Assignment – 1

2 $\quad yy' + 36x = 0$

$\therefore y\frac{dy}{dx} = -36x$

$\therefore y\,dy = -36x\,dx$

$\therefore \int y\,dy = -36\int x\,dx$

$\therefore \frac{y^2}{2} = -36\frac{x^2}{2} + c$

$\therefore y^2 + 36x^2 = 2c$

$\therefore \boxed{36x^2 + y^2 = c}$

$\int e^{ax} = \frac{e^{ax}}{a} \quad \int e^x \quad e^{2x} \quad e^{x}$ $\quad \frac{e^{2x}}{2}$

$\int \frac{dy}{y^2} = \int e^{(2x-1)}dx$

$= \int e^t \frac{dt}{2}$

$2x-1 = t$

$2dx = dt$

$d_1 = \frac{1}{2}e^t$

$\frac{1}{2}e^{2x-1}$

3 $\quad y' = e^{2x-1}y^2$

$\therefore \frac{dy}{dx} = e^{2x-1}y^2$

$\therefore \int \frac{1}{y^2}dy = \int e^{t}$

$\therefore \log|y^2| = \cancel{e^{2x-1}(2) + c}$

$\therefore \cancel{2\log y = 2e^{2x-1} + c}$

$\therefore \int y^{-2}dy = \int e^{t}\,dt$

$\therefore y^{(-2+1)} = e^{t}$

$\therefore -y^{-1} = 2e^{2x-1} + c$

$\therefore \boxed{-\frac{1}{y} = 2e^{2x-1} + c}$

suppose

$(2x-1) = t$

$\boxed{2dx = dt}$

$\therefore dt = 2dx$

$\therefore dx = \frac{dt}{2}$

$\therefore \int y^{-2}dy = \int e^{t}\frac{dt}{2}$

$\therefore -\frac{1}{y} = \frac{e^{2x-1}}{2} + c$

$\boxed{-\frac{1}{y} = e^{2x-1}(2) + c}$

$\therefore -\frac{1}{y} = 2e^{2x-1} + c$

a. $(1-x^2)\frac{dy}{dx} = 2y$

$\therefore \int \frac{1}{2y}\,dy = \int \frac{1}{1-x^2}\,dx$

$\therefore \cancel{\frac{1}{2}\log|y|} = \cancel{\int \frac{1}{(1-x)(1+x)}\,dx}$

$$\therefore \boxed{\frac{1}{2}\log|y| = \frac{1}{2(1)}\left|\frac{x+1}{x-1}\right| + c}$$

b. $y^3\frac{dy}{dx} = (y^4+1)\cos x$

$\therefore \int \frac{y^3}{y^4+1}\,dy = \int \cos x\,dx$

$\therefore \int \frac{4}{4}\frac{y^3}{(y^4+1)}\,dy = \int \cos x\,dx$ $\quad y^4+1 = t$

$\therefore \cancel{\frac{1}{4}\log|y^4+1| = 2\sin x + c}$ $\quad \therefore \boxed{4y^3 = dt}$

$\therefore \frac{1}{4}\int \frac{dt}{t} = \sin x + c$

$\therefore \frac{1}{4}\log t = \sin x + c$

$\therefore \frac{1}{4}\log(y^4+1) = \sin x + c$

Hiloy

Assignment - 1

c. $$\frac{dy}{dx} = \underline{1 + x + y} \; \underline{+ xy}$$

$$\therefore \frac{dy}{dx} = y(1+x) + (x+1)$$

$$\therefore \frac{dy}{dx} = [x+1](y+1)$$

$$\therefore \int \frac{dy}{y+1} = \int (x+1)\,dx$$

$$\therefore \log(y+1) = \frac{x^2}{2} + x + c$$

$$\therefore y+1 = e^{\left(\frac{x^2}{2} + x + c\right)}$$

$$\therefore \boxed{y+1 = e^{\left(\frac{x^2 + 2x + 2c}{2}\right)}}$$

12. $$y' = 1 + 4y^2 \qquad \text{with } y(1) = 0$$

$$\therefore \frac{dy}{dx} = 1 + 4y^2$$

$$\therefore \frac{dy}{4y^2+1} = dx$$

$$\cancel{\therefore \frac{1}{8}\frac{8\,dy}{4y^2+1} = dx} \qquad \cancel{4y^2+1 = t \quad \therefore 8y\,dy = dt}$$

b.

