissipation and power wastage takes place in the power transistors.
2. Heat sink is essential which makes the circuit bulky.
3. Poor efficiency (at the most equal to 25%).
4. The output impedance is high and there is no impedance matching technique used. Hence low impedance loads such as loud speaker cannot be driven efficiently.

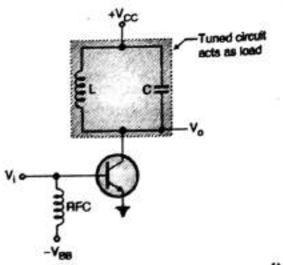
2.3.4 BJT as Class C Amplifiers :

MU : May 14

University Questions

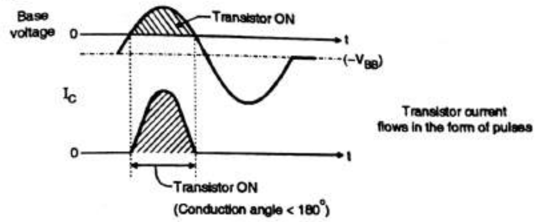
Q. 1 Explain class C BJT power amplifier in detail. Compare it with class A BJT power amplifier. **(May 14, 10 Marks)**

- Class C amplifiers cannot be used as audio amplifiers.
- However we can use the class C amplifier in tuned circuits used in communication.
- The circuit diagram of class C amplifier is as shown in Fig. 2.3.6.
- The LC tuned circuit acts as the load for the transistor Q.
- The dc source $- V_{BB}$ biases the transistor below the cut off. RFC is a Radio Frequency Choke. It is equivalent to a short circuit for the dc operating conditions but its reactance is very high at radio frequencies.



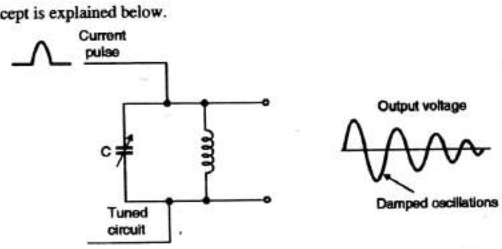
(F-882) **Fig. 2.3.6 : Class C amplifier circuit**

- The ac input voltage V_i is applied to the base of the transistor. This voltage is superimposed on $- V_{BB}$ as shown in Fig. 2.3.7.



(F-883) **Fig. 2.3.7 : Transistor biasing and its conduction angle**

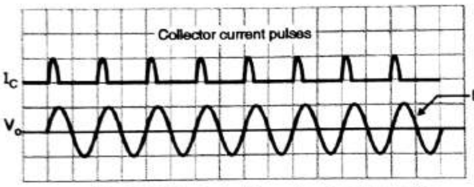
- The transistor will be forward biased and will conduct for an angle which is less than 180°.
- The transistor collector current flows in the form of pulses of duration less than 180°.
- When these current pulses are applied to the LC resonant circuit a full cycle of output signal is obtained at the resonant frequency of the tuned circuit. Thus the output voltage produced across the LC tuned circuit is a sinewave and its frequency is equal to the resonant frequency of the tuned circuit.
- This concept is explained below.



(F-884) **Fig. 2.3.8 : Figure showing the property of tuned circuit**

Property of a tuned circuit :

- To understand the class C amplifier, we must know an important property of a tuned circuit.
- The property states that if we apply a current pulse to a tuned circuit then it generates damped voltage oscillations at its output.
- This is as shown in Fig. 2.3.8. The amplitude of oscillations is proportional to the size of current pulse and the decay rate is proportional to the time constant. This is called as the flywheel effect in a tuned circuit.
- So if we apply a train of current pulses at frequency f_r which is equal to the resonant frequency of the tuned circuit, then we will obtain a sinusoidal signal at frequency f_r at the output as shown in Fig. 2.3.9.



(F-885) Fig. 2.3.9 : Output voltage of a class C amplifier

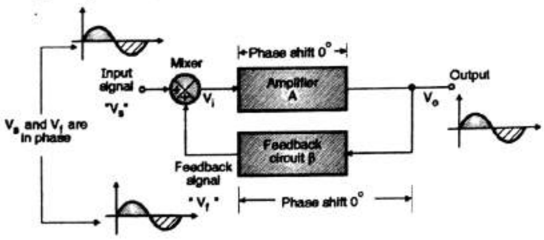
- This shows that the class C operation is limited to use at only one frequency i.e. resonant frequency f_r.

2.4 Oscillators :

- Oscillators are basically ac signal generators which you use in your laboratories.
- Oscillators generate sinusoidal voltage of desired magnitude, at desired frequency.
- The output voltage and frequency of an oscillator can be variable.
- The oscillator operates on a dc power supply + V volts and more importantly it produces an alternating output voltage without any alternating signal applied at its input.
- Oscillators operate on the principle of positive feedback. In this chapter, we are going to introduce the concept of positive feedback first and then discuss the principle of oscillators.

2.4.1 Concept of Positive Feedback :

- There are two types of feedback, namely negative and positive feedback.
- The positive feedback is used in oscillators. The concept of positive feedback can be explained with the help of Fig. 2.4.1.



(K-681) Fig. 2.4.1 : Principle of positive feedback

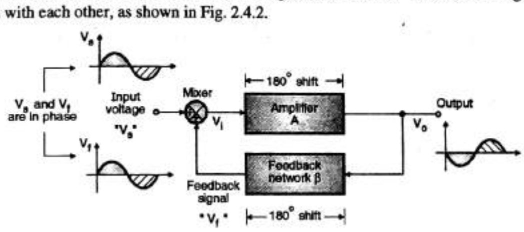
- **An oscillator is an amplifier with positive feedback incorporated in it.** A part of the output is fed back through the feedback network and mixer to the amplifier input. The feedback signal is

"in phase" with the original input signal as shown in Fig. 2.4.1 as the phase shift introduced by the feedback network is 0°. The amplifier in Fig. 2.4.1 is assumed to be a non-inverting amplifier with a voltage gain A which introduces a zero phase shift between its input and output.

- Let us assume that a sinusoidal input voltage V_s is being applied at the input of the non-inverting amplifier.
- As the non-inverting amplifier is being used, there is no phase difference between V_s and V_o. So V_o is an amplified version of V_s with a 0° phase shift. A part of this output voltage is fed back (V_f) to the input side.
- The feedback voltage V_f is in phase with the input voltage V_s as the feedback network does not introduce any phase shift. But the feedback voltage amplitude can be adjusted by changing the value of feedback factor β.

$$\therefore \; V_f = \beta V_o \qquad \text{...(2.4.1)}$$

- Note that if we use an inverting amplifier which introduces a 180° phase shift between V_s and V_o, then the feedback network should introduce a phase shift of 180° in order to bring V_s and V_f in phase with each other, as shown in Fig. 2.4.2.



(N-951) Fig. 2.4.2 : Principle of positive feedback

Conclusion :

Thus the feedback is called as a positive feedback if the input signal V_s and feedback signal V_f are in phase with each other.

2.4.2 Expression for the Gain with Positive Feedback (A_f) :

- It can be proved that the amplifier gain with feedback (A_f) is given by,

$$A_f = \frac{A}{1 - A\beta} \qquad \text{...(2.4.2)}$$

A = Open loop gain of the amplifier.

- This is the required expression for the gain with feedback. Note that Equation (2.4.2) is valid only for the positive feedback.

Conclusions :

1. In Equation (2.4.2), as Aβ is a positive quantity, (1 – Aβ) will be less than 1 and therefore $A_f > A$. **Thus positive feedback will increase the amplifier gain.**

2. If we increase the value of "β", keeping "A" constant then A_f will increase and at a particular value of β, the value of "A_f" will become ∞. This means that even without the input signal V_s, the amplifier will keep producing output voltage with the help of the feedback signal. This is where the amplifier starts acting as an oscillator.

2.4.3 Oscillator Principle :

MU : Dec. 15

University Questions

Q. 1 Give reason for Barkhausens criteria should be satisfied to get oscillations. **(Dec. 15, 5 Marks)**

- An oscillator is basically an "amplifier" which does not have any ac input but it operates on the principle of positive feedback to generate an ac signal on its own at its output.
- Thus it is clear that an amplifier can work as an oscillator if positive feedback is made to exist. However positive feedback not always guarantees oscillations.
- An amplifier will work as an oscillator if and only if it satisfies a set of conditions called the "Barkhausen Criterion".

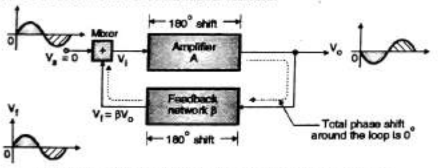
2.4.4 Barkhausen Criteria :

MU : May 14

University Questions

Q. 1 Explain Barkhausen's criteria for principle of oscillation. **(May 14, 5 Marks)**

- The Barkhausen criteria should be satisfied by an amplifier with positive feedback to ensure the sustained oscillations.
- For an oscillator circuit, there is no input signal "V_s", hence the feedback signal V_f itself should be sufficient to maintain the oscillations.
- Refer Fig. 2.4.3 to understand the Barkhausen criteria.



(N-952) **Fig. 2.4.3 : Block diagram of an oscillator**

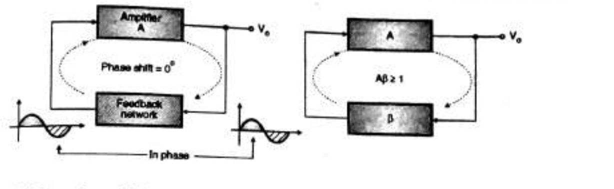
Statement of Barkhausen criterion :

The Barkhausen criterion states that :

1. An oscillator will operate at that frequency for which the total phase shift introduced, as measured from the input terminals, through the amplifier and feedback network and back again to the input is precisely equal to 0° or 360° or integral multiple of 360°.
2. At the oscillator frequency, the magnitude of the product of open loop gain of the amplifier A and the feedback factor β is equal to or greater than unity.

$$\therefore \ |A\beta| \geq 1$$

The product Aβ is called as the "loop gain". These conditions are diagrammatically illustrated in Figs. 2.4.4(a) and (b).



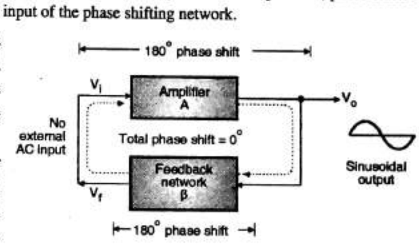
(a) The phase shift around the loop is 0° (b) Loop gain Aβ ≥ 1

(N-953) **Fig. 2.4.4 : Barkhausen criterion**

Block Diagram of an Oscillator :

The block diagram of an oscillator is shown in Fig. 2.4.5.

- An oscillator consists basically of an amplifier and a phase shifting network. The amplifier receives the output of the phase shifting network. The amplifier then amplifies it, phase shifts it through 180° and applies it to the input of the phase shifting network.
- The phase shifting network shifts the amplifier output through another 180° and attenuates it before applying it back to the amplifier input.
- Due to the total phase shift of 360°, the feedback becomes a positive feedback, which gives rise to the oscillations, if the Barkhausen criterion is satisfied.

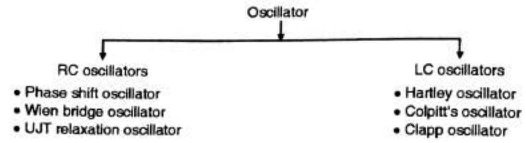


(N-949) **Fig. 2.4.5 : Block diagram of an oscillator**

- Note that there is no external ac input voltage applied to an oscillator still it produces a sinusoidal output voltage.
- An oscillator is an amplifier with positive feedback.

2.4.5 Classification of Oscillators :

- Depending on the components used for the feedback network, the oscillators are classified as RC oscillators and LC oscillators.
- The RC oscillators use only resistors (R) and capacitors (C) in their feedback network whereas the LC oscillators use inductors (L) and capacitors (C).
- Examples of RC oscillators are RC-phase shift oscillator and Wien bridge oscillator whereas Hartley, Colpitt's and Clapp oscillators are the well known examples of LC oscillators.



(F-788) **Fig. 2.4.6 : Classification based on the components used**

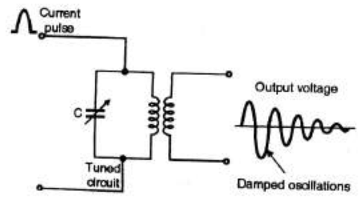
2.5 LC Oscillators :

- In LC oscillators the feedback network consists of inductors (L) and capacitors (C), instead of resistors and capacitors as in case of RC oscillators.
- These LC components determine the frequency of oscillations of the LC oscillator.
- The operating principle of LC oscillators is based on the Barkhausen conditions.
- These oscillator can operate at high frequencies typically from 200 kHz to few GHz. They are not suitable for low operating frequencies because the values of L and C will be large at low frequencies. Large value inductors and capacitors are bulky (large in size) and expensive as well.
- But as the operating frequency is increased we need small value inductors and capacitors which are small in size and less expensive.
- We are going to discuss the following types of LC oscillators in the subsequent sections :

  1. Hartley oscillator 2. Colpitts oscillator 3. Clapp oscillator

Property of tuned circuit :

- To understand the operation of LC oscillator, we must know an important property of a tuned circuit.

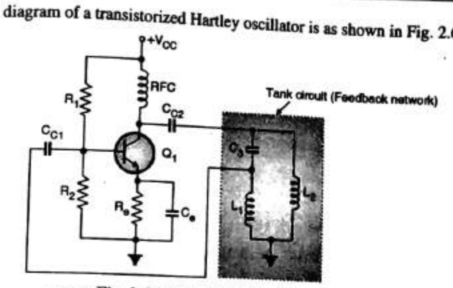


(B-1697) **Fig. 2.5.1 : The property of tuned circuit**

- This property states that if we apply a current pulse to a tuned circuit then it generates damped voltage oscillations at its output.
- This is as shown in Fig. 2.5.1. The amplitude of oscillations is proportional to the size of current pulse and the decay rate is proportional to the time constant.
- If the current pulses are applied at every alternate zero crossover point of the output waveform (at the beginning of every positive half cycle) then we will get the sustained oscillations at the output.

2.6 Hartley Oscillator :

- The circuit diagram of a transistorized Hartley oscillator is as shown in Fig. 2.6.1.



(F-810) **Fig. 2.6.1 : Transistorized Hartley oscillator**

Operation of the circuit :

- R_1, R_2 and R_e are the resistances for the biasing of the transistor. C_{C1} and C_{C2} are the coupling capacitors, and C_e bypasses R_e. The feedback circuit is formed by the components L_1, L_2 and L_3.
- The amplifier is in CE configuration. Hence it provides 180° phase shift between its input and output. The feedback circuit provides an additional 180° phase shift to satisfy the condition for the positive feedback.
- The frequency of oscillations is given by :

$$f = \frac{1}{2\pi\sqrt{C_3(L_1+L_2)}} \quad ...(2.6.1)$$

where $(L_1 + L_2)$ is the equivalent inductance.

$$\therefore \quad f = \frac{1}{2\pi\sqrt{C_3 L_{eq}}} \quad ...(2.6.2)$$

- Here we have not considered the mutual inductance between the two inductances L_1 and L_2. In practice the inductors L_1 and L_2 are wound on the same core so we can't neglect the mutual inductance M present between them. So considering the mutual inductance, the equivalent inductance L_{eq} is given by,

$$L_{eq} = L_1 + L_2 + 2M \quad ...(2.6.3)$$

- We have to then substitute this expression of L_{eq} into Equation (2.6.2).
- The oscillator frequency can be varied by varying the capacitor C_3. The frequency variation over a wide range can be easily obtained.

2.6.1 Advantages of Hartley Oscillator :

1. It is easy to tune.
2. It can operate over a wide frequency range typically from few Hz to several MHz.
3. It is easy to change the frequency by means of a variable capacitor.

2.6.2 Applications :

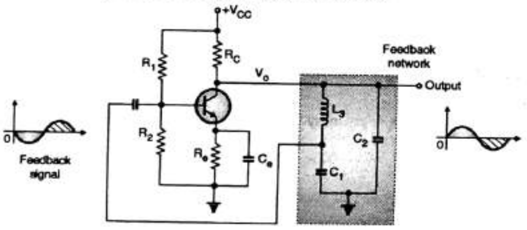
1. It is used as local oscillator in radio and TV receivers.
2. In the function generators.
3. In RF sources.

2.6.3 Disadvantage :

1. Poor frequency stability.

2.7 Colpitt's Oscillator :

- The Colpitt's oscillator using transistor is shown in Fig. 2.7.1.
- The resistors R_1, R_2 and R_e will provide the biasing for the transistor. C_{C1} and C_{C2} are the coupling capacitors and C_e is the bypass capacitor.
- The transistor is connected in the CE configuration. Therefore it introduces a phase shift of 180° between its input and output.
- The feedback network will provide additional 180° phase shift so as to make the total shift equal to zero. This will satisfy the condition for the positive feedback.



(F-817) **Fig. 2.7.1 : Colpitt's oscillator using a BJT**

- The frequency of oscillations is given by,

$$f = \frac{1}{2\pi\sqrt{L_3 C}} \qquad ...(2.7.1)$$

$$\text{where,} \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

- As in the case of transistor phase shift oscillator, the transistor current gain is important. The circuit analysis gives the minimum value of h_{fe} as,

$$h_{fe\,(min)} = \frac{C_1}{C_2} \qquad ...(2.7.2)$$

- The behaviour of the Colpitt's oscillator is very similar to that of the Hartley oscillator for the simple reason that both of them use the same basic LC oscillator except for the phase shifting network.

2.7.1 Frequency Stability :

It is expected that the oscillator frequency should remain constant. It should not be dependent on the factors such as changes in temperature or supply voltage and so on. Unfortunately the Hartley and Colpitt's oscillator circuits do not have a high frequency stability.

The reasons for poor frequency stability are as follows :

1. The collector base internal capacitance affects the value of capacitance in the feedback circuit. This will change the frequency of oscillations.
2. The transistor parameters depend on the temperature. Due to change in their values, the frequency of oscillations will change.

Remedy :

To improve the frequency stability we can take the following measures :

1. Use a temperature stabilized chamber to house the oscillator circuit. The temperature of chamber is maintained constant at the desired level. This will avoid any frequency changes due to drift in temperature.
2. Use voltage regulators to keep the supply voltage constant.
3. Use a special type of oscillator called "Clapp oscillator".

2.7.2 Advantages of Colpitt's Oscillator :

1. Simple construction.
2. It is possible to obtain oscillations at very high frequencies.

2.7.3 Disadvantages :

1. It is difficult to adjust the feedback as it demands change in capacitor values.
2. Poor frequency stability.

2.7.4 Application :

1. As a high frequency generator.

2.8 Applications of LC Oscillators :

The application of LC oscillators are as follows :

1. Used as local oscillator in radio and TV receivers.
2. In the function generators.
3. In RF sources.
4. As high frequency generator.
5. As special type of receivers.
6. In frequency synthesizers.

The LC oscillators are not preferred for the low frequency applications because at low frequencies the size of L and C becomes very large making the LC oscillator bulky.

2.9 Crystal Oscillators :

Principle of crystal oscillators :

- Certain materials such as quartz exhibit a unique property called "piezo electric" property.
- It states that if mechanical force is applied to a quartz crystal then it generates electric potential.
- Also if electric field is applied to a crystal it vibrates mechanically. If we apply mechanical vibrations to a quartz crystal then under proper operating conditions we can obtain electrical oscillations from it.

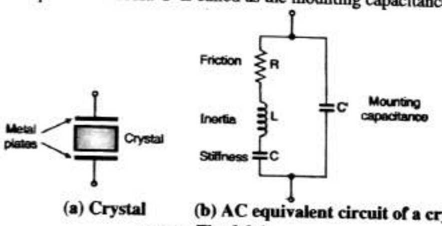
Frequency stability :

- The biggest advantage of using a crystal oscillator is its high frequency stability.
- The frequency of a crystal oscillator remains stable inspite of changes in temperature, voltage, humidity or other parameters.

2.9.1 Equivalent Circuit of a Crystal :

The ac equivalent circuit of a crystal is as shown in Fig. 2.9.1(b). It shows that the crystal is equivalent to a resonant circuit.

- In the ac equivalent circuit of a vibrating crystal, the internal frictional losses are represented by a resistance R.
- Mass of the crystal and hence its inertia is represented by L and stiffness under the vibrating condition is represented by capacitor C.
- Due to the mounting arrangement shown in Fig. 2.9.1(a), the crystal is equivalent to a capacitance denoted by C′ in the equivalent circuit. C′ is called as the mounting capacitance.



(a) Crystal (b) AC equivalent circuit of a crystal

(B-1700) Fig. 2.9.1

Resonant frequencies :

- There are two resonant circuits existing in the ac equivalent circuit of the crystal. RLC form a series resonant circuit and RLC in parallel with C′ will form a parallel resonant circuit. The resonant frequency of the series R-L-C series circuit is given by,

$$f_S = \frac{1}{2\pi\sqrt{LC}} \quad ...(2.9.1)$$

- This is with an assumption that quality factor Q is very large. The resonant frequency of the parallel resonant frequency formed by R-L-C and C′ is given by,

$$f_p = \frac{1}{2\pi\sqrt{L\,C_{eq}}} \quad ...(2.9.2)$$

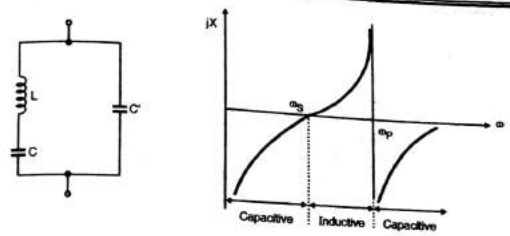
$$\text{Where,}\quad C_{eq} = \frac{CC'}{(C+C')}$$

The parallel resonant frequency f_p can be varied by varying the value of C′.

The parallel resonant frequency f_p is always higher than the series resonant frequency f_S.

Crystal impedance :

- To obtain the expression for crystal impedance let us use the approximate ac equivalent circuit of Fig. 2.9.2(a). The resistance "R" in the original equivalent circuit has been neglected.



(a) Approximate ac equivalent circuit (b) Variation in jX with ω

(B-1701) Fig. 2.9.2

- Let X be the reactance offered by the crystal. Then X is dependent on frequency as shown in Fig. 2.9.2(b).

1. If $\omega < \omega_S$: The reactance of the crystal i.e. "X" will be negative. i.e. capacitive.
2. If $\omega = \omega_S$: Reactance X = 0.
3. If $\omega_S < \omega < \omega_p$: X will be positive i.e. inductive.
4. If $\omega > \omega_p$: X will be negative i.e. capacitive.
5. If $\omega = \omega_p$: At $\omega = \omega_p$, $X = \infty$.

The variation of jX with ω is as shown in Fig. 2.9.2(b).

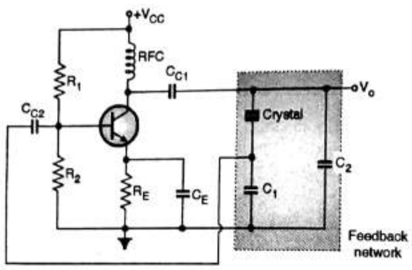
2.9.2 Types of Crystal Oscillators :

The two types of crystal oscillators which we are going to study are :

1. Pierce crystal oscillator 2. Miller crystal oscillator

2.9.3 Pierce Crystal Oscillator :

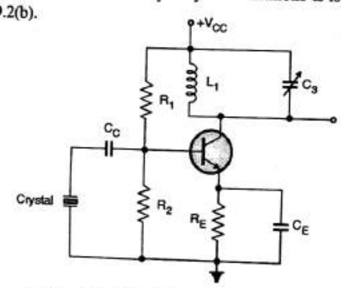
- If you compare this circuit with the Colpitt's oscillator circuit then you will find that both these circuits are identical except for one change. L_3 is replaced by the crystal here.
- The crystal is made to operate as an inductance. This is possible if the frequency of oscillations ω is adjusted between ω_S and ω_p as shown in Fig. 2.9.2(b).
- The basic operation of the pierce crystal oscillator is same as that of the Colpitt's oscillator.
- RFC is a radio frequency choke which connects the DC supply to the circuit but isolates the DC supply from the high frequency oscillations generated in the tank circuit (feedback network).



(B-1702) Fig. 2.9.3 : Pierce crystal oscillator

2.9.4 Miller Crystal Oscillator :

- If the Hartley oscillator circuit is modified by replacing one of the inductors by a crystal then we get the Miller crystal oscillator configuration shown in Fig. 2.9.4.
- The crystal acts like an inductor as the frequency of oscillations ω is adjusted between ω_s and ω_p as shown in Fig. 2.9.2(b).



(B-1703) Fig. 2.9.4 : Miller crystal oscillator

2.9.5 Advantages of Crystal Oscillator :

Following are some of the advantages of crystal oscillator :

1. Very high frequency stability.
2. Very low frequency drift due to change in temperature and other parameters.
3. It is possible to obtain very high, precise and stable frequency of oscillations.
4. The Q is very high.
5. It is possible to obtain frequencies, higher than the fundamental frequency by operating the crystal in the overtone mode.

2.9.6 Disadvantages :

1. These are suitable for high frequency applications.
2. Crystals of low fundamental frequencies are not easily available.

2.9.7 Applications of Crystal Oscillators :

1. As a crystal clock in microprocessors.
2. In the frequency synthesizers.
3. In the radio and TV transmitters.
4. In special types of receivers.

2.9.8 Comparison of LC and Crystal Oscillators :

Sr. No.	LC oscillators	Crystal oscillators
1.	Frequency of oscillations is dependent on values of L and C.	Frequency of oscillations depends on the dimensions of crystal.
2.	These are preferred at high frequencies.	Preferred at high frequencies.
3.	Hartley, Colpitt's and Clapp oscillators are the examples of LC oscillators.	Miller crystal oscillator and pierce crystal oscillator are the examples.
4.	Poor frequency stability except for the clapp oscillator.	Very high frequency stability.
5.	Used in radio, TV as high frequency sources, frequency synthesizers.	Crystal clock, frequency synthesizer, special type receivers are the applications.

Review Questions

Q. 1 Explain the concept of power amplifier.
Q. 2 State and explain the important features of power amplifier.
Q. 3 Explain the class A operation of power amplifiers.
Q. 4 State merits, demerits and applications of class A power amplifier.
Q. 5 Explain the class C operation.
Q. 6 State merits, demerits and applications of class C power amplifiers.
Q. 7 Define oscillator.
Q. 8 Explain the concept of positive feedback.
Q. 9 State the principle of oscillator.
Q. 10 Explain the Barkhausen criteria.
Q. 11 Explain the operation of Hartley oscillator and state its applications.
Q. 12 Explain the operation of Colpitt's oscillator and state its applications.
Q. 13 Explain the principle of crystal oscillators.
Q. 14 State advantages of crystal oscillators.
Q. 15 Compare LC and crystal oscillators.

2.10 University Questions and Answers :

Q. 1 Explain why crystal oscillators are considered to be more stable than other oscillators ?
(Dec. 13, 5 Marks)

Ans. :

- In a crystal oscillator, the frequency of oscillations depends on the physical dimensions of the crystal.
- As the dimensions remain constant the crystal oscillator frequency remains constant. Refer **Sections 2.9 and 2.9.1 for more details.**

Q. 2 Explain the fly wheel effect in class C amplifier. **(Dec. 2013, Dec. 2015, May 2016, 5 Marks)**

Ans. :

- When we briefly push a fly wheel, it absorbs this energy and continues to rotate for some time. This is known as a fly wheel effect.
- The same effect is being used in class C amplifier to produce an AM wave.
- Here the class C amplifier produces a short duration current pulse in each cycle of the carrier. This current pulse is applied to a tuned circuit. The tuned circuit will produce damped oscillations at its output. This is the fly wheel effect present in a class C amplifier.

Q. 3 Compare class A and class C power amplifiers. **(May 15, 5 Marks)**

Ans. :

Sr. No.	Parameter	Class A	Class C
1.	Conduction angle of collector current.	360° or full cycle.	Less than 180°.
2.	Position of Q point on the load line.	At the center.	Below the X-axis.
3.	Distortion in output voltage	No distortion.	More than A, B and AB.
4.	Efficiency.	Lowest 25% to 50%.	Very high (95%).
5.	Power dissipation in transistors	Very high	Very low

Q. 4 Compare Hartley and Colpitts oscillator alongwith neat diagrams. **(Dec. 16, 5 Marks)**

Ans. :

Table 1 : Comparison of different LC oscillators

Sr. No.	Parameter		Hartley	Colpitt's
1.	Circuit diagram		Refer Fig. 2.6.1	Refer Fig. 2.7.1
2.	Feedback network composition	Z_1	Inductor	Capacitor
		Z_2	Inductor	Capacitor
		Z_3	Capacitor	Inductor
		C_3	Absent	Absent
3.	Frequency of oscillations		$f = \frac{1}{2\pi\sqrt{(L_1 + L_2)\,C_3}}$	$f = \frac{1}{2\pi\sqrt{L_3 C}}$ $C = \frac{C_1 C_2}{C_1 + C_2}$
4.	Advantages		Suitable at HF. Small size and low cost.	Suitable at HF. Small size and low cost.
5.	Disadvantages		Poor frequency stability.	

□□□

CHAPTER 3

Operational Amplifier & its Applications

Module 3

Syllabus :

OP-amp-block diagram, parameters and characteristics, Applications-Inverting and non-inverting amplifier. Summing amplifier (numerical), Difference amplifier, Basic integrator and differentiator, Comparator, Zero crossing detector (only theory).

3.1 Introduction :

- Electronics field has many branches and each branch has its own application.
- Various branches of electronics are : Industrial electronics, Instrumentation, Communication, Power electronics etc.
- Various electronic circuits used in the applications of these branches are amplifiers, waveform generators, timers, various arithmetic circuits such as adder, subtractor, multipliers, Log-antilog amplifier etc.
- One electronic device which can be used to construct all the circuits mentioned above is called an operational amplifier or OP-AMP. This is the importance of an OP-AMP in the field of electronics. Following application list of OP-AMP will highlight this point.

3.1.1 DC Amplifier :

- As operational amplifier can amplify two types of signals dc and ac.
- A dc amplifier is the amplifier in which output signal changes in response to changes in dc input level.
- A dc amplifier can be inverting, non-inverting or differential.

(N-1) **Fig. 3.1.1 : DC amplifier**

3.1.2 Applications of OP-AMP :

- Some of the important applications of an OP-AMP are :

1. Amplifiers
2. Active filters
3. Arithmetic circuits
4. Log and antilog amplifier
5. Voltage comparators
6. Waveform generators
7. Precision rectifiers
8. Multipliers
9. Timers
10. Multivibrators
11. Regulated power supplies

3.2 OP-AMP :

- The term operational amplifier or OP-AMP was first used by John R. Ragazzini in 1947.
- He used it to indentify an amplifier which could be connected in various configurations to perform a variety of operations such as amplification, addition, subtraction, differentiation and integration.
- Hence the name operational amplifiers. During the initial days the OP-AMPs were being extensively used in the analog computers.
- OP-AMP is basically a multistage amplifier which uses a number of amplifier stages interconnected to each other in a complicated manner.
- These internal amplifiers use transistor or FET as an amplifying device. All this internal circuit with many transistors, FETs, resistors will occupy a space equivalent to a pin head.

3.2.1 Advantages of OP-AMP Over Conventional Amplifiers :

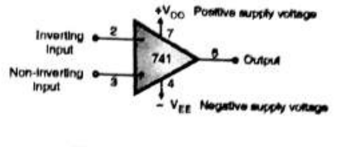
- The OP-AMP has following advantages over the conventional amplifier :
  1. It has smaller size.
  2. Its reliability is higher than conventional amplifier.
  3. Reduced cost as compared to its discrete circuit counterparts.
  4. Less power consumption.
  5. Easy to replace.
  6. Same OP-AMP can be used for different applications.
- The first operational amplifier in the integrated circuit (IC) form was brought in the market by the Fairchild company.
- They named this OP-AMP as µA 741 which became extremely popular and was used in a variety of applications. OP-AMP is a linear or more accurately an analog integrated circuit.
- In the subsequent years even though the number of OP-AMP families and manufacturers increased to a great extent, the 741 remained one of the most popular devices.

3.3 The Operational Amplifier Symbol and Terminals :

- The OP-AMP is basically a voltage amplifier with extremely high voltage gain. For example, the popular 741 OP-AMP has a typical gain of 2×10^5 or 10^6 dB.
- The large voltage gain of OP-AMP is an important factor which distinguishes it from the remaining amplifiers.

3.3.1 Symbol and Terminals :

- The circuit symbol of an OP-AMP with its different terminals is shown in Fig. 3.3.1(a) and its physical appearance is shown in Fig. 3.3.1(b).



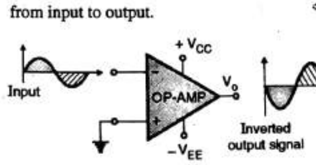
(B-332) (a) OP-AMP symbol and terminals

(B-333) (b) Physical appearance

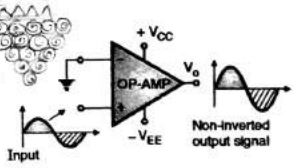
Fig. 3.3.1

A careful look at the circuit symbol of OP-AMP reveals the following things :

- An OP-AMP has two input terminals, one output terminal and two supply voltage terminals.
- We can apply the input voltage which is to be amplified to any one of the two input pins connecting the other pin to ground or the input signal can be connected "between" the two input pins differentially.
- The input terminal marked with a negative (–) sign is called as an "Inverting" (I) terminal. If we connect the input signal to this terminal then the amplified output signal is 180° out of phase with respect to input as shown in Fig. 3.3.2(a).
- The input terminal labelled with a positive (+) sign is called as "Non-inverting" (NI) terminal. If we connect the input signal to this terminal, then the amplified output signal is in phase with the input signal as shown in Fig. 3.3.2(b).
- The symbol of an OP-AMP appears like an arrowhead. The arrowhead signifies the signal flow from input to output.



(a) Input and output signals with 180° phase shift when the input signal is applied to the inverting (–) terminal



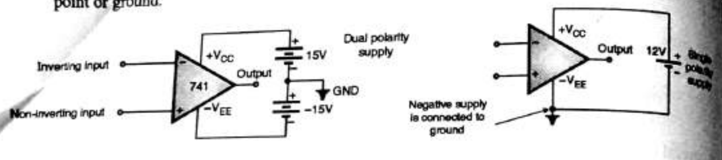
(b) Input and output signals with 0° phase shift when the input signal is applied to the non-inverting (+) terminal

(B-334) Fig. 3.3.2

3.3.2 DC Power Supply for an OP-AMP :

- A dual polarity dc power supply is essential for the operation of most of the OP-AMPs. The + V_{CC} and – V_{EE} terminals in Fig. 3.3.1(a) are the power supply terminals.

- The + V_{CC} pin of OP-AMP is connected to the positive terminal of one source and the – V_{EE} pin is connected to the negative terminal of the other source as shown in Fig. 3.3.3(a).
- The two sources are 15 V battery each. However in practice the supply voltage can range from ± 5 V to ± 22 V. The common terminal of + V_{CC} and – V_{EE} sources is connected to a reference point or ground.



(a) Dual polarity supply (b) Single polarity supply

(B-335) **Fig. 3.3.3 : Power supply connections for an OP-AMP**

- The OP-AMPs do not have a 0-volt ground terminal. Therefore as shown in Fig. 3.3.3(a), the ground reference is established externally by creating the common point of positive and negative power supplies.
- The OP-AMP IC 741 needs a dual polarity power supply as shown in Fig. 3.3.3(a) however there are some other OP-AMPs which can operate on a single polarity supply, as well.
- For single polarity OP-AMPs – V_{EE} terminal is connected to ground instead of connecting it to a negative supply voltage as shown in Fig. 3.3.3(b).

3.4 Block Diagram of a Typical OP-AMP :

MU : May 03, May 04, May 05, Dec. 05, May 08, May 09, Dec. 09, May 10, May 11

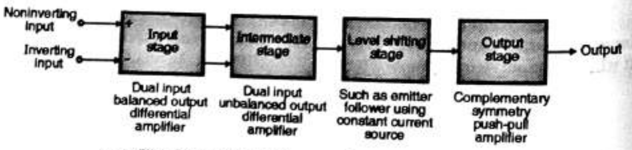
University Questions

Q. 1 Draw block diagram of typical OP-AMP. Explain function of each block. **(May 03, May 04, May 05, Dec. 05, 5 Marks, May 09, Dec. 09, 4 Marks, May 10, 6 Marks)**

Q. 2 Explain OP-AMP with block diagram. **(May 08, 5 Marks)**

Q. 3 Explain operational amplifier with a neat block diagram. **(May 11, 5 Marks)**

- The block diagram of a typical OP-AMP is shown in Fig. 3.4.1.



(B-337) **Fig. 3.4.1 : Block diagram of a typical OP-AMP**

- The OP-AMP is basically a differential amplifier i.e. it will amplify the voltage which is differentially present between its input terminals.

- **Input stage :**

  The input stage is a dual-input, balanced output differential amplifier. The two inputs are inverting and non-inverting input terminals. This stage provides most of the **voltage gain** of the OP-AMP and decides the value of input resistance R_i.

- **Intermediate stage :**

  This is usually another differential amplifier. The input stage drives the stage. This stage is a dual-input unbalanced output (single ended output) differential amplifier.

- **Level shifting stage :**

  Due to the direct coupling used between the first two stages, the input of level shifting stage is an amplified signal with some non-zero dc level. Level shifting stage is **used to bring this dc level to zero volts with respect to ground.**

- **Output stage :**

  This stage is normally a complementary output stage. It **increases the magnitude of voltage and raises the current supplying capability of the OP-AMP.** It also ensures that the output resistance of OPAMP is low.

3.5 Equivalent Circuit of an OP-AMP :

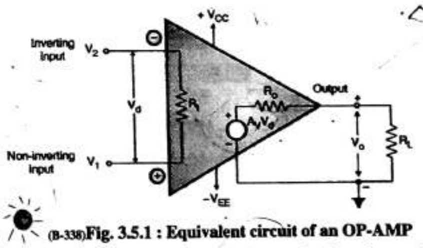
MU : Dec. 11, Dec. 12

University Questions

Q. 1 Draw and explain equivalent circuit of an ideal op-amp. **(Dec. 11, 10 Marks)**

Q. 2 Draw the equivalent circuit diagram of an op-amp and explain each term. **(Dec. 12, 4 Marks)**

- Fig. 3.5.1 shows the equivalent circuit of a practical OP-AMP. It includes important values such as A_V, R_i, R_o etc.
- Note that A_V, V_d is the equivalent Thevenin voltage source and R_o is the Thevenin equivalent resistance looking back into the output terminal of an OP-AMP.
- The value of input resistance R_i is finite but very high here and that of the output resistance R_o is non-zero because the OP-AMP is non-ideal.



(B-338) **Fig. 3.5.1 : Equivalent circuit of an OP-AMP**

- The equivalent circuit of Fig. 3.5.1 is made of the differential input resistance R_i, the voltage gain A_V and the output resistance R_o.
- As will be discussed later, the parameters R_i, A_V and R_o are called as the **open-loop parameters.**
- $V_d = (V_1 - V_2)$ is called as the differential input voltage, and A_V is called as the open loop gain. Hence the output voltage is given by,

$$V_o = A_V \times V_d = A_V (V_1 - V_2) \quad ...(3.5.1)$$

- V_1 and V_2 are the voltages at the non-inverting and inverting input terminals of the OP-AMP, with respect to ground.

- Since both the input terminals are allowed to be connected to independent potentials, with respect to ground, the input side of the OP-AMP is said to be of the **Double Ended** type. But the output is **single ended** type.
- Equation (3.5.1) tell us that the OP-AMP responds only to the differential input voltage V_d. That means it produces output voltage which is proportional only to the **difference** between the input voltages and not to the individual input voltages. **Hence OP-AMPs are also called as Difference amplifiers.**
- Repeating Equation (3.5.1),

$$V_o = A_V (V_1 - V_2)$$

- This shows that the polarity of output voltage depends on the polarity of the differential input signal V_d.
- The differential input voltage V_d is given by,

$$V_d = \frac{V_o}{A_V} \quad ...(3.5.2)$$

- This expression gives use of the value of only the differential input voltage V_d and does not give the individual input voltages V_1 and V_2.
- The open loop voltage gain A_V is of very large value. Hence the value of V_d even for maximum output voltage is extremely small. For example, to obtain $V_{o\,(max)} = 10$ V a 741 OP-AMP needs

$$V_d = \frac{10}{2 \times 10^5} = 50\ \mu V$$

Thus we need a very small differential input voltage V_d to obtain the maximum possible output voltage.

3.6 OP-AMP Input Modes :

MU : Dec. 02, May 04, Dec. 06, May 10, Dec. 10, Dec. 11, Dec. 12, May 14

University Questions

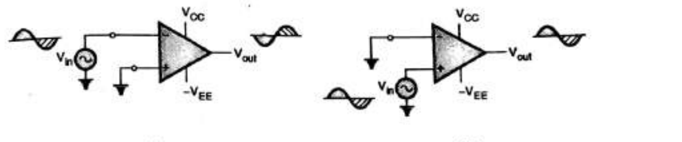
Q. 1	Explain CMRR used for OP-AMP.	(Dec. 02, Dec. 06, May 10, 2 Marks)
Q. 2	For OP-AMP explain CMRR.	(May 04, 3 Marks, 2 Marks)
Q. 3	What do you mean by CMRR ?	(Dec. 10, 2 Marks)
Q. 4	Explain the term CMRR.	(Dec. 11, Dec. 12, 3 Marks)
Q. 5	Explain input offset voltage, CMRR and SVRR for operational amplifier.	(May 14, 5 Marks)

Input signal modes :

- The input signal modes are determined by the differential amplifier input stage of the OP-AMP. Different input modes of OP-AMP are as follows :
  1. Single ended mode 2. Differential mode 3. Common mode

1. Single ended mode :

- If the input signal is applied only to one of the inputs and the other input terminal is connected to ground as shown in Fig. 3.6.1, the OP-AMP is said to be operating in the single ended mode.

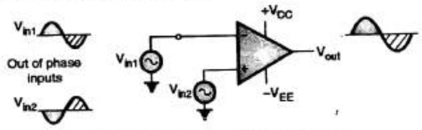


(B-2109) Fig. 3.6.1 : Single ended input mode

- The input and output signal are 180° out of phase as shown in Fig. 3.6.1(a) if the input signal is applied to the inverting input.
- The input and output signals are in phase with each other as shown in Fig. 3.6.1(b) if the input signal is applied to the non-inverting input.

2. Differential mode :

- In differential mode, two opposite polarity (out of phase) signals are applied to the two inputs of OP-AMP as shown in Fig. 3.6.2.

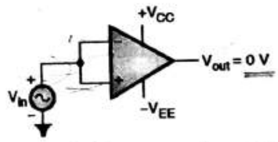


(B-2110) Fig. 3.6.2 : Differential input mode

- This type of operation is also known as the **double ended** operation. The difference between the input signal is amplified appears at the output.

3. Common mode :

- In the common mode of operation, the same input signal is applied to both the input terminals as shown in Fig. 3.6.3. Ideally a zero output voltage should be produced by the OP-AMP.
- Producing a zero output for a common mode signal is called as **common-mode rejection.**



(B-2111) Fig. 3.6.3 : Common-mode operation

Common mode rejection ratio (CMRR) :

- The signal which is present at both the input terminals of an OP-AMP is called as the common mode signal.
- The best example of a common mode signal in practice is "noise". The OP-AMP should produce a very small output voltage corresponding to the common mode signal. In other words it should be capable of "rejecting" the common mode signal.

- **Common Mode Rejection Ratio (CMRR)** is the ability of a differential amplifier to reject the common mode signal successfully. It is called as the figure of merit of the OP-AMP.
- Ideally the OP-AMP should produce vary high gain for the desired signal i.e. single ended or differential signals.
- The value of CMRR should be ideally infinite and practically as high as possible. The CMRR is defined as follows :

$$\text{CMRR} = \frac{A_V}{A_{cm}} \quad \text{...(3.6.1)}$$

where A_V = Open loop gain of OP-AMP

A_{cm} = Common mode gain of OP-AMP

- The open loop voltage gain is the voltage gain of an OP-AMP in the open loop mode (without any feedback) of OP-AMP and it is very high.
- Sometimes CMRR is expressed in decibels (dB) as follows :

$$\text{CMRR (dB)} = 20 \log_{10}\left[\frac{A_V}{A_{cm}}\right] \quad \text{...(3.6.2)}$$

3.6.1 Important Characteristics (Parameters) of an Ideal OP-AMP :

MU : Dec. 03, May 08, Dec. 08, May 09, May 10, Dec. 12, May 13, Dec. 15

University Questions

Q. 1 Give ideal characteristics of OP-AMP. **(Dec. 03, 8 Marks)**

Q. 2 List ideal OP-AMP characteristics. **(May 08, 5 Marks)**

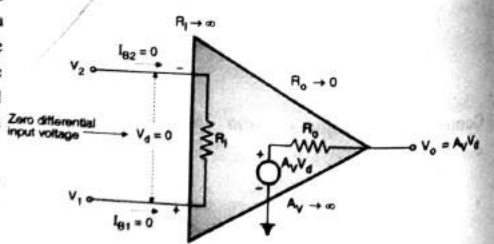
Q. 3 Explain all the features of Op-Amplifier. **(Dec. 08, 10 Marks)**

Q. 4 Write short note on properties of ideal Op-Amp. **(May 09, 5 Marks, May 10, 6 Marks, Dec. 12, May 13, 5 Marks)**

Q. 5 With respect to op-amp explain the ideal characteristics and concept of virtual ground. Explain how op-amp can be used as an averaging amplifier in inverting configuration. Also draw neat circuit diagrams to :

1. Convert sine wave to square wave using op-amp.
2. Detect the crossing of zero's in the generated square wave. **(Dec. 15, 10 Marks)**

- The equivalent circuit of an ideal OP-AMP is as shown in Fig. 3.6.4.
- We know that to minimise the loading on the input source, a well designed voltage amplifier must draw negligible current from the source and must present a negligible resistance to the output load. The OP-AMP being a voltage amplifier is no exception, to these requirements.



(B-339) **Fig. 3.6.4 : Equivalent circuit of an ideal OP-AMP**

- In order to define all the important characteristics of an ideal OP-AMP, refer to Fig. 3.6.4 in which V_1 and V_2 are the two input signals, V_o is the output voltage, A_V is the open loop voltage gain and R_i is the differential input resistance of the ideal OP-AMP.
- V_d is the differential input voltage and I_{B1} and I_{B2} are the currents flowing into the two input terminals.
- The important characteristics of an ideal OP-AMP are as follows :

1. Infinite voltage gain ($A_V = \infty$) :

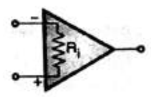
- The open loop (without any feedback) gain of an ideal OP-AMP is denoted by A_V. It is the differential voltage gain and its value for an ideal OP-AMP is ∞.

$$V_o = A_V \times V_d \quad \text{...(3.6.3)}$$

- As $A_V = \infty$, the differential voltage V_d required to obtain the maximum output voltage ($\pm V_{CC}$) will be negligible i.e. $V_d \approx 0V$.

2. Infinite input resistance ($R_i = \infty$) :

- The input resistance is also called as differential input resistance and it is defined as the total resistance measured between the two input terminals of an op-amp (see Fig. 3.6.5).

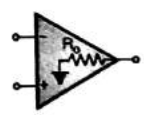


(B-2112) **Fig. 3.6.5 : Differential input resistance**

- The input resistance R_i of an ideal OP-AMP is infinite. Due to this, the current flowing in each input terminal will be zero i.e. $I_{B1} = I_{B2} = 0$.
- Due to infinite input resistance, almost any source can drive it and the source does not get loaded.

3. Zero output resistance ($R_o = 0$) :

- The output resistance of an OP-AMP is the resistance viewed from its output terminal as shown in Fig. 3.6.6.
- The output resistance R_o of an ideal OP-AMP is zero. Due to zero output resistance, the ideal OP-AMP can drive infinite number of other devices.
- Further there will not be any change in its output voltage due to change in load current. That means its voltage regulation will be good.



(B-2113) **Fig. 3.6.6 : Output resistance**

4. Zero offset voltage :

- In practical OP-AMPs a small output voltage is present even though both the inputs V_1 and V_2 are having a zero value.
- This voltage is called as the offset voltage. For ideal OP-AMPs the offset voltage is zero. That means output voltage will be zero when input voltage is zero.

5. **Infinite bandwidth :**

- Bandwidth of an amplifier is the range of frequencies over which all the signal frequencies are amplified almost equally.
- The bandwidth of an ideal OP-AMP is infinite. So it can amplify any frequency signal from 0 to ∞ Hz.
- Thus the gain of an ideal amplifier is constant from 0 frequency (dc signal) to ∞ Hz.

6. **Infinite CMRR :**

For an OP-AMP, the Common Mode Rejection Ratio (CMRR) is defined as the ratio of differential gain to common mode gain. CMRR is ∞ for an ideal OP-AMP. Thus the output voltage corresponding to the common mode noise is zero.

7. **Infinite slew rate (S = ∞) :**

The slew rate of an ideal OP-AMP is infinite so that the output voltage changes occur simultaneously with the input voltage changes. The slew rate definition and its significance have been explained later.

8. **Zero power supply rejection ratio (PSRR = 0) :**

- PSRR is a parameter which specifies the degree of dependence of the OP-AMP output, on the changes in power supply voltage.
- For an ideal OP-AMP, PSRR = 0. That means the output voltage does not change due to fluctuations in supply voltage.
- The characteristics of an ideal OP-AMP are tabulated in Table 3.6.1.

Table 3.6.1 : Characteristics of an ideal OP-AMP

Sr. No.	Characteristics	Ideal value
1.	Input resistance	$R_i = \infty\ \Omega$
2.	Output resistance	$R_o = 0\ \Omega$
3.	Voltage gain	$A_V = \infty$
4.	Bandwidth	$B = \infty$
5.	Common mode rejection ratio	CMRR = ∞
6.	Slew rate	∞
7.	Offset voltage	0
8.	Power supply rejection ratio (PSRR)	0

3.7 Practical OP-AMP Characteristics (Parameters) :

MU : May 12

University Questions

Q. 1 Explain two static and two dynamic parameters of OP-AMP. **(May 12, 4 Marks)**

- The OP-AMP characteristics are important in practice because we can use them to compare the performance of various OP-AMP ICs and select the best suitable from them for the required application.

- OP-AMP characteristics are classified into two categories namely DC characteristics and AC characteristics.
- The DC characteristics include input bias current, input offset current, input offset voltage and thermal drift whereas the AC characteristics include the frequency response, stability of OP-AMP, frequency compensation, slew rate etc.

3.7.1 Open Loop Gain (A_V) :

Open loop gain of a practical OP-AMP is not infinite. It is in the range of a few thousands. The open loop gain of IC 741 is 2×10^5.

3.7.2 Input Resistance (R_i) :

Input resistance of practical OP-AMP is few MΩ. For IC 741 the input resistance is 2 MΩ. For OP-AMPs having FET differential input stage the input resistance can be in the GΩ range ($1\ G\Omega = 1 \times 10^9\ \Omega$).

3.7.3 Output Resistance (R_o) :

Output resistance of a practical OP-AMP is few ohms. For IC 741 the output resistance is 75 Ω.

3.7.4 Bandwidth :

Practical OP-AMPs do not have infinite bandwidth. They have a bandwidth of few hundred kHz. Bandwidth of IC 741 is 1 MHz.

3.7.5 Input Offset Voltage (V_{ios}) :

MU : Dec. 02, May 04, May 10, May 14

University Questions

Q. 1 For OP-AMP explain input offset voltage. **(Dec. 02, May 10, 2 Marks, May 04, 3 Marks)**

Q. 2 Explain input offset voltage, CMRR and SVRR for operational amplifier. **(May 14, 5 Marks)**

- Ideally, for a zero input voltage, the OP-AMP output voltage should be zero.
- But practically it is not so. This is due to the unavoidable imbalances inside the OP-AMP, specially the imbalances in its differential input stage.
- So we have to apply a small differential voltage at the input of the OP-AMP to make the output voltage zero. This voltage is called as input offset voltage.
- The input offset voltage is denoted by V_{ios}. The input offset voltage is normally in a few mV range. The value of input offset voltage is temperature dependent. Ideally V_{ios} should be equal to zero.

3.7.6 Input Bias Current (I_B) :

MU : Dec. 02, May 04

University Questions

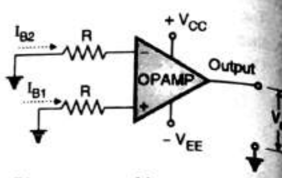
Q. 1 Explain input bias current used for OP-AMP. **(Dec. 02, 2 Marks)**

Q. 2 For OP-AMP explain bias current. **(May 04, 3 Marks)**

- Input bias current I_B is defined as the average of the currents flowing into the two input terminals of the OP-AMP i.e. I_{B1} and I_{B2} as shown in Fig. 3.7.1.

$\therefore$ Input bias current $I_B = \frac{I_{B1} + I_{B2}}{2}$ ...(3.7

- Ideally the currents I_{B1} and I_{B2} must be zero. But for practical OP-AMP they do exist due to the finite value of input resistance R_i. Due to slight difference in the characteristics of the transistors used in the input stage of an OP-AMP, the two currents I_{B1} and I_{B2} are not equal.



(B-340) Fig. 3.7.1 : Bias current and input offset current

- The maximum value of I_B is 50 nA for IC 741. It can be reduced to pA level using FET OP-AMPs. The value of input bias current is temperature dependent.

3.7.7 Input Offset Current (I_{ios}) :

MU : Dec

University Questions

Q. 1 For Op-Amp explain input offset current and give practical value. **(Dec. 05, 2 Ma**

- The input offset current I_{ios} of an OPAMP is defined as the algebraic difference between the currents flowing into its inverting and non-inverting terminals. Mathematically it is expressed as,

$$I_{ios} = | I_{B1} - I_{B2} | \quad ...(3.7.2)$$

Where I_{B1} is the current flowing into the non-inverting input and I_{B2} is the current flowing into the inverting terminal as shown in Fig. 3.7.1.

- Ideally, the input offset current must be zero and practically it should be as small as possible. The input offset current exists due to the unequal currents I_{B1} and I_{B2} flowing into the input terminals of the OP-AMP.
- The values of I_{B1} and I_{B2} are different from each other because the transistors used in the input stage of an OP-AMP are not exactly identical (or matched).
- The input offset currents for the BJT OP-AMPs is few tens or hundreds of nA. For IC 741C the maximum input offset current is 6 nA. For the FET OP-AMPs it is of the order of few pA.
- The input offset current will give rise to a finite output voltage even when the input voltage is zero. That is why it should be reduced to a lowest possible value. The input offset current is temperature dependent.

3.7.8 Common Mode Rejection Ratio (CMRR) :

MU : May 1

University Questions

Q. 1 List down various parameters of Op amp alongwith their typical values for IC741. Also explain what the significance of CMRR and slew rate is ? **(May 16, 10 Marks)**

CMRR of a practical OP-AMP is not ∞. However it is very high. For IC 741, the CMRR is 90 dB or 31622. Such a high CMRR helps to reject the common mode signals such as noise, successfully.

3.7.9 Power Supply Rejection Ratio (PSRR) :

MU : Dec. 02, Dec. 05, Dec. 06, Dec. 11, Dec. 12, May 14

University Questions

Q. 1 Explain PSRR used for OP-AMP and give typical values of each for 741 OP-AMP. **(Dec. 02, 2 Marks)**

Q. 2 For Op-Amp explain power supply rejection ratio and give practical value. **(Dec. 05, Dec. 06, 2 Marks)**

Q. 3 Explain the term PSRR. **(Dec. 11, Dec. 12, 3 Marks)**

Q. 4 Explain input offset voltage, CMRR and SVRR for operational amplifier. **(May 14, 5 Marks)**

- The change in an OP-AMPs input offset voltage (V_{ios}) due to variation in the supply voltage is called as Power Supply Rejection Ratio (PSRR). It is also called as Supply Voltage Rejection Ratio (SVRR) or Power Supply Sensitivity (PSS). Mathematically PSRR is expressed as,

$$PSRR = \frac{\Delta V_{ios}}{\Delta V} \quad ...(3.7.3)$$

where, ΔV_{ios} = Change in input offset voltage and ΔV = Change in the supply voltage.

- PSRR is expressed either in microvolts per volt or in decibels.
- For IC 741C, PSRR = 150 µV/V. The value of PSRR should ideally be equal to zero and practically it should be as small as possible.
- SVRR is specified assuming that both supply (positive and negative) magnitudes increasing or decreasing simultaneously.

3.7.10 Total Input Offset Voltage :

- Various parameters which affect the value of V_{ios} are temperature, CMRR, PSRR etc. The combined effect of these parameters on V_{ios} is expressed in the following equation :

$$V_{ios\,(total)} = V_{ios\,(initial)} + T.C.(V_{ios}) \times \Delta T + \frac{\Delta V_i}{CMRR} + \frac{\Delta V}{PSRR} + \frac{\Delta V_o}{A_d} \quad ...(3.7.4)$$

Where $V_{ios\,(initial)}$ = Initial input offset voltage (at ambient temperature)

3.7.11 Slew Rate :

MU : Dec. 02, Dec. 05, Dec. 06, May 16

University Questions

Q. 1 Explain the slew rate used for OP-AMP and give typical values of each for 741 OP-AMP. **(Dec. 02, 2 Marks)**

Q. 2 For Op-Amp explain slew rate and give practical values. **(Dec. 05, Dec. 06, 2 Marks)**

Q. 3 List down various parameters of Op amp alongwith their typical values for IC741. Also explain what the significance of CMRR and slew rate is ? **(May 16, 10 Marks)**

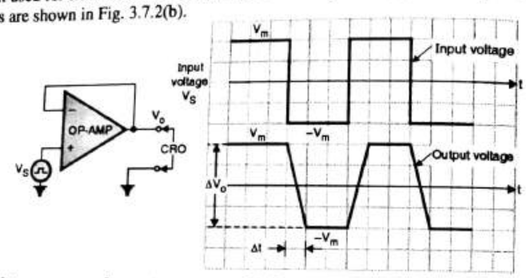
- Slew rate is defined as the maximum rate of change of output voltage per unit time. Mathematically it is expressed as follows :

$$S = \left.\frac{dV_o}{dt}\right|_{maximum} \text{Volts/µsec} \quad ...(3.7.5)$$

- The unit of slew rate are Volts/microseconds.

Importance of slew rate :

- Slew rate decides the capability of OP-AMP to change its output rapidly, hence it decides the highest frequency of operation of a given OP-AMP. The value of slew rate depends on change in voltage gain. Therefore it is generally specified at unity (+ 1) gain.
- Slew rate should be ideally ∞ and practically as high as possible. Slew rate of 741 OP-AMP is only 0.5 V/µS, which is its biggest drawback. Therefore it cannot be used for high frequency applications.
- The circuit used for slew rate measurement is shown in Fig. 3.7.2(a) and the input output voltage waveforms are shown in Fig. 3.7.2(b).



(a) Circuit to measure slew rate (b) Input and output voltage waveform

(K-143) **Fig. 3.7.2**

- The slew rate is measured from the output voltage waveform as :

$$\text{Slew rate } S = \frac{\Delta V_o}{\Delta t} \quad \text{...(3.7.6)}$$

- Ideally the slew rate should be infinite and practically it should be as large as possible.

Use of slew rate to calculate f_m :

- Assuming that the input signal V_S is a sinewave, we can obtain the value of maximum frequency for which the amplifier produces an undistorted output.

$$\text{Let, } V_S = V_m \sin \omega t \quad \text{...(3.7.7)}$$

- As we have used a unity gain non-inverting amplifier, the output is exactly equal to the input without any phase shift.

$$\therefore V_o = V_m \sin \omega t$$

Differentiating both sides we get,

$$\therefore \frac{dV_o}{dt} = \omega V_m \cos \omega t$$

$\frac{dV_o}{dt}$ will be maximum when $\cos \omega t = 1$ and maximum value of $\frac{dV_o}{dt}$ is nothing but slew rate S.

$$\therefore S = \frac{dV_o}{dt}(\max) = \omega V_m = 2\pi f_m V_m \text{ V/sec.}$$

$$\therefore f_m = \frac{S}{2\pi V_m} \quad \text{...(3.7.8)}$$

This is the maximum frequency f_m for which the amplifier produces an undistorted output. It is also called as full power bandwidth (FPB).

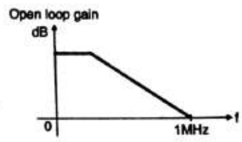
3.7.12 Gain Bandwidth Product :

MU : Dec. 05

University Questions

Q. 1 For Op-Amp explain unity gain bandwidth product and give practical values. **(Dec. 05, 2 Marks)**

- The gain bandwidth product (GB) is the bandwidth of the OP-AMP corresponding to the voltage gain of unity (1).
- The graph of open loop gain versus frequency of IC 741 is shown in Fig. 3.7.3. It shows that GB = 1 MHz.
- The other terms used for GB are closed loop bandwidth, unity gain bandwidth and small signal bandwidth.



(N-36) **Fig. 3.7.3 : A graph of open loop gain versus frequency of IC 741**

3.8 OP-AMP ICs 741 :

MU : May 03, May 05

University Questions

Q. 1 Give technical specifications of OP-AMP 741 with typical values of parameter. **(May 03, May 05, 5 Marks)**

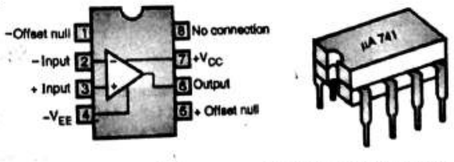
- This is one of the oldest and one of the most popular OP-AMP IC. It is a high performance monolithic operational amplifier.
- It can be used in various analog applications such as integrator, differentiator, summing amplifier etc. Features of this OP-AMP IC are as follows :

Features of IC 741 :

- No frequency compensation required.
- Short circuit protection has been provided.
- It has the offset voltage null capability.
- Large common mode and differential voltage ranges.
- There are no latch ups.

Pin configuration and pin functions :

- The pin configuration of OP-AMP IC 741 in the 8 pin mini DIP package is shown in Fig. 3.8.1(a).



(a) Pin configuration of IC 741 OP-AMP (b) 8 pin mini DIP package for IC 741

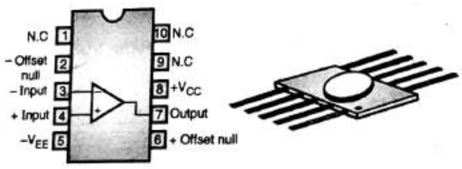
(N-36) **Fig. 3.8.1**

- This IC is an 8 pin IC in the dual in line package (DIP).
- Pin number 7 is for connecting the positive supply voltage $+V_{CC}$ while pin number 4 is to be connected to a negative supply voltage. Thus IC 741 needs a dual polarity power supply.

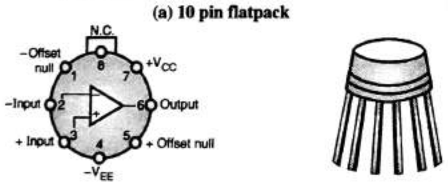
- Pin numbers 2 and 3 are inverting and non-inverting inputs respectively and we get the output voltage at pin number 6.
- Pin numbers 1 and 5 can be used to nullify the offset voltage and pin number 8 is a dummy pin which is not connected anywhere and hence should be left open (unconnected).
- Even though the mini DIP is the most popular package, the OP-AMP IC 741 is available in some other packages too. They are as follows :

Other packages of OP-AMP :

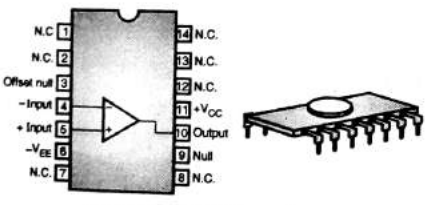
(a) 10 pin flatpack. (b) 8 pin metal can. (c) 14 pin DIP.



(a) 10 pin flatpack



(b) 8 pin-metal can package



(c) 14 lead dual in line package

(N-27)**Fig. 3.8.2 : Various packages of OP-AMP IC 741**

3.8.1 Important Characteristics of IC 741 :

MU : Dec. 02, Dec. 03, Dec. 11, May 16

University Questions

Q. 1 Give typical values of following terms for 741 OP-AMP :

1. CMRR
2. input offset voltage
3. input bias current
4. PSRR
5. Slew rate

(Dec. 02, 6 Marks)

Q. 2 Write the actual values of characteristics of OP-AMP IC 741.

Q. 3 Give typical practical values for IC 741 for the CMRR and PSRR. **(Dec. 03, 6 Marks)**

Q. 4 List down various parameters of Op amp alongwith their typical values for IC741. Also explain what the significance of CMRR and slew rate is ? **(Dec. 11, 5 Marks)** **(May 16, 10 Marks)**

The typical values of different important characteristics are listed in Table 3.8.1, alongwith the ideal values of those characteristics.

Table 3.8.1 : Important characteristics of IC 741 alongwith those of ideal OP-AMP

Sr. No.	Characteristics	Value for IC 741	Ideal value
1.	Input resistance R_i	2 MΩ	∞
2.	Output resistance R_o	75 Ω	0
3.	Voltage gain A_V	2×10^5	∞
4.	Bandwidth B.W.	1 MHz	∞
5.	CMRR	90 dB	∞
6.	Slew rate S	0.5 V/μS	∞
7.	Input offset voltage V_{ios}	2 mV	0
8.	PSRR	150 μV/V	0
9.	Input bias current I_B	50 nA	0
10.	Input offset current I_{ios}	6 nA	0

3.9 Voltage Transfer Characteristics of an Ideal OP-AMP :

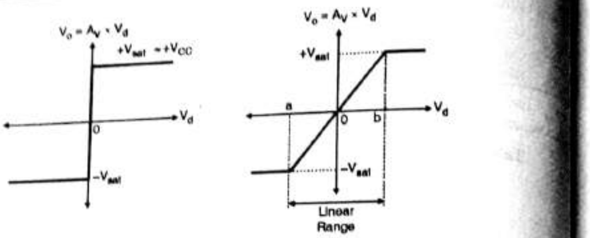
- A voltage transfer curve or characteristics of an OP-AMP is a graph of its output voltage V_o versus input voltage V_{in}.
- V_o is plotted on the Y-axis while V_{in} is plotted on the X-axis.
- The relation between the output voltage and input voltage is given by,

$$V_o = A_V \times V_d$$

where A_V = Open loop gain

V_d = Differential input voltage

- The maximum value of output voltage $V_{o\,(max)} = \pm V_{sat}$, where V_{sat} is called as the saturation voltage of OP-AMP.
- We know that the open loop gain of an ideal OP-AMP is infinite. Therefore the differential input voltage V_d required to obtain the maximum possible output voltage. i.e. $\pm V_{sat}$ is 0 volts where $\pm V_{sat} \approx + V_{CC}$.
- Therefore the ideal voltage transfer characteristics is as shown in Fig. 3.9.1(a).



(a) Ideal voltage transfer characteristics of an ideal OP-AMP **(b) Voltage transfer curve of a practical OP-AMP**

(N-37) **Fig. 3.9.1**

- Fig. 3.9.1(a) indicates that if the input voltage V_d becomes slightly positive, then V_o will swing to $+V_{sat}$ whereas if V_d is made slightly negative, then V_o will swing to $-V_{sat}$.

3.9.1 Voltage Transfer Characteristics of a Practical OP-AMP :

- The open loop voltage gain of a practical OP-AMP such as IC 741C is 2×10^5. Therefore even for very small magnitudes of V_d, the OP-AMP output will reach the positive or negative saturation voltage $+V_{sat}$ or $-V_{sat}$. The polarity of output is dependent on the polarity of V_d.
- If $\pm V_{sat} = \pm 12$ V, then let us calculate the required differential input voltage V_d to obtain $\pm V_{sat}$ at the output.

$$V_o = A_V \times V_d \qquad ...(3.9.1)$$

$$\therefore \pm V_{sat} = A_V \times V_d$$

$$\therefore V_d = \frac{\pm V_{sat}}{A_V} = \frac{\pm 12\text{ V}}{2 \times 10^5} = \pm 60\ \mu\text{V} \qquad ...(3.9.2)$$

- Equation (3.9.2) tells us that if V_d is greater than or equal to 60 μ V, then the output voltage will be $\pm V_{sat}$. This is shown in the voltage transfer characteristics of Fig. 3.9.1(b).
- **Range 0 – b :** If V_d is in the range of 0 to b, then the output voltage has linear relation with V_d. But if V_d is equal to or greater than point "b" then the OP-AMP goes into the positive saturation and $V_o = +V_{sat}$
- **Range 0 – a :** If V_d is in the range of 0 to a, then the output voltage increases linearly as shown in Fig. 3.9.1(b). But if V_d is equal to or greater than point "a" then the OP-AMP goes into the negative saturation i.e. $V_o = -V_{sat}$.

Linear Range (a to b) :

From Fig. 3.9.1(b) it is clear that the "linear relation" between the output voltage V_o and the differential voltage V_d exists only from point "a" to "b". The range of V_d over which the linear relation exists is very narrow.

3.10 Open Loop and Closed Loop Configurations of OP-AMP :

University Questions MU : Dec. 11

Q. 1 Draw :

1. Open loop configuration 2. Closed loop configuration

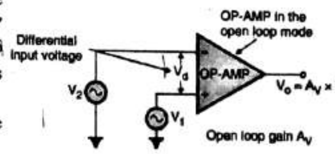
With respect to op-amp. Compare the above with respect to :

1. Feedback 2. Ideal and practical gain. **(Dec. 11, 10 Marks)**

- An OP-AMP can work in two different modes :
  1. Open loop mode 2. Closed loop mode.
- Let us understand the working of OP-AMP in these modes.

3.10.1 Open Loop Configuration of OP-AMP :

- All the ideal characteristics of OP-AMP discussed earlier are corresponding to the "open loop operation" of the OP-AMP.
- The meaning of "open loop operation" is that there is absolutely no feedback present from the output to input. That means no part of output gets connected to the input.
- The configuration of Fig. 3.10.1 shows the connections of OP-AMP in the "Open Loop" mode. Input signals V_1 and V_2 are applied to the non-inverting and inverting terminals respectively.
- The differential signal present between the two inputs is $V_d = (V_1 - V_2)$.



(N-38) **Fig. 3.10.1 : Open loop configuration**

- The open loop gain of the OP-AMP is A_V. Its value is very high. The output voltage is given by,

$$V_o = A_V \times V_d$$

$$= A_V (V_1 - V_2) = \pm V_{sat}$$

- Depending on the polarity of V_d, the output voltage will be either equal to $+V_{sat}$ or $-V_{sat}$.
- As A_V is very high, a very small value of V_d will drive the OP-AMP into positive or negative saturation. So the output voltage is not proportional to the differential input voltage.
- It is therefore not possible to use the open loop configuration for linear amplification.

Features of open loop configuration :

1. No feedback.	2. Very high voltage gain.
3. Can not be used as a linear amplifier.	4. Used in comparators.
5. Waveform distortion takes place.	6. Very high input resistance
7. Low output resistance	8. Large bandwidth

3.11 OP-AMP in Closed Loop Configurations :

MU : Dec. 11

University Questions

Q. 1 Draw :

1. Open loop configuration 2. Closed loop configuration

With respect to op-amp. Compare the above with respect to :

1. Feedback 2. Ideal and practical gain. **(Dec. 11, 10 Marks)**

In the closed loop configurations, some kind of "feedback" is introduced in the circuit. A part of output is returned back or fed back to the input.

Types of feedback :

The feedback can be one of the following two types :

1. Positive feedback or regenerative feedback OR
2. Negative feedback or degenerative feedback.

3.11.1 Positive Feedback :

- If the feedback signal and the original input signal are in phase with each other then it is called as the positive feedback.
- Positive feedback is used in the applications such as "Oscillators" and Schmitt triggers or regenerative comparators.

3.11.2 Negative Feedback :

- If the feedback signal and the original input signal are 180° out of phase with respect to each other, then it is called as negative feedback. Negative feedback is used in all the amplifiers.

3.12 Negative Feedback in OP-AMP :

MU : Dec. 11

University Questions

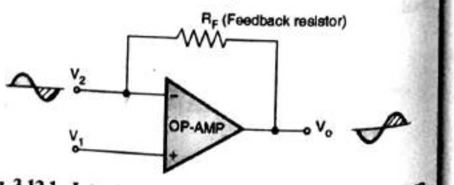
Q. 1 Draw :

1. Open loop configuration 2. Closed loop configuration

With respect to op-amp. Compare the above with respect to :

1. Feedback 2. Ideal and practical gain. **(Dec. 11, 10 Marks)**

- If the signal fed back to the input and the original input signal are 180° out of phase, then it is called as the negative feedback.
- In many applications of OP-AMP such as an amplifier, the negative feedback is used. In the amplifier circuits using OP-AMP, a feedback resistor R_F is connected between the output and inverting (–) input terminal as shown in Fig. 3.12.1 to introduce a negative feedback.



(K-27) **Fig. 3.12.1 : Introduction of negative feedback in OP-AMP**

- As the voltages V_2 and V_o are 180° out of phase, a fraction of output voltage feedback to the input via R_F will be 180° out of phase with the input. Hence R_F introduces a negative feedback. The voltage gain of OP-AMP decreases due to negative feedback.

Advantages of negative feedback :

Negative feedback is used in the amplifier circuits as they provide the following improvements in the operation of an amplifier :

1. It reduces and stabilizes the gain.
2. Reduces the distortion.
3. Increases the bandwidth.
4. Changes the values of input and output resistances. Input resistance increase and output resistance will decrease.
5. Reduces the effects of variations in temperature and supply voltage on the output of the OP-AMP.

Due to all these advantages, the negative feedback is used in the amplifier applications of OP-AMP.

