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Q2

$$Ax = y$$

$$\begin{bmatrix} 0 & 1 & 1 & 2 & -1 \\ 1 & 2 & 3 & 4 & -1 \\ 2 & 0 & 2 & 0 & 2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(i)
$$\left[\begin{array}{ccccc|c} 0 & 1 & 1 & 2 & -1 & 0 \\ 1 & 2 & 3 & 4 & -1 & 1 \\ 2 & 0 & 2 & 0 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 4 & 6 & -2 & 1 \\ 1 & 2 & 3 & 4 & -1 & 1 \\ 2 & 0 & 2 & 0 & 2 & 2 \end{array}\right] \quad R_2 + R_1$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 4 & 6 & -2 & 1 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & -6 & -6 & -12 & 6 & 0 \end{array}\right] \quad -1R_1+R_2,\ -2R_1+R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 4 & 6 & -2 & 1 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad -6R_2 + R_3$$

$\therefore S(A) = 2$

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(ii) Nullity (y) = no. of columns - $\rho(A)$

$= 5 - 2$

$= 3$

(iii) $x_1 + 3x_2 + 4x_3 + 6x_4 - 2x_5 = 0$

$-x_2 - x_3 - 2x_4 + x_5 = 0$

There are 5 variables and 2 eq^n.

So we will assume 3 variables

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow$ If $x_1 = 1$, $x_2 = 0$, $x_3 = 0$

we will get

$x_1 + 6x_4 - 2x_5 = 0$ | $1 + 6x_4 - 2x_5 = 0$

$(-2x_4 + x_5 = 0)\ 3$ | $-6x_4 + 3x_5 = 0$

Adding both we get

$1 + x_5 = 0 \quad \& \quad x_4 = 0$

$$\therefore x_5 = -1$$

(iii) If $x_1 = 0$, $x_2 = 1$, $x_3 = 0$ we will get

$3x_2 + 6x_4 - 2x_5 = 0$
$-x_2 - 2x_4 + x_5 = 0$

$3 + 6x_4 - 2x_5 = 0$
$(-1 - 2x_4 + x_5 = 0) \times 3$
$-3 - 6x_4 + 3x_5 = 0$

$\therefore 3 + 6x_4 = 0$

$$\therefore x_4 = \frac{-1}{2}$$

(iv) If $x_1 = 0$, $x_2 = 0$ & $x_3 = 1$ we will get

$4x_3 + 6x_4 - 2x_5 = 0$
$(-x_2 - 2x_4 + x_5 = 0) \times 3$

$$\therefore x_5 = -3$$

$$4 + 6x_4 + 6 = 0$$

$$\therefore x_4 = \frac{-10}{6} = \frac{-5}{3}$$

Set of solutions

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1/2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -5/3 \\ -3 \end{pmatrix}$$