

Q1

$$y(t) = \int_0^t \lambda e^{-(t-\lambda)} d\lambda$$

Using convolution theorem

$$L \{ (f * g)(t) \} = F(s) G(s)$$

Using Volterra integral equation

$$f * g = \int_0^t f(\lambda) g(t-\lambda) d\lambda$$

$$L \{ f * g \} = L \{ f \} L \{ g \}$$

$$L \left\{ \int_0^t \lambda e^{-(t-\lambda)} d\lambda \right\} = L \{ t \} L \{ e^{-t} \}$$

$$= \frac{1}{s^2} \cdot \frac{1}{s+1}$$

$$y(s) = \frac{1}{s^2(s+1)}$$

Taking Laplace inverse

$$y(t) = L^{-1} \left\{ \frac{-1}{s} + \frac{1}{s^2} - \frac{1}{s-1} \right\}$$

AMEY MAHENDRA THAKUR 1110107589

$$y(t) = -1 - t + e^t$$