3.13 Concept of Virtual Short and Virtual Ground :

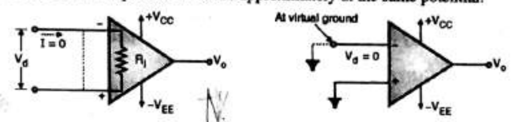
MU : Dec. 12

University Questions

Q. 1 Explain virtual short and virtual ground concept. **(Dec. 12, 4 Marks)**

- While analyzing different circuits that include OP-AMPs we need to use an important concept called "virtual short".
- According to virtual short concept, the potential difference between the two input terminals of an OP-AMP is almost zero.
- In other words both the input terminals are approximately at the same potential.

3.13 |



(a) Concept of virtual short **(b) Concept of virtual ground**

(B-349) **Fig. 3.13.1**

3.13.1 Virtual Short :

The concept of virtual short can be explained as follows :

- Assume that the OP-AMP is ideal one. The input impedance (R_i) of an OP-AMP is ideally infinite. Hence current "I" flowing from one input terminal to the other will be zero as shown in Fig. 3.13.1(a).
- Thus the voltage drop across R_i will be zero and both the input terminals will be at the same potential. In other words they are virtually shorted to each other.
- Other way of explaining this concept is as follows :
- The output voltage of an OP-AMP is given by,

$$V_o = A_V \cdot V_d \quad ...(3.13.1)$$

where, A_V = Open loop gain and V_d = Differential input voltage

$$\therefore \quad V_d = \frac{V_o}{A_V}$$

But $A_V = \infty$ for an ideal OP-AMP and $A_V = 2 \times 10^5$ = for IC 741.

$\therefore$ For ideal OP-AMP, $V_d = 0$. Thus the potential difference between the input terminals is zero.

- When we short circuit two points, they will have the same potential. Due to this reason, the two OP-AMP terminals which are almost equipotential are said to be virtually (not actually) short circuited. This is shown by a dotted line in Fig. 3.13.1(a).

3.13.2 Virtual Ground :

MU : Dec. 03. May 13. Dec. 15. Dec. 16

University Questions

Q. 1 Explain the concept of virtual ground. **(Dec. 03, 5 Marks)**

Q. 2 Write short note on : Virtual ground concept of OP-AMP. **(May 13, 5 Marks)**

Q. 3 With respect to op-amp explain the ideal characteristics and concept of virtual ground. Explain how op-amp can be used as an averaging amplifier in inverting configuration. Also draw neat circuit diagrams to :

1. Convert sine wave to square wave using op-amp.
2. Detect the crossing of zero's in the generated square wave. **(Dec. 15, 10 Marks)**

Q. 4 Explain the concept of virtual ground in operational amplifier. **(Dec. 16, 5 Marks)**

- If the non-inverting (+) terminal of OP-AMP is connected to ground as shown in Fig. 3.13.1(b), then due to the "virtual short" existing between the two input terminals, the inverting (–) terminal will also be at ground potential.
- Hence it is said to be at "virtual ground".
- Similarly if the inverting (–) terminal is connected to ground, then the non-inverting (+) terminal will be at "virtual ground" potential.
- The concept of virtual ground has been used extensively in analyzing various closed loop configurations, specially we use this concept in the inverting amplifier analysis.

3.13.3 Zero Input Current :

As the input resistance of the ideal OP-AMP is infinite, the current flowing into its input terminals is zero. Even for the practical OP-AMPs such as 741C, $R_{in} = 2\ M\Omega$ which is very large. Hence for all the practical purposes we will assume that the input current of an OP-AMP is zero.

3.14 Closed Loop Amplifier Configurations :

In this section we are going to discuss the following closed loop OP-AMP amplifier configurations :

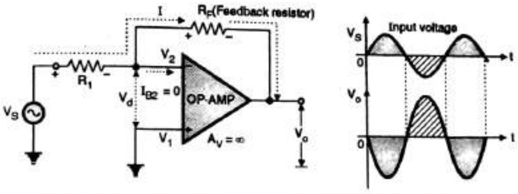
1. Inverting amplifier 2. Non-inverting amplifier 3. Voltage follower.

3.14.1 The Inverting Amplifier :

- The circuit diagram of an inverting amplifier is as shown in Fig. 3.14.1(a).
- The signal which is to be amplified is applied at the inverting (–) input terminal of the OP-AMP.
- The amplified output signal will be 180° out of phase with the input signal.
- In other words the output signal is "inverted" as shown in Fig. 3.14.1(b).
- Therefore this amplifier is known as the inverting amplifier.

Operation :

- The signal to be amplified (V_s) has been connected to the inverting terminal via the resistance R_1.
- The other resistor R_F, connected between the output and inverting input terminals is called as the feedback resistance. It introduces a negative feedback.
- The non-inverting (+) input terminal is connected to ground.
- As the OP-AMP is an ideal one, its open loop voltage gain $A_V = -\infty$ and input resistance $R_i = \infty$. The negative sign for A_V is due to the inverting configuration.
- The input and output voltage waveforms are as shown in Fig. 3.14.1(b). Output is an amplified and inverted version of the input signal V_S.



(a) Inverting amplifier **(b) Waveforms of inverting amplifier**

(K-29)**Fig. 3.14.1**

Expression for the closed loop voltage gain :

- Looking at Fig. 3.14.1(a) we can write that,

$$V_o = |A_V| \times V_d \quad ...(3.14.1)$$

$$\therefore \quad V_d = \frac{V_o}{|A_V|} \quad ...(3.14.2)$$

where, A_V = Open loop gain of OP-AMP.

- As we know A_V of an open loop OP-AMP is ∞.

$$\therefore \quad V_d = \frac{V_o}{\infty} = 0 \quad ...(3.14.3)$$

But, $V_d = V_1 - V_2$

$$V_1 - V_2 = 0 \quad ...(3.14.4)$$

- As the non-inverting (+) input terminal is connected to ground, $V_1 = 0$. Substituting this value in Equation (3.14.4) we get,

$$V_2 = 0$$

Thus V_2 is at ground potential.

- Since the input resistance $R_i = \infty$, the current going into the OP-AMP will be zero. Therefore the current "I" that passes through R_1 will also pass through R_F as shown in Fig. 3.14.1(a).

- As the input voltage V_S is being measured with respect to ground and as V_2 is at ground potential we can say that the input voltage V_S is voltage across R_1 and voltage across R_F is output voltage.
- The input voltage V_S is given by,

$$V_S = IR_1 \quad ...(3.14.5)$$

- And the output voltage is given by,

$$V_o = -IR_F \quad ...(3.14.6)$$

- We can write Equations (3.14.5) and (3.14.6) because V_2 is at approximately ground potential (virtual ground).
- Closed loop gain $A_{VF} = \frac{V_o}{V_S}$

Substituting the expressions for V_o and V_S we get,

$$A_{VF} = \frac{I R_F}{I R_1} = -\frac{R_F}{R_1} \quad ...(3.14.7)$$

$$\text{And} \quad V_o = A_{VF} \times V_S$$

Note : The negative sign indicates that there is a phase shift of 180° between the input and output voltages.

Conclusions from the expression for A_{VF} :

From Equation (3.14.7) we can draw the following important conclusions :

1. The value of closed loop voltage gain A_{VF} does not depend on the value of open loop voltage gain A_V.
2. Value of A_{VF} can be very easily adjusted by adjusting the values of the resistors R_F and R_1. Generally the feedback resistor R_F is a potentiometer to adjust the gain to its desired value.
3. The output voltage is an amplified inverted version of input voltage.

3.14.2 Non-Inverting Amplifier :

MU : Dec. 03

University Questions

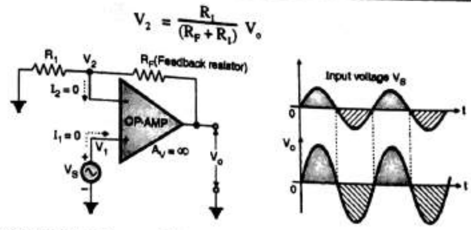
Q. 1 Derive the equation for voltage gain for a non-inverting amplifier. **(Dec. 03, 4 Marks)**

- The non-inverting amplifier using OP-AMP is as shown in Fig. 3.14.2(a).
- Here the signal which is to be amplified is applied to the non-inverting (+) input terminal of the OP-AMP and the inverting (–) input terminal is connected to ground via resistance R_1.
- As shown in Fig. 3.14.2(b), the input and output voltages are in phase with each other.
- The negative feedback is introduced in this circuit via the feedback resistor R_F which is connected between the output and inverting (–) input terminal of OP-AMP.

Expression for the closed loop voltage gain (A_{VF}) :

- As we are using an ideal OP-AMP, $R_i = \infty$. Therefore the currents entering into both the input terminals of OP-AMP will have a zero value. $(I_1 = I_2 = 0)$.
- Therefore voltage across R_1 is given by,

$$V_2 = \frac{R_1}{(R_F + R_1)} V_o \quad ...(3.14.8)$$



(a) Non-inverting amplifier

(b) Waveforms of non-inverting amplifier

(K-32)Fig. 3.14.2

- As per the virtual short concept discussed earlier

$$V_1 = V_S = V_2 \quad ...(3.14.9)$$

Substituting the expression for V_2 form Equation (3.14.8) we get,

$$\therefore \quad V_S = \frac{R_1}{(R_F + R_1)} V_o$$

- Therefore the closed loop voltage gain A_{VF} is given as,

$$A_{VF} = \frac{V_o}{V_S} = \frac{R_1 + R_F}{R_1}$$

$$\therefore \quad A_{VF} = 1 + \frac{R_F}{R_1}$$

$$\text{and} \quad V_o = A_{VF} \times V_S \quad ...(3.14.10)$$

Conclusions from the expressions for A_{VF} :

1. The positive sign of Equation (3.14.10) indicates that the input and output are in phase with each other.
2. The closed loop control gain is always greater than unity (1).
3. A_{VF} is adjustable and its value can be adjusted by varying the values of R_F and R_1. Generally a variable resistor is used in place of R_F to adjust the closed loop gain to its desired value.
4. A_{VF} is independent of the open loop gain of the OP-AMP. It depends only on the values of R_F and R_1.

3.14.3 Comparison of the Amplifier Configurations :

Table 3.14.1 : Comparison of inverting and non-inverting amplifiers

Sr. No.	Parameter	Inverting amplifier	Non-inverting amplifier
1.	Voltage gain.	$A_{VF} = -R_F / R_1$	$A_{VF} = 1 + \frac{R_F}{R_1}$
2.	Phase relation between input and output voltages.	180° out of phase.	In phase.

Sr. No.	Parameter	Inverting amplifier	Non-inverting amplifier
3.	Value of voltage gain.	Can be greater than, less than or equal to unity.	Always greater than or equal to unity.
4.	Input resistance.	Equal to R_1.	Very large.

3.14.4 Solved Examples on Inverting and Non-inverting Amplifiers :

Ex. 3.14.1 : The OP-AMP is used in the inverting and non-inverting mode with R_1 = 2 kΩ and R_F = 100 kΩ. If V_{CC} = ± 15 V and rms input voltage V_i = 20 mV, calculate the output voltage in each case.

Soln. :

Given : $R_1 = 2\ k\Omega,\ R_F = 100\ k\Omega,\ V_i = 20\ mV,\ V_{CC} = \pm 15\ V.$

1. Output voltage for an inverting amplifier :

For an inverting amplifier the output voltage,

$$V_o = A_{VF} \times V_i$$

$$\therefore\ V_o = \frac{-R_F}{R_1} \times V_i$$

Substituting the values we get, $V_o = \frac{-100}{2} \times 20 \times 10^{-3} = -1V$ ...Ans.

2. Output voltage for the non-inverting amplifier :

For the non-inverting amplifier the expression for output voltage is,

$$V_o = A_{VF} \times V_i = \left[1 + \frac{R_F}{R_1}\right] \times V_i$$

Substituting the values we get,

$$V_o = \left[1 + \frac{100}{2}\right] \times 20 \times 10^{-3} = 1.02V$$ ...Ans.

Ex. 3.14.2 : For an inverting amplifier the input voltage is a sinewave with a peak voltage of 1 volt and frequency of 1 kHz. Determine the value of output voltage and draw the input and output voltage waveforms. Assume R_1 = 1 kΩ, R_F = 10 kΩ and the supply voltage to be ± 15 V.

Soln. :

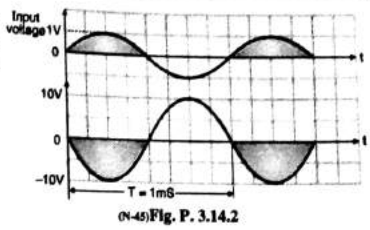
- Gain of the inverting amplifier is given by,

$$A_{VF} = \frac{-R_F}{R_1} = -\frac{10}{1} = -10 \quad ...(1)$$

- Output voltage = $A_{VF} \times V_{in} = -10 \times 1 = -10$ V peak ...Ans.

- Waveforms of input and output voltage are as shown in Fig. P. 3.14.2. Note that as f = 1 kHz the period of 1 cycle is given by,

$$T = \frac{1}{f} = \frac{1}{1\ kHz} = 1\ msec.$$



(N-45)Fig. P. 3.14.2

Note : As the amplifier is inverting type there is a phase shift of 180° between the input and output voltage waveforms. Thus the positive half cycle of the input coincides with the negative half cycle of the output (shaded portion of Fig. P. 3.14.2).

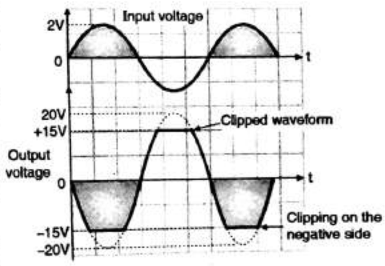
Ex. 3.14.3 : For the same amplifier of Ex. 3.14.2, if the input voltage is increased to 2 Volt peak draw the input and output voltage waveforms and explain them.

Soln. :

- The input and output voltage waveforms are as shown in Fig. P. 3.14.3. Gain A_{VF} = – 10.
- Here we expect that the peak output voltage will be :

$$V_{o\,(peak)} = -10 \times 2 = -20\ V$$

- However it does not happen so. The maximum output voltage that we can get is restricted to $\pm V_{(sat)}$ i.e. the saturation voltage.
- If we assume that $V_{(sat)} = V_{CC}$ then the maximum output voltage is restricted to $\pm V_{CC}$ i.e. ± 15 V.
- Hence the output waveform gets clipped on both the sides as shown in Fig. P. 3.14.3.
- Also note that the input and output waveforms are out of phase with respect to each other.



(N-46)Fig. P. 3.14.3

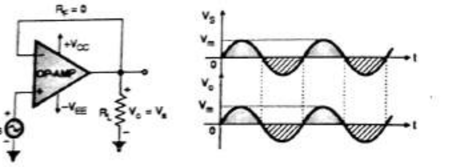
3.15 The Voltage Follower (Unity Gain Buffer) : MU : May 09, May 11

University Questions

Q. 1 With the circuit diagram explain voltage-follower and draw input-output waveforms. **(May 09, 4 Marks, May 11, 5 Marks)**

- When $R_1 = \infty$ and $R_F = 0$ the non-inverting amplifier gets converted into a voltage follower or unity gain amplifier.

- When the non-inverting amplifier is configured so as to obtain a gain of 1, it is called as a voltage follower or unity gain non-inverting buffer.
- The schematic diagram for a voltage follower is as shown in Fig. 3.15.1(a).
- The voltage follower configuration of Fig. 3.15.1(a) is obtained by short circuiting R_F and open circuiting R_1 connected in the usual non-inverting amplifier configuration of Fig. 3.14.2(a).
- Thus all the output voltage is fedback to the inverting input of the OP-AMP. Therefore the feedback factor of this circuit i.e. B = 1.



(a) Voltage follower circuit **(b) Waveforms of voltage follower**

(K-35)**Fig. 3.15.1**

Closed loop gain (A_{VF}) :

- Consider the expression for the closed loop gain of a non-inverting amplifier, that is,

$$A_{VF} = 1 + \frac{R_F}{R_1}$$

- In this equation, substitute the values of $R_F = 0$ and $R_1 = \infty$ to get the closed loop gain of the voltage follower as :

$$A_{VF} = 1 \quad ...(3.15.1)$$

- Therefore the output voltage will be equal to and in phase with the input voltage, as shown in Fig. 3.15.1(b). Thus voltage follower is a non-inverting amplifier with a voltage gain of unity.
- The unity gain amplifier does not behave like a conventional voltage amplifier but it acts as a **Resistance Transformer.**

Features of a Voltage Follower Circuit :

The important features of the voltage follower circuit are as follows :

1. Closed loop voltage gain equal to 1 i.e. output is equal to input with no phase shift.
2. Very high input impedance
3. Very low output impedance
4. Large bandwidth.

3.16 Summing Amplifier or Adder : MU : May 10, Dec. 10, May 12, Dec. 12, Dec. 13

University Questions

Q. 1 Explain how an Op-Amp can be used as : Summing amplifier. **(May 10, 5 Marks, Dec. 12, 4 Marks)**

Q. 2 Explain summing amplifier. **(Dec. 10, 10 Marks)**

Q. 3 Explain OP-AMP as summer. **(May 12, 5 Marks)**

Q. 4 Explain how op-amp can be used as a summing, scaling and averaging amplifier in the inverting configuration ? **(Dec. 13, 10 Marks)**

- It is possible to apply more than one input signal to an inverting amplifier. This circuit will then add all these input signals to produce their addition at the output.
- Such a circuit will then be called as an adder or a summing amplifier.
- Depending on the polarity or sign of the output voltage the adder circuits can be classified into two categories as :

1. Inverting adder and
2. Non-inverting adder.

3.16.1 Inverting Adder or Inverting Summing Amplifier : MU : Dec. 08, Dec. 09, Dec. 15

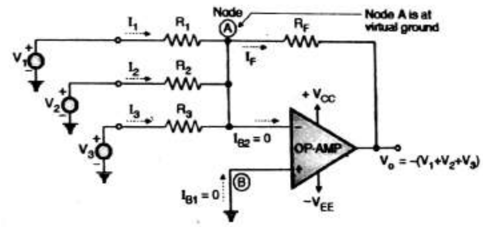
University Questions

Q. 1 With the circuit diagram explain inverting summing amplifier. **(Dec. 08, 4 Marks)**

Q. 2 Explain how OP-AMP can be used as summing amplifier in inverting configuration. **(Dec. 09, 4 Marks)**

Q. 3 Explain how operational amplifier can be used for addition of two AC signals with one DC signal ? **(Dec. 15, 5 Marks)**

- Fig. 3.16.1 shows the "inverting summing amplifier" configuration with three inputs V_1, V_2 and V_3.
- Depending on the relation between the feedback resistor R_F and the three input resistances R_1, R_2 and R_3 we can use the same circuit shown in Fig. 3.16.1 as a summing amplifier, scaling amplifier or averaging amplifier.



(N-66)**Fig. 3.16.1 : An inverting adder**

- V_1, V_2 and V_3 are three input signals applied simultaneously to the inverting terminal of the OP-AMP through resistors R_1, R_2 and R_3 respectively.
- V_1, V_2 and V_3 are measured with respect to ground. R_F is the feedback resistor connected between the output terminal and the inverting input terminal of OP-AMP. The non-inverting input terminal is connected to ground.
- So the configuration of Fig. 3.16.1 is basically an inverting amplifier with three inputs.
- Let the currents through the resistors R_1, R_2 and R_3 be I_1, I_2 and I_3 respectively..

For the analysis of this circuit we assume that the OP-AMP is ideal. Hence its input resistance is $R_i = \infty$. Therefore the currents I_{B1} and I_{B2} are zero. In addition to this, Node A is at virtual ground potential.

Expression for the output voltage :

- Apply KCL at node A of Fig. 3.16.1 to write,

$$I_1 + I_2 + I_3 = I_{B2} + I_F \quad ...(3.16.1)$$

- But as R_i of the OP-AMP is ideally infinite, $I_{B2} = 0$ and $V_A = V_B = 0$ due to virtual ground concept.

$$\text{Hence,}\quad I_1 + I_2 + I_3 = I_F \quad ...(3.16.2)$$

$$\text{On the input side,}\quad I_1 = \frac{V_1 - V_A}{R_1} = \frac{V_1}{R_1} \text{ as } V_A = 0 \quad ...(3.16.3)$$

$$\text{Similarly,}\quad I_2 = \frac{V_2 - V_A}{R_2} = \frac{V_2}{R_2} \text{ and } I_3 = \frac{V_3 - V_A}{R_3} = \frac{V_3}{R_3} \quad ...(3.16.4)$$

- And on the output side,

$$I_F = \frac{V_A - V_o}{R_F} = -\frac{V_o}{R_F} \quad ...(3.16.5)$$

- Substituting these values in Equation (3.16.2) we get,

$$\left.\begin{array}{c} \dfrac{V_1}{R_1} + \dfrac{V_2}{R_2} + \dfrac{V_3}{R_3} = -\dfrac{V_o}{R_F} \\ \text{OR} \\ V_o = -\left[\dfrac{R_F}{R_1} V_1 + \dfrac{R_F}{R_2} V_2 + \dfrac{R_F}{R_3} V_3\right] \end{array}\right\} \quad ...(3.16.6)$$

- In Equation (3.16.6) if we substitute $R_F = R_1 = R_2 = R_3 = R$ then we get,

$$V_o = -(V_1 + V_2 + V_3) \quad ...(3.16.7)$$

Thus output voltage is the negative sum of the input voltage. Therefore this circuit is called as "Inverting adder" or "Inverting summing amplifier". Similarly we can add any number of inputs.

3.16.2 Scaling or Weighted Amplifier :

MU : Dec. 09, Dec. 12, Dec. 13

University Questions

Q. 1 Explain how OP-AMP can be used as scaling amplifier in inverting configuration. **(Dec. 09, Dec. 12, 3 Marks)**

Q. 2 Explain how op-amp can be used as a summing, scaling and averaging amplifier in the inverting configuration ? **(Dec. 13, 10 Marks)**

- The same circuit shown in Fig. 3.16.1 can also be used as a scaling amplifier.
- This is possible if each input voltage is amplified by a different factor. This can be done if resistors R_1, R_2, R_3 and R_F are not equal to each other but of different values.
- Repeating Equation (3.16.6) we can write that,

$$V_o = -\left[\frac{R_F}{R_1} V_1 + \frac{R_F}{R_2} V_2 + \frac{R_F}{R_3} V_3\right] \quad ...(3.16.8)$$

where $R_1 \neq R_2 \neq R_3 \neq R_F$

- Thus it is possible to "scale" each of the input terminals as per our requirements.

3.16.3 Averaging Circuit :

MU : Dec. 09, Dec. 12, Dec. 13, Dec. 15, May 16

University Questions

Q. 1 Explain how OP-AMP can be used as averaging amplifier in inverting configuration. **(Dec. 09, Dec. 12, 3 Marks)**

Q. 2 Explain how op-amp can be used as a summing, scaling and averaging amplifier in the inverting configuration ? **(Dec. 13, 10 Marks)**

Q. 3 With respect to op-amp explain the ideal characteristics and concept of virtual ground. Explain how op-amp can be used as an averaging amplifier in inverting configuration. Also draw neat circuit diagrams to :

1. Convert sine wave to square wave using op-amp.
2. Detect the crossing of zero's in the generated square wave. **(Dec. 15, 10 Marks)**

Q. 4 Explain how operational amplifier can be used for taking average of three signals. **(May 16, 5 Marks)**

- The inverting adder circuit of Fig. 3.16.1 can be used as an averaging circuit by setting $R_1 = R_2 = R_3 = R$ and $R_F = R / 3$. Substitute these values into Equation (3.16.6) to get,

$$V_o = \frac{-R_F}{R}(V_1 + V_2 + V_3) = \frac{-R/3}{R}(V_1 + V_2 + V_3)$$

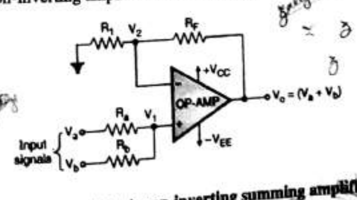
$$\therefore\ V_o = \frac{-(V_1 + V_2 + V_3)}{3} \quad ...(3.16.9)$$

- Thus the magnitude of output voltage is equal to the average of the three input voltages. This principle can be extended for n number of inputs by setting $R_F = R / n$ and $R_1 = R_2 = ... = R_n = R$.
- Then the output voltage is given by,

$$V_o = \frac{-(V_1 + V_2 + V_3 + ... + V_n)}{n}$$

3.16.4 Non-Inverting Adder :

- An adder circuit which can produce the addition of its input signals without inversion (without negative sign) is called as the non-inverting adder or non-inverting summing amplifier.
- Such a configuration with two inputs V_a and V_b is shown in Fig. 3.16.2. Note that the input signals and their resistors are connected to the non-inverting (+) input terminal of the OP-AMP.
- In short this is a non-inverting amplifier with two inputs.



(K-41)Fig. 3.16.2 : A non-inverting summing amplifier

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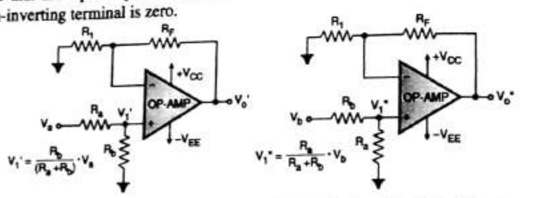
Expression for output voltage (V_o) :

- Assume that the input impedance of OP-AMP is very large. Therefore the current entering into the non-inverting terminal is zero.



(a) Equivalent circuit for $V_b = 0$ **(b) Equivalent circuit for $V_a = 0$**

(K-42) **Fig. 3.16.3**

- Let us use the superposition theorem to obtain the voltage V_1. So consider only one input at a time making the other input zero. Consider V_a only and make $V_b = 0$ i.e. short circuit it to ground. The equivalent circuit is as shown in Fig. 3.16.3(a).
- The voltage at the non-inverting terminal due to only V_a is given by,

$$\therefore \; V_1' = V_a \cdot \frac{R_b}{(R_a + R_b)} \qquad ...(3.16.10)$$

Assume $$R_a = R_b = R \; \therefore \; V_1' = \frac{V_a}{2} \qquad ...(3.16.11)$$

Now consider V_b only and make $V_a = 0$. The equivalent circuit is as shown in Fig. 3.16.3(b).

$$\therefore \; V_1'' = \frac{R_a}{(R_a + R_b)} \cdot V_b = \frac{V_b}{2} \qquad ...(3.16.12)$$

$$\therefore \; V_1 = V_1' + V_1'' \; \therefore \; V_1 = \frac{V_a + V_b}{2} \qquad ...(3.16.13)$$

As the amplifier is non-inverting type, its gain is given by,

$$A_{VF} = 1 + \frac{R_F}{R_1} \qquad ...(3.16.14)$$

$$\text{If} \quad R_F = R_1 = R \quad \text{then} \quad A_{VF} = 1 + 1 = 2 \qquad ...(3.16.15)$$

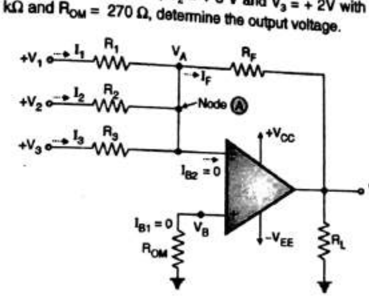
The output voltage, $$V_o = A_{VF} \times V_1 = 2 \times \frac{(V_a + V_b)}{2}$$

$$\therefore \; V_o = V_a + V_b \qquad ...(3.16.16)$$

- This expression for output voltage shows that the output voltage is the algebraic addition of the two input voltages.
- The positive sign of output voltage indicates that the inversion does not take place in this adder circuit and therefore it is called as the non-inverting adder.

3.16.5 Solved Examples on Inverting and Non-Inverting Adders :

Ex. 3.16.1 : In Fig. P. 3.16.1, if $V_1 = +1V$, $V_2 = +3$ V and $V_3 = +2V$ with $R_1 = R_2 = R_3 = 2$ kΩ and $R_F = 3$ kΩ and $R_{OM} = 270$ Ω, determine the output voltage.



(B-358) **Fig. P. 3.16.1**

Soln. :

- From the circuit diagram of Fig. P. 3.16.1 it is clear that it is an inverting summing amplifier. The expression for its output voltage is given by Equation (3.16.6) as :

$$V_o = -\left[\frac{R_F}{R_1} V_1 + \frac{R_F}{R_2} V_2 + \frac{R_F}{R_3} V_3\right] \qquad ...(1)$$

But $R_1 = R_2 = R_3 = 2$ kΩ and $R_F = 3$ kΩ

$$\therefore \; V_o = -\frac{3\text{ k}\Omega}{2\text{ k}\Omega}(V_1 + V_2 + V_3) \qquad ...(2)$$

$$= \frac{-3}{2}(1+3+2)$$

$$\therefore \; V_o = -9\text{ V} \qquad \textbf{...Ans.}$$

- Here R_{om} resistor is used for bias compensation and it does not affect the operation of inverting summer. Hence the equation for output voltage remains same.

Ex. 3.16.2 : What modification will you make in the summing amplifier of Fig. P. 3.16.1 to convert it into an inverting averaging amplifier ?

Soln. : In order to convert the inverting summing amplifier of Fig. P. 3.16.1 into an inverting averaging amplifier, the expression for output voltage should get modified to

$$V_o = -\frac{1}{3}(V_1 + V_2 + V_3)$$

Now compare this expression with Equation (1) of the Ex. 3.16.1 i.e.

$$V_o = -\frac{R_F}{R}(V_1 + V_2 + V_3)$$

So comparing these equations we get,

$$\frac{R_F}{R} = \frac{1}{3}$$

Assume $R_1 = R_2 = R_3 = R = 3\ k\Omega$.

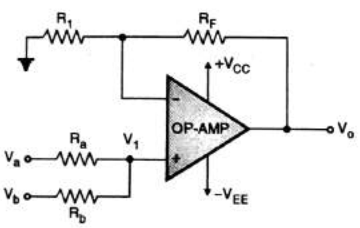
$$\therefore \quad R_F = \frac{R}{3} = 1\ k\Omega$$

Thus the resistor values should be modified as follows to get an inverting averaging amplifier :

$$R_F = 1\ k\Omega \text{ and } R_1 = R_2 = R_3 = 3\ k\Omega$$

...Ans.

Ex. 3.16.3 : In Fig. P. 3.16.3, if $V_a = +2$ V and $V_b = +4$V, $R_a = R_b = R_1 = 1\ k\Omega$ and $R_F = 3\ k\Omega$ determine the voltage V_1 at the non-inverting terminal of OP-AMP and the output voltage V_o.



(B-359) **Fig. P. 3.16.3**

Soln. :

1. **Voltage V_1 at the non-inverting terminal :**

Using the superposition theorem we can obtain the expression for V_1. This has already been done in section 3.16.4. So referring to Equation (3.16.13) we can write that,

$$V_1 = \frac{V_a + V_b}{2} = \frac{2+4}{2} = 3\text{V}$$

...Ans.

2. **Output voltage V_o :**

$$V_o = A_{VF} \times V_1 = \left[1 + \frac{R_F}{R_1}\right] \times V_1$$

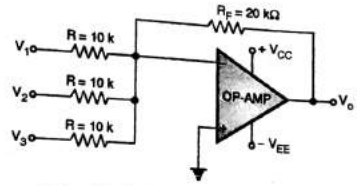
$$\therefore \quad V_o = \left[1 + \frac{3}{1}\right] \times 3 = 12\text{V}$$

...Ans.

Ex. 3.16.4 : Design a summing amplifier to add three input voltages. The output of this circuit must be equal to 2 times the negative sum of the inputs.

Soln. :

The required summing amplifier circuit is as shown in Fig. P. 3.16.4.



(F-2963) **Fig. P. 3.16.4 : Summing amplifier**

- The expression for output voltage of this circuit is given by,

$$V_o = \frac{-R_F}{R}(V_1 + V_2 + V_3) \qquad ...(1)$$

- We want the output to be equal to 2 times the negative sum of inputs. That means,

$$V_o = -2\,(V_1 + V_2 + V_3) \qquad ...(2)$$

$$\therefore \quad \frac{R_F}{R} = 2\ R_F \text{ or } = 2R$$

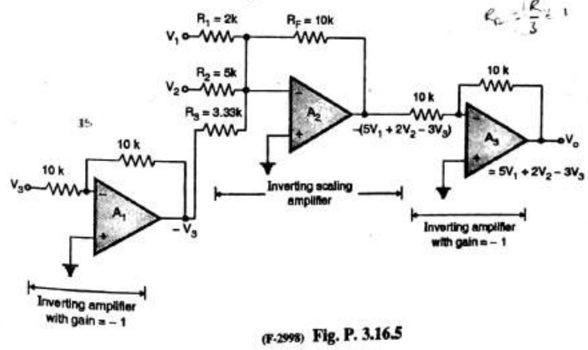
Select $R = 10\ k\Omega \qquad \therefore\ R_F = 20\ k\Omega$

...Ans.

Ex. 3.16.5 : Using practical Op-Amp like IC 741 with values of component realize the expression. $V_o = 5\ V_1 + 2\ V_2 - 3\ V_3$

Dec. 04, 6 Marks

Soln. : The realization is shown in Fig. P. 3.16.5.



(F-2998) **Fig. P. 3.16.5**

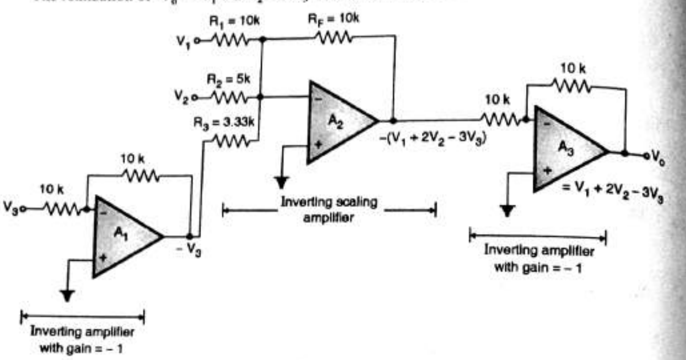
Ex. 3.16.6 : Using OP-AMP draw circuit diagram with the values of resistors to realise the following expressions.

$V_o = V_1 + 2\ V_2 - 3\ V_3$

May 05, 5 Marks

Soln. :

The realization of $V_o = V_1 + 2V_2 - 3V_3$ is shown in Fig. P. 3.16.6.

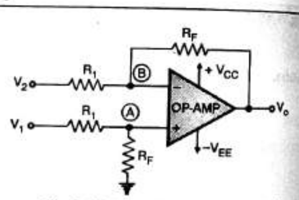


(F-2999) **Fig. P. 3.16.6**

3.17 Difference Amplifier and Subtractor :

3.17.1 Difference Amplifier :

- The difference amplifier and subtractor circuits are used to obtain the subtraction of two input voltages.
- The circuit diagram of the difference amplifier using OP-AMP is shown in Fig. 3.17.1.



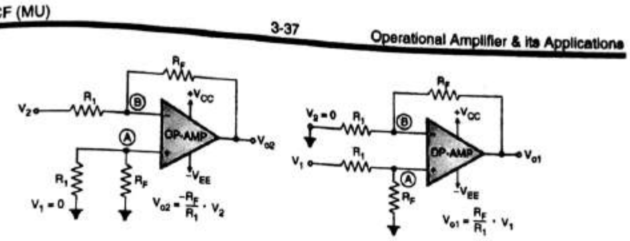
(K-44) **Fig. 3.17.1 : A difference amplifier**

Expression for output voltage :

- Let us use the superposition theorem to obtain the expression for output voltage.
- Consider only V_2 connected and assume V_1 to be short circuited i.e. connected to ground. Then the difference amplifier circuit reduces to an inverting amplifier with input V_2 as shown in Fig. 3.17.2(a).
- The output voltage due to only V_2 be denoted by V_{o2}.

  The output voltage V_{o2} is given by, $V_{o2} = \frac{-R_F}{R_1} \times V_2$ ...(3.17.1)

  Where $- R_F/ R_1$ is the gain of inverting amplifier shown in Fig. 3.17.2(a).



(a) Equivalent circuit with $V_1 = 0$ (b) Equivalent circuit with $V_2 = 0$

(K-45) **Fig. 3.17.2**

- Now assume $V_2 = 0$. That means V_2 is short circuited to ground and only V_1 connected. Then the difference amplifier circuit gets modified to the equivalent circuit of Fig. 3.17.2(b).
- It is a non-inverting amplifier with output voltage V_{o1}.
- The output voltage V_{o1} due to only V_1 is given by, $V_{o1} = A_{VF} \times V_A$

  But $A_{VF} = 1 + \frac{R_F}{R_1}$ and $V_A = \frac{R_F}{R_1 + R_F} \cdot V_1$

  $$\therefore \quad V_{o1} = \left[1 + \frac{R_F}{R_1}\right] \times \left[\frac{R_F}{(R_F + R_1)}\right] V_1 = \frac{(R_1 + R_F)}{R_1} \times \frac{R_F}{(R_F + R_1)} \cdot V_1$$

  $$\therefore \quad V_{o1} = \frac{R_F}{R_1} \cdot V_1 \qquad ...(3.17.2)$$

- Combining these two outputs we get the output voltage of the difference amplifier as,

  $$V_o = V_{o1} + V_{o2} = -\frac{R_F}{R_1} \cdot V_2 + \frac{R_F}{R_1} \cdot V_1$$

  $$\therefore \quad V_o = \frac{R_F}{R_1}(V_1 - V_2) \qquad ...(3.17.3)$$

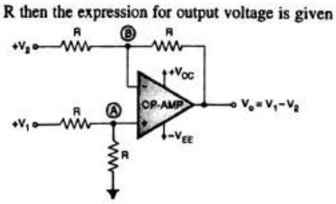
- Thus the output voltage is proportional to the difference between the two inputs. (R_F/R_1) is called as the "gain of the difference amplifier".

3.17.2 Subtractor :

- In Equation (3.17.3) if we substitute $R_F = R_1 = R$ then the expression for output voltage is given by,

  $$V_o = (V_1 - V_2) \qquad ...(3.17.4)$$

- And the difference amplifier gets transformed into a subtractor. The subtractor circuit is as shown in Fig. 3.17.3. Note that all the resistors are of same value R. Therefore its gain is 1.



(K-46) **Fig. 3.17.3 : Subtractor using OP-AMP**

3.17.3 Examples on Difference Amplifier and Subtractor :

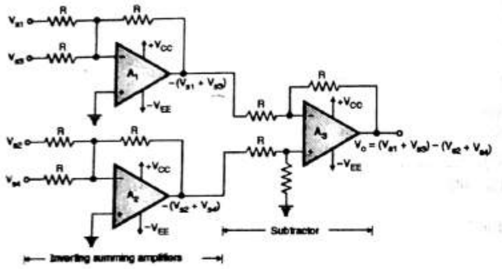
Ex. 3.17.1 : Design a circuit with OP-AMP to produce the output V_o given by,

$V_o = (V_{s1} + V_{s3}) - (V_{s2} + V_{s4})$

May 13, 05 Marks

Soln. :

The required circuit is shown in Fig. P. 3.17.1.



(B-368) **Fig. P. 3.17.1**

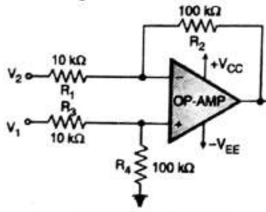
Ex. 3.17.2 : A circuit is desired in which the difference between two signals V_1 and V_2 is to be amplified by a factor of 10. Design a circuit to perform this.

Soln. :

The mathematical expression for the output voltage is given by,

$$V_o = 10\,(V_1 - V_2)$$

- The circuit that we need to implement the above expression is a subtractor with a gain of 10. The subtractor circuit is as shown in Fig. P. 3.17.2.



(F-2964) **Fig. P. 3.17.2**

For this circuit, if $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

then $V_o = \frac{R_2}{R_1}(V_1 - V_2)$

Assume $R_1 = 10\ k\Omega$

$\therefore\quad R_2 = 10\,R_1 = 100\ k\Omega$ $\quad ...\left[\because \frac{R_2}{R_1} = 10\right]$

Also $R_4 = R_2 = 100\ k\Omega$ and $R_3 = R_1 = 10\ k\Omega$.

With these component values it is possible to get,

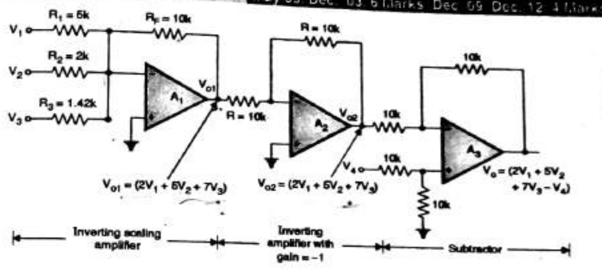
$$V_o = 10\,(V_1 - V_2)$$

Ex. 3.17.3 : Give the circuit diagram with values of components to realize following relation.

$V_o = 2\,V_1 + 5\,V_2 + 7\,V_3 - V_4$ (Use OP-AMP 741)

May 03, Dec. 03, 6 Marks, Dec. 09, Dec. 12, 4 Marks

Soln. :



(F-2997) **Fig. P. 3.17.3**

Description :

The given expression can be realised by using three blocks namely the inverting scaling amplifier, a unity gain inverting amplifier and a subtractor.

1. **Inverting scaling amplifier :**

For the inverting scaling amplifier the expression for output voltage is

$$V_{o1} = \left[\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V_2 + \frac{R_F}{R_3}V_3\right] \quad ...(1)$$

Comparing it with the given expression we get,

$$\frac{R_F}{R_1} = 2,\quad \frac{R_F}{R_2} = 5 \quad \text{and} \quad \frac{R_F}{R_3} = 7$$

Let $R_F = 10\ k\Omega$

$\therefore\quad R_1 = 5\ k\Omega,\ R_2 = 2\ k\Omega$ and $R_3 = 1.42\ k\Omega$

2. **Unity gain inverting amplifier :**

The gain of this block is $A_{VF} = -\frac{R_F}{R_1}$

We want it to be – 1.

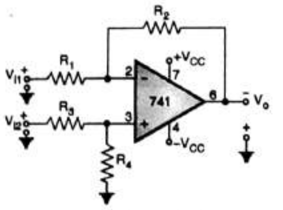
$\therefore\quad -R_F / R_1 = -1 \qquad \therefore\ R_F = R_1$

Choose $R_F = R_1 = 10\ k\Omega$

3. Subtractor :

For a subtractor, all the resistors should be of the same value. Hence choose all the resistors of 10 kΩ value.

Ex. 3.17.4 : **A difference amplifier is to be designed to amplify the difference between two voltages by a factor of 20. The inputs each approximately equal to 2 V. Determine suitable resistor values for the circuit shown in Fig. P. 3.17.4 using a 741 opamp.** Dec. 16, 5 Marks



(F-4765) **Fig. P. 3.17.4**

Soln. :

The gain of the difference amplifier is given by,

$$\text{Gain} = \frac{R_2}{R_1}$$

$$\therefore\ 20 = \frac{R_2}{R_1} \text{ or } R_2 = 20\,R_1.$$

Let $R_1 = 1\ k\Omega$ ...Ans.

$\therefore$ $R_2 = 20\ k\Omega$ ...Ans.

For proper operation of the circuit,

$R_4 = R_2 = 20\ k\Omega$ ...Ans.

and $R_3 = R_1 = 1\ k\Omega$ ...Ans.

3.18 Integrator :

MU : Dec. 09, Dec. 13

University Questions

Q. 1 **Write short note on : OP-AMP as an Integrator.** **(Dec. 09, 4 Marks)**

Q. 2 **Sketch an op-amp integrating circuit together with the circuit waveforms. Explain in brief the circuit operation.** **(Dec. 13, 5 Marks)**

The integrators are broadly categorized into two categories as :

1. Passive integrators
2. Active integrators.

But we are going to discuss only about the active integrator using OP-AMP.

3.18.1 Active Integrator using OP-AMP :

MU : Dec. 13

University Questions

Q. 1 **Sketch an op-amp integrating circuit together with the circuit waveforms. Explain in brief the circuit operation.** **(Dec. 13, 5 Marks)**

The circuit in which the output voltage waveform is the "integration" of the input voltage waveform is called as an integrator or integrating amplifier. In this section, we are going to discuss two integrator circuits i.e. ideal and practical integrators.

3.18.2 Basic Integrator Circuit :

MU : Dec. 03, May 10, May 12, May 13, Dec. 13

University Questions

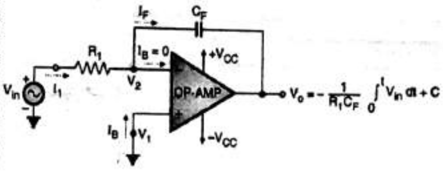
Q. 1 **Explain the working of integrator circuit.** **(Dec. 03, 6 Marks)**

Q. 2 **Explain how an Op-Amp can be used as : Integrator** **(May 10, May 13, 5 Marks)**

Q. 3 **Compare ideal and practical integrator circuit.** **(May 12, 10 Marks)**

Q. 4 **Sketch an op-amp integrating circuit together with the circuit waveforms. Explain in brief the circuit operation.** **(Dec. 13, 5 Marks)**

- The ideal integrator circuit is as shown in Fig. 3.18.1. This circuit is obtained by replacing the feedback resistor R_F in the inverting amplifier configuration by "C_F". Assume that the OP-AMP used here is an ideal one. V_2 is at virtual ground potential.



(K-52) **Fig. 3.18.1 : An ideal integrator circuit**

Expression for the output voltage :

- Referring to the Fig. 3.18.1, apply the Kirchhoff's current law at node V_2 to write,

$$I_1 = I_B + I_F \quad ...(3.18.1)$$

- Due to high input impedance R_i of the OP-AMP, I_B will be negligible as compared to I_F.

$$\therefore\ I_1 \approx I_F \quad ...(3.18.2)$$

- The basic relation between the current through and voltage across a capacitor is as follows :

$$I_C = C \cdot \frac{d\,V_c}{dt} \quad ...(3.18.3)$$

- But in case of an integrator, $I_F = I_{CF}$. Therefore Equation (3.18.3) gets modified as :

$$I_1 = C_F\,\frac{dV_c}{dt} \quad ...(3.18.4)$$

But, $I_1 = \dfrac{V_{in} - V_2}{R_1}$ and $V_c = (V_2 - V_0)$.

- Substituting these values into Equation (3.18.4) we get,

$$\frac{V_{in} - V_2}{R_1} = C_F\,\frac{d}{dt}(V_2 - V_0). \quad ...(3.18.5)$$

- Using the concept of "virtual ground" we can write that,

$$V_2 = 0 \qquad ...(3.18.6)$$

- Substituting this value into Equation (3.18.5) we get,

$$\frac{V_{in}}{R_1} = C_F \frac{d}{dt}(-V_o) \qquad ...(3.18.7)$$

- The output voltage can be obtained by integrating the above equation as :

$$V_o = -\frac{1}{R_1 C_F}\int_0^t V_{in}\,dt + C \qquad ...(3.18.8)$$

Where C is the constant of integration and it is proportional to the output voltage V_o at t = 0 seconds.

Conclusion :

Equation (3.18.8) indicates that the output voltage is negative integration of the input voltage. The negative sign is introduced due to the fact that the basic configuration used is an inverting amplifier. This circuit is therefore called as an inverting integrator.

3.18.3 Input and Output Waveforms : MU : Dec. 13

University Questions

Q. 1 Sketch an op-amp integrating circuit together with the circuit waveforms. Explain in brief the circuit operation. **(Dec. 13, 5 Marks)**

- The expression for output voltage of a basic integrator is given by,

$$V_o = -\frac{1}{R_1 C_F}\int_0^t V_{in}\,dt + C \qquad ...(3.18.9)$$

- But C = Integration constant = $V_o(0)$

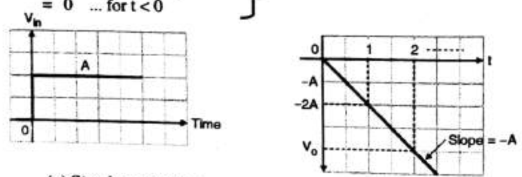
Where $V_o(0)$ is the output voltage at t = 0.

$$\therefore \quad V_o = -\frac{1}{R_1 C_F}\int_0^t V_{in}\,dt + V_o(0) \qquad ...(3.18.10)$$

Output for a step input :

- Let us obtain the output voltage for a step input voltage of amplitude A as shown in Fig. 3.18.2(a). This voltage is denoted mathematically as follows :

$$\left.\begin{aligned} V_{in} &= A \quad ... \text{for } t \geq 0 \\ \text{and} \quad &= 0 \quad ... \text{for } t < 0 \end{aligned}\right\} \qquad ...(3.18.11)$$



(a) Step input voltage (b) Output voltage

(K-53) Fig. 3.18.2

- Assume that $R_1 C_F = 1$ and substitute the expression of V_{in} into Equation (3.18.10) to write,

$$\text{Output voltage} \qquad V_o = -1\int_0^t A\,dt + V_o(0) \qquad ...(3.18.12)$$

- Assume that the initial value of the output voltage $V_o(0)$ is equal to zero.

$$\therefore \quad V_o = -1\int_0^t A\,dt$$

$$\therefore \quad V_o = -At \quad (\text{for } t \geq 0) \qquad ...(3.18.13)$$

- This is the equation of a straight line with a slope equal to – A and Y intercept equal to zero (passing through the origin). Hence the output voltage is as shown in Fig. 3.18.2(b). It shows that the output changes linearly with time.

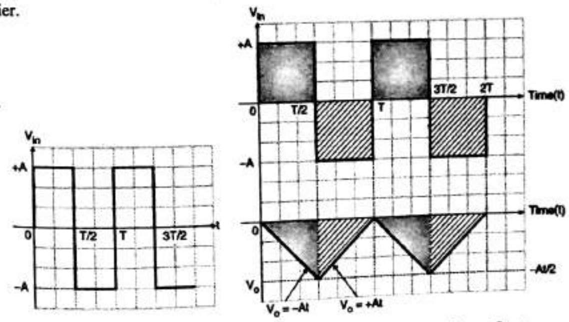
Output for a square wave input :

- Let us now apply the square wave shown in Fig. 3.18.3(a) at the input of an ideal integrator and obtain the corresponding output voltage.
- The square wave input is mathematically represented as follows :

$$\left.\begin{aligned} V_{in} &= A \quad ... \text{for } 0 \leq t \leq T/2 \\ \text{and} \quad &= -A \quad ... \text{for } T/2 < t \leq T \end{aligned}\right\} \qquad ...(3.18.14)$$

This expression is valid only for the first cycle.

- This expression shows that the square wave is made of many step signals shifted in time. Hence for $V_{in} = A$ for $0 \leq t \leq T/2$ the output voltage will be a straight line with a slope – A as discussed earlier.



(a) Square wave input (b) Input output voltage waveforms for a square wave input

(K-54) Fig. 3.18.3

- For $V_{in} = -A$ for $T/2 < t \leq T$, the output will be again a straight line with a slope of + A as shown in Fig. 3.18.3(b). Thus for one cycle, the output voltage can be mathematically expressed as :

$$V_o = -At \quad ...0 \le t \le T/2$$
and $$= At \quad ...T/2 < t \le T \quad ...(3.18.15)$$
with $R_1 C_F = 1$.

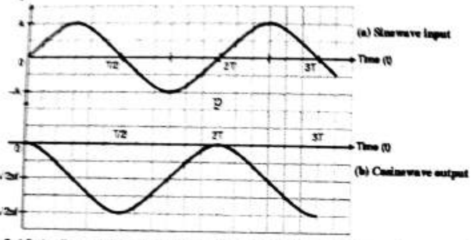
- This is plotted as shown in Fig. 3.18.3(b).

Output for a sinewave input :

- Here the sinewave input with a peak voltage of A volts and frequency of f Hz is used. It is as shown in Fig. 3.18.4(a) and mathematically represented by,

$$V_{in} = A \sin(2\pi ft) \quad ...(3.18.16)$$

where $f = \frac{1}{T}$ and T = One cycle period.



(K-55) **Fig. 3.18.4 : Input and output voltage waveforms for a sinewave input**

- Assuming the value of $R_1C_F = 1$, the output voltage is given by,

$$V_o = 1\int_{-0}^{t} A \sin(2\pi ft)dt = A \times -\frac{\cos(2\pi ft)}{2\pi f}$$

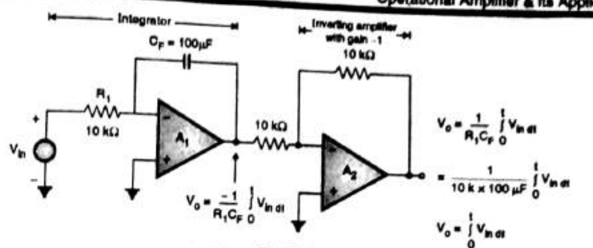
$$= \frac{-A}{2\pi f}[\cos(2\pi ft)] \quad ...(3.18.17)$$

- This output voltage is plotted in Fig. 3.18.4(b). It shows that the output of the integrator is a cosine wave when the input is a sinewave.

Ex. 3.18.1 : Using OP-AMP draw circuit diagram with the values of resistors to realise the following expression. $V_o = \int_0^t V_{in} dt$ May 05, 5 Marks

Soln. :

The realization of $V_o = \int_0^t V_{in} dt$ is as shown in Fig. P. 3.18.1.



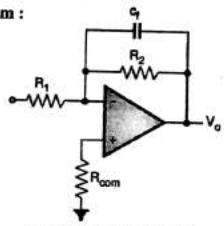
(F-3000) **Fig. P. 3.18.1**

Ex. 3.18.2 : Find R_1 and R_2 in the lossy integrator so that the peak gain is 20 dB and the gain is 3 dB down from its peak when ω = 10,000 rad/s. Use capacitance of 0.01μF. Dec. 14, 10 Marks

Soln. :

Given : Maximum gain = 20 dB, 3 dB frequency ω_a = 10000 rad/s, c_f = 0.01 μF.

Step 1 : Draw the circuit diagram :



(D-1545) **Fig. P. 3.18.2(a)**

Step 2 : Find the component values :

Maximum gain is nothing but dc gain.

$$\therefore \text{DC gain} = 20\text{ dB} = 20\log_{10}|A_V|$$
$$\therefore |A_V| = 10$$
$$\text{But } |A_V| = \frac{R_2}{R_1}$$
$$\therefore R_2 = 10 R_1 \quad ...(1)$$
$$\text{Also } \omega_a = 10000 \text{ rad/sec.}$$
$$\therefore f_a = \frac{10000}{2\pi} = 1591.55 \text{ Hz}$$
$$\text{But } f_a = \frac{1}{2\pi R_2 c_f}$$
$$c_f = 0.01\ \mu\text{F given}$$

$$\therefore\ R_2 = \frac{1}{2\pi f_a c_f} = \frac{1}{10000 \times 0.01 \times 10^{-6}}$$

$$\therefore\ R_2 = 10\ k\Omega \qquad \text{...Ans.}$$

Substituting in Equation (1) we get,

$$10\ k\Omega = 10\ R_1$$

$$\therefore\ R_1 = 1\ k\Omega \qquad \text{...Ans.}$$

3.18.4 Advantages of Active Integrator :

MU : Dec. 10

University Questions

Q. 1 Explain advantages of practical integrator over a simple integrator. **(Dec. 10, 4 Marks)**

1. Low distortion.
2. Better linearity.
3. Gain can be controlled.
4. Sharp frequency response.
5. Less effect of noise.
6. This circuit is highly stable, so less possibility of oscillations.

3.18.5 Disadvantages of Active Integrator :

1. It can operate as an integrator over a short frequency range.
2. Outside this frequency range the output is distorted.
3. OP-AMP parameters affect the output waveform and voltage.
4. Gain reduces with increase in frequency.
5. Errors may get introduced due to bias current, input bias voltage etc.

3.18.6 Applications of an Integrator :

Some of the important applications of an integrator are as follows :

1. In the triangular wave or ramp generators.
2. In the analog to digital (A to D) converters.
3. In the integral type controllers used in a closed loop control system.
4. In analog computers to solve differential equations.
5. As a low pass filter.
6. In the communication circuits for recovering the modulating signal.

3.19 Differentiator :

MU : Dec. 04, May 10, Dec. 10, May 13

University Questions

Q. 1 Write note on : Differentiator. **(Dec. 04, 6 Marks)**

Q. 2 Explain how an Op-Amp can be used as : Differentiator. **(May 10, May 13, 5 Marks)**

Q. 3 Write a short note on : Differentiator. **(Dec. 10, 5 Marks)**

Differentiator can be classified into two categories as :

1. Passive differentiator 2. Active differentiator.

3.19.1 Differentiator using OP-AMP (Active Differentiator) :

A differentiator circuit produces differentiated version of the input voltage applied to it. This process is exactly opposite to integration. Therefore the components connected in the integrator (Fig. 3.18.1) have interchanged their positions to produce a differentiator circuit of Fig. 3.19.1. (R_1 and C_F have interchanged their positions).

In this section we are going to discuss two types of differentiator circuits namely :

- Basic or ideal differentiator
- Practical differentiator.

3.19.2 Basic Differentiator Circuit :

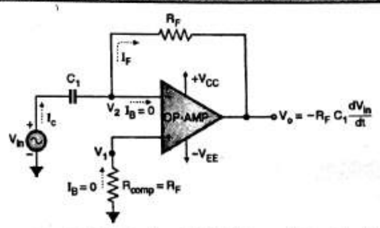
MU : Dec. 03, Dec. 16

University Questions

Q. 1 Explain the working of differentiator circuit. **(Dec. 03, 6 Marks)**

Q. 2 With suitable waveforms explain how op-amp can be used as differentiator ? **(Dec. 16, 10 Marks)**

- The differentiator or differentiating amplifier is as shown in Fig. 3.19.1.
- This circuit will perform the mathematical operation of differentiation on the input voltage waveform and the output is a derivative of the input signal.
- The differentiator can be constructed from the basic inverting amplifier by replacing resistance R_1 by capacitor C_1 as shown in Fig. 3.19.1.



(K-59) **Fig. 3.19.1 : Basic or Ideal differentiator circuit**

- The resistance R_{comp} connected from the non-inverting terminal to ground provides the bias compensation.

Expression for the output voltage :

- Assume that an ideal OP-AMP is being used. Hence $V_2 = 0, I_B = 0$

$$V_2 = 0, I_B = 0$$

- Referring to Fig. 3.19.1 we can write that,

$$I_C = I_B + I_F \qquad \text{...(3.19.1)}$$

- But since $I_B \approx 0$ the above equation gets modified to,

$$I_C = I_F \qquad \text{...(3.19.2)}$$

$$\text{We know that, } I_C = C\frac{d\,V_c}{dt} \qquad \text{...(3.19.3)}$$

$$I_C = C\frac{d\,V_c}{dt} \qquad \text{...(3.19.4)}$$

- The voltage across C_1 is given by,

$$V_c = V_{in} - V_2 \qquad \text{...(3.19.5)}$$

- Substituting this into Equation (3.19.3) we get,

$$I_f = C_1 \frac{d}{dt}(V_{in} - V_2) \quad \text{...(3.19.6)}$$

- Now let us obtain the expression for the current I_F.

$$I_F = \frac{V_2 - V_o}{R_F} \quad \text{...(3.19.7)}$$

- But I_C and I_F are equal because $I_B = 0$ as shown in Fig. 3.19.1. Therefore, equating the Equations (3.19.6) and (3.19.7) we get,

$$C_1 \frac{d}{dt}(V_{in} - V_2) = \frac{V_2 - V_o}{R_F} \quad \text{...(3.19.8)}$$

- Using the concept of virtual ground we can write that,

$$V_1 = V_2 = 0 \quad \text{...(3.19.9)}$$

- Substituting $V_1 = V_2 = 0$ into Equation (3.19.8) we get,

$$C_1 \frac{d}{dt}(V_{in}) = \frac{-V_o}{R_F}$$

$$\therefore V_o = -R_F C_1 \frac{d}{dt}(V_{in}) \quad \text{...(3.19.10)}$$

- Thus the output is $-R_F C_1$ times the time derivative of the input voltage.

3.19.3 Input and Output Voltage Waveforms :

MU : Dec. 16

University Questions

Q. 1 With suitable waveforms explain how op-amp can be used as differentiator ? **(Dec. 16, 10 Marks)**

- The expression for instantaneous output voltage of an ideal differentiator is ;

$$V_o = -R_F C_1 \frac{d}{dt}(V_{in})$$

- Assume $R_F C_1 = 1$

$$\therefore V_o = -\frac{dV_{in}}{dt} \quad \text{...(3.19.11)}$$

Output for a step input :

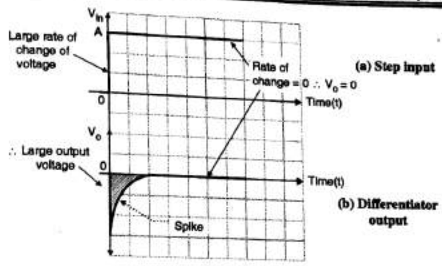
- Equation (3.19.11) tells us that the output voltage is equal to the **rate of change of input voltage** with respect to time. Now refer to Fig. 3.19.2(a) which shows the step input voltage.
- The step input is mathematically expressed as,

$$V_{in} = A \quad \text{...} t \geq 0$$
$$= 0 \quad \text{...} t < 0$$

- Substituting this into Equation (3.19.11) we get,

$$V_o = -\frac{d}{dt}(A) = 0$$

- Thus output voltage is zero when the step input voltage is constant at "A", i.e. when the rate of change is zero as shown in Fig. 3.19.2(a).

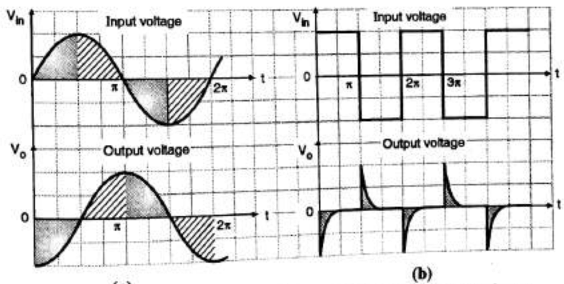


(K-1097) **Fig. 3.19.2**

- But the step input changes suddenly from 0 to A at instant t = 0. So there is a large rate of change of voltage associated at this instant of time.
- Therefore a large output voltage is produced at t = 0 and this voltage is negative as the differentiator is an inverting differentiator. The output voltage is a negative spike as shown in Fig. 3.19.2(b).

Output voltage for a square wave input :

- The square waveform of Fig. 3.19.3(b) is actually made up of many step signal. So we have to extend the same principle that we have used for step input voltage.
- Thus the output voltage is in the form of spikes corresponding to the rising and falling edges of the square wave and the output voltage is zero when the input is constant at ± A. These waveforms are as shown in Fig. 3.19.3(b).



(K-61) **Fig. 3.19.3 : Input and output voltage waveforms of a differentiator**

Output voltage for a sinewave input :

- Let the sinusoidal input signal be expressed as,

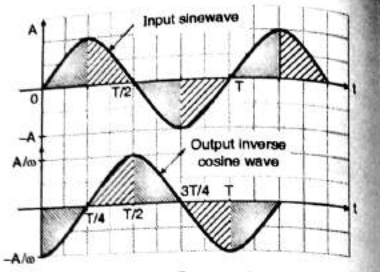
$$V_{in} = A \sin \omega t \quad ...(3.19.12)$$

- The corresponding output voltage, assuming $R_F C_1 = 1$ is given by,

$$V_o = \frac{-A \cdot d(\sin \omega t)}{dt}$$

$$\therefore \quad V_o = \frac{-A \cos \omega t}{\omega}$$

$$= -\frac{A}{\omega} \cos \omega t \quad ...(3.19.13)$$



(K-42) **Fig. 3.19.3(c) : Input output voltage waveform for sinewave**

- Thus the output voltage is an inverse cosine wave as shown in Fig. 3.19.3(c). The following table tells you how to plot this wave.

t	0	T/4	T/2	3T/4	T
V_o	$-A/\omega$	0	A/ω	0	$-A/\omega$

Problems associated with the basic differentiator :

- The gain of basic differentiator is (R_F / X_{C1}). As X_{C1} decreases with increase in frequency, the gain will increase with increase in frequency.
- This increase in gain at high frequencies will make the circuit unstable. Also the input impedance which is equal to X_{C1} reduces with increase in frequency.
- This makes the circuit very much susceptible to high frequency noise. The amplified noise at the output can completely override the output signal.

3.19.4 Steps to Design a Practical Differentiator :

Following are the steps to be followed for design of a practical differentiator.

Steps to be followed :

Step 1 : Select f_a equal to the highest frequency of the input signal.

Step 2 : Assume the value of C_1 and calculate R_F from the expression of f_a.

Step 3 : Select $f_b = 20 f_a$ and calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

Ex. 3.19.1 : Design a differentiator to differentiate an input signal that varies in frequency from 10 Hz to about 1 kHz. (use practical circuit).

Soln. : The steps to be followed to design a differentiators are :

Step 1 : Select "f_a" equal to the highest frequency of the input signal.

Step 2 : Assume a value of $C_1 < 1\ \mu F$ and calculate R_F.

Step 3 : Select $f_b = 20 f_a$ and calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

Step 1 : Select "f_a" and calculate R_F :

$f_a = 1$ kHz as the highest input frequency is 1 kHz.

Let $C_1 = 0.1\ \mu F$.

But $$f_a = \frac{1}{2\pi R_F C_1}$$

$$\therefore \quad R_F = \frac{1}{2\pi f_a C_1}$$

$$= \frac{1}{2\pi \times 1 \times 10^3 \times 0.1 \times 10^{-6}}$$

$$\therefore \quad R_F = 1.59\ k\Omega \quad \text{...Ans.}$$

Step 2 : Calculate R_1 and C_F :

$$f_b = 20 f_a = 20\ kHz$$

But $$f_b = \frac{1}{2\pi R_1 C_1}$$

$$\therefore \quad R_1 = \frac{1}{2\pi f_b C_1}$$

$$= \frac{1}{2\pi \times 20 \times 10^3 \times 0.1 \times 10^{-6}}$$

$$\therefore \quad R_1 = 79.5\ \Omega \quad \text{...Ans.}$$

Let us use the standard value of 82 Ω.

$$\text{As } R_1 C_1 = R_F C_F$$

$$C_F = \frac{82 \times 0.1 \times 10^{-6}}{1.59 \times 10^3} = 0.00515\ \mu F \quad \text{...Ans.}$$

Let C_F be 0.005 μF. And finally,

$$R_{comp} = R_F = 1.59\ k\Omega \approx 1.5\ k\Omega.$$

Thus the component values are as follows :

$R_1 = 82\ \Omega$

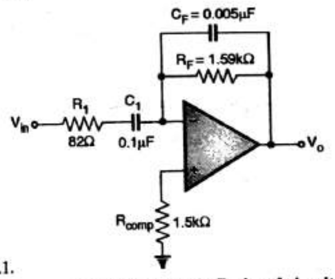
$R_{comp} = 1.5\ k\Omega$

$C_F = 0.005\ \mu F$

$R_F = 1.59\ k\Omega$

$C_1 = 0.1\ \mu F$

The circuit diagram is as shown in Fig. P. 3.19.1.



(F-2972) **Fig. P. 3.19.1 : Designed circuit**

Ex. 3.19.2 : Design an op-amp differentiator that will differentiate an input signal with f_{max} = 100 Hz. Draw the output waveform for a sine wave of 1 V peak at 100 Hz applied to the differentiator. Also repeat it for square wave input. **May 15. 10 Marks**

Soln. :

Part I : Design of the differentiator :

Given : $f_{max} = 1$ kHz.

The steps to be followed to design a differentiators are :

Step 1 : Select "f_a" equal to the highest frequency of the input signal.

Step 2 : Assume a value of $C_1 < 1\ \mu F$ and calculate R_F.

Step 3 : Select $f_b = 20 f_a$ and calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

Step 1 : Select "f_a" and calculate R_F :

$f_a = 1$ kHz as the highest input frequency is 1 kHz.

Let $C_1 = 0.1\ \mu F$.

But $f_a = \dfrac{1}{2\pi R_F C_1}$

$\therefore \quad R_F = \dfrac{1}{2\pi f_a C_1} = \dfrac{1}{2\pi \times 1 \times 10^3 \times 0.1 \times 10^{-6}}$

$\therefore \quad R_F = 1.59\ k\Omega$ ...Ans.

Step 2 : Calculate R_1 and C_F :

$f_b = 20 f_a = 20$ kHz

But $f_b = \dfrac{1}{2\pi R_1 C_1}$

$\therefore \quad R_1 = \dfrac{1}{2\pi f_b C_1} = \dfrac{1}{2\pi \times 20 \times 10^3 \times 0.1 \times 10^{-6}}$

$\therefore \quad R_1 = 79.5\ \Omega$ ...Ans.

Let us use the standard value of 82 Ω.

As $R_1 C_1 = R_F C_F$

$C_F = \dfrac{82 \times 0.1 \times 10^{-6}}{1.59 \times 10^3} = 0.00515\ \mu F$ ...Ans.

Let C_F be 0.005 μF. And finally,

$R_{comp} = R_F = 1.59\ k\Omega \approx 1.5\ k\Omega$.

Thus the component values are as follows :

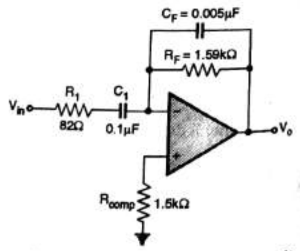
$R_1 = 82\ \Omega$

$R_{comp} = 1.5\ k\Omega$

$C_F = 0.005\ \mu F$

$R_F = 1.59\ k\Omega$

$C_1 = 0.1\ \mu F$



The circuit diagram is as shown in Fig. P. 3.19.2(a). (F-2972) **Fig. P. 3.19.2(a) : Designed circuit**

Part II : To draw the waveforms :

For an inverting differentiator,

$$V_o = -R_F C_1 \frac{dV_{in}}{dt} \quad ...(1)$$

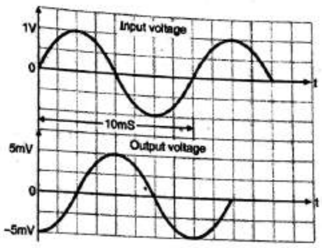
But, $V_{in} = 1 \sin(2\pi \times 100\, t)$

$= 2 \sin (200\pi t)$ ...(2)

From the Fig. P. 3.19.2(b) values of R_F and C_1 are :

$R_F = 1.59\ k\Omega$ and

$C_1 = 0.005 \times 10^{-6}$ F



(B-2744) **Fig. P. 3.19.2(b) : Waveforms for a differentiator**

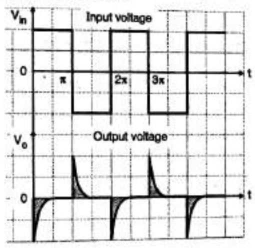
Substituting these values into Equation (1) we get,

$$V_o = -1.59 \times 10^{+3} \times 0.005 \times 10^{-6} \cdot \frac{d}{dt}[1 \sin (200\pi t)]$$

$$= 7.95 \times 10^{-6} \times 200\pi [\cos (200\pi t)]$$

$$= -5 \times 10^{-3} \cos (200\pi t)$$ ...Ans.

This is a cosine wave with a peak voltage of 5 mV and frequency of 100 Hz. The input and output waveforms are as shown in Fig. P. 3.19.2(b).



(K-61) **Fig. P. 3.19.2(c) : Input and output voltage waveforms of a differentiator for a square wave input**

3.19.5 Advantages of Active Differentiator :

MU : Dec. 07

University Questions

Q. 1 Give the advantages of basic differentiator. **(Dec. 07, 3 Marks)**

1. Sharp output
2. Gain can be controlled
3. Sharp frequency response.

3.19.6 Disadvantages of Active Differentiator :

1. This circuit is badly affected by noise.
2. It is less stable. There is a possibility of oscillations.
3. Gain increases with increase in frequency.
4. Output is affected by the parameter of OP-AMP.

3.19.7 Applications of a Differentiator :

1. In the P-I-D controllers
2. As a high pass filter
3. In the wave shaping circuits to generate narrow pulses corresponding to any sharp change in the input signal.

3.19.8 Comparison of Active Integrator and Differentiator :

MU : May 06

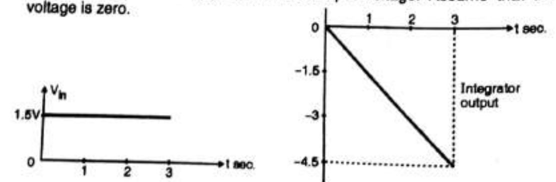
University Questions

Q. 1 Explain the difference between the integrator and differentiator and give one application of each. **(May 06, 8 Marks)**

Sr. No.	Parameter	Active Integrator	Active Differentiator
1.	Output voltage	$V_o = -\frac{1}{R_1 C_F}\int_1^t V_{in}\,dt + k$	$V_o = -R_F C_1 \frac{d}{dt} V_{in}$
2.	Feedback element	Capacitor	Resistor
3.	Gain	Decreases with increase in frequency	Increases with increase in frequency
4.	Acts as	Low pass filter	High pass filter
5.	Effect of noise	Less	More
6.	Stability	More	Less
7.	Applications	A/D converter, PID controllers, filters, waveform generator	Logic circuits, pulse shaping filters.

Examples on integrator and differentiators :

Ex. 3.19.3 : For an integrator circuit, $R_1 = 100\ k\Omega$ and $C_F = 10\ \mu F$. The input is a step (dc) voltage as shown in Fig. P. 3.19.3(a). Sketch the output voltage. Assume that the output offset voltage is zero.



