$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y} \quad T/F \checkmark$$

$$4\sec(4y) = \frac{\sin^{-1}(16x+c)}{4}$$

$$8\sec(8y) \Rightarrow \frac{\sin^{-1}(64x+c)}{8}$$

1. $\int \frac{dy}{dx} y' = \int 6+6y^2\, dx \qquad \frac{1}{6}\int \frac{dy}{1+y^2} = \int dx$

Ans.

$\frac{1}{6} \frac{y'\, dy}{1+y^2} = \int dx \qquad \frac{1}{6}\tan^{-1}(y) = x+c$

$= 6x+c$

$\frac{1}{6}\left[\tan^{-1}(y)\right] = x + C$

$\tan^{-1}(y) = 6(x+c) \qquad y = \tan(6x+c)$

$~~y = \tan\left(\frac{x+c}{6}\right)~~$

$$\boxed{y = \tan(6x+c)}$$

2. $y' = 8\sec(8y)$

Ans.

$\int \frac{dy}{\sec(8y)} = \int 8\, dx$

$\int \cos(8y)\, dy = 8x+c$

$\frac{\sin(8y)}{8} = 8x+c$

$\sin(8y) = 64x + C$

$$\boxed{y = \frac{\sin^{-1}(64x+c)}{8}}$$

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(3.) $e^{4x} y' = 4(x+8) y^5$ $\quad y(0) = \frac{1}{\sqrt[4]{33}}$

$$\frac{y'}{y^5} = \frac{4(x+8)}{e^{4x}}$$

$$y^{-5} y' = (4x+32) e^{-4x}$$

$$\Rightarrow 4x e^{-4x} + 32 e^{-4x}$$

$$\int y^{-5} \frac{dy}{dx} = 4\int x e^{-4x} + 32 \int e^{-4x}$$

$$\boxed{\frac{y^{-4}}{-4} = 4\int x e^{-4x} + 32 \int e^{-4x}} \text{ — (1)}$$

$$4 \int x e^{-4x}$$

$$\left\{ \int u \frac{dv}{dx} \Rightarrow uv - \int v \frac{du}{dx} \cdot dx \right.$$

$$u = x$$
$$\frac{dv}{dx} = e^{-4x}$$

$$v = \int e^{-4x} \Rightarrow \frac{-1}{4} e^{-4x}$$

$$v = -\tfrac{1}{4} e^{-4x}$$

$$\int v \frac{du}{dx} = uv - \int v \frac{du}{dx} \cdot dx$$

when $= u = x$

$v = \frac{-e^{-4x}}{4}$ $\quad \frac{du}{dx} = 1$ $\quad \left\{ \int e^{ax} = \frac{e^{ax}}{a} \right\}$

$$\Rightarrow -\frac{x e^{-4x}}{4} + \frac{1}{4} \times \frac{e^{-4x}}{-4}$$

$$\Rightarrow \boxed{-\frac{1}{4} x e^{-4x} - \frac{1}{16} e^{-4x}} \text{ — (2)}$$

Put - (2) in (1).

$$\frac{y^{-4}}{-4} = 4\left[-\frac{1}{4}xe^{-4x} - \frac{e^{-4x}}{16}\right] + 32\left[\frac{e^{-4x}}{-4}\right] + c$$

$$\frac{y^{-4}}{-4} = -xe^{-4x} - \frac{e^{-4x}}{4} - \frac{32\, e^{-4x}}{4}$$

$$\frac{y^{-4}}{4} = xe^{-4x} + \frac{e^{-4x}}{4} + 8e^{-4x}$$

$$\frac{1}{y^4} = 4xe^{-4x} + e^{-4x} + 32e^{-4x}$$

$$\frac{1}{y^4} = 4xe^{-4x} + 33e^{-4x}$$

$$y^{\cancel{4}} = \frac{e^{x}}{[4x+33]^{1/4}}$$

$$\boxed{y = \frac{e^{x}}{(4x+33)^{1/4}}}$$

Direct formula for UV Rule

$$\boxed{\int [x e^{ax}]\, dx = \frac{1}{a^2}(ax-1)e^{ax}}$$

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④ $y' x \ln(x) = y$; $y(5) = \ln(625)$

$$\int \frac{dy}{y} = \int \frac{dx}{x\ln(x)}$$

$t = \ln x$
$dt = \frac{1}{x}dx$ ⇒ $\int \frac{dy}{dy} = \int \frac{1}{t} dt$

$\cancel{\ln y = \ln(\ln(x))}$

⇒ $\ln y = \ln t + \ln c$

$\ln y = \ln(t \cdot c)$

$y = t \cdot c$

$\cancel{y = \ln(x)}$

⇒ $\boxed{y = \ln x \cdot c}$ $\cancel{\ln y} =$

$4\ln(x)$

$y(5) = \ln(5^4)$

$y(5) = 4\ln(5)$

$\boxed{y(x) = 4\ln(x)}$

⑤ $\left[\frac{-y}{x^2} + 4\cos(4x)\right]dx + \left[\frac{1}{x} - 4\sin(4y)\right]dy = 0$

