B.J.T ~~modelling~~ modeling

The transistor model is a combination of circuit elements, properly chosen, that the best approximate the actual behaviour of a transistor under specific operating condition. The transistor behaviour is different at low frequencies and high frequencies. At low frequencies the reactance of junction capacitances of the transistor are very high and can be ignored, but this is not the case at high frequencies Therefore we use different transistor models to analyse the transistor at low and high frequencies
At low frequencies we use the model of hybrid equivalent model
and
at high frequencies we use hybrid - π model

The r_e transistor Model.

The r-parameter (resistance parameter) are perhaps easier to work with than h-parameters. r parameter model for transistor is as shown

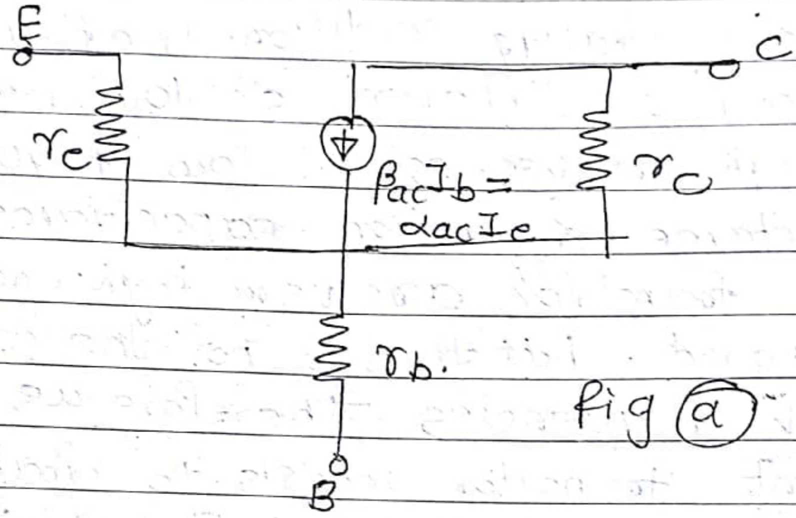


fig (a)

r-parameter model for BJT

It consists of five r-parameter as listed

r-parameter.	Description
α_{ac}	a.c alpha (I_c/I_e)
& β_{ac}	a.c beta (I_c/I_b)
r_e	a.c emitter resistance
r_c	a.c collector resistance
r_b	a.c base resistance

Current source is represented by

$I_c = \alpha_{ac} I_e = \beta_{ac} I_b$

Relationship of r-parameters and h-parameters

Data sheets often specify the common emitter h-parameters of the transistor. In such situations, we have to convert these parameters to analyze transistor amplifier using r-parameters. The conversion formulae are as follows.

$$\alpha_{ac} = h_{fb} = \frac{-h_{fe}}{1+h_{fe}} \rightarrow (1)$$

$$\beta_{ac} = h_{fe} \rightarrow (2)$$

h_{fb} - Forward transfer current gain CB ampl

h_{fe} - " - for CE ampl

$$r_e = \frac{h_{re}}{h_{oe}}$$

h_{re} = Reverse voltage gain

$$r_c = \frac{h_{re}+1}{h_{oe}}$$

h_{oe} = o/p admittance

$$r_b = h_{ie} - \frac{h_{re}(1+h_{fe})}{h_{oe}}.$$

Simplified r-parameter Model for BJT

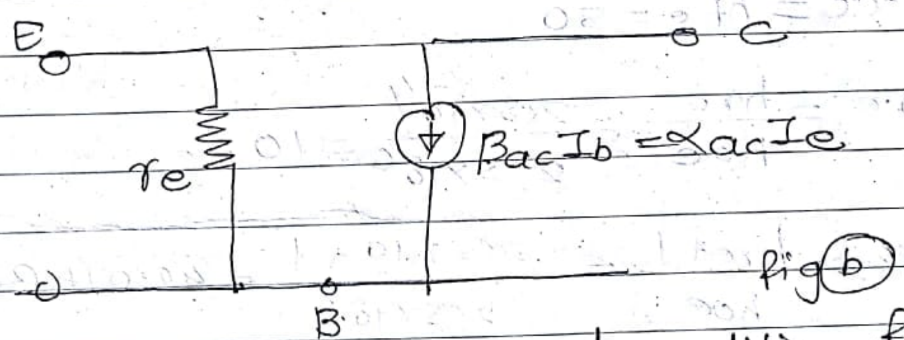


fig (b)

In a forward biased condition of base emitter junction r_b is very small and its effect is also small enough to neglect it. Thus it can be replaced by a short circuit. The collector base junction is always reverse biased in the active region. In this biasing state, a.c collector resistance r_c is usually several hundred kilo-ohms and hence it can be replaced by an open-cct. This gives the simplified r-parameter model for BJT as shown in figure (b)

Example -

Determine the r-parameter of a transistor if h-parameter are $h_{ie} = 1.1 k\Omega$, $h_{fe} = 50$, $h_{oe} = 25 \mu A/V$ and $h_{re} = 2.5 \times 10^{-4}$

→ Referring table, eqn (1) & (5)

$$\alpha_{ac} = h_{fb} = \frac{-h_{fe}}{1+h_{fe}} = \frac{-50}{1+50} = -0.98$$

$$\beta_{ac} = h_{fe} = 50$$

$$\beta_{ac} = h_{fe} = 50$$

$$r_e = \frac{h_{re}}{h_{oe}} = \frac{2.5 \times 10^{-4}}{2.5 \times 10^{-6}} = 10$$