(a) (b) Integrator output

(F-2973) **Fig. P. 3.19.3**

Soln. :

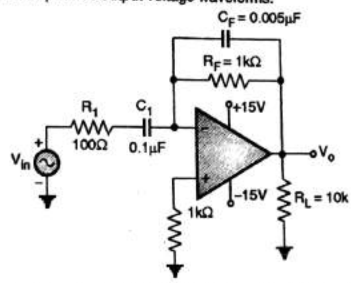
From the Fig. P. 3.19.3(a) it is clear that the input voltage is constant equal to 1.5 Volts for $0 \le t \le 3$. Therefore the output voltage is given by,

$$V_o = -\int_0^{t=3} \frac{1}{R_1 C_F} \cdot 1.5\,dt = \frac{-1}{100 \times 10^3 \times 10 \times 10^{-6}} \int_{t=0}^{t=3} 1.5\,dt$$

$$= -1.5 \times 3 = -4.5 \text{ Volts.}$$

The output voltage waveform is as shown in Fig. P. 3.19.3(b). The slope of this waveform is -1.5V/sec. Thus when we apply a constant voltage at the integrator input, it gives a ramp at its output.

Ex. 3.19.4 : A differentiator circuit is as shown in Fig. P. 3.19.4(a). If a sinewave of peak amplitude of 2 V and frequency of 1 kHz is applied at the input, obtain the output voltage and sketch the input and output voltage waveforms.



(F-2974) **Fig. P. 3.19.4(a) : An inverting differentiator**

Soln. : For an inverting differentiator,

$$V_o = R_F C_1 \frac{d V_{in}}{dt} \qquad ...(1)$$

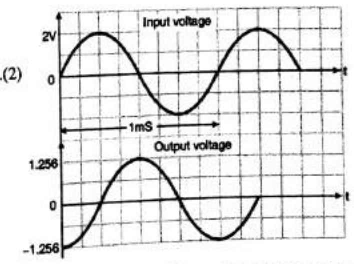
But, $V_{in} = 2 \sin (2\pi \times 1 \times 10^3 t)$

$= 2 \sin (6.28 \times 10^3 t)$ ...(2)

From the Fig. P. 3.19.4(a) values of R_F and C_1 are :

$R_F = 1\ k\Omega$ and

$C_1 = 0.1 \times 10^{-6}$ F



(F-2975) **Fig. P. 3.19.4(b) : Waveforms for a differentiator**

Substituting these values into Equation (1) we get,

$$V_o = -1 \times 10^{+3} \times 0.1 \times 10^{-6} \cdot \frac{d}{dt} [2 \sin (6.28 \times 10^3 t)]$$

$$= -1 \times 10^{-4} \times 2 \times 6.28 \times 10^3 [\cos (6.28 \times 10^3 t)]$$

$$= -1.256 \cos [6.28 \times 10^3 t] \quad \text{...Ans.}$$

This is a cosine wave with a peak voltage of 1.256 volts and frequency of 1 kHz. The input and output waveforms are as shown in Fig. P. 3.19.4(b).

Ex. 3.19.5 : Design a practical integrator circuit to integrate a square wave of frequency 10 kHz. The dc gain of the integrator should be adjusted to 12.

Soln. :

Given : f = 10 kHz and G = 12.

Follow the steps given below :

Step 1 : From the value of gain, calculate R_1 and R_F.

Step 2 : Calculate f_a by assuming $f_b = f = 10$ kHz.

Step 3 : Calculate C_F from the value of f_a.

Step 4 : Calculate $R_{comp} = R_1 \| R_F$

Step 1 : To calculate R_F and R_1 :

The dc gain of an integrator is its gain at f = 0. At f = 0, the capacitor C_F acts as an open circuit. Hence the dc gain is decided by R_F and R_1.

$$\therefore \quad G = \frac{R_F}{R_1} = 12 \quad \text{...(1)}$$

Select $R_1 = 10\ k\Omega$

$$\therefore \quad R_F = 120\ k\Omega \quad \text{...Ans.}$$

Step 2 : To calculate "f_a" :

Assuming $f_b = f = 10$ kHz we have,

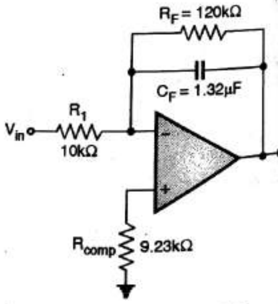
$$f_a = \frac{f_b}{10} = 1000\ \text{Hz} \quad \text{...(2)}$$

Step 3 : To calculate C_F :

$$f_a = \frac{1}{2 \pi R_F C_F}$$

$$\therefore \quad C_F = \frac{1}{2\pi f_a R_F} = \frac{1}{2\pi \times 1000 \times 120 \times 10^3}$$

$$\therefore \quad C_F = 1.32\ \text{nF} \quad \text{...Ans.}$$



(F-2976) **Fig. P. 3.19.5**

Step 4 : To calculate R_{comp} :

$$R_{comp} = R_1 \| R_F$$

$$= \frac{10 \times 120}{(10 + 120)} = 9.23\ k\Omega \quad \text{...Ans.}$$

Hence the required practical integrator circuit is as shown in Fig. P. 3.19.5.

Ex. 3.19.6 : Design a differentiator to differentiate an input signal that varies in frequency from 10 Hz to about 500 Hz.

Soln. :

Given : Frequency range : 10 Hz to 500 Hz.

Peak value of sinewave = 2 V, f = 500.

Part I : Design of differentiator :

Step 1 : Select f and calculate R_F :

Let f_a = 500 Hz (highest frequency) and $C_1 = 0.1\ \mu F$

$$\text{But} \quad f_a = \frac{1}{2 \pi R_F C_1}$$

$$\therefore \quad R_F = \frac{1}{2\pi C_1 f_a} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 500}$$

$$\therefore \quad R_F = 3.183\ k\Omega \approx 3.2\ k\Omega$$

Step 2 : Calculate R_1 and C_F :

$$f_b = 20 f_a = 10\ \text{kHz}$$

$$\text{But} \quad f_b = \frac{1}{2 \pi R_1 C_1}$$

$$\therefore \quad R_1 = \frac{1}{2 \pi f_b C_1}$$

$$= \frac{1}{2\pi \times 10 \times 10^3 \times 0.1 \times 10^{-6}}$$

$$\therefore \quad R_1 = 159.15\ \Omega \approx 160\ \Omega$$

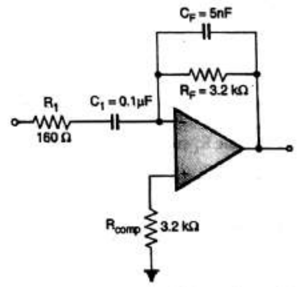
$$\text{As} \quad R_1 C_1 = R_F C_F$$

$$C_F = \frac{R_1 C_1}{R_F}$$

$$= \frac{160 \times 0.1 \times 10^{-6}}{3.2 \times 10^3} = 5\ \text{nF}$$

$$R_{comp} = 3.2\ k\Omega$$

The designed circuit in shown in Fig. P. 3.19.6.



(F-2977) **Fig. P. 3.19.6 : Designed differentiator circuit**

Ex. 3.19.7 : The input to the differentiator circuit is a sinusoidal voltage of peak value 4 mV and frequency 1 kHz. Find the output voltage, if R = 50 kΩ and C = 1 μF.

Soln. :

Given : $V_m = 4 \times 10^{-3}$ $\quad f = 1$ kHz

$R_F = 50$ kΩ $\quad C_1 = 1$ μF

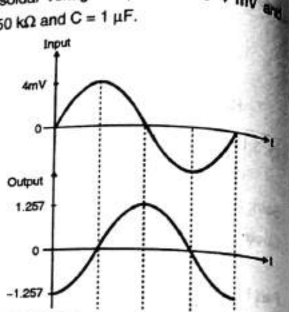
Output voltage

$$V_o = -R_F C_1 \frac{d V_i}{dt}$$

But $V_{in}(t) = V_m \sin(2\pi t) = 4 \times 10^{-3} \sin(2000\,\pi t)$

$$V_o = -50 \times 10^3 \times 1 \times 10^{-6}$$

$$= \frac{d}{dt}[4 \times 10^{-3} \sin(2000\,\pi t)]$$



(F-2979) **Fig. P. 3.19.7 : Input/output waveforms**

$$= -50 \times 10^{-3} \times 4 \times 10^{-3} \times 2000\,\pi \cos(2000\,\pi t)$$

$$V_o = -1.257 \cos(2000\,\pi t)$$

...Ans.

The input and output voltages are plotted in Fig. P. 3.19.7.

Ex. 3.19.8 : Design the practical differentiator for the frequency 5 kHz.

Dec. 08, 10 Marks, Dec. 12, 4 Marks

Soln. :

Step 1 : Calculate R_F :

$f_a = 5$ kHz ...given,

Let $C_1 = 0.01\,\mu F$ $\quad f_a = \frac{1}{2\pi R_F C_1}$

$$\therefore \quad R_F = \frac{1}{2\pi f_a C_1} = \frac{1}{2\pi \times 5 \times 10^3 \times 0.01 \times 10^{-6}} = 3.183 \text{ k}\Omega$$

Step 2 : Calculate R_1 and C_F :

$f_b = 20 f_a = 20 \times 5$ kHz $= 100$ kHz. $\quad$ But $\quad f_b = \frac{1}{2\pi R_1 C_1}$

$$\therefore \quad R_1 = \frac{1}{2\pi f_b C_1} = \frac{1}{2\pi \times 100 \times 10^3 \times 0.01 \times 10^{-6}} = 159.15\ \Omega$$

Use standard value of 150 Ω

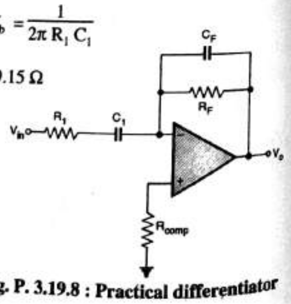
$$R_1 C_1 = R_F C_F$$

$$\therefore \quad 150 \times 0.01 \times 10^{-6} = 3.183 \times 10^3\, C_F$$

$$\therefore \quad C_F = 471.25 \text{ pF.}$$

$$R_{comp} = R_F = 3.183 \text{ k}\Omega$$

The designed practical differentiator is shown in Fig. P. 3.19.8.



(F-3001) **Fig. P. 3.19.8 : Practical differentiator**

Ex. 3.19.9 : Design a practical integrator for the output frequency of 5 kHz. Draw the input-output waveforms.

May 09, 10 Marks

Soln. :

Given : f = 5 kHz, Assume gain = 10

Step 1 : Calculate R_F and R_1 :

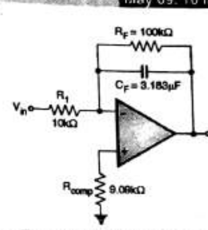
$$G = \frac{R_F}{R_1} = 10 \;\therefore\; R_F = 10\, R_1$$

Select $R_1 = 10$ kΩ $\therefore R_F = 10 \times 10k = 100$ kΩ

Step 2 : Calculate f_a :

Assuming $f_b = 5$ kHz, We have

$$f_a = \frac{f_b}{10} = 500 \text{ Hz}$$



(F-3002) **Fig. P. 3.19.9(a) : Designed circuit**

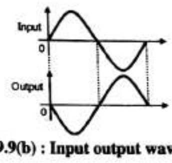
Step 3 : Calculate C_F and R_{comp} :

$$f_a = \frac{1}{2\pi R_F C_F}$$

$$\therefore \quad C_F = \frac{1}{2\pi f_a R_F} = \frac{1}{2\pi \times 500 \times 100 \times 10^3}$$

$$\therefore \quad C_F = 3.183 \text{ nF}$$

$$R_{comp} = R_1 \,\|\, R_F = 10k \,\|\, 100k = 9.09k\ \Omega$$



(F-3003) **Fig. P. 3.19.9(b) : Input output waveforms**

Fig. P. 3.19.9 shows the designed circuit and waveforms.

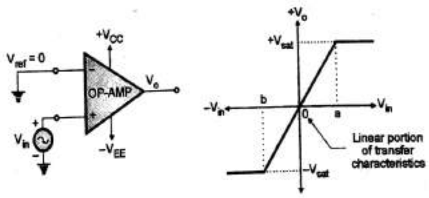
3.20 OP-AMP as a Comparator :

MU : May 07, Dec. 11, May 12

University Questions

Q. 1	Write short note on : OP-AMP as comparator.	(May 07, 10 Marks)
Q. 2	Draw comparator and its input and output waveforms.	(Dec. 11, 5 Marks)
Q. 3	Explain OP-AMP as comparator.	(May 12, 5 Marks)

- A comparator in its simplest form is nothing but an open loop OP-AMP with two inputs and one output.
- It compares a signal voltage applied to one input of the OP-AMP, with a known voltage called the reference voltage applied to the other input.



(a) A basic comparator circuit **(b) Transfer characteristics of a comparator**

(K-435) **Fig. 3.20.1**

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- The output of a comparator is either positive or negative saturation voltage ($\pm V_{(sat)}$), depending on which input is larger.
- Fig. 3.20.1(a) shows a non-inverting comparator. This is called as non-inverting comparator because the signal voltage V_{in} is applied to the non-inverting (+) terminal of the OP-AMP.
- The inverting (–) terminal is connected to ground. So the reference voltage here is $V_{ref} = 0V$.
- As no feedback is being used, the OP-AMP operates in the open loop mode with a large open loop gain A_V.

3.20.1 Transfer Characteristics :

- Fig. 3.20.1(b) shows the voltage transfer characteristics (VTC) of a comparator. Transfer characteristics is a graph of input voltage versus output voltage.
- When the input voltage V_{in} is positive but very small and in the range 0 to "a", the output voltage $V_o = A_V \times V_{in}$, where A_V is the open loop gain. As A_V is very very high, a very small positive input voltage beyond point "a" will be required to drive the output voltage to $+V_{(sat)}$. Note that $+V_{(sat)} \approx +V_{CC}$.
- Similarly when V_{in} is negative but small and in the range 0 to "b", the relationship between V_{in} and V_o is linear. But beyond point "b", the negative input voltage will drive the output to $-V_{(sat)}$ where $-V_{(sat)} \approx -V_{EE}$.
- The transfer characteristics of the basic comparator is very similar to that of an open loop OP-AMP.

3.20.2 Types of Comparator :

Depending on which input terminal receives the signal input, the comparators are classified into two categories. They are :

1. Non-inverting comparator.
2. Inverting comparator.

3.20.3 Comparator Applications :

Important applications of OP-AMP are :

1. Zero crossing detector
2. Window comparator
3. Level detector
4. Phase detector
5. Schmitt trigger
6. Peak detector
7. A to D converter

3.21 Non-Inverting Comparator :

MU : May 12, Dec. 14

University Questions

Q. 1 Explain OP-AMP as comparator. **(May 12, 5 Marks)**

Q. 2 Draw and explain op-amp non inverting comparator. Draw input and output waveforms for V_{ref} positive and also for V_{ref} negative. **(Dec. 14, 5 Marks)**

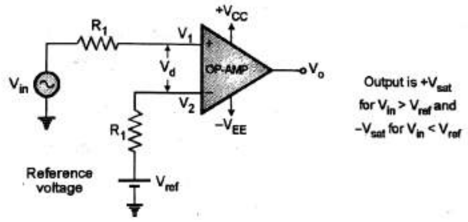
Circuit diagram :

- The schematic diagram of a non-inverting comparator is as shown in Fig. 3.21.1(a).
- A positive dc reference voltage is applied to the inverting terminal (–) and an ac sinusoidal signal is applied to the non-inverting (+) terminal.
- As the ac signal is connected to the non-inverting (+) terminal, this comparator is called as non-inverting comparator.

Operation of the circuit :

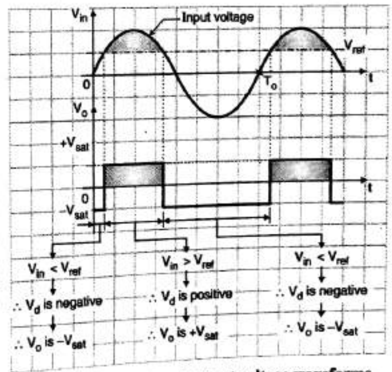
- As the current through resistors R_1 is almost zero, the voltage drop across them will be equal to zero. Hence $V_1 = V_{in}$ and $V_2 = V_{ref}$. Hence the differential input voltage V_d is given by,

$$V_d = V_1 - V_2 = V_{in} - V_{ref} \quad ...(3.21.1)$$



(K-436)**Fig. 3.21.1(a) : A non-inverting comparator**

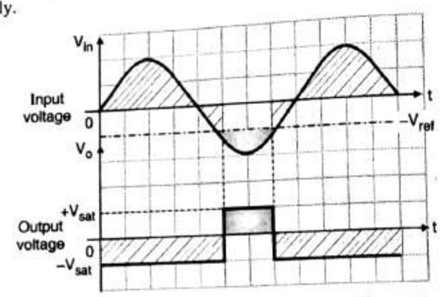
- When V_{in} is less than V_{ref} the inverting (–) terminal is at higher potential than the non-inverting (+) terminal. Therefore the differential input voltage V_d is negative and the OP-AMP output will swing to the $-V_{(sat)}$ Volts.
- When V_{in} is greater than V_{ref} the non-inverting (+) terminal is at higher potential than the inverting (–) terminal. Therefore the differential input voltage V_d is positive and the OP-AMP output will swing to $+V_{(sat)}$ Volts as shown in Fig. 3.21.1(b).



(K-437)**Fig. 3.21.1(b) : Input and output voltage waveforms**

- If the reference voltage is made negative ($-V_{ref}$) and applied to the inverting (–) terminal then the input and output voltage waveforms are modified as shown in Fig. 3.21.2.

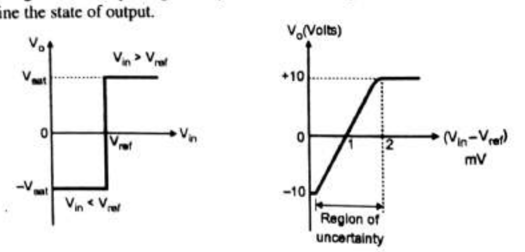
- If the OP-AMP is capable of operating on the single polarity supply i.e. only on + V_{CC} then the output waveform of such a comparator will be unipolar. Then we will get either + V_{sat} or GND at the comparator output. OP-AMP ICs such as IC LM 324 is capable of working on a single polarity supply.



(K-438)**Fig. 3.21.2 : Waveforms for a non-inverting comparator with negative reference voltage**

3.21.1 Transfer Characteristics of Non-Inverting Comparator :

- Transfer characteristics is a graph relating the input voltage (x-axis) and output voltage (y-axis).
- The ideal transfer characteristic of a non-inverting comparator for positive reference voltage is as shown in Fig. 3.21.3(a).
- It shows that the output voltage is equal to $-V_{sat}$ for $V_{in} < V_{ref}$ and it switches to $+V_{sat}$ as soon as V_{in} is slightly greater than V_{ref}.
- The differential voltage needed to switch the state of output is ideally zero.
- The practical transfer characteristic of the commercial OP-AMPs is as shown in Fig. 3.21.3(b).
- The change in output state takes place when the differential voltage is about 2 mV.
- The state changeover takes place gradually and there is a region of uncertainty where we cannot exactly define the state of output.



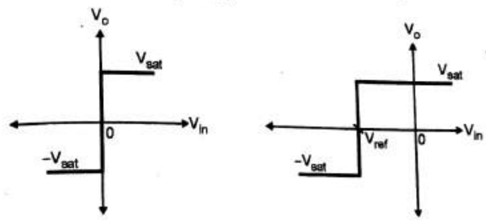
(a) Ideal transfer characteristic of a non-inverting comparator for positive V_{ref}

(b) Practical transfer characteristic of a non-inverting comparator

(K-439)**Fig. 3.21.3**

3.21.2 Transfer Characteristics for Zero or Negative Reference :

Assuming the OP-AMP to be ideal, the transfer characteristics of a non-inverting comparator for zero or negative reference is shown in Fig. 3.21.4.



(a) For $V_{ref} = 0$

(b) For negative V_{ref}

(K-1124)**Fig. 3.21.4 : Transfer characteristics of non-inverting comparator**

3.22 Inverting Comparator :

MU : Dec. 11, May 12, May 15

University Questions

Q. 1	Draw the ideal inverting comparator.	**(Dec. 11, 5 Marks)**
Q. 2	Explain OP-AMP as comparator.	**(May 12, 5 Marks)**
Q. 3	Draw and explain op-amp inverting comparator. Draw input and output waveforms for $V_{ref} > 0$ and also for $V_{ref} < 0$.	**(May 15, 5 Marks)**

- The schematic diagram of an inverting comparator is as shown in Fig. 3.22.1(a) and the relevant waveforms are as shown in Fig. 3.22.1(b).
- Note that the reference voltage V_{ref} has now been applied to the non-inverting (+) terminal and the input voltage is applied to the inverting (–) terminal.

3.22.1 Operation of the Circuit :

MU : May 15

University Questions

Q. 1	Draw and explain op-amp inverting comparator. Draw input and output waveforms for $V_{ref} > 0$ and also for $V_{ref} < 0$.	**(May 15, 5 Marks)**

- Assume the OP-AMP to have a very high input resistance. Hence the currents flowing through the resistors R_1 and R of Fig. 3.22.1(a) is very small. Hence the voltage drop across them is close to zero.

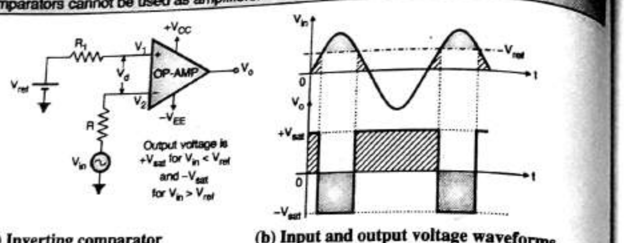
$$\therefore \quad V_1 = V_{ref} \text{ and } V_2 = V_{in}$$

$$\therefore \text{ Differential input voltage } V_d = V_1 - V_2 = (V_{ref} - V_{in})$$

- When V_{in} is less than V_{ref}, the voltage at the non-inverting (+) terminal is higher than the voltage at the inverting (–) terminal. This makes the differential input voltage V_d positive and the OP-AMP output will swing to $+V_{(sat)}$.
- When V_{in} is greater than V_{ref}, the voltage at the inverting (–) terminal is greater than the voltage at the non-inverting (+) terminal. Therefore the differential input voltage V_d is negative and the OP-AMP output will be $-V_{(sat)}$.

(PLL)

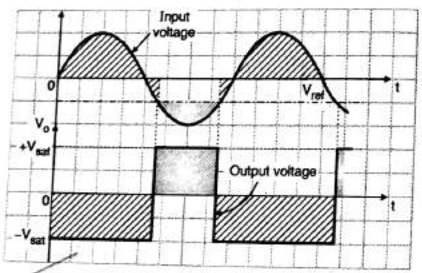
Note : It is important to note that the comparator output "switches" from + $V_{(sat)}$ to − $V_{(sat)}$ or vice-versa. It does not remain in-between the above mentioned extreme values. Therefore comparators cannot be used as amplifiers.



(a) Inverting comparator (b) Input and output voltage waveforms

(K-440)**Fig. 3.22.1**

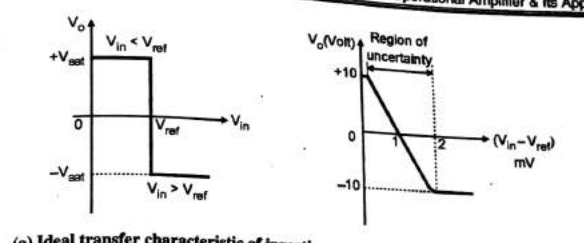
- If the reference voltage is made negative (− V_{ref}) then the input and output voltage waveforms gets modified as shown in Fig. 3.22.2.



(K-441)**Fig. 3.22.2 : Waveforms of an inverting comparator for a negative reference voltage**

3.22.2 Transfer Characteristics of an Inverting Comparator :

- Fig. 3.22.3(a) shows the ideal transfer characteristic of an inverting comparator for a positive V_{ref}. It shows that the output voltage is equal to + V_{sat} for V_{in} less than V_{ref}.
- It switches instantaneously from + V_{sat} to − V_{sat} when V_{in} becomes just slightly greater than V_{ref}.
- The differential voltage needed for switching the output state is very small, ideally it is zero volt.
- The commercial OP-AMPs have the transfer characteristic as shown in Fig. 3.22.3(b). The change in output state takes place when the differential voltage is about 2 mV.
- The state change take place gradually and there is a region of uncertainty where we cannot directly define the output.

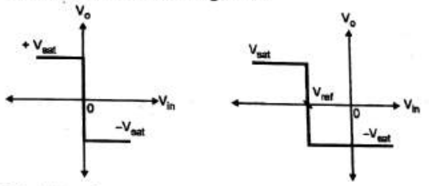


(a) Ideal transfer characteristic of inverting comparator for positive V_{ref} (b) Practical transfer characteristic of an inverting comparator

(K-442)**Fig. 3.22.3**

3.22.3 Transfer Characteristics for Zero and Negative Reference Voltage :

Assuming the comparator to be ideal, the transfer characteristics of an inverting comparator for zero and negative reference voltage are shown in Fig. 3.22.4.



(a) For $V_{ref} = 0$ (b) For negative V_{ref}

(K-1125)**Fig. 3.22.4 : Transfer characteristics of an inverting comparator**

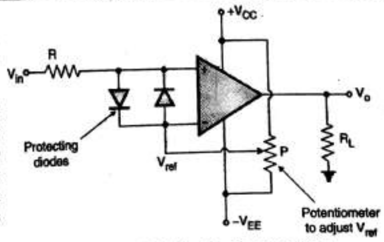
3.23 Practical Comparator :

MU : May 12

University Questions

Q. 1 Explain OP-AMP as comparator. **(May 12, 5 Marks)**

- The comparators discussed so far are the ideal comparators. The practical comparator circuit is as shown in Fig. 3.23.1.
- The potentiometer P connected between the + V_{CC} and − V_{EE} is used to adjust the reference voltage applied to the inverting (−) terminal of the comparator. The reference voltage can be adjusted to be positive or negative or zero volts.



(K-443)**Fig. 3.23.1 : Practical comparator**

- The input voltage V_{in} is applied to the non-inverting input terminal (+) through a resistor R. So this is a non-inverting comparator.
- Two diodes are connected back to back between the two input terminals of the comparator. They will not allow the differential voltage V_d to exceed $\pm$ 0.7 Volts. Thus difference voltage V_d will be clamped to $\pm$ 0.7 Volts if input voltage is higher than $\pm$ 0.7 V. Hence these diodes are called as the protecting diodes.
- Resistance R is used for limiting the current through the diodes to a safe value.

3.24 Zero Crossing Detector :

MU : May 03, Dec. 06, Dec. 07, May 09, May 11, Dec. 12, May 13, Dec. 15

University Questions

Q. 1 Write short note on : zero crossing detector using OP-AMP working and applications. **(May 03, 8 Marks)**

Q. 2 Explain working and applications of ZCD using op-amp. **(Dec. 06, 8 Marks)**

Q. 3 Write short note on : Zero-crossing detector using OP-AMP working and applications. **(Dec. 07, 10 Marks)**

Q. 4 Write short note on : ZCD. **(May 09, May 11, Dec. 12, May 13, 5 Marks)**

Q. 5 With neat diagram explain any one application of zero-crossing detector. **(Dec. 15, 5 Marks)**

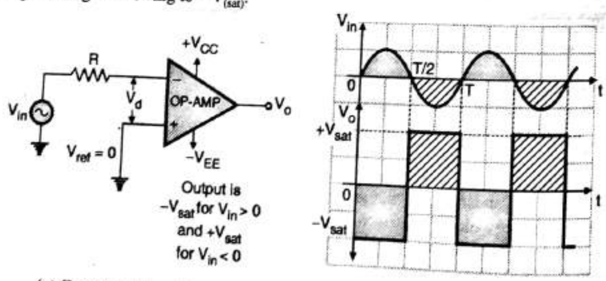
Q. 6 With respect to op-amp explain the ideal characteristics and concept of virtual ground. Explain how op-amp can be used as an averaging amplifier in inverting configuration. Also draw neat circuit diagrams to :

1. Convert sine wave to square wave using op-amp.
2. Detect the crossing of zero's in the generated square wave. **(Dec. 15, 10 Marks)**

- Zero crossing detector is as shown in Fig. 3.24.1(a). As you can see, zero crossing detector is nothing but the basic comparator circuit with a zero reference voltage applied to the non-inverting terminal.

Circuit operation :

- Refer to the waveforms shown in Fig. 3.24.1(b). When the input sinewave crosses zero and becomes positive at instant t = 0, the differential input voltage V_d becomes negative and the output voltage will swing to $-V_{(sat)}$.



(a) Zero crossing detector **(b) Input and output voltage waveforms**

(K-446) **Fig. 3.24.1**

- When the sinewave again crosses zero and becomes negative at instant t = T/2, the differential input voltage becomes positive and the output voltage will swing to + $V_{(sat)}$ as now the (+) terminal is more positive than the (–) terminal.
- The zero crossing detector thus switches its output from one state to the other everytime when the input voltage crosses the zero.
- The zero crossing detector is also known as a sinewave to square wave converter.

Demerit of zero crossing detector :

The noise present on the input side can cause false switching.

Applications :

1. Square wave generators.
2. In the mains supply synchronizing circuit.
3. Microprocessor based triggering circuit for thyristors.

Review Questions

Q. 1 Explain any six characteristics of an ideal OP-AMP.

Q. 2 Draw the equivalent circuit with significance of each component of a practical OP-AMP.

Q. 3 Explain the use of equivalent circuit.

Q. 4 Define the characteristics of a practical OP-AMP :

1. Input offset voltage 2. CMRR 3. PSRR 4. Slew rate

Q. 5 Compare the ideal and practical values of characteristics of OP-AMP.

Q. 6 Draw the block diagram of an OP-AMP and explain the purpose of using each block.

Q. 7 Explain the operation of an OP-AMP in open loop mode, with the help of the voltage transfer characteristics.

Q. 8 Explain why OP-AMP is not used as amplifier in the open loop mode.

Q. 9 What is the difference between positive and negative feedback ?

Q. 10 Explain why is negative feedback preferred in OP-AMP used as amplifier ?

Q. 11 Write a short note on virtual short and virtual ground.

Q. 12 What type of feedback is introduced due to the feedback resistor R_F ?

Q. 13 What is the maximum value of output voltage obtainable from an OP-AMP ?

Q. 14 Give the symbols for operational amplifiers and give characteristics of an ideal Op-Amp.

Q. 15 State the applications of Op-Amp.

Q. 16 Define an ideal differential amplifier and explain ?

Q. 17 Why D.C. level shifting is required in LIC ? Draw and explain various level shifting circuits

Q. 18 How is negative feedback introduced in an amplifier ? Explain.

Q. 19 Write short note on virtual short and virtual ground.

Q. 20 State important characteristics of IC 741 and compare their values with those of an ideal OP-AMP.

Q. 21 What are the advantages of using feedback ?

Q. 22 Define the following characteristics of a practical OP-AMP :

1. Input offset voltage
2. CMRR
3. PSRR
4. Slew rate
5. Input bias current
6. Input offset current.

Q. 23 What are the factors affecting the input offset voltage, input bias current and input offset current?

Q. 24 Draw the pin configuration of IC 741. State the typical values of following parameters.

1. Supply voltage
2. Power consumption.
3. Input resistance
4. Output resistance.

Q. 25 What is slew rate ? What are its causes ? Derive an expression for maximum frequency of operation for a desired output swing in terms of slew rate ?

Q. 26 What is the significance of slew rate ?

Q. 27 Explain how OP-AMP use as a summing amplifier.

Q. 28 Write short note on difference amplifier.

Q. 29 Explain with the help of circuit diagram the operation of an OP-AMP, non-inverting amplifier. Derive an expression for the voltage gain of this amplifier. Where and why it is required ?

Q. 30 Explain with the help of circuit diagram the operation of an OP-AMP inverting amplifier. Derive an expression for the voltage gain of this amplifier ?

Q. 31 Draw a neat circuit diagram of OP-AMP as a unity gain amplifier ?

Q. 32 Draw and explain differential amplifier with one Op-Amp.

Q. 33 With the help of circuit diagram, explain inverting adder using an Op-Amp.

Q. 34 Draw the circuit of non-inverting adder using an Op-Amp.

Q. 35 Explain how the Op-Amp works as a subtractor or difference amplifier.

Q. 36 Draw and explain the circuit of adder subtractor using Op-amp.

Q. 37 Explain differentiator circuit. Draw the output when input is square wave.

Q. 38 Explain integrator circuit . Draw the output when input is square wave.

Q. 39 Draw the circuit of differentiator using Op-Amp. What are the problems associated with this circuit ? How are they overcome ?

Q. 40 Draw and explain the circuit of practical active differentiator.

Q. 41 Draw the circuit of basic integrator using an Op-Amp. What are the problems associated with this configuration ? How they are overcome ?

Q. 42 What are the drawbacks of integrator using Op-Amp? Draw and explain the circuit of integrator.

Q. 43 Explain with the help of circuit diagram the operation of an OP-AMP comparator. Also draw the transfer characteristics of the comparator.

Q. 44 What are the types of comparator ? Explain the operation of an inverting comparator. Draw input and output voltage waveforms.

Q. 45 What are the types of comparator ? Explain the operation of a non-inverting comparator.Draw input and output voltage waveforms.

Q. 46 Is it possible to use the comparator as an amplifier ? Why ?

Q. 47 State the applications of comparator and explain any one of them.

Q. 48 Explain the operation of a zero crossing detector.

Q. 49 What are the applications of comparator. Draw the circuit for "zero crossing detector". Draw the input and output waveforms.

Q. 50 Explain the use of OPAMP as a non-inverting amplifier.

Q. 51 With the help of circuit diagram explain the operation of inverting amplifier.

Q. 52 With neat waveforms, describe how an OPAMP can be used as the basic integrator.

Q. 53 Explain the operation of the basic differentiator.

Q. 54 Describe the operation of zero crossing detector (ZCD) using OPAMP.

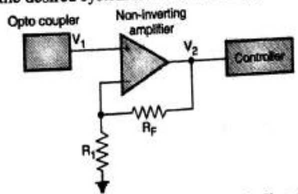
3.25 Solved University Examples :

Ex. 3.25.1 : Optical sensor of a package counting system create 1.5 V DC signal when it detects the seal. This voltage need to be amplified and sent to a controller with an input resistance of 5 kΩ. Draw block diagram for the system. Choose correct amplifier to drive this low resistance load and select the resistor values to set the gain. The input voltage range for the controller is +10 V to +30 V DC. **Dec. 13. 5 Marks**

Soln. :

Step 1 : Draw the block diagram of the system :

The block diagram of the desired system is as shown in Fig. P. 3.25.1.



(F-4326) **Fig. P. 3.25.1 : System block diagram**

Step 2 : Calculate R_1 and R_F :

The amplifier selected is non-inverting amplifier. The input V_1 to this amplifier is + 1.5 V from the opto-coupler output and its output V_2 should be in the range + 10 V to + 30 V.

$$\text{Let} \quad V_2 = 12\ V$$

- Gain $A_V = \frac{V_2}{V_1} = \frac{12}{1.5} = 8$
- But $A_V = 1 + \frac{R_F}{R_1}$

$$\therefore \quad 1 + \frac{R_F}{R_1} = 8$$

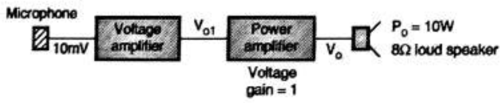
$$\text{Let} \quad R_1 = 10\ k\Omega \qquad \text{...Ans.}$$

$$\therefore \quad R_F = 7 R_1 = 70\ k\Omega \qquad \text{...Ans.}$$

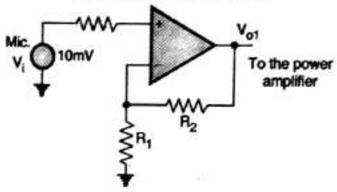
Ex. 3.25.2 : A public address system is connected to a microphone that has a maximum output voltage of 10 mV. The microphone is connected to a 10 Watts audio amplifier system that is driving an 8 Ohm speaker. The voltage amplifier is a non inverting op-amp circuit. Calculate the maximum voltage gain for the voltage amplifier stage and determine the resistor values to obtain the desired gain. Assume the power amplifier stage has a voltage gain of 1. **Dec. 14, 5 Marks**

Soln. :

Step 1 : Draw circuit diagram :



(a) Public address system



(b) Voltage amplifier

(D-1543) **Fig. P. 3.25.2**

Step 2 : Calculate V_o and V_{o1} :

Output voltage of the power amplifier is given by,

$$P_o = \frac{V_o^2}{R_L}$$

$$\therefore \quad V_o = \sqrt{P_o \times R_L} = \sqrt{10 \times 8} = 8.94\ \text{Volts} \qquad \text{...(1)}$$

Voltage gain of the power amplifier is 1. Hence the output voltage of the voltage amplifier is given by,

$$V_{o1} = \frac{V_o}{\text{Voltage gain of P.A.}} = \frac{V_o}{1}$$

$$\therefore \quad V_{o1} = V_o = 8.94\ \text{Volts}$$

Step 3 : Find R_1 and R_2 and A_V :

Voltage gain of voltage amplifier is,

$$A_V = 1 + \frac{R_2}{R_1}$$

$$\text{and } V_{o1} = A_V \times V_i$$

$$\therefore \quad A_V = \frac{V_{o1}}{V_i} = \frac{8.94}{10 \times 10^{-3}} = 894$$

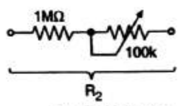
$$\therefore \quad 894 = 1 + \frac{R_2}{R_1}$$

$$\therefore \quad \frac{R_2}{R_1} = 893.4$$

$$R_2 = 893.4\ R_1$$

$$\text{Let} \quad R_1 = 1.2\ k\Omega \qquad \text{...Ans.}$$

$$\therefore \quad R_2 = 1071.6\ k\Omega \qquad \text{...Ans.}$$



(D-1544) **Fig. P. 3.25.2(c)**

We will use R_2 as a combination of a 1 MΩ fix value resistance and a 100 kΩ preset as shown in Fig. P. 3.25.2(c).

3.26 University Questions and Answers :

Q. 1 With neat diagram explain any one application of zero-crossing detector. **(May 2016, 5 Marks)**

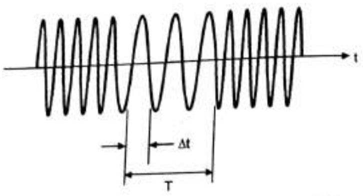
Ans. :

Zero crossing detector as frequency demodulator :

- The zero crossing detector operates on the principle that the instantaneous frequency of an FM wave is approximately given by,

$$f_i = \frac{1}{2\Delta t} \qquad \text{...(1)}$$

- Where Δt is the time difference between the adjacent zero crossover points of the FM wave as shown in Fig. 1(a).



(D-311) **Fig. 1(a) : Definitions of T and Δt for an FM wave**

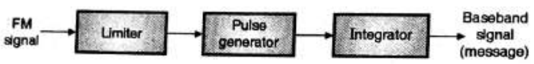
- Consider a time duration T as shown in Fig. 1(a). The time T is chosen such that it satisfies the following two conditions :
  1. T should be small as compared to (1 / W) where W is the bandwidth of the message signal.
  2. T should be large as compared to ($1 / f_c$) where f_c is the carrier frequency of the FM wave.
- Let the number of zero crossings during interval T be denoted by n_o. Hence Δt i.e. the time between the adjacent zero crossing points is given by,

$$\Delta t = \frac{T}{n_o} \quad ...(2)$$

Therefore the instantaneous frequency is given by,

$$f_i = \frac{1}{2\,\Delta t} = \frac{n_o}{2T} \quad ...(3)$$

- By definition of the instantaneous frequency, we know that there is a linear relation between f_i and message signal x (t). Hence we can recover x (t) if n_o is known.
- This can be achieved by using a zero crossing detector of Fig. 1(b).



(D-312) **Fig. 1(b) : Block diagram of zero crossing detector**

❑❑❑

CHAPTER 4 Analog Communication

Module 4

Syllabus :

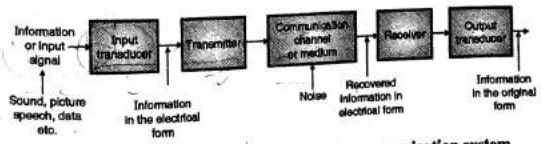
Block diagram and elements of analog communication systems, Theory of amplitude modulation and types of AM (Numerical) Generation of DSB SC using diode based balanced modulator, Generation of SSB using phase shift method, Introduction of FM, and its mathematical representation, Statement of Carson's Rule Comparison of AM, FM, Block diagram of AM transmitter (HLM and LLM) Block diagram of AM Superheterodyne receiver.

4.1 Introduction :

- The communication branch is the oldest branch of the electronics field. Telecommunication means communicating at a distance. A communication system is the means of conveying the information from one place to the other. This information can be of different types such as sound, picture, music, computer data etc.
- The field of communication engineering started developing rapidly in the nineteenth century when the telegraph, telephone and then the radio were invented. The development was still faster in the twentieth century when first the black and white and then colour TVs were brought in use. Then came the age of satellite communication, cable TV, mobile telephones etc.
- In order to understand the subject, it is necessary to understand the basic concepts in communication engineering such as, modulation, noise, demodulation, information theory etc.
- The communication systems can be broadly categorised into two categories.
  1. Analog communication systems and 2. Digital communication systems
- In this chapter we will discuss only the analog systems.

4.2 Block Diagram and Elements of Analog Communication System :

- The block diagram of the simplest possible analog communication system is as shown in Fig. 4.2.1.



(D-1) **Fig. 4.2.1 : Block diagram of the basic communication system**

- As seen from the Fig. 4.2.1, the elements of a basic communication system are transmitter, a communication medium (channel) and the receiver.
- When the transmitted signal is travelling from the transmitter to the receiver over a communication channel, noise gets added to it.
- The elements of basic communication system are as follows :
  1. Information or input signal
  2. Input transducer
  3. Transmitter
  4. Communication channel or medium
  5. Noise
  6. Receiver
  7. Output transducer

1. Information or Input signal :

- The communication systems have been developed for communicating useful information from one place to the other.
- This information can be in the form of a sound signal like speech or music, or it can be in the form of pictures (TV signals) or it can be data information coming from a computer.

2. Input transducer :

- The information in the form of sound, picture or data signals cannot be transmitted as it is.
- First it has to be converted into a suitable electrical signal. The input transducer block does this job.
- The input transducers commonly used in the communication systems are microphones, TV camera etc.

3. Transmitter :

- The function of the transmitter block is to convert the electrical equivalent of the information to a suitable form.
- In addition to that it increases the power level of the signal. The power level should be increased in order to increase the range of transmitted signal.
- The transmitter consists of the electronic circuits such as amplifier, mixer, oscillator and power amplifier.

4. Communication channel or medium :

The communication channel is the path used for transmission of electronic signal from one place to the other. The communication medium can be conducting wires, cables, optical fibre or free space. Depending on the type of communication medium, two types of communication systems will exist. They are :

- Wired communication or line communication
- Wireless communication or radio communication

Line communication :

- The line communication systems use the communication mediums like the simple wires or cables or optical fibers.
- The examples of such systems, are telegraph and telephone systems, cable T.V. etc.
- Due to physical connection from one point to the other, these systems cannot be used for the communication over long distances.

Radio communication :

- The radio communication systems use the free space as their communication medium. They do not need the wires for sending the information from one place to the other.
- The radio or TV broadcasting, satellite communication are the examples of the wireless communication. These systems transmit the signal using a transmitting antenna in the free space.
- The transmitted signal is in the form of electromagnetic waves. A receiving antenna will pick up this signal and feed it to the receiver.
- Radio communication can be used for the long distance communication such as from one country to the other or even from one planet to the other.

5. Noise :

- Noise is an unwanted electrical signal which gets added to the transmitted signal when it is travelling towards the receiver.
- Due to noise, the quality of the transmitted information will degrade. Once added, the noise cannot be separated out from the information.
- Hence noise is a big problem in the communication systems. (Specially analog communication systems).
- The noise can be either natural or man made. The sources of natural noise are lightning or radiation from the sun and stars etc.
- The man made noise includes the noise produced by electrical ignition systems of the automobiles, welding machines, electric motors etc.
- Eventhough noise cannot be completely eliminated, its effect can be reduced by using various techniques.

6. Receiver :

- The process of reception is exactly the opposite process of transmission. The received signal is amplified, demodulated and converted into a suitable form.
- The receiver consists of electronic circuits like mixer, oscillator, detector, amplifier etc.

7. Output transducers :

- The output transducer converts the electrical signal at the output of the receiver back to the original form i.e. sound or TV pictures etc.
- The typical examples of the output transducers are loud speakers, picture tubes, computer monitor etc.

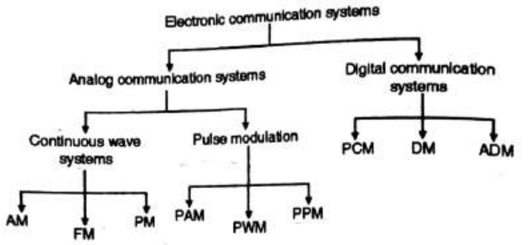
4.3 Principle of Analog Communication : MU : Dec. 12

University Questions

Q. 1 Distinguish between analog and digital modulation. **(Dec. 12, 5 Marks)**

Fig. 4.3.1 shows another way of classifying the electronic communication systems. They are classified into two categories namely :

1. Analog communication systems.
2. Digital communication systems.



(D-6) Fig. 4.3.1 : Classification based on analog or digital communication

4.3.1 Analog Communication :

- **Definition :** The modulation systems or techniques in which one of the characteristics of the carrier is changed in proportion with the instanteous value of modulating signal is called as analog modulation system.
- If the carrier is sinusoidal then it's amplitude, frequency or phase is changed in accordance with the modulating signal to obtain AM, FM or PM respectively. These are continuous wave modulation systems.
- Analog modulation can be pulsed modulation as well. Here the carrier is in the form of rectangular pulses. The amplitude, width (duration) or position of the carrier pulses is varied in accordance with the modulating signal to obtain the PAM, PWM or PPM outputs.

Examples of Analog Modulation :

Following are the examples of analog modulation systems :

1. Amplitude Modulation (AM)
2. Frequency Modulation (FM)
3. Phase Modulation (PM)
4. Pulse Amplitude Modulation (PAM)
5. Pulse Width Modulation (PWM)
6. Pulse Position Modulation (PPM).

Advantages of Analog Communication :

Some of the advantages of analog communication are as follows :

1. Transmitters and receivers are simple.
2. Low bandwidth requirement
3. FDM (frequency division multiplexing) can be used.

Disadvantages of Analog Communication :

Some of the disadvantages are :

1. Noise affects the signal quality.
2. It is not possible to separate noise and signal.
3. Repeaters can not be used between transmitters and receivers.
4. Coding is not possible.
5. It is not suitable for the transmission of secret information.

Applications :

1. Radio broadcasting (AM and FM).
2. TV broadcasting
3. Telephones

4.3.2 Digital Communication :

- **Definition :** The modulation system or technique in which the transmitted signal is in the form of digital pulses of constant amplitude, constant frequency and phase is called as digital modulation system.
- **Examples :** Pulse Code Modulation (PCM), Delta Modulation (DM), Differential PCM (DPCM) and Adaptive Delta Modulation (ADM) are the examples of digital modulation.
- In the PCM and DM, a train of digital pulses is transmitted by the transmitter.
- All the pulses are of constant amplitude, width and position. The information is contained in the combination of the transmitted pulses.

Advantages of Digital Communication :

Some of the advantages of digital communication are as follows :

1. Due to the digital nature of the transmitted signal, the interference of additive noise does not introduce many errors. So digital communication has a better noise immunity.
2. Due to the channel coding techniques used in digital communication, it is possible to detect and correct the errors introduced during the data transmission.
3. Repeaters can be used between transmitter and receiver to regenerate the digital signal. This improves the noise immunity further.
4. Due to the digital nature of the signal, it is possible to use the advanced data processing techniques such as digital signal processing, image processing, data compression etc.
5. TDM (Time Division Multiplexing) technique can be used to transmit many voice channels over a single common transmission channel.
6. Digital communication is useful in military applications where only a few permitted receivers can receive the transmitted signal.
7. Digital communication is becoming simpler and cheaper as compared to the analog communication due to the invention of high speed computers and integrated circuits (ICs).

Disadvantages of Digital Communication :

Some of the important disadvantages of digital communication are :

1. The bit rates of digital systems are high. Therefore they require a larger channel bandwidth as compared to analog systems.
2. Digital modulation needs synchronization in case of synchronous modulation.

Applications of Digital Communications :

1. Long distance communication between earth and space ships.
2. Satellite communication.
3. Military communications which needs coding.
4. Telephone systems.
5. Data and computer communications.

4.4 Need of Modulation :

MU : May 13, May 16

University Questions

Q. 1 What is the need of modulation in communication. **(May 13, 5 Marks)**

Q. 2 Explain the necessity and significance of modulation in communication. **(May 16, 5 Marks)**

- A question may be asked as, when the baseband signals can be transmitted directly why to use the modulation ?
- The answer is that the baseband transmission has many limitations which can be overcome using modulation. It is as explained below.
- In the process of modulation, the baseband signal is "translated" i.e. shifted from low frequency side to high frequency side of the frequency spectrum.
- This frequency shift is proportional to the frequency of carrier. The modulation process has the following advantages :

Advantages of modulation :

1. Reduction in the height of antenna
2. Avoids mixing of signals
3. Increases the range of communication
4. Multiplexing becomes possible
5. Improves quality of reception.

1. Reduction in height of antenna :

- For transmission of radio signals, the antenna height must be a multiple of ($\lambda/4$). Here λ is the wavelength. $\lambda = c/f$ where c is velocity of light and f is the frequency of the signal to be transmitted.
- The minimum antenna height required to transmit a baseband signal of f = 10 kHz is calculated as follows :

$$\text{Minimum antenna height} = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 10 \times 10^3}$$

$$= 7500 \text{ meters i.e. } 7.5 \text{ km}$$

The antenna of this height is practically impossible to install.

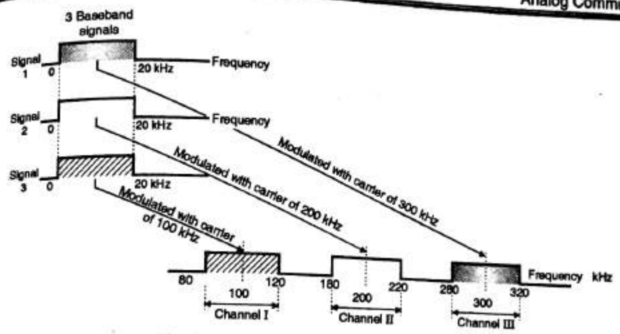
- Now consider a modulated signal at f = 1 MHz. The minimum antenna height is given by,

$$\text{Minimum antenna height} = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3 \times 10^8}{4 \times 10 \times 10^6} = 75 \text{ meters}$$

- This antenna can be easily installed practically. Thus modulation reduces the height of the antenna.

2. Avoids mixing of signals :

- If the baseband sound signals are transmitted without using the modulation by more than one transmitter, then all the transmitted signal by multiple transmitters will be in the same frequency range i.e. 0 to 20 kHz.
- Therefore the signals from different stations get mixed together and a receiver cannot separate them from each other.
- So if each baseband sound signal is used to modulate a different carrier which corresponds to a different station then they will occupy different slots in the frequency spectrum (different channels).
- This is as shown in Fig. 4.4.1. **Thus modulation avoids mixing of signals.**



(D-10) **Fig. 4.4.1 : Modulation avoids mixing of signals**

3. Increases the range of communication :

- The frequency of baseband signals is low, and the low frequency signals can not travel a long distance when they are transmitted. They get attenuated (suppressed) quickly.
- The attenuation reduces with increase in frequency of the transmitted signals, and they travel longer distance.
- The modulation process increases the frequency of the signal. Hence it increases the range of communication.

4. Multiplexing becomes possible :

- Multiplexing is a process in which two or more signals can be transmitted over the same communication channel simultaneously.
- This is possible only with modulation. The multiplexing allows the same channel to be used by many signals.
- So many TV channels can use the same frequency range, without getting mixed with each other. OR different frequency signals can be transmitted at the same time.

5. Improves quality of reception :

With frequency modulation (FM), and the digital communication techniques like PCM, the effect of noise is reduced to a great extent. This improves quality of reception.

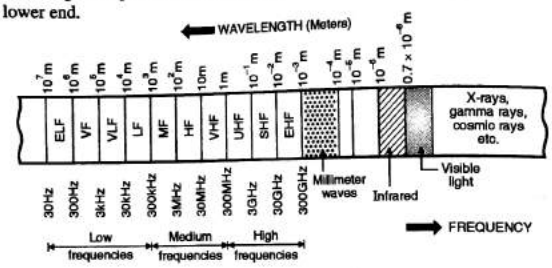
4.5 Demodulation or Detection :

- The modulated signals are transmitted by the transmitter via air medium or wire medium. These signals then reach the receivers by travelling over the communication medium.
- At the receiver, the original information signal is separated from the carrier. This process is called as demodulation or detection. Detection is exactly the opposite process of modulation.

4.6 The Electromagnetic Spectrum :

- The information signal should be first converted into an electromagnetic signal before transmission because the wireless transmission takes place using the electromagnetic waves.

- The electromagnetic waves consist of both electric and magnetic fields. The electromagnetic waves can travel a long distance through space.
- The electromagnetic signals are also called as radio frequency (RF) waves.
- The EM waves oscillate, they are sinusoidal and their frequency is measured in Hz.
- The frequency of EM signal can be very low or it can be extremely high. This entire range of frequencies of EM waves is called as **Electromagnetic spectrum.**
- The electromagnetic spectrum consists of signals such as 50 Hz line frequency and voice signals at the lower end.



(D-16) **Fig. 4.6.1 : Complete electromagnetic (EM) spectrum**

- The radio frequencies which are used for the two way communication reside at the center of the EM spectrum. These frequencies are used for the applications such as radio or TV broadcasting as well.
- The infrared and visible light are at the upper end of the EM spectrum.
- Fig. 4.6.1 shows the entire electromagnetic spectrum.
- The short forms used in the EM spectrum of Fig. 4.6.1 have the following meanings.

Table 4.6.1 : Segments of the electromagnetic spectrum

Sr. No.	Name	Frequency	Wavelength
1.	Extremely low frequency (ELF)	30-300 Hz	10^7 to 10^6 m
2.	Voice frequencies (VF)	300-3000 Hz	10^6 to 10^5 m
3.	Very low frequencies (VLF)	3-30 kHz	10^5 to 10^4 m
4.	Low frequencies (LF)	30-300 kHz	10^4 to 10^3 m
5.	Medium frequencies (MF)	300 kHz - 3 MHz	10^3 to 10^2 m
6.	High frequencies (HF)	3-30 MHz	10^2 to 10 m
7.	Very high frequencies (VHF)	30-300 MHz	10 to 1 m
8.	Ultra high frequencies (UHF)	300 MHz-3GHz	1 to 10^{-1} m
9.	Super high frequencies (SHF)	3-30 GHz	10^{-1} to 10^{-2} m
10.	Extremely high frequencies (EHF)	30-300 GHz	10^{-2} to 10^{-3} m
11.	Infrared	–	0.7 to 10 μm
12.	Visible light	–	0.4 μm to 0.8 μm

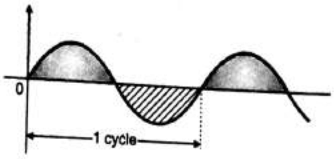
4.6.1 Frequency and Wavelength :

In the EM spectrum, we have used frequency as well as wavelength in order to define various segments. So let us define these terms and the relation between them.

Frequency :

- Frequency is defined as the number of cycles of a waveform per second. It expressed in hertz (Hz).
- The units used to measure higher frequencies are kilohertz (kHz), Megahertz (MHz) and Gigahertz (GHz). Their relation with the basic unit Hz is as follows :

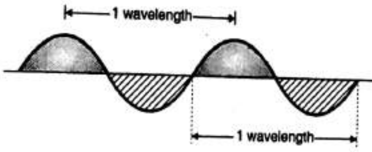
$$1 \text{ kHz} = 1000 \text{ Hz}$$
$$1 \text{ MHz} = 1000 \text{ kHz} = 1 \times 10^6 \text{ Hz.}$$
$$1 \text{ GHz} = 1000 \text{ MHz} = 1 \times 10^9 \text{ Hz.}$$



(D-17) **Fig. 4.6.2 : One cycle**

Wavelength (λ) :

- Wavelength is defined as the distance travelled by an electromagnetic wave during the time of one cycle. Refer Fig. 4.6.3 for the concept of wavelength.



(D-18) **Fig. 4.6.3 : Definition of wavelength**

- Since EM waves travel at the speed of light in the free space or vaccum, their wavelength is given by,

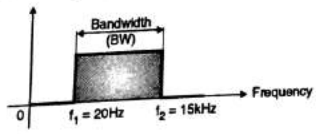
$$\lambda = \frac{\text{Speed of light}}{\text{Frequency}} = \frac{3 \times 10^8 \text{ m/s}}{f} \quad ...(4.6.1)$$

- Hence wavelength decreases with increase in frequency.

4.7 Bandwidth :

- Bandwidth is defined as the portion of the electromagnetic spectrum occupied by a signal.
- We may also define the bandwidth as the frequency range over which an information signal is transmitted.
- Bandwidth is the difference between the upper and lower frequency limits of the signal.
- We already know different types of baseband signals such as voice signal, music signal, TV signal etc. Each of these signals will have its own frequency range. This frequency range of a signal is known as its bandwidth.
- For example, the range of music signal is 20 Hz to 15 kHz. Therefore as shown in Fig. 4.7.1 the bandwidth is $(f_2 - f_1)$.

$$\therefore \quad BW = f_2 - f_1 = 15000 - 20$$
$$= 14980 \text{ Hz.}$$



(D-19) **Fig. 4.7.1 : Bandwidth of music signal**

The bandwidths of different signals are as listed in Table 4.7.1.

Table 4.7.1

Sr. No.	Type of the signal	Range of frequency in Hz	Bandwidth in Hz
1.	Voice signal (speech) for telephony	300 – 3400	3,100
2.	Music signal	20 – 15000	14,980
3.	TV signals (picture)	0 – 5 MHz	5 MHz
4.	Digital data	* 300 – 3400 (If it is using the telephone line for its transmission).	3,100

***Note :** Actually the required bandwidth in the data transmission depends on the rate at which the data is being transmitted. The BW increases with increase in the rate of data transmission.

4.7.1 Frequency Spectrum :

- Frequency spectrum is the representation of a signal in the frequency domain. It can be obtained by using either Fourier series or Fourier transform.
- It consists of the amplitude and phase spectrums of the signal. The frequency spectrum indicates the amplitude and phase of various frequency components present in the given signal.
- The frequency spectrum enables us to analyze and synthesize a signal.

4.8 Introduction to Amplitude Modulation :

MU : May 16

University Questions

Q. 1 Explain the concept of amplitude modulation. **(May 16, 10 Marks)**

- We know that in order to transmit the information or message signal over a bandpass communication channel such as satellite channel or radio channel, some type of modulation is essential.
- The modulation process is associated with shift of frequency range from low frequency side to high frequency side of the spectrum.
- Modulation is the process by which some characteristics of a carrier is varied in accordance with a modulating wave.
- The three characteristics of the carrier are amplitude, frequency and phase. So one of these characteristics is varied in proportion with the modulating signal (i.e. message or information signal).
- The message signal is referred to as the modulating signal and the result of modulation process is referred to a **modulated signal**.
- It is important to note that all the information is contained in the "varying characteristics" of the carrier but the carrier itself does not contain any information.

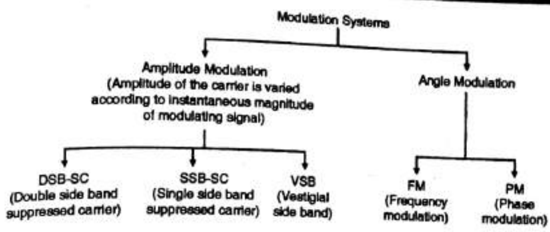
4.8.1 Types of Analog Modulation Systems :

MU : May 16

University Questions

Q. 1 Explain the concept of amplitude modulation. **(May 16, 10 Marks)**

- Depending on which characteristics of the carrier is being changed, the modulation systems are classified as shown in Fig. 4.8.1. Note that the amplitude modulation that we are going to discuss in this chapter is also called as Double Sideband Full Carrier (DSBFC) modulation.



(D-34) **Fig. 4.8.1 : Classification of modulation systems**

4.8.2 Principle of Amplitude Modulation (AM) or DSB FC-Modulation :

MU : May 11, Dec. 15, May 16

University Questions

Q. 1 Define Amplitude modulation. Draw a neat waveform for an AM wave. **(May 11, 5 Marks)**

Q. 2 Explain amplitude modulation for more than one modulating signal in the following cases :

1. Mathematical equation
2. AM waveform
3. AM amplitude and power spectrum
4. Modulation coefficient
5. Transmission power

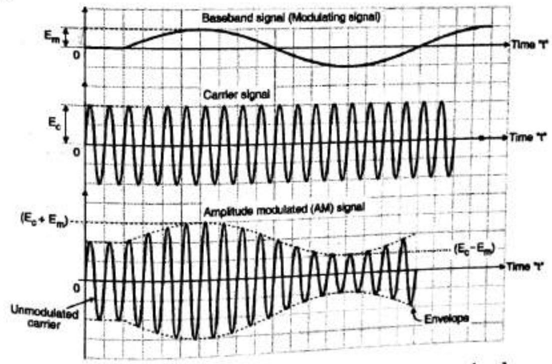
(Dec. 15, 10 Marks)

Q. 3 Explain the concept of amplitude modulation. **(May 16, 10 Marks)**

Amplitude modulation (AM) or Amplitude Modulation with Full Carrier (AM-FC) is the process of changing the amplitude of a high frequency sinusoidal carrier signal in proportion with the instantaneous value of modulating signal.

How does an AM wave look like ?

- Fig. 4.8.2 shows the amplitude modulated wave when the modulating signal is a sinusoidal signal.



(D-12) **Fig. 4.8.2 : AM waveform for sinusoidal modulating signal**

Observations :

1. The frequency of the sinusoidal carrier is much higher than that of the modulating signal.
2. In AM the instantaneous amplitude of the sinusoidal high frequency carrier is changed in proportion to the instantaneous amplitude of the modulating signal. This is the principle of AM.
3. The time domain display of AM signal is as shown in Fig. 4.8.2. This AM signal is transmitted by a transmitter. The information in the AM signal is contained in the amplitude variations of the carrier of the envelope shown by dotted lines in Fig. 4.8.2.
4. Note that the frequency and phase of the carrier remain constant.
5. AM is used in the applications such as radio transmission, TV transmission etc.

Note : The modulating signal in practice may or may not be purely sinusoidal. Most of the times it will have a complex shape.

The AM Envelope :

- Although there are several types of amplitude modulation, the double sideband full carrier (DSBFC) is the most commonly used.
- The DSBFC is also called as conventional AM. Fig. 4.8.2 shows the AM waveform and the shape of modulated waveform is called as **envelope.**
- The repetition rate of the envelope is equal to the frequency of the modulating signal and the envelope shape is identical to the shape of modulating signal.

4.9 Mathematical Representation of an AM Wave :

The mathematical representation of an AM wave can be divided into two parts namely :

1. Time domain description. 2. Frequency domain description.

4.9.1 Time Domain Description (AM Voltage Distribution) :

MU : Dec. 04, May 11, Dec. 15

University Questions

Q. 1 Derive the equation for an Amplitude modulated wave. **(Dec. 04, 3 Marks)**

Q. 2 Derive expression for an AM wave. **(May 11, 3 Marks)**

Q. 3 Explain amplitude modulation for more than one modulating signal in the following cases :

1. Mathematical equation
2. AM waveform
3. AM amplitude and power spectrum
4. Modulation coefficient
5. Transmission power

(Dec. 15, 10 Marks)

Assumptions :

- Let the modulating signal be sinusoidal and be represented as,

$$e_m = E_m \cos \omega_m t \quad ...(4.9.1)$$

where "e_m" is the instantaneous amplitude of the modulating signal, E_m is the peak amplitude, $\omega_m = 2\pi f_m$ and f_m = Frequency of the modulating signal.

- Let the carrier signal also be sinusoidal at a much higher frequency than that of the modulating signal. The instantaneous carrier signal e_c is given by,

$$e_c = E_c \cos \omega_c t \quad ...(4.9.2)$$

where E_c = Peak carrier amplitude,

f_c = Carrier frequency and $\omega_c = 2\pi f_c$.

- The AM wave is expressed by the following expression,

$$e_{AM} = A \cos (2\pi f_c t) \quad ...(4.9.3)$$

where A = Envelope of AM wave

- Where A represents the instantaneous value of the envelope. The modulating signal either adds or gets subtracted from the peak carrier amplitude E_c as shown in Fig. 4.8.2. Hence we can represent the instantaneous value of envelope as,

$$A = E_c + e_m$$
$$= E_c + E_m \cos (2\pi f_m t) \quad ...(4.9.4)$$

- Hence the AM wave is given by,

$$e_{AM} = A \cos (2\pi f_c t)$$
$$= [E_c + E_m \cos (2\pi f_m t)] \cos (2\pi f_c t)$$
$$\therefore \quad e_{AM} = E_c \left[1 + \frac{E_m}{E_c} \cos (2\pi f_m t) \right] \cos (2\pi f_c t)$$

- Let $m = E_m / E_c$ be the modulation index.

$$\therefore \quad e_{AM} = E_c [1 + m \cos (2\pi f_m t)] \cos (2\pi f_c t) \quad ...(4.9.5)$$

This expression represents the time domain representation of an AM signal.

Note : It is not necessary to always consider the cosine waves to obtain the mathematical expression. We can even use the sine waves to obtain the mathematical expression for AM.

Ex. 4.9.1 : Obtain the mathematical expression for an A.M. wave if the modulating signal is $e_m = E_m \sin (2\pi f_m t)$ and the carrier is $e_c = E_c \sin (2\pi f_c t)$.

Soln. :

Step 1 : Modulating and carrier signals :

- Given that the modulating signal is,

$$e_m = E_m \sin (2\pi f_m t) \quad ...(1)$$

- And the carrier signal is,

$$e_c = E_c \sin (2\pi f_c t) \quad ...(2)$$

Step 2 : Express the envelope of A.M. wave mathematically :

- Let the A.M. wave be given by,

$$e_{AM} = A \sin (2 \pi f_c t) \quad ...(3)$$

where A is the envelope of AM wave. It is given by,

$$A = E_c + e_m = E_c + E_m \sin (2 \pi f_m t) \quad ...(4)$$

Step 3 : Obtain expression for A.M. wave :

- Substitute Equation (4) into Equation (3) to get,

$$e_{AM} = [E_c + E_m \sin (2 \pi f_m t)] \sin (2 \pi f_c t) \quad ...(5)$$

$$= E_c \left[1 + \frac{E_m}{E_c} \sin (2 \pi f_m t) \right] \sin (2 \pi f_c t)$$

- But $\frac{E_m}{E_c} = m$ (modulation index)

$$\therefore \quad e_{AM} = E_c [1 + m \sin (2 \pi f_m t)] \sin (2 \pi f_c t)$$

This is the required expression for AM wave in terms of sine wave.

4.9.2 Modulation Index or Modulation Factor and Percentage Modulation :

MU : Dec. 15

University Questions

Q. 1 Explain amplitude modulation for more than one modulating signal in the following cases :
1. Mathematical equation
2. AM waveform
3. AM amplitude and power spectrum
4. Modulation coefficient
5. Transmission power

(Dec. 15, 10 Marks)

- In AM wave the modulation index (m) is defined as the ratio of amplitudes of the modulating and carrier waves as follows :

$$m = \frac{E_m}{E_c} \quad ...(4.9.6)$$

- When $E_m \le E_c$ the modulation index "m" has values between 0 and 1 and no distortion is introduced in the AM wave. But if $E_m > E_c$ then m is greater than 1. This will distort the shape of AM signal. The distortion is called as "over modulation."
- The modulation index is also called as modulation factor, modulation coefficient or degree of modulation. However if modulation index is expressed as percentage it is called as "percentage modulation."

$$\therefore \ \% \text{ Modulation} = \frac{E_m}{E_c} \times 100 \quad ...(4.9.7)$$

- Note that "m" is a dimensionless quantity.

4.9.3 Frequency Spectrum of the AM Wave (Frequency Domain Description) :

MU : Dec. 06, Dec. 09, Dec. 14, May 15, Dec. 15

University Questions

Q. 1 Explain frequency spectrum of the AM wave. **(Dec. 06, Dec. 09, 5 Marks)**

Q. 2 Draw the spectrum of an amplitude modulated wave and explain its components. **(Dec. 14, May 15, 5 Marks)**

Q. 3 Explain amplitude modulation for more than one modulating signal in the following cases :
1. Mathematical equation
2. AM waveform
3. AM amplitude and power spectrum
4. Modulation coefficient
5. Transmission power

(Dec. 15, 10 Marks)

- The frequency spectrum is a graph of amplitude on Y axis versus frequency on X axis. The frequency spectrum of AM wave tells us about which frequency components are present in the AM wave and what are their amplitudes. So consider the equation for AM wave.

$$e_{AM} = (E_c + E_m \cos \omega_m t) \cos \omega_c t$$

$$E_c = \left[1 + \frac{E_m}{E_c} \cos \omega_m t \right] \cos \omega_c t$$

- As per the definition of the modulation index, $m = E_m / E_c$.

$$\therefore \quad e_{AM} = E_c (1 + m \cos \omega_m t) \cos \omega_c t \quad ...(4.9.8)$$

Simplifying we get,

$$e_{AM} = E_c \cos \omega_c t + m E_c \cos \omega_m t \cos \omega_c t \quad ...(4.9.9)$$

- For the second term in the above expression use the following standard identity :

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$$

- Therefore Equation (4.9.9) gets simplified as follows :

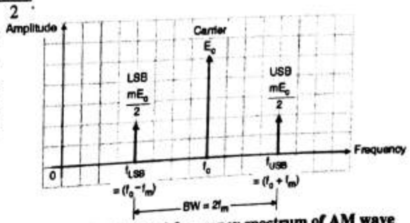
$$e_{AM} = \underbrace{E_c \cos \omega_c t}_{\text{Carrier}} + \underbrace{\frac{m E_c}{2} \cos (\omega_c + \omega_m) t}_{\text{Upper sideband}} + \underbrace{\frac{m E_c}{2} \cos (\omega_c - \omega_m) t}_{\text{Lower sideband}} \quad ...(4.9.10)$$

Observations :

The expression for the AM wave shows that it consists of three terms :

1. First term is nothing else but the unmodulated carrier signal.
2. The second term is a sinusoidal signal at frequency ($f_c + f_m$). This is called as the upper sideband (USB). Its amplitude is $\frac{m E_c}{2}$.
3. The third term represents a sinusoidal signal at frequency ($f_c - f_m$). It is called as the lower sideband (LSB). Its amplitude is $\frac{m E_c}{2}$.

- Hence the frequency spectrum of an A.M. wave is as shown in Fig. 4.9.1. Note that it is a single sided spectrum i.e. the spectrum plotted for only the positive values of frequency.



(D-37) **Fig. 4.9.1 : Single sided frequency spectrum of AM wave**

4.9.4 Bandwidth of AM Wave :

The bandwidth of the AM signal is obtained by the subtraction of the highest and the lowest frequency component in the frequency spectrum. Therefore :

$$BW = f_{USB} - f_{LSB} = (f_c + f_m) - (f_c - f_m)$$

$$BW = 2 f_m \quad ...(4.9.11)$$

Ex. 4.9.2 : **A modulating signal 10 sin (2 π × 10³ t) is used to modulate a carrier signal 20 sin (2 π × 10⁴ t). Find the modulation index, percentage modulation, frequencies of the sideband components and their amplitudes. What is the bandwidth of the modulated signal ? Also draw the spectrum of the AM wave.** May 13, 10 Marks

Soln. :

- The modulating signal $e_m = 10 \sin (2\pi \times 10^3 t)$. So comparing this with the expression

$$e_m = E_m \sin (2\pi f_m t) \text{ we get}$$

$$E_m = 10 \text{ volt}, f_m = 1 \times 10^3 \text{ Hz} = 1 \text{ kHz}$$

- The carrier signal $e_c = 20 \sin (2\pi \times 10^4 t)$.

Comparing this with the expression $e_c = E_c \sin (2\pi f_c t)$ we get :

$$E_c = 20 \text{ Volts}, f_c = 1 \times 10^4 \text{ Hz} = 10 \text{ kHz}.$$

w = 2πf

1. **Modulation index and percentage modulation :**

$$m = \frac{E_m}{E_c} = \frac{10}{20} = 0.5 \text{ and \% modulation} = 0.5 \times 100 = 50\%.$$ 2π×10³ = 2π

2. **Frequencies of sideband components :**
   1. Upper sideband $f_{USB} = f_c + f_m = (10 + 1) = 11$ kHz
   2. Lower sideband $f_{LSB} = f_c - f_m = (10 - 1) = 9$ kHz
3. **Amplitudes of sidebands :**

   The amplitudes of upper as well as the lower sideband is given by,

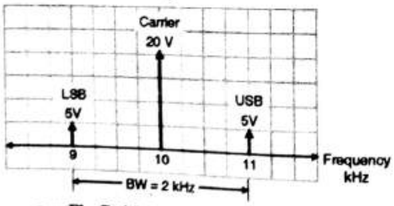
$$\text{Amplitude of each sideband} = \frac{m E_c}{2} = \frac{0.5 \times 20}{2} = 5 \text{ Volts}$$

4. **Bandwidth :**

$$\text{Bandwidth} = 2 f_m = 2 \times 1 \text{ kHz} = 2 \text{ kHz}$$ ...Ans.