$\underbrace{\qquad}_{m}$ $\underbrace{\qquad}_{n}$

$u(x,y) = \frac{y}{x} + \cos(4y) + \sin(4x)$

$\frac{\partial M}{\partial y} = \frac{-1}{x^2}$ $\frac{\partial N}{\partial x} = \frac{1}{x^2}$

$\boxed{\frac{\partial M}{\partial y} = \frac{\partial n}{\partial x}}$ → exact equ

$\cancel{\left[\frac{-y}{x^2} + 4\cos(4x)\right] dx = -\left[\frac{1}{x} - 4\sin(4y)\right] dy}$

⇒ $\cancel{\frac{dy}{dx} = \left[\frac{3y}{x^3} + 16\cos(4x)\right] - \left[\frac{y}{x} - 16\cos(4y)\right]}$

$\cancel{16(\cos 4y - \sin 4x) = \frac{3y}{x^3} - \frac{y}{x}}$

$\frac{y}{x}\left[\frac{3}{x^2} - 1\right]$

so,

$$\int m\,dx + \int n\,dy = c$$

$$\int\left[\frac{-y}{x^2} + 4\cos(4x)\right]dx + \int -4\sin(4y)\,dy = c$$

$$-y\left(-\frac{1}{x}\right) + \frac{4\sin(4x)}{4} + \frac{-4[-\cos(4y)]}{4} = c$$

$$\boxed{y + \sin(4x) + \cos(4x) = c}$$

(6.) $-y\,dx + x\,dy = 0$

$y(x) = c(x)$

$m = -y \,;\; n = x$

$\frac{\partial m}{\partial y} = -1 \,;\; \frac{\partial n}{\partial x} = 1$

$\frac{\partial m}{\partial y} \neq \frac{\partial n}{\partial x}$ [Not. Exact Equ]

$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = f(y) \Rightarrow \frac{1+1}{-y} \Rightarrow -\frac{2}{y}$

I.F $= e^{\int f(y)dy} \Rightarrow e^{\int -2/y\, dy} \Rightarrow e^{-2\ln y}$

I.F $= e^{\ln(y^{-2})} \Rightarrow y^{-2}$

$\boxed{\text{I.F} = y^{-2}}$

So, $\int m\,dx + \int n\,dy = c \Rightarrow \int -\frac{y}{y^2}\,dx + 0 = c$

$\Rightarrow -\frac{1}{y}x = c$

$\boxed{y = x\left[-\frac{1}{c}\right]}$ So, $\boxed{y(x) = c(x)}$

(7.) $-11\,y\,dx + 10\,x\,dy = 0$, $F(x,y) = \frac{y^9}{x^{12}}$

$y(x) = c(x)^{11/10}$

$m = -11y \,;\; n = 10x$

$\frac{\partial m}{\partial y} = -11 \,;\; \frac{\partial n}{\partial x} = 10$

$\frac{\partial m}{\partial y} \neq \frac{\partial n}{\partial x}$ not exact equ.

$\frac{\frac{\partial m}{\partial y} - \frac{\partial n}{\partial x}}{N} = f(x) = \frac{-11-10}{10x} \Rightarrow \frac{-21}{10x}$

I.F $= e^{\int p\,dx} \Rightarrow e^{\int\left(\frac{-21}{10x}\right)dx}$

I.F $= e^{-21/10 \ln x} \Rightarrow e^{\ln(x)^{-21/10}}$

$\boxed{\text{I.F} = x^{-21/10}}$

* both side I.F

$-11\,y\,x^{-21/10}\,dx + 10\,x\,x^{-21/10}\,dy = 0.$

$-11\,y\,x^{-21/10}\,dx + 10\,x^{-11/10}\,dy = 0.$

$\int m\,dx + \int N\,dy = c.$

$\int -11\,y\,x^{-21/10}\,dx + 0 = C$

$-11\,y\left(x^{-11/10}\right)/\frac{21}{10} = C$ constant!

