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Q. 3

$$y(k+2) = y(k+1) + y(k)$$

Applying z-transformation on both sides

$$z^2 y(z) - z^2 y(0) - z(y)(1)$$

$$= z y(z) - z y(0) + y(z)$$

$$(z^2 - z - 1)\, y(z) = z^2 y(0) + z y(1) - z y(0) \rightarrow (1)$$

Given, $x_1(0) = 0$; $y(k) = x_1(k)$

$$y(0) = x^2(0) = 0$$

$$y(k+1) = x_2(k) \text{ and } x_2(0) = 1$$

$$y(1) = y(0+1) = x_2(0) = 1$$

$\therefore$ Substituting $y(0) = 0$ and $y(1) = 1$ in eq^n (1)

$$(z^2 - z - 1)\, y(z) = 0 + z - 0 = z$$

$$y(z) = \frac{z}{z^2 - z - 1}$$

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$m^2 - m - 1 = 0$

Roots $= m_1, _2 = \frac{1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2}$

$m_1, _2 = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$

$\alpha, \beta = \frac{1}{2} \pm \sqrt{\frac{5}{2}}$

$y(z) = \frac{z}{(z-\alpha)(z-\beta)}$

where $\alpha = \frac{1}{2} + \sqrt{5}/2$,

$\beta = \left(\frac{1}{2} - \sqrt{5}/2\right.$

$= \frac{a}{z-\alpha} + \frac{b}{z-\beta}$

$a(z-\beta) + b(z-\alpha) = 1$

$az - a\beta + bz - b\alpha = 1$

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$a + b = 1$

$-a\beta - b\alpha = 1$

$a(\alpha - \beta) = 1$

$$\boxed{a = \frac{1}{\alpha - \beta}}$$

$$\boxed{b = \frac{-1}{(\alpha - \beta)}}$$

$$y(z) = a\left(\frac{z}{z-\alpha}\right) + b\left(\frac{z}{z-\beta}\right)$$

Apply inverse transform

$$y(k) = a(\alpha)^k + b(\beta)^k$$

$$y(k) = \frac{\alpha^k - \beta^k}{\alpha - \beta}$$

$$x_1(k) = y(k)$$

$$\boxed{x_1(k) = \frac{\alpha^k - \beta^k}{\alpha - \beta}}$$

$$x_2(k) = y(k+1)$$

$$\boxed{x_2(k) = \frac{\alpha^{k+1} - \beta^{k+1}}{\alpha - \beta}}$$

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where $\alpha = \frac{1}{2} + \frac{\sqrt{5}}{2}$ and

$$\beta = \frac{1}{2} - \frac{\sqrt{5}}{2}$$

2, $$\frac{y(k+1)}{y(k)} = \left(\frac{\alpha^{k+1} - \beta^{k+1}}{\cancel{\alpha - \beta}}\right) \left(\frac{\cancel{\alpha - \beta}}{\alpha^{k} - \beta^{k}}\right)$$

$$= \frac{\alpha^{k+1} - \beta^{k+1}}{\alpha^{k} - \beta^{k}}$$

$$= \frac{\alpha^{k+1}\left(1 - (\beta/\alpha)^{k+1}\right)}{\alpha^{k}\left(1 - (\beta/\alpha)^{k}\right)}$$

$$= \alpha\left(\frac{1 - (\beta/\alpha)^{k+1}}{1 - (\beta/\alpha)^{k}}\right)$$

As $k \to \infty$; $\frac{y(k+1)}{y(k)}$ will approach to α

which is $\boxed{\frac{1}{2} + \frac{\sqrt{5}}{2}}$