$$\therefore \frac{dy}{4\left(\frac{1}{4}+y^2\right)} = dx$$

$$\therefore \frac{dy}{\left(\frac{1}{4}+y^2\right)} = 4\,dx$$

$$\therefore \int \frac{dy}{\left(\left(\frac{1}{2}\right)^2+y^2\right)} = \int 4\,dx$$

$$\therefore \frac{1}{\left(\frac{1}{2}\right)} \tan^{-1}\frac{y}{\left(\frac{1}{2}\right)} = 4x + c$$

$$\therefore 2\tan^{-1}2y = 4x + \frac{c}{2} \qquad \boxed{\frac{c}{2} = c'}$$

$$\therefore \tan^{-1}2y = 2x + c'$$

$$\therefore 2y = \tan(2x + c')$$

$$\therefore \boxed{y = \frac{\tan(2x+c')}{2}}$$

$\therefore$ ~~$y(1) = \tan$~~

$$\therefore \tan^{-1}2y = 2x + \frac{c}{2} \qquad (y(1) = 0,\; x = 1,\; y = 0)$$

$$\therefore \tan^{-1}(2(0)) = 2(1) + \frac{c}{2}$$

$$\therefore 0 = 2 + \frac{c}{2}$$

$$\therefore -2 = \frac{c}{2}$$

$$\therefore \boxed{c = -4}$$

$$\therefore \boxed{y = \frac{\tan(2x - 2)}{2}}$$

Hilroy

d. $y' + 4y = 0$ with $y(-1) = 0$

$\therefore \frac{dy}{dx} = -4y$

$\therefore \int \frac{dy}{y} = -4\int dx$

$\therefore \log y = -4x + c$

$\therefore \boxed{y = e^{-4x+c}}$

$y(-1) = 0$

$\therefore x = -1$

$y = 0$

$\therefore \log(0) = -4(-1) + c$

$\therefore 1 = +4 + c$

$\therefore \boxed{c = -3}$

$\therefore \boxed{y = e^{-4x-3}}$

Part :- 2

Test for exactness. If exact solve if not, use integrating factor to solve.

1. $2xy\,dx + x\,dy = 0$

$\therefore M = 2xy$

$N = x^2$

$\therefore \frac{\partial M}{\partial y} = 2x$, $\frac{\partial N}{\partial x} = 2x$

$\therefore$ exact equation

$f(x,y) = c \quad \ldots (1)$

$\therefore \frac{\partial f}{\partial x} = M, \ldots (2) \quad \frac{\partial f}{\partial y} = N \quad \ldots (3)$

$\therefore \frac{\partial f}{\partial x} = 2xy \ldots (4) \quad \frac{\partial f}{\partial y} = x^2 \ldots (5)$

integrate (4) w.r. to x

$\therefore \int \frac{\partial f}{\partial x} dx = \int 2xy \, dx$

$\therefore \int \frac{\partial f}{\partial x} dx = 2y \int x \, dx$

$= 2y \left[\frac{x^2}{2}\right] + g(y)$

$\therefore \boxed{\frac{\partial f}{\partial x} = x^2 y + g(y)} \quad \ldots (6)$

$\frac{\partial f}{\partial x}$

eq. (6) par. differentiate w.r. y

$\therefore \frac{\partial f}{\partial y} = x^2 + g'(y) \quad \ldots (7)$

put value of (5) to eq. (7)