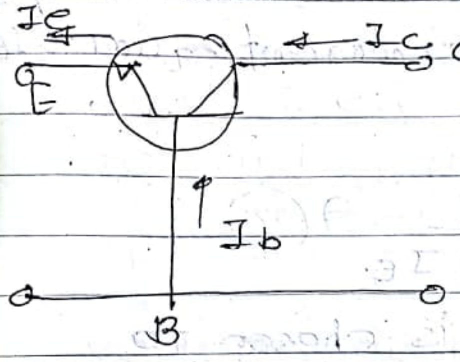
$$r_c = \frac{h_{re}+1}{h_{oe}} = \frac{2.5 \times 10^{-4}+1}{2.5 \times 10^{-6}} = 40.01 k\Omega$$

$$r_b = h_{ie} - \frac{h_{re}(1+h_{fe})}{h_{oe}}$$

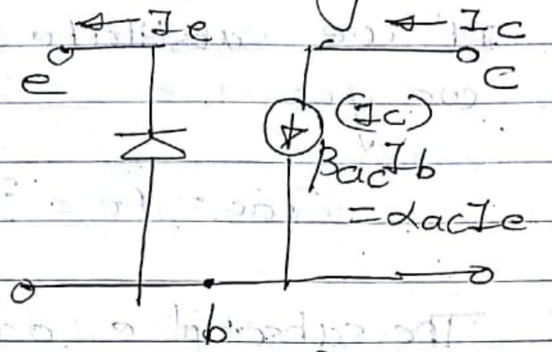
$$= 1.1 \times 10^{3} - \frac{2.5 \times 10^{-4}(1+50)}{25 \times 10^{-6}}$$

$$r_b = 590 \Omega$$

r_e parameter Model for CB configuration



Common Base transistor config



re model for common base transistor config. fig (c)

The re model employs a diode and controlled current source to duplicate the behaviour of a transistor in the active region. ~~is replaced~~ Forward biased base emitter junction in the active region is replaced by the PN junction diode. The forward biased base emitter junction causes collector current to flow which inturn depends upon the base current. This input, output current relationship is represented by a controlled current source whose value is $\beta_{ac} I_b$ or $\alpha_{ac} I_e$. Because $I_c = \beta_{ac} I_b$ or $I_c = \alpha_{ac} I_e$

We know that, ac resistance of a diode can be determined by the eq^n

$$r_{ac} = \frac{26mV}{I_D} \quad \text{--- (6)}$$

where I_D is the dc current through the diode at the Q point. This same eq^n can be used to find the ~~ac~~

a.c. resistance of the diode as shown in fig ©
If we substitute diode current equal to I_E
we get

$$r_{ac} = r_e = \frac{26\,mV}{I_D \text{ or } I_E} \quad \text{——} (7)$$

The subscript e of r_e is chosen to emphasize that it is the d.c. level of emitter current that determines the a.c. resistance of the diode shown in fig ©.
Replacing the diode by its eqv resistance we get r parameter model for the ckt as shown in fig (D)

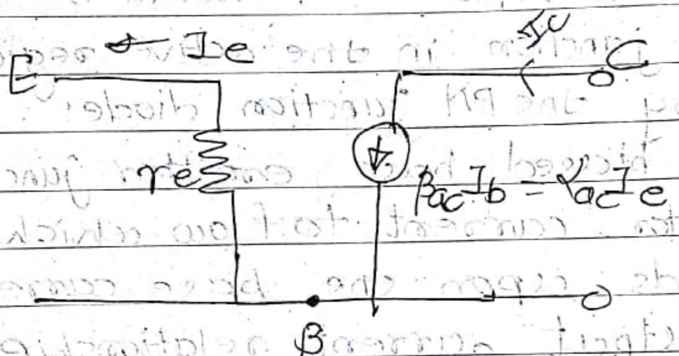


fig (D) r parameter model for CB circuit

Input resistance → Due to isolation that exists between input and output circuit of fig (B) , it is fairly obvious that input resistance for common base config of a transistor is simply r_e. That is

$$\boxed{R_i = r_e} \quad \text{——} (8)$$

Typical value of R_i for CB range from few ohms to a maxm of about 50Ω output resistance :- To determine the o/p resistance we have to make signal zero i.e. $I_e = 0$, αI_e is also zero and output is open circuited.

$\therefore \boxed{R_o = \infty}$ ——→ (9)

voltage gain — is given as

$$A_V = \frac{V_o}{V_i}$$

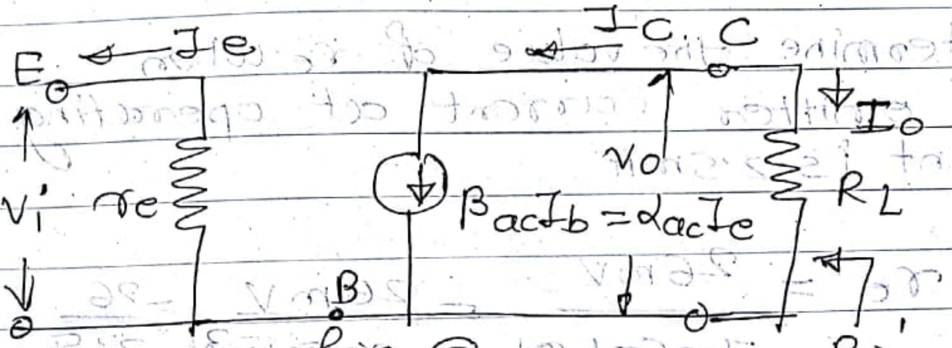


fig ©

It can be determined by connecting load at the o/p terminals as shown in fig ©

We have

$V_i = -I_e r_e$ and

$V_o = I_o R_o' = -I_c R_o' = -\alpha_{ac} I_e R_o'$

$$A_V = \frac{V_o}{V_i} = \frac{-\alpha_{ac} I_e R_o'}{-I_e r_e} = \frac{\alpha_{ac} R_o'}{r_e} \longrightarrow (10)$$

$$A_V = \frac{R_o'}{r_e}$$

$\because \alpha_{ac} = 1$

$\therefore R_o' = R_L$

$$\boxed{A_V = \frac{R_L}{r_e}} \longrightarrow (11)$$

Current Gain

$$A_i = \frac{I_o}{I_i} = \frac{-I_c}{I_i}$$

$$= \frac{-\alpha_{ac} I_e}{-I_e}$$

$$= +\alpha_{ac} = \alpha.$$

as $\alpha \to 1$

$$\boxed{A_i = 1}$$

$\to$ (12)

problem (a)

