* Exact Equation *

Ex: Solve the ~~so~~ equation:

$$\underbrace{(2x^3 - xy^2 - 2y + 3)}_{M}\,dx - \underbrace{(x^2y - 2x)}_{N}\,dy = 0$$

Step:01 Find M and Find N.

Step:02 Find $\frac{\partial M}{\partial y}$ and $\frac{\partial N}{\partial x}$

Step:03 $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\rightarrow$ The equation is exact equation.

Step:04 Find function F ; where,

$$\frac{\partial F}{\partial x} = M = \_\_\_$$

$$\frac{\partial F}{\partial y} = N = \_\_\_$$

> Our Main goal is to Find the function F.
> $\therefore$ F = C

Step:05 To find F; intigrate one of the above (w.r.t. x)

$$\int \frac{\partial F}{\partial x} = \int \_\_\_$$

$$F = \_\_\_ + R(y) \quad \cdots\cdots\cdots \ (1)$$

Step:06 To find R(y); again derivate F w.r.t y $\left(\frac{\partial F}{\partial y}\right)$

($\because$ bcoz we have the value of $\partial F/\partial y$)

Step:07 Equate both $\partial F/\partial y$

$\rightarrow$ you will find out value of R(y)

Step:08 put the value of R(y) in equation (1)

F = C = your solution.

* If the eqn is not exact *

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ → you have to go with Intigrating Factor Method.

* Intigruting Factor Method : Linear Equation *

Step:01 Bring the equation in standard form.

$$y' + P(x)\,y = Q(x)$$

or $x' + P(y)\,x = Q(y)$

* Our main goal is to find y. (in the form of equation)

Step:02 $I.F. = e^{\int P\,dx}$

Step:03 Multiply the given eqn by. I.F.

Step:04 simplify it (—— + ——) = this can be written as.

$$\downarrow$$

$$\frac{d}{dt}(x\cdot y) = \text{——}$$

Step:05 To Find y. Intigrate both the side.

Step:06 Simplify the equation.

Step:07 Make the subjet : y

Step:08 The eqn of y is the solution you want.

* Integration Factor Method *

Non-Linear

Ex: Solve the solution of

$$y(x+y+1)\,dx + x(x+3y+2)\,dy = 0 \quad \text{—— (1)}$$

Step:01 Check it is non-Linear. $\because$ $x\,dx$, $y\,dy$.

Step:02 If it is non-linear, we have to go for case1 and case 2.

Step:03 Find $\frac{\partial M}{\partial y}$ & $\frac{\partial N}{\partial x}$ then $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$

Step:04 check: Case 1 - $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = F(x)$ only ?

case-2 - $-\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = g(y)$ only ?

Step:05 Find Integration Factor. Case 1 - $I.F. = e^{\int f(x)\,dy}$

case 2 - $I.F = e^{-\int g(y)\,dy}$

Step:06 Multiply eq(1) with I.F.

Step:07 $\frac{\partial F}{\partial x} = M = ----$ $\Rightarrow$ $F = \int ---- = \_\_\_\_ + R$

$\frac{\partial F}{\partial y} = N = ----$ $\Rightarrow$ $F = \int ---- = \_\_\_\_ + R'$

Step:08 Find $F = c$

which is the solution.

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* Bernoulli's Equation *

Step:01 Bring the equation in standard form.

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n \quad [n = \text{Real number}]$$

Step:02 Find n. $\Rightarrow$ Divide eq^n by y^n

$n = 3 \rightarrow$ divide by y^3

$n = 9 \rightarrow$ divide by y^9.

Step:03 Let $r = y^{abc.} \Rightarrow \frac{dr}{dy} = \_\_$

$\Rightarrow dr = \_\_ (dy)$

$\Rightarrow dy = \_\_$

Step:04 put the value of r and dy in the equation.

Step:05 Get a standard form.

$$dr + r.\_\_ P dx = \_\_ Q dx. \quad \text{— (2)}$$

Step:06 Find. $I.F. = e^{\int P dx}$.

Step:07 Multiply eq (2) with I.F.

solve it. by puting value of $r = \_\_$ in it

Step:08 Find sol^n. (Final). (in the form of c)