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Q2 $\frac{dy}{dx} + y = xy^3$

It is of bernoulli's form

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$$

$$\frac{dy}{dx} + y = xy^n$$

$$\frac{dy}{dx} + y = xy^3$$

Let $r = y^{-n+1}$

$\therefore \quad r = y^{-3+1}$

$$\boxed{r = y^{-2}}$$

$$\therefore \frac{dr}{dx} + (1-n)\,Pr = (1-n)\,Q$$

$$\frac{dr}{dx} + (1-3)\,Pr = (1-3)\,Q$$

$$\frac{dr}{dx} + (-2)\ Pr = (-2)\ Q$$

$$\frac{dr}{dx} + (-2)(1)r = (-2)x$$

$$\frac{dr}{dx} + (-2)r = -2x$$

$$IF = e^{\int -2\,dx}$$

$$= e^{-2x}$$

$$e^{-2x}\frac{dr}{dx} + (-2)e^{-2x}r = -2xe^{-2x}$$

$$\frac{d(re^{-2x})}{dx} = -2xe^{-2x}$$

$$d(re^{-2x}) = -2xe^{-2x}\,dx$$

Integrating both the sides

$$re^{-2x} = -\int(2xe^{-2x})\,dx$$

$$re^{-2x} = -2\int xe^{-2x}\,dx$$

(Applying Integral uv formula)

$$u = x, \quad v = -e^{-2x}$$

$$re^{-2x} = -2\left[x\int e^{-2x}dx - \int\left(\frac{dx}{dx}\cdot\int e^{-2x}dx\right)dx\right]$$

$$+ c$$

$$re^{-2x} = -2\left[\frac{xe^{-2x}}{-2} - \int\frac{1\cdot e^{-2x}}{-2}dx\right] + c$$

$$re^{-2x} = -2\left[\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{(-2)(-2)}\right] + c$$

$$re^{-2x} = -2\left[\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{4}\right] + c$$

$$re^{-2x} = xe^{-2x} + \frac{e^{-2x}}{2} + c$$

Resubstituting

$$r = y^{-2}$$

$$\therefore (y^{-2})\,e^{-2x} = xe^{-2x} + \frac{e^{-2x}}{2} + c$$

$$2(y^{-2})\,e^{-2x} = 2xe^{-2x} + e^{-2x} + 2c$$

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$$\frac{2e^{-2x}}{2xe^{-2x} + e^{-2x} + 2c} = y^2$$

$$\therefore y = \pm\sqrt{\frac{2e^{-2x}}{2xe^{-2x} + e^{-2x} + 2c}}$$

$$y = \pm\sqrt{\frac{2e^{-2x}}{e^{-2x}(2x+1) + 2c}}$$

$$y = \pm\sqrt{\frac{1}{\frac{1}{2}(2x+1) + ce^{2x}}}$$