Determine the value of r_e when dc emitter current at operating point is 2.5mA

$$r_e = \frac{26mV}{I_E(\text{at } Q)} = \frac{26mV}{2.5 \times 10^{-3}} = \frac{26}{2.5}$$

$$\boxed{r_e = 10.4\Omega}$$

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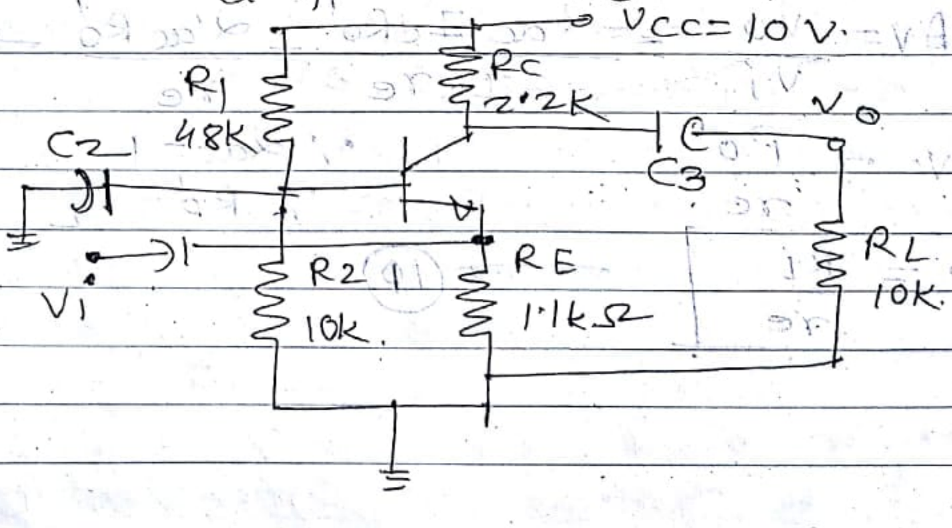
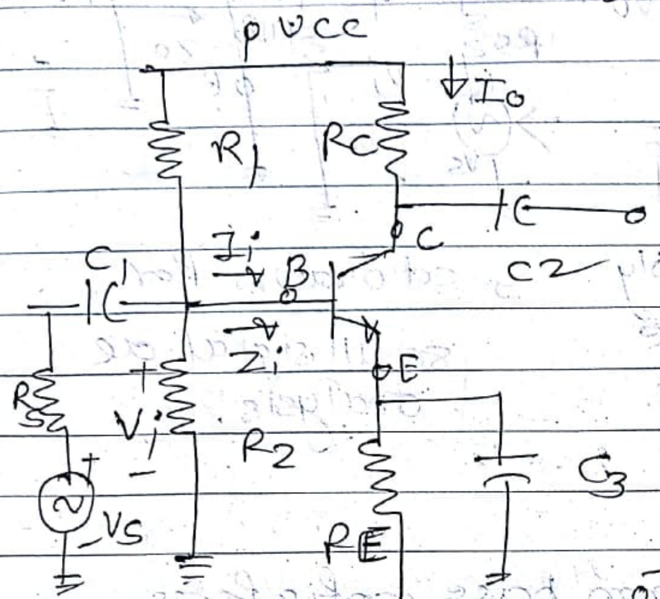
problem (b)

Determine R_i, R_i', A_V, A_i, R_o, R_o' and power gain for the common base amplifier as shown.



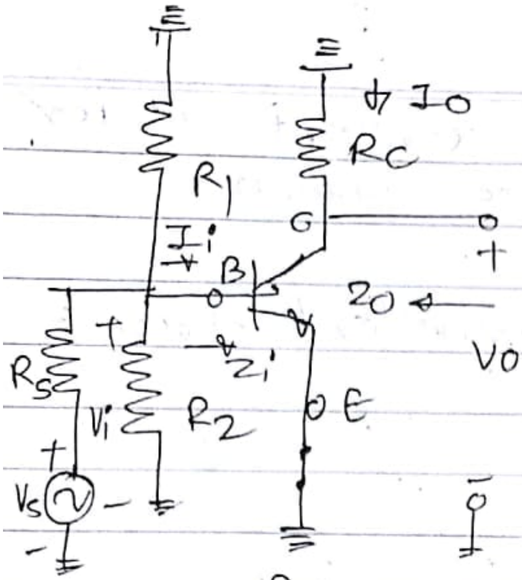
The r parameter equivalent cct for a given ampr is as shown

** How to draw ac equivalent circuit

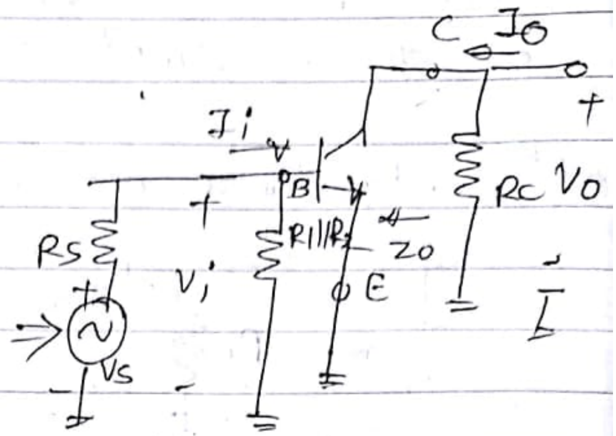


ac eqv cct is obtained by

1. Setting all dc sources to zero and replacing them by short cct eqv
2. Replacing all capacitors by a short cct eqv
3. Removing all elements bypassed by the short cct equivalents introduced in steps 1 and 2
4. Redrawing the network in a more convenient and logical form



Removal of dc supply and insertion of the short cct eq^v for the capacitors.



cct drawn for small signal ac analysis.