5. **Spectrum :**

   The spectrum of AM wave is shown in Fig. P. 4.9.2.



(D-40) **Fig. P. 4.9.2 : Spectrum of the AM wave**

Ex. 4.9.3 : Consider the message signal x(t) = 20 cos (2 π t) volts and carrier wave c(t) = 50 cos (100 π t) volts. Derive an expression for the resulting AM wave for 75% modulation.

Soln. :

1. **Given :** $x(t) = 20 \cos (2\pi t)$

$$\therefore \quad f_m = 1 \text{ Hz and } E_m = 20 \text{ V}$$

2. $c(t) = 50 \cos (100 \pi t)$

$$\therefore \quad f_c = 50 \text{ Hz and } E_c = 50 \text{ V}$$

and Modulation index m = 0.75

3. $\therefore$ AM wave $s(t) = E_c [1 + m \cos (2\pi t)] \cos (2\pi f_c t)$

$$= 50 [1 + 0.75 \cos (2\pi t)] \cos (100 \pi t)$$ ...Ans.

Ex. 4.9.4 : A carrier frequency of 8 MHz with peak value of 6V is amplitude modulated by a 10 kHz sine wave signal with amplitude 4 Volts. Determine the modulation index and draw the amplitude spectrum.

Soln. :

Given : Carrier frequency $f_c = 8$ MHz = 8000 kHz, Peak voltage value $E_c = 6$ V

Modulating frequency $f_m = 10$ kHz, Amplitude $E_m = 4$ Volts

Step 1 : Modulation index (m) :

Modulation index m is given by,

$$m = \frac{E_m}{E_c} = \frac{4V}{6V} = \frac{2}{3}$$

Step 2 : Amplitude of sidebands :

Amplitude of sideband at $(f_c \pm f_m)$ is given by $\frac{m E_c}{2}$

Substituting the values, we get $= \frac{\frac{2}{3} \cdot 6}{2} = \frac{4}{2} = 2V$

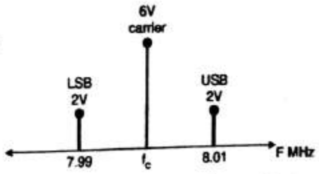
Step 3 : Spectrum of the AM wave :

$$(f_c + f_m) = (8000 + 10) \text{ kHz}$$

$$= 8010 \text{ kHz} = 8.01 \text{ MHz}$$

$$(f_c - f_m) = (8000 - 10) \text{ kHz}$$

$$= 7990 \text{ kHz} = 7.99 \text{ MHz}.$$



(D-1269) **Fig. P. 4.9.4 : Spectrum of AM wave**

Ex. 4.9.5 : **One input to a conventional AM modulator is a 500 kHz carrier with an amplitude of 20 V_p. The second input is a 10 kHz modulating signal that is sufficient amplitude to cause a change in the output wave of ± 7.5 V_p. Determine :**

1. **Side frequencies and modulation index.**
2. **Peak amplitude of the modulated carrier and the upper and lower side frequency voltages.**
3. **Maximum and minimum amplitudes of the envelope.**
4. **Expression for the modulated wave.**
5. **Draw the output spectrum and output envelope.**

Dec. 07, 12 Marks, Dec. 13, Dec. 16, 10 Marks

Soln. :

(a) Upper and lower sideband frequencies :

$$f_{USB} = f_c + f_m = 500 \text{ kHz} + 10 \text{ kHz} = 510 \text{ kHz} \quad \text{...Ans.}$$

$$f_{LSB} = f_c - f_m = 500 \text{ kHz} - 10 \text{ kHz} = 490 \text{ kHz} \quad \text{...Ans.}$$

(b) Modulating coefficient and percent modulation :

From the given description the peak modulating voltage $V_m = 7.5$ V and peak carrier voltage $E_c = 20$ V.

$$\therefore \text{ Modulating coefficient } m = \frac{E_m}{E_c} = \frac{7.5}{20} = 0.375 \quad \text{...Ans.}$$

$$\% \text{ Modulation} = m \times 100\% = 0.375 \times 100 = 37.5\% \quad \text{...Ans.}$$

(c) Amplitude of sidebands :

$$\text{Peak amplitude of modulated carrier} = E_c + E_m = 20 + 7.5 = 27.5 \text{ V} \quad \text{...Ans.}$$

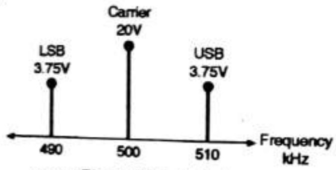
$$\text{Amplitude of sidebands} = \frac{m E_c}{2} = 0.375 \times \frac{20}{2} = 3.75 \text{ V} \quad \text{...Ans.}$$

(d) Expression for modulated wave :

$$e_{AM} = E_c [1 + m \sin (2\pi f_m t)] \sin (2\pi f_c t)$$

$$= 20 [1 + 0.375 \sin (2\pi \times 10 \times 10^3 t)] \sin (2\pi \times 500 \times 10^3 t) \quad \text{...Ans.}$$

(e) Output spectrum :



(F-4795) **Fig. P. 4.9.5 : Output spectrum**

Ex. 4.9.6 : A modulating signal 5 cos 2π × 15 × 10³t, angle modulates a carrier A cos ωct with deviation 75 kHz. Find the modulation index and bandwidth for FM. May 07, 4 Marks

Soln. :

Given :

$$\delta = 75 \text{ kHz}$$

$$\text{Modulating frequency } f_m = 15 \times 10^3 = 15 \text{ kHz}$$

1. Modulation index :

$$m_f = \frac{\delta}{f_m} = \frac{75 \text{ kHz}}{15 \text{ kHz}} = 5 \quad \text{...Ans.}$$

2. Bandwidth BW :

$$BW = 2 [\delta + f_m] = 2 [75 + 15] \text{ kHz} = 180 \text{ kHz} \quad \text{...Ans.}$$

4.10 Linear Modulation and Overmodulation :

- Depending on the value of percentage modulation (m) the AM wave can be classified into two categories :

  1. Linear modulation 2. Overmodulation.

4.10.1 Linear Modulation :

- If $m \leq 1$ or if the percentage modulation is less than 100% then the type of amplitude modulation is linear amplitude modulation.
- The waveforms of AM waves with linear modulation are in Figs. P. 4.10.1(a) and (b) respectively (Refer Ex. 4.10.1).

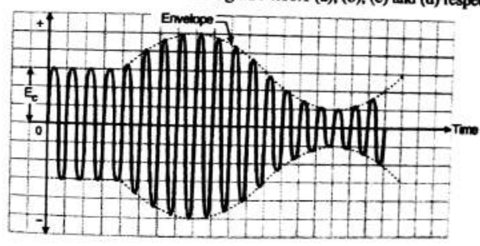
4.10.2 Overmodulation :

- If $m > 1$ i.e. if the percentage modulation is greater than 100% then the type of amplitude modulation is called as overmodulation.
- For $m > 1$ the envelope can sometimes reverse the phase as shown in Fig. P. 4.10.1(c) in Ex. 4.10.1.
- Overmodulation introduces envelope distortion. Hence it should be avoided.

Ex. 4.10.1 : Draw the AM waveforms for less than 100 %, with 100 %, more than 100 % and with 0 % percentage modulation. Assume that the modulating signal is a pure sine wave.

Soln. :

The required waveforms are shown in Figs. P. 4.10.1 (a), (b), (c) and (d) respectively.

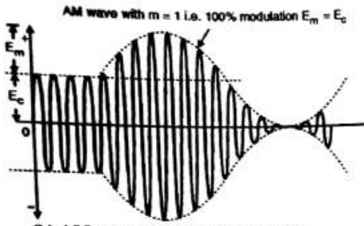


(D-41) **Fig. P. 4.10.1(a) : AM wave for percentage modulation less than 100 %**

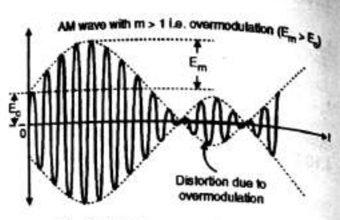
Comparison of waveforms :

Some important conclusions drawn from the waveforms of the previous example are :

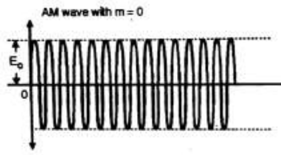
1. The envelope of the AM wave has exactly the same shape are one-to-one correspondence with the message signal if the percentage modulation is less than or equal to 100 %.
2. No such correspondence is observed when the percentage modulation is greater than 100 %. Therefore envelope distortion takes place and the AM wave is said to be overmodulated.



(b) AM wave with 100% modulation



(c) AM with over modulation



(d) AM wave with m = 0

(D-42) **Fig. P. 4.10.1**

Why should the shape of envelope be same as that of modulating signal ?

- The shape of the AM wave envelope should be same as that of the modulating signal because this reduces the complexity of the demodulator circuit to a great extent.

How to achieve this ?

- This can be achieved if the following conditions are satisfied :
  1. The percentage modulation should be less than 100 % to avoid the envelope distortion.
  2. The bandwidth "W" of the modulating signal should be small as compared to f_c (carrier frequency). This will help to visualize the envelope a(t) satisfactorily.

4.11 AM Power Distribution :

MU : Dec. 04, May 11

University Questions

Q. 1 Explain the power relations for AM. **(Dec. 04, 3 Marks)**

Q. 2 Derive expression of total transmitted power of AM wave. **(May 11, 3 Marks)**

- In practice, the AM wave is a voltage or current wave.
- An AM wave consists of carrier and two sidebands. Hence the AM wave will contain more power than the power contained by an unmodulated carrier.
- The amplitudes of the two sidebands are dependent on the modulation index "m". Hence the power contained in the sidebands depends on the value of m. Hence the total power in an AM wave is a function of the value of modulation index m.

4.11.1 The Total Power in AM :

MU : Dec. 15

University Questions

Q. 1 Explain amplitude modulation for more than one modulating signal in the following cases :
1. Mathematical equation
2. AM waveform
3. AM amplitude and power spectrum
4. Modulation coefficient
5. Transmission power

(Dec. 15, 10 Marks)

The total power in an AM wave is given by,

$$P_t = [\text{Carrier Power}] + [\text{Power in USB}] + [\text{Power in LSB}] \quad ...(4.11.1)$$

$$\therefore P_t = \frac{E_{carr}^2}{R} + \frac{E_{USB}^2}{R} + \frac{E_{LSB}^2}{R} \quad ...(4.11.2)$$

Where E_{carr}, E_{USB} and E_{LSB} are the rms values of the carrier and sideband amplitudes respectively and R is the characteristic resistance of antenna in which the total power is dissipated. The total transmitted power in AM is thus the rms power.

4.11.2 Carrier Power (P_c) :

The carrier power is given by,

$$P_c = \frac{E_{carr}^2}{R}$$

$$\text{But } E_{carr} = \text{Rms value of carrier} = \frac{E_c}{\sqrt{2}}$$

$$\therefore P_c = \frac{[E_c/\sqrt{2}]^2}{R} = \frac{E_c^2}{2R} \quad ...(4.11.3)$$

where E_c = Peak carrier amplitude

As E_c is constant, the carrier power P_c also will be constant. It does not depend on the modulation index m.

4.11.3 Power in the Sidebands :

The power in each of the two sidebands is given as,

$$P_{USB} = P_{LSB} = \frac{E_{LSB}^2}{R}$$

Where E_{LSB} = Rms value of sideband magnitude.

As we know the peak amplitude of each sideband is $\frac{m E_c}{2}$.

$$\therefore P_{USB} = P_{LSB} = \frac{[m E_c/2\sqrt{2}]^2}{R}$$

$$= \frac{m^2 E_c^2}{8R} \quad ...(4.11.4)$$

$$\therefore P_{USB} = P_{LSB} = \frac{m^2}{4} \times \frac{E_c^2}{2R}$$

In the above equation $\frac{E_c^2}{2R} = P_c$

$$\text{Hence } P_{USB} = P_{LSB} = \frac{m^2}{4} P_c \quad ...(4.11.5)$$

Thus the power contained by the sidebands is directly proportional to the square of modulation index (m).

4.11.4 Total Power (P_t) :

The total power is given by,

$$P_t = P_c + P_{USB} + P_{LSB}$$

Substituting Equation (4.11.5) into this equation we get,

$$P_t = P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c$$

$$\therefore P_t = \left[1 + \frac{m^2}{2}\right] P_c \quad ...(4.11.6)$$

$$\text{or } \frac{P_t}{P_c} = 1 + \frac{m^2}{2} \quad ...(4.11.7)$$

Equation (4.11.7) tells us about the relation between the total power of AM wave and the power contents of an unmodulated carrier. With increase in the value of "m", total power also increases. P_t will be maximum for m = 1 and it will be 1.5 P_c.

4.11.5 Modulation Index in Terms of P_t and P_c :

Consider the Equation (4.11.7),

$$\frac{P_t}{P_c} = 1 + \frac{m^2}{2}$$

$$\therefore m^2 = 2\left[\frac{P_t}{P_c} - 1\right]$$

$$\therefore m = \left[2\left(\frac{P_t}{P_c} - 1\right)\right]^{1/2} \quad ...(4.11.8)$$

4.11.6 Transmission Efficiency (η) :

- Transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e. the total sideband power) to the total transmitted power.
- Only the sidebands contain all the information. A carrier does not contain any information.

$$\therefore \eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\left[\frac{m^2}{4} P_c + \frac{m^2}{4} P_c\right]}{\left[1 + \frac{m^2}{2}\right] P_c}$$

$$\therefore \eta = \frac{m^2/2}{1 + \frac{m^2}{2}}$$

$$\therefore \eta = \frac{m^2}{2 + m^2} \quad ...(4.11.9)$$

- The percent transmission efficiency is given by,

$$\eta = \frac{m^2}{2 + m^2} \times 100\,\% \quad ...(4.11.10)$$

Why should "m" be as high as possible ?

- The higher percentage of modulation is preferred for strong and more intelligible received signal, because higher percentage of modulation means higher value of modulation index m. With increased value of m, the transmitted power P_t by the AM transmitter will increase as,

$$P_t = P_c\left(1 + \frac{m^2}{2}\right)$$

- Thus it is ensured that the received signal will be strong by having a higher percentage of modulation. Moreover a strong signal is less likely to get contaminated or lost in the noise, hence becomes more intelligible.

1.	Equation for an AM wave	:	$e_{AM} = E_c (1 + m \cos \omega_m t) \cos \omega_c t$
2.	Modulation index	:	$m = \frac{E_m}{E_c}$
3.	Amplitude of each sideband	:	$m E_c/2$
4.	Frequency of sidebands	:	$f_{USB} = (f_c + f_m), f_{LSB} = (f_c - f_m)$
5.	Bandwidth of AM wave	:	$BW = 2 f_m$
6.	Carrier power	:	$P_c = E_c^2 / 2R$
7.	Power in each sideband	:	$P_{USB} = P_{LSB} = m^2 P_c/4$
8.	Total power in AM	:	$P_t = P_c + P_{USB} + P_{LSB}$
9.	Total power in AM	:	$P_t = \left[1 + \frac{m^2}{2}\right] P_c$
10.	Modulation index	:	$m = \left[2\left(\frac{P_t}{P_c} - 1\right)\right]^{1/2}$
11.	Transmission efficiency	:	$\eta = \frac{m^2}{2 + m^2}$

Ex. 4.11.1 : Derive the efficiency η of ordinary AM and show that for a single tone AM, η_{max} = 33.3 % at $\mu = 1$.

Soln. : Transmission efficiency is given by

$$\eta = [m^2 / (2 + m^2)] \times 100\,\%$$

$$\text{At } m = 1 \text{ we get}$$

$$\eta_{max} = [1 / (2 + 1)] \times 100\,\% = 33.33\,\% \quad \text{...Ans.}$$

4.11.7 AM Current Calculations :

- The total power P_t of an AM wave and the carrier power P_c can be expressed in terms of currents. Assume I_c to be the rms current corresponding to the unmodulated carrier and I_t to be the rms current for AM wave.
- Then if these currents flow in the characteristic impedance R of an antenna, we can write,

$$P_c = I_c^2 R$$

$$\text{and} \quad P_t = I_t^2 R$$

$$\therefore \quad \frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \times \frac{R}{R} = \left[\frac{I_t}{I_c}\right]^2 \quad \text{...(4.11.11)}$$

$$\text{But} \quad \frac{P_t}{P_c} = \left[1 + \frac{m^2}{2}\right]$$

$$\therefore \left[\frac{I_t}{I_c}\right]^2 = 1 + \frac{m^2}{2} \quad \text{...(4.11.12)}$$

$$\therefore \quad I_t = I_c\left[1 + \frac{m^2}{2}\right]^{1/2} \quad \text{...(4.11.13)}$$

4.11.8 Modulation Index in Terms of Currents :

From Equation (4.11.12),

$$1 + \frac{m^2}{2} = \left[\frac{I_t}{I_c}\right]^2$$

$$\therefore \quad m = \left[2\left(\left(\frac{I_t}{I_c}\right)^2 - 1\right)\right]^{1/2} \quad \text{...(4.11.14)}$$

Ex. 4.11.2 : If the rms value of the aerial current before modulation is 12.5 A and during modulation is 14 A. Calculate the percentage of modulation employed, assuming no distortion.

Soln. :

Given : $I_c = 12.5$ A, $I_t = 14$ A

To find : Percentage modulation

We know that,

$$\left[\frac{I_t}{I_c}\right]^2 = 1 + \frac{m^2}{2}$$

$$\therefore \left[\frac{14}{12.5}\right]^2 = 1 + \frac{m^2}{2}$$

$$\therefore \quad m = 0.7133$$

$$\therefore \quad \text{Percentage modulation} = 71.33\ \%$$ ...Ans.

Ex. 4.11.3 : A broadcast radio transmitter radiates 5 kW power when the modulation percentage is 60%. How much is the carrier power ?

Soln. :

Given : $P_t = 5$ kW, $m = 0.6$

To find : Carrier power P_c

$$P_t = \left[1 + \frac{m^2}{2}\right] P_c$$

$$\therefore \quad 5 \times 10^3 = \left[1 + \frac{(0.6)^2}{2}\right] P_c$$

$$\therefore \quad P_c = 4.237 \times 10^3 \text{ W or } 4.237 \text{ kW}$$ ...Ans.

Ex. 4.11.4 : The AM transmitter developes an unmodulated power output of 400 Watts. across a 50 Ω resistive load. The carrier is modulated by a sinusoidal signal with a modulation index of 0.8. Assuming $f_m = 5$ kHz and $f_c = 1$ MHz.

1. Obtain the value of carrier amplitude V_c and hence write the expression for AM signal.
2. Find the total average power of the modulator output.
3. Find the power efficiency of the modulator.

Soln. :

$P_c = 400$ W, $R_L = 50\ \Omega$, $m = 0.8$, $f_m = 5$ kHz, $f_c = 1$ MHz.

1. **Carrier amplitude V_c :**

$$\text{The carrier power } P_c = \frac{V_c^2}{2 R_L}$$

$$\therefore \quad V_c = \sqrt{2 R_L P_c} = \sqrt{2 \times 50 \times 400} = 200 \text{ Volts}$$

Thus peak carrier voltage is 200 V.

2. **Expression for AM wave :**

$$e_{AM} = E_c [1 + m \cos (2 \pi f_m t)] \cos (2 \pi f_c t)$$

Substituting $E_c = 200$ V, $m = 0.8$, $f_m = 5$ kHz, $f_c = 1$ MHz we get,

$$e_{AM} = 200 [1 + 0.8 \cos (2 \pi \times 5 \times 10^3 t)] \cos (2 \pi \times 1 \times 10^6 t)$$

$$\therefore e_{AM} = 200 [1 + 0.8 \cos (10^4 \pi t)] \cos (2\pi \times 10^6 t)$$ ...Ans.

3. **Total average output modulator power :**

$$\text{Modulator output} = \text{Total sideband power} = \frac{m^2}{2} \times P_c$$

$$= \frac{(0.8)^2}{2} \times 400 = 128 \text{ W}$$ ...Ans.

Ex. 4.11.5 : Prove that in AM, maximum average power transmitted by an antenna is 1.5 times the carrier power.

Soln. :

Let P_c = Unmodulated carrier power

P_t = Transmitted AM power,

m = Modulation index

In section 4.11.4 we have already proved that,

$$P_t = \left[1 + \frac{m^2}{2}\right] P_c \quad \text{...(1)}$$

The maximum value of "m" without introducing distortion in the modulated wave is m = 1. Substitute this value in Equation (1) to get,

$$P_t (\text{max}) = \left[1 + \frac{1}{2}\right] P_c = 1.5\, P_c$$ ...Proved.

Ex. 4.11.6 : A sinusoidal carrier has an amplitude of 10 V and a frequency of 100 kHz. It is amplitude modulated by a sinusoidal voltage of amplitude 3 V and frequency 500 Hz. Modulated voltage is developed across a 75 Ω resistance.

1. Write the equation for the modulated wave
2. Determine the modulation index.
3. Draw the spectrum of modulated wave.
4. Calculate the total average power.
5. Calculate the power carried by sidebands.

Dec. 03, 12 Marks

Soln. :

Given : $E_c = 10V$, $f_c = 100$ kHz, $E_m = 3V$, $f_m = 500$ Hz, $R = 75\ \Omega$

1. Modulation index :

$$m = \frac{E_m}{E_c} = \frac{3}{10} = 0.3$$

2. Equation of modulated wave :

$$e_{AM} = E_c (1 + m \cos \omega_m t) \cos 2\pi f_c t = 10 [1 + 0.3 \cos (2\pi \times 500\, t)] \cos (2\pi \times 100 \times 10^3)$$

$$e_{AM} = 10 [1 + 0.3 \cos 1000\, \pi t] \cos (2\pi \times 10^5 t)$$ ...Ans.

3. Spectrum of AM wave :

Various spectral components are as follows :

1. Carrier : $f_c = 100$ kHz
2. USB : $(f_c + f_m) = 100.05$ kHz
3. LSB : $(f_c - f_m) = 99.95$ kHz

Amplitudes of these spectral components are as follows :

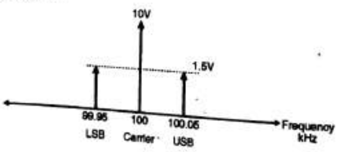
1. Carrier : $E_c = 10V$
2. LSB and USB : $mE_c / 2 = \frac{0.3 \times 10}{2} = 1.5V$

Spectrum is as shown in Fig. P. 4.11.6.

4. Total average power :

$$P_t = \left(1 + \frac{m^2}{2}\right) P_c = \left(1 + \frac{m^2}{2}\right) \frac{E_c^2}{2R} = \left(1 + \frac{(0.3)^2}{2}\right) \frac{(10)^2}{2 \times 75}$$

$$= 0.6967\ W$$ ...Ans.



(D-1300) Fig. P. 4.11.6 : Spectrum of AM wave

5. Total sideband power :

$$P_{SB} = \frac{m^2}{2} \times P_c = \frac{(0.3)^2}{2} \times \frac{(10)^2}{2 \times 75} = 0.03\ W$$ ...Ans.

Ex. 4.11.7 : An AM transmitter supplies 10 kW of carrier power to a 50 Ω load. It operates at a carrier frequency of 1.2 MHz and is 80 % modulated by a 3 kHz sinewave.

1. Sketch the signal in frequency domain with frequency and power scales. Show the power in dBW.
2. Calculate the total average power in the signal in watts and dBW.
3. Calculate the RMS voltage of the signal.
4. Calculate the peak voltage of the signal.

May 03, 8 Marks

Soln. :

Given : $P_c = 10$ kW, $R_L = 50\ \Omega$, $f_c = 1.2$ MHz, $m = 0.8$, $f_m = 3$ kHz.

Frequency domain display of the AM signal :

1. Carrier power = 10 kW

Convert it into dBW by considering 1W as reference power.

$$\therefore\ P_c \text{ in dBW} = 10 \log_{10}\left[\frac{10\text{ kW}}{1W}\right]$$

$$\therefore\ P_c = 10 \log_{10}\left[\frac{10 \times 10^3\ W}{1W}\right] = 40\text{ dBW}$$

2. Power contained in each sideband, $P_{USB} = P_{LSB} = \frac{m^2}{4} P_c = \frac{(0.8)^2}{4} \times 10kW = 1.6\ kW$

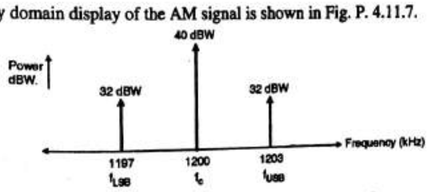
Convert into dBW.

$$\therefore\ P_{USB(dBW)} = P_{LSB(dBW)} = 10 \log_{10}\left[\frac{1.6 \times 10^3}{1}\right] = 32\text{ dBW}$$

3. $f_{USB} = 1.2\text{ MHz} + 3\text{ kHz} = 1203$ kHz

$f_{LSB} = 1.2\text{ MHz} - 3\text{ kHz} = 1197$ kHz and $f_c = 1200$ kHz

The frequency domain display of the AM signal is shown in Fig. P. 4.11.7.



(D-1086) Fig. P. 4.11.7 : Frequency domain display

Total average power :

1. Total average power in watt is given by,

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$$P_t = P_c\left[1+\frac{m^2}{2}\right] = 10\times10^3\left[1+\frac{(0.8)^2}{2}\right] = 13.2\times10^3 \text{ Watts}$$ ...Ans.

2. Total average power in dBW is given by,

$$P_{t(dBW)} = 10\log_{10}P_t = 10\log_{10}(13.2\times10^3) = 41.2 \text{ dBW}$$ ...Ans.

RMS value of AM signal :

$$P_t = (V_{rms})^2/R_L$$

$$\therefore \quad V_{rms} = \sqrt{P_t\times R_L} = \sqrt{13.2\times10^3\times50} = 812.4 \text{ Volts}$$ ...Ans.

Peak value of the AM signal :

1. $$P_c = \frac{E_c^2}{2R_L}$$

$$\therefore \quad 10\times10^3\times2\times50 = E_c^2$$

$$\therefore \quad E_c = 1000 \text{ Volts}$$ ... This is the peak carrier voltage.

2. $$m = \frac{E_m}{E_c},$$

$$\therefore \quad E_m = m\,E_c = 0.8\times1000 = 800 \text{ Volts.}$$ ...Ans.

3. Hence peak value of AM signal is given by,

$$E_{max} = E_c + E_m = 1000 + 800 = 1800 \text{ Volts}$$ ...Ans.

Ex. 4.11.8 : An AM signal appears across a 50 Ω load and has the following equation :
$v(t) = 12(1+\sin 12.566\times10^3 t)\sin 18.85\times10^6 t$ volts.
1. Sketch the envelope of this signal in time domain.
2. Calculate modulation index, sideband frequencies, total power and bandwidth.

Dec. 04, May 10, 10 Marks

Soln. :

Given : $R = 50\,\Omega$, $v(t) = 12(1+\sin 12.566\times10^3 t)\sin 18.85\times10^6 t$ Volts.

Step 1 : Compare the given expression with standard AM :

A standard AM signal is given by,

$$e_{AM} = E_c[1+m\sin\omega_m t]\sin\omega_c t$$

Comparing this with the given expression we get

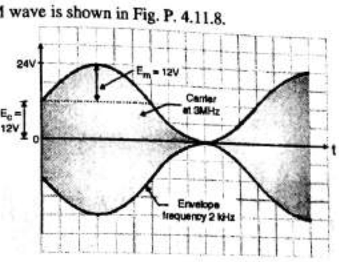
$E_c = 12$ V, $m = 1$ so $E_m = E_c = 12$ V

$$f_c = \frac{18.85\times10^6}{2\pi} \approx 3 \text{ MHz.}$$

$$f_m = \frac{12.566\times10^3}{2\pi} \approx 2 \text{ kHz.}$$

Step 2 : Sketch the envelope :

The envelope of AM wave is shown in Fig. P. 4.11.8.



(D-1268) **Fig. P. 4.11.8 : Envelope of AM signal in the time domain**

Step 3 : m, f_{USB}, f_{LSB}, P_t and BW :

- We have already calculated m = 1
- $f_{USB} = f_c + f_m = 3002$ kHz
- $f_{LSB} = f_c - f_m = 2998$ kHz
- $P_t = P_c\left[1+\frac{m^2}{2}\right] = P_c\left[1+\frac{1}{2}\right] = 1.5\,P_c$

$$\therefore \quad P_t = \frac{1.5\left[\frac{E_c}{\sqrt{2}}\right]^2}{R} = \frac{1.5\times(12)^2}{2\times50} = 2.16 \text{ W}$$

- $BW = 2f_m = 4$ kHz.

Ex. 4.11.9 : A sinusoidal carrier has amplitude of 10 V and frequency 30 kHz. It is amplitude modulated by a sinusoidal voltage of amplitude 3V and frequency 1 kHz. Modulated voltage is developed across a 50 Ω resistance.
1. Write the equation for modulated wave.
2. Plot the modulated wave showing maxima and minima of waveform.
3. Determine the modulation index.
4. Draw the spectrum of modulated wave.
5. Calculate the total average power.
6. Calculate the power carried by the sidebands.

May 02, 12 Marks

Soln. :

Given : $E_c = 10$ V, $E_m = 3$ V, $f_c = 30$ kHz, $f_m = 1$ kHz, $R_L = 50\,\Omega$

1. **Modulation index,** $$m = \frac{E_m}{E_c} = \frac{3}{10} = 0.3$$ ...Ans.

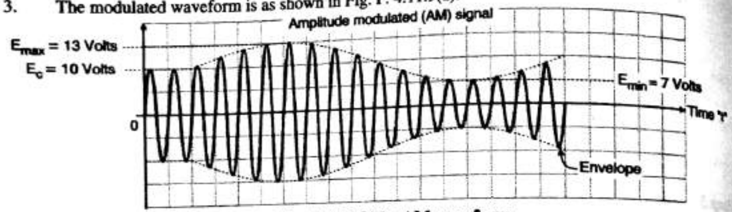
2. Equation for modulated wave :

$$s(t) = E_c(1 + m \cdot \cos \omega_m t)\cos \omega_c t$$

$$= 10\,[1 + 0.3\cos(2\pi \times 1 \times 10^3 t)]\cos(2\pi \times 30 \times 10^3 t)$$

$$\therefore \quad s(t) = 10\,[1 + 0.3\cos(2\pi \times 10^3 t)]\cos(6\pi \times 10^4 t) \qquad \text{...Ans.}$$

3. The modulated waveform is as shown in Fig. P. 4.11.9(a).



(D-47) **Fig. P. 4.11.9(a) : AM waveform**

4. Spectrum of modulated wave :

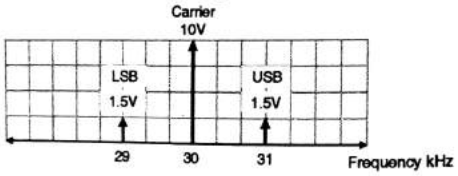
The sideband frequencies are :

$$f_{USB} = f_c + f_m = 30 + 1 = 31 \text{ kHz}$$

$$f_{LSB} = f_c - f_m = 30 - 1 = 29 \text{ kHz}$$

$$\text{Amplitude of each sideband} = \frac{m}{2} \times E_c = \frac{0.3 \times 10}{2} = 1.5 \text{ Volts}$$

The spectrum of AM wave is as shown in Fig. P. 4.11.9(b).



(D-48) **Fig. P. 4.11.9(b)**

5. Total average power of AM wave :

$$P_t = P_c\left(1 + \frac{m^2}{2}\right) = \frac{E_c^2}{2R_L}\left(1 + \frac{m^2}{2}\right) = \frac{(10)^2}{2 \times 50}\left[1 + \frac{(0.3)^2}{2}\right] = 1.045 \text{ W}$$

6. Total sideband power :

$$P_{SB} = \frac{m^2}{2} \times P_c = \frac{(0.3)^2}{2} \times \frac{(10)^2}{2 \times 50} = 0.045 \text{ W} \qquad \text{...Ans.}$$

Ex. 4.11.10 : **The AM transmitter develops an unmodulated power output of 400 watts across a 50 Ω resistive load. The carrier is modulated by a sinusoidal signal with a modulation index of 0.8. Assuming f_m = 5 kHz and f_c = 1 MHz.**

1. **Obtain the value of carrier amplitude V_c and hence write the expression for AM signal.**
2. **Find the total average power of the modulator output.**
3. **Find the power efficiency of the modulator.**

Dec. 06. 10 M...

Soln. :

P_c = 400 W, R_L = 50Ω, m = 0.8, f_m = 5 kHz, f_c = 1 MHz.

1. Carrier amplitude V_c :

$$\text{The carrier power } P_c = \frac{V_c^2}{2R_L}$$

$$\therefore \quad V_c^2 = 2R_L P_c = 2 \times 50 \times 400 = 200 \text{ Volts}$$

Thus peak carrier voltage is 200 V.

2. Expression for AM wave :

$$e_{AM} = E_c\,[1 + m\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

Substituting E_c = 200 V, m = 0.8, f_m = 5 kHz, f_c = 1 MHz we get,

$$e_{AM} = 200\,[1 + 0.8\cos(2\pi \times 5 \times 10^3 t)]\cos(2\pi \times 1 \times 10^6 t)$$

$$\therefore \quad e_{AM} = 200\,[1 + 0.8\cos(10^4 \pi t)]\cos(2\pi \times 10^6 t) \qquad \text{...Ans.}$$

3. Total average output modulator power :

$$\text{Modulator output} = \text{Total sideband power}$$

$$= \frac{m^2}{2} \times P_c = \frac{(0.8)^2}{2} \times 400 = 128 \text{ W} \qquad \text{...Ans.}$$

Ex. 4.11.11 : **The antenna current of an AM broadcast transmitter, modulated to a depth of 40% by an audio sine wave is 11 A. It increases to 12 A as result of simultaneous modulation by another audio sine wave. What is the modulation index due to this second wave?**

May 14. 10 Marks

Soln. :

Given : I_{t1} = 11 A, m_1 = 0.4, I_{t2} = 12 A

To find : Modulation index due to second wave.

Step 1 : Find I_c :

$$\left[\frac{I_{t1}}{I_c}\right]^2 = 1 + \frac{m_1^2}{2}$$

$$\therefore \quad I_c = \frac{I_{t1}}{\left[1 + \frac{m_1^2}{2}\right]^{1/2}} = \frac{11}{\left[1 + \frac{(0.4)^2}{2}\right]^{1/2}} = 10.58 \text{ A}$$

Step 2 : Find m_t :

After modulating with the second signal,

$$I_{t2}^2 = I_c^2\left[1+\frac{m_t^2}{2}\right]$$

$$m_t = \sqrt{2\left[\left(\frac{I_{t2}}{I_c}\right)^2 - 1\right]} = \sqrt{2\left[\left(\frac{12}{10.58}\right)^2 - 1\right]}$$

$$\therefore m_t = 0.756$$

Step 3 : Find m_2 :

$$m_t = \left[m_1^2 + m_2^2\right]^{1/2}$$

$$\therefore \quad m_2 = \left[m_t^2 - m_1^2\right]^{1/2} = [(0.756)^2 - (0.4)^2]^{1/2}$$

$$\therefore \quad m_2 = 0.6415 = 64.15\%$$

...Ans.

Ex. 4.11.12 : When a broadcast AM transmitter is 50% modulated, its antenna current is 12 A. What will be the current, when the modulation depth is increased to 0.9 ? **Dec. 16, 5 Marks**

Soln. :

Step 1 : Find I_C :

Given : $I_{t1} = 12$ A, $m_1 = 0.5$

$$I_{t1} = I_c\left[1+\frac{m_1^2}{2}\right]^{1/2}$$

$$\therefore 12 = I_c\left[1+\frac{(0.5)^2}{2}\right]^{1/2}$$

$$\therefore I_c = 11.32 \text{ Amp} \quad ...(1)$$

Step 2 : Find I_{t2} :

Given : $m_2 = 0.9$

$$I_{t2} = I_c\left[1+\frac{m_2^2}{2}\right]^{1/2} = 11.32\left[1+\frac{(0.9)^2}{2}\right]^{1/2}$$

$$\therefore I_{t2} = 13.42 \text{ Amp}$$

...Ans.

4.11.9 Modulation by a Complex Modulating Signal :

MU : May 14

University Questions

Q. 1 Explain amplitude modulation for more than one modulating signal in following cases :

1. Mathematical equation
2. AM waveform
3. Am amplitude and power spectrum
4. Modulation coefficient
5. Transmission power

(May 14, 10 Marks)

- Uptill now we have assumed that only one sinusoidal modulating signal is present. But in practice more than one modulating signals will be present. This is due to the fact that a practical modulating signal is a non-sinusoidal complex signal which consists of a mixture of many sinusoidal modulating signal in different proportions.
- Let us see first how to express the AM wave when more than one modulating signals are simultaneously used.

1. Let us assume that there are two modulating signals.

$$x_1(t) = E_{m1}\cos\omega_{m1} t$$

$$\text{and} \quad x_2(t) = E_{m2}\cos\omega_{m2} t$$

The total modulating signal will be the sum of these two in the time domain.

$$\therefore \text{Total modulating signal} = x_1(t) + x_2(t) = E_{m1}\cos\omega_{m1} t + E_{m2}\cos\omega_{m2} t$$

- Let the carrier be $c(t) = E_c \cos\omega_c t$
- The instantaneous value of the envelope of AM wave is,

$$A = E_c + x_1(t) + x_2(t) = E_c + E_{m1}\cos\omega_m t + E_{m2}\cos\omega_{m2} t$$

- Therefore the AM wave is given by,

$$e_{AM} = A\cos\omega_c t$$

- It can be proved that the expression for the DSBFC AM wave is given by,

$$e_{AM} = E_c\cos\omega_c t + \frac{m_1 E_c}{2}\cos(\omega_c + \omega_{m1})t + \frac{m_1 E_c}{2}\cos(\omega_c - \omega_{m1})t$$

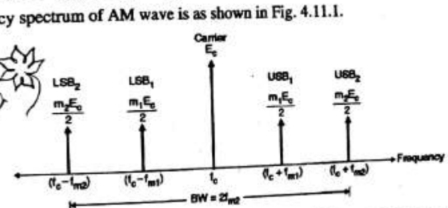
$$+ \frac{m_2 E_c}{2}\cos(\omega_c + \omega_{m2})t + \frac{m_2 E_c}{2}\cos(\omega_c - \omega_{m2})t \quad ...(4.11.15)$$

- Where m_1 and m_2 are the modulation indices corresponding to the two modulating signals $x_1(t)$ and $x_2(t)$ respectively

4. Equation (4.11.15) shows that in the AM wave along with carrier there are four sideband components.

There are two USB components at frequencies $(f_c + f_{m1})$ and $(f_c + f_{m2})$ and two LSB components at frequencies $(f_c - f_{m1})$ and $(f_c - f_{m2})$.

The frequency spectrum of AM wave is as shown in Fig. 4.11.1.



(D-66) **Fig. 4.11.1 : Frequency spectrum of AM wave with two modulating signals**

Note : Thus for every modulating signal two sidebands are produced. The amplitude of the sidebands is proportional to the corresponding modulation index.

Total Power (P_t) :

The total power in AM wave with a complex modulating signal is given by,

$$P_t = P_c\left[1+\frac{m_1^2}{2}+\frac{m_2^2}{2}\cdots\cdots+\frac{m_n^2}{2}\right]$$

Effective modulation index (m_t) :

In general the total modulation index due to the simultaneous modulation by "n" modulating signals is given by,

$$m_t = [\, m_1^2 + m_2^2 + \ldots .. m_n^2 \,]^{1/2} \qquad \ldots(4.11.16)$$

Ex. 4.11.13 : A complex modulating waveform consisting of a sine-wave of amplitude 3 V peak and frequency 1kHz and a cosine wave of amplitude 5 V peak and frequency 3 kHz amplitude modulates a 500 kHz and 10 V peak carrier voltage. Determine and plot the spectrum of the modulated wave and determine the average power when the modulated wave is fed into 50 Ω load.

Soln. :

Given : $E_{m1} = 3V$, $E_{m2} = 5V$, $f_{m1} = 1$ kHz, $f_{m2} = 3$ kHz

$E_c = 10V$, $f_c = 500$ kHz, $R_L = 50\,\Omega$

To find :
1. Spectrum of AM wave.
2. Average power of AM wave.

Step 1 : Calculate modulation index :

- Modulation index by the first modulating wave is,

$$m_1 = E_{m1} / E_c = 3/10 = 0.3$$

- Modulation index by the second modulating wave is,

$$m_2 = E_{m2} / E_c = 5/10 = 0.5$$

- Net modulation index $m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{(0.3)^2 + (0.5)^2} = 0.583$

Step 2 : Spectrum of AM wave :

The frequency components in the AM wave are,

$f_c = 500$ kHz

$$\text{Lower sidebands} : (f_c - f_{m1}) = 500 - 1 = 499 \text{ kHz}$$
$$(f_c - f_{m2}) = 500 - 3 = 497 \text{ kHz}$$
$$\text{Upper sidebands} : (f_c + f_{m1}) = 501 \text{ kHz}$$
$$(f_c + f_{m2}) = 503 \text{ kHz}$$

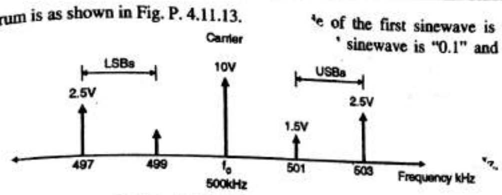
The amplitude of various frequency components are as follows.

Table P. 4.11.13

Sr. No.	Frequency component	Amplitude
1.	f_c	$E_c = 10V$
2.	$(f_c + f_{m1})$ and $(f_c - f_{m1})$	$m_1 E_c / 2 = 0.3 \times 10 / 2 = 1.5V$
3.	$(f_c + f_{m2})$ and $(f_c - f_{m2})$	$m_2 E_c / 2 = 0.5 \times 10 / 2 = 2.5V$

The spectrum is as shown in Fig. P. 4.11.13.

e of the first sinewave is "0.2" and its sinewave is "0.1" and its frequency



(D-1418) **Fig. P. 4.11.13 : Frequency spectrum of AM**

Step 3 : Power in AM wave :

$$\text{Carrier power } P_c = \frac{(E_c/\sqrt{2})^2}{R_L} = \frac{(10/\sqrt{2})^2}{50} = 1W$$

Total power in AM wave

$$P_t = P_c\left[1 + \frac{m_t^2}{2}\right] = 1\left[1 + \frac{(0.583)^2}{2}\right] = 1.1699 \text{ W} \qquad \text{...Ans.}$$

4.11.10 Bandwidth with a Complex Modulating Signal :

Refer to the spectrum of AM wave shown in Fig. 4.11.1. The bandwidth of this wave is given by,

$$\text{B.W.} = f_{USB2} - f_{LSB2} = f_c + f_{m2} - (f_c - f_{m2})$$
$$= 2 f_{m2} \qquad \ldots(4.11.17)$$

Hence the bandwidth of an AM system is equal to twice the maximum modulating frequency is present in the modulating signal.

Ex. 4.11.14 : A transmitter radiates 10 kW power with the carrier unmodulated and 10.5 kW when the carrier is modulated by one sinusoidal signal. Calculate the modulation index. If another modulating signal corresponding to 30 % modulation is transmitted simultaneously determine the total radiated power.

Soln. :

1. It is given that $P_c = 10$ kW and $P_t = 10.5$ kW and $m_2 = 0.3$

$$\therefore \frac{P_t}{P_c} = 1 + \frac{m_1^2}{2}, \qquad \therefore m_1 = 0.32$$

Thus modulation index corresponding to the first modulating signal is 0.32.

2. Total modulation index $m_t = [\, m_1^2 + m_2^2 \,]^{1/2} = [\,(0.32)^2 + (0.3)^2\,]^{1/2} = 0.44$

3. Total radiated power $P_t = P_c\left[1 + \frac{m_t^2}{2}\right] = 10\left[1 + \frac{(0.44)^2}{2}\right] = 10.968$ kW ...Ans.

Ex. 4.11.15 : The antenna current of an AM transmitter is 10 Amp when it is modulated to a depth of 30 % by an audio signal. It increases to 11 Amp when another signal modulates the carrier. What is the modulation index due to second wave ?

Soln. :

1. It is given that $I_{t1} = 10$ Amp, $m_1 = 0.3$, $I_{t2} = 11$ Amp

$$\left[\frac{I_{t1}}{I_c}\right]^2 = 1 + \frac{m_1^2}{2}$$

$$\therefore \quad I_c = \frac{I_{t1}}{\left[1+\frac{m_1^2}{2}\right]^{1/2}} = \frac{10}{\left[1+\frac{(0.3)^2}{2}\right]^{1/2}} = 9.78 \text{ Amp} \qquad ...(1)$$

2. After modulating with the second signal,

$$I_{t2}^2 = I_c^2\left[1+\frac{m_t^2}{2}\right]$$

$$m_t = \sqrt{2\left[\left(\frac{I_{t2}}{I_c}\right)^2 - 1\right]} = \sqrt{2\left[\left(\frac{11}{9.78}\right)^2 - 1\right]}$$

$$\therefore \quad m_t = 0.73 \qquad ...\text{Ans.}$$

3. But $m_t = [m_1^2 + m_2^2]^{1/2}$

$$\therefore \quad m_2 = [m_t^2 - m_1^2]^{1/2} = [(0.73)^2 - (0.3)^2]^{1/2} = 0.66 \text{ or } 66\% \qquad ...\text{Ans.}$$

Ex. 4.11.16 : A 400 W carrier is simultaneously modulated by two audio waves with modulation percentage of 50 and 60 respectively. What is the total sideband power radiated ?

Dec. 03, 8 Marks

Soln. :

Given : $P_c = 400$ W, $m_1 = 0.5$, $m_2 = 0.6$

1. **Effective modulation index (m_t) :**

$$m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{(0.5)^2 + (0.6)^2} = 0.78$$

2. **Total sideband power :**

$$P_{SB} = \frac{m_t^2}{2} \times P_c = \frac{(0.78)^2}{2} \times 400 = 122 \text{ Watts} \qquad ...\text{Ans.}$$

Ex. 4.11.17 : A carrier wave $e_c = 4 \sin (2\pi \times 500 \times 10^3 t)$ is amplitude modulated by an audio wave $V_m = 0.2 \sin 3 [(2\pi \times 500t)] + 0.1 \sin 5 [2\pi \times 500t]$. Determine the upper and lower sidebands and sketch the complete spectrum of the modulated wave. Estimate the total power in the sidebands.

May 06, 10 Marks

Soln. :

Given :

- The carrier is, $e_c = 4 \sin (2\pi \times 500 \times 10^3 t)$.

  Comparing this equation with $e_c = E_c \sin (2\pi f_c t)$ we get,

$$E_c = 4\text{V and } f_c = 500 \times 10^3 \text{ Hz}$$

- The modulating signal is,

$$e_m = 0.2 \sin 3 [2\pi \times 500t] + 0.1 \sin 5 [2\pi \times 500t]$$

So it consists of two sinewaves. The peak amplitude of the first sinewave is "0.2" and its frequency is $f_{m1} = 1500$ Hz. The peak amplitude of the second sinewave is "0.1" and its frequency $f_{m2} = 2500$ Hz.

(a) USB and LSB of the AM signal :

- The USB and LSB corresponding to first modulating signal are at,

$$USB_1 = (f_c + f_{m1}) = 500 \text{ kHz} + 1.5 \text{ kHz} = 501.5 \text{ kHz.}$$

$$LSB_1 = (f_c - f_{m1}) = 500 \text{ kHz} - 1.5 \text{ kHz} = 498.5 \text{ kHz.}$$

- The USB and LSB corresponding to the second modulating signal are at,

$$USB_2 = (f_c + f_{m2}) = 500 \text{ kHz} + 2.5 \text{ kHz} = 502.5 \text{ kHz}$$

$$LSB_2 = (f_c - f_{m2}) = 500 \text{ kHz} - 2.5 \text{ kHz} = 497.5 \text{ kHz.}$$

(b) Modulation index of individual modulating signals :

- Modulation index for the first signal, $m_1 = \frac{E_{m1}}{E_c} = \frac{0.2}{4} = 0.05$
- Modulation index for second signal, $m_2 = \frac{E_{m2}}{E_c} = \frac{0.1}{4} = 0.025.$

(c) Sideband amplitudes :

- Amplitudes of USB_1 and LSB_1 will be : $(m_1/2) E_c$

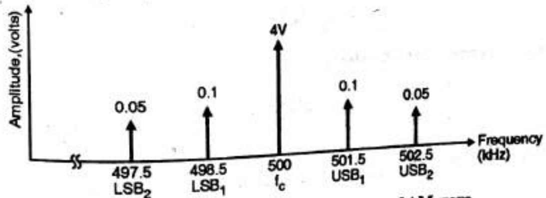
$$= (0.05/2) \times 4 = 0.1 \text{ Volts.}$$

- Amplitudes of USB_2 and LSB_2 will be : $(m_2/2) E_c$

$$= \left(\frac{0.025}{2}\right) \times 4 = 0.05 \text{ Volts}$$

(d) Complete spectrum of AM signal :

- Therefore the complete spectrum of AM wave is as shown in Fig. P. 4.11.17.



(D-71) **Fig. P. 4.11.17 : Complete spectrum of AM wave**

(e) Total power in the sidebands :

As we know, the total power in the sidebands is given by :

$$P_{SB} = (m_t^2/2) P_c = P_{USB} + P_{LSB}$$

However, m_t = Total modulation index = $[m_1^2 + m_2^2]^{1/2}$

$$\therefore \quad m_t = [(0.05)^2 + (0.025)^2]^{1/2}$$

$$\therefore \quad m_t = 0.0559$$

Hence total power in the sidebands $P_{SB} = \left[\frac{(0.0559)^2}{2}\right] P_c$

$$\therefore \quad P_{SB} = 1.56 \times 10^{-3} P_c$$

$$\text{But} \quad P_c = \frac{E_c^2}{2R} = \frac{(4)^2}{2R} = \frac{8}{R}$$

$$\therefore \quad P_{SB} = \frac{0.0125}{R} \text{ Watts.}$$

...Ans.

Ex. 4.11.18 : A 5 kW unmodulated carrier is simultaneously modulated by two audio signals with modulations index 80% and 50%. Find the transmitted power and effective modulation index. Also find the antenna current with and without modulating signal assuming antenna resistance of 50 Ω. **May 07, 10 Marks**

Soln. :

Given : $P_c = 5\text{ kW}, m_1 = 0.8, m_2 = 0.5, R = 50\ \Omega$

Step 1 : Effective modulation index (m_t) :

$$m_t = \left(m_1^2 + m_2^2\right)^{1/2} = [(0.8)^2 + (0.5)^2]^{1/2} = 0.9434$$

...Ans.

Step 2 : Transmitted power :

$$P_t = \left(1 + \frac{m_t^2}{2}\right) P_c = \left[1 + \frac{(0.9434)^2}{2}\right] \times 5\text{ kW} = 7.225\text{ kW}$$

...Ans.

Step 3 : Antenna current without modulating signal :

$$P_c = I_c^2 \times R$$

$$\therefore \quad 5 \times 10^3 = I_c^2 \times 50 \qquad \therefore \quad I_c = 10\text{ A}$$

...Ans.

Step 4 : Antenna current with modulation :

$$I_t = I_c\left[1 + \frac{m_t^2}{2}\right]^{1/2} = 10\left[1 + \frac{(0.9434)^2}{2}\right]^{1/2} = 12\text{ A}$$

...Ans.

4.12 Generation of AM :

- Till now we have studied the principles of AM. In this section let us see the different methods to generate A.M.
- The generating circuits for AM wave are called as amplitude modulator circuits.
- The modulator circuits are classified into two categories :

1. **Low level modulation :**
   - The generation of AM wave takes place at a low power level.

   - The generated AM signal is then amplified using a chain of linear amplifiers such as class A, B or AB amplifiers.
   - The linear amplifiers are required in order to avoid any waveform distortion. The efficiency of low level modulator is low as linear amplifiers are not very efficient.

2. **High level modulation :**
   - In this method, the generation of AM wave takes place at high power levels.
   - The carrier and the modulating signal both are amplified first to an adequate power level and the modulation takes place in the last RF amplifier stage of the transmitter.
   - Highly efficient class C amplifiers are used in high level modulation. Hence the efficiency of high level modulators is higher than that of low level modulation.

4.13 AM Transmitters : MU : Dec. 07

University Questions

Q. 1 Draw the block diagram of AM transmitter and explain each block in brief. **(Dec. 07, 8 Marks)**

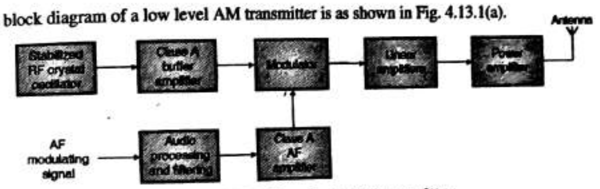
The AM transmitters can be of two types :

1. High level modulated transmitter or
2. Low level modulated transmitter.

Let us see their operation one by one.

4.13.1 Low Level Modulated AM Transmitter :

- The block diagram of a low level AM transmitter is as shown in Fig. 4.13.1(a).



(D-92) **Fig. 4.13.1(a) : Low level AM transmitter**

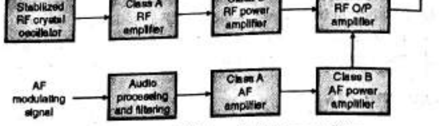
- The RF oscillator produces the carrier signal. The RF oscillator is stabilized in order to maintain the carrier frequency deviation within a prescribed limit. The carrier frequency is equal to the transmitter frequency and it should remain very stable.
- The amplified modulating signal is applied to the modulator along with the carrier. At the output of the modulator we get the AM wave.
- This AM signal is then amplified using a chain of linear amplifiers to raise its power level.
- The linear amplifiers can be class A, AB or B type amplifiers. The linear amplifiers are used in order to avoid the waveform distortion in AM wave. However these amplifiers posses a low efficiency.
- The amplitude modulated signal is then transmitted using transmitting antenna.
- The transistorized modulator circuits can be used for low level modulator due to the low power which is to be handled.
- The low level transmitter does not require a large AF modulator power so its design is simplified.
- However the overall efficiency is much lower compared to high level modulation. This is due to the use of less efficient linear amplifiers.

4.13.2 High Level Modulated AM Transmitter : MU : May 06

University Questions

Q. 1 Write short note on : High level AM transmitter. **(May 06, 5 Marks)**

Many of the AM transmitters use the high level modulation technique. The block diagram of high level AM transmitter is as shown in Fig. 4.13.1(b).



(D-93) **Fig. 4.13.1(b) : High level AM transmitter**

Operation :

- Here the carrier generated by the stabilized crystal oscillator is first amplified to the adequate power level using class C RF power amplifiers.
- The modulating signal also is amplified to a high power level before modulation takes place. If we want 100 % modulation then the power of modulating signal must be 33 % of the total power. So if 1500 W total power is to be transmitted, the modulating power will be 500 W. This highlights the need to amplify the modulating signal to an adequate power level.
- The modulation takes place in the last class C RF amplifier. The modulator output is AM wave which can be directly transmitted.
- The collector modulated transistorized circuit or plate modulated vacuum tube modulator is used as modulator stage.
- The advantage of high level modulation is its high efficiency due to the use of highly efficient class C amplifiers.
- The disadvantage is that a large AF power amplifier is needed to raise the modulating signal to the adequate power level.

4.13.3 Comparison of High Level and Low Level Modulation : MU : May 08

University Questions

Q. 1 Differentiate between low level modulation and high level modulation. **(May 08, 5 Marks)**

Sr. No.	Parameter	High level modulation	Low level modulation
1.	Modulation takes place at	High power level.	Low power level.
2.	Types of amplifiers	Highly efficient class C amplifiers are used.	Linear amplifiers (A, AB or B) are used after modulation.
3.	Efficiency	Very high.	Lower than high level modulators.
4.	Devices used	Vacuum tubes or transistors for medium power applications.	Transistors, JFET, OP-AMPs.
5.	Design of AF power amplifier	Complex due to very high power involved.	Easy as it is to be done at low power.
6.	Applications	High power broadcast transmitters.	Sometimes used in TV transmitters (IF modulation).
7.	Power handling capacity	High.	Low.

4.14 Advantages, Disadvantages and Applications of AM :

4.14.1 Disadvantages of AM (DSBFC) : MU : May 03

University Questions

Q. 1 Amplitude modulation is a waste of power and bandwidth. Justify. **(May 03, 4 Marks)**

The AM signal is also called as "Double Sideband Full Carrier (DSBFC)" signal. The three main disadvantages of this technique are :

1. Power wastage takes place (carrier does not contain any information).
2. AM needs larger bandwidth.
3. AM wave gets affected due to noise.

These are explained as follows :

- The carrier signal in the DSBFC system does not convey any information.
- The information is contained in the two sidebands only. But the sidebands are images of each other and hence both of them contain the same information.
- Thus all the information can be conveyed by only one sideband.

Power wastage due to DSBFC transmission :

- As we know, the total power transmitted by an AM wave is given by :

$$P_t = P_c + P_{USB} + P_{LSB} \quad ...(4.14.1)$$

$$\therefore \quad P_t = P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c \quad ...(4.14.2)$$

- Out of the three terms in Equation (4.14.1), carrier component does not contain any information and one sideband is redundant.
- So out of the total power $P_t = \left[1 + \frac{m^2}{2}\right] P_c$ the wasted power is given by :

$$\text{Power wastage} = P_c + \frac{m^2}{4} P_c = \left[1 + \frac{m^2}{4}\right] P_c \quad ...(4.14.3)$$

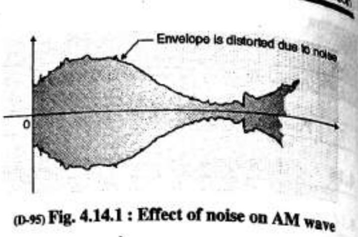
Bandwidth requirement of DSBFC :

- The BW of DSBFC system is $2f_m$. This is due to the simultaneous transmission of both the sidebands, out of which only one is sufficient to convey all the information.
- Thus the BW of DSBFC is "double" than actually required. Therefore DSBFC is a "bandwidth inefficient" system.

Effect of Noise :

- When the AM wave travels from the transmitter to receiver over a communication channel, noise gets added to it.

- The noise will change the amplitude of the envelope of AM in a random manner.
- As the information is contained in the amplitude variations of the AM wave, the noise will contaminate the information contents in the AM.
- Hence the performance of AM is very poor in presence of noise.
- The waveform of AM wave contaminated by noise is shown in Fig. 4.14.1.



(D-95) **Fig. 4.14.1 : Effect of noise on AM wave**

4.14.2 Advantages of AM :

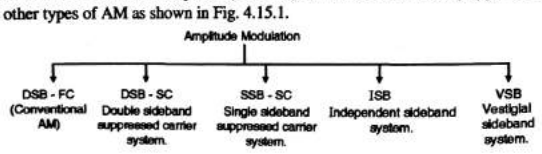
1. AM transmitters are less complex.
2. AM receivers are simple, detection is easy.
3. AM receivers are cost efficient. Hence even a common person can afford to buy it.
4. AM waves can travel a longer distance.
5. Low bandwidth.

4.14.3 Applications of AM :

1. Radio broadcasting. 2. Picture transmission in a TV system.

4.15 Other Types of AM :

- Earlier in this chapter we have learnt about the amplitude modulation AM. It is also called as the **Double side-band full carrier system (DSB-FC).** But this is not the only type of AM. There are some other types of AM as shown in Fig. 4.15.1.



(D-105) **Fig. 4.15.1 : Different types of AM**

- The DSB-FC or conventional AM system has different disadvantages. In order to overcome some of them, the other types of AM have been developed.

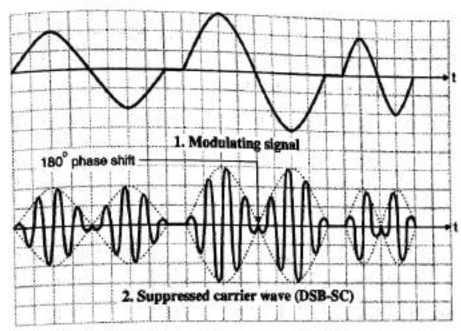
4.15.1 DSB-SC or DSB or AM-SC Signal :

MU : Dec. 14, May 15

University Questions

Q. 1 What is DSBSC wave ? Explain its generation using balanced modulator.

(Dec. 14, May 15, 10 Marks)

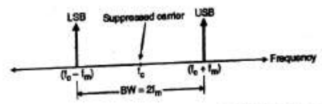


(D-106) **Fig. 4.15.2 : Time domain display of DSB-SC signal**

- From the AM wave (DSB-FC) if only the carrier component is suppressed then the remaining signal is called as DSBSC (double sideband suppressed carrier) signal.
- The time domain display of DSB-SC signal is as shown in Fig. 4.15.2 and the spectrum of this signal is as shown in Fig. 4.15.3.
- If m = 1 then the percent power saving in DSBSC is given by,

$$\% \text{ Power saving } = \frac{[m^2/2]\, P_c \times 100}{\left[1+\frac{m^2}{2}\right] P_c} = \frac{0.5}{1.5} \times 100 \times = 66.667\%$$

- Thus DSB-SC is a power efficient system as compared to DSB-FC.
- However as both the sidebands are being transmitted, the BW of DSB-SC system is still 2 f_m i.e. same as that of AM wave.



(D-107) **Fig. 4.15.3 : Frequency spectrum of DSB-SC signal**

Important points about DSB-SC signal :

1. The spectrum of DSB signal contains only two sidebands but the carrier is not present. Hence some power saving does take place.
2. But the bandwidth of DSB signal is same as that of DSB-FC signal i.e. **BW = 2 f_m**. Hence DSB-SC system is equally bad when it comes to the bandwidth requirement is concerned.

How will you differentiate between DSB-FC and DSB-SC signals ?

- The time domain displays of DSB-FC signal with m = 100 % and DSB-SC signal looks exactly the same.
- The only difference between them is that in the time domain display of DSB-SC (See Fig. 4.15.2) the carrier undergoes 180° phase shift. This is how we can identify the DSB-SC signal.

4.15.2 Power Saving :

- Due to the suppression of carrier, a lot of power saving takes place in DSB-SC. At 100% modulation, m = 1 the percent power saving is given by (P_c / 1.5 P_c) i.e. 66.66%.
- The concept of power saving will become clear after solving the following example.

Ex. 4.15.1 : Calculate the percent power saving for a DSB-SC signal for the percent modulation of (a) 100% and (b) 50%.

Soln. : The total power in AM wave, $P_t = P_c\left[1+\frac{m^2}{2}\right]$ ...(1)

At 100 % depth of modulation m = 1

$\therefore \quad P_t = 1.5\,P_c$

$\therefore \quad$ % power saving $= \left[\frac{P_c}{1.5\,P_c}\right] = 66.66\,\%$ ...Ans.

At 50 % depth of modulation m = 0.5

$\therefore \quad P_t = 1.125\,P_c$

$\therefore \quad$ % power saving $= \frac{P_c}{1.125\,P_c} = 88.88\,\%$ ...Ans.

Ex. 4.15.2 : A DSBSC signal is expressed as $\cos \omega_c t \cdot \cos \omega_m t$ where f_c = 1000 kHz, f_m = 10 kHz. What is the bandwidth of the signal ?

Soln. : We know that

Bandwidth of DSBSC $= 2\,f_m$

B.W. $= 2 \times 10\text{ kHz} = 20\text{ kHz}$ ...Ans.

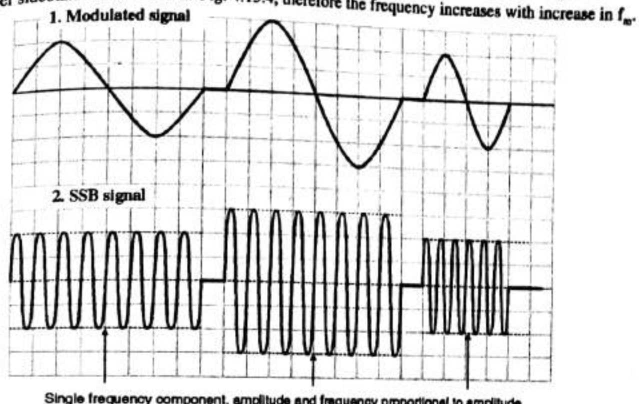
4.15.3 Single Sideband Signal (SSB) :

- We know that the transmission bandwidth of standard AM as well as DSB-SC modulated wave is 2W Hz or 2 f_m Hz i.e. twice the message bandwidth W.
- Hence both these systems are bandwidth inefficient systems.
- In both these systems, one half of the transmission bandwidth is occupied by the upper sideband (USB) and the other half is occupied by the lower sideband (LSB).
- But the most important point to be noted is that the information contained in the USB is exactly identical to that carried by the LSB. So by transmitting both the sidebands we are transmitting the same information twice.
- Hence we can transmit only one sideband (USB or LSB) without any loss of information. And it is possible to suppress the carrier and one sideband completely.
- When only one sideband is transmitted, the modulation is referred to as **single sideband modulation.** It is also called as SSB or SSB-SC modulation.
- The DSB-FC signal is power efficient but bandwidth inefficient. So the next step is to suppress the redundant sideband. The time domain display of SSB is as shown in Fig. 4.15.4.

Observations :

- The important observations from the Fig. 4.15.4 are :

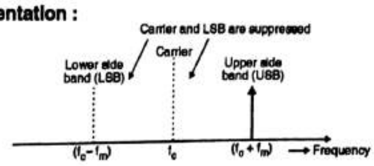
1. The SSB wave is a single frequency wave. Its frequency is equal to the sideband frequency ($f_c + f_m$) or ($f_c - f_m$).
2. The amplitude of the SSB wave is proportional to the amplitude of the modulating signal (f_m).
3. The frequency of the SSB wave varies with the frequency of the modulating signal (f_m). The upper sideband is shown in the Fig. 4.15.4, therefore the frequency increases with increase in f_m.



(D-113) **Fig. 4.15.4 : Time domain display of SSB signal (upper sideband)**

4.15.4 Frequency Domain Representation :

The frequency spectrum of the SSB signal is as shown in Fig. 4.15.5. This shows that the bandwidth of the SSB signal will be "f_m" instead of 2 f_m.



(D-114) **Fig. 4.15.5 : Frequency spectrum of SSB signal**

4.15.5 Advantages of SSB over DSB-FC :

MU : Dec. 14

University Questions

Q. 1 Give advantages of SSB over full carrier DSB amplitude modulated wave. **(Dec. 14, 3 Marks)**

The advantages of SSB over DSB-FC signal are :

1. Less bandwidth requirement as SSB requires a BW of f_m. This will allow more number of signals to be transmitted in the same frequency range.
2. Lots of power saving. This is due to the transmission of only one sideband component. At 100 % modulation, the percent power saving is 83.33 %.

3. Reduced interference of noise. This is due to the reduced bandwidth. As the bandwidth increases, the amount of noise added to the signal will increase.

4.15.6 Disadvantages of SSB :

MU : Dec. 14

University Questions

Q. 1 Give and disadvantages of SSB over full carrier DSB amplitude modulated wave. **(Dec. 14, 5 Marks)**

Even though the SSB system has many advantages it has the following disadvantages :

1. The generation and reception of SSB signal is complicated as discussed later on.
2. The SSB transmitter and receiver need to have an excellent frequency stability. A slight change in frequency will hamper the quality of transmitted and received signal. SSB therefore is not generally used for the transmission of good quality music. It is used for speech transmission.

4.15.7 Transmission Bandwidth of SSB-SC :

- Since we are transmitting the frequencies only in the range $(f_c + f_m)$ or $(f_c - f_m)$, the transmission bandwidth for the SSB-SC will be,

$$\left.\begin{aligned} \text{Bandwidth B} &= (f_c + f_m) - f_c = f_m \text{ Hz} \\ \text{OR} \quad B &= f_c - (f_c - f_m) = f_m \text{ Hz} \end{aligned}\right\} \quad ...(4.15.1)$$

- This is exactly half the bandwidth of the DSB-FC or DSB-SC modulated waves.

4.15.8 Applications of SSB :

SSB transmission is used in the applications where the power saving and low bandwidth requirements are important. The application areas are land and air mobile communication, telemetry, military communications, navigation and amateur radio. Many of these applications are point to point communication applications.

Ex. 4.15.3 : Calculate the percent power saving an SSB signal if the AM wave is modulated to a depth of (a) 100 % and (b) 50 %. **Dec. 04, 3 Marks**

Soln. :

Carrier and one sideband are suppressed. Therefore only one sideband is transmitted.

$$\therefore \quad \%\text{ power saving} = \frac{\text{Power in carrier + Power in one sideband}}{\text{Total power}}$$

$$= \frac{P_c\left[1 + \frac{m^2}{4}\right]}{P_c\left[1 + \frac{m^2}{2}\right]} = \frac{[1 + (m^2/4)]}{[1 + (m^2/2)]} \quad .....(1)$$

At 100 % modulation, $m = 1$

$$\%\text{ power saving} = \frac{1.25}{1.5} = 83.33\ \% \quad ...\text{Ans.}$$

At 50 % modulation, $m = 0.5$

$$\therefore \quad \%\text{ power saving} = \frac{1.0625}{1.125} = 94.44\ \% \quad ...\text{Ans.}$$

Ex. 4.15.4 : An AM signal appear across a 50 Ω load and has the following equation
$v(t) = 10\,(1 + \sin 2\pi \times 10 \times 10^3\, t) \sin 4\pi \times 10^6\, t$

1. Calculate the modulation index, sideband frequencies, total power and bandwidth.
2. Sketch the envelope of SSB signal in time domain. Also draw the spectrum of SSB signal.

Dec. 11, 10 Marks

Soln. :

Given : $R = 50\ \Omega$, $v(t) = 10\,(1 + \sin 2\pi \times 10 \times 10^3\, t) \sin 4\pi \times 10^6\, t$

To find : m, f_{USB}, P_t and BW

Step 1 : Compare the given expression with standard AM :

A standard AM signal is given by,

$$e_{AM} = E_C\,[1 + m \sin \omega_m t] \sin \omega_c t$$

Compare this with the given expression we get,

$$E_C = 10, \quad m = 1 \text{ so } E_m = E_C = 10 \text{ V}$$

$$f_c = \frac{4\pi \times 10^6}{2\pi} \approx 2 \text{ MHz}$$

$$f_m = \frac{2\pi \times 10 \times 10^3}{2\pi} \approx 10 \text{ kHz}$$

Step 2 : Find m, f_{USB}, f_{LSB}, P_t and BW :

$$m = 1 \quad ...\text{Ans.}$$

$$f_{USB} = f_c + f_m = 2010 \text{ kHz} \quad ...\text{Ans.}$$

$$f_{LSB} = f_c - f_m = 1990 \text{ kHz} \quad ...\text{Ans.}$$

$$P_t = P_c\left[1 + \frac{m^2}{2}\right] = P_c\left[1 + \frac{1}{2}\right] = 1.5\,P_c$$

$$\therefore \quad P_t = \frac{1.5\left[\frac{E_c}{\sqrt{2}}\right]^2}{R} = \frac{1.5 \times (10)^2}{2 \times 50} = 1.5 \text{ W} \quad ...\text{Ans.}$$

$$\text{B.W.} = 2 f_m = 2 \times 10 = 20 \text{ kHz} \quad ...\text{Ans.}$$

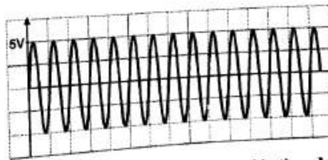
Step 3 : Sketch the envelope and spectrum of SSB signal :

Spectrum of modulated wave :

$$\text{Amplitude of each sideband} = \frac{m}{2} \times E_c = \frac{1 \times 10}{2} = 5 \text{ V}$$

$$f_{USB} = 2010 \text{ kHz}$$

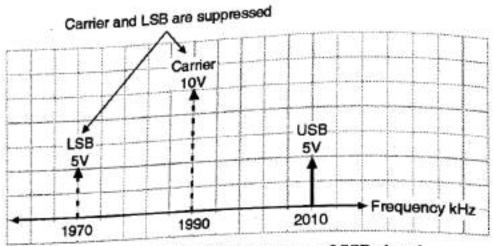
$$f_{LSB} = 1990 \text{ kHz}$$



(D-1298)**Fig. P. 4.15.4(a) : Envelope of SSB signal in time domain**

The spectrum of SSB signal is shown in Fig. P. 4.15.4(b).

Carrier and LSB are suppressed



(D-1299) **Fig. P. 4.15.4(b) : Spectrum of SSB signal**

4.16 Generation of DSB-SC using Balanced Modulator :

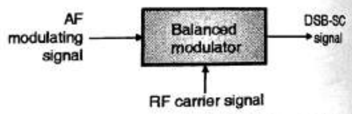
MU : Dec. 14, May 15, Dec. 16

University Questions

Q. 1 What is DSBSC wave? Explain its generation using balanced modulator. **(Dec. 14, May 15, 10 Marks)**

Q. 2 Explain the generation of DSBSC using balanced modulator. **(Dec. 16, 10 Marks)**

- The nonlinear modulators are also known as the balanced modulators.
- The balanced modulators are used to suppress the unwanted carrier in an AM wave.
- The carrier and modulating signals are applied to the inputs of the balanced modulator and we get the DSB signal with suppressed carrier at the output of the balanced modulator.
- Thus the output consists of the upper and lower sidebands only.



(D-120) **Fig. 4.16.1 : Block diagram of balanced modulator**

Principle of operation :

- The principle of operation of a balanced modulator states that if two signals at different frequencies are passed through a "nonlinear resistance" then at the output we get an AM signal with suppressed carrier.
- The device having a nonlinear resistance can be a diode or a JFET or even a bipolar transistor.