$\frac{y}{x^{11/10}} = \frac{10}{11\times 21}C$

So, $\boxed{y = C(x)^{11/10}}$

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(8.) $15x\,dx + 4y\,dy = 0$ $\quad \frac{15x^2}{2} + 2y^2$

$$\int 4y\,dy = -\int 15x\,dx$$

$$\overset{2}{\cancel{4}}\frac{y^2}{\cancel{2}} = -\frac{15}{2}x^2$$

$$2y^2 = -\frac{15}{2}x^2$$

$$u(x,y) = \frac{15}{2}x^2 + 2y^2$$

(9.) $e^{-y}dx + e^{-x}(-e^{-y}+9)dy = 0 \quad F = e^{x+y}$ $\quad \cancel{e^{x+y}}$

$e^{x} + 9e^{y} - y$

$$e^{-y}(dx - e^{-x}) + 9e^{-x}dy = 0$$

$$e^{-y}dx + 9e^{-x}dy = e^{-(x+y)}$$

$$e^{-y}dx + 9e^{-x}dy = \frac{1}{F}$$

$$\frac{dx}{e^{y}} + \frac{9\,dy}{e^{x}} = e^{-(x+y)}$$

$$\frac{e^{x}dx + 9e^{y}dy}{e^{x}.e^{y}} = e^{-(x+y)}$$

$$e^{x}dx + 9e^{y}dy = e^{\cancel{(x+y)} - \cancel{(x+y)}} \Rightarrow e^{0}$$

$$\int e^{x}dx + 9e^{y}dy = \int 1\,dy$$

$$\boxed{u(x,y) = e^{x} + 9e^{y} - y}$$

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(10.) $y' + \frac{y}{x^2} = 4x^3 e^{\frac{1}{x}}$, $y(1) = 139.18$. $e^{1/x}(x^4 + 50.2)$

$\cancel{\frac{dy}{dx} + \frac{y}{x^2} = 4x^3 e^{1/x}}$ $P = \frac{1}{x^2}$

$I.F = e^{\int P dx} \Rightarrow e^{\int \frac{1}{x^2} dx} \Rightarrow e^{\int \frac{1-2}{x^{2-1}} dx} \Rightarrow e^{-1/x}$

$e^{-1/x} \frac{dy}{dx} + e^{-1/x} \frac{y}{x^2} = 4x^3 e^{1/x} e^{-1/x}$

$\frac{d}{dx} y(e^{-1/x}) = 4x^3$

$y e^{-1/x} = \int 4x^3 \, dx$

$= \frac{\cancel{4} x^4}{\cancel{4}} + C.$

$y e^{-1/x} = x^4 + C$

$y = [x^4 + C] e^{1/x}$

$y(1) = 139.18$

$139.18 \, e^{-1/1} = 1 + C$

$\frac{139.18}{e} - 1 = C$

$\boxed{C = 50.20146}$

$\therefore y = \frac{x^4}{e^{1/x}} + \frac{50.20146}{e^{1/x}}$

$\boxed{y = e^{1/x} [x^4 + 50.2]}$

11. $y' + 2.25y = 9 \quad ; \quad y(0) = 11.3$

$\cancel{4 + \frac{7.3}{e^{2.25x}}}$

$$\frac{dy}{dx} + 2.25y = 9$$

$$\cancel{dy + 2.25y\,dx = 9\,dx}$$

$$e^{2.25x}\frac{dy}{dx} + 2.25y\,e^{2.25x} = 9e^{2.25x}$$

$$\frac{d}{dx}\int \left(y e^{2.25x}\right) = \int 9e^{2.25x}$$

$$y e^{2.25x} = \frac{9e^{2.25x}}{2.25} + C$$

$$11.3(1) = \frac{9}{2.25} + C$$

$$11.3 - 4 = C$$

$$\boxed{7.3 = C}$$

$$y e^{2.25x} = \frac{9e^{2.25x}}{2.25} + 7.3$$

$$\therefore \boxed{y = \frac{9}{2.25} + \frac{7.3}{e^{2.25x}}}$$

$$\boxed{y = 4 + \frac{7.3}{e^{2.25x}}}$$

$x + C$

$\frac{C}{e^{kx}} - \frac{1}{4ke^{5kx}}$

12. $y' + ky = e^{-5kx}$

$\frac{dy}{dx} + ky = e^{-5kx}$

$\frac{dy}{dx} = e^{-5kx} - ky$

$P = k$ $\quad \frac{dy}{dx} = \frac{1}{e^{5kx}} - ky$

$I.F = e^{\int P dx} = e^{\int k dx} \Rightarrow e^{kx}$

$e^{kx} \cdot \frac{dy}{dx} + e^{kx} ky = e^{kx - 5kx} \Rightarrow e^{-4kx}$

$\int \frac{d}{dx}(y e^{kx}) = \int e^{-4kx}$

$y e^{kx} = \frac{-e^{-4kx}}{4k} + C$

$\boxed{y = \frac{C}{e^{kx}} - \frac{1}{4k e^{5kx}}}$ // $\Rightarrow k \neq 0$

if. $k = 0$

$\frac{d}{dx}(y e^{kx}) = \int e^{-4kx}$

$\left[\frac{d}{dx}(y e^{0})\right] = \int e^{0} \, dx$

$\boxed{y \Rightarrow x + C}$ // $\Rightarrow k = 0$