For a common base config for as shown in fig (c) $I_E = 4mA$, $\alpha = 0.991$ and an ac signal of 3mV applied b/w base and emitter terminal

(1) Determine the I/P impedance

(2) Calculate the voltage gain if a load of 610Ω is connected to the output terminal

(3) Find the O/P impedance and current gain

sol^n (a) $r_e = \frac{26mV}{I_E} = \frac{26\times10^{-3}}{4\times10^{-3}} = 6.5\Omega$

$\boxed{r_e = 6.5\Omega}$

(b) $I_i = I_e = \frac{V_i}{Z_i}$ $A_V = \frac{\alpha_{ac} R_L}{r_e} = \frac{0.991\times610}{6.5}$

$\boxed{A_V = 93}$

Current gain

$$A_i = \frac{I_o}{I_i} = \frac{-I_c}{I_i} = \frac{-\alpha_{ac} I_e}{-I_e} = \alpha_{ac}$$

$= \alpha_{ac} = 0.991$

$\boxed{A_i = 0.991}$

o/p Impedance

$\boxed{Z_o = \infty\ \Omega}$

or

For voltage gain (Not necessary)

$$I_i = -I_e = \frac{V_i}{Z_i \text{ or } r_e} = \frac{3\,mV}{6.5} = 461.54\ \mu A$$

$$V_o = I_o R_L = -I_c R_L = -\alpha_{ac} I_e R_L$$

$= -(0.991) \times (461.54) \times 10^{-6} \times 610$

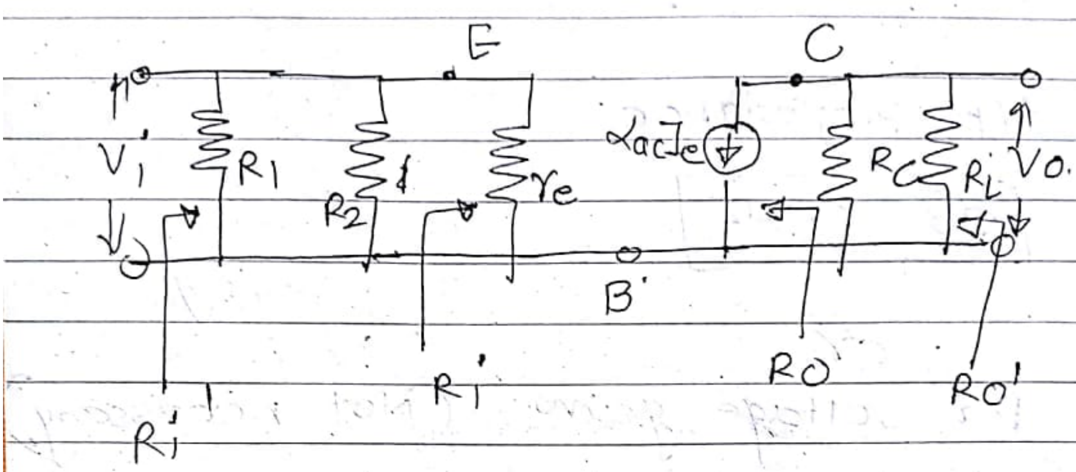
$= 279\,mV$

$$A_V = \frac{V_o}{V_i} = \frac{279\,mV}{3\,mV} = 93$$

$\boxed{A_V = 93}$

back to problem. (c):

r parameter eq^v cct is drawn by short circuiting dc source and all capacitors and replacing transistor by its r-parameter model.



To determine r_e, we have to first find I_E, It is calculated as follows

$$V_B = \left(\frac{R_2}{R_1+R_2}\right) V_{CC} = \left(\frac{10k}{10k+48K}\right) 10$$

$$\boxed{V_B = 1.72V}$$

$$V_E = V_B - 0.7 = 1.72 - 0.7 = 1.02 \text{ Volt}$$

$$I_E = \frac{V_E}{R_E} = \frac{1.02}{1.1k} = 0.927 mA$$

$$\therefore \quad r_e = \frac{26mV}{I_D \text{ or } I_E} = \frac{26mV}{0.927mA} = 28\Omega$$

$$\boxed{r_e = 28\Omega}$$

$\boxed{R_i = r_e = 28\,\Omega}$

$R_i' = R_1 \| R_2 \| R_i$

$= 48k \| 10k \| 28\,\Omega$

$\boxed{R_i' = 27.9\,\Omega}$

$R_o = \infty$

$R_o' = R_o \| R_C \| R_L$

$= \infty \| 2.2k \| 10k$

$= 1.8k.$?

$A_V = \frac{R_o'}{r_e} = \frac{1.8K}{28} = 64.28$

$\boxed{A_V = 64.28}$

$A_V = \frac{\alpha_{ac} R_o'}{r_e}$

$= \frac{0.991 \times 1.8 \times 10^3}{28}$

$= 63.7071$

$= 63.71\,\Omega$

$\boxed{A_i \approx 1}$

power gain $A_p = A_V A_I$

$= 64.28 \times 1$

$\boxed{A_p = 64.28}$

Merits of r_e model:-

- The parameters of r_e model can be determined for any region of operation within the active region.
- It is simple and less elaborate model of transistor.
- These parameters can be obtained easily from the h parameters which are specified by the manufacturer.

