What is the type of equation ?

↓

Are you able to see M and N clearly.

$M\,dx + N\,dy = 0.$

Yes.

↓

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$?

exact equation.

↓

* $\frac{\partial F}{\partial x} = M.$

  $\frac{\partial F}{\partial y} = N.$ — ①

↓

* $\int \frac{\partial F}{\partial x} = \int M$

  $F = \underline{abc} + R(y)$

↓

* $\frac{\partial F}{\partial y} = \underline{abc}$ — ②

↓

* equate eqn ① & ②

  $\underline{abc} = \underline{def}.$

  $R(y) = \underline{g(y)}$

↓

* put R(y) in F
* solution.

No. or $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

↓

Intigrating Factor Method.

↓

Linear.

1) $y' + P(x)y = Q(x)$
2) $I.F. = e^{\int P\,dx}$
3) Multiply I.F. to the equation.
4) $(\quad + \quad) = —$

   ↓

   $(\quad) = —$
5) Make y as a subject
6) solution.

Non-Linear.

1) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$?
2) Case:1 $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$?

   $\Rightarrow I.F. = e^{\int f(x)\,dx}$

   Case:2 $\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = g(y)$?

   $\Rightarrow I.F. = e^{-\int g(y)\,dy}.$
3) Multiply I.F. to the equation
4) $\frac{\partial F}{\partial x} = M.$

   $\frac{\partial F}{\partial y} = N$ — ①
5) $F = — + R.$

   $\frac{\partial F}{\partial y} = —$ — ②
6) eqn ① & ②

   ~~R(y) =~~
7) Find $F = C$
8) solution.

* Is it

$y' + P(x)y = Q(x)y^n$

$\Rightarrow$ Bernoulli's equation.

BY - Parth.