4.16.1 Balanced Modulator using Diodes :

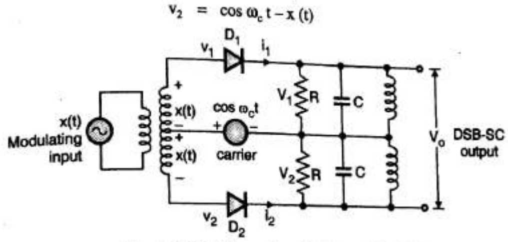
- Fig. 4.16.2 shows the balanced modulator using diode as nonlinear device.
- The modulating signal x(t) is applied at the primary of a center tapped AF transformer. At its secondary windings.
- We get two equal voltages with a phase shift of 180°.
- Thus the two diodes receive the modulating signal x(t) of same magnitude but opposite phase.
- The carrier is applied to the center tap of the secondary.

- So input voltage to D_1 is given by,

$$v_1 = \cos \omega_c t + x(t) \qquad ...(4.16.1)$$

- And the input voltage to D_2 is given by,

$$v_2 = \cos \omega_c t - x(t) \qquad ...(4.16.2)$$



(D-123) **Fig. 4.16.2 : Balanced modulator using diodes**

- The parallel RLC circuits on the output side form the tuned band pass filters.

Analysis :

- The diode current i_1 and i_2 are given by,

$$\begin{aligned} i_1 &= a v_1 + b v_1^2 \\ &= a [x(t) + \cos \omega_c t] + b [x(t) + \cos \omega_c t]^2 \\ &= a x(t) + a \cos \omega_c t + b x^2(t) + 2 b x(t) \cos \omega_c t + b \cos^2 \omega_c t \qquad ...(4.16.3) \end{aligned}$$

- Similarly

$$\begin{aligned} i_2 &= a v_2 + b v_2^2 = a [\cos \omega_c t - x(t)] + b [\cos \omega_c t - x(t)]^2 \\ &= a \cos \omega_c t - a x(t) + b x^2(t) - 2 b x(t) \cos \omega_c t + b \cos^2 \omega_c t \qquad ...(4.16.4) \end{aligned}$$

- The output voltage is given by,

$$v_o = v_1 - v_2$$

$$v_o = i_1 R - i_2 R$$

- Substituting the expression for i_1 and i_2 from Equations (4.16.3) and (4.16.4) we get,

$$v_o = R [2 a x(t) + 4 b x(t) \cos \omega_c t]$$

$$\therefore v_o = \underbrace{2aR\, x(t)}_{\text{Modulating signal}} + \underbrace{4bR\, x(t) \cos \omega_c t}_{\text{DSB-SC signal}} \qquad ...(4.16.5)$$

(D-124)

- So the output voltage contains a modulating signal term and the DSB-SC signal. The modulating signal term is eliminated by the bandpass filters and the second term is allowed to pass through to the output by the LC band pass filter section. ...(4.16.6)

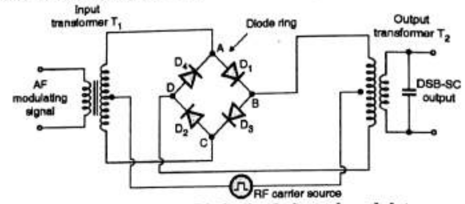
$$\therefore \text{Final output} = 4 b R x(t) \cos \omega_c t \qquad ...(4.16.7)$$

$$= K x(t) \cos \omega_c t$$

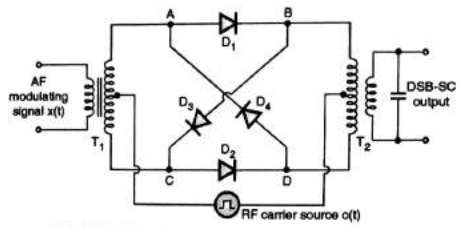
- Thus the diode balanced modulator produces the DSB-SC signal at its output.

4.16.2 Diode Ring Modulator :

- The diode ring modulator is also called as the lattice modulator.
- The circuit configurations of these modulators are as shown in Figs. 4.16.2(a) and 4.16.2(b).
- In both these circuits an AF transformer T_1 and RF transformer T_2 are used. Four diodes are connected in the bridge or lattice configurations. If you see carefully, both the circuits are identical. Only the way they are drawn is different.
- The RF carrier source is connected between the center taps of the two transformers. The AF modulating signal is applied to the primary winding of input transformer T_1. The output is obtained across the secondary of the output transformer T_2.



(D-129) **Fig. 4.16.2(a) : Diode ring balanced modulator**



(D-130) **Fig. 4.16.2(b) : Lattice type diode balanced modulator**

Operation of the circuit :

- The operation is explained with the assumptions that the diodes act as perfect switches and that they are switched on and off by the RF carrier signal. This is because the amplitude and frequency of the carrier is higher than that of the modulating signal.
- The operation can be divided into different modes without the modulating signal and with modulating signal as follows :

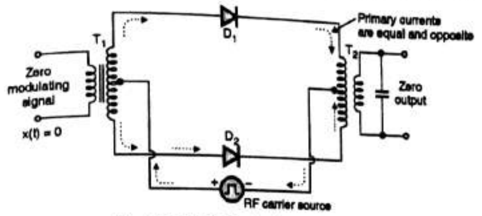
Mode I : Carrier suppression :

To understand how carrier suppression takes place, we assume that the modulating signal is absent and only the carrier is applied. Hence x (t) = 0.

Operation in the positive half cycle of carrier :

- In this mode, let us assume that the modulating signal is zero, and only the carrier signal is applied. The equivalent circuit for this mode of operation is as shown in Fig. 4.16.3(a).

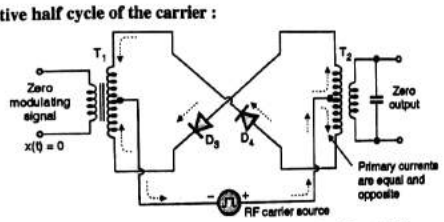
- As shown, the diodes D_1 and D_2 are forward biased. Diodes D_3 and D_4 are reverse biased.



(D-131) **Fig. 4.16.3(a) : Equivalent circuit of mode I**

- See the directions of currents flowing through the primary windings of output transformer T_2, they are equal and opposite to each other.
- Therefore the magnetic fields produced by these currents are equal and opposite and cancel each other. Hence the induced voltage in secondary winding is zero.
- Thus the carrier is suppressed in the positive half cycle.

Operation in negative half cycle of the carrier :



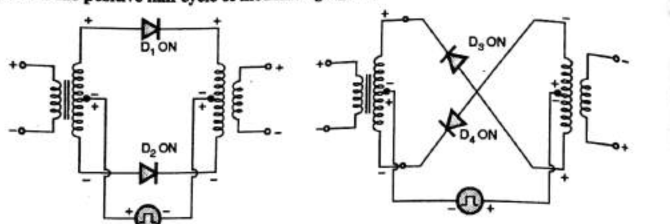
(D-132) **Fig. 4.16.3(b) : Equivalent circuit of mode II**

- In this mode also we assume that the modulating signal is zero. In the negative half cycle of the carrier the diodes D_3 and D_4 are forward biased and the diodes D_1 and D_2 are reverse biased.
- See Fig. 4.16.3(b), the currents flowing in the upper and the lower halves of the primary winding of T_2 are again equal and in opposite directions.
- This is going to cancel the magnetic fields as explained in mode I. Thus the output voltage in this mode also is zero. In this way the carrier is suppressed in the negative half cycle as well.
- It is important to note that the perfect cancellation of the carrier will take place if and only if the characteristics of the diodes are perfectly matched and the center tap is placed exactly at the center of the primary of transformer T_2.

Mode II : Operation in presence of modulating signal :

Now let us discuss the operation when RF carrier and the modulating signal both are applied.

Operation in the positive half cycle of modulating signal :

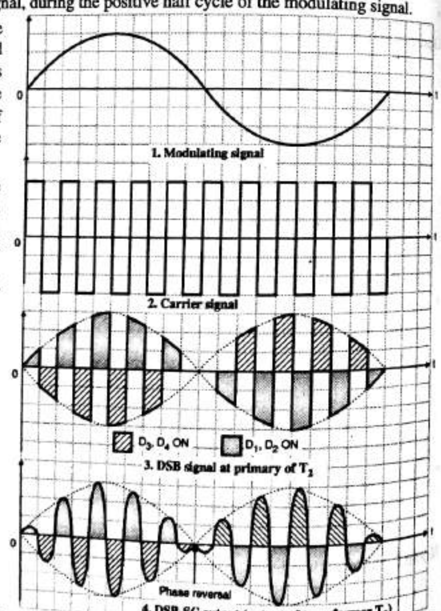


(a) Equivalent circuit in the positive half cycle of modulating signal with carrier negative

(b) Equivalent circuit in the negative half cycle of modulating signal with carrier positive

(D-133) **Fig. 4.16.4**

- As we apply the low frequency modulating signal through the input audio transformer T_1, there are many cycles of the carrier signal, during the positive half cycle of the modulating signal.
- In the positive half cycle of the carrier, D_1 and D_2 are on and secondary of T_1 is applied as it is across the primary of T_2. Hence during the positive half cycle of carrier the output of T_2 is positive as shown in Fig. 4.16.4(a).
- In the negative half cycle of the carrier, D_3 and D_4 are turned on and the secondary of T_1 is applied in a reversed manner across the primary of T_2 as shown in equivalent circuit of Fig. 4.16.4(b). Thus the primary voltage of T_2 is negative and output voltage also becomes negative.



(D-134) **Fig. 4.16.4(c) : Waveforms of the lattice type balanced modulator**

Operation in the negative half cycle of modulating signal :

When modulating signal reverses the polarities the operation of the circuit is same as that in the positive half cycle discussed earlier. The only difference is now the diode pair D_3 D_4 will produce a positive output voltage whereas D_1 D_2 will produce a negative output voltage as shown in the waveforms of Fig. 4.16.4(c).

4.17 Generation of SSB :

There are three practical methods of suppressing the unwanted sideband. They are as follows :

1. Filter method 2. The phase shift method 3. The third method.

- In all these methods balanced modulators are used to suppress the carrier but each method uses a different technique to remove the unwanted sideband.
- All the three systems can remove the upper or lower sideband as desired, to get a SSB signal from the DSB-SC signal. Let us understand the methods one by one.

4.17.1 Generation of SSB using Phase Shift Method :

MU : May 03, Dec. 13, Dec. 15, May 16

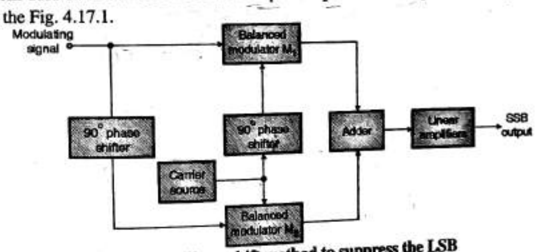
University Questions

Q. 1 Explain the phase shift method to generate lower sidebands. **(May 03, 4 Marks)**

Q. 2 Draw the block diagram of a phase cancellation SSB generation and explain how the carrier and unwanted sideband are supposed. What change is necessary to suppress the other side band ? **(Dec. 13, 10 Marks)**

Q. 3 Explain generation of SSB using phase shift method. **(Dec. 15, May 16, 10 Marks)**

- The block diagram for the phase shift method of SSB generation is as shown in Fig. 4.17.1. This system is used for the suppression of lower sideband.
- This system uses two balanced modulators M_1 and M_2 and two 90° phase shifting networks as shown in the Fig. 4.17.1.



(D-137) **Fig. 4.17.1 : Phase shift method to suppress the LSB**

Operation :

The operation of the phase shift method is as follows :

- The balanced modulator M_1 has two inputs, the modulating signal without any phase shift and the RF carrier with a 90° phase shift.
- The other balanced modulator M_2 receives the modulating signal with a 90° phase shift and carrier without any phase shift.
- At the output of both the balanced modulators we DSB-SC signal consisting of both sidebands. The carrier is completely removed.

- The upper sidebands (USBs) at the outputs of both the balanced modulators lead the carrier by 90°. But LSB at the output of M_1 leads the carrier by 90° and the LSB at the output of M_2 lags behind the carrier by 90°. Thus the LSBs are out of phase.
- So when the outputs of M_1 and M_2 are applied to the adder, the LSBs are cancelled out and the output of the adder consists of only the upper sideband.
- The linear amplifiers will follow the adder. They are class B or AB type amplifiers used to amplify the USB without introducing any distortion.

Mathematical proof of sideband suppression :

- Refer Fig. 4.17.1. The inputs to the balanced modulator M_1 are

$$\text{Inputs to } M_1 \begin{cases} \cos \omega_m t & \text{.....Modulating signal as it is} \\ \cos (\omega_c t + 90°) & \text{......90° phase shifted carrier} \end{cases}$$

- And the inputs to balanced modulator M_2 are

$$\text{Inputs to } M_2 \begin{cases} \cos (\omega_m t + 90°) & \text{....90° shifted modulating signal} \\ \cos \omega_c t & \text{....carrier as it is} \end{cases}$$

- So the output of $M_1 = \cos (\omega_c t) \cdot \cos \omega_m t$ ...(4.17.1)

$$= \underbrace{\frac{1}{2} \cos [\omega_c t + \omega_m t + 90°]}_{\text{USB with 90° advance}} + \underbrace{\frac{1}{2} \cos [\omega_c t - \omega_m t + 90°]}_{\text{LSB with 90° delay}} \quad ...(4.17.2)$$

- And output of $M_2 = \cos \omega_c t \cdot \cos (\omega_m t + 90°)$

$$= \underbrace{\frac{1}{2} \cos [\omega_c t + \omega_m t + 90°]}_{\text{USB with 90° advance}} + \underbrace{\frac{1}{2} \cos [\omega_c t - \omega_m t - 90°]}_{\text{LSB with 90° delay}} \quad ...(4.17.3)$$

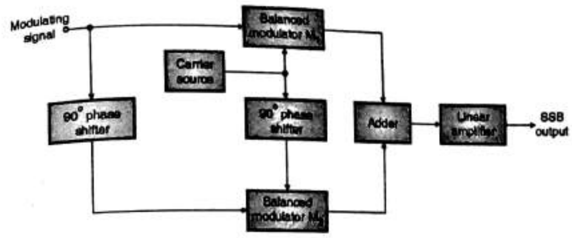
- Output of the adder = Output of M_1 + Output of M_2

$$= \cos (\omega_c t + \omega_m t + 90°) \quad ...(4.17.4)$$

- This output is obtained by adding Equations (4.17.2) and (4.17.3).
- The LSBs in the outputs of M_1 and M_2 are 180° out of phase with respect to each other. Hence they are cancelled out when added.
- So the adder output contains only the upper sideband.

Suppression of the upper sideband :

- We can suppress the USB and generate the SSB signal consisting of the LSB by arranging the blocks in the phase shift method as shown in Fig. 4.17.2.
- Here the modulating and the carrier signals are applied to the upper balanced modulator M_1 directly (without any phase shift). Whereas both these signals are 90° phase shifted and then applied to the lower balanced modulator M_2.



(D-138) **Fig. 4.17.2 : Phase shift method to suppress the USB**

Mathematical proof for USB suppression :

1. Inputs to M_1 are : $\cos \omega_m t$ and $\cos \omega_c t$
2. Inputs to M_2 are : $\cos (\omega_m t + 90°)$ and $\cos (\omega_c t + 90°)$
3. Output of $M_1 = \cos \omega_m t \cos \omega_c t$

$$= \frac{1}{2} \cos (\omega_c t + \omega_m t) + \frac{1}{2} \cos (\omega_c t - \omega_m t)$$

$$= \underbrace{\frac{1}{2} \cos (\omega_c + \omega_m) t}_{\text{USB with 0° phase shift}} + \underbrace{\frac{1}{2} \cos (\omega_c - \omega_m) t}_{\text{LSB with 0° phase shift}} \quad ...(4.17.5)$$

4. Output of $M_2 = \cos (\omega_c t + 90°) \cdot \cos (\omega_m t + 90°)$

$$= \underbrace{\frac{1}{2} \cos (\omega_c t + \omega_m t + 180°)}_{\text{USB with 180° phase shift}} + \underbrace{\frac{1}{2} \cos (\omega_c t - \omega_m t)}_{\text{LSB with 0° phase shift}} \quad ...(4.17.6)$$

5. Adder output $= \cos (\omega_c - \omega_m) t$ ...(4.17.7)

- This output is obtained by adding outputs of M_1 and M_2.
- The USB in the output of M_1 and that in the output of M_2 are 180° out of phase. So in the adder they cancel each other and only the LSB is obtained.

Advantages of phase shift method :

The advantages of the phase shift method are :

1. It can generate the SSB at any frequency so the frequency up converter stage is not required.
2. It can use the low audio frequencies as modulating signal. (In filter method this is not possible.)
3. It is easy to switch from one sideband to the other.

Disadvantage :

The disadvantage is that the design of the 90° phase shifting network for the modulating signal is extremely critical. This network has to provide a correct phase shift of 90° at all the modulating frequencies which is practically extremely difficult to achieve.

4.18 Comparison between DSB-FC, DSB-SC and SSB : MU : Dec. 03

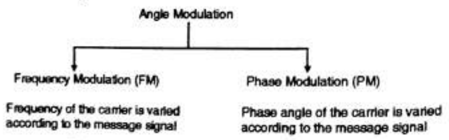
University Questions

Q. 1 Compare the following amplitude modulated systems for transmission/reception efficiencies.
1. DSB with carrier 2. DSB / SC 3. SSB. **(Dec. 03, 8 Marks)**

Sr. No.	Parameter	DSB-FC	DSB-SC	SSB
1.	Carrier suppression	N.A.	Fully	Fully
2.	Sideband suppression	N.A.	N.A.	One S.B. completely
3.	Bandwidth	$2 f_m$	$2 f_m$	f_m
4.	Transmission efficiency	Minimum	Moderate	Maximum
5.	No. of modulating inputs	1	1	1
6.	Application	Radio broadcasting	Radio broadcasting	Point to point mobile communication
7.	Power requirement to cover same area	High	Medium	Very small
8.	Complexity	Simple	Simple	Complex

4.19 Introduction to Angle Modulation :

- In the preceding section of this chapter we have discussed the various types of A.M. systems in which the carrier amplitude is charged in accordance with the variation in the message signal amplitude.
- There is another method of modulating a sinusoidal carrier namely the angle modulation. In angle modulation either frequency or phase of the carrier is varied in proportion with the message signal amplitude, but the carrier amplitude remains constant.
- Thus angle modulation systems can be classified as follows :

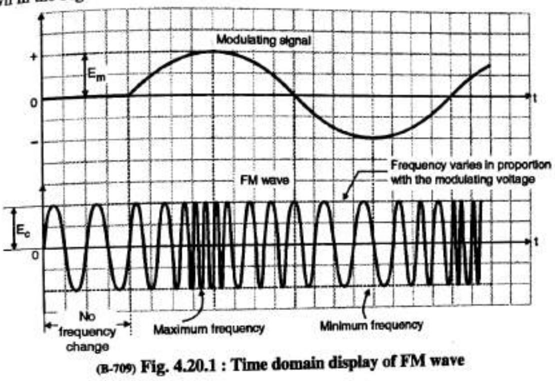


(B-708) **Fig. 4.19.1 : Classification of angle modulation**

- Frequency modulation as well as phase modulation are the forms of **angle modulation.**
- Angle modulation has several advantages over the amplitude modulation such as noise reduction, improved system fidelity and more efficient use of transmitter output power.
- But there are some disadvantages too such as increased bandwidth and need for the use of more complex circuits.
- Angle modulation is being used for the following applications :
  1. Radio broadcasting
  2. TV sound transmission
  3. Two way mobile radio
  4. Cellular radio
  5. Microwave communication
  6. Satellite communication.

4.20 Frequency Modulation (Sinusoidal Signals) :

- In sinusoidal Frequency Modulation (FM), the modulating signal $x(t) = E_m \cos(2\pi f_m t)$ is a pure sinusoidal signal. The carrier signal c (t) is also a sinewave at much higher frequency.
- FM is a system of modulation in which the instantaneous frequency of the carrier is varied in proportion with the amplitude of the modulating signal. The amplitude of the carrier signal remain constant. Thus the information is conveyed via frequency changes.
- FM was first practically tried in 1936 as an alternative to AM. As will be shown later on, FM transmission is more resistant to noise than A.M. The time domain display of FM wave is as shown in the Fig. 4.20.1.



(B-709) **Fig. 4.20.1 : Time domain display of FM wave**

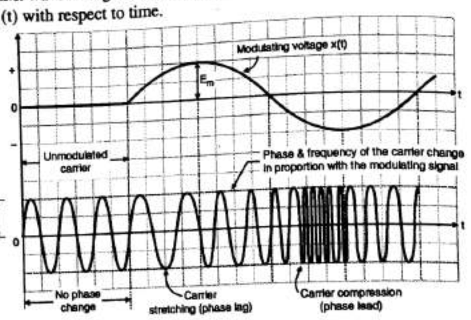
The observation from the Fig. 4.20.1 are as follows :

- The amount by which the carrier frequency deviates from its unmodulated value is called as **"deviation".** The deviation (δ) is made proportional to the instantaneous value of modulating voltage.
- The rate at which these frequency variations or oscillations takes place in the FM wave is equal to the modulating frequency (f_m).
- The amplitude of the FM wave always remains constant. This is the biggest advantage of FM.

4.20.1 Sinusoidal Phase Modulation (PM) :

- Phase modulation is very similar to the frequency modulation. The only difference is that the phase of the carrier is varied instead of varying the frequency. The amplitude of the carrier remains constant.
- As shown in Fig. 4.20.2, as the modulating signal goes positive, the amount of phase lag increases with the amplitude of the modulating signal. The effect of this is that the carrier signal is stretched or its frequency is reduced.
- When the modulating signal goes negative, the phase shift becomes leading. This causes the carrier wave to be effectively compressed. The effect of this is as if the carrier frequency is increased.
- Thus phase modulation is always associated with frequency modulation and vice versa.

- Note that the P.M. wave of Fig. 4.20.2 is the same as the F.M. wave produced by $dx(t) / dt$ i.e. the derivative of x (t) with respect to time.



(L-45) **Fig. 4.20.2 : Time domain display of PM wave**

- So in Fig. 4.20.2 we have plotted the derivative of x (t) which is original x (t) shifted by 90°.
- From the discussion it is clear that the difference between F.M. and P.M. waves can be made only by comparing with the original modulating wave.

4.21 Important Definitions in Frequency Modulation :

- For the single tone F.M. the modulating signal x (t) be a sinusoidal signal of amplitude E_m and frequency f_m.

$$\therefore \quad x(t) = E_m \cos(2\pi f_m t) \qquad ...(4.21.1)$$

1. The unmodulated carrier is represented by the expression,

$$e_c = A \sin(\omega_c t + \phi)$$

2. **Instantaneous frequency of an FM wave :**

In FM, the frequency f of the FM wave varies in accordance with the modulating voltage. The instantaneous frequency of the FM wave is denoted by $f_i(t)$ and is given by,

$$f_i(t) = f_c[1 + k_f x(t)] = f_c[1 + k_f \cdot E_m \cos(2\pi f_m t)]$$

$$= f_c + \delta \cos(2\pi f_m t) \qquad ...(4.21.2)$$

Where $\delta = k_f E_m E_c$ and it is called as **frequency deviation,** where k_f is a constant with units Hz/Volts.

4.21.1 Frequency Deviation (δ) or (Δf) :

MU : May 03

University Questions

Q. 1 In relation with FM, explain : Maximum frequency deviation. (May 03, 2 Marks)

- **Frequency deviation** δ represents the maximum departure of the instantaneous frequency $f_i(t)$ of the FM wave from the carrier frequency f_c.
- Since $\delta = k_f E_m$, the **frequency deviation** is proportional to the amplitude of modulating voltage (E_m) and it is independent of the modulating frequency f_m.

Maximum and minimum frequency of FM wave :

The maximum frequency of FM wave is,

$$f_{max} = f_c + \delta \qquad ...(4.21.3)$$

The minimum frequency of a FM wave is $f_{min} = (f_c - \delta)$.

4.21.2 Mathematical Expression for F.M. :

MU : Dec. 05, May 07

University Questions

Q. 1 Find the mathematical expression of FM signal. (Dec. 08, 10 Marks)

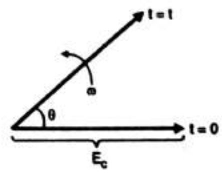
Q. 2 Derive an expression for frequency modulated waveform. (May 07, 6 Marks)

- We know that the FM wave is a sinewave having a constant amplitude and a variable instantaneous frequency. As the instantaneous frequency is changing continuously, the angular velocity "ω" of an FM wave is the function of ω_c and ω_m.
- Therefore the FM wave is represented by,

$$e_{FM} = s(t) = E_c \sin[F(\omega_c, \omega_m)] \qquad ...(4.21.4)$$

$$= E_c \sin\theta(t) \qquad ...(4.21.5)$$

where $\theta(t) = F(\omega_c, \omega_m)$ ...(4.21.6)



(D-160) **Fig. 4.21.1 : Frequency modulated vector**

- As shown in Fig. 4.21.1, $E_c \sin\theta(t)$ is a rotating vector. If "E_c" is rotating at a constant velocity "ω" then we could have written that $\theta(t) = \omega t$. But in FM this velocity is not constant. In fact it is changing continuously. The angular velocity of FM wave is given as,

$$\omega = \omega_c[1 + k_f E_m \cos\omega_m t] \qquad ...(4.21.7)$$

- Hence to find "θ (t)" we must integrate "ω" with respect to time.

$$\therefore \quad \theta(t) = \int \omega\, dt = \int \omega_c[1 + k_f E_m \cos\omega_m t]\, dt \qquad ...(4.21.8)$$

$$\therefore \quad \theta(t) = \omega_c \int [1 + k_f E_m \cos\omega_m t]\, dt$$

$$= \omega_c\left[t + \frac{k_f E_m \sin\omega_m t}{\omega_m}\right] = \omega_c t + \frac{k_f E_m \omega_c \sin\omega_m t}{\omega_m}$$

$$\therefore \quad \theta(t) = \omega_c t + \frac{k_f E_m f_c \sin\omega_m t}{f_m} \qquad ...(4.21.9)$$

- As per the definition, $\delta = k_f E_m f_c$ ...(4.21.10)

$$\therefore \quad \theta(t) = \omega_c t + \frac{\delta \sin\omega_m t}{f_m}$$

- Substitute this value of θ (t) in Equation (4.21.5) to get the equation for the FM wave as,

$$e_{FM} = s(t) = E_c \sin\left[\omega_c t + \frac{\delta}{f_m}\sin\omega_m t\right] \qquad ...(4.21.11)$$

- But $\frac{\delta}{f_m} = m_f$ i.e. the modulation index of FM wave. Hence the equation for FM wave is given as,

$$e_{FM} = E_c \sin[\omega_c t + m_f \sin\omega_m t] \qquad ...(4.21.12)$$

- This is the expression for a FM wave, where m_f represents the modulation index.

Meaning of mathematical representation :

- The mathematical expression for a FM wave is as follows :

$$e_{FM} = \underbrace{E_c}_{\text{Peak amplitude of FM wave is constant and equal to the peak amplitude of the carrier}} \underbrace{\sin}_{\text{FM wave is a sine wave}} \underbrace{[\omega_c t + m_f \sin \omega_m t]}_{\text{Frequency of FM wave varies according to the modulating signal}}$$

(D-1311)

- The amplitude of FM wave is constant and equal to the amplitude of the carrier i.e. E_c.
- FM wave is sinusoidal i.e. it has a shape of sine or cosine wave.
- The frequency of FM wave is not constant. It varies continuously, above and below the carrier frequency f_c.

4.21.3 Modulation Index :

MU : May 03, Dec. 14, May 15

University Questions

Q. 1 In relation with FM, explain : Modulation index. **(May 03, 2 Marks)**

Q. 2 Discuss the factors that influence modulation index of an FM wave. **(Dec. 14, May 15, 5 Marks)**

- The modulation index of an FM wave is defined as :

$$m_f = \frac{\text{Maximum frequency deviation}}{\text{Modulating frequency}} \quad \text{...(4.21.13)}$$

$$\therefore \quad m_f = \frac{\delta}{f_m} \quad \text{...(4.21.14)}$$

- The modulation index (m_f) is very important in FM because it decides the bandwidth of the FM wave (discussed later on).
- The modulation index also decides the number of sidebands having significant amplitudes.
- In AM the maximum value of the modulation index m is 1. But for FM the modulation index can be greater than 1. The modulation index m_f is measured in radians.

4.21.4 Deviation Ratio :

In FM broadcasting the maximum value of deviation is limited to 75 kHz. The maximum modulating frequency is also limited to 15 kHz. The modulation index corresponding to the maximum deviation and maximum modulating frequency is called as the "deviation ratio".

$$\text{Deviation ratio} = \frac{\text{Maximum deviation}}{\text{Maximum modulating frequency}} \quad \text{...(4.21.15)}$$

4.21.5 Percentage Modulation of FM Wave :

The percent modulation is defined as the ratio of the actual frequency deviation produced by the modulating signal to the maximum allowable frequency deviation.

$$\therefore \quad \%\ \text{Modulation} = \frac{\text{Actual frequency deviation}}{\text{Maximum allowed deviation}} \quad \text{...(4.21.16)}$$

4.21.6 Frequency Spectrum of FM Wave :

MU : May 03

University Questions

Q. 1 In relation with FM, explain : Frequency spectrum. **(May 03, 1 Mark)**

- The expression for the FM wave is not simple. It is complex since it is sine of sine function. The only way to solve this equation is by using the Bessel functions.
- By using the Bessel functions the equation for FM wave can be expanded as follows :

$$\begin{aligned} e_{FM} &= s(t) = E_c\{ J_0(m_f) \sin \omega_c t + J_1(m_f) \\ &\quad [\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t] + J_2(m_f) \\ &\quad [\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t] + J_3(m_f) \\ &\quad [\sin(\omega_c + 3\omega_m)t - \sin(\omega_c - 3\omega_m)t + J_4(m_f) \\ &\quad [\sin(\omega_c + 4\omega_m)t + \sin(\omega_c - 4\omega_m)t]......\} \quad \text{...(4.21.17)} \\ &= \text{Carrier + Infinite number of sidebands} \end{aligned}$$

- Expanding Equation (4.21.17) we get,

$$e_{FM} = \underbrace{J_0(m_f) E_c \sin \omega_c t}_{\text{Carrier}} + \underbrace{J_1(m_f) E_c [\sin(\omega_c \pm \omega_m)t]}_{\text{Pair of first sidebands}} + \text{.........}$$

Observations :

Looking at Equation (4.21.17) we can draw the following conclusions :

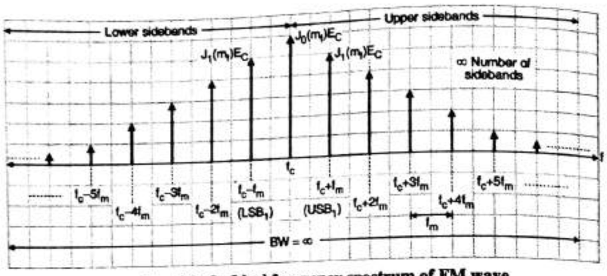
1. The FM wave consists of carrier. The first term in Equation (4.21.17) represents the carrier.
2. The FM wave ideally consists of infinite number of sidebands. All the terms except the first one are sidebands.
3. The amplitudes of the carriers and sidebands are dependent on the J coefficients. For example, amplitude of carrier component depends on $J_0(m_f)$, that of the first pair of sidebands depends on $J_1(m_f)$ etc.
4. The values of these J coefficients can be obtained from Table 4.21.2. For example $J_1(m_f)$ denotes the value of J_1 for the particular value of m_f written inside the bracket.

FM spectrum :

- From Equation (4.21.17) we get the carrier and infinite number of sidebands.
- Table 4.21.1 enlists all these terms, their amplitudes and frequencies and Fig. 4.21.2 is the frequency domain representation or spectrum of the FM wave.

Table 4.21.1

Sr. No.	Term in Equation (4.21.17)	Name	Amplitude	Frequency
1.	$J_0(m_f) E_c \sin \omega_c t$	Carrier	$J_0(m_f) E_c$	f_c
2.	$J_1(m_f) E_c \sin(\omega_c + \omega_m)t$	USB 1	$J_1(m_f) E_c$	$(f_c + f_m)$
3.	$J_1(m_f) E_c \sin(\omega_c - \omega_m)t$	LSB 1	$J_1(m_f) E_c$	$(f_c - f_m)$
4.	$J_2(m_f) E_c \sin(\omega_c + 2\omega_m)t$	USB 2	$J_2(m_f) E_c$	$(f_c + 2f_m)$
5.	$J_2(m_f) E_c \sin(\omega_c - 2\omega_m)t$	LSB 2	$J_2(m_f) E_c$	$(f_c - 2f_m)$
	⋮ Infinite terms	⋮	⋮	⋮



(D-166) **Fig. 4.21.2 : Ideal frequency spectrum of FM wave**

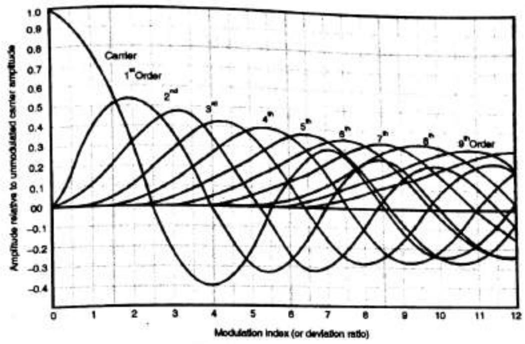
- In FM, sum and difference sideband frequencies are produced.
- In addition, theoretically infinite number of pairs of upper and lower sidebands are also generated.
- Hence the spectrum of FM signal is generally wider than the spectrum of AM.
- Note that the adjacent sidebands are spaced from each other by a frequency equal to modulating signal frequency f_m.

Important points about FM spectrum :

1. As the values of J coefficients are dependent on the modulation index m_f, the modulation index (m_f) determines the number of sideband components that will have significant amplitudes. Ideally there are infinite number of sidebands, but practical bandwidth depends on the number of significant sidebands and hence on the modulation index value.
2. Some of the J coefficients can be negative. Therefore there is a 180° phase shift for that particular pair of sidebands. This is denoted by a negative sign attached to the amplitude of that sideband.
3. The carrier component does not remain constant. As $J_0(m_f)$ is varying, the amplitude of the carrier will also vary. However the amplitude of FM wave will remain constant.
4. For certain values of modulation index the carrier component will disappear completely. Their values are called "eigen values".
5. In FM, the total transmitted power always remains constant. It is not dependent on the modulation index. The reason for this is that the amplitude of the FM signal i.e. E_c is always constant and the power transmitted is given by,

$$P_t = (E_c/\sqrt{2})^2/R = \frac{E_c^2}{2R} \quad \ldots(4.21.18)$$

$$\text{if } R = 1\Omega, \quad \therefore P_t = \frac{E_c^2}{2}$$



(D-165) **Fig. 4.21.3 : Bessel functions**

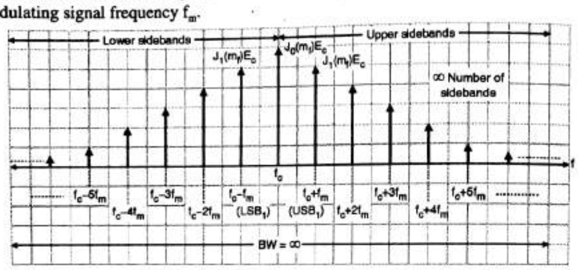
Table 4.21.2 : Bessel functions of the first kind

	ORDER OF THE FUNCTION																
m_f	$J_0 m_f$	$J_1 m_f$	$J_2 m_f$	$J_3 m_f$	$J_4 m_f$	$J_5 m_f$	$J_6 m_f$	$J_7 m_f$	$J_8 m_f$	$J_9 m_f$	$J_{10} m_f$	$J_{11} m_f$	$J_{12} m_f$	$J_{13} m_f$	$J_{14} m_f$	$J_{15} m_f$	$J_{16} m_f$
0.00	1.00	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.25	0.98	0.12	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.5	0.94	0.24	0.03	–	–	–	–	–	–	–	–	–	–	–	–	–	–
1.0	0.77	0.44	0.11	0.02	–	–	–	–	–	–	–	–	–	–	–	–	–
1.5	0.51	0.56	0.23	0.06	0.01	–	–	–	–	–	–	–	–	–	–	–	–
2.0	0.22	0.58	0.35	0.13	0.03	–	–	–	–	–	–	–	–	–	–	–	–
2.5	– 0.05	0.50	0.45	0.22	0.07	0.02	–	–	–	–	–	–	–	–	–	–	–
3.0	– 0.26	0.34	0.49	0.31	0.13	0.04	0.01	–	–	–	–	–	–	–	–	–	–
4.0	– 0.4	– 0.07	0.36	0.43	0.28	0.13	0.05	0.02	–	–	–	–	–	–	–	–	–
5.0	– 0.18	– 0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	–	–	–	–	–	–	–	–
6.0	0.15	– 0.28	– 0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	–	–	–	–	–	–	–
7.0	0.3	0.00	– 0.30	– 0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	–	–	–	–	–	–
8.0	0.17	0.23	– 0.11	– 0.29	– 0.10	0.19	0.34	0.32	0.22	0.13	0.06	00.3	–	–	–	–	–
9.0	– 0.09	0.24	0.14	– 0.18	– 0.27	0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03	–	–	–	–
10.0	– 0.25	0.04	– 0.25	0.06	– 0.22	– 0.23	– 0.01	0.22	0.31	0.29	0.20	0.12	0.07	0.03	–	–	–
12.0	0.05	– 0.22	– 0.08	0.20	0.18	– 0.07	– 0.24	– 0.17	0.06	0.23	0.30	0.27	0.20	0.12	0.7	–	–
15.0	– 0.01	0.21	0.04	– 0.19	– 0.12	0.13	0.21	0.03	– 0.17	– 0.22	– 0.09	0.10	0.24	0.28	0.25	0.18	–

This is the value of J_1 (15)

4.21.7 Sidebands and Modulation Index :

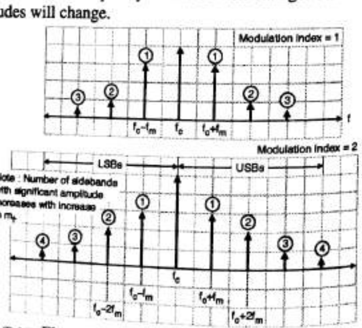
- We know that any modulation process produces sidebands.
- We have seen that in AM that two sidebands are produced with frequencies equal to $f_c + f_m$ and $f_c - f_m$.
- In FM and PM also the sideband frequencies similar to those in AM are produced.
- In addition, theoretically infinite number of pairs of upper and lower sidebands are also generated.
- Hence the spectrum of FM or PM signal is generally wider than the spectrum of AM.
- Fig. 4.21.4 shows the example of a typical spectrum of an ideal F.M. signal.
- Note that the sidebands are spaced from the carrier f_c and from each other by a frequency equal to modulating signal frequency f_m.



(D-166) **Fig. 4.21.4 : Ideal frequency spectrum of FM wave**

Effect of modulation index : Factors that influence modulation index of FM wave,,

- As the amplitude of modulating signal varies, the frequency deviation will change. The number of sidebands produced and their amplitudes will change.
- Fig. 4.21.5 illustrate the effect of modulation index on the frequency spectrum of FM.
- From the Figs. 4.21.4 and 4.21.5 we can conclude that theoretically the bandwidth of FM is infinite.
- But practically it depends on the number of significant sidebands.
- The number of sidebands having significant amplitudes will increase with increase in the value of modulation index m_f.



(D-167) **Fig. 4.21.5 : Effect of modulation index on the significant number of sidebands**

Bandwidth requirement of FM :

- Bandwidth of an FM wave is defined as the frequency difference between the highest pair of sidebands.
- Ideally the bandwidth of FM is infinite, because its spectrum consists of infinite number of upper and lower sidebands.
- But practically the bandwidth depends on the number of significant sidebands.
- The number of sidebands having significant amplitudes will increase with increase in the value of modulation index m_f. Hence the bandwidth increases with increase in the value of m_f.

4.21.8 Bandwidth Requirement of FM :

- Bandwidth of an FM wave is defined as the frequency difference between the highest pair of sidebands.
- Ideally the bandwidth of FM is infinite, because its spectrum consists of infinite number of upper and lower sidebands.
- But practically the bandwidth depends on the number of significant sidebands.
- The number of sidebands having significant amplitudes will increase with increase in the value of modulation index m_f. Hence the bandwidth increases with increase in the value of m_f.

4.21.9 Practical Bandwidth : MU : May 03

University Questions

Q. 1 In relation with FM, explain : Bandwidth. **(May 03, 1 Mark)**

- Theoretically the bandwidth of the FM wave is infinite. But practically it is calculated based on how many sidebands have significant amplitude.
- The simplest method to calculate the bandwidth is as follows :

$$BW = 2 f_m \times \text{Number of significant sidebands} \quad ...(4.21.19)$$

- With increase in modulation index, the number of significant sidebands increase. This will increase the bandwidth. The bandwidth of FM is higher than that of AM.

Carson's Rule :

- The second method to find the practical bandwidth is a rule of thumb (Carson's rule). It states that the bandwidth of FM wave is equal to twice the sum of the deviation and the highest modulating frequency.

$$BW = 2[\delta + f_{m\,(max)}] \quad ...(4.21.20)$$

- The Carson's rule gives correct results if the modulation index is greater than 6.

4.21.10 Types of F.M. : MU : May 04, Dec. 04, Dec. 05

University Questions

Q. 1 Explain narrow band and wideband FM. **(May 04, Dec. 04, Dec. 05, 5 Marks)**

- The FM systems are basically classified into two types :
  1. Narrowband FM
  2. Broadband FM

1. **Narrowband FM (NBFM):**
   - A narrowband FM is the FM wave with a small bandwidth. The modulation index m_f of narrowband FM is small as compared to one radian.

- Hence the spectrum of narrowband FM consists of the carrier and upper sideband and a lower sideband.
- For small values of m_f the values of the J coefficients are,

$$\left.\begin{aligned} J_0(m_f) &= 1, J_1(m_f) = m_f/2 \\ J_n(m_f) &= 0 \text{ for } n > 1 \end{aligned}\right\} \qquad ...(4.21.21)$$

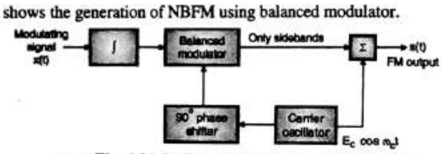
- Therefore a narrowband FM wave can be expressed mathematically as follows,

$$e_{FM} = s(t) = \underbrace{E_c \sin \omega_c t}_{\text{Carrier}} + \underbrace{\frac{m_f E_c}{2} \sin(\omega_c + \omega_m) t}_{\text{USB}} - \underbrace{\frac{m_f E_c}{2} \sin(\omega_c - \omega_m) t}_{\text{LSB}} \qquad ...(4.21.22)$$

- The (–) sign associated with the LSB magnitude represents a phase shift of 180°.
- Practically the narrowband FM systems have m_f less than 1. The maximum permissible frequency deviation is restricted to about 5 kHz.
- This system is used in FM mobile communications such as police wireless, ambulances, taxicabs etc.

Generation of NBFM :

- Fig. 4.21.6 shows the generation of NBFM using balanced modulator.

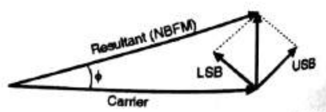


(D-168) **Fig. 4.21.6 : Generation of narrow band FM**

- The modulating signal is integrated and applied to a balanced modulator.
- The other input to the balanced modulator is a 90° phase shifted carrier signal.
- The output of balanced modulator contains only the sidebands because it suppresses the carrier.
- These sidebands are added with the carrier to obtain the NBFM signal.

Phasor diagram of NBFM :

- Fig. 4.21.7 shows the phasor diagram of NBFM.
- This phasor diagram has been drawn by referring to Equation (4.21.22). The carrier phasor is represented by $E_c \sin \omega_c t$. It is drawn along the horizontal axis and remains fixed always.



(D-1274) **Fig. 4.21.7 : Phasor diagram of NBFM**

- The upper sideband is represented by $\frac{1}{2} m_f E_c \sin(\omega_c + \omega_m)$. It is drawn as a phasor rotating in counter clockwise direction at an angular velocity ω_m.
- Similarly LSB is represented by $\frac{1}{2} m_f E_C \sin(\omega_c - \omega_m)$. This phasor rotates in the clockwise direction at an angular velocity of ω_m.

- The resultant of the two sidebands is always perpendicular to the carrier phasor and the resultant phasor that represents the NBFM signal makes an angle ϕ with the carrier vector.

2. **Wideband FM (WBFM) :**

- As discussed earlier, for large values of modulation index m_f, the FM wave ideally contains the carrier and an infinite number of sideband located symmetrically around the carrier.
- Such a FM wave has infinite bandwidth and hence called as wideband FM.
- The modulation index of wideband FM is much higher than 1.
- The maximum permissible deviation is 75 kHz and it is used in the entertainment broadcasting applications such as FM radio, TV etc.

Comparison of Wideband and Narrowband FM :

Sr. No.	Parameter/Characteristics	Wideband FM	Narrowband FM
1.	Modulation index	Greater than 1	Less than or slightly greater than 1
2.	Maximum deviation	75 kHz	5 kHz
3.	Range of modulating frequency	30 Hz to 15 kHz	30 Hz to 3 kHz
4.	Maximum modulation index	5 to 2500	Slightly greater than 1
5.	Bandwidth	Large, about 15 times higher than BW of narrowband FM.	Small. Approximately same as that of AM
6.	Applications	Entertainment broadcasting (can be used for high quality music transmission)	FM mobile communication-like police wireless, ambulance etc. (This is used for speech transmission).
7.	Pre-emphasis and De-emphasis	Needed	Needed

Constant average power :

- We know that the envelope of FM wave always has a constant magnitude. Therefore the average power of such a wave dissipated in 1 Ω resistance will always be constant.
- According to Equation (4.21.18) the transmitted power in F.M. is given by,

$$P_t = \frac{E_c^2}{2 R_L} \qquad [\text{ since } A = E_c]$$

So substituting $R_L = 1$ we get,

$$P_t = \frac{1}{2} E_c^2 \qquad ...(4.21.23)$$

- Since E_c is constant, P_t also will be constant. It is possible to express the transmitted power in the form of series expansion as follows,

$$P_t = \frac{1}{2} E_c^2 \sum_{n=-\infty}^{\infty} J_n^2 (m_f) \quad ...(4.21.24)$$

$$\text{But} \quad \sum_{n=-\infty}^{\infty} J_n^2 (m_f) = 1 \qquad \text{Hence } P_t = \frac{1}{2} E_c^2 \quad ...(4.21.25)$$

Ex. 4.21.1 : Define the term "percent modulation" and determine the percent modulation for an FM wave with a frequency deviation of 10 kHz if the maximum deviation allowed is 25 kHz.

Soln. :

The percent modulation is defined as the ratio of the actual frequency deviation produced by the modulating signal to the maximum allowable frequency deviation.

$$\therefore \quad \%\ \text{Modulation} = \frac{\text{Actual frequency deviation}}{\text{Maximum allowed deviation}} \quad ...(1)$$

In the example it is given that $\delta = 10$ kHz and $\Delta f_{max} = 25$ kHz

$$\therefore \quad \%\ \text{Modulation} = \frac{10\ \text{kHz}}{25\ \text{kHz}} = 40\% \quad ...\text{Ans.}$$

Ex. 4.21.2 : In an F.M. system, if the maximum value of deviation is 75 kHz and the maximum modulating frequency is 10 kHz calculate the deviation ratio and bandwidth of the system using Carson's rule.

Soln. :

Given : $\Delta f_{max} = 75$ kHz, $f_{m\,(max)} = 10$ kHz

1. Deviation ratio D $= \frac{\Delta f_{max}}{f_{m(max)}} = \frac{75\ \text{kHz}}{10\text{kHz}} = 7.5$ ...**Ans.**

2. System bandwidth B $= 2\,[\Delta f_{max} + f_{m(max)}]$

$= 2\,(75 + 10) = 170$ kHz ...**Ans.**

Why is FM a constant BW system ?

- It is said that the FM system is a constant bandwidth system. Let us prove it by taking an example.
- Consider an FM system with deviation $\delta = 75$ kHz, $f_c = 100$ MHz, and the maximum modulating frequency $f_m = 500$ Hz. The bandwidth can be calculated using Carson's rule.

$$\therefore \quad BW = 2\,[75 + 0.5] = 151\ \text{kHz} \quad ...(4.21.26)$$

- Now keeping everything else unchanged, assume $f_{m(max)} = 5$ kHz and calculate the new bandwidth.

$$BW = 2[75 + 5] = 160\ \text{kHz} \quad ...(4.21.27)$$

- Compare Equations (4.21.26) and (4.21.27). You will find that corresponding to the ten times increase in the modulating frequency, the percent change in the BW is only about 6%. This is why FM is called as constant BW system.

Ex. 4.21.3 : The carrier frequency of a broadcast signal is 100 MHz. The maximum frequency deviation is 75 kHz. If the highest modulating frequency is limited to 15 kHz, then what is the approximate bandwidth of the modulated signal ?

Soln. :

Given : $f_c = 100$ MHz, $f_{m\,(max)} = 15$ kHz, $\Delta f = 75$ kHz

To find : Bandwidth

To find the approximate bandwidth, we use the Carson's formulae.

$$BW = 2\,[\Delta f + f_{m\,(max)}]$$

$$= 2\,[75\ \text{kHz} + 15\ \text{kHz}]$$

$$= 2 \times 90\ \text{kHz} = 180\ \text{kHz} \quad ...\text{Ans.}$$

Ex. 4.21.4 : Consider an angle modulated signal $x_c(t) = 10\cos(\omega_c t + 3\sin\omega_m t)$. Assume FM and $f_m = 1$ kHz. Calculate the modulation index and find the bandwidth when 1. f_m is doubled 2. f_m is decreased by one half.

Soln. :

Given : An FM wave $x_c(t) = 10\cos(\omega_c t + 3\sin\omega_m t)$

$f_m = 1$ kHz

To find : Modulation index m_f and BW.

Step 1 : Calculate deviation Δf :

From the given expression modulation index is

$$m_f = 3 \qquad \text{But } m_f = \frac{\Delta f}{f_m}$$

$$\therefore \quad \Delta f = m_f \times f_m = 3 \times 1\ \text{kHz} = 3\ \text{kHz}$$

We assume that the deviation remains constant

Step 2 : Calculate m_f and BW for $f_m = 2$ kHz :

$$m_f = \frac{\Delta f}{f_m} = \frac{3\text{kHz}}{2\text{kHz}} = 1.5 \quad ...\text{Ans.}$$

Using Carson's rule

$$BW = 2\,[\Delta f + f_m] = 2\,[3 + 2]\ \text{kHz} = 10\ \text{kHz} \quad ...\text{Ans.}$$

Step 3 : Calculate m_f and BW for $f_m = 0.5$ kHz :

$$m_f = \frac{3\ \text{kHz}}{0.5\ \text{kHz}} = 6 \quad ...\text{Ans.}$$

$$BW = 2[3 + 0.5] = 7\ \text{kHz} \quad ...\text{Ans.}$$

Ex. 4.21.5 : The equation of an angle modulated voltage is $e = 10\sin(10^8 t + 3\sin 10^4 t)$. What form of angle modulation is this ? Calculate the carrier and modulating frequencies, the modulation index and deviation and the power dissipated in 100 Ω resistor.

Soln. :

- This is an FM wave.
- The standard expression for FM wave is,

$$e_{FM} = A\sin[(2\pi f_c t) + m_f \sin(2\pi f_m t)]$$

- Compare this expression with the given expression to get the required quantities.

Carrier frequency f_c :

$$2\pi f_c = 10^8 \qquad \therefore \quad f_c = 10^8 / 2\pi = 15.91\ \text{MHz} \quad ...\text{Ans.}$$

Modulating frequency f_m :

$$2\pi f_m = 10^4 \qquad \therefore \quad f_m = 10^4 / 2\pi = 1591.5 \text{ Hz}$$

$$= 1.5915 \text{ kHz}$$ ...Ans.

Modulation index, deviation :

$$m_f = 3$$ ...Ans.

$$\Delta f = m_f \times f_m = 3 \times 1591.5 = 4.774 \text{ kHz.}$$ ...Ans.

Power dissipation in 100 Ω resistance :

$$P = \frac{(A/\sqrt{2})^2}{R}$$

But A = 10 V $\quad \therefore \quad P = \frac{(10/\sqrt{2})^2}{100} = \frac{100}{2 \times 100} = 0.5$ Watts. ...Ans.

Ex. 4.21.6 : Explain how Carson's rule of bandwidth in FM is obtained. For an FM modulator with peak frequency deviation Δf = 5 kHz modulating frequency f_m = 5 kHz, with amplitude of carrier 5V and frequency 500 kHz, determine the bandwidth using Carson's rule.

Soln. :

Given : Frequency deviation Δf = 5 kHz

Modulating frequency f_m = 5 kHz

By Carson's rule bandwidth is given by,

$$BW = 2[\Delta f + f_{m\,(max)}] = 2[5 \text{ kHz} + 5 \text{ kHz}]$$

$$= 20 \text{ kHz}$$

Bandwidth = 20 kHz ...Ans.

Ex. 4.21.7 : The maximum deviation allowed in a FM broadcast system is 75 kHz. If the modulating signal is a single tone sinusoidal of frequency 15 kHz, find the bandwidth of the FM signal. How does the bandwidth change if the modulating frequency is doubled ?

Dec. 14, 5 Marks

Soln. :

Given : Δf = 75 kHz, f_m = 15 kHz

To find :
1. Bandwidth of FM signal.
2. Bandwidth of FM signal if modulating frequency is doubled.

Step 1 : Calculate bandwidth :

$$BW = 2[\Delta f + f_m] = 2[75 + 15] = 180 \text{ kHz}$$ ...Ans.

Step 2 : Calculate BW if modulating frequency is doubled :

If f_m = 30 kHz

$$\therefore BW = 2[\Delta f + f_m] = 2[75 + 30] = 210 \text{ kHz}$$ ...Ans.

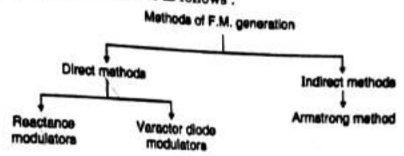
4.22 Generation of FM Waves :

MU : May 04

University Questions

Q. 1 List the different methods of FM generation. **(May 04, 2 Marks)**

- There are two basic methods of generating the FM waves as follows :
  1. Direct methods and
  2. Indirect methods
- The classification of FM generation methods is shown in Fig. 4.22.1.



(D-183) **Fig. 4.22.1 : Classification of FM generation methods**

4.23 Generation of FM by Armstrong Method :

MU : May 05, May 08, Dec. 11, Dec. 15, May 16, Dec. 16

University Questions

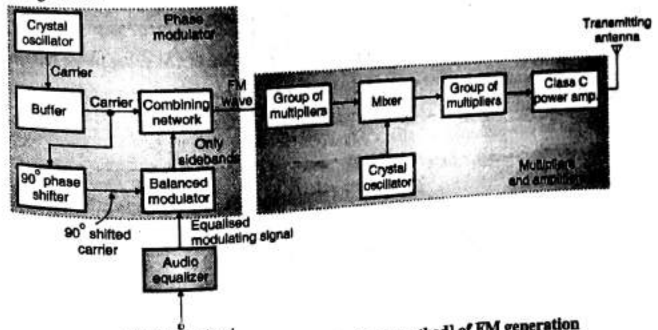
Q. 1 Draw the block diagram of Armstrong frequency modulation system and explain the functions of mixer and multiplier. In what circumstances can the mixer be dispensed with ? **(May 05, 10 Marks)**

Q. 2 With reference to FM generation, explain with the aid of neat figure, the indirect method of FM generation. **(May 08, 10 Marks)**

Q. 3 Draw the block diagram of Armstrong Frequency Modulation System and explain its working. **(Dec. 11, 10 Marks)**

Q. 4 Write short note on generation of FM by Armstrong method. **(Dec. 15, May 16, Dec. 16, 5 Marks)**

- In the direct methods of generation of FM, LC oscillators are to be used. The crystal oscillators cannot be used.
- The LC oscillators are not stable enough for the communications or broadcast purpose.
- Thus the direct methods cannot be used for the broadcast applications.
- The alternative method is to use the indirect method called as the Armstrong method of FM generation.
- In this method the FM is obtained through phase modulation. A crystal oscillator can be used hence the frequency stability is very high.
- This method is widely used in practice. The block diagram of the Armstrong method is as shown in Fig. 4.23.1.



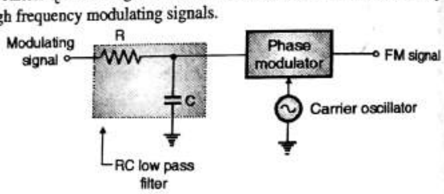
(D-202) **Fig. 4.23.1 : Indirect method [Armstrong method] of FM generation**

- The Armstrong method uses the phase modulation to generate frequency modulation.
- This method can be understood by dividing it into four parts.

Part 1 : How to obtain FM from phase modulator ?
Part 2 : Implementation of phase modulator.
Part 3 : Combining parts 1 and 2 to obtain the indirect method.
Part 4 : Use of frequency multipliers and amplifiers.

Part 1 : How to generate FM from PM ?

- In PM along with the phase variation, some frequency variation also takes place. Higher modulating voltages produce greater phase shift which in turn produces greater frequency deviation.
- And higher modulating frequencies produce a faster rate of change of modulating voltage hence they also produce greater frequency deviation.
- Thus in PM the carrier frequency deviation is proportional to modulating voltage as well as the modulating frequency.
- But in FM the frequency deviation is only proportional to the modulating voltage regardless of its frequency.
- To correct this problem the modulating signal is passed through a low pass RC filter as shown in Fig. 4.23.2. Due to this the high frequency modulating signals are attenuated but there is no change in the amplitudes of low frequency modulating signals.
- The filter output is then applied to a phase modulator alongwith the carrier as shown in Fig. 4.23.2.
- Hence the extra deviation in the carrier f_c due to higher modulating frequency is compensated by reducing the amplitude of the high frequency modulating signals.
- Hence the frequency deviation at the output of the phase modulator will be effectively proportional only to the modulating voltage and we obtain an FM wave at the output of phase modulator.



(D-203) **Fig. 4.23.2 : Generation of FM using phase modulation**

Due to this arrangement the frequency deviation at the output of phase modulator, corresponding to higher modulating frequencies is reduced. The result is FM produced by a phase modulator.

- This can be proved mathematically as follows :

Consider the expression for a PM wave,

$$e_{PM} = A \sin [\omega_c t + m_p \sin \omega_m t] \quad ...(4.23.1)$$

$$\text{Let } e_{PM} = A \sin \theta \text{ where } \theta = [\omega_c t + m_p \sin \omega_m t] \quad ...(4.23.2)$$

The instantaneous angular frequency of the PM wave is defined as,

$$\omega_i = \frac{d\theta}{dt} = \frac{d}{dt} [\omega_c t + m_p \sin \omega_m t] = \omega_c + m_p \cos \omega_m t \times \omega_m$$

$$\therefore f_i = f_c + m_p f_m \cos \omega_m t \quad ...(4.23.3)$$

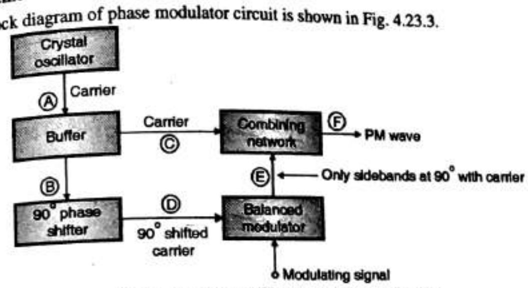
The second term in the RHS of Equation (4.23.3) is the frequency deviation. The maximum deviation is given by,

$$\delta_{max} = f_m m_p \quad ...(4.23.4)$$

As m_p is proportional to the modulating voltage, the frequency f_i in the Equation (4.23.3) will vary in proportion with the modulating voltage. Thus frequency modulation can be obtained using PM. Now let us see how to implement the phase modulator.

Part 2 : Implementation of the phase modulator :

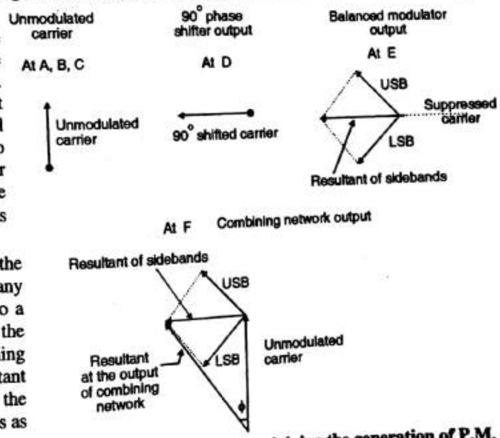
The block diagram of phase modulator circuit is shown in Fig. 4.23.3.



(D-204) **Fig. 4.23.3 : Phase modulator circuit**

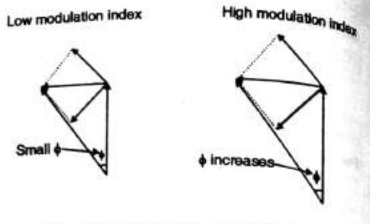
Operation :

- The crystal oscillator produces a stable unmodulated carrier which is applied to the "90° phase shifter" as well as the "combining network" through a buffer.
- The 90° phase shifter produces a 90° phase shifted carrier. It is applied to the balanced modulator alongwith the modulating signal. Thus the carrier used for modulation is 90° shifted with respect to the original carrier.
- At the output of the balanced modulator we get DSB-SC signal i.e. A.M. signal without carrier. This signal consists of only two sidebands with their resultant in phase with the 90° shifted carrier as shown in Fig. 4.23.4.
- The two sidebands and the original carrier without any phase shift are applied to a combining network. At the output of the combining network we get the resultant of vector addition of the carrier and two sidebands as shown in Fig. 4.23.4.



(D-205) **Fig. 4.23.4 : Phasors explaining the generation of P.M.**

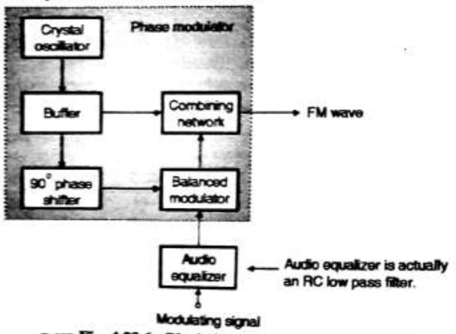
- Now as the modulation index is increased, the amplitude of sidebands will also increase. Hence the amplitude of their resultant increases. This will increase the angle "ϕ" made by the resultant with unmodulated carrier. The angle "ϕ" decreases with reduction in modulation index as shown in Fig. 4.23.5. Thus the resultant at the output of the combining network is phase modulated. Hence the block diagram of Fig. 4.23.5 operates as a phase modulator.



(D-206) **Fig. 4.23.5 : Effect of modulation index on ϕ**

Part 3 : Combine parts 1 and 2 :

- Combining parts 1 and 2 we get the block diagram of the Armstrong method of FM generation as shown in Fig. 4.23.6.
- The audio equalizer block shown in Fig. 4.23.6 is nothing but an RC low pass filter. The role of RC filter has already been discussed in part - 1.
- The modulating signal is passed through the audio equalizing circuit and applied to the phase modulator circuit.
- We get the FM wave at the output of the combining network. Thus in the indirect method of FM generation we use phase modulation to obtain FM.



(D-207) **Fig. 4.23.6 : Block diagram of indirect method**

Part 4 : Use of Frequency Multipliers, Mixer and Amplifier :

The FM signal produced at the output of phase modulator has a low carrier frequency and low modulation index. They are increased to an adequately high value with the help of frequency multipliers and mixer. The power level is raised to the desired level by the amplifier.

Summary of operation of the Armstrong Method :

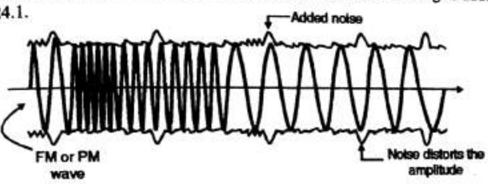
Now refer to Fig. 4.23.1. The operation of the Armstrong method is as follows :

- The crystal oscillator generates the carrier at low frequency typically at 1 MHz. This is applied to the combining network and a 90° phase shifter.

- The modulating signal is passed through an audio equalizer to boost the low modulating frequencies, for the reason discussed earlier. The modulating signal is then applied to a balanced modulator.
- The balanced modulator produces two sidebands such that their resultant is 90° phase shifted with respect to the unmodulated carrier.
- The unmodulated carrier and 90° shifted sidebands are added in the combining network.
- As discussed earlier, at the output of the combining network we get FM wave. This FM wave has a low carrier frequency f_c and low value of the modulation index m_f.
- The carrier frequency and the modulation index are then raised by passing the FM wave through the first group of multipliers. The carrier frequency is then raised by using a mixer and then the f_c and m_f both are raised to the required high values using the second group of multipliers. The effect of multiplication and mixing is as discussed earlier.
- The FM signal with high f_c and high m_f is then passed through a class C power amplifier to raise the power level of the FM signal.

4.24 Effect of Noise on the Angle Modulated Wave :

- As the F.M. or P.M. wave travels from transmitter to receiver, the noise gets added to it as shown in Fig. 4.24.1.



(D-208) **Fig. 4.24.1 : Effect of noise on FM and PM**

- Due to noise the amplitude of the FM or PM wave gets distorted.
- But FM or PM waves do not contain the information in their amplitude variations at all.
- Hence the noise cannot distort the information contained in the FM or PM wave.
- So FM and PM waves are more immune to noise or more noise resistant.

4.24.1 Advantages of FM :

1. Improved noise immunity.
2. Low power is required to be transmitted to obtain the same quality of received signal at the receiver.
3. F.M. transmission covers a larger area with the same amount of transmitted power.
4. Transmitted power remains constant.
5. All the transmitted power is useful.

4.24.2 Disadvantages of FM :

1. Very large bandwidth is required.
2. Since the space wave propagation is used, the radius of transmission is limited by the line of sight.
3. FM transmission and reception equipments are complex.

4.24.3 Applications of FM :

Some of the applications of FM are :

1. Radio broadcasting (Vividh Bharti, Radio Mirchi).
2. Sound broadcasting in T.V.
3. Satellite communication.
4. Police wireless.
5. Point to point communication.

4.25 Comparison of FM and AM Systems :

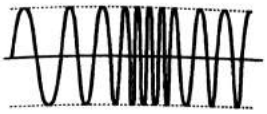
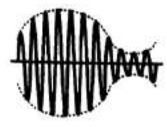
MU : May 03, Dec. 03, May 04, Dec. 05, Dec. 09, Dec. 12, May 14

University Questions

Q. 1 Compare with proper justification AM and FM with reference to power requirements, noise immunity, bandwidth and applications. **(May 03, 8 Marks)**

Q. 2 Compare AM with FM. **(Dec. 03, 4 Marks, May 04, Dec. 05, Dec. 09, May 14, 5 Marks)**

Q. 3 Distinguish between AM and FM. **(Dec. 12, 5 Marks)**

Sr. No.	FM	AM
1.	Amplitude of FM wave is constant. It is independent of the modulation index.	Amplitude of AM wave will change with the modulating voltage.
2.	Hence transmitted power remains constant. It is independent of m_f.	Transmitted power is dependent on the modulation index.
3.	All the transmitted power is useful.	Carrier power and one sideband power are useless.
4.	FM receivers are immune to noise.	AM receivers are not immune to noise.
5.	It is possible to decrease noise further by increasing deviation.	This feature is absent in AM.
6.	Bandwidth = $2 [\delta + f_m]$. The bandwidth depends on modulation index.	BW = $2f_m$. It is not dependent on the modulation index.
7.	BW is large. Hence wide channel is required.	BW is much less than FM.
8.	Space wave is used for propagation. So radius of transmission is limited to line of sight.	Ground wave and sky wave propagation is used. Therefore larger area is covered than FM.
9.	Hence it is possible to operate several transmitters on same frequency.	Not possible to operate more channels on the same frequency.
10.	FM transmission and reception equipment are more complex.	AM equipments are less complex.
11.	The number of sidebands having significant amplitudes depends on modulation index m_f.	Number of sidebands in AM will be constant and equal to 2.
12.	The information is contained in the frequency variation of the carrier.	The information is contained in the amplitude variation of the carrier.
13.	FM wave : 	AM wave : (1-46) 
14.	**Applications :** Radio, TV broadcasting, police wireless, point to point communications.	**Applications :** Radio and TV broadcasting.

4.26 Functions of a Receiver :

- We have learnt about the different types of AM, their generation, and transmission. Now let us see how to receive them.
- At the receiver, signals from various transmitters at different frequencies are present. In addition to this noise is also present. The receiver is expected to receive the desired signal from this crowd of the signals.
- Therefore the functions that a receiver must perform in order to receive the desired signal are as follows :
  1. Select the desired signal from all the other unwanted signals.
  2. Amplify the desired signal.
  3. Demodulate (process opposite to modulation) the amplified signal.
  4. After demodulation, the original modulating signal is obtained which must be amplified.
  5. Apply the amplified demodulated signal to the loudspeaker.

4.27 Superheterodyne Receivers :

MU : May 05, May 07, May 09, May 13, Dec. 13, May 14, Dec. 14, May 16, Dec. 16

University Questions

Q. 1 Draw the block diagram of a superheterodyne receiver. Explain its working for reception of medium wave band AM signals, giving relevant frequencies. **(May 05, 10 Marks)**

Q. 2 What do you mean by superheterodyne receiver? Explain its working with neat waveform. **(May 07, 10 Marks)**

Q. 3 Draw and explain superheterodyne receiver for amplitude modulation. **(May 09, 5 Marks)**

Q. 4 Explain in detail superheterodyne AM receiver with waveforms at various points. **(May 13, 10 Marks)**

Q. 5 Explain in brief the principle of superheterodyne receiver. **(Dec. 13, 5 Marks)**

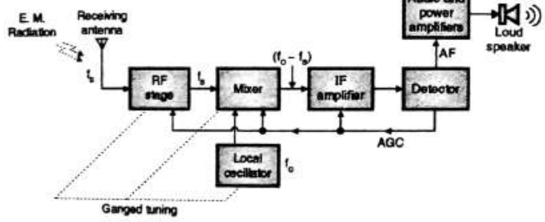
Q. 6 Explain with block diagram AM superheterodyne receiver. **(May 14, 10 Marks)**

Q. 7 Explain the working of a superheterodyne receiver with the help of a neat block diagram. Show the waveforms at the output of each block. **(Dec. 14, 10 Marks)**

Q. 8 Explain superheterodyne receiver in detail alongwith the waveforms at each stage. **(May 16, 10 Marks)**

Q. 9 With neat diagram and waveforms, explain the principle of operation of super heterodyne receiver. **(Dec. 16, 10 Marks)**

- The problems in the TRF receiver are solved in this receiver by converting every selected RF signal (station) to a fixed lower frequency called as the "intermediate frequency (IF)".
- This frequency contains the same modulation as the original carrier. The IF signal is then amplified and detected to get back the modulating signal. The intermediate frequency is lower than the lowest frequency that is to be received i.e. 530 kHz.
- As the "IF" is lower than the lowest RF signal frequency, the possibility of oscillations and instability is minimized.
- Also the required value of Q for constant BW does not depend on the frequency of desired input signal, because the "IF" is constant and same for all the incoming RF signals.
- Thus the superheterodyne receiver solves all the problems associated with the TRF receiver.
- The radio and TV receivers operate on the principle of **superheterodyning.** The block diagram of a superheterodyne radio receiver is shown in Fig. 4.27.1.



(D-230)**Fig. 4.27.1 : The superheterodyne receiver**

Operation :

- The DSBFC or AM signal transmitted by the transmitter travels through the air and reaches the receiving antenna. This signal is in the form of electromagnetic waves. It induces a very small voltage (few μV) into the receiving antenna.
- **RF stage :** The RF stage is an amplifier which is used to select the desired signal and reject other out of many, present at the antenna. It also reduces the effect of noise. At the output of the RF amplifier we get the desired signal at frequency "f_s".
- **Mixer :** The mixer receives signals from the RF amplifier at frequency (f_s) and from the local oscillator at frequency (f_o) such that $f_o > f_s$.
- **Intermediate frequency (IF) :** The mixer will mix these signals to produce signals having frequencies f_s, f_o, $(f_o + f_s)$ and $(f_o - f_s)$. Out of these the difference of frequency component i.e. $(f_o - f_s)$ is selected and all others are rejected. This frequency is called as the intermediate frequency (IF).

$$\therefore \ \text{I.F.} = (f_o - f_s) \qquad ...(4.27.1)$$

This frequency contains the same modulation as the original signal f_s.

- In order to maintain a constant difference between the local oscillator frequency and the incoming frequency, ganged tuning is used. This is simultaneous tuning of RF amplifier, mixer and local oscillator and it is achieved by using ganged tuning capacitors (Tuning control knob in radio set).
- This intermediate frequency signal is then amplified by one or more IF amplifier stages. IF amplifiers satisfy most of the gain (and hence sensitivity) and the bandwidth requirements of the receiver. Therefore the sensitivity and selectivity of this receiver do not change much with changes in the incoming frequency.
- The amplified IF signal is detected by the detector to recover the original modulating signal. This is then amplified and applied to the loudspeaker.
- AGC means automatic gain control. This circuit controls the gains of the RF and IF amplifiers automatically to maintain a constant output voltage level even when the signal level at the receiver input is fluctuating. This is done by feeding a controlling dc voltage to the RF and IF amplifiers. The amplitude of this dc voltage is proportional to the detector output.
- The sequence of operation of a superheterodyne receiver can be summarized as follows :

Summary of superheterodyne principle : (D-231)

- Select the desired station at frequency f_s by tuning the RF amplifier and local oscillator.