Demerits of r_e model.

- r_e model does not account for the output impedance level of a device and feedback effect from the output to input.
- This model is sensitive to the dc level of operation of the amp^r. Therefore the input resistance will vary with the dc operating point.

r parameter model for CE configuration

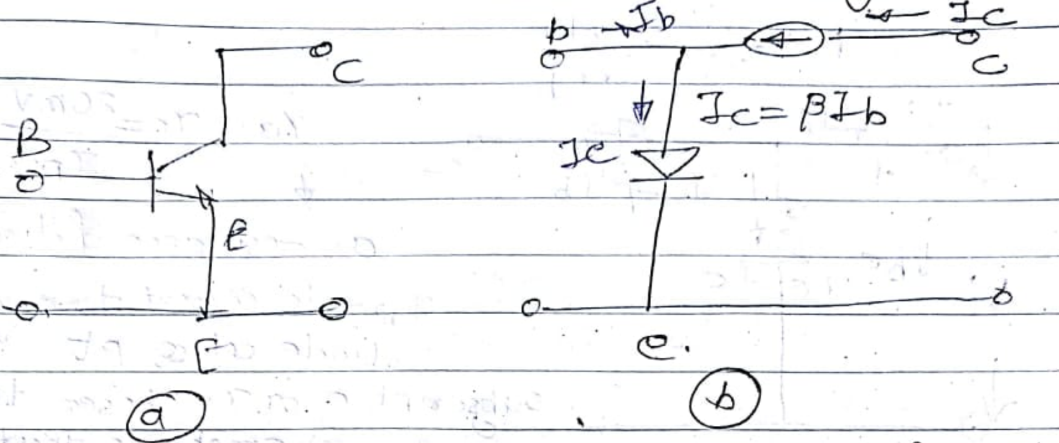


Fig (a) shows common emitter config and (b) shows its r_e model.
Here r_e model employs a diode and controlled current source to duplicate the behaviour of a transistor in the active region.

* Forward biased base emitter junction in the active region is represented by the diode and the controlled current. Source is still connected between the collector and the base terminals.

Looking at the model we have current through diode equal to

$$I_e = I_b + \beta I_b = (1+\beta) I_b$$

$$\boxed{I_e = \beta I_b} \longrightarrow (1) \quad \because \beta \gg 1$$

The input resistance is given by

$$R_i = \frac{V_i}{I_i} = \frac{V_{be}}{I_b}$$

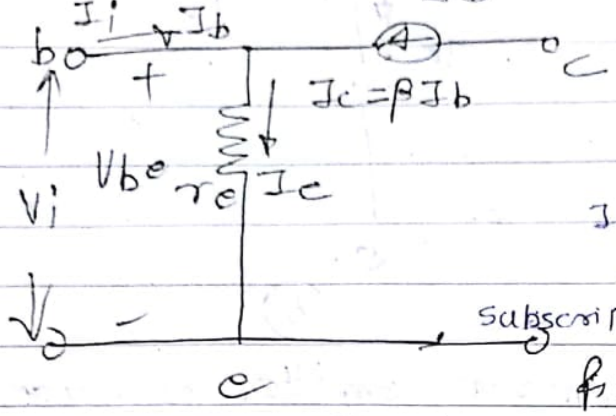


fig (C)

$r_{ac} = r_e = \frac{26\,mV}{I_D \text{ or } I_E}$

↓

ac resistance of diode

I_D = dc current through diode at Q pt

subscript e or r_e chosen to emphasis that dc level of emitter current determine ac resistance of diode

The voltage V_{be} is across the diode resistance as shown.
The level of r_e is still determined by the dc current I_E. Using Ohm's law gives

$$V_i = V_{be} = I_e r_e \approx \beta I_b r_e$$

$$\therefore R_i = \frac{V_{be}}{I_b} = \frac{\beta I_b r_e}{I_b} = \beta r_e$$

$$\boxed{R_i = \beta r_e} \quad \text{--- eq (2)}$$

The typical value of R_i for CE config range from a few hundred ohms to maximum about 6-7 kΩs.

Output Resistance -

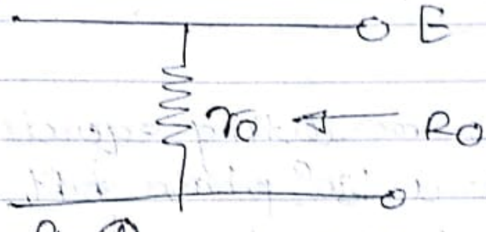


fig (d)

The re model shown in fig (c) does not include an o/p resistance. However, if it is available from a graphical analysis or from data sheets it can be included as shown in fig (d)

When I/P is short circuited $I_b = 0$ and ~~Ic=0~~ $I_c = 0$ and o/p resistance is given as

$$\boxed{R_O = r_O}$$

If r_O is ignored from re model of CE the o/p resistance is ∞

For CE config typical values of Ro is in the range 40kΩ to 50kΩ

Voltage Gain

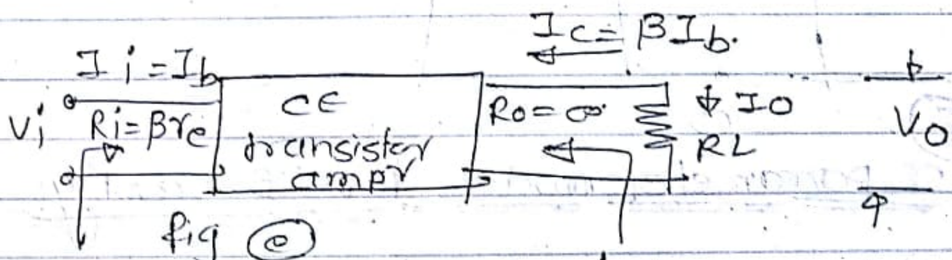


fig (e)

It can be determined considering the load connected at the output terminals and o/p resistance ∞, as shown in fig.

$$V_O = I_O R_L = -I_C R_L = -\beta I_b R_L$$

$$V_i = I_b \beta r_e$$

$$A_V = \frac{V_O}{V_i} = \frac{-\beta I_b R_L}{I_b \beta r_e} = \frac{-R_L}{r_e}$$

$$\boxed{A_V = \frac{-R_L}{r_e}}$$

The −ve sign in the resulting equation for Av indicates that a 180° phase shift occurs between the input and output signals.