$\therefore x^2 = x^2 + g'(y)$

$\therefore g'(y) = 0$

$\therefore$ integrate w.r. to y

$\therefore g(y) = 0 \quad \ldots (8)$

Hilroy

Put $g(y) = 0$ in eq. 6

$\therefore \dfrac{\partial f}{\partial x} = x^2 y + 0$

$\therefore \boxed{\dfrac{\partial f}{\partial x} = x^2 y}$

$\therefore f(x,y) = x^2 y$

$\therefore f(x,y) = c$

$\therefore \boxed{x^2 y = c} = f(x,y)$

2. $x^3 dx + y^3 dy = 0$

$\therefore M = x^3, \quad N = y^3$

$\therefore \dfrac{\partial M}{\partial y} = 0, \quad \dfrac{\partial N}{\partial x} = 0$

$\therefore$ this equation is exact

$\therefore \dfrac{\partial f}{\partial x} = M \quad \ldots 1, \quad \dfrac{\partial f}{\partial y} = N \quad \ldots 2$

$\therefore \dfrac{\partial f}{\partial x} = x^3 \quad \ldots 3, \quad \dfrac{\partial f}{\partial y} = y^3 \quad \ldots 4$

$\therefore$ integrate eq. 3 w.r.to x

$\therefore \int \dfrac{\partial f}{\partial x} dx = \int x^3 dx$

$\therefore \dfrac{\partial f}{\partial x} = \dfrac{x^4}{4} + g(y) \quad \ldots 5$

Partial. diff. w.r. to y to eq.(5)

$\therefore \frac{\partial f}{\partial y} = 0 + g'(y)$

put eq. (4) value.

$\therefore y^3 = g'(y) \quad \ldots (6)$

$\therefore$ Integrate. w.r. to y

$\therefore \boxed{\frac{y^4}{4} = g(y)}$

$\therefore$ put value in eq. (5)

$\therefore \frac{\partial f}{\partial x} = \frac{x^4}{4} + \frac{y^4}{4} \quad \ldots (7)$

$\therefore f(x,y) = c = \frac{\partial f}{\partial x} = \frac{x^4}{4} + \frac{y^4}{4}$

6. $3(y+1)\,dx = 2x\,dy$

$\therefore M = 3y+3 \quad , \quad N = 2x$

$\therefore \frac{\partial M}{\partial y} = 3 \quad \quad \frac{\partial N}{\partial x} = 2$

$\therefore$ this equation is not exact.

$\therefore M^* dx + N^* dy = 0 \quad \ldots (1)$

$M^* = IF\ M$

$N^* = IF\ N$

Hilroy

$3(y+1)\,dx = 2x\,dy$

IF $\left(3(y+1)dx - 2x\,dy = 0 \right)$

$M = 3(y+1)$

$N = -2x$

$3 \neq -2$

$2x\,dy - 3(y+1)dx = 0$

$\frac{dy}{dx} - \frac{3(y+1)}{2x} = 0$

$\frac{dy}{dx} - \frac{3y}{2x} - \frac{3}{2x} = 0$

$\frac{dy}{dx} - \frac{3y}{2x} = \frac{3}{2x} \Rightarrow Q(x)$

$P(x) = \frac{-3}{2x}$

$e^{\int \frac{-3}{2x} dx} = P(x)$

$e^{\frac{-3}{2}\int \frac{1}{x} dx}$

$e^{\frac{-3}{2} \ln|x|}$

$e^{\ln|x|^{-3/2}}$

IF $= x^{-3/2}$

$M\,dx + N\,dy = 0$ (Differential form)

$\frac{dM}{dy} = \frac{dN}{dx}$ } equal → exact differ.

