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Question 1

$$5y'' + 6y' + 45y = 50\cos 5t$$

— Characteristic polynomial

$$5m^2 + 6m + 45 = 0$$

Here, $a = 5$, $b = 6$ & $c = 45$

$$\therefore m_1, m_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm 29.39i}{10}$$

$$\therefore m_1, m_2 = -0.6 \pm 2.94i$$

$$\therefore \boxed{y_c = e^{-0.6t}\left(C_1 \cos 2.94t + C_2 \sin 2.94t\right)}$$

— Now, for particular solution,

$$y_p(t) = A\cos 5t + B\sin 5t$$

$$\therefore y_p' = (-A\sin 5t)(5) + (5)B\cos 5t$$

$$\therefore y_p'' = (-A\cos 5t)(5)^2 + -(5)^2 B\sin 5t$$

$$\therefore y_p'' = -25A\cos 5t - 25B\sin 5t$$

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$\therefore 5(-8.64A\cos 2.94t - 8.64B\sin 2.94t)$

$+6(-2.94A\sin 2.94t + 2.94B\cos 2.94t)$

$+45(A\cos 2.94t + B\sin 2.94t) =$

$\therefore 5(-25A\cos 5t - 25B\sin 5t) + 6(-5A\sin 5t$
$+5B\cos 5t) + 45(A\cos 5t + B\sin 5t) = 50\cos 5t$

$\therefore -125A\cos 5t - 125B\sin 5t + 30A\sin 5t$
$+30B\cos 5t + 45A\cos 5t + 45B\sin 5t = 50\cos 5t$

$\therefore \cos 5t(-125A + 30B + 45A) + \sin 5t(-125B - 30A$
$+45B) = 50\cos 5t$

$\therefore -125A + 30B + 45A = 50 \Rightarrow 30B - 80A = 50$
& $-125B - 30A + 45B = 0 \qquad -30A - 80B = 0$

$\therefore A = \frac{-80B}{30} = \frac{-8B}{3}$

$\therefore 30B - 80\left(\frac{-8B}{3}\right) = 50$

$\therefore 30B + \frac{640}{3}B = 50$

$\therefore 90B + 640B = 150 \Rightarrow B = 0.205$

$\therefore A = -0.547$

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$$\therefore \boxed{y_p = -0.547\cos 5t + 0.205\sin 5t}$$

$$y = y_c + y_p$$

$$\therefore \boxed{y = e^{-0.6t}\left(c_1\cos 2.94t + c_2\sin 2.94t\right) - 0.547\cos 5t + 0.205\sin 5t}$$

For y_c :

$$y_c' = -0.6e^{-0.6t}\left(-2.94c_1\sin 2.94t + 2.94c_2\cos 2.94t\right)$$

$$y_c'' = \mp 0.36\left(-8.64c_1\cos 2.94t - 8.64c_2\sin 2.94t\right)$$

$$y_c(0) = \frac{-3}{13} \Rightarrow c_1 = -3/13 = \boxed{c_1 = -0.23}$$

$$y'(0) = \frac{153}{13} \Rightarrow (-0.6)(2.94c_2) = \frac{153}{13}$$

$$\therefore \boxed{c_2 = -6.67}$$

$$\therefore \boxed{y = e^{-0.6t}\left(-0.23\cos 2.94t - 6.67\sin 2.94t\right) - 0.547\cos 5t + 0.205\sin 5t}$$

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→ Transient solution is

$$e^{-0.6t}\left[-0.23\cos 2.94t - 6.67\sin 2.94t\right].$$

→ Steady-state solution is

$$-0.547\cos 5t + 0.205\sin 5t.$$

→ Now, $y_{ss} = \sqrt{0.3 + 0.042}\ \cos(5t - \alpha)$

$$\therefore y_{ss} = 0.584\cos(5t - 0.3578)$$

$$K(\omega) = \frac{50}{\sqrt{(45 - 5\omega^2)^2 + (6\omega)^2}} = \frac{50}{\sqrt{2025 - 450\omega + 25\omega^2 + 36\omega^2}}$$

$$\therefore K(\omega) = \frac{50}{\sqrt{61\omega^2 - 450\omega + 2025}}$$

$$\therefore K'(\omega) = \frac{\left(-\frac{1}{2}\right)50\left(122\omega - 450\right)}{\left(61\omega^2 - 450\omega + 2025\right)^{3/2}} = 0$$

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$$\therefore -25\left(1220\omega - 450\right) = 0$$

$$\therefore 1220\omega = 450$$

$$\therefore \boxed{\omega^* = 3.688 \text{ Hz}}$$

Practical resonance occurs at

$$\boxed{\omega^* = 3.688 \text{ Hz}}$$