Current Gain — CE

$$A_i = \frac{I_o}{I_i} = \frac{-I_c}{I_b} = \frac{-\beta I_b}{I_b}$$

$$\boxed{A_i = -\beta}$$

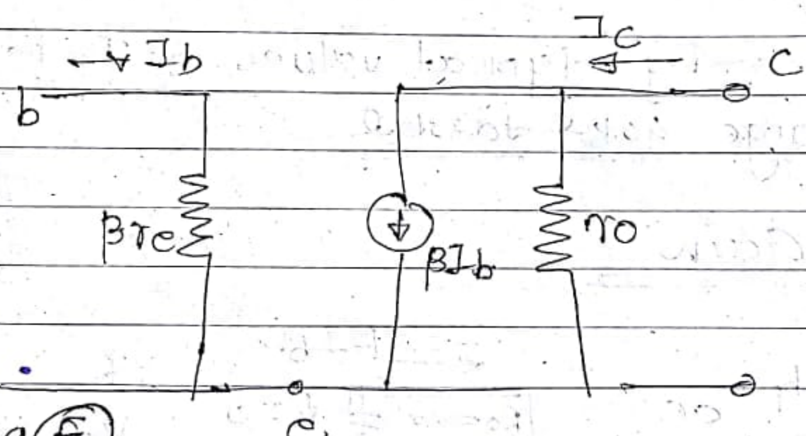


Fig (f) r parameter model for CE config.

Using the derived values of input resistance (βr_e), the collector current (βI_b), and the output resistance (r_o), the equivalent model for common emitter config is as shown in fig (f)

Given $\beta = 150$, $I_E = 3.2\,mA$ for CE config with $r_o = \infty\,\Omega$, determine

(a) Z_i
(b) A_v if a load of $2k\Omega$ is applied
(c) A_i with the $2k\Omega$ load.

Solution :-

(a) $r_e = \frac{26mV}{I_E} = \frac{26mV}{3.2mA} = 8.125\,\Omega$

$\boxed{r_e = 8.125\,\Omega}$

(b) $Z_i = R_i = \beta r_e = 150 \times 8.125 = 1218.75\,\Omega$

$\boxed{Z_i = 1.22\,k\Omega}$

(c) $A_V = \frac{-R_L}{r_e} = \frac{2k\Omega}{8.125\,\Omega} = -246.15$

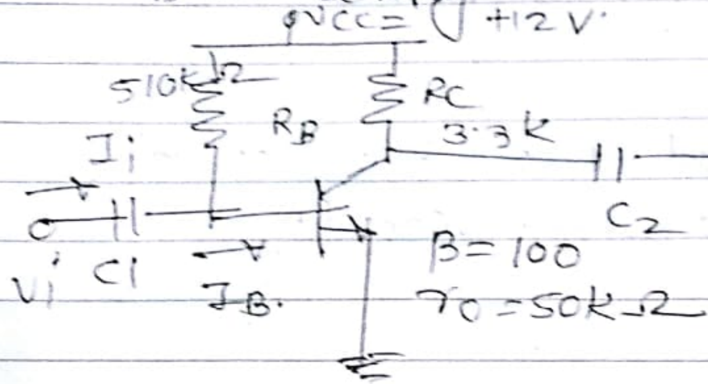
$\boxed{A_V = -246.15}$

(d) $A_i = \frac{I_o}{I_i} = \frac{-I_c}{I_b} = \frac{-\beta I_b}{I_b} = -\beta$

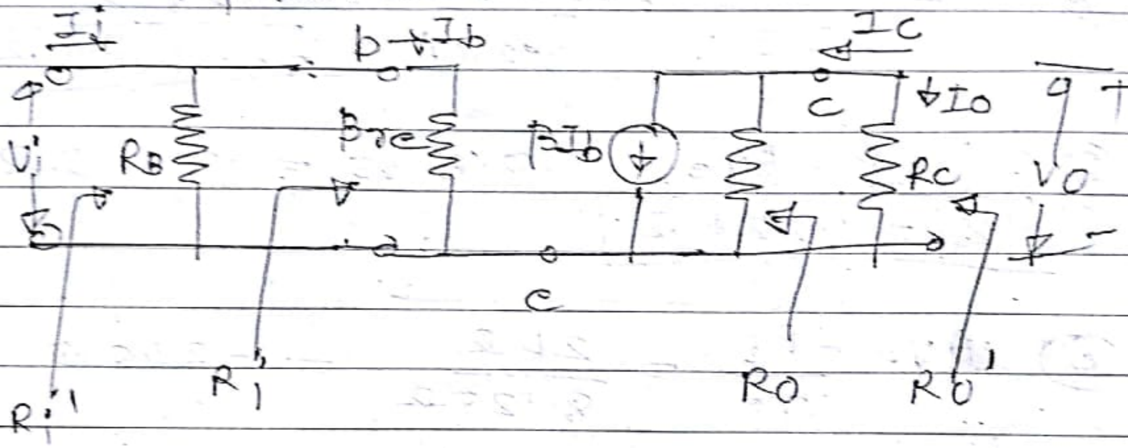
$A_i = -\beta = -150$

$\boxed{A_i = -150}$

Analysis of common Emitter Fixed Bias config.



