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Q1 $y^2 dx + (3xy - 1)\, dy = 0$

$\frac{\partial M}{\partial y} = 2y$, $\frac{\partial N}{\partial x} = 3y$

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{3xy-1}(2y - 3y)$

$= \frac{1}{3xy-1}(-y)$

$= \frac{-y}{3xy-1}$

Note that the above right side is not a function of x only.

$\therefore \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y^2}(-y)$

$= \frac{-1}{y}$

Note that the above right side is a function of y only.

$\therefore$ Rule 2 applies

$IF = e^{-\int g(y)\,dy}$

$= e^{-\int -\frac{1}{y}\,dy}$

$= e^{\ln(y)}$

$= y$

Multiplying IF to main eq^n.

$y^3\,dx + (3xy^2 - y)\,dy = 0$

$\frac{\partial F}{\partial x} = m = y^3$

$\partial F = y^3\,\partial x$

$dF = y^3\,dx$

Integrating both sides wrt x.

$F = y^3 x + R(y) \quad - (1)$

$\frac{\partial F}{\partial y} = N = 3xy^2 - y$

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$$\frac{\partial}{\partial y}\left((y^3 x) + R(y)\right) = 3xy^2 - y$$

$$\cancel{3xy^2} + R'(y) = \cancel{3xy^2} - y$$

$$R'(y) = -y$$

Integrating both sides wrt to y

$$R(y) = \frac{-y^2}{2}$$

Putting the value of $R(y)$ in eq^n 1

$$F = y^3 x - \frac{y^2}{2}$$

$$\frac{1}{2} c = y^3 x - \frac{y^2}{2}$$

$$c = 2y^3 x - y^2$$

$$c = (2xy - 1) y^2$$