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Q1

$$f(t) = \begin{cases} t & 0 \le t < 1 \\ 2 - t & 1 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$

a) Find Laplace Transform

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^1 e^{-st}(t)\, dt + \int_1^2 e^{-st}(2-t)\, dt + \int_2^\infty e^{-st}(0)\, dt$$

Following ILATE in integrals

$$= t\int_0^1 e^{-st}\, dt - \int_0^1 \left(\frac{dt}{dt} \cdot \int_0^1 e^{-st}\, dt\right) dt + (2-t)$$

$$\int_1^2 e^{-st}\, dt - \int_1^2 \left(\frac{d(2-t)}{dt} \cdot \int_1^2 e^{-st}\, dt\right) dt + 0$$

$$= t\left[\frac{e^{-st}}{-s}\right]_0^1 - \int\left(\frac{e^{-st}}{-s}\right) dt + (2-t)\left[\frac{e^{-st}}{-s}\right]_1^2$$

$$- \int_1^2 \left(\frac{(-1)(e^{-st})}{-s}\right) dt$$

$$= t\left[\frac{e^{-s(1)}}{-s} - \frac{e^{-s(0)}}{-s}\right] - \left[\frac{e^{-st}}{s^2}\right]_0^1 + (2-t)\left[\frac{e^{-st}}{-s}\right]_1^2$$

$$+ \left[\frac{e^{-st}}{s^2}\right]_1^2$$

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$$= t\left[\frac{e^{-s}}{-s} + \frac{1}{s}\right] - \left[\frac{e^{-s}}{s^2} - \frac{e^{0}}{s^2}\right] + (2-t)$$

$$\left[\frac{e^{-s2}}{-s} + \frac{e^{-s}}{s}\right]$$

$$= t\left[\frac{1-e^{-s}}{s}\right] - \left[\frac{e^{-s}}{s^2} - \frac{1}{s^2}\right] + (2-t)$$

$$\left[\frac{-e^{-2s}}{s} + \frac{e^{-s}}{s}\right]$$

$$= t\left[\frac{1-e^{-s}}{s}\right] + \left[\frac{1-e^{-s}}{s^2}\right] + (2-t)$$

$$e^{-s}\left[\frac{1-e^{-s}}{s}\right]\left[\frac{1-e^{-s}}{s}\right]$$

$$\left[t + \frac{1}{s} + (2-t)\,e^{-s}\right]$$

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b) $y(0) = 1$, $y'(0) = 0$

$y''t + y'(t) + y(t) = f(t)$

Taking laplace transform on both sides

$\therefore s^2 y(s) + s y(s) + y(s) = f(s)$

$\therefore (s^2 + s + 1)\, y(s) = F(s)$ — (i)

3 cases

Case i : $f(t) = t$; $0 \le t < 1$

$\therefore F(s) = \frac{1}{s^2}$

$\therefore$ Eqn (1) $\Rightarrow$

$(s^2 + s + 1)\, y(s) = \frac{1}{s^2}$

$y(s) = \frac{1}{s^2(s^2 + s + 1)}$

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$$y(t) = L^{-1}\left\{\frac{1}{s^2(s^2+s+1)}\right\}$$

$$= L^{-1}\left\{\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^2+s+1}\right\}$$

$$= L^{-1}\left\{\frac{-1}{s} + \frac{1}{s^2} + \frac{s}{s^2+s+1}\right\}$$

$$= L^{-1}\left\{\frac{-1}{s} + \frac{1}{s^2} + \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} - \frac{1}{2} \cdot \frac{1}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}\right\}$$

$$L = -1 + t + e^{-t/2}\cos\left(\frac{\sqrt{3}\,t}{2}\right) - \frac{1}{2}e^{-t/2}\,\frac{2}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right) \quad (0 \le t < 1)$$

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Case 2 :

$$f(t) = (2-t)$$

$$F(s) = \frac{2}{s} - \frac{1}{s^2}.$$

eqn (i) $\Rightarrow$

$$(s^2+s+1) \; y(s) = f(s)$$

$$y(s) = \frac{\left(\frac{2}{s} - \frac{1}{s^2}\right)}{(s^2+s+1)}$$

$$y(t) = L^{-1}\left\{ \frac{\left(\frac{2}{s} - \frac{1}{s^2}\right)}{s^2+s+1} \right\}.$$

$$= 2L^{-1}\left\{ \frac{\left(\frac{1}{s}\right)}{s^2+s+1} \right\} - L^{-1}\left\{ \frac{\frac{1}{s^2}}{s^2+s+1} \right\}$$

$$= 2\left(1 - e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}}\right.$$

$$-1 + t \; T e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}}$$

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$$= 2\left(1 - e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}\right) - \frac{e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}}\right)$$

$$- \left(-1 + t + e^{-t/2}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{e^{-t/2}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}}\right)$$

$$(1 \le t < 2$$

Case 3:

When $t \geq 2$

$$F(t) = 0$$

$$F(s) = 0$$

$$\therefore y(t) = 0$$