→ r parameter eq^v cct



To find r_e we have to first determine I_E

Dc analysis:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B} = \frac{12 - 0.7}{510 \times 10^3}$$

$$\boxed{I_B = 22.15 \mu A}$$

$$I_E = (1+\beta) I_B = (1+100) \times 22.15 \times 10^{-6}$$
$$= 2.23 mA$$

$$\boxed{I_E = 2.23 mA}$$

$$r_e = \frac{26 mV}{I_E} = \frac{26 mV}{2.23 mA} = 11.66 \Omega$$

$$\boxed{r_e = 11.66 \Omega}$$

*

④ Voltage gain $A_V = \frac{V_O}{V_i}$

∵ Rc and ro are in parallel.

$$\therefore V_O = I_O (r_o \| R_C)$$
$$= -I_C (r_o \| R_C)$$
$$= -\beta I_B (r_o \| R_C)$$
$$V_i = I_b \beta r_e$$
$$A_V = \frac{V_O}{V_i} = \frac{-\beta I_b (r_o \| R_C)}{I_b \beta r_e}$$

$$A_V = \frac{-(r_o \| R_C)}{r_e}$$

$$A_V = \frac{-(50 \| 3.3)}{11.66} = \frac{3.09 k\Omega}{11.66\Omega} = -265$$

$$\boxed{A_V = -265}$$

*

② Input Resistance

$$R_i = \beta r_e = 100 \times 11.66 = 1.166 k\Omega$$

$$\boxed{R_i = 1.166 k\Omega}$$

$$R_i' = R_i \| R_B = 1.166 \| 510 = 1.163 k\Omega$$

$$\boxed{R_i' = 1.163 k\Omega}$$

③ Output resistance

$$\boxed{R_O = r_o = 50 k\Omega}$$

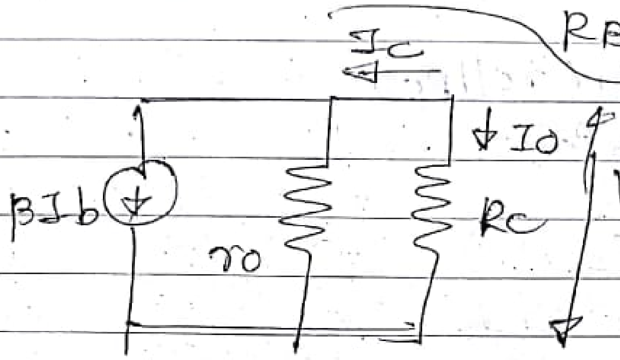
$$R_O' = r_o \| R_C = 50 k\Omega \| 3.3 k\Omega$$

$$\boxed{R_O' = 3.09 k\Omega}$$

(5) Current Gain

$$A_i = \frac{I_o}{I_i} = \frac{-I_c}{I_i} = \frac{-I_c}{I_b} \times \frac{I_b}{I_i} \rightarrow (A)$$

where $I_b = I_i \times \frac{R_B}{R_B+\beta r_e}$ (current divider rule)



$$\boxed{\frac{I_b}{I_i} = \frac{R_B}{R_B+\beta r_e}} \rightarrow (B)$$

$$I_i = \frac{(R_B+\beta r_e) I_b}{R_B}$$

Looking at fig we have

$$I_c = \beta I_b + \frac{V_o}{r_o}$$

$$= \beta I_b + \frac{I_o R_c}{r_o}$$

$$I_c = \beta I_b - \frac{I_c R_c}{r_o}$$

$$I_c + \frac{I_c R_c}{r_o} = \beta I_b$$

$$\frac{I_c (r_o + R_c)}{r_o} = \beta I_b$$

$$\frac{I_c}{I_b} = \frac{\beta r_o}{\cancel{1+R}(r_o+R_c)} \rightarrow (C)$$

$$I_o = -I_c$$

$$= \frac{-\beta I_b \times r_o}{r_o+R_c}$$

$$I_o/I_i = \frac{-\beta \cancel{I_b} \times r_o}{r_o+R_c} \times \frac{R_B}{(R_B+\beta r_e)\cancel{I_b}} \; *$$

$$I_i = I_b$$

$$\boxed{I_b = I_i \times \frac{R_B}{R_B+\beta r_e}}$$

$$A_i = \frac{I_o}{I_i} = \frac{-\beta I_b \times r_o}{r_o+R_c} \times \frac{1}{I_i}$$

$$= \frac{I_b r_o}{r_o+R_c} \times \frac{\cancel{I_i} \times R_B}{R_B+\beta r_e} \times \frac{I_i}{\cancel{I_i}}$$

This will do

$$A_i = \frac{I_o}{I_i} = \frac{-I_c}{I_b} \times \frac{I_b}{I_i} \rightarrow (A)$$

put (B) & (C) in eqⁿ (A).