⬇

- Local oscillator is tuned to frequency f_o with $f_o > f_s$.

⬇

- Mixer produces IF. Note that IF = $(f_o - f_s)$

⬇

- Output of mixer is an AM signal with two sidebands and carrier equal to IF. The IF amplifier amplifies this signal

⬇

- Detector will demodulate this signal to recover the modulating signal. (AF signal)

⬇

- The audio amplifier and power amplifier will amplify the AF signal and apply it to the loud speaker

Definition of heterodyning :

The process of mixing two signals having different frequencies to produce a new frequency is called as heterodyning.

4.27.1 Waveforms at Various Points of a Superheterodyne Receiver :

The waveforms at different points of a superheterodyne receivers are as shown in Fig. 4.27.2.

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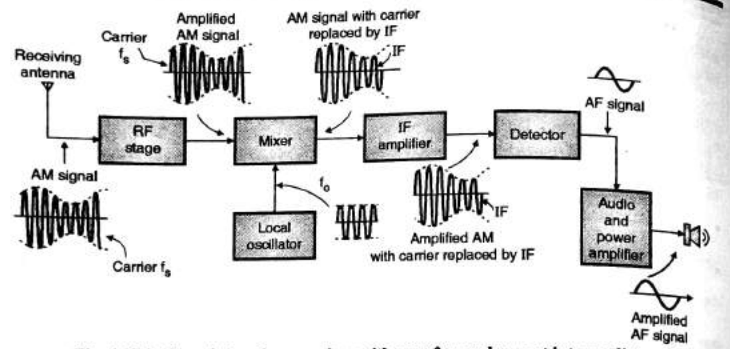


(D-232)Fig. 4.27.2 : Superheterodyne receiver with waveforms shown at intermediate points

- The selected input signal is amplified by the RF amplifier. Hence the waveform at its output is an amplified AM signal, with a carrier frequency f_s.
- Local oscillator produces a sinusoidal signal of frequency f_o, where $f_o = f_s + IF$.
- At the mixer output we get an AM signal with IF acting as a carrier. The same signal is then available in the amplified form at the output of the IF amplifier. Note that the shape of AM envelope at the input and output of IF amplifier is same as that of the input signal.
- This waveform is applied to a detector, which removes the IF and produces an AF signal at the detector output.
- This AF signal is amplified by the audio and power amplifier. Hence we get the amplified version of the AF signal.

4.27.2 Frequency Spectrums at Various Points of a Superheterodyne Receiver :

MU : May 07, May 13, Dec. 14, May 16, Dec. 16

University Questions

Q. 1 What do you mean by superheterodyne receiver? Explain its working with neat waveform. **(May 07, 10 Marks)**

Q. 2 Explain in detail superhetrodyne AM receiver with waveforms at various points. **(May 13, 10 Marks)**

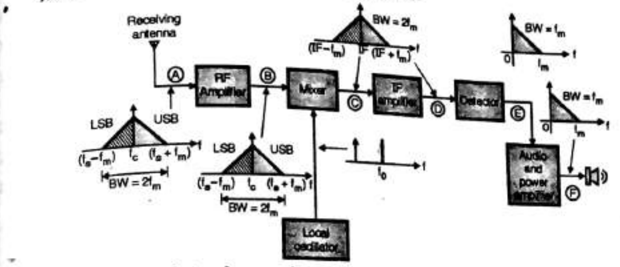
Q. 3 Explain the working of a superheterodyne receiver with the help of a neat block diagram. Show the waveforms at the output of each block. **(Dec. 14, 10 Marks)**

Q. 4 Explain superheterodyne receiver in detail alongwith the waveforms at each stage. **(May 16, 10 Marks)**

Q. 5 With neat diagram and waveforms, explain the principle of operation of super heterodyne receiver. **(Dec. 16, 10 Marks)**

- The waveforms drawn at various points in the previous section are in the time domain.
- But time domain representation does not give us any information about the frequencies present inside or the bandwidth of the signal.
- All this information can be obtained by drawing the frequency spectrums at various points in a receiver as shown in Fig. 4.27.3.

- Note that all these spectrums are **single sided spectrums.**



(D-233)Fig. 4.27.3 : Superheterodyne receiver with centre frequencies and bandwidth of each block

4.27.3 Advantages of Superheterodyning :

1. No variation in bandwidth. The BW remains constant over the entire operating range.
2. High sensitivity and selectivity. The selectivity also remains constant irrespective of the signal frequency.
3. High adjacent channel rejection.

4.27.4 Frequency Parameters of AM Receiver :

- The AM receiver has the following frequency parameters.
- Two frequency bands : Medium wave (MW) band
  Short wave (SW) band
- RF carrier range (MW band) : 535 kHz to 1650 kHz
  (SW band) : 5 to 15 MHz
- Intermediate frequency IF : 455 kHz
- IF bandwidth B : 10 kHz

Review Questions

Q. 1 What is line communication ?

Q. 2 What is radio communication ?

Q. 3 Why is "Modulation" used ?

Q. 4 Define the "bandwidth" of a baseband signal.

Q. 5 What is the function of a carrier in a modulation system ?

Q. 6 List different types of analog modulation systems.

Q. 7 How is the "information" transmitted using AM ?

Q. 8 Define modulation index of an AM wave.

Q. 9 What is the maximum value of "m" for distortion free transmission ?

Q. 10 Write down the expression for an AM wave.

Q. 11 What is the minimum bandwidth of an AM wave ?

Q. 12 When does overmodulation take place and what is its effect ?

Q. 13 Write down the expression for carrier power.

Q. 14 Write the expression for sideband power in AM.

Q. 15 What is the maximum value of sideband power in AM ?

Q. 16 What is the maximum transmitted power in AM ?

Q. 17 What is the other name of AM ?

Q. 18 Why is an AM signal get severely affected due to noise ?

Q. 19 What are the disadvantages of AM ?

Q. 20 State advantages of AM.

Q. 21 State applications of AM.

Q. 22 Define the term deviation in context with the frequency modulation.

Q. 23 How to vary the deviation ?

Q. 24 What is the role of modulating frequency f_m in frequency modulation ?

Q. 25 Define modulation index for FM. How is it different from that of AM ?

Q. 26 Why FM is called as a type of "angle modulation" ?

Q. 27 Express the FM signal mathematically and explain each terms

Q. 28 How to calculate the amplitudes of various sidebands in the spectrum of FM ?

Q. 29 Justify the total transmitted power in FM always remains constant.

Q. 30 State Carson's rule for bandwidth of FM wave.

Q. 31 What is the effect of m_f on the bandwidth of FM ?

Q. 32 With neat schematic diagram explain the operation of the basic communication system.

Q. 33 Explain the need of modulation.

Q. 34 Write a note on : Electromagnetic spectrum and its applications in communication field.

Q. 35 Define the term "Modulation" and explain the concept of amplitude modulation.

Q. 36 Give the classification of AM systems.

Q. 37 Derive the mathematical expression for the spectrum of an AM wave and plot it.

Q. 38 For the single tone AM, derive the expression for total transmitted power P_t in terms of P_c and m.

Q. 39 Define FM and draw the necessary waveforms to explain it.

Q. 40 Derive an equation for FM wave.

Q. 41 Compare AM and FM.

Q. 42 Write short note on : Frequency Spectrum of FM wave.

Q. 43 Compare AM with FM with special reference to power requirements signed to noise ratio and bandwidth required.

Q. 44 Derive the formula for instantaneous value of an FM voltage and define modulation index.

Q. 45 What do you understand by the term baseband signal ?

4.28 University Questions and Answers :

Q. 1 Derive the equation for an Amplitude modulated wave and explain the power relations. Calculate the percentage power saving when the carrier and upper side band are suppressed in an AM wave modulated to a depth of 100 percent and 50 percent. **(Dec. 2004, 10 Marks)**

Ans. :

1. For derivation of AM wave refer section 4.9.1.
2. For power relations refer section 4.11.
3. For the solution of problem refer Ex. 4.15.3.

□□□

CHAPTER 5

Analog Pulse Modulation

Module 5

Syllabus :

Statement of sampling theorem. Generation and detection of PAM, PPM, PWM.

5.1 Pulse Modulation :

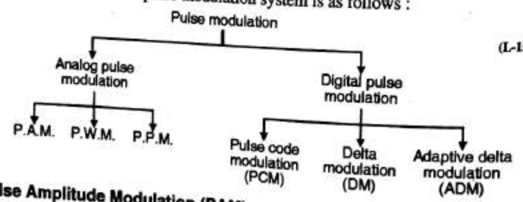
MU : May 07, Dec. 11, Dec. 14, May 15

University Questions

Q. 1 What are the various pulse modulation techniques ? **(May 07, Dec. 11, 5 Marks)**

Q. 2 Draw the PAM, PWM and PPM waveforms in time domain assuming a sinusoidal modulating signal. Explain them in brief. **(Dec. 14, May 15, 10 Marks)**

- In pulse modulation, the carrier is in the form of train of periodic rectangular pulses.
- Pulse modulation can be either analog or digital.
- In the analog pulse modulation, the amplitude, width or position of the rectangular carrier pulses is changed in accordance with the modulating signal.
- This will result in PAM (pulse amplitude modulation), PWM (pulse width modulation) or PPM (pulse position modulation) respectively.
- PAM, PWM and PPM the examples of analog pulse modulation.
- The pulse modulation can be digital as well. The well known examples of digital pulse modulation are pulse code modulation (PCM), delta modulation (DM), adaptive delta modulation (ADM), etc.
- The classification of the pulse modulation system is as follows :



(L-153)

1. **Pulse Amplitude Modulation (PAM) :**

The amplitude of a constant width, constant position rectangular carrier is varied in proportion with the instantaneous magnitude of the modulating signal as shown in Fig. 5.1.1(c).

2. **Pulse Width Modulation (PWM) :**

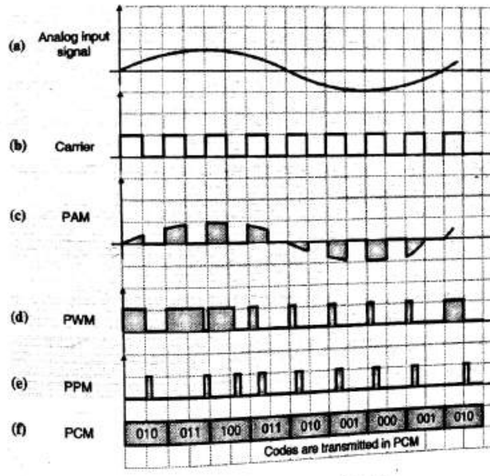
- The width of carrier pulses is made to vary in proportion with the instantaneous magnitude of the modulating signal as shown in Fig. 5.1.1(d).
- PWM is also called as pulse duration modulation (PDM) or pulse length modulation (PLM).

3. **Pulse Position Modulation (PPM) :**

- In PPM the amplitude and width of the pulses is kept constant but the position of each pulse is varied in accordance with the amplitudes of the sampled values of the modulating signal. The position of the pulses is changed with respect to the position of reference pulses.
- The PPM pulses can be derived from the PWM pulses as shown in Fig. 5.1.1(e). Note that with increase in the modulating voltage the PPM pulses shift further with respect to reference.

4. **Pulse Code Modulation (PCM) :**

- The analog message signal is sampled and converted to a fixed length, serial binary number as shown in Fig. 5.1.1(f).



(L-154) **Fig. 5.1.1 : Pulse modulation**

- In other words a binary code is transmitted. Hence the name pulse code modulation.
- The PAM, PWM and PPM are called as the analog pulse communication systems whereas PCM, delta modulation (DM) are the examples of digital pulse communication systems.

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What is the difference between analog pulse communication and digital pulse communication ?

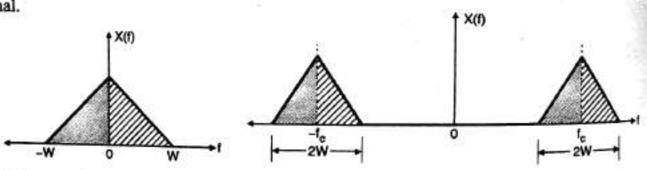
- For analog as well as digital pulse communication systems, the transmitted signal is a discrete time signal.
- In analog pulse communication, the information is transmitted in the form of change in amplitude, width or position of the rectangular carrier pulses. So the transmitted pulsed signal is still an analog signal.
- In digital pulse communication, the information is transmitted in the form of codes. Codeword are formed by grouping the digital pulses.
- Note that for digital pulse communication we do not change amplitude, frequency or phase of the transmitted signal. Thus the transmitted signal in digital pulse communication is a digital signal.

Practical use :

- PAM does not have a good noise immunity. So its practical use is restricted.
- PWM and PPM are used for some military applications but are not used for commercial communication applications.
- PCM is the most useful method of all.

5.1.1 Low Pass and Band Pass Signals :

- A low pass signal is defined as the signal which has a frequency spectrum extending right from 0 Hz to W Hz, if a single sided spectrum is plotted and it extends from – W to + W, if a double sided spectrum is plotted as shown in Fig. 5.1.2(a).
- A video signal which has a frequency spectrum from 0 to 5 MHz is an example of low pass signal.



(a) Spectrum of a low pass signal **(b) Spectrum of a band pass signal**

(D-406) **Fig. 5.1.2**

- But the nature of the spectrum of a bandpass signal is completely different. It is as shown in Fig. 5.1.2(b). The bandwidth is 2W Hz but it is centered about a frequency $\pm f_c$ rather than zero. We assume that $f_c > W$, therefore $(f_c - W) > 0$.
- Thus bandpass signals are the signals having their spectrum extending from f_1 to f_2 with $f_2 > f_1$ and both f_1 and f_2 being of nonzero value. For example, the voice signal which has a spectrum extending from 20 Hz to 3.4 kHz is a bandpass signal.

5.2 Sampling Process :

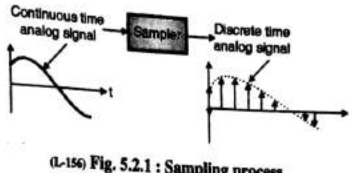
MU : Dec. 03, Dec. 12

University Questions

Q. 1 State and explain the significance of sampling theorem in pulse modulation schemes. **(Dec. 03, 8 Marks)**

Q. 2 Explain sampling technique principles. **(Dec. 12, 4 Marks)**

- In the pulse modulation and digital modulation systems, the signal to be transmitted must be in the discrete time form.
- If the message signal is coming from a digital source (e.g. a digital computer) then it is in the proper for a digital communication system to be processed.



(L-156) **Fig. 5.2.1 : Sampling process**

- But this is not always the case. The message signal can be analog in nature (e.g. speech or video signal).
- In such a case it has to be first converted into a discrete time signal. We use the "sampling process" to do this.
- Thus using the sampling process we convert a continuous time signal into a discrete time signal.
- For the sampling process to be of practical utility it is necessary to choose the sampling rate properly. The sampling process should satisfy the following requirements :
  1. Sampled signal should represent the original signal faithfully.
  2. We should be able to reconstruct the original signal from its sampled version.
- Fig. 5.2.1 summarizes the sampling process.

Thus sampling is the process of converting a continuous analog signal to a discrete analog signal and the sampled signal is the discrete time representation of the original analog signal.

5.3 Sampling Theorem for Low Pass Signals :

MU : Dec. 03, May 04, May 06, Dec. 07, May 10, May 11, Dec. 11, Dec. 12, May 13, May 14

University Questions

Q. 1 State and explain the significance of sampling theorem in pulse modulation schemes. **(Dec. 03, 8 Marks)**

Q. 2 State and prove the sampling theorem for low pass band limited signal. **(May 04, 10 Marks, May 06, Dec. 07, 6 Marks)**

Q. 3 Sate and prove the sampling theorem for Low pass signal. **(May 10, May 11, May 13, 10 Marks)**

Q. 4 Write short notes on : Sampling theorem for low pass bandlimited signal. **(Dec. 11, 7 Marks)**

Q. 5 Explain sampling theorem for low pass and band pass filters. **(Dec. 12, 6 Marks)**

Q. 6 State sampling theorem. What happens if the sampling is done at less than 2 f_{max}. **(May 14, 10 Marks)**

- In order to represent the original message signal "faithfully" (without loss of information), it is necessary to take as many samples of the original signal as possible.
- Higher the number of samples, closer is the representation.
- The number of samples depends on the "sampling rate" and the maximum frequency of the signal to be sampled.
- Sampling theorem was introduced to the communication theory in 1949 by Shannon. Therefore this theorem is also called as **"Shannon's sampling theorem"**.
- The statement of sampling theorem in time domain, for the bandlimited signals of finite energy is as follows :

Statement :

1. If a finite energy signal x (t) contains no frequencies higher than "W" Hz (i.e. it is a band limited signal) then it is completely determined by specifying its values at the instants of time which are spaced (1/2W) seconds apart.
2. If a finite energy signal x (t) contains no frequency components higher than "W" Hz then it may be completely recovered from its samples which are spaced (1/2W) seconds apart.

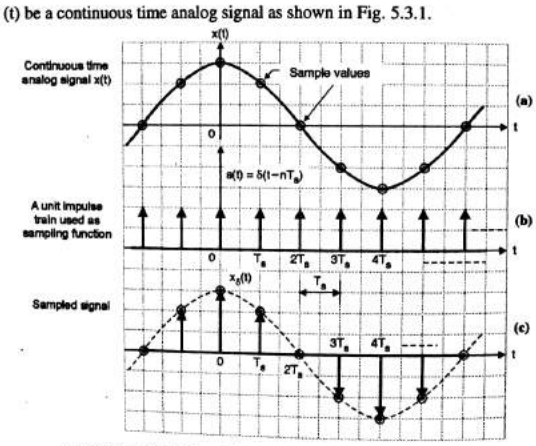
Combined statement of sampling theorem : A continuous time signal x (t) can be completely represented in its sampled form and recovered back from the sampled form if the sampling frequency $f_s \geq 2W$ where "W" is the maximum frequency of the continuous time signal x (t).

5.3.1 Proof of Sampling Theorem :

Let us now prove the sampling theorem in time domain. The assumptions made for this proof are as follows :

Assumptions :

Let x (t) be a continuous time analog signal as shown in Fig. 5.3.1.



(D-403) **Fig. 5.3.1 : Sampling of a continuous time signal x (t)**

- Let x (t) be a signal with finite energy and infinite duration.
- Let x (t) be a strictly bandlimited signal. That means it does not contain any frequency components above "W" Hz.
- Let s (t) be the sampling function as shown in Fig. 5.3.1. It is a train of unit impulses, spaced by a period of T_s seconds. This sampling function samples the original signal at a rate of "f_s" samples per second. Therefore "T_s" represents the sampling period such that,

$$T_s = \frac{1}{f_s} = \text{Sampling period} \qquad ...(5.3.1)$$

and $f_s = \frac{1}{T_s}$ = Sampling rate.

Procedure to be followed :

We are going to follow the steps given below to prove the sampling theorem :

- Step 1 : Represent the sampling function s (t) mathematically.
- Step 2 : Represent the sampled signal x_δ (t) mathematically.
- Step 3 : Obtain the Fourier transform of the sampled signal.
- Step 4 : Prove that the sampled signal x_δ (t) completely represents x (t).
- Step 5 : Represent x (t) as summation of sinc functions (interpolation).
- Step 6 : Graphical representation of the interpolation process.
- Step 7 : Actual recovery of x (t) using an ideal low pass filter.

Part 1 : Sampling theorem :

Spectrum of the sampled signal :

Step 1 : Represent the sampling function s (t) mathematically :

- Fig. 5.3.1 shows the sampling function s (t) which is a train of unit impulses.
- The spacing between the adjacent unit impulses is T_s seconds, therefore the frequency of the sampling function is equal to the sampling frequency f_s.
- The sampled signal is denoted by x_δ (t) and it is as shown in Fig. 5.3.1.
- The sample function s (t) can be represented mathematically as follows :

$$s(t) = \text{.........}\delta(t+2T_s)+\delta(t+T_s)+\delta(t)+\delta(t-T_s)+\delta(t-2T_s)+\text{.....}$$

$$\therefore\ s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \qquad ...(5.3.2)$$

Step 2 : Represent the sampled signal x_δ (t) mathematically :

- Fig. 5.3.1 shows the sampled signal x_δ (t) graphically. It is present only at the sampling instants i.e. T_s, 2 T_s etc. and its instantaneous amplitude is equal to the amplitude of original signal x (t) at the sampling instants.
- This is shown by the encircled points in Fig. 5.3.1. Let us represent the instantaneous amplitude of x (t) at the various sampling points t = n T_s as x (n T_s). This is the amplitude of the encircled points of Fig. 5.3.1.
- Looking at the sampled signal x_δ (t) we can say that the sampled signal is obtained by multiplying x (t) and s (t). ...(5.3.3)

$$\therefore\ x_\delta(t) = x(t)\times s(t) = x(nT_s)\times s(t)$$

- Substituting the expression for s (t) from Equation (5.3.2) we get the mathematical expression for the sampled signal x_δ (t) as,

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\cdot\delta(t-nT_s) \qquad ...(5.3.4)$$

Step 3 : Obtain the Fourier transform of the sampled signal :

- The fourier transform of a train of impulses (dirac delta function) is given by,

$$X(f) = f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$$

- Here we have the similar pulse train as sampling function s (t). Therefore the Fourier transform of the sampling function is given by,

$$S(f) = f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad ...(5.3.5)$$

- Note that f_0 has been replaced by f_s in the above equation.
- The sampled signal in the time domain is represented as product of x (t) and s (t).

i.e. $$x_\delta(t) = x(t) \times s(t) \quad ...(5.3.6)$$

- Taking the Fourier transform of both the sides we get,

i.e. $$X_\delta(f) = X(f) * S(f) \quad ...(5.3.7)$$

- This is because the Fourier transform of the product of two signals in the time domain is the convolution of their Fourier transforms. Substituting the value of S(f) from Equation (5.3.5) we get,

$$X_\delta(f) = X(f) * \left[f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right] \quad ...(5.3.8)$$

where * denotes convolution. Interchanging the orders of convolution and summation results in :

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s) \quad ...(5.3.9)$$

- From the properties of delta function, we find that the convolution of X (f) and $\delta(f - nf_s)$ is equal to $X(f - nf_s)$. Hence the above equation can be simplified as follows :

F.T. of the sampled signal, $$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \quad ...(5.3.10)$$

where X (f) = Fourier transform of the original signal x (t).

Conclusion from Equation (5.3.10) :

1. The term $X(f - nf_s)$ in Equation (5.3.10) represents the shifted version of the spectrum X (f) of the original signal x (t). Thus depending on the value of "n" (which extends from $-\infty$ to $+\infty$) we will get infinite number of original spectrums X (f) centered at frequencies $0, \pm f_s, \pm 2f_s, \pm 3f_s, \pm 4f_s$ ...etc. In other words,

$$X(f - nf_s) = X(f) \text{ at } f = 0, \pm f_s, \pm 2f_s, \pm 3f_s \quad ...(5.3.11)$$

2. This concept will be clear if we open Equation (5.3.10) and write the terms separately as shown below.

Now open the summation sign in Equation (5.3.10) to get,

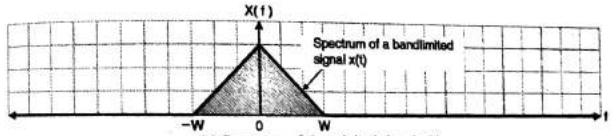
(D-409)

$$X_\delta(f) = \ldots\ldots + \underbrace{f_s X(f + 2f_s)}_{X(f) \text{ shifted left by } 2f_s} + \underbrace{f_s X(f + f_s)}_{X(f) \text{ shifted left by } f_s} + \underbrace{f_s X(f)}_{\text{Spectrum } X(f)} + \underbrace{f_s X(f - f_s)}_{X(f) \text{ shifted right by } f_s} + \ldots\ldots$$

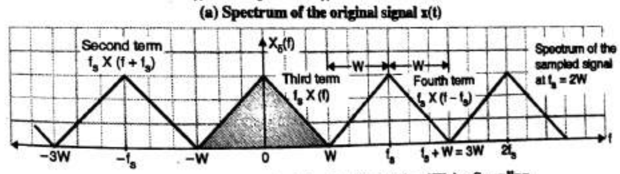
The spectrum $X_\delta(f)$ of the sampled signal is plotted as shown in Fig. 5.3.2.

Equation (5.3.10) can also be written as :

$$X_\delta(f) = f_s X(f) + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} f_s X(f - nf_s) \quad ...(5.3.12)$$



(a) Spectrum of the original signal x(t)



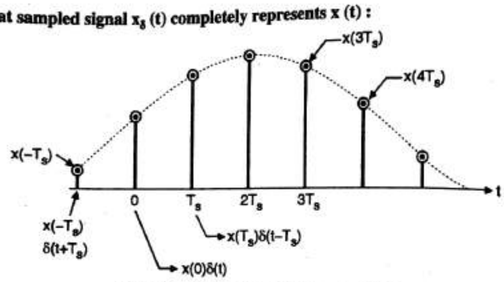
(b) Spectrum of the sampled signal $x_\delta(t)$ with $f_s = 2W$, i.e. Sampling is done exactly at Nyquist rate

(D-410) **Fig. 5.3.2**

Comment :

From Equation (5.3.12) we conclude that the process of uniform sampling of a signal in the time domain results in a periodic spectrum in the frequency domain with a period equal to the sampling rate f_s.

3. Prove that sampled signal $x_\delta(t)$ completely represents $x(t)$:



(D-411) **Fig. 5.3.3 : Sampled signal $x_\delta(t)$**

- $x_\delta(t)$ can be represented in the summation form as follows (Refer Fig. 5.3.3).

$$x_\delta(t) = \ldots x(-T_s)\,\delta(t+T_s) + x(0)\,\delta(t) + x(T_s)\,\delta(t-T_s) + \ldots \qquad \ldots(5.3.12(a))$$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\cdot\delta(t-nT_s)$$

- We can obtain another useful expression for the fourier transform $X_\delta(f)$ by taking the fourier transform of both the sides of the equation stated above as,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-j2\pi n f T_s} \qquad \ldots(5.3.13)$$

- This equation is the fourier transform of a discrete time signal $x_\delta(t)$. Therefore it is called as the discrete fourier transform (DFT). Compare it with the definition of fourier transform of a continuous time signal. i.e.

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

- As the signal is discrete, the integration sign has been replaced by the summation sign and "t" has been replaced by "nT_s".
- Now consider Equation (5.3.12)

$$X_\delta(f) = f_s X(f) + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} f_s X(f-nf_s)$$

$$\therefore\ X(f) = \frac{1}{f_s} X_\delta(f) - \sum X(f-nf_s)$$

- But in the range $-W \le f \le W$ the second term of the above expression will not be present

$$\therefore\ X(f) = \frac{1}{f_s} X_\delta(f) \qquad \ldots(5.3.14)$$

- Substitute $f_s = 2W$ and $X_\delta(f)$ from Equation (5.3.13) to get,

$$X(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} x(nT_s)\cdot e^{-j2\pi n f T_s} \qquad \ldots(5.3.15)$$

- This is the frequency spectrum of $x(t)$ in terms of $x(nT_s)$ i.e. the sampled signal. Substitute $T_s = 1/2W$ to get,

$$X(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} x(n/2W)\cdot e^{-j2\pi n f/2W} \quad \ldots -W \le f \le W \qquad \ldots(5.3.16)$$

- This equation shows that the spectrum of $x(t)$ is same as the spectrum of $x_\delta(t)$ in the frequency range $-W$ to $+W$. Hence the sampled signal represents the original signal $x(t)$ successfully.

Thus if the sample values $x(n/2W)$ of the signal $x(t)$ are specified for all time, then the Fourier transform $X(f)$ of the original signal is uniquely determined by using the Equation (5.3.16). Because $x(t)$ is related to $X(f)$ by the inverse Fourier transform, it follows that the signal $x(t)$ is itself uniquely determined by the sample values $x(n/2W)$ for $-\infty \le n \le \infty$. Or in other words the sequence of samples $\{x(n/2W)\}$ contains all the information of $x(f)$.

- Thus we have proved first part of the sampling theorem.

Part 2 of the sampling theorem :

4. Reconstruction of signal from samples :

- This is the second part of the sampling theorem. From Equation (5.3.16) we can obtain $x(t)$ by taking the inverse fourier transform (IFT).

$$x(t) = \text{IFT}\{X(f)\}$$

$$= \text{IFT}\left\{\frac{1}{2W}\sum_{n=-\infty}^{\infty} x(n/2W).\, e^{-j\pi f n/W}\right\}$$

- Using the definition of inverse Fourier transform,

$$x(t) = \int_{-W}^{W} \frac{1}{2W}\sum_{n=-\infty}^{\infty} x(n/2W)\cdot e^{-j\pi f n/W}\, e^{j2\pi f t}\, df$$

- Interchanging the order of summation and integration we get,

$$x(t) = \sum_{n=-\infty}^{\infty} x(n/2W)\frac{1}{2W}\int_{-W}^{W} e^{j2\pi f\left(t-\frac{n}{2W}\right)}\,df$$

$$x(t) = \sum_{n=-\infty}^{\infty} x(n/2W)\cdot\frac{1}{2W}\times\frac{1}{j2\pi\left[t-\frac{n}{2W}\right]}\cdot\left[e^{j2\pi f(t-n/2W)}\right]_{-W}^{W}$$

$$x(t) = \sum_{n=-\infty}^{\infty} x(n/2W)\,\frac{1}{j4\pi W\left[t-\frac{n}{2W}\right]}\cdot\left[e^{j2\pi W(t-n/2W)} - e^{-j2\pi W(t-n/2W)}\right]$$

$$= \sum_{n=-\infty}^{\infty} x(n/2W)\cdot\left[\frac{e^{j2\pi W(t-n/2W)} - e^{-j2\pi W(t-n/2W)}}{j4\pi W\left[t-\frac{n}{2W}\right]}\right]$$

- The term inside the square bracket is a "sine" function.

$$\therefore\; x(t) = \sum_{n=-\infty}^{\infty} x(n/2W)\frac{\sin(2\pi Wt-n\pi)}{(2\pi Wt-n\pi)} \qquad ...(5.3.17)$$

- We can simplify the equation above by using the definition of the "sinc function". The sinc function is defined as :

$$\text{sinc}\, x = \frac{\sin(\pi x)}{\pi x} \qquad ...(5.3.18)$$

- Therefore Equation (5.3.17) can be written as :

$$x(t) = \sum_{n=-\infty}^{\infty} x(n/2W)\,\text{sinc}\,(2Wt-n) \qquad ...(5.3.19)$$

- Equation (5.3.19) provides an interpolation formula for reconstructing the original signal x (t) from the sequence of sample values $\{x(n/2W)\}$. The "sinc" function plays the role of an interpolation function. Each sample x (n / 2W) is multiplied by a delayed version of the interpolation function i.e. sinc function. Then all these resulting waveforms are added to obtain x (t).

5. **Graphical representation of the interpolation process :**

Let us re-arrange Equation (5.3.19) as follows :

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\text{sinc}\,2W\left(t-\frac{n}{2W}\right)$$

This is because $\frac{1}{2W} = T_s$

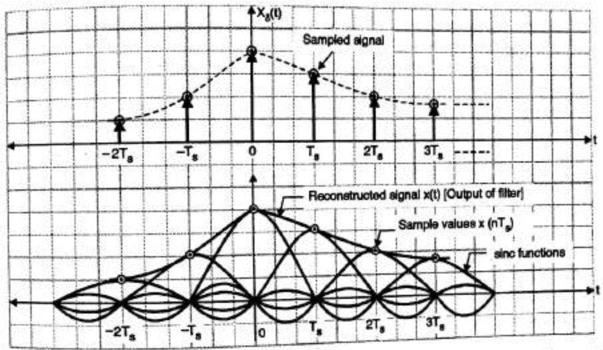
$$\therefore\; x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\text{sinc}\,2W(t-nT_s) \qquad ...(5.3.20)$$

Let us expand this equation to write,

$$x(t) = x(0)\,\text{sinc}\,2Wt + x(\pm T_s)\,\text{sinc}\,2W(t\pm T_s) + x(\pm 2T_s)\,\text{sinc}\,2W(t\pm 2T_s) + \dots \qquad ...(5.3.21)$$

(a) **First term : x (0) sinc 2Wt :**

- This will have a maximum amplitude at t = 0. The maximum amplitude is equal to the sample value x (0) at t = 0. This sinc function will pass through zeros at t = ± 1/2 W, ± 1/4 W ...etc. This is as shown in Fig. 5.3.4.



(D-412) **Fig. 5.3.4 : Reconstruction of the original signal x (t) from its samples using the interpolation**

(b) **Second term : x (± T_s) sinc 2W (t ± T_s) :**

- This sinc function will have maximum amplitude at t = ± T_s. The maximum amplitude is equal to the sample value x (± T_s) at t = ± T_s respectively. Thus sinc 2W (t ± T_s) represents shifted sinc function i.e. "sinc 2Wt" by a period ± T_s. This is as shown in Fig. 5.3.4.
- Similarly the third term, x (± $2T_s$) sinc 2W (t ± $2T_s$) represents shifted sinc function "sinc 2Wt" by a period of ± $2T_s$ and so on. We can plot all these sinc functions along with the sampled signal x_δ (t) as shown in Fig. 5.3.4. Note that the peak amplitude of any sinc function is equal to the corresponding sample value x (nT_s).

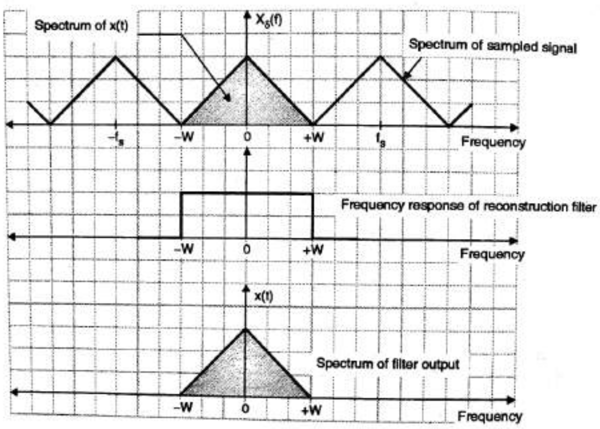
6. **Actual reconstruction of the original signal by using a low pass filter :**

- Thus the peaks of the sinc pulses represent the amplitudes of the samples.

- The signal x (t) expressed in Equation (5.3.19) is then passed through an ideal low pass filter to recover the original signal x (t). This low pass filter is therefore called as the reconstruction filter. This is shown in the graphical representation of Fig. 5.3.5(a).

(D-413) **Fig. 5.3.5(a) : Reconstruction filter**

- Assume that the cut-off frequency of the ideal low pass filter is adjusted precisely to W Hz. The frequency response of the reconstruction filter is shown in Fig. 5.3.5(b).



(D-414) **Fig. 5.3.5(b) : Operation of reconstruction**

- When the sampled signal $x_\delta(t)$ is applied at the input, this filter will allow only the shaded portion in the spectrum of x_δ (t) to pass through to the output and will block all other frequency components.
- Thus the frequency components only corresponding to x (t) will be passed through to the output and the original signal x (t) is recovered.

5.3.2 Aliasing or Foldover Error :

MU : Dec. 04, Dec. 07, May 14

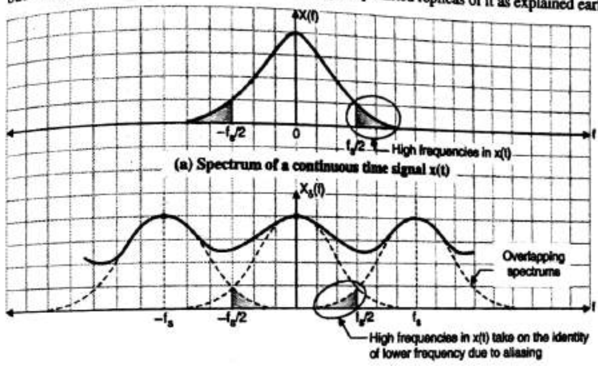
University Questions

Q. 1 Explain the following term : Aliasing effects. **(Dec. 04, 3 Marks)**

Q. 2 Explain aliasing error. **(Dec. 07, 2 Marks)**

Q. 3 State sampling theorem. What happens if the sampling is done at less than $2 f_{max}$. **(May 14, 10 Marks)**

- If the signal x (t) is not strictly bandlimited and / or if the sampling frequency f_s is less than 2W, then an error called **aliasing** or **foldover error** is observed. The adjacent spectrums overlap if $f_s < 2W$. This is shown in Fig. 5.3.6(b).
- The signal x (t) is not strictly bandlimited. The spectrum of signal x (t) is shown in Fig. 5.3.6(b).
- The spectrum X_δ (f) of the discrete time signal x_δ (t) is shown in Fig. 5.3.6(b) which is nothing but the sum of X (f) and infinite number of frequency shifted replicas of it as explained earlier.



(a) Spectrum of a continuous time signal x(t)

(b) Spectrum of the sampled version of x(t) with $f_s < 2W$

(D-415) **Fig. 5.3.6**

- Consider the two replicas of X (f) which are centered about the frequencies f_s and $-f_s$.
- If we use a reconstruction filter with its pass-band extending from $-f_s/2$ to $+f_s/2$ then its output will not be an undistorted version of the original signal x (t). Some distortion will be present in the filter output.
- The distortion occurs due to the overlapping of the adjacent spectrums as shown in Fig. 5.3.6(b). Due to this overlapping, it is seen that the portions of the frequency shifted replicas are "folded over" inside the desired spectrum.
- Due to this "fold over", high frequencies in X (f) are reflected into low frequencies in X_δ (f). This can be understood by comparing the shaded portions of the spectra shown in Fig. 5.3.6(a) and (b).
- **Aliasing :** This phenomenon of a high frequency in the spectrum of the original signal x (t), taking on the identity of lower frequency in the spectrum of the sampled signal x_δ (t) is called as aliasing or fold over error.

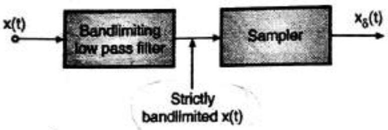
Effect of aliasing :

Due to aliasing some of the information contained in the original signal x (t) is lost in the process of sampling.

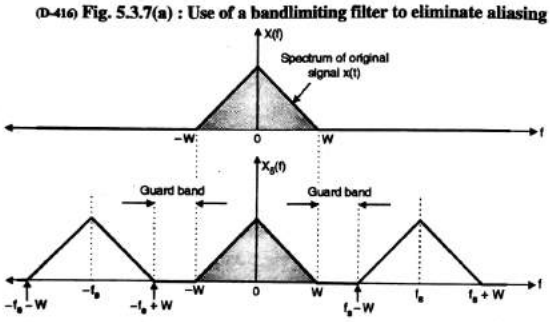
How to eliminate aliasing ?

Aliasing can be completely eliminated if we take the following action :

- Use a bandlimiting low pass filter and pass the signal x (t) through it before sampling as shown in Fig. 5.3.7(a).
- This filter has a cutoff frequency at $f_c = W$, therefore it will strictly bandlimit the signal x (t) before sampling takes place. This filter is also called as antialiasing filter or prealias filter.



(D-416) **Fig. 5.3.7(a) : Use of a bandlimiting filter to eliminate aliasing**



(D-417) **Fig. 5.3.7(b) : Spectrum of a sampled signal for $f_s > 2W$**

- Increase the sampling frequency f_s to a great extent i.e. $f_s >>> 2W$. Due to this, even though x (t) is not strictly bandlimited, the spectrums will not overlap. A guard band is created between the adjacent spectrums as shown in Fig. 5.3.7(b).

Thus aliasing can be prevented by :

1. Using an antialiasing or prealiasing filter and
2. Using the sampling frequency $f_s > 2W$.

5.3.3 Nyquist Rate and Nyquist Interval :

MU : May 03, Dec. 06, May 16

University Questions

Q. 1 What is meant by "Nyquist rate" in sampling ?

Q. 2 Explain Nyquist criteria. **(May 03, 3 Marks, Dec. 06, 4 Marks) (May 16, 5 Marks)**

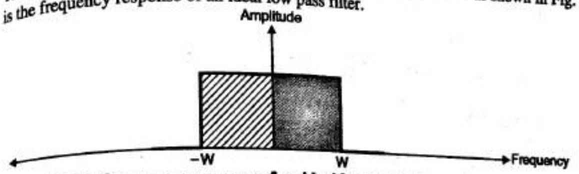
- The minimum sampling rate of "2W" samples per second for a signal x (t) having maximum frequency of "W" Hz is called as "Nyquist rate". The reciprocal of Nyquist rate i.e. 1/2W is called as the Nyquist interval.

$$\text{Nyquist rate} = 2W \text{ Hz}$$

$$\text{Nyquist interval} = 1/2W \text{ seconds}$$

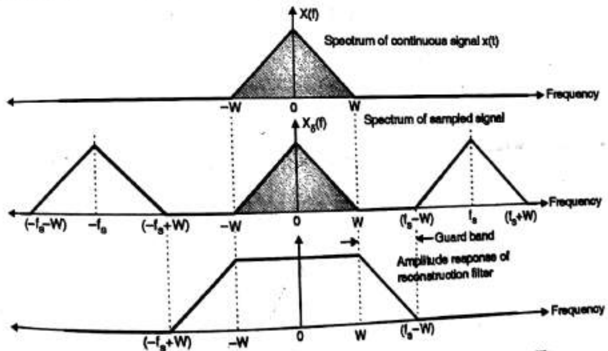
5.3.4 Effect of Nonideal Filter :

- As mentioned earlier the reconstruction filter is a low pass filter. It is expected to pass all the frequencies in the range of (– W to + W) Hz.
- This is because the original signal x (t) is bandlimited to "W" Hz.
- Therefore the frequency response of a reconstruction filter should be as shown in Fig. 5.3.8. This is the frequency response of an ideal low pass filter.



(D-418) **Fig. 5.3.8 : Frequency response of an ideal low pass filter used as a reconstruction filter**

- But it is not possible to practically realize an ideal low pass filter. Therefore a practical low pass filter with a frequency response as shown in Fig. 5.3.9 is used.
- It is possible to use the practical low pass filter without introducing any distortion due to the presence of the guard bands between the adjacent frequency spectrums as shown in Fig. 5.3.8. That is why it is necessary to have $f_s > 2W$.



(D-419) **Fig. 5.3.9 : Amplitude response of a practical reconstruction filter**

5.3.5 Examples on Sampling Theorem for Low Pass Signals :

Ex. 5.3.1 : Find the Nyquist rate and Nyquist interval for each of the following signals :

(a) $x(t) = 5 \cos 1000 \pi t \cos 4000 \pi t$. (b) $x(t) = \dfrac{\sin 200 \pi t}{\pi t}$.

Soln. :

(a) The given signal x (t) is in the form of product of cosine term. So let us use the following standard expression :

$$2 \cos A \cos B = \cos (A + B) + \cos (A - B)$$

$$\therefore 5 \cos 1000 \pi t \cos 4000 \pi t = 2.5 \cos 5000 \pi t + 2.5 \cos 3000 \pi t$$

$$\therefore \quad x(t) = 2.5 \cos 5000 \pi t + 2.5 \cos 3000 \pi t \quad ...(1)$$

From Equation (1) it is clear that the maximum frequency component present in the signal x (t) is of 2500 Hz. In other words x (t) is bandlimited to 2.5 kHz (W = 2.5 kHz).

$$\therefore \text{ Nyquist rate } = 2W = 2 \times 2.5 \text{ kHz } = 5 \text{ kHz}$$

...Ans.

$$\text{and Nyquist interval } = \frac{1}{2W} = \frac{1}{5 \times 10^3} = 0.2 \text{ msec}$$

...Ans.

(b) $x(t) = \dfrac{\sin 200 \pi t}{\pi t}$

- In order to calculate the Nyquist rate we need to calculate the maximum frequency component present in its spectrum. The spectrum of x (t) can be obtained by taking its fourier transform.
- Multiply numerator and denominator of x (t) by 200 to get,

$$x(t) = \frac{200 \sin (200\pi t)}{(200\pi t)}$$

$$\text{But } \frac{\sin \pi t}{\pi t} = \text{sinc } t$$

$$\therefore \frac{\sin (200\pi t)}{(200\pi t)} = \text{sinc } (200 t)$$

$$\therefore \quad x(t) = 200 \text{ sinc } (200 t)$$

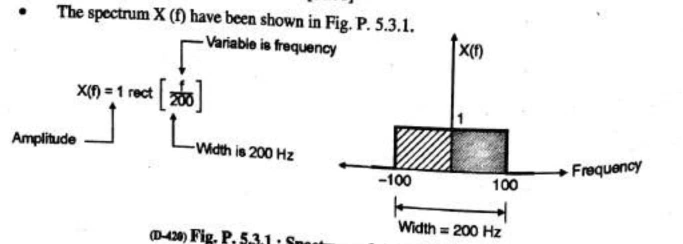
- We know that $A \text{ sinc } 2 Wt \overset{F}{\leftrightarrow} \dfrac{A}{2W} \text{ rect}\left[\dfrac{f}{2W}\right]$

$$\therefore \quad 200 \text{ sinc } (200 t) \overset{F}{\leftrightarrow} \frac{200}{200} \text{ rect}\left[\frac{f}{200}\right]$$

$$\therefore \quad 200 \text{ sinc } (200 t) \overset{F}{\leftrightarrow} \text{rect}\left[\frac{f}{200}\right]$$

$$\therefore \quad X(f) = \text{rect } [f/200]$$

- The spectrum X (f) have been shown in Fig. P. 5.3.1.



(D-420) **Fig. P. 5.3.1 : Spectrum of the given signal**

- From Fig. P. 5.3.1 the maximum frequency in the frequency spectrum is 100 Hz.

$$\text{Nyquist rate } = 2 \times 100 = 200 \text{ Hz}$$

...Ans.

$$\text{And Nyquist interval } = 1/200 = 5 \text{ mS}$$

...Ans.

5.3.6 Sampling Theorem for Bandpass Signals :

MU : Dec. 10, May 12

University Questions

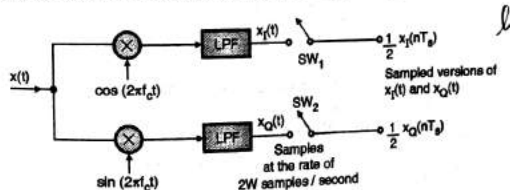
Q. 1 State and prove the sampling theorem for band pass filters. (Dec. 10, May 12, 10 Marks)

The sampling theorem for the bandpass signals can be stated as follows :

A bandpass signal x (t), having a maximum bandwidth of 2W Hz can be completely represented in its sampled form and recovered back from the sampled form if it is sampled at a rate which is at least twice the maximum bandwidth. (i.e. $f_s \geq 4W$.)

5.3.6.1 Quadrature Sampling of Bandpass Signals :

- In this section, we consider a scheme called "quadrature sampling" for the uniform sampling of bandpass signals. This scheme is actually a natural extension of the sampling of low pass signals. The scheme is as follows :
- In this scheme, we do not sample the bandpass signal directly. Instead, before sampling we represent the bandpass signal x (t) in terms of its "in-phase" and "quadrature" components, $x_I(t)$ and $x_Q(t)$ respectively.
- The in-phase and quadrature components can be obtained by multiplying the bandpass signal x (t) by cos $(2\pi f_c t)$ and sin $(2\pi f_c t)$ respectively and then by suppressing the sum frequency components by means of low pass filters as shown in Fig. 5.3.10(a).



hey hey hey hey hey Hey

(D-423) **Fig. 5.3.10(a) : Generation of in-phase and quadrature samples from the bandpass signal x (t)**

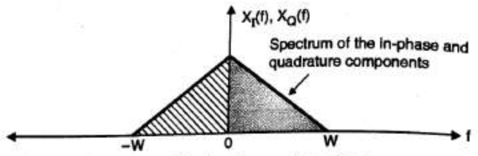
- If $x_I(t)$ = In-phase component and $x_Q(t)$ = Quadrature component.

Then we can express the bandpass signal x (t) in terms of $x_I(t)$ and $x_Q(t)$ as follows :

$$x(t) = x_I(t) \cos (2\pi f_c t) - x_Q(t) \sin (2\pi f_c t) \quad ...(5.3.22)$$

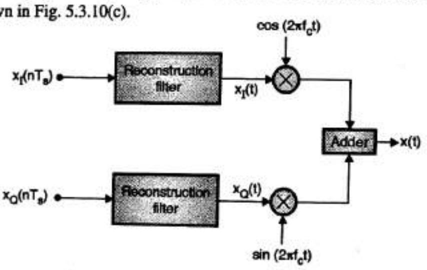
- Under the assumption of $f_c > W$, it is found that $x_I(t)$ and $x_Q(t)$ both are "low pass signals" extending from – W to + W as shown in Fig. 5.3.10(b).

hey



(D-424) **Fig. 5.3.10(b) : Spectrum of the in-phase and quadrature components of x (t)**

- Then both the in-phase and quadrature components are separately sampled at a rate of 2W samples per second by the switches SW_1 and SW_2 as shown in Fig. 5.3.10(a) to obtain the sampled versions of $x_I(t)$ and $x_Q(t)$.
- In order to reconstruct the original bandpass signal from its quadrature sampled version, we first reconstruct the in-phase component $x_I(t)$ and quadrature component $x_Q(t)$ from their respective sampled versions $x_I(nT_s)$ and $x_Q(nT_s)$ by means of reconstruction filters. Then multiply $x_I(t)$ and $x_Q(t)$ by $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ respectively and add the result. The reconstruction process of x (t) is shown in Fig. 5.3.10(c).



(D-425) **Fig. 5.3.10(c) : Reconstruction of the bandpass signal x(t)**

Ex. 5.3.2 : The given signal is $m(t) = 10 \cos 2000 \pi t \cos 8000 \pi t$

(a) What is the minimum sampling rate based on the low pass uniform sampling theorem ?

(b) Repeat (a) based on the bandpass sampling theorem.

Soln. :

(a)

$$m(t) = 10 \cos 2000 \pi t \cos 8000 \pi t$$

$$\therefore \; m(t) = 5 \cos 10000 \pi t + 5 \cos 6000 \pi t \quad ...(1)$$

Thus the highest frequency present in m (t) is W = 5000 Hz. Therefore as per the low pass sampling theorem, the minimum sampling rate is given by,

$$f_{s\,min} = 2W = 2 \times 5000 = 10 \text{ kHz} \quad \text{...Ans.}$$

(b) Looking at Equation (1) it is clear that the given signal m (t) contains two frequency components which are,

$$f_1 = 3000 \text{ Hz and } f_2 = 5000 \text{ Hz}$$

The bandwidth of m (t) is,

$$B = f_2 - f_1 = 2000 \text{ Hz}$$

When f_1 and f_2 are not harmonically related to the sampling frequency f_s, the bandpass sampling theorem stated in section 5.3.6 is stated in a more generalized form as follows :

If a bandpass signal x (t) has a bandwidth "B" and the highest frequency "f_M" then x (t) can be recovered from its sampled version if $f_s = \frac{2f_M}{k}$ where k is the largest integer not exceeding $\frac{f_M}{B}$. All higher sampling rates are not necessarily usable unless they exceed $2f_M$.

$$\text{Thus } f_M = f_2 = 5 \text{ kHz}$$

$$\text{and } B = 2 \text{ kHz}$$

$$\therefore \; k = \frac{5}{2} = 2.5. \text{ Therefore the value of k is 2}$$

$$\therefore \; f_s = \frac{2 \times 5 \text{ kHz}}{2} = 5 \text{ kHz} \quad \text{...Ans.}$$

Ex. 5.3.3 : A bandpass signal has a spectral range that extends from 20 kHz to 82 kHz. Find the sampling frequency f_s.

Soln. :

$$f_1 = 20 \text{ kHz and } f_2 = 82 \text{ kHz.}$$

$$\therefore \; \text{Bandwidth } B = f_2 - f_1 = 82 - 20 = 62 \text{ kHz} \quad ...(1)$$

$$\text{Let us assume that } f_s = 2B = 2 \times 62 = 124 \text{ kHz} \quad ...(2)$$

From Equations (1) and (2) we observe that neither f_1 nor f_2 is harmonically related to f_s. Hence we have to use the general bandpass sampling theorem stated in the preceding example.

$$\therefore \; k = \frac{f_M}{B} = \frac{82}{62} = 1.32 \rightarrow 1$$

$$\therefore \; f_s = \frac{2f_M}{k} = \frac{2 \times 82}{1} = 164 \text{ kHz} \quad \text{...Ans.}$$

5.4 Pulse Amplitude Modulation (PAM) :

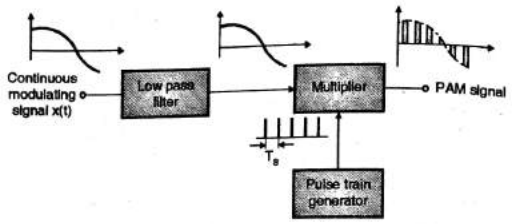
MU : Dec. 03, Dec. 14, May 15

University Questions

Q. 1 With proper waveforms explain principles of PAM system of modulation. **(Dec. 03, 3 Marks)**

Q. 2 Draw the PAM, PWM and PPM waveforms in time domain assuming a sinusoidal modulating signal. Explain them in brief. **(Dec. 14, May 15, 10 Marks)**

- In the PAM system, the amplitude of the pulsed carrier is changed in proportion with the instantaneous amplitude of the modulating signal x(t). So the information is contained in the amplitude variation of PAM signal.
- The carrier is in the form of train of narrow pulses as shown in Fig. 5.4.2.
- If you compare the PAM system with the sampling process, you will find that these two processes are identical.
- The PAM signal is then sent by either wire or cable or it is used to modulate a carrier.



(L-181) **Fig. 5.4.1 : Generation of PAM**

Types of PAM :

There are two types of PAM :

1. Natural PAM 2. Flat top PAM

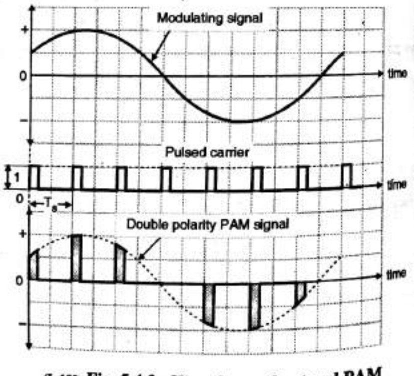
5.4.1 Generation of Natural PAM :

MU : May 07, Dec. 11, Dec. 12

University Questions

Q. 1	Give one method for the generation of PAM.	(May 07, Dec. 11, 5 Marks)
Q. 2	Explain generation of PAM.	(Dec. 12, 5 Marks)

- Refer Fig. 5.4.1 to understand the generation of natural PAM.
- The continuous modulating signal x(t), is passed through a low pass filter. The LPF will bandlimit this signal to f_m. That means all the frequency components higher than the frequency f_m are removed. Bandlimiting is necessary to avoid the "aliasing" effect in the sampling process.
- The pulse train generator generates a pulse train at a frequency f_s, such that $f_s \geq 2f_m$. Thus the Nyquist criteria is satisfied.
- The rectangular narrow carrier pulses generated by the pulse train generator would carry out the uniform "sampling" in the multiplier block, to generate the PAM signal as shown in Fig. 5.4.2. The samples are placed T_s seconds away from each other.

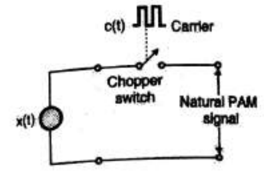


(L-182) **Fig. 5.4.2 : Waveform of natural PAM**

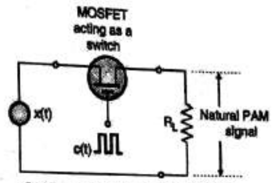
- The "information" in the modulating signal is contained in the "Amplitude variations" of the pulsed carrier. Therefore this system is similar to the AM system discussed earlier.

- Natural PAM is sometimes called as chopper sampled PAM because the waveform of the sampled signal appears to be chopped off from the continuous time signal x(t).
- The chopper arrangement is as shown in Fig. 5.4.3 where the chopper switch is being operated by the pulsed carrier "c (t)".

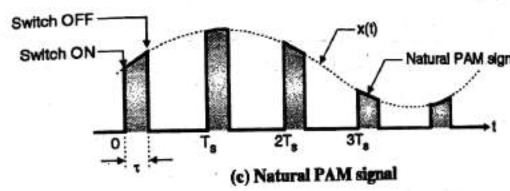
Circuit arrangement for natural PAM :



(a) Natural PAM



(b) Natural PAM with a MOSFET acting as a switch



(c) Natural PAM signal

(L-183) **Fig. 5.4.3**

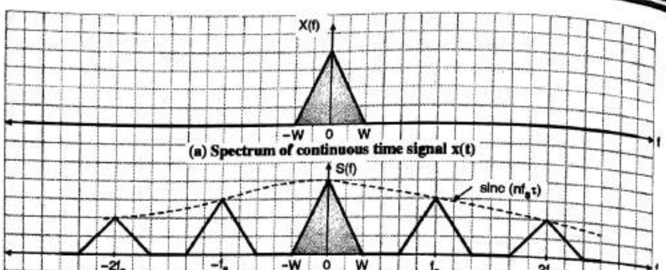
- The natural PAM signal is same as the naturally sampled signal. Hence the spectrum of natural PAM signal is same as that of the naturally sampled signal. It is given by,

Spectrum of natural PAM signal,

$$S(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(n f_s \tau)\, X(f - n f_s) \quad ...(5.4.1)$$

Conclusions :

1. The term $X(f - n f_s)$ represents the shifted version of the frequency spectrum X (f). The spectrum S (f) consists of X (f) and its shifted replicas as shown in Fig. 5.4.4(b).
2. These shifted replicas are observed at frequencies $f = \pm f_s, \pm 2 f_s, \pm 3 f_s$ ...etc.
3. The spectrum of x (t) is periodic in f_s and weighted by the sinc function. [See the term $\frac{\tau A}{T_s}$ sinc $(n f_s \tau)$ in Equation (5.4.1)]. Therefore the amplitude of the spectrum of natural PAM signal reduces on both sides of Y axis as shown in Fig. 5.4.4.



(a) Spectrum of continuous time signal x(t)

(b) Spectrum of natural PAM signal

(D-447) Fig. 5.4.4

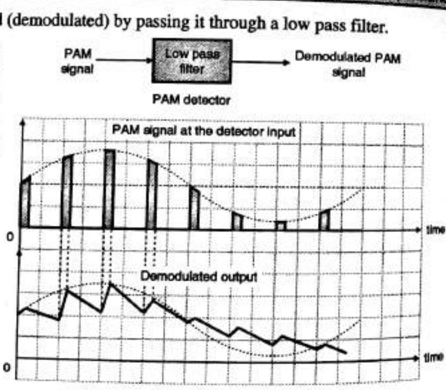
5.4.2 Detection of Natural PAM :

MU : Dec. 12

University Questions

Q. 1 Explain detection of PAM. **(Dec. 12, 5 Marks)**

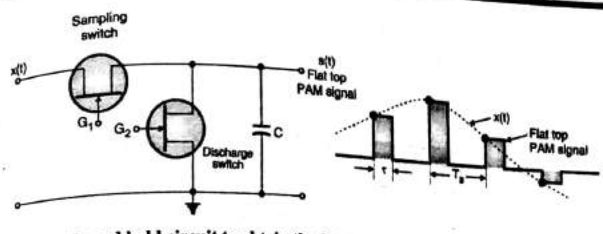
- The PAM signal can be detected (demodulated) by passing it through a low pass filter.
- The low pass filter cutoff frequency is adjusted to f_m so that all the high frequency ripple is removed and the original modulating signal is recovered back.
- The PAM detection and the corresponding waveforms are as shown in Fig. 5.4.5.
- From the waveforms, it is seen that the demodulated output signal is close to the original modulating signal x (t).



(L-185) Fig. 5.4.5 : Detection of PAM and waveforms

5.4.3 Flat Top PAM :

- The natural PAM is rarely employed in practice. Instead the flat top PAM is employed in practice.
- In the flat top PAM technique, the analog signal x(t) is sampled instantaneously at the rate $f_s = \frac{1}{T_s}$ and the duration of each sample is lengthened to a duration "τ" as shown in Fig. 5.4.6.



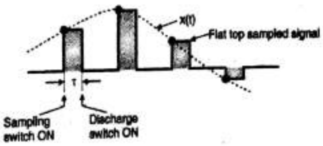
(a) Sample and hold circuit to obtain the flat top PAM (b) Flat top PAM signal

(D-449) Fig. 5.4.6

- Thus the amplitudes of these pulses are constant and equal to the corresponding sampled values.
- The flat top PAM can be obtained by using the sample and hold circuit shown in Fig. 5.4.6(a).

Operation of the sample and hold circuit :

- The sample and hold circuit consists of two FET switches and a capacitor as shown in Fig. 5.4.6(a). The analog signal x (t) is applied at the input of this circuit and the flat topped PAM signal s (t) is obtained across the capacitor.
- A gate pulse will be applied to gate G_1 at the instant of sampling, for a very short time. The sampling switch will turn on and the capacitor charges through it to the sample value x (nT_s). This is instantaneous value of x (t) at instant $t = nT_s$ where n = 0, 1, 2.... The sampling switch is then turned off. Both the FETs will remain OFF for a duration of "τ" seconds and the capacitor will hold the voltage across it constant for this period. Thus the pulse is stretched to "τ" seconds and we get a pulse with a flat top.
- At the end of the pulse interval (τ), a pulse is applied to G_2 i.e. gate terminal of discharge FET. This will turn on the discharge FET and short circuit the capacitor. The output voltage then reduces to zero. This is as shown in Fig. 5.4.7.



(D-450) Fig. 5.4.7 : Operation of sample and hold circuit

Principle of generating the flat top PAM pulses :

- Flat top PAM is same as flat top sampled signal. So the methods of generation, detection, spectrum, aperture effect etc will be exactly the same.

5.4.4 Spectrum of Flat Top PAM Signal :

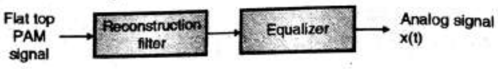
The spectrum of flat top PAM signal is same as that of the flat top sampled signal. It is given by :

Spectrum of flat top PAM signal

$$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - n f_s) \cdot H(f) \qquad ...(5.4.2)$$

5.4.5 Reconstruction of Original Signal x (t) :

- Due to the aperture effect discussed earlier, an amplitude distortion as well as a delay is introduced in the flat top sampled signal.
- This distortion can be corrected by connecting an equalizer after the reconstruction filter (low pass filter) as shown in Fig. 5.4.8.



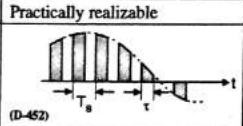
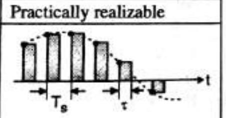
(D-451) **Fig. 5.4.8 : Reconstruction of x (t)**

5.4.6 Merits and Demerits of Flat Top PAM :

1. Better SNR due to increased signal power. This is due to the finite width "τ" of the pulses.
2. Generation is easy.
3. Practical filters can be used for reconstruction.
4. Aperture effect introduces distortion.

5.4.7 Comparison of PAM Techniques :

Table 5.4.1 : Comparison of PAM techniques

Sr. No.	Parameter	Natural PAM	Flat top PAM
1.	Nature of the sampling function	Train of finite duration pulses	Train of finite duration pulses
2.	Circuit arrangement	Uses a chopper	Uses a sample and hold circuit
3.	Practical realizability	Practically realizable	Practically realizable
4.	Waveforms	 (D-452)	
5.	Sampling rate	Satisfies Nyquist criteria	Satisfies Nyquist criteria
6.	Mathematical representation in time domain	$s(t) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} x(t)\,\text{sinc}(nf_s\tau)\, e^{j2\pi nf_s t}$	$s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s)$
7.	Frequency spectrum	$S(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(nf_s\tau)\, X(f - nf_s)$	$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)\, H(f)$
8.	Signal power	Increases with increase in the pulse width τ.	Increases with increase in pulse width τ.
9.	Bandwidth requirement	Increases with the reduction in pulse width.	Increases with the reduction in pulse width.
10.	Effect of noise	Moderate.	Moderate.

5.4.8 Transmission Bandwidth of PAM Signal :

- Let us assume that "τ" is the width of each pulse in a flat top sampled PAM and "T_s" is the duration between adjacent samples.
- We assume that the pulse duration "τ" is very small as compared to T_s.

$$\therefore \tau \ll T_s \qquad ...(5.4.3)$$

- But $T_s = \frac{1}{f_s}$ where f_s = Sampling frequency
- If W is the maximum frequency in x (t), then $f_s \geq 2W$.
- Hence $T_s \leq \frac{1}{2W}$. Substituting this in Equation (5.4.3) we get,

$$\tau \ll \frac{1}{2W} \qquad ...(5.4.4)$$

- For adequate pulse resolution i.e. to transmit and receive this PAM signal without much signal distortion, the transmission bandwidth B_T needs to satisfy the following equation,

$$B_T \geq \frac{1}{2\tau} \gg W \qquad ...(5.4.5)$$

- Thus the transmission bandwidth is inversely proportional to the pulse width "τ" of the PAM pulses. Transmission bandwidth should be as small as possible.
- It can be reduced by increasing the pulse width "τ". But this will increase the "aperture error" as discussed earlier.
- The transmission bandwidth B_T is much larger than the maximum frequency content in x (t). i.e. $B_T \gg W$
- Due to changes in amplitudes of PAM pulses, the transmitted power does not remain constant.
- With increase in the pulse width "τ" the aperture error increases.

Ex. 5.4.1 : For a PAM transmission of a voice signal with W = 3 kHz, calculate the transmission bandwidth B_T if the width of each pulse, $\tau = 0.1T_s$ and the sampling frequency f_s = 8 kHz.

Soln. : The sampling time T_s is given by,

$$T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} \text{ sec.}$$

$$\text{and } \tau = 0.1\,T_s = \frac{0.1}{8 \times 10^3} \text{ sec.}$$

From Equation (5.4.5), the transmission bandwidth B_T is given as,

$$B_T \geq \frac{1}{2\tau}$$

$$\therefore B_T \geq \frac{1}{(0.2/8 \times 10^3)}$$

$$\therefore B_T = 40 \text{ kHz} \qquad ...\text{Ans.}$$

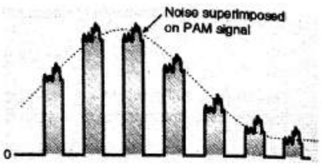
5.4.9 Advantages and Disadvantages of PAM :

There are not many advantages of a PAM system except for the simplicity of generation and detection. But there are many disadvantages. They are as follows :

1. The amplitude of PAM signal changes according to the amplitude of modulating signal. Therefore like AM, the effect of additive noise is maximum in PAM. The added noise cannot be removed easily.
2. The transmission bandwidth required for a PAM signal is too large as compared to the maximum frequency content in x (t).
3. Due to the changes in amplitudes of PAM pulses, the transmitted power is not constant.

5.4.10 Effect of Noise on PAM :

- The amplitude of the pulsed carrier is being changed in proportion with the amplitude of modulating signal in PAM.
- Hence all the "information" is contained in the amplitude variation of the PAM signal.
- When PAM signal travels over a communication channel, noise gets added to it as shown in Fig. 5.4.9.
- Note that the noise distorts the amplitude of PAM signal. Since the information is contained in the amplitude, the noise will contaminate the information.



(L-190) **Fig. 5.4.9 : Effect of noise on PAM signal**

- Therefore the noise performance of PAM system is very poor.
- The PWM and PPM systems have a better noise performance.

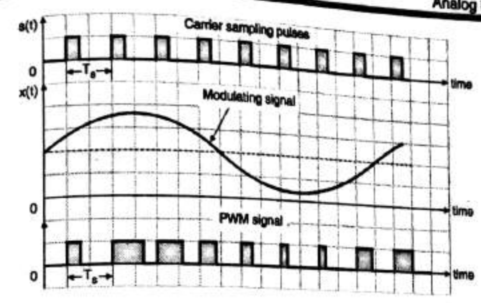
5.5 Pulse Width Modulation (PWM) or Pulse Duration Modulation (PDM) :

MU : Dec. 03, Dec. 10, Dec. 14, May 15

University Questions

Q. 1	With proper waveforms explain principles of PWM system of modulation.	(Dec. 03, 3 Marks)
Q. 2	Explain PWM.	(Dec. 10, 5 Marks)
Q. 3	Draw the PAM, PWM and PPM waveforms in time domain assuming a sinusoidal modulating signal. Explain them in brief.	(Dec. 14, May 15, 10 Marks)

- The other type of a pulse analog modulation is the pulse width modulation (PWM). In PWM, the width of the carrier pulses varies in proportion with the amplitude of modulating signal. The waveforms of PWM are as shown in Fig. 5.5.1.
- As seen from the waveforms, the amplitude and the frequency of the PWM wave remains constant. Only the width changes.
- That is why the "information" is contained in the width variation. This is similar to FM. As the noise is normally "additive" noise, it changes the amplitude of the PWM signal.
- At the receiver, it is possible to remove these unwanted amplitude variations very easily by means of a limiter circuit.
- As the information is contained in the width variation, it is unaffected by the amplitude variations introduced by the noise. Thus the PWM system is more immune to noise than the PAM signal.



(D-454) **Fig. 5.5.1 : PWM signal [Trail edge modulated signal]**

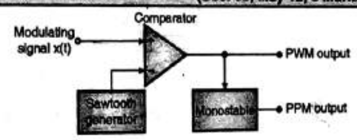
5.5.1 Generation of PWM Signal :

MU : Dec. 09, May 12

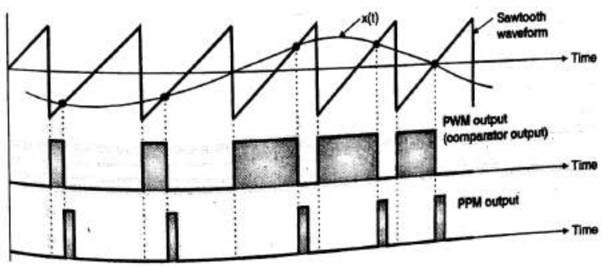
University Questions

Q. 1	Explain pulse width modulation.	(Dec. 09, May 12, 5 Marks)

- The block diagram of Fig. 5.5.2(a) can be used for the generation of PWM as well as PPM.



(D-455) **Fig. 5.5.2(a) : PWM and PPM generator**



(D-456) **Fig. 5.5.2(b) : Waveforms**

- A sawtooth generates a sawtooth signal of frequency f_s, therefore the sawtooth signal in this case is a sampling signal. It is applied to the inverting terminal of a comparator.
- The modulating signal x (t) is applied to the non-inverting terminal of the same comparator.
- The comparator output will remain high as long as the instantaneous amplitude of x (t) is higher than that of the ramp signal.
- This gives rise to a PWM signal at the comparator output as shown in Fig. 5.5.2(b).

- Note that the leading edges of the PWM waveform coincide with the falling edges of the ramp signal.
- Thus the leading edges of PWM signal are always generated at fixed time instants. However the occurrence of its trailing edges will be dependent on the instantaneous amplitude of x (t).
- Therefore this PWM signal is said to be trail edge modulated PWM.

5.5.2 Detection of PWM Signal :

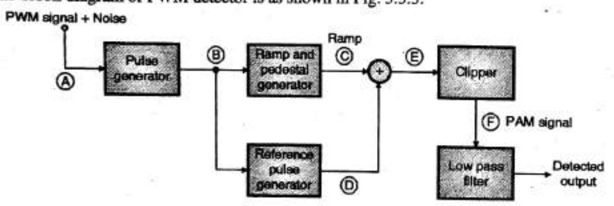
MU : May 03, Dec. 09, May 12

University Questions

Q. 1 Draw the block diagram of a PWM detector. Explain the working giving waveforms at the output of each block. **(May 03, 8 Marks)**

Q. 2 Explain pulse width demodulation. **(Dec. 09, May 12, 5 Marks)**

The block diagram of PWM detector is as shown in Fig. 5.5.3.

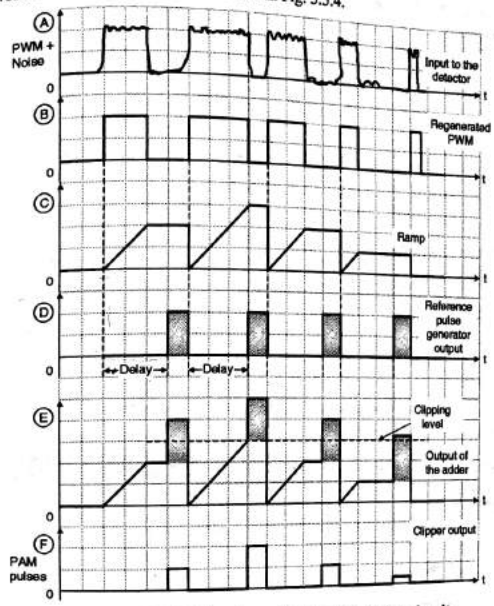


(D-457) **Fig. 5.5.3 : PWM detection circuit**

Operation :

- The PWM signal received at the input of the detection circuit is contaminated with noise. This signal is applied to pulse generator circuit which regenerates the PWM signal. Thus some of the noise is removed and the pulses are squared up.
- The regenerated pulses are applied to a reference pulse generator. It produces a train of constant amplitude, constant width pulses. These pulses are synchronized to the leading edges of the regenerated PWM pulses but delayed by a fixed interval.
- The regenerated PWM pulses are also applied to a ramp generator. At the output of it we get a constant slope ramp for the duration of the pulse. The height of the ramp is thus proportional to the widths of the PWM pulses. At the end of the pulse a sample and hold amplifier retains the final ramp voltage until it is reset at the end of the pulse.
- The constant amplitude pulses at the output of reference pulse generator are then added to the ramp signal. The output of the adder is then clipped off at a threshold level to generate a PAM signal at the output of the clipper.
- A low pass filter is used to recover the original modulating signal back from the PAM signal.