$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^0$ → first order linear

IF $e^{\int P(x)\,dx}$

$y(IF) = \int Q(x)\,IF\,dx$

$2x\,x^{-3/2}dy - 3(y+1)x^{-3/2}dx = 0$

$2x^{-1/2}dy - \underbrace{3(y+1)x^{-3/2}}\,dx = 0$

$-3x^{-\frac{3}{2}} = \frac{dM}{dy}$

$2 \times \frac{-1}{2} \times x^{-1/2-1}$

$-1\,x^{-\frac{3}{2}}$

DE Par $= \frac{\partial u}{\partial x}$ }

order

$\frac{\partial^2 u}{\partial x^2}$ } $u = f(x, y)$

$y = mx$ — linear

$y = mx^2$ — non linear

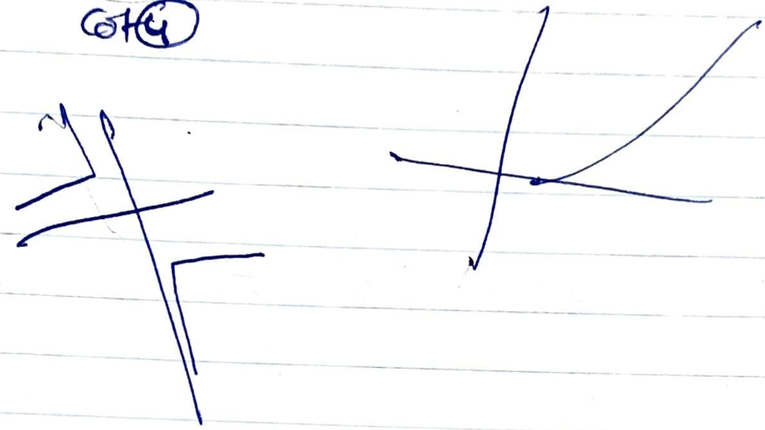
$\frac{du}{dt}$ $\quad t\frac{du}{dt} \quad x\frac{dy}{dx}$

non linear

linear

e^u sin u tan cot(u)

sin u



$y' = 1 + 4y^2 \quad y(1) = 0$

$\frac{dy}{dx} = 1 + 4y^2$

$\frac{dy}{1+4y^2}$

$y' + 4y = 0 \quad y(-1) = 0$

$\frac{dy}{dx} = -4y$

$\int \frac{dy}{y} = \int -4\,dx + c$

$e^{a+b} = e^a e^b$

$\ln|y| = -4x + c$

$y = e^c e^{-4x}$

$y = Ke^{-4x}$

$0 = Ke^{4}$

zero

$y = e^{c-4x} = e^c e^{-4x}$

$0 = e^{c+4}$

$= Ke^{-4x}$

$0 = Ke^4$

$0 =$

6

$3(y+1)dx - 2x\,dy = 0$

$3 \neq -2$

$\cancel{dx}$

$$\frac{3(y+1)}{-2x} + \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} - \frac{3y}{2x} - \frac{3}{2x} = 0$$

$$\frac{dy}{dx} - \frac{3}{2x}y = \frac{3}{2x}$$

$$IF = e^{\int -\frac{3}{2x}dx}$$

$$= e^{-\frac{3}{2}\int \frac{1}{x}dx}$$

$$= e^{-\frac{3}{2}\ln(x)}$$

$$IF = x^{-3/2} = \boxed{\frac{1}{x^{\frac{3}{2}}}}$$

$IF =$

$$y(IF) = \int Q(x)(IF)dx + C$$

$$yx^{-3/2} = \int \frac{3}{2x} x^{-3/2}dx + C$$

$$yx^{-3/2} = \frac{3}{2}\int x^{-5/2}dx + C$$

$$yx^{-3/2} = \frac{3}{2}\frac{x^{-5/2+1}}{-5/2+1} + C$$

$$yx^{-3/2} = \frac{3}{\cancel{2}}\frac{x^{-3/2}}{-3/\cancel{2}} + C$$

$$yx^{-3/2} = -x^{-3/2} + C$$

$$x^{-3/2}(y+1) = C$$

$1 - \frac{3}{2}$

assi. 1 (9)