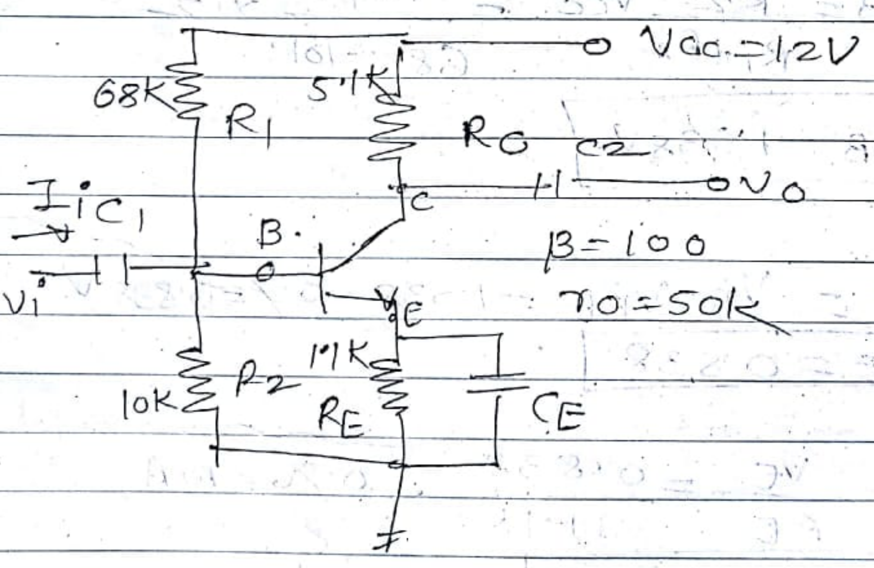
$$\frac{I_C}{I} \; A_i = \frac{-\beta r_o}{(r_o+R_C)} \times \frac{R_B}{R_B+\beta r_e}$$

$$= \frac{-\beta r_o \times R_B}{(r_o+R_C)(R_B+\beta r_e)}$$

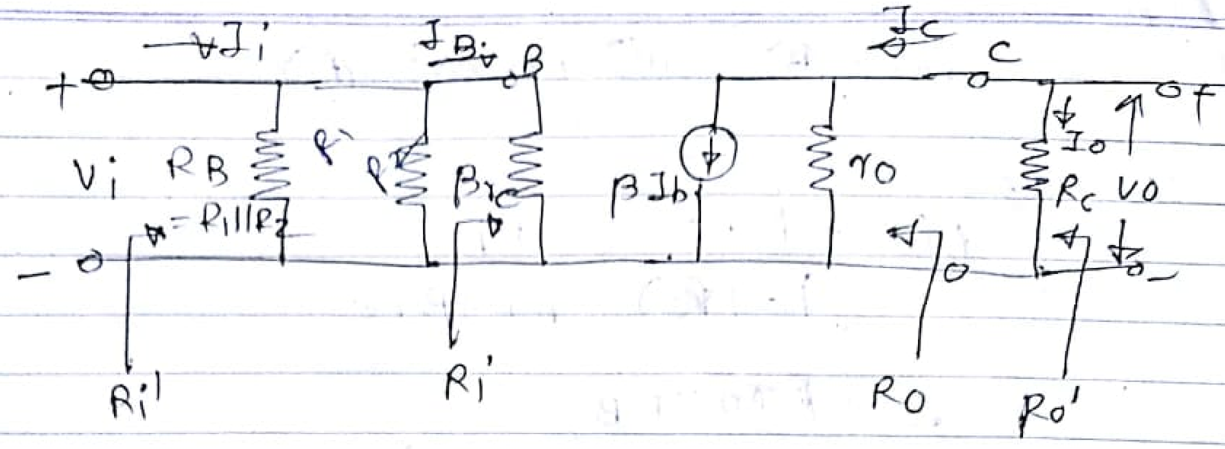
$$= \frac{-100 \times 50k \times 510k}{(50k+3.3k)\left[510k+(100)(11.60)\right]}$$

$$\boxed{A_i = -93.59}$$

Analysis of common Emitter voltage divider Bias configuration



By short circuiting C1, C2, CE and dc power supply, and replacing transistor with its eqv r-parameter eqv model we get the r-parameter eqv cct for given transistor ampr as shown.



To find the re we have to first determine the I_E. It can be calculated as follows

DC analysis: If $\beta R_E > 10 R_2$ we can use approximate approach.

$$100 \times 1.1K > 10 \times 10$$

$$110K > 100k$$

Using the approximate approach we have,

$$V_B = \frac{R_2}{R_1 + R_2} V_{cc} = \frac{10k}{68k + 10k} \times 12$$

$$\boxed{V_B = 1.538V}$$

$$V_E = V_B - V_{BE} = 1.538 - 0.7 = 0.838 V$$

$$\boxed{V_E = 0.838}$$

$$I_E = \frac{V_E}{R_E} = \frac{0.838}{1.1K} = 0.762 mA$$

$$\boxed{I_E = 0.762 mA}$$

$$r_e = \frac{26mV}{I_E} = \frac{26mV}{0.762mA} = 34.12\Omega$$

$$\boxed{\therefore r_e = 34.12\Omega}$$

Input resistance $R_i = \beta r_e = 100 \times 34.12$

$R_i = 3.412k\Omega$

$$\boxed{R_i = \cancel{3.412}\ 3.412k\Omega}$$

$R_i' = R_i \| R_B$

$R_B = R_1 \| R_2 = 68k \| 10k = 8.71k$

$R_i' = 3.412k \| 8.71k$

$$\boxed{R_i' = 2.45k\Omega}$$

o/p Resistance $R_o = r_o = 50k\Omega$

$R_o' = R_o \| R_C = 50k \| 5.1k$

$$\boxed{R_o' = 4.62k\Omega}$$

This Eq^v cct is same as eq^v cct for CE fixed bias config hence we have

Voltage gain $A_v = \frac{V_o}{V_i} = \frac{-(r_o \| R_C)}{r_e}$

$= \frac{-50k \| 5.1k}{34.12} = \frac{-4.62k}{34.12} = -135.4$

$$\boxed{A_v = -135.4}$$

current Gain $\rightarrow A_i = \frac{I_o}{I_i} = \frac{-\beta r_o R_B}{(r_o + R_C)(R_B + \beta r_e)}$

$= \frac{-100 \times 50k \times 8.71k}{(50k + 5.1k)(8.71k + 100(34.12))}$

$$\boxed{A_i = -65.2}$$