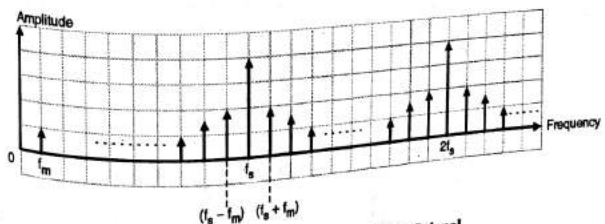
- The waveforms for this circuit are as shown in Fig. 5.5.4.



(D-458) **Fig. 5.5.4 : Waveforms for PWM detection circuit**

5.5.3 Frequency Spectrum of PWM Wave :

- Fig. 5.5.5 shows the spectrum of a PWM signal for a sinusoidal modulating signal with a frequency f_m. The spectrum shows that the modulating frequency f_m and many of its sidebands are present.



(D-459) **Fig. 5.5.5 : Spectrum of PWM signal**

5.5.4 Advantages of PWM :

MU : May 03

University Questions

Q. 1 List the advantages of PWM. **(May 03, 1 Mark)**

1. Less effect of noise i.e. very good noise immunity.
2. Synchronization between the transmitter and receiver is not essential. (Which is essential in PPM).
3. It is possible to reconstruct the PWM signal from a noise contaminated PWM, as discussed in the detection circuit. Thus it is possible to separate out signal from noise (which is not possible in PAM).

5.5.5 Disadvantages of PWM :

MU : May 03

University Questions

Q. 1 List the disadvantages of PWM. **(May 03, 1 Mark)**

1. Due to the variable pulse width, the pulses have variable power contents. So the transmitter must be powerful enough to handle power corresponding to the maximum width pulse. The average power transmitted can be as low as 50% of this maximum power.
2. In order to avoid any waveform distortion, the bandwidth required for the PWM communication is large as compared to BW of PAM.

5.6 Pulse Position Modulation (PPM) :

MU : Dec. 03, Dec. 10, Dec. 14, May 15

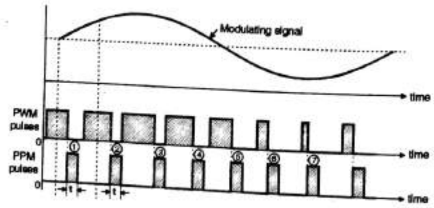
University Questions

Q. 1 With proper waveforms explain principle of PPM system of modulation. **(Dec. 03, 3 Marks)**

Q. 2 Explain PPM. **(Dec. 10, 5 Marks)**

Q. 3 Draw the PAM, PWM and PPM waveforms in time domain assuming a sinusoidal modulating signal. Explain them in brief. **(Dec. 14, May 15, 10 Marks)**

- In PPM the amplitude and width of the pulsed carrier remains constant but the position of each pulse is varied in proportion with the amplitudes of the sampled values of the modulating signal. The position of the pulses is changed with respect to the position of reference pulses.
- The PPM pulses can be derived from the PWM pulses as shown in Fig. 5.6.1. Note that with increase in the modulating voltage the PPM pulses shift further with respect to reference.
- The vertical dotted lines drawn in Fig. 5.6.1 are treated as reference lines to measure the shift in position of PPM pulses. The leading edge of each PPM pulse coincides with the trailing pulse of a PWM pulse.



(D-460) **Fig. 5.6.1 : PPM pulses generated from PWM signal**

- The PPM pulses marked 1, 2 and 3 etc. in Fig. 5.6.1 go away from their respective reference lines.
- This is corresponding to increase in the modulating signal amplitude. Then as the modulating voltage decreases the PPM pulses 4, 5, 6, 7 come progressively closer to their respective reference lines.

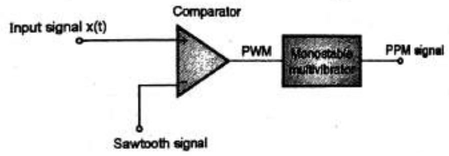
5.6.1 Generation of PPM Signal :

MU : Dec. 04

University Questions

Q. 1 Explain the following term : Explain how PPM is obtained from PWM. **(Dec. 04, 3 Marks)**

- The PPM signal can be generated from PWM signal as shown in Fig. 5.5.2(a). The same block diagram has been repeated in Fig. 5.6.2 as shown.
- The PWM pulses obtained at the comparator output are applied to a monostable multivibrator. The monostable is negative edge triggered.
- Hence corresponding to each trailing edge of PWM signal, the monostable output goes high. It remains high for a fixed time decided by its own RC components.
- Thus as the trailing edges of the PWM signal keep shifting in proportion with the modulating signal x(t), the PPM pulses also keep shifting as shown in Fig. 5.6.1.

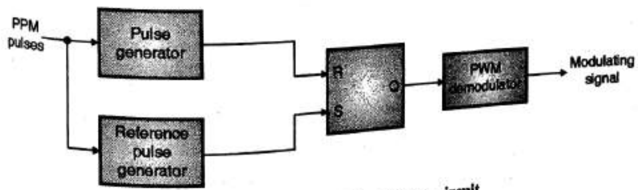


(D-461) **Fig. 5.6.2 : Generation of PPM signal**

- Note that all the PPM pulses have the same width and amplitude. The information is conveyed via changing position of the pulses.

5.6.2 Demodulation of PPM :

- The PPM demodulator block diagram is as shown in Fig. 5.6.3.



(D-462) **Fig. 5.6.3 : PPM demodulator circuit**

The operation of the demodulator circuit is explained as follows :

- The noise corrupted PPM waveform is received by the PPM demodulator circuit.

- The pulse generator develops a pulsed waveform at its output of fixed duration and apply these pulses to the reset pin (R) of a SR flip-flop.
- A fixed period reference pulse is generated from the incoming PPM waveform and the SR flip-flop is set by the reference pulses.
- Due to the set and reset signals applied to the flip-flop, we get a PWM signal at its output. The PWM signal can be demodulated using the PWM demodulator. This is same as the one discussed in section 5.5.

5.6.3 Advantages of PPM :

MU : May 03

University Questions

Q. 1 List the advantages of PPM. **(May 03, 1 Mark)**

1. Due to constant amplitude of PPM pulses, the information is not contained in the amplitude. Hence the noise added to PPM signal does not distort the information. Therefore it has good noise immunity. This is same as that explained for PWM in section 5.5.
2. It is possible to reconstruct PPM signal from the noise contaminated PPM signal. This is also possible in PWM but not possible in PAM.
3. Due to constant amplitude of pulses, the transmitted power always remains constant. It does not change as it used to, in PWM.

5.6.4 Disadvantages of PPM :

MU : May 03

University Questions

Q. 1 List the disadvantages of PPM. **(May 03, 1 Mark)**

1. As the position of the PPM pulses is varied with respect to a reference pulse, a transmitter has to send synchronizing pulses to operate the timing circuits in the receiver. Without them the demodulation won't be possible to achieve.
2. Large bandwidth is required to ensure transmission of undistorted pulses.

5.7 Comparison of PAM, PWM and PPM Systems :

MU : May 06, Dec. 07, May 11, Dec. 13, Dec. 16

University Questions

Q. 1 Compare PAM and PWM. **(May 06, 4 Marks)**

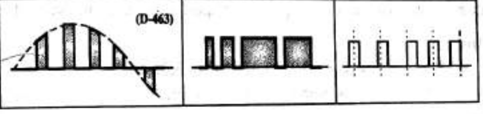
Q. 2 Write a short note : Comparison of PWM, PAM and PPM. **(Dec. 07, 10 Marks)**

Q. 3 Compare PAM, PWM and PPM. **(May 11, 10 Marks)**

Q. 4 Compare PAM, PPM and PWM. **(Dec. 13, 5 Marks)**

Q. 5 Compare various pulse modulation techniques. **(Dec. 16, 5 Marks)**

Sr. No.	Parameter	PAM	PWM	PPM
1.	Type of carrier	Train of pulses	Train of pulses	Train of pulses
2.	Variable characteristic of the pulsed carrier	Amplitude	Width	Position
3.	Bandwidth requirement	Low	High	High

Sr. No.	Parameter	PAM	PWM	PPM
4.	Noise immunity	Low	High	High
5.	Information is contained in	Amplitude variations	Width variation	Position variation
6.	Transmitted power	Varies with amplitude of pulses	Varies with variation in width	Remains constant
7.	Need to transmit synchronizing pulses	Not needed	Not needed	Necessary
8.	Complexity of generation and detection	Complex	Easy	Complex
9.	Similarity with other modulation systems	Similar to AM	Similar to FM	Similar to PM
10.	Output waveforms	 (D-463)		

Review Questions

Q. 1 Define the sampling process and explain its necessity in communication system.

Q. 2 State and prove the sampling theorem for a low pass bandlimited signal.

Q. 3 What do you understand by the word bandlimited ?

Q. 4 Explain the term aliasing and its effects.

Q. 5 How can we avoid aliasing ?

Q. 6 State the sampling theorem for a bandpass signal.

Q. 7 What is the difference between ideal and practical sampling ?

Q. 8 Explain the natural sampling and draw the spectrum of a naturally sampled signal.

Q. 9 Explain the flat top sampling, state its advantages over natural sampling and draw the spectrum of flat top sampled signal.

Q. 10 Draw and explain the generation of flat top sampled signal.

Q. 11 Explain the effects of the finite pulse width of the sampling function.

Q. 12 What is "aperture effect" and explain how to reduce it.

Q. 13 Compare the ideal, natural and flat top sampling techniques.

Q. 14 Define PAM and explain its generation and detection.

Q. 15 Why is PAM not used for communication applications ?

Q. 16 State the disadvantages of PAM.

Q. 17 Explain the generation and detection of a PWM signal.

Q. 18 State and explain the merits and demerits of PWM transmission.

Q. 19 With the help of neat circuit diagram explain the generation and detection of a PPM signal.

Q. 20 State the merits and demerits of a PPM transmission.

Q. 21 Compare PAM, PWM and PPM systems.

5.8 University Questions and Answers :

Q. 1 What is meant by "Nyquist rate" in sampling ? What is the standard sampling frequency for speech signals ? **(May 2003, 4 Marks, Dec. 2006, Dec. 2013, 5 Marks)**

Ans. :

Please refer section 5.3.3 for Nyquist rate.

The standard sampling frequency for speech signals is 8 kHz.

Q. 2 What happens when a signal is sampled at less than the Nyquist rate ? **(Dec. 2004, 4 Marks)**

Ans. :

Please refer section 5.3.2 for Aliasing or Foldover Error.

When a signal is sampled at a rate less than Nyquist rate, the distortion called as aliasing takes place.

Q. 3 What is sampling theorem and state its significance in communication. What is the standard frequency for speech signal ? **(Dec. 2015, 5 Marks)**

Ans. :

For sampling theorem and significance in communication refer sections 5.2 and 5.3.

The standard sampling frequency for speech signal is 8 kHz.

□□□

CHAPTER 6

Module 5

Digital Pulse Modulation

Syllabus :

Generation and detection of PCM, DM and ADM.

6.1 Introduction :

- Digital transmission is defined as the transmission of digital signals between two or more points in a communication system. The transmitted signal is the form of a digital signal.
- The digital signals can be of different types such as binary, octal, hexadecimal etc.
- The original information can be analog or digital. If it is analog then it is converted to digital.
- The communication medium can be a coaxial cable or optical fiber link.
- Examples of digital pulse modulation are PCM, DM and ADM.

6.1.1 Advantages of Digital Transmission :

MU : May 04, Dec. 04

University Questions

Q. 1 Explain the following : Advantages of digital communication. **(May 04, 3 Marks)**

Q. 2 What are the advantages of digital communications ? **(Dec. 04, 2 Marks)**

The digital representation of a signal has following advantages :

1. Immunity to transmission noise and interference.
2. Regeneration of the coded signal along the transmission path is possible.
3. Communication can be kept "private" and "secured" through the use of encryption.
4. It is possible to use a uniform format for different kinds of baseband signals.
5. It is possible to store the signal and process it further.
6. Digital signals are better suited for processing and multiplexing.
7. Digital transmission systems are more immune to noise.
8. Measurement and evaluation of digital signals is simpler.
9. It is possible to evaluate error performance of digital systems.

6.1.2 Disadvantages :

MU : May 04, Dec. 04

University Questions

Q. 1 Explain the following : Disadvantages of digital communication. **(May 04, 3 Marks)**

Q. 2 What are the disadvantages of digital communications ? **(Dec. 04, 2 Marks)**

1. The required bandwidth is increased due to digital technology.
2. System complexity is increased.
3. In order to convert the analog signal to digital prior to transmission and then from digital to analog at the receiver, we need to use the additional encoders and decoder circuits.
4. Synchronization is necessary for the digital systems (between transmission and receiver clocks).
5. Digital transmission systems are not compatible to the older analog transmission systems.

6.2 Pulse Code Modulation (PCM) :

- PCM is a type of pulse modulation like PAM, PWM or PPM but there is an important difference between them. PAM, PWM or PPM are "analog" pulse modulation systems whereas PCM is a "digital" pulse modulation system.
- That means the PCM output is in the coded digital form. It is in the form of digital pulses of constant amplitude, width and position.
- The information is transmitted in the form of "code words". A PCM system consists of a PCM encoder (transmitter) and a PCM decoder (receiver).
- The essential operations in the PCM transmitter are sampling, quantizing and encoding.
- All these operations are usually performed in the same circuit called as analog-to-digital (A to D) converter.
- It should be understood that the PCM is not modulation in the conventional sense.
- Because in modulation, one of the characteristics of the carrier is varied in proportion with the amplitude of the modulating signal. Nothing of that sort happen in PCM.

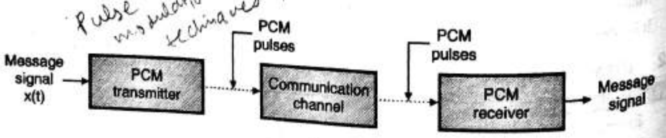
6.3 Pulse Code Modulation (PCM) System : MU : Dec. 12

University Questions

Q. 1 Explain PCM in detail. **(Dec. 12, 10 Marks)**

- Fig. 6.3.1 shows the simplified block diagram of a PCM system. It consists of a transmitter and receiver.
- The transmitter converts the message signal x (t) into a series of coded pulses and sends it over the communication channel.
- The transmitter is also called as an encoder. PLL as FM detector
- The receiver performs exactly in the reverse way as compared to the transmitter. It will convert the received encoded PCM pulses back into the message signal.

Pulse modulation techniques



(D-771) **Fig. 6.3.1**

Application

6.3.1 PCM Transmitter (Encoder) : MU : Dec. 04, Dec. 07, May 11, Dec. 11

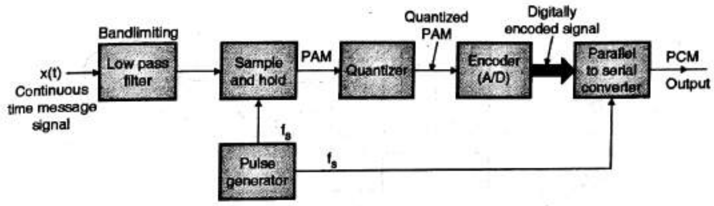
University Questions

Q. 1 Draw the block diagram of PCM and explain it. **(Dec. 04, 4 Marks)**

Q. 2 Explain the pulse code modulation (Diagram, working and waveforms). **(Dec. 07, 7 Marks)**

Q. 3 Draw a neat block diagram and waveforms for PCM transmitter and explain the working. **(May 11, Dec. 11, 5 Marks)**

- Block diagram of the PCM transmitter is as shown in Fig. 6.3.2.



(L-221) **Fig. 6.3.2 : PCM transmitter (Encoder)**

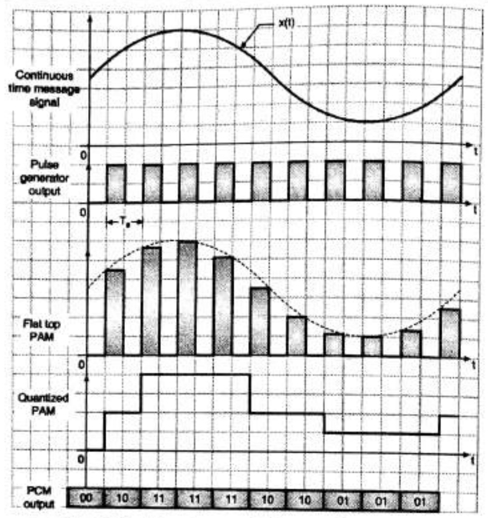
Operation of PCM transmitter :

Operation of the PCM transmitter is as follows :

- The analog signal x (t) is passed through a bandlimiting low pass filter, which has a cut-off frequency f_c = W Hz. This will ensure that x (t) will not have any frequency component higher than "W". This will eliminate the possibility of aliasing.
- The band limited analog signal is then applied to a sample and hold circuit where it is sampled at adequately high sampling rate. Output of sample and hold block is a flat topped PAM signal.
- These samples are then subjected to the operation called "Quantization" in the "Quantizer". Quantization process is the process of approximation as will be explained later on. The quantization is used to reduce the effect of noise. The combined effect of sampling and quantization produces the quantized PAM at the quantizer output.
- The quantized PAM pulses are applied to an encoder which is basically an A to D converter. Each quantized level is converted into an N bit digital word by the A to D converter. The value of N can be 8, 16, 32, 64 etc.
- The encoder output is converted into a stream of pulses by the parallel to serial converter block. Thus at the PCM transmitter output we get a train of digital pulses.
- A pulse generator produces a train of rectangular pulses with each pulse of duration "τ" seconds. The frequency of this signal is "f_s" Hz. This signal acts as a sampling signal for the sample and hold block. The same signal acts as "clock" signal for the parallel to serial converter. The frequency "f_s" is adjusted to satisfy the Nyquist criteria.

Waveforms :

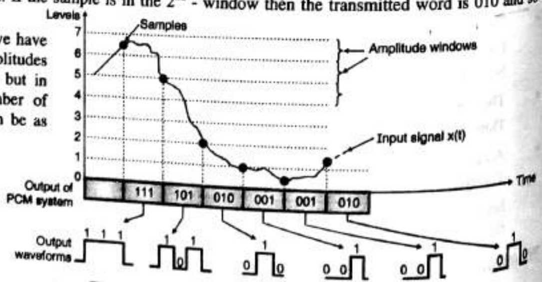
The waveforms at various points in the PCM transmitter are as shown in Fig. 6.3.3.



(L-222) **Fig. 6.3.3 : Waveforms at different points in PCM transmitter**

6.3.2 Shape of the PCM Signal (A to D Conversion Concept) :

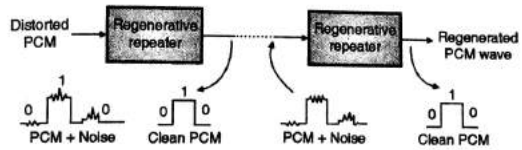
- Fig. 6.3.4 shows input to and output of a PCM system. It is important to understand that the output is in the form of binary codes. Each transmitted binary code represents a particular amplitude of the input signal. Hence the "information" is contained in the "code" which is being transmitted.
- The range of input signal magnitudes is divided into 8-equal levels. Each level is denoted by a three bit digital word between 000 and 111.
- Input signal x (t) is sampled. If the sample is in the 5^{th} - window of amplitude then a digital word 101 is transmitted. If the sample is in the 2^{nd} - window then the transmitted word is 010 and so on.
- In this example we have converted the amplitudes into 3 bit codes, but in practice the number of bits per word can be as high as 8, 9 or 10.



(L-223) **Fig. 6.3.4 : Input and output waveforms of a PCM system**

6.3.3 PCM Transmission Path :

- The path between the PCM transmitter and PCM receiver over which the PCM signal travels is called as PCM transmission path and it is as shown in Fig. 6.3.5.



(D-472) **Fig. 6.3.5 : PCM transmission path**

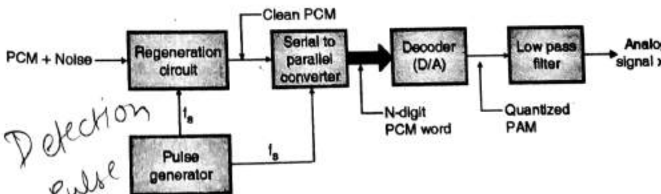
- The most important feature of PCM system lies in its ability to control the effects of distortion and noise when the PCM wave travels on the channel.
- PCM accomplishes this capacity by means of using a chain of regenerative repeaters as shown in Fig. 6.3.5.
- Such repeaters are spaced close enough to each other on the transmission path.
- The regenerative repeater performs three basic operations namely equalization, timing and decision making.
- So each repeater actually reproduces the clean noise free PCM signal from the PCM signal distorted by the channel noise. This improves the performance of PCM in presence of noise.

6.3.4 PCM Receiver (Decoder) : MU : Dec. 04, Dec. 07, May 11, Dec. 11, Dec. 16

University Questions

Q. 1	Draw the block diagram of PCM and explain it.	(Dec. 04, 3 Marks)
Q. 2	Explain the pulse code modulation (Diagram, working and waveforms).	(Dec. 07, 7 Marks)
Q. 3	Draw a neat block diagram and waveforms for PCM receiver and explain the working.	(May 11, Dec. 11, 5 Marks)
Q. 4	Explain the detection of pulse code modulation.	(Dec. 16, 5 Marks)

- Fig. 6.3.6 shows the block diagram of a PCM receiver.



Detection pulse code modulation

(L-224) **Fig. 6.3.6 : PCM receiver (Decoder)**

Operation of PCM receiver :

- A PCM signal contaminated with noise is available at the receiver input.
- The regeneration circuit at the receiver will separate the PCM pulses from noise and will reconstruct the original PCM signal.
- The pulse generator has to operate in synchronization with that at the transmitter. Thus at the regeneration circuit output we get a "clean" PCM signal.
- The reconstruction of PCM signal is possible due to the digital nature of PCM signal. The reconstructed PCM signal is then passed through a serial to parallel converter.
- Output of this block is then applied to a decoder.
- The decoder is a D to A converter which performs exactly the opposite operation of the encoder.
- The decoder output is the sequence of a quantized multilevel pulses. The quantized PAM signal is thus obtained, at the output of the decoder.
- This quantized PAM signal is passed through a low pass filter to recover the analog signal, x (t).
- The low pass filter is called as the reconstruction filter and its cut off frequency is equal to the message bandwidth W.

6.3.5 Quantization Process :

MU : Dec. 06, May 12

University Questions

Q. 1	Explain what is meant by quantization noise.	(Dec. 06, 4 Marks)
Q. 2	Write short note on quantization.	(May 12, 5 Marks)

- Quantization is a process of approximation or rounding off. The sampled signal in PCM transmitted is applied to the quantizer block.
- Quantizer converts the sampled signal into an approximate quantized signal which consists of only a finite number of predecided voltage levels.
- Each sampled value at the input of the quantizer is approximated or rounded off to the nearest standard predecided voltage level.
- These standard levels are known as the "quantization levels". Refer to Fig. 6.3.7 to understand the process of quantization.

The quantization process takes place as follows :

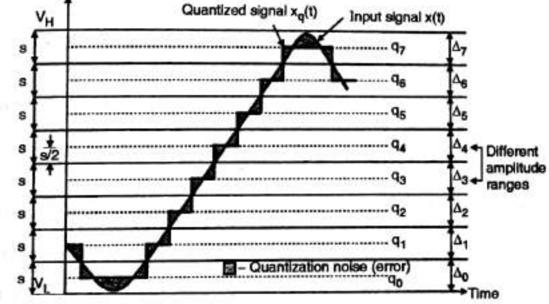
- The input signal x (t) is assumed to have a peak to peak swing of V_L to V_H volts. This entire voltage range has been divided into "Q" equal intervals each of size "s".
- "s" is called as the step size and its value is given as,

$$s = \frac{V_H - V_L}{Q} \qquad ...(6.3.1)$$

In Fig. 6.3.7, the value of Q = 8

- At the center of these ranges, the quantization levels $q_0, q_1, ... q_7$ are placed. Thus the number of quantization levels is Q = 8. The quantization levels are also called as decision thresholds.
- x_q (t) represents the quantized version of x (t). We obtain x_q (t) at the output of the quantizer.
- When x (t) is in the range Δ_0, then corresponding to any value of x (t), the quantizer output will be equal to "q_0".
- Similarly for all the values of x (t) in the range Δ_1, the quantizer output is constant equal to "q_1".

- Thus in each range from Δ_0 to Δ_7, the signal x (t) is rounded off to the nearest quantization level and the quantized signal is produced.
- The quantized signal x_q (t) is thus an approximation of x (t). The difference between them is called as **quantization error or quantization noise.**
- This error should be as small as possible.
- To minimize the quantization error we need to reduce the step size "s" by increasing the number of quantization levels Q.



(L-225) **Fig. 6.3.7 : Process of quantization**

Why is quantization required ?

- If we do not use the quantizer block in the PCM transmitter, then we will have to convert each and every sampled value into a unique digital word.
- This will need a large number of bits per word (N). This will increase the bit rate and hence the bandwidth requirement of the channel.
- To avoid this, if we use a quantizer with only 256 quantization levels then all the sampled values will be finally approximated into only 256 distinct voltage levels.
- So we need only 8 bits per word to represent each quantized sampled value.
- Thus the number of bits per word can be reduced. This will eventually reduce the bit rate and bandwidth requirement.

Quantization error or quantization noise ∈ :

- The difference between the instantaneous values of the quantized signal and input signal is called as quantization error or quantization noise.

$$\epsilon = x_q(t) - x(t) \qquad ...(6.3.2)$$

- The quantization error is shown by shaded portions of the waveform in Fig. 6.3.7.
- The maximum value of quantization error is ± s / 2 where s is step size.
- Therefore to reduce the quantization error we have to reduce the step size by increasing the number of quantization levels i.e. Q.
- The mean square value of the quantization is given by,

Mean square value of quantization error $= \frac{s^2}{12}$ ...(6.3.3)

- The relation between the number of quantization levels Q and the number of bits per word (N) in the transmitted signal can be found as follows :
- Because each quantized level is to be converted into a unique N bit digital word, assuming a binary coded output signal,
- The number of quantization levels Q = Number of combinations of bits/word.

$$\therefore \quad Q = 2N \qquad ...(6.3.4)$$

- Thus if N = 4 i.e. 4 bits per word then the number of quantization levels will be 2^4 i.e.16.

Signal to quantization noise ratio (SNR_q) :

- This ratio is the figure of merit for the PCM systems. The signal to quantization noise ratio with a sinusoidal input signal to the PCM system is expressed as,

$$\frac{S_i}{N_q} = [1.8 + 6N] \text{ dB} \qquad \textbf{For a sinusoidal signal} \qquad ...(6.3.5)$$

- This equation shows that the signal to quantization noise ratio is solely dependent on the number of bits per word i.e. N.
- This ratio should be as high as possible, which can be achieved by increasing N. But this increases the bit rate and hence bandwidth of the PCM system.
- Therefore the number of bits per word is a compromise between high SNR_q and bandwidth requirements.

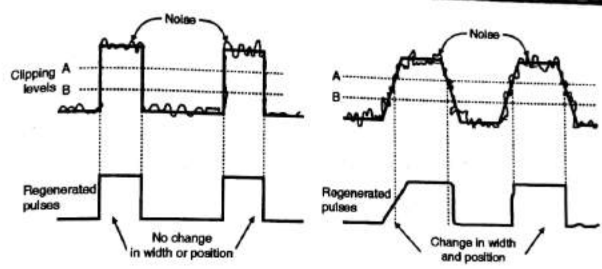
6.4 Effect of Noise on the PCM System :

MU : May 03, Dec. 04

University Questions

Q. 1 Why is PCM more noise resistant than other forms of pulse modulation ?
(May 03, 8 Marks, Dec. 04, 2 Marks)

- Look at the two Figs. 6.4.1(a) and 6.4.1(b) which illustrate the effect of noise on the transmitted pulses.
- Consider Fig. 6.4.1(a) first. Due to the noise superimposed on the pulses, only the PAM system will be affected.
- However the PWM, PPM and PCM systems will remain unaffected. The regeneration of the pulses is achieved by using a clipper circuit with reference levels A and B.
- Now consider Fig. 6.4.1(b). Here the sides of the transmitted pulse are not perfectly vertical. In practice the transmitted pulses usually have slightly sloping sides (edges).
- As the noise is superimposed on them, the width and the position of the regenerated pulses is changed.
- Now this is going to distort the information contents in the PWM and PPM signals.
- But PCM is still unaffected as it does not contain any information in the width or the position of the pulses.
- Thus PCM has much better noise immunity as compared to PAM, PWM and PPM systems.



(a) (b)

(L-229) **Fig. 6.4.1 : Effect of noise on PCM**

6.5 Robust Quantization (Nonuniform Quantization) :

- In the previous section for uniform quantizer with a step size "s" it was shown that the variance of quantization noise or the normalized quantization noise power $N_q = \epsilon^2 = s^2/12$.
- Thus the quantization noise is independent of the size of input signal. It is constant.
- As a result of this the signal to quantization noise ratio SNR_q decreases with decrease in the input signal power level. This is highly objectionable and unacceptable.
- In certain applications where PCM is used for the transmission of speech or music signals, this problem is very serious.
- Because the same quantizer has to accommodate the input signals of varying power levels. This happens because the range of voltages covered by a speech signals from maximum to minimum has a ratio of the order of 1000 : 1.
- Therefore the weak speech signals will have a small value of SNR_q and hence the PCM performance will degrade.
- Therefore it is desirable that SNR_q should remain essentially constant over a wide range of input power level.
- A quantizer that satisfies all these requirements is called as a **Robust Quantizer.**
- Such a robust performance can be obtained by using a **nonuniform quantization.**

6.5.1 Nonuniform Quantization :

- If the quantizer characteristics is nonlinear and the step size is not constant instead if it is kept variable, dependent on the amplitude of input signal then the quantization is called as non uniform quantization.
- In non-uniform quantization, the step size is reduced with reduction in signal level. For weak signals (P << 1), the step size is small, therefore the quantization noise reduces, to improve the signal to quantization noise ratio for weak signals.
- The step size is thus varied according to the signal level to keep the signal to noise ratio adequately high. This is non-uniform quantization.

- The non-uniform quantization is practically achieved through a process called "companding". We will discuss companding in the next section.

Need of non-uniform quantization for speech signals :

- Non-uniform quantization is generally used for the speech and music signals.
- To understand the need of non-uniform quantization for the speech and music signals it is necessary to define an important parameter called "crest factor".
- Crest factor is defined as the ratio of peak amplitude to the rms amplitude of a signal.

$$\therefore \text{ Crest factor } = \frac{\text{Peak value}}{\text{rms value}} \quad ...(6.5.1)$$

- The value of crest factor is very high for the speech and music signals. Now let us see the effect of this high crest factor on the normalized power P.
- The destination signal power P is defined as,

$$P = \frac{\text{Mean square value of the signal}}{R}$$

$$\therefore \quad P = \frac{x^2(t)}{R} \quad ...(6.5.2)$$

where, $x^2(t)$ = Mean square value of the signal.

- The normalized signal power is obtained by substituting R = 1 in Equation (6.5.2).

$\therefore$ Normalized signal power $P = x^2(t)$ ...(6.5.3)

- $$\text{The crest factor } = \frac{\text{Peak value}}{\text{rms value}}$$

$$= \frac{x_{max}}{[x^2(t)]^{1/2}} \quad ...(6.5.4)$$

- But $x^2(t) = P$

$$\therefore \quad CF = \frac{x_{max}}{\sqrt{P}} \quad ...(6.5.5)$$

- Now if we normalize the signal i.e. if $x_{max} = 1$, then

$$CF = \frac{1}{\sqrt{P}}$$

$$\text{or} \quad P = \frac{1}{CF^2} \quad ...(6.5.6)$$

- The maximum possible value of the normalized power P is 1. Equation (6.5.6) shows that the normalized power P for the speech and music signal will be much less than 1 (which is its maximum possible value). This happens due to the high value of the crest factor.
- Equation (6.5.2) states that the signal to quantization noise ratio, for nonsinusoidal signals is given by,

$$\frac{S}{N_q} = 3 \times 2^{2N} \times P$$

- Hence if P <<< 1 then the signal to quantization noise ratio will reduce drastically. Thus for the speech and music signal having high crest factor, the signal to quantization noise ratio is poor which leads to degradation in the quality of sound.

- This problem can be overcome by use of non-uniform quantization. This is because in non-uniform quantization, the step size reduced with reduction in signal level.
- For weak signals (P << 1), the step size is small, therefore the quantization noise reduces, to improve the signal to quantization noise ratio for weak signals.
- The step size is thus varied according to the signal level to keep the signal to noise ratio adequately high. This is non-uniform quantization.
- The non-uniform quantization is practically achieved through a process called "companding". We will discuss companding in the next section.

6.5.2 Dynamic Range (D.R.) :

- Dynamic range is defined as the ratio of the largest possible magnitude to the smallest possible magnitude that can be decoded by the D to A converter in PCM.
- Mathematically the dynamic range can be defined as,

$$D.R. = \frac{V_{max}}{V_{min}}$$

- The dynamic range is also expressed in decibels as follows :

$$D.R. \text{ in dB} = 20 \log \frac{V_{max}}{V_{min}}$$

- The number of bits used for a PCM code is dependent on the dynamic range.

6.6 Companding : MU : Dec. 03, May 06, Dec. 06, May 08, May 10, Dec. 10, May 13

University Questions

Q. 1	What is companding ?	(Dec. 03, 4 Marks, May 06, Dec. 06, May 13, 5 Marks)
Q. 2	Write short note on companding.	(May 08, 7 Marks, May 10, Dec. 10, 5 Marks)

- Companding is non-uniform quantization. It is required to be implemented to improve the signal to quantization noise ratio of weak signals.
- The quantization noise is given by,

$$N_q = s^2 / 12$$

- This shows that in the uniform quantization once the step size is fixed, the quantization noise power remains constant.
- But the signal power is not constant. It is proportional to the square of signal amplitude. Hence signal power will be small for weak signals, but quantization noise power is constant.
- Therefore the signal to quantization noise ratio for the weak signals is very poor. This will affect the quality of signal. The remedy is to use **companding.**
- Companding is a term derived from two words, compression and expansion.

$$\text{Companding} = \text{Compressing} + \text{Expanding}$$

- Practically it is difficult to implement the non-uniform quantization because it is not known in advance about the changes in the signal level.
- Therefore a trick is used. The weak signals are amplified and strong signals are attenuated before applying them to a uniform quantizer.
- This process is called as "compression" and the block that provides it is called as a "compressor".
- At the receiver exactly opposite process is followed which is called expansion. The circuit used for providing expansion is called as an "expander".

- The compression of signal at the transmitter and expansion at the receiver is combined to be called as "companding".
- The process of companding is shown in the block diagram form in Fig. 6.6.1.

(D-479) **Fig. 6.6.1 : Model of companding**

6.6.1 Types of Companding :

- There are two possible types of companding :
  1. Analog companding 2. Digital Companding

6.7 Advantages, Disadvantages and Applications of PCM :

- The PCM is considered to be the best modulation scheme to transmit the voice and video signals.
- All the advantages of PCM are due to the fact that it uses coded pulses for the transmission of information.

6.7.1 Advantages of PCM :

1. Very high noise immunity.
2. Due to digital nature of the signal, repeaters can be placed between the transmitter and the receivers. The repeaters actually regenerate the received PCM signal. This is not possible in analog systems. Repeaters further reduce the effect of noise.
3. It is possible to store the PCM signal due to its digital nature.
4. It is possible to use various coding techniques so that only the desired person can decode the received signal. This makes the communication secure.
5. The increased channel bandwidth requirement for PCM is balanced by the improved SNR.
6. There is a **uniform format** used for the transmission of different types of base band signals. Hence it is easy to integrate all these signals together and send them on the common network.
7. It is easy to drop or reinsert the message sources in a PCM-TDM system.

6.7.2 Disadvantages of PCM :

1. The encoding, decoding and quantizing circuitry of PCM is complex.
2. PCM requires a large bandwidth as compared to the other systems.

6.7.3 Applications of PCM :

Some of the applications of PCM are as follows :

1. In digital telephone systems.
2. In the space communication, space craft transmits signals to earth. Here the transmitted power is very low (10 to 15W) and the distances are huge (a few million km). Still due to the high noise immunity, only PCM systems can be used in such applications.

6.8 Linear Delta Modulation (D.M.) :

MU : Dec. 04, May 06, May 08, May 09, Dec. 09, Dec. 10, May 11, Dec. 12, Dec. 16

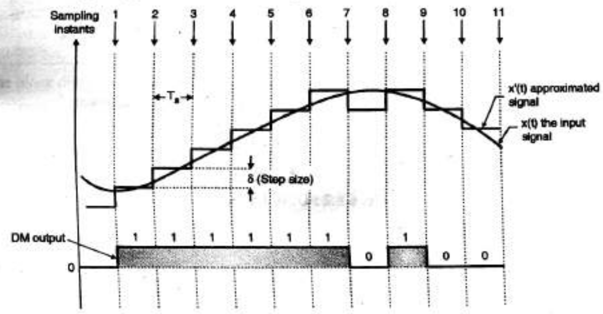
University Questions

Q. 1	Write short note on : Delta modulation.	(Dec. 04, 6 Marks, May 11, 2 Marks)
Q. 2	Explain : Delta Modulation.	(May 06, 5 Marks)
Q. 3	Explain linear delta modulation.	(May 08, 3 Marks)
Q. 4	Explain delta modulation.	(May 09, Dec. 09, Dec. 10, 4 Marks)
Q. 5	What is delta modulation ?	(Dec. 12, 5 Marks)
Q. 6	Discuss delta modulation and adaptive delta modulation.	(Dec. 16, 5 Marks)

- In PCM system, N number of binary digits are transmitted per quantized sample. Hence the signaling rate and transmission channel bandwidth of the PCM system are very large.
- These disadvantages can be overcome by using the delta modulation.

Principle of operation :

- Delta modulation transmits only one bit per sample instead of N bits transmitted in PCM. This reduces its signaling rate and bandwidth requirement to a great extent.



(L-235) **Fig. 6.8.1 : D.M. Waveforms**

- In the basic or linear D.M., a staircase approximated version of the sampled input signal is produced as shown in Fig. 6.8.1.
- The original signal and its staircase representation are then compared to produce a difference signal.
- And this difference signal is quantized into only two levels namely $\pm \delta$ corresponding to positive and negative difference respectively.
- That means if the approximated signal $x'(t)$ lies below $x(t)$ at the sampling instant, then the approximated signal is increased by "δ". (See instants 1, 2, 3, 4, 5 and 6 in Fig. 6.8.1.)
- Whereas if $x'(t)$ is greater than $x(t)$ at the sampling instant, then $x'(t)$ is decreased by "δ" (see instants 7, 9 and 10 in Fig. 6.8.1.)

D.M. output :

- As shown in Fig. 6.8.1, the D.M. output is 1 if the staircase signal $x'(t)$ is increased by "δ" i.e. at sampling instants 1, 2, 3, 4, 5 and 6.
- Whereas D.M. output is 0 if $x'(t)$ is decreased by "δ" i.e. at sampling instants 7, 9 and 10.
- In delta modulation, the present sample value $x(t)$ is compared with the approximate value $x'(t)$ and the result of this comparison is transmitted.
- Thus we are sending the information of whether the present sample value is higher than or lower than the approximate value. Note that the actual sampled value is not being transmitted.

6.8.1 Delta Modulator Transmitter :

MU : May 03, May 04, May 07, Dec. 11

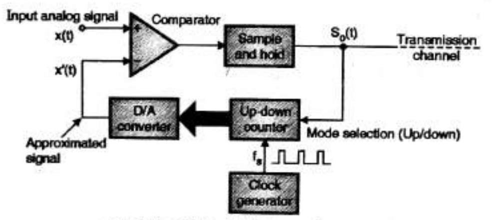
University Questions

Q. 1 Draw a neat block diagram of a delta modulator system. Explain the working with waveforms at the output of each block. **(May 03, 4 Marks)**

Q. 2 Explain the following : Delta modulation : Diagram, working. **(May 04, 7 Marks)**

Q. 3 Draw neat block diagram of Delta modulator and explain its working. **(May 07, Dec. 11, 6 Marks)**

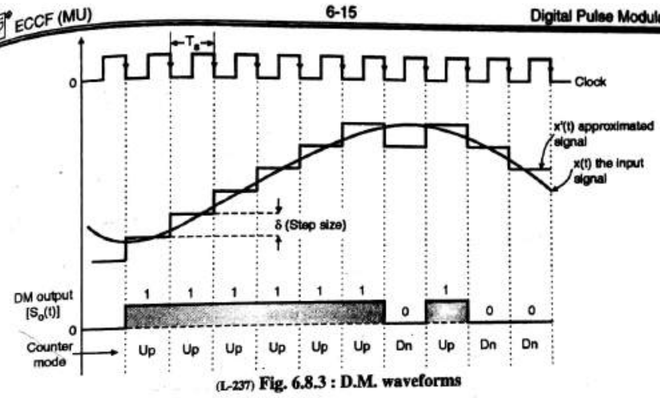
The block diagram of a delta modulator transmitter is as shown in the Fig. 6.8.2.



(L-236) **Fig. 6.8.2 : D.M. transmitter**

The operation of the circuit is as follows :

- $x(t)$ is the analog input signal and $x'(t)$ is the quantized (approximated) version of $x(t)$. Both these signals are applied to a comparator.
- The comparator output goes high if $x(t) > x'(t)$ and it goes low if $x(t) < x'(t)$. Thus the comparator output is either 1 or 0. The sample and hold circuit will hold this level (0 or 1) for the entire clock cycle period.
- The output of the sample and hold circuit is transmitted as the output of the DM system. Thus in DM, the information which is transmitted is only whether $x(t) > x'(t)$ or vice versa. Also note that one bit per clock cycle is being sent. This will reduce the bit rate and hence the BW.
- The transmitted signal is also used to decide the mode of operation of an up/down counter. The counter output increments by 1 if $S_o(t) = 1$ and it decrements by 1 if $S_o(t) = 0$, at the falling edge of each clock pulse. This is as shown in the waveform in the Fig. 6.8.3.



(L-237) **Fig. 6.8.3 : D.M. waveforms**

- The counter output is converted into analog signal by a D to A converter. Thus we get the approximated signal $x'(t)$ at the output of the D to A converter.

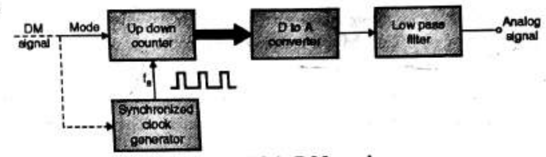
6.8.2 D.M. Receiver :

MU : May 03

University Questions

Q. 1 Draw a neat block diagram of a delta demodulator system. Explain the working with waveforms at the output of each block. **(May 03, 4 Marks)**

- The block diagram of the D.M. receiver is as shown in Fig. 6.8.4.
- Compare it with the transmitter block diagram, you will find that it is identical to the chain of blocks producing the signal $x'(t)$ i.e. the approximated signal.
- The original modulating signal can be recovered back by passing this signal through a low pass filter.



(L-238) **Fig. 6.8.4 : D.M. receiver**

6.8.3 Comparison of D.M. and DPCM :

- The comparison of D.M. and DPCM systems revels that except for an output low pass filter, they are identical 60 D.M. is actually a special case of DPCM.

6.8.4 Features of D.M. :

- The output codeword consist of only one bit. Hence no need of framing.
- Simplicity of design for transmitter and receiver.
- Less bit rate and lower bandwidth.

6.8.5 Quantization Noise (Distortions) in the DM System :

MU : May 03, Dec. 06, Dec. 07

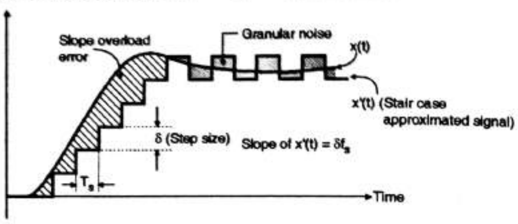
University Questions

Q. 1 Explain slope overload error and hunting error (granular noise) in Delta Modulation. Derive the condition to avoid slope overload error. **(May 03, 8 Marks, Dec. 06, 7 Marks)**

Q. 2 Explain the granular noise. **(Dec. 07, 8 Marks)**

The DM system is subjected to two types of quantization error or distortions :

1. Slope overload distortion and 2. Granular noise.



(L-239) **Fig. 6.8.5 : Distortions in D.M.**

1. Slope overload distortion :

- Look at the Fig. 6.8.5. Due to small step size (δ), the slope of the approximated signal $x'(t)$ will be small.

$$\text{The slope of } x'(t) = \frac{\delta}{T_s} = \delta f_s \quad ...(6.8.1)$$

- If slope of the input analog signal $x(t)$ is much higher than that of $x'(t)$ over a long duration then $x'(t)$ will not be able to follow the variations in $x(t)$, at all.
- The difference between $x(t)$ and $x'(t)$ is called as the slope overload distortion.
- Thus the slope overload error occurs when slope of $x(t)$ is much larger than slope of $x'(t)$.
- The slope overload error can be reduced by increasing slope of the approximated signal $x'(t)$.
- Slope of $x'(t)$ can be increased and hence the slope overload error can be reduced by either increasing the step size "δ" or by increasing the sampling frequency f_s.
- However with increase in δ the granular noise increases and if f_s is increased, signaling rate and bandwidth requirements will go up. Thus reducing the slope overload error is not easy.

2. Granular noise :

- When the input signal $x(t)$ is relatively constant in amplitude, the approximated signal $x'(t)$ will fluctuate above and below $x(t)$ as shown in Fig. 6.8.5. The difference between $x(t)$ and $x'(t)$ is called as granular noise.
- The granular noise is similar to the quantization noise in the PCM system.

- It increases with increase in the step size δ. To reduce the granular noise, the step size should be as small as possible.
- However this will increase the slope overload distortion.
- In the linear delta modulator the step size δ is not variable. If it is made variable then the slope overload distortion and granular noise both can be controlled.
- A system with a variable step size is known as the adaptive delta modulator (ADM).

6.8.6 D.M. Bit Rate (Signaling Rate) :

- D.M. bit rate (r) = Number of bits transmitted / second

$$= \text{Number of samples/sec} \times \text{Number of bits/sample} = f_s \times 1 = f_s \quad ...(6.8.2)$$

- Thus the D.M. bit rate is (1/N) times the bit rate of a PCM system, where N is the number of bits per transmitted PCM codeword.
- Hence the channel bandwidth for the D.M. system is reduced to a great extent as compared to that for the PCM system.

6.8.7 Advantages of Delta Modulation :

MU : May 08

University Questions

Q. 1 Give advantages of delta modulation. **(May 08, 2 Marks)**

1. Low signaling rate and low transmission channel bandwidth, because in delta modulation, only one bit is transmitted per sample.
2. The delta modulator transmitter and receiver are less complicated to implement as compared to PCM.

6.8.8 Disadvantages of Delta Modulation :

MU : Dec. 03, May 04, May 07, May 08, Dec. 11, Dec. 13

University Questions

Q. 1 What are the drawbacks of delta modulation ? How can they be minimized. **(Dec. 03, 6 Marks)**

Q. 2 Explain delta modulation and limitations. **(May 04, 3 Marks)**

Q. 3 What are the drawbacks of Delta modulator and how are they overcome by ADM ? **(May 07, Dec. 11, Dec. 13, 5 Marks)**

Q. 4 What are the various problems associated with DM ? **(May 08, 2 Marks)**

1. The two distortions discussed earlier i.e. slope overload error and granular noise are present.
2. Practically the signaling rate with no slope overload error will be much higher than that of PCM.

The slope overload error can be reduced by using another type of delta modulation, called as adaptive delta modulation (ADM).

6.8.9 Applications of D.M. :

- For some types of digital communications.
- For digital voice storage.

6.8.10 Condition for Avoiding the Slope Overload Error : MU : May 03, Dec. 06

University Questions

Q. 1 Derive the condition to avoid slope overload error. **(May 03, 8 Marks, Dec. 06, 7 Marks)**

Refer the following example to derive the condition for avoiding the slope overload error.

Ex. 6.8.1 : Consider a sinusoidal signal $x(t) = A \cos(\omega_m t)$ applied to a delta modulator with a step size δ. Show that the slope overload distortion will occur if

$$A > \frac{\delta}{\omega_m T_s} = \frac{\delta}{2\pi}\left(\frac{f_s}{f_m}\right)$$

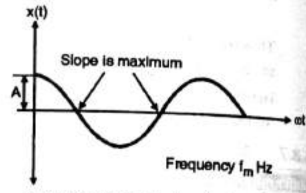
where T_s is the sampling period.

Soln. :

- Let the input signal be sinusoidal with amplitude A volts and frequency f_m Hz as shown in Fig. P. 6.8.1.

  The given signal is $x(t) = A \cos \omega_m t$

- The slope of this signal will be maximum when derivative of $x(t)$ with respect to time is maximum.



(D-494) **Fig. P. 6.8.1 : Input signal x (t)**

$$\therefore \quad \text{Slope of } x(t) = \frac{dx(t)}{dt} = -A\,\omega_m \sin \omega_m t$$

$$\text{The maximum value of the slope of } x(t) = -A\,\omega_m \quad \ldots(1)$$

$$\text{Slope of the staircase approximated signal } x'(t) = \frac{\delta}{T_s} \quad \ldots(2)$$

- To avoid the slope overload distortion, it is necessary that the maximum slope of x (t) be less than the slope of x' (t).

$$\therefore \quad \left|\frac{dx(t)}{dt}\right|_{max} \le \frac{\delta}{T_s}$$

$$\therefore \quad A\,\omega_m \le \frac{\delta}{T_s}$$

$$\therefore \quad A \le \frac{\delta}{\omega_m T_s}$$

- This is the condition for avoiding the slope overload distortion. Therefore the slope overload distortion will occur if this condition is not satisfied i.e.

$$\text{if } A > \frac{\delta}{\omega_m T_s} \quad \ldots(6.8.3)$$

6.8.11 Maximum Output Signal to Noise Ratio :

- It can be proved that the maximum signal to noise ratio of a D.M. system is given by,

$$\frac{S}{N_q} = \frac{3}{8\pi^2 f_m^2 f_M T_s^3} \quad \ldots(6.8.4)$$

where f_M = Cutoff frequency of the low pass filter in the D.M. receiver.

Ex. 6.8.2 : A sinusoidal voice signal $x(t) = \cos(6000\,\pi\, t)$ is to be transmitted using either PCM or DM. The sampling rate for PCM system is 8 kHz and for the transmission with DM, the step size δ is decided to be of 31.25 mV. The slope overload error is to be avoided. Assume that the number quantization levels for a PCM system is 64. Calculate the signaling rates of both these systems and comment on the result.

Soln. :

1. **Signaling rate of a PCM system :**

$$r = N f_s$$

$$\text{But } Q = 2^N$$

$$\therefore \quad N = \log_2 Q = \log_2 64 = 6$$

$$\therefore \quad \text{Signaling rate of PCM} = r = 6 \times 8 \text{ kHz} = 48 \text{ kHz}$$ ...Ans.

2. **Signaling rate of DM system :**

- The signaling rate of a DM system is equal to its sampling rate f_s because in DM only one bit is transmitted per sample. We know that the condition to avoid the slope overload distortion is given by,

$$A \le \frac{\delta}{\omega_m T_s} \quad \text{or} \quad A \le \frac{\delta f_s}{2\pi f_m}$$

- We want to calculate f_s

$$\therefore \quad f_s \ge \frac{2\pi f_m A}{\delta}$$

- Substitute values to get

$$f_s \ge \frac{2\pi \times 3 \times 10^3 \times 1}{31.25 \times 10^{-3}}$$

$$\therefore \quad f_s \ge 603.18 \text{ kHz}$$

$$\therefore \quad \text{Signaling rate of DM} \ge 603.18 \text{ kHz}$$ ...Ans.

Comment : To transmit the same voice signal, the DM needs a very large signaling rate as compared to PCM. This is the biggest disadvantage of DM, which makes it an impractical system.

6.9 Adaptive Delta Modulation (ADM) :

MU : Dec. 03, May 06, May 07, May 09, Dec. 09, Dec. 10, Dec. 11, Dec. 12, Dec. 13, Dec. 14, May 15, Dec. 16

University Questions

Q. 1 What are the drawbacks of delta modulation ? How can they be minimized ? **(Dec. 03, 6 Marks)**

Q. 2 How is adaptive delta modulation better than linear delta modulation ? Draw block diagram of adaptive delta modulation and explain each block in detail. **(May 06, 10 Marks)**

Q. 3 What are the drawbacks of Delta modulator and how are they overcome by ADM. **(May 07, Dec. 11, Dec. 13, 5 Marks)**

Q. 4 Explain adaptive delta modulation. **(May 09, Dec. 09, Dec. 10, Dec. 12, 5 Marks)**

Q. 5 How is adaptive delta modulation superior to delta modulation ? **(Dec. 14, May 15, 5 Marks)**

Q. 6 Discuss delta modulation and adaptive delta modulation. **(Dec. 16, 5 Marks)**

- In the ADM system, the step size is not constant. Rather when the slope overload occurs the step size becomes progressive larger and therefore x′ (t) will catch up with x (t) more rapidly.
- Whenever the slope of input signal is large, the step size of the staircase approximated signal x′ (t) is increased.
- On the other hand when the input signal is varying slowly the step size is reduced.
- Thus the step size is adapted as per the level of input signal.

6.9.1 Types of ADM :

- There are various types of ADM systems available depending on the type of scheme used for adjusting the step size.
- In one type a discrete set of values is provided for the step size whereas in another type a continuous range of step size variation is provided.
- We will discuss the first type here.

6.9.2 Adaptive Delta Modulation (Transmitter) :

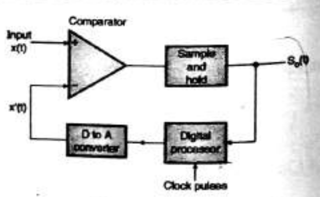
MU : May 06, May 10

University Questions

Q. 1 How is adaptive delta modulation better than linear delta modulation ? Draw block diagram of adaptive delta modulation and explain each block in detail. **(May 06, 10 Marks)**

Q. 2 Explain the transmitter for the adaptive delta modulation system. **(May 10, 5 Marks)**

- In the ADM system, the step size is not constant. Rather when the slope overload occurs the step size becomes progressive larger and therefore x′ (t) will catch up with x(t) more rapidly.
- The ADM transmitter is as shown in Fig. 6.9.1.



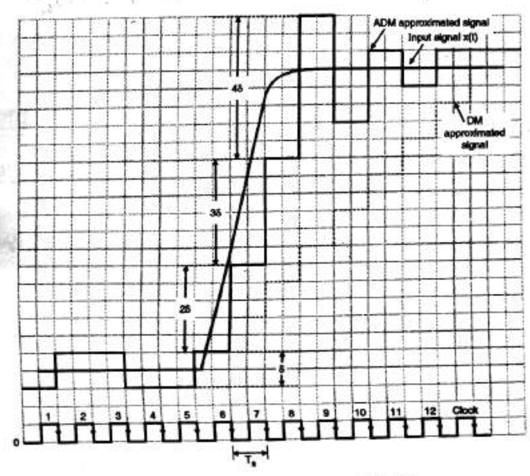
(D-495) **Fig. 6.9.1 : ADM transmitter**

- If you compare this block diagram with that of the linear delta modulator, then you will find that except for the counter being replaced by the digital processor, the remaining blocks are identical. Let us understand the operation of the digital processor. For that carefully see the waveforms of Fig. 6.9.2.

Operation :

- In response to the k^{th} clock pulse trailing edge, the processor generates a step which is equal in magnitude to the step generated in response to the previous i.e. $(k-1)^{th}$ clock edge.
- If the direction of both the steps is same, then the processor will increase the magnitude of the present step by "δ". If the directions are opposite then the processor will decrease the magnitude of the present step by "δ".
- $S_o(t)$ in the Fig. 6.9.1, i.e. the output of the ADM system is given as,

$$S_o(t) = +1 \text{ if } x(t) > x'(t) \text{ just before the } k^{th} \text{ clock edge.}$$



(E-711) **Fig. 6.9.2 : Waveforms of ADM**

and $S_o(t) = -1$ if $x(t) < x'(t)$ just before the k^{th} clock edge.

- Then the step size at the sampling instant k is given by,

$$\underset{\substack{\text{Step size}\\ \text{at } k^{th} \text{ clock}\\ \text{edge}}}{\delta(k)} = \underset{\substack{\text{Step size}\\ \text{at } (k-1)^{th}\\ \text{clock edge}}}{[\delta(k-1)]} \; \underset{\substack{\text{Output at}\\ k^{th} \text{ edge}}}{S_o(k)} + \underset{\substack{\text{Basic}\\ \text{step}\\ \text{size}}}{\delta} \; \underset{\substack{\text{Output at } (k-1)^{th}\\ \text{clock edge}}}{S_o(k-1)} \quad \ldots(6.9.1)$$

- Let us take an example :

  Refer to the waveforms of Fig. 6.9.2. Let us assume k = 6, i.e. consider the 6^{th} clock edge.

$$\therefore \quad k-1 = 5$$

$$\therefore \quad \delta(k-1) = \delta(5) = \delta$$

$$S_o(k) = S_o(6) = +1$$

$$S_o(k-1) = S_o(5) = +1$$

- Substitute in Equation (6.9.1) to get,

$$\delta(6) = \delta + \delta = 2\delta \quad ...(6.9.2)$$

  Look at the Fig. 6.9.2, the step size at the 6^{th} clock edge is 2δ.

- As shown in Fig. 6.9.2, due to variable step size, the slope overload error is reduced. But quantization error is increased. Due to the adjustable step size, the slope overload problem is solved. Hence ADM system has a low bit rate than the PCM system. Therefore the BW required is also less than a comparable PCM system.

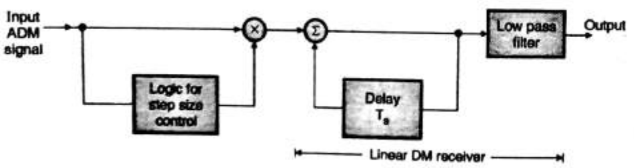
6.9.3 ADM Receiver :

MU : May 10

University Questions

Q. 1	**Explain the receiver for the adaptive delta modulation system.**	**(May 10, 5 Marks)**

The block diagram of ADM receiver is shown in Fig. 6.9.3.



(D-497) **Fig. 6.9.3 : ADM receiver**

- The ADM signal is first converted into a D.M. signal with the help of the step size control logic and then applied to a D.M. receiver.
- At the output of low pass filter we get the original signal back.

6.9.4 Advantages of Adaptive Delta Modulation :

The advantages of ADM over DM are as follows :

1. Reduction in slope overload distortion and granular noise.
2. Improvement in signal to noise ratio.
3. Wide dynamic range due to variable step size.
4. Better utilization of bandwidth as compared to delta modulation.
5. Low signaling rate.
6. Simplicity of implementation.

6.9.5 Disadvantages :

For a relatively constant magnitude input signal x (t), the ADM will produce a high granular noise.

6.10 Comparison of Digital Pulse Modulation Systems :

MU : May 04, May 09, Dec. 10, May 13

University Questions

Q. 1	**Compare pulse code modulation and delta modulation.**	**(May 04, 5 Marks)**
Q. 2	**Compare delta modulation and adaptive delta modulation.**	**(May 09, Dec. 10, 2 Marks)**
Q. 3	**Compare PCM, DM and ADM.**	**(May 13, 10 Marks)**

The PCM, DM, ADM and DPCM all are digital pulse modulation systems. Table 6.10.1 shows the comparison of these systems.

Table 6.10.1 : Comparison of PCM, DM and ADM

Sr. No.	Parameter	PCM	DM	ADM
1.	Number of bits per sample	N can be 4, 8, 16, 32, 64 etc.	N = 1	N = 1
2.	Step size	Depends on the number of Q levels.	Step size is fixed	Step size is variable
3.	Distortions / errors	Quantization error	Slope overload and granular noise	Granular noise
4.	Signaling rate and bandwidth	Highest	Low, if the input is slow varying	Lowest
5.	System complexity	Complex	Simple	Simple
6.	Feedback from output	No feedback	Feedback is present	Feedback is present
7.	Noise immunity	Very good	Very good	Very good
8.	Use of repeaters	Possible	Possible	Possible

Review Questions

Q. 1 How is the "information" transmitted in a PCM system ?

Q. 2 What is quantization ?

Q. 3 What is the relation between number of quantization levels and the number of bits per word?

Q. 4 What is quantization error ? What is its maximum value ?

Q. 5 How to reduce the quantization error ?

Q. 6 What are the advantages of PCM ?

Q. 7 What are the disadvantages of PCM ?

Q. 8 What is the signaling rate in terms of N and f_s ?

Q. 9 Why is companding used ?

Q. 10 How bandwidth required for the DM signal is less than that of PCM ?

Q. 11 What information do you transmit in DM system ?

Q. 12 What is the cause of slope overload error in DM ?

Q. 13 How to reduce the slope overload error ?

Q. 14 What is the signaling rate of a DM system ?

Q. 15 What is granular noise ?

Q. 16 How to reduce granular noise ?

Q. 17 Can we use DM practically as an alternative to PCM ?

Q. 18 How is slope overload reduced in ADM ?

Q. 19 What are the advantages of digital representation of a signal ?

Q. 20 Is the signal and noise separable in PCM ?

Q. 21 What is the relation between the quality of PCM signal and the number of digits per word ?

Q. 22 What is the maximum value of signal to quantization noise of a PCM system ?

Q. 23 What is the bandwidth of a PCM system ?

Q. 24 What is the main disadvantage of a PCM system ?

Q. 25 How is this disadvantage overcome using the linear delta modulation.

Q. 26 What is the condition for avoiding the slope overload error ?

Q. 27 What is the difference between DM and ADM ?

Q. 28 State the advantages of ADM over DM.

Q. 29 What is the operating principle of differential PCM system ?

Q. 30 Explain why is PCM system more immune to noise as compared to the other pulse modulation systems.

Q. 31 Explain the Robust quantization process.

Q. 32 What is companding ? State its advantages.

Q. 33 Explain the transmitter and receiver for the adaptive delta modulation (ADM) system.

Q. 34 How does ADM overcome the problem of slope overload ?

Q. 35 Compare PCM, DM and ADM systems.

Q. 36 Explain the operation of D-M System.

□□□

CHAPTER 7

Multiplexing

Module 5

Syllabus :

Principle of TDM using PCM and FDM.

7.1 Introduction to Multiplexing :

MU : May 03, May 04, May 05, Dec. 05, May 06, Dec. 06, Dec. 09, Dec. 13, May 14, Dec. 14, May 15, May 16

University Questions

Q. 1 What is multiplexing in communication systems ?
(May 03, 4 Marks, May 04, May 05, Dec. 05, May 06, Dec. 06, 5 Marks)

Q. 2 What is need of multiplexing ? **(Dec. 09, 5 Marks)**

Q. 3 What is multiplexing in communication system ? Draw black diagram of TDM-PCM system and explain ? **(Dec. 13, May 14, 10 Marks)**

Q. 4 What do you understand by signal multiplexing ? Explain TDM and FDM with suitable examples. **(Dec. 14, May 15, 10 Marks)**

Q. 5 Explain with suitable example what do you understand by signal multiplexing ? **(May 16, 5 Marks)**

- Multiplexing is the process of simultaneously transmitting two or more individual signals over a single communication channel.
- Due to multiplexing it is possible to increase the number of communication channels so that more information can be transmitted.
- The typical applications of multiplexing are in telemetry and telephony or in the satellite communication.

7.2 Concept of Multiplexing :

MU : May 03, May 04, May 05, Dec. 05, May 06, Dec. 06, Dec. 09, Dec. 13, May 14, Dec. 14, Dec. 15, May 16

University Questions

Q. 1 What is multiplexing in communication systems ?
(May 03, 4 Marks, May 04, May 05, Dec. 05, May 06, Dec. 06, 5 Marks)

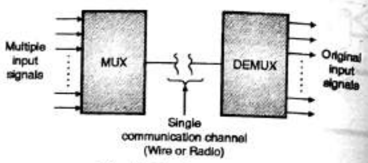
Q. 2 What is need of multiplexing ? **(Dec. 09, 3 Marks)**

Q. 3 What is multiplexing in communication system ? Draw block diagram of TDM-PCM system and explain. **(Dec. 13, May 14, 10 Marks)**

Q. 4 What do you understand by signal multiplexing ? Explain TDM and FDM with suitable examples. **(Dec. 14, May 15, 10 Marks)**

Q. 5 Explain with suitable example what do you understand by signal multiplexing ? **(Dec. 15, May 16, 5 Marks)**

- The concept of a simple multiplexer is illustrated in Fig. 7.2.1.
- The multiplexer receives a large number of different input signals.
- Multiplexer has only one output which is connected to the single communication channel.
- The multiplexer combines all input signals into a single composite signal and transmits it over the communication medium.



(L-105) **Fig. 7.2.1 : Concept of multiplexing**

- Sometimes the composite signal is used for modulating a carrier before transmission.
- At the receiving end, of communication link, a demultiplexer is used to separate out the signals into their original form.
- The operation of demultiplexer is exactly opposite to that of a multiplexer. Demultiplexing is the process which is exactly opposite to that of multiplexing.

7.2.1 Types of Multiplexing :

MU : May 16

University Questions

Q. 1 Explain with suitable example what do you understand by signal multiplexing ? **(May 16, 5 Marks)**

- There are three basic types of multiplexing. They are :
  1. Frequency division multiplexing (FDM)
  2. Time division multiplexing (TDM).
  3. Wavelength division multiplexing (WDM).

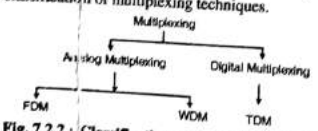
7.2.2 Classification of Multiplexing :

MU : May 09, Dec. 10

University Questions

Q. 1 Define and explain various multiplexing techniques used in communication systems. **(May 09, Dec. 10, 10 Marks)**

- The multiplexing techniques can be broadly classified into two categories namely analog and digital.
- Analog multiplexing can be either FDM or WDM and digital multiplexing is TDM.
- Fig. 7.2.2 shows the classification of multiplexing techniques.



(L-106) **Fig. 7.2.2 : Classification of multiplexing techniques**

- Generally the FDM and WDM systems are used to deal with the analog information whereas the TDM systems are used to handle the digital information.
- In FDM many signals are transmitted simultaneously where each signal occupies a different frequency slot within a common bandwidth.
- In TDM the signals are not transmitted at a time, instead they are transmitted in different time slots.

7.3 Frequency Division Multiplexing (FDM) :

MU : Dec. 03, Dec. 06, May 08, May 09, Dec. 09, May 10, Dec. 10, Dec. 12, May 13, May 14, Dec. 14, May 15, May 16

University Questions

Q. 1 Explain FDM. **(Dec. 03, Dec. 06, 3 Marks, May 10, Dec. 12, May 13, 5 Marks)**

Q. 2 Explain FDM in detail. **(May 08, 5 Marks, Dec. 09, 7 Marks)**

Q. 3 Define and explain various multiplexing techniques used in communication systems. **(May 09, Dec. 10, 10 Marks)**

Q. 4 Explain principle of FDM. **(May 14, 5 Marks)**

Q. 5 What do you understand by signal multiplexing ? Explain TDM and FDM with suitable examples. **(Dec. 14, May 15, 10 Marks)**

Q. 6 Explain with suitable example what do you understand by signal multiplexing ? **(May 16, 5 Marks)**

- The operation of FDM is based on sharing the available bandwidth of a communication channel among the signals to be transmitted.
- That means many signals are transmitted simultaneously with each signal occupying a different frequency slot within the total available bandwidth.
- Each signal to be transmitted modulates a different carrier. The modulation can be AM, SSB, FM or PM.
- The modulated signals are then added together to form a composite signal which is transmitted over a single channel.
- The spectrum of composite FDM signal is shown in Fig. 7.3.1(a).
- Generally the FDM systems are used for multiplexing the analog signals.

(L-107) **Fig. 7.3.1(a) : Spectrum of FDM signal**

7.3.1 FDM Transmitter :

MU : Dec. 14, May 15

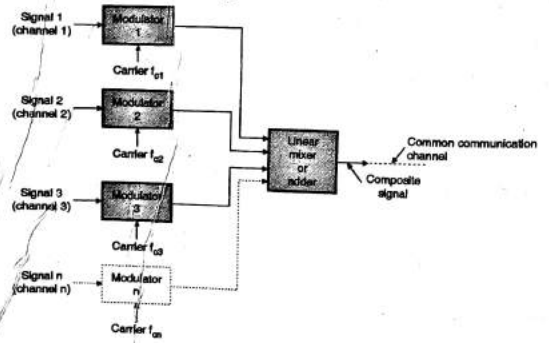
University Questions

Q. 1 What do you understand by signal multiplexing ? Explain TDM and FDM with suitable examples. **(Dec.14, May 15, 10 Marks)**

- Fig. 7.3.1(b) shows the block diagram of an FDM transmitter. The signals which are to be multiplexed will each modulate a separate carrier.
- The type of modulation can be AM, SSB, FM or PM.
- The modulated signals are then added together to form a complex signal which is transmitted over a single channel.

Operation :

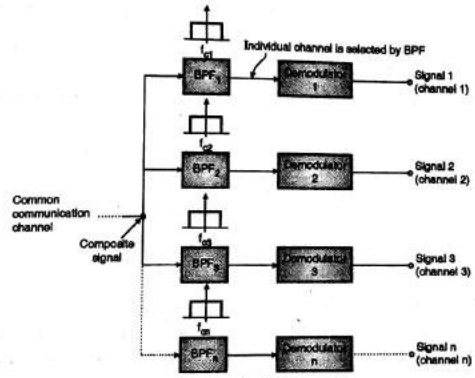
- Each signal modulates a separate carrier. The modulator outputs will contain the sidebands of the corresponding signals.
- The modulator outputs are added together in a linear mixer or adder. The linear mixer is different from the normal mixers. Here the sum and difference frequency components are not produced. But only the algebraic addition of the modulated outputs will take place.
- Different signals are thus added together in the time domain but they have their own separate identity in the frequency domain. This is as shown in the Fig. 7.3.1(a).
- The composite signal at the output of mixer is transmitted over the single communication channel as shown in Fig. 7.3.1(b). This signal can be used to modulate a radio transmitter if the FDM signal is to be transmitted through air.



(L-108) **Fig. 7.3.1(b) : The FDM transmitter**

7.3.2 FDM Receiver :

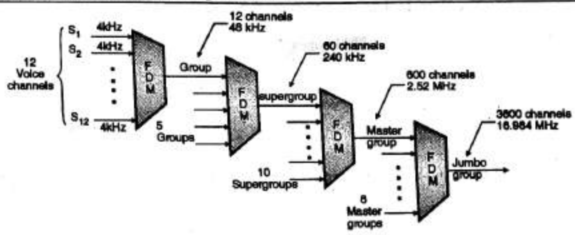
- The block diagram of an FDM receiver is as shown in Fig. 7.3.1(c). The composite signal is applied to a group of band pass filters (BPF).



(L-109) **Fig. 7.3.1(c) : FDM receiver**

- Each BPF has a center frequency corresponding to one of the carriers used in the transmitter i.e. $f_{c1}, f_{c2}, \ldots f_{cn}$ etc.
- The BPFs have an adequate bandwidth to pass all the channel information without any distortion.
- Each filter will pass through only its channel and reject all the other channels. Thus all the multiplexed channels are separated out.
- The channel demodulator then removes the carrier and recovers the original signal back.

7.4 Multiplexing Hierarchy in FDM :



(L-110) **Fig. 7.4.1 : FDM hierarchy**

The levels of multiplexing is also called as multiplexing hierarchy.

- The different levels of multiplexing which is also called multiplexing hierarchy is as follows :

  Level (1) : Basic Group. [12 voice channels multiplexed together].

  ↓

  Level (2) : Super Group. [Upto 5 basic groups multiplexed together i.e. upto 60 voice channels].

  ↓

  Level (3) : Master Group. [Upto 10 super groups multiplexed together i.e. upto 600 voice channels].

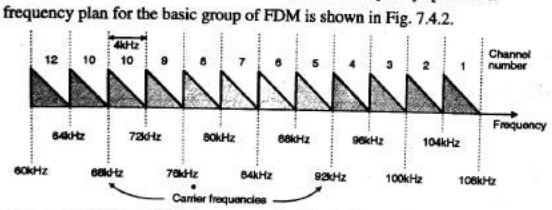
  ↓

  Level (4) : Jumbo Group. [Upto 6 master groups multiplexed together i.e upto 3600 voice channels].

- This hierarchy is used by AT and T and shown in Fig. 7.4.1.

Basic Group [12 voice channels] :

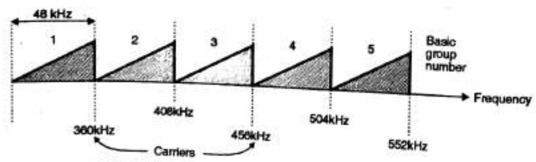
- The frequency plan for the typical basic group is as shown in Fig. 7.4.2. Here the 12 voice channels such as telephone channels modulate the carrier frequencies in the range of 60 to 108 kHz range. The carrier frequencies are spaced at 4 kHz from each other.
- SSB modulation technique is used to save the bandwidth. Each voice channel is applied to a balanced modulator along with a carrier. The output of a balanced modulator consists of the upper and lower sidebands.
- Frequency plans of groups of FDM are nothing but the frequency spectrums.
- The frequency plan for the basic group of FDM is shown in Fig. 7.4.2.