$e^{2x}(2\cos y\, dx - \sin y\, dy) = 0, \quad y(0) = 0$ ... (1)

$M = e^{2x}\, 2\cos y \qquad N = -e^{2x}\sin y$

$\frac{\partial M}{\partial y} = 2 e^{2x} - \sin y \qquad \frac{\partial N}{\partial x} = -e^{2x}(2)\sin y$

this is ~~not~~ exact

~~eq. (1) divide by e^{2x} $2\cos x$~~ $e^{2x}\, 2\cos x$

$\cancel{\frac{1}{N}\left[\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right]}$

$f(x, y) = c$ ... (1)

$\frac{\partial f}{\partial x} = M$ ... (2) & $\frac{\partial f}{\partial y} = N$ ... (3)

$\therefore \frac{\partial f}{\partial x} = e^{2x}\, 2\cos y$ ... (4) $\qquad \frac{\partial f}{\partial y} = -e^{2x}\sin y$ ... (5)

integrate (4) w.r. to x

$\therefore \int \frac{\partial f}{\partial x}\, dx = 2\int e^{2x}\cos y\, dx$

$G'(y) = 0$

$G(y) = C$

$-e^{2x}\sin y = \int G'(y)$

$\frac{dF}{dx} = e^{2x} \times 2 \times \cos y$

$\int dF = \int 2e^{2x}\cos y\, dx$

$-e^{2x}\cos y = G(y)$

$F = \frac{2\cos y\, e^{2x}}{2} + G(y)$

$F = \cos y\, e^{2x} + G(y)$

$\frac{dF}{dy} = -\sin y\, e^{2x} + G'(y)$

(9) $x\,dy + y \ln y\, dx = 0$ ... (1)

missing Q

$M = y \ln y$ $\qquad N = x$

$\therefore \frac{\partial M}{\partial y} = \cancel{y}\frac{1}{\cancel{y}} + \ln y$ $\qquad \frac{\partial N}{\partial x} = 1$

$= 1 + \ln y$

this is non exact equation

divide eq. (1) by $x\,dx$

$\therefore$ ~~$\frac{dy}{dx} + \frac{y}{x} \ln y = 0$~~

$\left\{ \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x) \right.$

$\therefore$ ~~$\frac{dy}{dx}+$~~ $\qquad \left. IF = e^{\int f(x)\,dx} \right\}$

$\therefore$

$\left\{ \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = g(y) \right\}$

$\left. IF = e^{\int -g(y)\,dy} \right\}$

~~(1) $F(x) = \frac{1}{x}\left[\left(1 + \log y\right) - 1\right]$~~

~~$= \boxed{\frac{1}{x}(\log y)}$~~

~~$If = e^{\int F(x)\,dx}$~~

~~$= e^{\int t\,dt}$~~

~~$\log x = t$~~

~~$\therefore \frac{1}{x}dx = dt$~~

~~If multiply with eq. ①~~

~~$\therefore x^2 dy + xy \log y \, dx = 0$~~

~~$\therefore \frac{\partial M}{\partial y} = y\frac{1}{y} + \log y$~~

~~$= 1 + \log$~~

$x\,dy + y \ln y \, dx = 0$

$1 + \ln y \neq 1$

$\frac{1}{N}(1 + \ln y - 1)$

$\frac{\ln y}{x} \neq f(x)$

$\frac{1}{M}(1 + \ln y - 1)$

$\frac{\ln y}{y \ln y}$

$= \frac{1}{y} = f(y)$

$IF = e^{\int -f(y) dy}$

$= e^{\int -\frac{1}{y} dy}$

$= e^{-\ln|y|}$

$= y^{-1}$

$= \frac{1}{y}$

$\left(\frac{x}{y}\right) dy + \ln y \, dx = 0$

$\frac{\partial F}{\partial x} = \ln y$

$\frac{\partial F}{\partial y} = \frac{x}{y}$