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Q3 $(D^2+1)\, y(x) = \sin x + x \cos x$

$y(0) = 1$

$y'(0) = \frac{3}{4}$

We can write given eqⁿ as

$D^2 y + y = \sin x + x \cos x$

$y'' + y = \sin x + x \cos x$

consider the auxillary eqⁿ.

$D^2 + 1 = 0$

$D = \pm \sqrt{-1}$

$D = \pm i$

So, complimentary solⁿ is

$y_c = c_1 \cos x + c_2 \sin x$

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Now, we can take

$$y_p = A\sin x + B\cos x + ((x+D)\,e^0\sin x + (Ex+F)\,e^0\cos x$$

$$= A\sin x + B\cos x + [(x+D)]\sin x + [Ex+F]\cos x$$

$$\cancel{y_p' = A(\cos x + B(-\sin x) + [((x+D)\cos x + \sin x(1)] + [(Ex+F)(-\sin x) + \cos x\,[E]}$$

$$y_p' = A\cos x + B(-\sin x) + (1)\sin x + ((x+D)\cos x + (E)\cos x + (Fx+F)(-\sin x)$$

$$y_p' = -A\sin x - B\cos x + C\cos x + C\cos x - ((x+D)\sin x - E\sin x - E\sin x - (Ex+F)\cos x$$

$$y_p'' = -A\sin x - B\cos x + 2C\cos x - ((x+D)\sin x - 2E\sin x - (Ex+F)\cos x$$

$$y'' + y = \sin x + x\cos x$$

$$-A\sin x - B\cos x + 2C\cos x - ((x+D)\sin x - 2E\sin x - (Ex+F)\cos x + A\sin x + B\cos x + ((x+D)\sin x + (Fx+F)\cos x = \sin x + x\cos x$$

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$\therefore -2E = 1$

$\therefore E = \frac{-1}{2}$

$2C = x$

$\therefore C = \frac{x}{2}$

$\therefore A, B, D, F = 0$

$\Rightarrow y_p = \left(\left(\frac{x}{2}\right)x + 0\right)\sin x + \left(\left(\frac{-1}{2}\right) + 0\right)\cos x$

$y_p = \frac{x^2}{2}\sin x - \frac{1}{2}\cos x$

$y = y_c + y_p$

$y = c_1 \cos x + c_2 \sin x + \frac{x^2}{2}\sin x - \frac{1}{2}\cos x$

$y(0) = 1$

$1 = c_1 - \frac{1}{2}$

$c_1 = \frac{1}{2}$

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$$y' = C_1(-\sin x) + C_2 \cos x + \frac{x^2}{2}(\cos x)$$

$$+ \frac{2x}{2} \sin x + \frac{1}{2} \sin x$$

$$C_2 = \frac{3}{4}$$

$$y = y_c + y_p$$

$$= \frac{1}{2} \cos x + \frac{3}{4} \sin x + \frac{x^2}{2} \sin x$$

$$- \frac{1}{2} \cos x$$

$$y = \sin x \left(\frac{3}{4} + \frac{x^2}{2} \right)$$