(L-111) **Fig. 7.4.2 : Frequency plan for the basic group of FDM**

Super group :

The frequency plan for a super group is as shown in Fig. 7.4.3. A super group consists of at the most 60 voice channels.



(L-112) **Fig. 7.4.3 : Frequency plan for a super group of FDM**

7.5 Application of FDM in Telephone System :

- One of the important applications of the FDM system is the telephone system.
- Here each telephone signal is in the range of 300 to 3000 Hz. These voice channels modulate different subcarriers.
- These modulated subcarriers are then added together. As the number of telephone channels are very large, the multiplexing process is repeated at several levels.

7.6 Advantages, Disadvantages and Applications of FDM :

7.6.1 Advantages of FDM :

1. A large number of signals (channels) can be transmitted simultaneously.
2. FDM does not need synchronization between its transmitter and receiver for proper operation.
3. Demodulation of FDM is easy.
4. Due to slow narrow band fading only a single channel gets affected.

7.6.2 Disadvantages of FDM :

1. The communication channel must have a very large bandwidth.
2. Intermodulation distortion takes place.
3. Large number of modulators and filters are required.
4. FDM suffers from the problem of crosstalk.
5. All the FDM channels get affected due to wideband fading.

7.6.3 Applications of FDM :

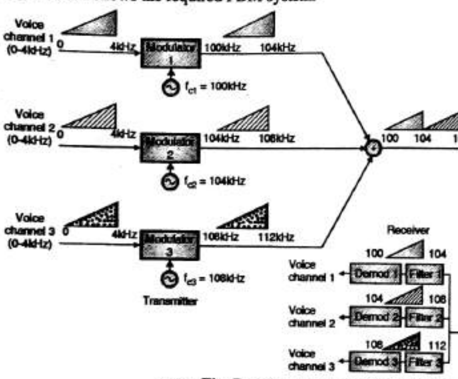
Some of the important applications of FDM are :

1. Telephone systems.
2. AM (amplitude modulation) and FM (frequency modulation) radio broadcasting.
3. TV broadcasting
4. First generation of cellular phones used FDM.

Ex. 7.6.1 : Draw the FDM system to combine three voice channels. Each voice channel occupies a bandwidth of 4 kHz. The common voice channel has a bandwidth of 12 kHz from 100 kHz to 112 kHz.

Soln. :

Fig. P. 7.6.1 shows the required FDM system.

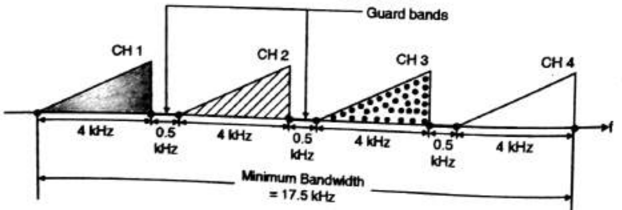


(D-508) **Fig. P. 7.6.1 : Required FDM system**

Ex. 7.6.2 : **4 voice channels each having a bandwidth of 4 kHz are to be multiplexed using FDM. A guardband of 500 Hz is to be inserted between the adjacent channels. Calculate the minimum bandwidth of the link.**

Soln. :

- The frequency spectrum of the FDM signal is shown in Fig. P. 7.6.2.
- The minimum bandwidth is equal to 17.5 kHz as shown in Fig. P. 7.6.2.
- The bandwidth without guardbands would have been 16 kHz.



(D-509) **Fig. P. 7.6.2**

Conclusion :

Guardbands increase the bandwidth of FDM signal still they should be included in order to avoid interference between the adjacent channels.

7.7 Time Division Multiplexing (TDM) :

MU : Dec. 03, Dec. 06, May 07, May 08, May 09, May 10, Dec. 10, Dec. 11, May 12, Dec. 12, Dec. 14, May 15, Dec. 15

University Questions

Q. 1 Explain TDM. **(Dec. 03, Dec. 06, 3 Marks, May 10, Dec. 12, 5 Marks)**

Q. 2 What do you mean by TDM ? **(May 07, 5 Marks)**

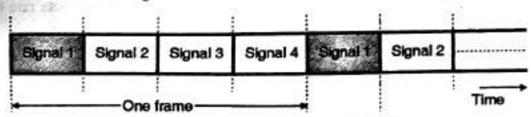
Q. 3 Explain TDM in detail. **(May 08, Dec. 11, May 12, 5 Marks)**

Q. 4 Define and explain various multiplexing techniques used in communication systems. **(May 09, Dec. 10, 10 Marks)**

Q. 5 What do you understand by signal multiplexing ? Explain TDM and FDM with suitable examples. **(Dec. 14, May 15, 10 Marks)**

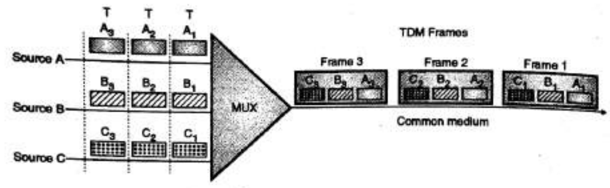
Q. 6 Explain with suitable example what do you understand by signal multiplexing ? **(Dec. 15, 5 Marks)**

- The process called multiplexing is used in order to utilize common transmission channel or medium to transmit more than one signals simultaneously.
- TDM is a digital multiplexing process.
- In TDM all the signals to be transmitted are not transmitted simultaneously. Instead, they are transmitted one-by-one.
- Thus each signal will be transmitted for a very short time. One cycle or frame is said to be complete when all the signals are transmitted once on the transmission channel. The TDM principle is illustrated in Fig. 7.7.1.



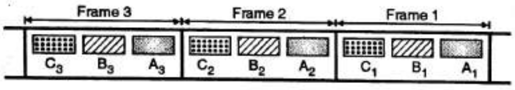
(L-122) **Fig. 7.7.1 : Principle of TDM**

- As shown in the Fig. 7.7.1 one transmission of each channel completes one cycle of operation called as a "Frame".
- The TDM system can be used to multiplex analog or digital signals, however it is more suitable for the digital signal multiplexing.
- The concept of TDM will be more clear if you refer to Fig. 7.7.2.
- The data flow of each source (A, B or C) is divided into units (say A_1, A_2 or B_1, C_1 etc.)
- Then one unit from each source is taken and combined to form one frame. The size of each unit such as A_1, B_1 etc. can be 1 bit or several bits.



(L-123) **Fig. 7.7.2 : TDM system**

- Fig. 7.7.3 shows the frames of TDM signal. For 3 inputs being multiplexed, a frame of TDM will consist of 3 units i.e. one unit from each source.
- Similarly for n number of inputs, each TDM frame will consist of n units.



(L-124) **Fig. 7.7.3 : TDM frames**

- The TDM signal in the form of frames is transmitted on the common communication medium.

Data rate :

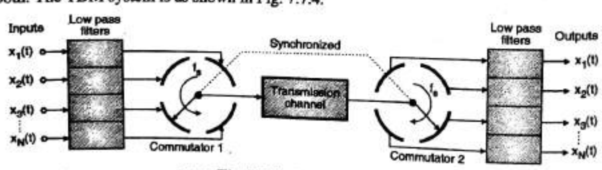
- For a TDM, the data rate of the multiplexed signal is always n times the data rate of individual sources, where n is the number of sources.
- So if three sources are being multiplexed, then the data rate of the TDM signal is three times higher than the individual data rate.
- Naturally the duration of every unit (A_1 or B_1 etc.) in TDM signal is n times shorter than the unit duration before multiplexing.

7.7.1 PAM - TDM System : MU : Dec. 04

University Questions

Q. 1 Write short note on : PAM/TDM systems. (Dec. 04, 6 Marks)

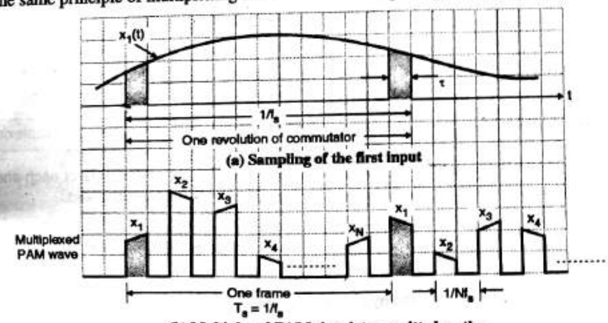
- The TDM system which is going to be discussed now, combines the concepts of PAM and TDM both. The TDM system is as shown in Fig. 7.7.4.



(L-125) **Fig. 7.7.4 : PAM/TDM system**

The operation of the system is as follows :

- The multiplexer here is a single pole rotating switch or commutator. It can be a mechanical switch or an electronic switch. It rotates at f_s rotations per second.
- As the switch arm rotates, it is going to make contact with the position 1, 2, 3 or N for a short time. To these contacts are connected the N analog signals which are to be multiplexed.
- Thus the switch arm will connect these N input signals one by one to the communication channel.
- The waveform of a TDM signal which is being transmitted is as shown in Fig. 7.7.5. It shows that the rotary switch samples each channel during each of its rotations. Each rotation corresponds to one frame. Hence 1 frame is completed in T_s seconds where $T_s = 1/f_s$.
- At the receiver, there is one more rotating switch or commutator used for demultiplexing.
- It is important to note that this switch must rotate at the same speed as that of the commutator 1 at the transmitter and its position must be synchronized with commutator 1 in order to ensure proper demultiplexing.
- The same principle of multiplexing can be used for multiplexing more number of signals.



(b) Multiplexed PAM signal, transmitted on the transmission channel

(L-126) **Fig. 7.7.5**

Ex. 7.7.1 : 3 signals having a data rate of 2 kbps are grouped together by means of time division multiplexing. Each unit consists of 1 bit. Calculate :

1. The bit duration before multiplexing.
2. The transmission rate of TDM.
3. The duration of each time slot in TDM.
4. The duration of one TDM frame.

Soln. :

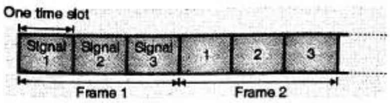
Step 1 : Duration of a bit before multiplexing :

- Each signal has a data rate of 2 kbps. That means 2000 bits per second.
- Hence the duration of each bit is,

$$T_b = \frac{1}{2000} = 0.5 \text{ mS} \qquad \text{...Ans.}$$

Step 2 : Transmission rate of TDM :

- The TDM frame is shown in Fig. P. 7.7.1.



(D-522) **Fig. P. 7.7.1 : TDM frames**

- As discussed earlier the transmission rate of TDM is n times higher than the bit rate of each source.

$$\text{Transmission rate of TDM} = n \times 2000 = 3 \times 2000 = 6000 \text{ bps or } 6 \text{ kbps} \quad \text{...Ans.}$$

Step 3 : Duration of time slot in TDM :

$$\text{Duration of each time slot in TDM} = \frac{1}{6000} = 166.67 \mu S \quad \text{...Ans.}$$

Step 4 : Frame duration :

$$\text{Duration of 1 frame} = n \times \text{duration of one slot}$$

$$= 3 \times 166.67 \mu S = 0.5 \text{ mS} \quad \text{...Ans.}$$

Note : The duration of a TDM frame is always equal to the duration of one unit before multiplexing.

Ex. 7.7.2 : Three channels are to be multiplexed using TDM technique. The rate of each channel is 150 bytes per second. In TDM, one byte per channel is to be multiplexed.

Calculate : 1. Frame size 2. Frame duration 3. Frame rate and 4. Bit rate of the TDM signal.

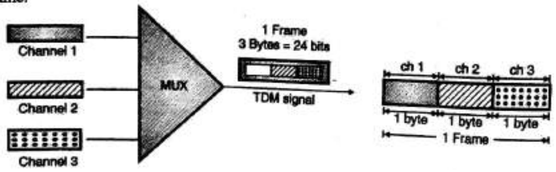
Soln. :

Given : Bit rate of each channel 150 bytes per sec, n = 3.

1 byte per channel to be multiplexed.

Step 1 : Draw the system block diagram :

Fig. P. 7.7.2(a) shows the block diagram of the TDM system and Fig. P. 7.7.2(b) shows one TDM frame.



(a) TDM system **(b) One frame of TDM**

(D-523) **Fig. P. 7.7.2**

Step 2 : Frame size :

Each frame consists of one byte from each channel. So frame size is 3 bytes or 24 bits.

Step 3 : Frame duration and frame rate :

- The duration of a TDM frame is always equal to the duration of one unit before multiplexing.
- Here one unit before multiplexing is 1 byte i.e. 8 bits.

$$\therefore \quad \text{Frame duration} = 1 \text{ byte duration}$$

- But each channel transmits at 150 bytes/sec.

$$\therefore \quad \text{Frame duration} = \frac{1}{150} = 6.666 \text{ mS} \quad \text{...Ans.}$$

$$\text{Frame duration} = \frac{1}{\text{Frame duration}} = \frac{1}{6.666 \times 10^{-3}} = 150 \text{ frames/sec.} \quad \text{...Ans.}$$

Step 4 : Bit rate of TDM signal :

$$\text{Bit rate of TDM} = \text{Number of bits per frame} \times \text{Number of frames per second.}$$

$$= 24 \times 150 = 3600 \text{ bps} \quad \text{...Ans.}$$

Interleaving :

- On the multiplexer side the commutator-1 opens in front of a connection, that connection has the opportunity to send its bit on to the channel.
- This process is called as interleaving.

7.7.2 Signaling Rate (r) :

The signaling rate of a TDM system is defined as the number of pulses transmitted per second. It is denoted by "r". Let us now derive the expression for the signaling rate of the PAM-TDM system.

- Let W = Maximum frequency of all the input signals x_1 to x_N.
- Therefore as per Nyquist criteria, the sampling frequency $f_s \geq 2$ W. Therefore the speed of rotation of the commutators is f_s rotations per second with $f_s \geq 2W$.
- As shown in Fig. 7.7.6, one revolution of commutators corresponding to one frame contains one sample from each input signal.

$$\therefore \quad 1 \text{ Revolution} \Rightarrow 1 \text{ frame} \Rightarrow N \text{ pulses} \quad \text{...(7.7.1)}$$

- 1 frame period is $(1/f_s)$ i.e. T_s seconds. Therefore in "T_s" seconds "N" number of pulses are transmitted. Hence the pulse to pulse spacing within the frame is given by,

$$\text{Pulse to pulse spacing} = \frac{T_s}{N} = \frac{1}{Nf_s} \quad \text{...(7.7.2)}$$

- As the period of one pulse (ON + OFF) is $(1/Nf_s)$ seconds, the number of pulses per second is given by,

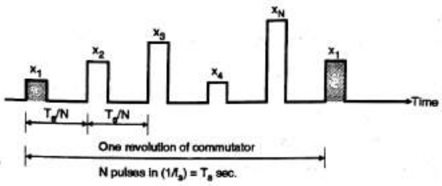
$$\text{Number of pulses per second} = Nf_s$$

- This is nothing but the signaling rate.

$\therefore$ Signaling rate of a TDM system = r = Nf_s pulses/second. But as $f_s \geq 2W$.

$$\text{Signaling rate of a TDM system} = r \geq 2NW \text{ pulses/second} \quad \text{...(7.7.3)}$$

- A TDM system is supposed to have its signaling rate as high as possible. It is evident from the expressions above that the signaling rate can be increased by increasing the sampling rate f_s and/or the number of input signals N.



(L-129) **Fig. 7.7.6 : Calculation of number of pulses per second for PAM-TDM system**

7.7.3 Transmission Bandwidth of a TDM Channel :

- The minimum transmission bandwidth of a PAM-TDM channel is given by,

$$B_T = \frac{1}{2} \text{signaling rate}$$

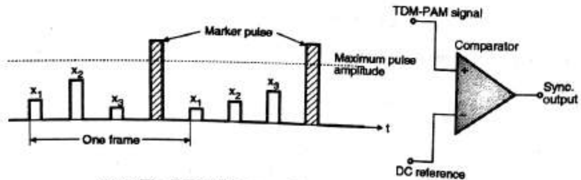
$\therefore$ Minimum transmission bandwidth $B_T \geq \frac{1}{2} \times 2\,NW$

$\therefore$ Minimum transmission bandwidth $B_T = NW$ ...(7.7.4)

7.7.4 Synchronization in TDM System :

Synchronization in TDM PAM system :

- The multiplexed PAM signals can be received properly if and only if the transmitter and receiver commutators are synchronized to each other in terms of the speed and the position.
- In order to ensure synchronization, a marker pulse is introduced at the end of each frame in the transmitted signal as shown in Fig. 7.7.7.
- The amplitude of this pulse is higher than the maximum permissible amplitude of the multiplexed channels.
- At the receiver the received signal is compared with a DC reference level. The comparator responds to only the marker pulse to produce output.
- Thus the marker pulse is separated from the remaining multiplexed channels.
- Due to the introduction of synchronizing pulse, only three signals instead of four can now be transmitted.



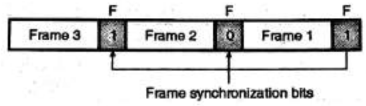
(D-516) **Fig. 7.7.7 : Frame synchronization and detection**

Synchronization in digital TDM system :

- In digital TDM, the inputs are digital bit streams. All the digital pulses are of same amplitudes. So the synchronizing techniques for TDM-PAM system cannot be used here.
- If the synchronization is lost then a bit belonging to one channel may be received by a wrong channel.

Synchronization bit :

- In order to establish synchronization between the transmitter and receiver, one synchronization bit is added at the beginning of each TDM frame as shown in Fig. 7.7.8.



(D-524) **Fig. 7.7.8 : Frame synchronization in TDM**

- These bits are called frame synchronizing bits or simply framing bits.
- The framing bits will follow a pattern frame to frame. For example the pattern shown in Fig. 7.7.8 is 101.
- The framing bit pattern will allow the demux to synchronize itself to the mux.

7.7.5 Advantages of TDM :

1. Full available channel bandwidth can be utilized for each channel.
2. Intermodulation distortion is absent.
3. TDM circuitry is not very complex.
4. The problem of crosstalk is not severe.

7.7.6 Disadvantages of TDM :

1. Synchronization is essential for proper operation.
2. Due to slow narrowband fading, all the TDM channels may get wiped out.

7.7.7 Comparison of FDM and TDM Systems :

MU : May 07, May 11, Dec. 11

University Questions

Q. 1 Compare TDM and FDM. **(May 07, May 11, Dec. 11, 5 Marks)**

Sr. No.	FDM	TDM
1.	The signals which are to be multiplexed are added in the time domain. But they occupy different slots in the frequency domain.	The signals which are to be multiplexed can occupy the entire bandwidth but they are isolated in the time domain.
2.	FDM is usually preferred for the analog signals.	TDM is preferred for the digital signals.
3.	Synchronization is not required.	Synchronization is required.
4.	The FDM requires a complex circuitry at the transmitter and receiver.	TDM circuitry is not very complex.

Sr. No.	FDM	TDM
5.	FDM suffers from the problem of crosstalk due to imperfect band pass filters.	In TDM the problem of crosstalk is not severe.
6.	Due to wideband fading in the transmission medium, all the FDM channels are affected.	Due to fading only a few TDM channels will be affected.
7.	Due to slow narrowband fading taking place in the transmission channel only a single channel may be affected in FDM.	Due to slow narrowband fading all the TDM channels may get wiped out.

7.7.8 Applications of TDM :

1. Multiplexing of digital signals.
2. Digital telephony
3. Satellite communications.
4. Fiber optic communication
5. Wireless communication applications.

7.8 PCM-TDM System (Multiplexing the PCM Signals) :

MU : May 03, May 04, May 05, Dec. 05, May 06, Dec. 06, Dec. 07, Dec. 13, May 14, Dec. 15

University Questions

Q. 1 Draw the block diagram of a TDM-PCM system to transmit five voice channels. **(May 03, 4 Marks, May 05, 5 Marks)**

Q. 2 Draw the block diagram of TDM-PCM system and explain each block. Also calculate the bit rate at the output of this system. **(May 04, Dec. 05, May 06, Dec. 06, 5 Marks)**

Q. 3 Write short note on : TDM-PCM system. **(Dec. 07, 10 Marks)**

Q. 4 What is multiplexing in communication system ? Draw block diagram of TDM-PCM system and explain. **(Dec. 13, May 14, 10 Marks)**

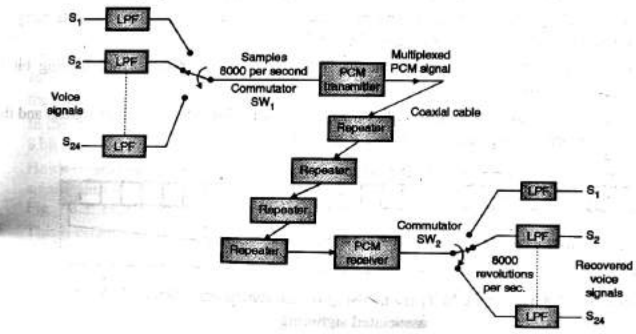
Q. 5 With block diagram explain TDM-PCM system. **(Dec. 15, 5 Marks)**

- When a large number of PCM signals are to be transmitted over a common channel, multiplexing of these PCM signals is required.
- Fig. 7.8.1 shows the basic time division multiplexing scheme for PCM voice channels called as the **T_1 digital system.**
- This system is used to convey a number of voice signals over telephone lines using wideband coaxial cable. Thus the communication medium used is a coaxial cable.

Operation of the T_1 system :

- The operation of the PCM-TDM system shown in Fig. 7.8.1 is as follows :
- This system has been designed to multiplex 24 voice channels marked as S_1 to S_{24}. Each signal is bandlimited to 3.3 kHz, and the sampling is done at a standard rate of 8 kHz. This sampling rate is higher than the Nyquist rate. The sampling is done by the commutator switch SW_1.
- These voice signals are selected one by one and connected to a PCM transmitter by the commutator switch SW_1, as it completes its rotation. The commutator switch remains in contact with each voice channel for a short time. Thus it samples each of the 24 channels.

- Each sampled signal is then applied to the PCM transmitter which converts it into a digital signal by the process of A to D conversion and companding. Each sampled voice signal is converted into an 8-bit PCM word.
- The resulting digital waveform is transmitted over a co-axial cable. This waveform is called as the PCM-TDM signal.
- Periodically, after every 6000 ft., the PCM-TDM signal is regenerated by amplifiers called "Repeaters". They eliminate the distortion introduced by the channel and remove the superimposed noise and regenerate a clean noise free PCM-TDM signal at their output. This ensures that the received signal is free from the distortions and noise.
- At the destination the signal is companded, decoded and demultiplexed, using a PCM receiver. The PCM receiver output is connected to different low pass filters via the commutator switch SW_2. The LPF outputs are applied to the destination receivers (subscribers).
- Synchronization between the transmitter and receiver commutators SW_1 and SW_2 is essential in order to ensure proper communication.



(G-1312) **Fig. 7.8.1 : Block diagram of a basic PCM-TDM system**

Bits/Frame :

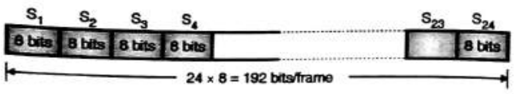
- The commutators sweep continuously from S_1 to S_{24} and back to S_1 at the rate of 8000 revolutions per second (Sampling rate = 8000 samples/sec.).
- This will generate 8000 samples per second of each signal (S_1 to S_{24}). Each sample is then encoded (converted) into an eight bit digital word. One complete revolution of commutator switches corresponds to generation of one frame which consists of all 24 voice channels.
- Thus the digital signal generated during one complete sweep (revolution) of the commutator is given by :

$$1 \text{ Frame} \equiv 1 \text{ revolution} = 24 \text{ channels}$$

$$= 24 \times 8 \text{ bits} = 192 \text{ bits}$$

- One frame of PAM-TDM is shown in Fig. 7.8.2. Each voice signal from S_1 to S_{24} is encoded into eight bits.

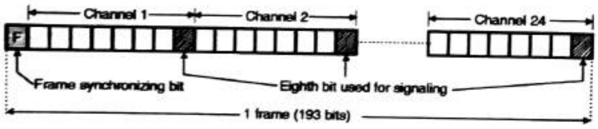
- One frame corresponds to one revolution which is the time taken to transmit each signal once. Hence 1-frame corresponds to one-revolution of the commutator.



(G-1313) **Fig. 7.8.2 : One frame and bits per frame**

7.8.1 Frame Synchronization :

- As we have already seen, the synchronization between the transmitter and receiver commutators is essential.
- Without such synchronization the receiver cannot know which received bits were generated by whom at the transmitter and are meant for which subscriber on the receiving side.
- To provide such synchronization, an extra bit is transmitted preceding the 192 data bits carrying the information in each frame, as shown in Fig. 7.8.3.
- This bit is called as the frame synchronizing bit "F". Thus one frame synchronizing bit is transmitted per frame.
- This makes the total number of bits per frame to be 193. The time slots for the 24 signals and the extra frame synchronizing bit is as shown in Fig. 7.8.3.



(G-1314) **Fig. 7.8.3 : The PCM T_1 frame using frame synchronization and channel associated signaling**

- Twelve successive F slots are used to transmit a 12 bit code. The code is 1101 1100 1000.
- This code is transmitted repeatedly once every 12 frames and it is used at the receiver to achieve synchronization between the transmitter and receiver commutators.

Bit rate :

- Bit rate means number of bits transmitted by a system per second. In the T_1 system; as each signal is sampled 8000 times per second :

  1 frame (1 revolution of commutator) = 1/8000 = 125 μsec.
- But 1 frame consists of 193 bits.

$\therefore$ 193 bits are transmitted in 125μsec.

$$\therefore \text{ Number of bits in 1 sec.} = \frac{193}{125 \times 10^{-6}} = 1.554 \times 10^{6}$$

$$\therefore \text{ Bit rate of } T_1 \text{ system} = 1.544 \text{ Mbits/sec.}$$

Bandwidth of T_1 system :

$$\text{Minimum bandwidth } B_T = \frac{1}{2} \text{bit rate} = \frac{1}{2} \times 1.544 \times 10^6 = 772 \text{ kHz}$$

Duration of each bit :

$$193 \text{ bits} = 125\,\mu S$$

$$1 \text{ bit} = (125/193)\,\mu S = 0.6476\,\mu S$$

7.8.2 Channel Associated Signaling :

- When the PCM-TDM system is being used for the telephony, it is expected to transmit certain control signals along with the voice information. The control information is of two types : signalling and supervisory.
- The signaling information consists of the signals such as a call is being initiated or a call is being terminated, or the address of calling party etc.
- In analog system such a signaling information is transmitted over a separate channel other than the voice channel. But in the T_1 system which is a digital system, a separate channel is not used.
- In T_1 system the signaling information is sent using the same data bit slots which are used to send the voice information. The technique used is "bit slot sharing".
- In the "bit slot sharing" method, for the first five frames, all the 24 channels are encoded into an 8 bit digital code. That means all the 8-bits in each PCM word will carry the voice information.
- However in the sixth frame, all the channels are coded into a 7 bit code and the LSB (least significant bit) of each channel is used to transmit the signaling information. This is as shown in Fig. 7.8.3. That means MSB 7-bits carry voice and the LSB bit carries the signalling information.
- This is called as "channel associated signalling". This pattern is repeated after every six frames.

Review Questions

Q. 1 With the help of block schematic, explain the principle of FDM.

Q. 2 Compare FDM and TDM methods of multiplexing ?

Q. 3 Explain the principles of Time Division Multiplexing.

Q. 4 Why is it necessary to use time division multiplexing while transmitting PAM signals ?

Q. 5 Why is synchronization needed in TDM system ?

Q. 6 Describe how transmission distortion of a TDM signal can cause cross-talk between two adjacent channels.

Q. 7 Describe the multiplexing hierarchy for an FDM system.

Q. 8 Explain the PCM-TDM system.

Q. 9 What do you understand by the channel associated signaling ?

Q. 10 State the applications of PCM-TDM.

Q. 11 How is synchronization achieved in PCM-TDM system ?

Q. 12 State advantages and disadvantages of TDM system.

❑❑❑

CHAPTER 8

Module 6

Information Theory

Syllabus :

Amount of information, average information, information rate, Statement of Shannon's theorem, channel capacity (Numericals).

8.1 Introduction :

- Any communication system needs to be efficient as well as reliable.
- After the World War II, the field of communication started expanding very rapidly and engineers started designing more efficient and more reliable systems.
- Great scientist Shannon published his paper on information theory in 1948. It was refined later on by many other researchers.
- Shannon suggested that efficient communication from a source to destination is possible using **source coding.**
- Whereas reliable communication over a noisy channel can be achieved by using **error control coding.**
- The performance of communication systems is dependent on three factors namely :
  1. The available signal power.
  2. The background noise present on the channel
  3. Limited bandwidth.
- Till now we have used the signal theory for the analysis of the noise on transmitted signal. But the limitation of signal theory is that, it cannot explain the fundamental communication process of the "information transfer".
- Therefore, a new approach called "information theory" has been adopted.
- The information theory deals with three basic concepts namely :
  1. Amount of source.
  2. The capacity of a channel.
  3. Use of coding for utilizing channel capacity for information transfer.
- **Information theory** is a broadly based mathematical discipline and it is applicable to various fields such as communications, computer science, statistics and probability.
- In the field of communications the information theory is used for mathematical modelling and analysis of a communication system.

- The information theory is used to find answers to many questions related to a communication system. But the most important questions that it answers are as follows :
- The answers to these questions can be obtained from the entropy of the source and the capacity of a channel.

8.2 Uncertainity :

- The words uncertainity, surprise and information are all related to each other. Before an event occurs, there is an uncertainty, when the event actually takes place there is an amount of surprise and after the event has taken place there is a gain of information.
- The concept of information is related to the "uncertainity". This can be explained using the following sentences :
  1. Earth revolves around sun.
  2. The rainfall is likely in the evening today.
  3. India may win the world cup.
- The first sentence does not have any uncertainity or surprise element. Hence the information content in it is minimum.
- But look at the other two sentences. They are full of uncertainity or element of doubt. Therefore they carry more information.
- Extending this concept we can say that if a source transmits a message of probability p_k then the information carried by the message will increase as its probability p_k decreases, i.e. as the message becomes less likely. So higher the uncertainity higher the information.

8.2.1 Definition of Information (Measure of Information) :

- The quantitative measure of the information is dependent on our notion of the word "information". To explain what is meant by information let us take an example.
- Consider a communication system transmitting messages $m_1, m_2, m_3,.....$ Let the probabilities of occurrence of these messages be p_1, p_2...etc. respectively. Then the amount of "information" transmitted through the signal m_k is given as,

$$I_k = \log_2\left(\frac{1}{p_k}\right) \quad ...(8.2.1)$$

where, p_k = Probability of occurrence of m_k.

- This is the definition of the information.

Unit of information :

- The information I_k defined in Equation (8.2.1) is actually a dimensionless quantity. But by convention the unit attached to it is "bits". This is, when the base of the logarithm is 2.
- If we change the base of the logarithm, then the unit of information will also change.
- For the natural logarithm (base "e") the unit of information is "nat", and for base 10 the unit is "Harley" or "decit".
- Previously the term "bit" was used to represent a binary digit. So hence-forth we are going to use the term "bit" as unit of information and represent the binary digit by the term "binit".

8.2.2 Properties of Information :

The important properties about the amount of information conveyed by a message are as follows :

1. The information contained in an absolutely certain event is zero. That means,

$$I_k = 0 \quad ... \text{for } p_k = 1 \qquad ...(8.2.2)$$

2. The information contents of a message increases with decrease in the value of its probability of occurrence (p_k).

   That means the most unexpected event (which has the least probability of occurrence) will contain maximum information. (see Ex. 8.2.6).

3. The occurance of an event, either provides some information or no information, but it is never associated with a loss of information. That means

$$I_k \geq 0 \quad ...\text{for } 0 \leq p_k \leq 1 \qquad ...(8.2.3)$$

4. I_k is a continuous function of p_k.

5. The total information of two or more mutually independent message signals or events is equal to the sum of the information contents of the individual messages. i.e.

$$I_T = I_1 + I_2 + I_3 + ... \qquad ...(8.2.4)$$

where, I_T = Total information

$I_1, I_2, ...$ are the information in individual messages.

Significance of the amount of information in practice :

- Let us see how the concept of measure of information is useful in our day to day life. Suppose that you are planning a trip to Bangalore.
- To determine what clothes to pack if you telephone the weather bureau and hear one of the following forecasts :
  1. The sun will rise.
  2. There will be scattered rain storms.
  3. There will be a tornado.
- Now look at the information contents of the above messages. The first message conveys virtually zero information.
- The second message does give some information which is not known and the third one gives us maximum information. This is because tornado is a rare and unexpected event.

This shows that the most unexpected event or message is going to convey maximum information and the most expected event or message will convey minimum information.

Ex. 8.2.1 : A message signal m_k is transmitted by a transmitter. The probability of occurrence of this signal is 1/4. Calculate the information conveyed by it in terms of bits, nats and decit.

Soln. :

1. It is given that $p_k = \frac{1}{4}$ hence using the definition of information we get,

$$I_k = \log_2 [1/p_k] = \frac{\log_{10} 4}{\log_{10} 2} = 2 \text{ bits} \qquad ...\text{Ans.}$$

2. Information in terms of nats : (log base "e")

$$I_k = \log_e [1/p_k]$$

$$= \log_e 4 = 1.386 \text{ nats} \qquad ...\text{Ans.}$$

This shows that 1 bit = 0.693 nat

3. Information in terms of decit (log base 10)

$$I_k = \log_{10} [1/p_k] = \log_{10} [4] = 0.6 \text{ decit}$$

This shows that 1 bit = 0.3 decit ...**Ans.**

Ex. 8.2.2 : In a binary PCM system the binits 0 and 1 are transmitted. If they have the probabilities of $\frac{3}{4}$ and $\frac{1}{4}$ respectively then calculate the information conveyed by each one of them and comment on the result.

Soln. :

$$p_0 = \frac{3}{4} \text{ and } p_1 = \frac{1}{4}$$

$\therefore$ Information conveyed by binit 0 $= \log_2 (4/3) = \frac{\log_{10} (4/3)}{\log_{10} 2}$

$= 0.415$ bits.

Information conveyed by the binit 1 $= \log_2 (4) = 2$ bits.

This example proves that the information content of a message having less probability of occurrence is higher.

Ex. 8.2.3 : For the same system in the previous example calculate the amount of information conveyed by the binits if they are equally likely to be transmitted.

Soln. :

The binits 0 and 1 are equally likely to be transmitted.

$$\therefore \quad p_0 = p_1 \text{ and} \qquad ...(1)$$

$$p_0 + p_1 = 1 \qquad ...(2)$$

$\therefore$ here p_0 = Probability of "0" being transmitted.

and p_1 = Probability of "1" being transmitted.

$$\therefore \quad p_0 = p_1 = \frac{1}{2}$$

$\therefore$ Information conveyed by binit 0 $= I_0 = \log_2 2 = 1$ bit and

Information conveyed by binit 1 $= I_1 = \log_2 2 = 1$ bit.

Ex. 8.2.4 : For a system that transmits M equally likely message signals where $M = 2^N$ (N is an integer), prove that the information in each message is N bits.

Soln. :

It is given that there are M message signals $m_1, m_2 \dots\dots\dots m_M$

And all are equally likely, i.e. probability of each message will be the same. Now,

$$p_1 + p_2 + \dots + p_M = 1 \quad \dots(1)$$

$$\text{Let} \quad p_1 = p_2 = \dots\dots = p_M = p$$

$$\therefore \quad p + p + \dots + p = 1$$

$$\therefore \quad M \times p = 1$$

$$p = \frac{1}{M} = \frac{1}{2^N} \quad \dots(2)$$

This is the probability of occurrence of each message.

$\therefore$ Information in each message

$$I = \log_2 [1/p] = \log_2 [2^N] = N \text{ bits} \quad \dots(3)$$

This is a PCM system with "N" number of binits used in each transmitted message. The information content per message is also N bits. Thus the number of binary digits (binits) per message will be numerically equal to the bits of information.

Ex. 8.2.5 : For two independent messages m_1 and m_2 prove that the total amount of information conveyed is the sum of the information associated with each message individually.

Soln. :

Let the probabilities of occurrence of the messages m_1 and m_2 be p_1 and p_2 respectively. Therefore the individual information contents will be :

$$I_1 = \log_2 (1/p_1) \quad \dots(1)$$

$$\text{and} \quad I_2 = \log_2 (1/p_2) \quad \dots(2)$$

As the two messages are mutually independent, the probability of their simultaneous occurrence is given as,

$$p = p_1 \times p_2 \quad \dots(3)$$

Therefore the corresponding information is given as,

$$I_{1,2} = \log_2 [1/p] = \log_2 [1/p_1 p_2]$$

$$I_{1,2} = \log_2 [1/p_1] \times \log_2 [1/p_2] \quad \dots(4)$$

$$\therefore \quad I_{1,2} = I_1 + I_2 \quad \dots(5)$$

Ex. 8.2.6 : If a transmitter is supposed to transmit only one message m_k always, prove that the information conveyed by this message is zero.

Soln. :

As only one message is being transmitted its probability of occurrence will be maximum i.e. unity.

$$\therefore \quad p_k = 1.$$

Substitute this value in the equation of information.

$$I_k = \log_2 [1/1] = \frac{\log_{10} 1}{\log_{10} 2}$$

$$\therefore \quad I_k = 0 \text{ bits.}$$

Thus the information carried by a "certain or sure" event is zero.

8.3 Average Information or Entropy :

- The "Entropy" is defined as the average information per message. It is denoted by H and its units are bits/message.
- The entropy must be as high as possible in order to ensure maximum transfer of information.
- We will prove that the entropy depends only on the probabilities of the symbols that are being produced by the source.

8.3.1 Expression for Entropy :

- Follow the steps given below to obtain the expression for entropy.

Steps to be followed :

Step 1 : Let there be M different messages $m_1, m_2 \dots m_M$. Let their probabilities of occurrences be $p_1, p_2 \dots p_M$.

Step 2 : Let there be total L messages. Therefore there are p_1 L number of m_1 messages, p_2 L number of m_2 messages etc.

Step 3 : Calculate the information conveyed by message m_1 as $I_1 = \log_2 [1/p_1]$

Step 4 : Calculate the total information conveyed by m_1 as $I_{1(Total)} = p_1 L \times I_1$.

Step 5 : Similarly calculate the total information for all the other messages, i.e. $I_{2(Total)}, I_{3(Total)}, \dots$ etc.

Step 6 : Add all these information to obtain the total information $I_{(Total)} = I_{1(Total)} + I_{2(Total)} + \dots$

Step 7 : Divide the total information obtained in step 6 to obtain the expression for entropy.

- Suppose that a transmitter is transmitting M different and independent messages $m_1, m_2, m_3 \dots$ Let their probabilities of occurrence be $p_1, p_2, p_3 \dots$ respectively.
- Suppose that during a long period of transmission a sequence of L messages is generated.

1. Then if L is very very large we can expect that in the L message sequence,

   p_1 L messages of m_1 are transmitted

   p_2 L messages of m_2 are transmitted

   p_3 L messages of m_3 are transmitted

   ⋮

   p_M L messages of m_N are transmitted.

2. The information conveyed by the message m_1 is given as,

$$I_1 = \log_2 [1/p_1]$$

However there are p_1 L number of messages of m_1. Therefore the information conveyed by p_1 L number of messages will be

$$I_{1(Total)} = p_1 L \log_2 [1/p_1] \quad ...(8.3.1)$$

Similarly the total information conveyed by p_2 L number of m_2 messages is given as :

$$I_{2(Total)} = p_2 L \log_2 [1/p_2] \quad ...(8.3.2)$$

Similar expression can be written for the remaining messages.

3. As we already know, the total information of more than one mutually independent message signals is equal to the sum of the information content of individual messages. i.e.

$$I_{(Total)} = I_{1(Total)} + I_{2(Total)} + I_{3(Total)} + \dots \quad ...(8.3.3)$$

Substitute the values of $I_{1(Total)}$, $I_{2(Total)}$ ...etc. from the Equations (8.3.1) and (8.3.2), to get,

$$I_{(Total)} = p_1 L \log_2 [1/p_1] + p_2 L \log_2 [1/p_2] + p_3 L \log_2 [1/p_3] + \dots \quad ...(8.3.4)$$

$$\therefore I_{(Total)} = L [p_1 \log_2 (1/p_1) + p_2 \log_2 (1/p_2) + p_3 \log_2 (1/p_3) + \dots\dots] \quad ...(8.3.5)$$

4. The "Entropy" is defined as the average information per message interval. It is represented by the symbol "H". Therefore from Equation (8.3.5), we can write that,

$$H = \frac{I_{(Total)}}{L} = p_1 \log_2 (1/p_1) + p_2 \log_2 (1/p_2) + \dots\dots \quad ...(8.3.6)$$

$$\therefore \text{Entropy : } H = \sum_{k=1}^{M} p_k \log_2 (1/p_k) \quad ...(8.3.7)$$

This is the expression for Entropy.

This expression indicates that the entropy of a source is dependent only on the probabilities of the symbols that are being produced by the source.

Ex. 8.3.1 : Prove that the entropy of extremely likely and extremely unlikely message is zero.

Soln. :

1. In case of the "extremely likely" message, there is only one single possible message m_k to be transmitted. Therefore its probability $p_k = 1$. The entropy of a most likely message m_k is given as,

$$H = p_k \log_2 (1/p_k) = 1 \log_2 (1)$$

$$= \frac{\log_{10} 1}{\log_{10} 2} = 0 \quad ...(1)$$

2. For an extremely unlikely message m_k, its probability $p_k \to 0$

$$\therefore H = p_k \log_2 (1/p_k)$$

$$= 0 \quad ...(2)$$

Thus the average information or entropy of the most likely and most unlikely messages is zero.

8.4 Information Rate (R) :

If the source of the messages generates "r" number of messages per second then the information rate is given as,

$$R = r \times H \quad ...(8.4.1)$$

where, r = Number of messages/sec, and H = Average information/message. (Entropy)

Units of information rate :

$$R = \left[r \frac{\text{messages}}{\text{sec}} \right] \times \left[H \frac{\text{information}}{\text{message}} \right]$$

$\therefore$ R = Average information per second expressed in bits/sec.

Ex. 8.4.1 : An analog signal is bandlimited to 4 kHz. It is sampled at the Nyquist rate and the samples are quantized into 4 levels. The quantization levels Q_1, Q_2, Q_3 and Q_4 are independent messages and have the probabilities $p_1 = p_2 = \frac{1}{8}$ and $p_3 = p_4 = \frac{3}{8}$. Find the information rate of the source.

Soln. :

Data :

1. It is given that there are 4 messages Q_1, Q_2, Q_3 and Q_4 with probabilities of $\frac{1}{8}$, $\frac{1}{8}$, $\frac{3}{8}$ and $\frac{3}{8}$ respectively.
2. f_m = 4 kHz. Therefore sampling rate = 2 × 4 = 8 kHz.

(a) **Value of H :**

The average information per message is found as,

$$H = p_1 \log_2 (1/p_1) + p_2 \log_2 (1/p_2) + p_3 \log_2 (1/p_3) + p_4 \log_2 (1/p_4)$$

$$= \frac{1}{8}\log_2 (8) + \frac{1}{8}\log_2 (8) + \frac{3}{8}\log_2 (8/3) + \frac{3}{8}\log_2 (8/3)$$

$$\therefore H = 1.8 \text{ bits/message} \quad ...(1)$$

(b) Rate of message generation (r) [messages/sec] :

As the sampling rate is 8 kHz, we have 8000 samples per second. Each sample is converted to one of the four quantization levels Q_1 to Q_4 (messages). Therefore we have 8000 messages/sec.

$$\therefore \quad r = 8000 \text{ messages/sec} \qquad ...(2)$$

(c) The information rate :

$$R = H \times r = 1.8 \times 8000$$

$$\therefore \quad R = 14400 \text{ bits/sec} \qquad \textbf{...Ans.}$$

Ex. 8.4.2 : Assuming that the messages Q_1, Q_2, Q_3 and Q_4 in Ex. 8.4.1 are to be transmitted using a binary PCM, calculate the information rate.

Soln. :

The four messages can be identified by using binary code as indicated in the following table :

Message OR quantization level	Probability	Binary code
Q_1	1/8	0 0
Q_2	1/8	0 1
Q_3	3/8	1 0
Q_4	3/8	1 1

1. Number of messages per second = 8000

   But here number of binits per message = 2

   Hence number of binits per second = $8000 \times 2 = 16000$ binits/sec.
2. We have already seen that each binit is capable of conveying an average information (H) of 1 bit, if the binits are equally likely (having same probability).
3. Therefore the rate of information will be,

$$R = r \times H = 16{,}000 \times 1 = 16{,}000 \text{ bits/sec} \qquad \textbf{...Ans.}$$

Ex. 8.4.3 : Consider a telegraph source having two symbols, dot and dash. The dot duration is 0.2 seconds; and the dash duration is 3 times the dot duration. The probability of the dot occurring is twice that of the dash, and the time between symbols is 0.2 seconds. Calculate the information rate of the telegraph source.

Soln. :

Given that :

1. Dot duration : 0.2 sec.
2. Dash duration : $3 \times 0.2 = 0.6$ sec.
3. P (dot) = P (dot) = 2 P (dash).
4. Space between symbols is 0.2 sec.

   Information rate = ?

1. **Probabilities of dots and dashes :**

Let the probability of a dash be "P". Therefore the probability of a dot will be "2P". The total probability of transmitting dots and dashes is equal to 1.

$$\therefore \quad P(\text{dot}) + P(\text{dash}) = 1$$

$$\therefore \quad P + 2P = 1 \qquad \therefore P = 1/3$$

$$\therefore \quad \left.\begin{array}{l} \text{Probability of dash} = 1/3 \\ \text{and probability of dot} = 2/3 \end{array}\right\} \qquad ...(1)$$

2. **Average information H (X) per symbol :**

$$\therefore \quad H(X) = P(\text{dot}) \cdot \log_2 [1/P(\text{dot})] + P(\text{dash}) \cdot \log_2 [1/P(\text{dash})]$$

$$\therefore \quad H(X) = (2/3) \log_2 [3/2] + (1/3) \log_2 [3]$$

$$\therefore \quad H(X) = 0.3899 + 0.5283 = 0.9182 \text{ bits/symbol.}$$

3. **Symbol rate (Number of symbols/sec.) :**

The total average time per symbol can be calculated as follows :

$$\text{Average symbol time } T_s = [T_{DOT} \times P(DOT)] + [T_{DASH} \times P(DASH)] + T_{space}$$

$$= [0.2 \times 2/3] + [0.6 \times 1/3] + 0.2$$

$$\therefore \quad T_s = 0.5333 \text{ sec./ symbol.}$$

Hence the average rate of symbol transmission is given by :

$$r = 1/T_s = 1.875 \text{ symbols/sec.}$$

4. **Information rate (R) :**

$$R = r \times H(X) = 1.875 \times 0.9182$$

$$\therefore \quad R = 1.72 \text{ bits/sec.} \qquad \textbf{...Ans.}$$

Ex. 8.4.4 : The voice signal in a PCM system is quantized in 16 levels with the following probabilities:

$P_1 = P_2 = P_3 = P_4 = 0.1$

$P_5 = P_6 = P_7 = P_8 = 0.05$

$P_9 = P_{10} = P_{11} = P_{12} = 0.075$

$P_{13} = P_{14} = P_{15} = P_{16} = 0.025$

Calculate the entropy and information rate. Assume $f_m = 3$ kHz.

Soln. :

It is given that,

1. The number of levels = 16. Therefore number of messages = 16.
2. $f_m = 3$ kHz.

(a) To find the entropy of the source :

The entropy is defined as,

$$H = \sum_{k=1}^{M} p_k \log_2 (1/p_k) \quad ...(1)$$

As $M = 16$, Equation (1) gets modified to,

$$H = \sum_{k=1}^{16} p_k \log_2 (1/p_k)$$

$$= 4 [0.1 \log_2 (1/0.1)] + 4 [0.05 \log_2 (1/0.05)]$$

$$+ 4 [0.075 \log_2 (1/0.075)] + 4 [0.025 \log_2 (1/0.025)]$$

$$\therefore \quad H = 0.4 \log_2 (10) + 0.2 \log_2 (20) + 0.3 \log_2 (13.33) + 0.1 \log_2 (40)$$

$$= 0.4 \frac{\log_{10} 10}{\log_{10} 2} + \frac{0.2 \log_{10} 20}{\log_{10} 2} + 0.3 \frac{\log_{10}(13.33)}{\log_{10} 2} + \frac{0.1 \log_{10} 40}{\log_{10} 2}$$

$$\therefore \quad H = 3.85 \text{ bits/message} \quad ...(2) \text{ ...Ans.}$$

(b) To find the message rate (r) :

The minimum rate of sampling is Nyquist rate

Therefore $f_s = 2 \times f_m$

$$= 2 \times 3 \text{ kHz} = 6 \text{ kHz} \quad ...(3)$$

Hence there are 6000 samples/sec. As each sample is converted to one of the 16 levels, there are 6000 messages/sec.

$$\therefore \quad \text{Message rate } r = 6000 \text{ messages/sec} \quad ....(4)$$

(c) To find the information rate (R) :

$$R = r \times H = 6000 \times 3.85 = 23100 \text{ bits/sec} \quad \text{...Ans.}$$

Ex. 8.4.5 : A message source generates one of four messages randomly every microsecond. The probabilities of these messages are 0.4, 0.3, 0.2 and 0.1. Each emitted message is independent of other messages in the sequence :

1. What is the source entropy ?
2. What is the rate of information generated by this source in bits per second ?

Soln. :

It is given that,

1. Number of messages, $M = 4$, let us denote them by m_1, m_2, m_3 and m_4.
2. Their probabilities are $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.2$ and $p_4 = 0.1$.
3. One message is transmitted per microsecond.

$$\therefore \quad \text{Message transmission rate } r = \frac{1}{1 \times 10^{-6}} = 1 \times 10^6 \text{ messages/sec.}$$

(a) To obtain the source entropy (H) :

$$H = \sum_{k=1}^{4} p_k \log_2 (1/p_k)$$

$$\therefore \quad H = p_1 \log_2 (1/p_1) + p_2 \log_2 (1/p_2) + p_3 \log_2 (1/p_3) + p_4 \log_2 (1/p_4)$$

$$= 0.4 \log_2 (1/0.4) + 0.3 \log_2 (1/0.3) + 0.2 \log_2 (1/0.2) + 0.1 \log_2 (1/0.1)$$

$$\therefore \quad H = 1.846 \text{ bits/message} \quad \text{...Ans.}$$

(b) To obtain the information rate (R) :

$$R = H \times r = 1.846 \times 1 \times 10^6$$

$$\therefore \quad R = 1.846 \text{ M bits/sec} \quad \text{...Ans.}$$

Ex. 8.4.6 : A source consists of 4 letters A, B, C and D. For transmission each letter is coded into a sequence of two binary pulses. A is represented by 00, B by 01, C by 10 and D by 11. The probability of occurrence of each letter is $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{4}$ and $P(D) = \frac{3}{10}$. Determine the entropy of the source and average rate of transmission of information.

Soln. :

The given data can be summarised as shown in the following table :

Message	Probability	Code
A	1/5	0 0
B	1/4	0 1
C	1/4	1 0
D	3/10	1 1

Assumption : Let us assume that the message transmission rate be $r = 4000$ messages/sec.

(a) To determine the source entropy :

$$H = \frac{1}{5} \log_2 (5) + \frac{1}{4} \log_2 (4) + \frac{1}{4} \log_2 (4) + 0.3 \log_2 (10/3)$$

$$\therefore \quad H = 1.9855 \text{ bits/message} \quad \text{...Ans.}$$

(b) To determine the information rate :

$$R = r \times H = [4000 \text{ messages/sec}] \times [1.9855 \text{ bits/message}]$$

$$R = 7942.3 \text{ bits/sec} \quad \text{...Ans.}$$

(c) Maximum possible information rate :

$$\text{Number of messages/sec} = 4000$$

$$\text{But here the number of binary digits/message} = 2$$

$$\therefore \quad \text{Number of binary digits (binits)/sec.} = 4000 \times 2 = 8000 \text{ binits/sec.}$$

We know that each binit can convey a maximum average information of 1 bit

$$\therefore \quad H = 1 \text{ bit/binit}$$

$$\therefore \text{ Maximum rate of information transmission} = [8000 \text{ binits/s}] \times [H_{max} / \text{binit}]$$

$$= 8000 \times 1 \text{ bits/sec}$$

...Ans.

8.5 Some Important Definitions :

Channel capacity (C) :

entropy 1/m rate m/s

- The channel capacity is defined as the maximum data rate at which the digital data can be transmitted over the channel reliably.
- The various other concepts related to channel capacity are as follows :
  1. Data rate 2. Bandwidth
  3. Noise 4. Error rate

Data rate :

- It is defined as the number of bits transmitted by the transmitter per second. It indicates how fast a signal can be transmitted reliably over the given medium.
- This capability depends on the following factors :

  b.its/sec

  1. The amount of energy put into transmitting each signal.
  2. Distance to be travelled.
  3. Noise.
  4. Channel bandwidth

Channel bandwidth :

- The bandwidth of the communication medium should be large enough to transmit the digital signal reliably.
- An inadequate bandwidth will distort the signal and introduce errors into the received signal.

Noise : This is the average level of noise over the communication path.

Error rate : It is defined as the rate at which errors occur in the received (or detected) signal.

8.6 Data Rate Limits :

- In data communication a large data is required to be transferred from one place to the other.
- It is necessary to transfer it as quickly as possible. In other words the data rate in bits per second over a channel should be as high as possible.
- The data rate is decided by the following factors :
  1. The maximum bandwidth.
  2. The signal level.
  3. The noise presented by the channel.

- Two theorems were developed to calculate the data rate and we can use them on the basis of the type of channel as follows :
  1. A noiseless channel : Nyquist theorem 2. A noisy channel : Shannon's theorem

8.6.1 Noiseless Channel : Nyquist Bit Rate :

- As we know a transmission channel is a medium over which the electrical signals from a transmitter travel to the receiver. Two important characteristics of a transmission channel are :
  1. Signal to Noise ratio (SNR) and 2. Channel bandwidth.
- These two characteristics will ultimately decide the maximum capacity of a channel to carry information.
- Nyquist and Shannon worked on finding the maximum channel capacity of a bandlimited channel.
- Nyquist's theorem states that if the bandwidth of a transmission channel is "B" which carries a signal having "L" number of levels, then the maximum data rate "R" on this channel is given by.

$$R = 2B \log_2 L \qquad ...(8.6.1)$$

- As maximum data rate for reliable transmission is defined as channel capacity C, the above expression gets modified as :

$$C = 2B \log_2 L \qquad ...(8.6.2)$$

- This expression indicates that the data rate can be increased by increasing the number of different signal elements (L).

8.6.2 Noisy Channel : Shannon's Channel Capacity Theorem :

- A noiseless channel is not possible in the real world, so Shannon introduced a theorem called Shannon's capacity theorem to determine the highest possible data rate on the noisy channel.
- We have seen the Nyquist bandwidth earlier in this book.
- Shannon extended Nyquist's work. He included the effect of noise present on the transmission channel.
- According to Shannon's theorem, if (S/N) is the signal to noise ratio then the maximum data rate is given by

$$C = R_{max} = B \log_2 \left[1 + \frac{S}{N}\right] \text{ bits/sec} \qquad ... (8.6.3)$$

- Shannon's theorem puts a limit on the maximum number of levels for a given (S/N) ratio and bandwidth. This expression shows that the maximum data rate for a communication channel is dependent on the channel bandwidth B and signal to noise ratio (S/N).
- It is important to note that the Shannon's formula does not indicate the signal level. It says no matter how much is the signal level, it is not possible to achieve a data rate (R) which is greater than the capacity of the channel (C).

Importance of channel bandwidth :

- Bandwidth of the communication channel should be higher than the bandwidth of the signal that is to be transmitted over it.
- This is essential in order to preserve the shape of the signal being transmitted.
- If the channel bandwidth is less than the signal bandwidth then the signal shape will be distorted when it travels over this channel.

8.6.3 Solved Examples :

Ex. 8.6.1 : A channel has a bandwidth of 5 kHz and a signal to noise power ratio 63. Determine the bandwidth needed if the S/N power ratio is reduced to 31. What will be the signal power required if the channel bandwidth is reduced to 3 kHz ?

Soln. :

1. To determine the channel capacity :

It is given that B = 5 kHz and $\frac{S}{N} = 63$. Hence using the Shannon Hartley theorem the channel capacity is given by,

$$C = B \log_2 \left[1 + \frac{S}{N}\right] = 5 \times 10^3 \log_2 [1 + 63]$$

$$\therefore \quad C = 30 \times 10^3 \text{ bits/sec.} \quad ...(1)$$

2. To determine the new bandwidth :

The new value of $\frac{S}{N} = 31$. Assuming the channel capacity "C" to be constant we can write,

$$30 \times 10^3 = B \log_2 [1 + 31]$$

$$\therefore \quad B = \frac{30 \times 10^3}{5} = 6 \text{ kHz} \quad ...(2)$$

3. To determine the new signal power :

Given that the new bandwidth is 3 kHz. We know that noise power $N = N_o B$.

Let the noise power corresponding to a bandwidth of 6 kHz be $N_1 = 6 N_o$ and the noise power corresponding to the new bandwidth of 3 kHz be $N_2 = 3 N_o$.

$$\therefore \quad \frac{N_1}{N_2} = \frac{6 N_o}{3 N_o} = 2 \quad ...(3)$$

The old signal to noise ratio $= \frac{S_1}{N_1} = 31$

$$\therefore \quad S_1 = 31 N_1 \quad ...(4)$$

The new signal to noise ratio $= \frac{S_2}{N_2}$. We do not know its value, hence let us find it out.

$$30 \times 10^3 = 3 \times 10^3 \log_2 \left(1 + \frac{S_2}{N_2}\right)$$

$$\therefore \quad \frac{S_2}{N_2} = 1023$$

$$\therefore \quad S_2 = 1023 N_2 \quad ...(5)$$

But from Equation (3), $N_2 = \frac{N_1}{2}$, substituting we get,

$$\therefore \quad S_2 = 1023 \frac{N_1}{2} \quad ...(6)$$

Dividing Equation (6) by Equation (4) we get,

$$\therefore \quad \frac{S_2}{S_1} = \frac{1023 N_1}{2 \times 31 N_1} = 16.5$$

$$\therefore \quad S_2 = 16.5 S_1 \quad \text{...Ans.}$$

Thus if the bandwidth is reduced by 50% then the signal power must be increased 16.5 times i.e. 1650% to get the same capacity.

Ex. 8.6.2 : Calculate the maximum bit rate for a channel having bandwidth 3100 Hz and S/N ratio 20 dB.

Soln. :

Given : B = 3100 Hz

$$\frac{S}{N} = 20 \text{ dB.}$$

$$\text{But } 20 \text{ dB} = 10 \log (S/N)$$

$$\therefore \quad S/N = 100$$

The maximum bit rate is given by,

$$R_{max} = B \log_2 \left[1 + \frac{S}{N}\right] = 3100 \log_2 [1 + 100]$$

$$= \frac{3100 \log_{10} 101}{\log_{10} 2} = 20{,}640 \text{ bits/sec.} \quad \text{...Ans.}$$

Ex. 8.6.3 : Calculate the maximum bit rate for a channel having bandwidth 3100 Hz and S/N ratio 10 dB.

Soln. :

Given : B = 3100 Hz

$$\left(\frac{S}{N}\right)_{dB} = 10$$

$$\therefore \quad 10 = 10 \log_{10} \left(\frac{S}{N}\right)$$

$$\therefore \quad \frac{S}{N} = 10$$

$$\therefore \quad \text{Maximum bit rate} = R_{max} = B \log_2 \left[1 + \frac{S}{N}\right]$$

$$= 3100 \log_2 (1 + 10) = \frac{3100 \log_{10} 11}{\log_{10} 2}$$

$$= 10{,}724 \text{ bits/s} \quad \text{...Ans.}$$

Ex. 8.6.4 : Calculate the maximum bit rate for a channel having bandwidth 1600 Hz if :

(a) S/N ratio is 0 dB (b) S/N ratio is 20 dB.

Soln. :

Given : B = 1600 Hz

(a) R_{max} for S/N = 0 dB :

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N}\right)$$

$$\therefore \frac{S}{N} = 1$$

$$\therefore R_{max} = B \log_2 (1 + S/N) = 1600 \log_2 (1 + 1)$$

$$= 1600 \text{ bits/sec}$$

...Ans.

(b) R_{max} for S/N = 20 dB :

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N}\right)$$

$$\therefore 20 = 10 \log_{10} (S/N) \qquad \therefore \frac{S}{N} = 100$$

$$\therefore R_{max} = B \log_2\left(1 + \frac{S}{N}\right) = 1600 \log_2 (101) = \frac{1600 \log_{10}(101)}{\log_{10}(2)}$$

$$R_{max} = 10{,}654 \text{ bits/sec}$$

...Ans.

Using both the limits :

In practice we have to use both the methods to calculate the required bandwidth and signal level. consider the following example for the same.

Ex. 8.6.5 : The bandwidth of a channel is 2 MHz and its signal to noise ratio is 63. Calculate the appropriate bit rate and signal level.

Soln. :

Step 1 : Calculate the upper limit using Shannon's theorem :

$$C = B \log_2\left(1 + \frac{S}{N}\right) = 2 \times 10^6 \log_2 (1 + 63)$$

$$= 12 \text{ M bits / sec.}$$

This is the upper limit. For ensuring a better performance we select a somewhat lower value say 8 Mbps.

Step 2 : Calculate number of signal levels using Nyquist theorem :

$$C = 2 B \log_2 L$$

$$\therefore 8 \times 10^6 = 2 \times 2 \times 10^6 \log_2 L$$

$$\therefore 2 = \log_2 L$$

$$\therefore L = 4$$

...Ans.

Ex. 8.6.6 : Calculate the maximum bit rate of channel having bandwidth 1200 Hz if :

1. S/N ratio is 0 dB 2. S/N ratio is 20 dB

Soln. :

Given : B = 1200 Hz.

1. R_{max} for S/N = 0 dB

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N}\right)$$

$$\frac{S}{N} = 1$$

$$\therefore R_{max} = B \log_2\left(1 + \frac{S}{N}\right)$$

$$= 1200 \log_2 (1 + 1)$$

$$= 1200 \text{ bits/sec}$$

...Ans.

2. R_{max} for S/N = 20 dB

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N}\right)$$

$$20 = 10 \log_{10}\left(\frac{S}{N}\right)$$

$$\therefore \frac{S}{N} = 100$$

$$\therefore R_{max} = B \log_2\left(1 + \frac{S}{N}\right) = 1200 \log_2 (101)$$

$$= 1200 \frac{\log_{10}(101)}{\log_{10}(2)}$$

$$= 7990 \text{ bits/sec}$$

...Ans.

Maximum bit rate = 1200 bits/sec for 0 dB

Maximum bit rate = 7990 bits/sec for 20 dB

Ex. 8.6.7 : Find the number of coding or symbol levels if C = 31000 bits/s and B is 3100 Hz.

Soln. : C is the channel capacity while B is the bandwidth.

According to Shannon's theorem,

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

where S/N is the signal to noise ratio.

$$\therefore 31000 = 3100 \log_2\left(1 + \frac{S}{N}\right)$$

$$\therefore \log_2\left(1 + \frac{S}{N}\right) = 10$$

$$\therefore \frac{S}{N} = 1023 \text{ or } 30\text{dB}$$

$$\left(\frac{S}{N}\right) \text{dB} = 1.8 + 6 \text{ NdB}$$

$$\therefore 30 = 1.8 + 6N$$

where N = Number of bits per word.

$$\therefore N = 4.72 \approx 5$$

$$\text{Number of symbol levels } Q = 2^N = 2^5 = 32$$

...Ans.

Ex. 8.6.8 : Calculate the channel capacity for a noisy channel having bandwidth = 5 kHz and SNR = 0 using appropriate formula.

Soln. :

Given : $B = 5\text{ kHz},\ \frac{S}{N} = 0$

Find : Channel capacity.

1. To determine channel capacity :

$$C = B\log_2\left[1+\frac{S}{N}\right] = 5\times 10^3 \log_2 [1+0]$$

$$C = 0 \text{ bits/sec.}$$

...Ans.

Ex. 8.6.9 : An analog signal has a bit rate of 8000 bps and a baud rate of 1000 baud. How many data elements are carried by each signal element ? How many signal elements do we need ?

Soln. :

Given : Bit rate (n) = 8000 bps, Baud rate = 1000 baud.

To find :
1. Data elements carried by each signal element (R)
2. Total signal elements (L)

Step 1 : Calculate R :

$$\text{Bit rate} = \text{Numbers of data elements per signal} \times \text{baud rate}$$

$$\therefore \text{Number of data elements per signal (R)} = \frac{\text{Bit rate (n)}}{\text{baud rate}}$$

$$\therefore \quad R = \frac{8000}{1000} = 8 \text{ bits/ baud}$$

...Ans.

Step 2 : Calculate L :

$$\text{Total signal elements (L)} = 2^R = 2^8 = 256$$

...Ans.

Ex. 8.6.10 : State and explain the Nyquist theorem and Shannon capacity and solve the following example :
Calculate the maximum bit rate for noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels.

Soln. :

Given : B = 3000 Hz, L = 2

To find : Maximum bit rate

$$\text{Maximum bit rate (R)} = 2B\log_2 L$$

$$= 2\times 3000 \log_2 (2) = 2 \times 3000 \times \left(\frac{\log_{10}(2)}{\log_{10} 2}\right)$$

$$\therefore \quad R = 6000 \text{ bits/sec.}$$

...Ans.

Ex. 8.6.11 : The channel capacity is given by,

$$C = B\log_2\left[1+\frac{S}{N}\right]$$

In the presence of white Gaussian noise, with a constant signal power the channel capacity reaches its upper limit with increase in the bandwidth B. Prove that this upper limit of C is given by, $C_\infty = 1.44\frac{S}{N_0}$

Soln. :

Consider the equation $C = B\log_2\left[1+\frac{S}{N}\right]$

1. As the noise present is white Gaussian noise, the noise power N can be expressed as,

$$N = N_0 B \qquad ...(1)$$

This is because the power spectral density of white Gaussian noise is ($N_0/2$). Substitute this value of N into the equation for C to get,

$$C = B\log_2\left[1+\frac{S}{N_0 B}\right] \qquad ...(2)$$

2. Rearranging the Equation (2) as follows :

$$C = \frac{S}{N_0}\times\frac{N_0}{S}\cdot B\log_2\left[1+\frac{S}{N_0 B}\right]$$

$$= \frac{S}{N_0}\cdot\log_2\left[1+\frac{S}{N_0 B}\right]^{N_0 B/S}$$

$$= \frac{S}{N_0}\log_2\left[1+\frac{S}{N_0 B}\right]^{1/(S/N_0 B)} \qquad ...(3)$$

3. Now as the bandwidth B approaches ∞, the channel capacity will approach its upper limit denoted by "C_∞".

$$\therefore \quad C_\infty = \lim_{B\to\infty} C$$

$$= \lim_{B\to\infty}\frac{S}{N_0}\log_2\left[1+\frac{S}{N_0 B}\right]^{1/(S/N_0 B)} \qquad ...(4)$$

4. Let us substitute $x = \frac{S}{N_0 B}$ in Equation (4). Also as $B\to\infty,\ x\to 0$

$$\therefore \quad C_\infty = \lim_{x\to 0}\frac{S}{N_0}\log_2[1+x]^{1/x}$$

$$\therefore \quad C_\infty = \frac{S}{N_0}\lim_{x\to 0}\log_2[1+x]^{1/x} \qquad ...(5)$$

Let us use the standard relation stating that,

$$\therefore \quad 44.812 \times 10^3 \leq B \log_2 [1 + 10000]$$

$$\therefore \quad B \geq \frac{44.812 \times 10^3}{13.287}$$

$$\therefore \quad B \geq 3.372 \text{ kHz}$$

...Ans.

Trade off between bandwidth and SNR : As the signal to noise ratio is increased from 30 dB to 40 dB, the bandwidth will have to be decreased.

Ex. 8.6.14 : A 2 kHz channel has signal to noise ratio of 24 dB :

(a) Calculate maximum capacity of this channel.

(b) Assuming constant transmitting power, calculate maximum capacity when channel bandwidth is : 1. halved 2. reduced to a quarter of its original value.

Soln. :

Given : B = 2 kHz and (S/N) = 24 dB.

The SNR should be converted from dB to power ratio.

$$\therefore \quad 24 = 10 \log_{10} (S/N)$$

$$\therefore \quad \frac{S}{N} = 251 \qquad ...(1)$$

(a) To determine the channel capacity :

$$C = B \log_2 \left[1 + \frac{S}{N}\right] = 2 \times 10^3 \log_2 [1 + 251]$$

$$= 2 \times 10^3 \frac{\log_{10} 252}{\log_{10} 2}$$

$$\therefore \quad C = 15.95 \times 10^3 \text{ bits/sec}$$

...Ans.

(b) 1. Value of C when B is halved :

The new bandwidth $B_2 = 1$ kHz, let the old bandwidth be denoted by $B_1 = 2$ kHz.

We know that the noise power $N = N_0 B$.

$$\therefore \text{ Noise power with old bandwidth} = N_1 = N_0 B_1 \qquad ...(2)$$

$$\text{and Noise power with new bandwidth} = N_2 = N_0 B_2 \qquad ...(3)$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{N_0 B_2}{N_0 B_1} = \frac{B_2}{B_1} = \frac{1}{2}$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{1}{2} \qquad ...(4)$$

As the signal power remains constant, the SNR with new bandwidth is,

$$\frac{S}{N_2} = \frac{S}{N_1/2} = 2\frac{S}{N_1}$$

$$\text{But we know that } \frac{S}{N_1} = 251 \qquad \text{...See Equation (1)}$$

$$\therefore \quad \frac{S}{N_2} = 2 \times 251 = 502 \qquad ...(5)$$

Hence the new channel capacity is given by,

$$C = B_2 \log_2 \left[1 + \frac{S}{N_2}\right] = 2 \times 10^3 \log_2 [503]$$

$$= 2 \times 10^3 \frac{\log_{10} (503)}{\log_{10} 2}$$

$$\therefore \quad C = 17.94 \times 10^3 \text{ bits/sec}$$

...Ans.

2. Value of C when B is reduced to 1/4 of original value :

Equation (4) gets modified to,

$$\frac{N_3}{N_1} = \frac{1}{4} \qquad ...(6)$$

$$\therefore \quad \frac{S}{N_3} = 4\frac{S}{N_1} = 4 \times 251 = 1004 \qquad ...(7)$$

Hence new channel capacity is given by,

$$C = B_3 \log_2 \left(1 + \frac{S}{N_3}\right) = 500 \log_2 (1004)$$

$$\therefore \quad C = 4.99 \times 10^3 \text{ bits/sec}$$

...Ans.

Review Questions

Q. 1 What is uncertainity ?

Q. 2 Define information.

Q. 3 State the units of information.

Q. 4 State and explain various properties of information.

Q. 5 Explain the significance of information.

Q. 6 Define and explain the term information rate. State the relation between information rate and entropy.

Q. 7 What is information capacity ?

Q. 8 What is Shannon's limit ?

Q. 9 Write a note on : Trade off between bandwidth and signal-to noise ratio.

Q. 10 Define entropy.

Q. 11 Define channel capacity.

Q. 12 Explain Shannon's theorem and state its importance.

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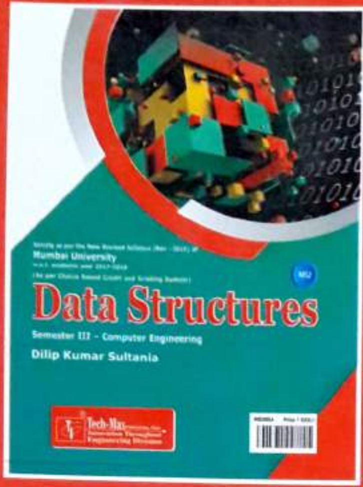

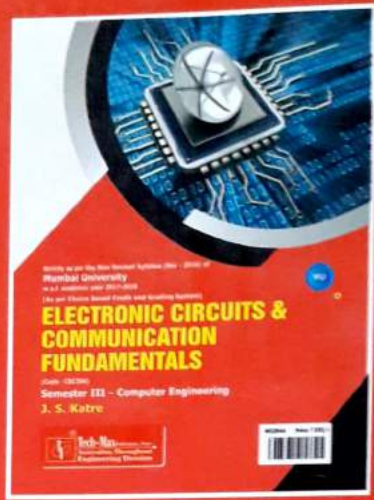

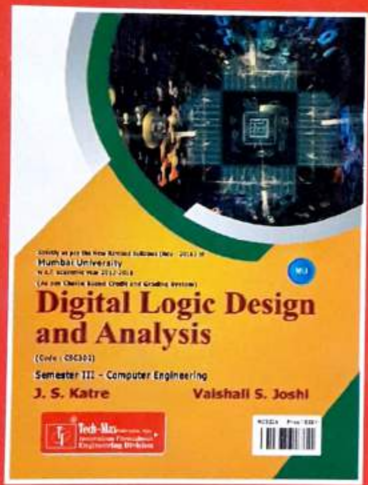

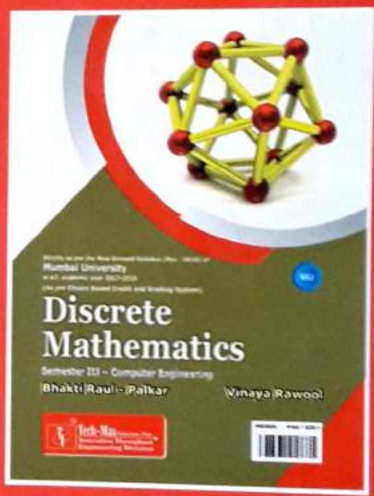

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