COMPUTER ENGINEERING DEPARTMENT

ASSIGNMENT NO-07

Sub: Theory of Computer Science

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Tutorial 7

1. Give the context-free grammar which generates a string containing only a's
2. Give CFG for generating an alternating sequence of 0 and 1
3. Write a CFG to generate the language of all strings that have more a's than b's (not necessary only on more, but any number of more a's than b's)
4. Convert the following grammar in GNF

   $S \rightarrow AB$
   $A \rightarrow BSB \mid BB \mid b$
   $B \rightarrow a$

5. Convert the following grammar in CNF

   $S \rightarrow bA \mid aB$
   $A \rightarrow bAA \mid aS \mid a$
   $B \rightarrow aBB \mid bS \mid b$

Q.1 Give the context free grammar which generates a string containing only a's.

Ans:

Context Free Grammar is defined by 4 tuples as

$G = \{ V, \Sigma, S, P \}$.

where,

V = Set of Variables or Non-Terminal symbols

Σ = Set of Terminal symbols.

S = Start symbol.

P = Production Rule.

Now, CFG which generates a string containing only a's.

$G = \{ (S), (a), (S), (S \rightarrow aS \mid a) \}$

$S \rightarrow aS$

$\rightarrow aaS \quad (S \rightarrow aS)$

$\rightarrow aaa \quad (S \rightarrow a)$

Q.2. Give CFG for generating an alternating sequence of 0 and 1.

Ans:

$G = \{(S,A,B), (0,1), (S), (S \rightarrow 0A \mid 1B, A \rightarrow 1B \mid \epsilon, B \rightarrow 0A \mid \epsilon)\}$

$S \rightarrow 0A$		$S \rightarrow 1B$	
$\rightarrow 01B$	$(A \rightarrow 1B)$	$\rightarrow 10A$	$(B \rightarrow 0A)$
$\rightarrow 010A$	$(B \rightarrow 0A)$	$\rightarrow 101B$	$(A \rightarrow 1B)$
$\rightarrow 0101B$	$(A \rightarrow 1B)$	$\rightarrow 1010A$	$(B \rightarrow 0A)$
$\rightarrow 0101$	$(B \rightarrow \epsilon)$	$\rightarrow 1010$	$(A \rightarrow \epsilon)$

Q.3. Write a CFG to generate the language of all strings that have more a's than b's (not necessary only on more, but any number of more a's than b's).

Ans:

$G = \{(S, A), (a, b), (S), (S \to aSb \mid A,\ A \to aA \mid \epsilon)\}$

$S \to aSb$

$\to aAb \quad (S \to A)$

$\to aaAb \quad (A \to aA)$

$\to aa\epsilon b \quad (A \to \epsilon)$

Q.4. Convert the following grammar in GNF

$S \rightarrow AB$

$A \rightarrow BSB \mid BB \mid b$

$B \rightarrow a$

Ans:

Greibach Normal Form (GNF)

A CFG is in GNF if the productions are -

(i) $A \rightarrow b$

(ii) $A \rightarrow bC_1C_2 \ldots\ldots C_n$.

where $A, C_1, \ldots - C_n$ are Non-Terminals and b is a Terminal.

(i) Since S appears in RHS, we add a new state S' and $S' \rightarrow S$ is added to the production

P: $S' \rightarrow S$

$S \rightarrow AB$

$A \rightarrow BSB \mid BB \mid b$

$B \rightarrow a$

(ii) Remove the Null Productions.

(Note: No Null Productions)

P: $S' \rightarrow S$

$S \rightarrow AB$

$A \rightarrow BSB \mid BB \mid b$

$B \rightarrow a$

(iii) Remove the Unit Productions : $S' \rightarrow S$

After removing $S' \rightarrow S$: P: $S' \rightarrow AB$,

$S \rightarrow AB$,

$A \rightarrow BSB \mid BB \mid b$

$B \rightarrow a$

(iv) Now find out the productions that has more than TWO variables in RHS.

$A \rightarrow BSB$

After removing this, we get: P: $S' \rightarrow AB$

$S \rightarrow AB$

$A \rightarrow BZ \mid BB \mid b$

$B \rightarrow a$

$Z \rightarrow SB$

(v) Now the grammar is in CNF

P: $S' \rightarrow AB$

$S \rightarrow AB$

$A \rightarrow BZ \mid BB \mid b$

$B \rightarrow a$

$Z \rightarrow SB$

(vi) Change the names of the Non Terminal symbols into some A_i in ascending order of i.

$S \rightarrow AB$ Replace S with A_1

$A \rightarrow BZ \mid BB \mid b$ A with A_2

$B \rightarrow a$ B with A_3

$Z \rightarrow SB$ Z with A_4

We get:

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_4 \mid A_3 A_3 \mid b$

$A_3 \rightarrow a$

$A_4 \rightarrow A_1 A_3$

(VII) Alter the rules so that the non-Terminals are in ascending order., such that,
If the production is of the form $A_i \rightarrow A_j x$, then, $i < j$ and should never be $i \geq j$

$A_4 \rightarrow A_1 A_3$

$A_4 \rightarrow A_2 A_3 A_3$ ($\because A_1 \rightarrow A_2 A_3$)

$A_4 \rightarrow A_3 A_4 A_3 A_3 \mid A_3 A_3 A_3 A_3 \mid b A_3 A_3$

$A_4 \rightarrow a A_4 A_3 A_3 \mid a A_3 A_3 A_3 \mid b A_3 A_3$

(VIII) Now the grammar is

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_4 \mid A_3 A_3 \mid b$

$A_3 \rightarrow a$

$A_4 \rightarrow a A_4 A_3 A_3 \mid a A_3 A_3 A_3 \mid b A_3 A_3$

$A_1 \rightarrow b A_3$

$A_2 \rightarrow a A_4 \mid a A_3 \mid b$

$A_3 \rightarrow a$

$A_4 \rightarrow a A_4 A_3 A_3 \mid a A_3 A_3 A_3 \mid b A_3 A_3$

Which is the required Greibach Normal Form for the given CFG.

Q5. Convert the following grammar in CNF.

$S \rightarrow bA \mid aB$,

$A \rightarrow bAA \mid aS \mid a$,

$B \rightarrow aBB \mid bS \mid b$

Ans:

Chomsky Normal Form (CNF)

A CFG is in CNF if the productions are -

(i) $A \rightarrow a$

(ii) $A \rightarrow BC$

Where A, B and C are non-terminal and a is terminal.

(i) Since S appears in RHS, we add a new state S' and $S' \rightarrow S$ is added to the production.

P: $S' \rightarrow S$,

$S \rightarrow bA \mid aB$,

$A \rightarrow bAA \mid aS \mid a$,

$B \rightarrow aBB \mid bS \mid b$

(ii) Remove the Null Productions.

(Note: No Null Productions)

P: $S' \rightarrow S$,

$S \rightarrow bA \mid aB$,

$A \rightarrow bAA \mid aS \mid a$,

$B \rightarrow aBB \mid bS \mid b$

(iii) Remove the Unit Productions : $S' \rightarrow S$

After removing $S' \rightarrow S$: P: $S' \rightarrow bA \mid aB$,

$S \rightarrow bA \mid aB$,

$A \rightarrow bAA \mid aS \mid a$,

$B \rightarrow aBB \mid bS \mid b$

(IV) Now find out the productions that has more than TWO variables in RHS.

$A \rightarrow bAA$ and $B \rightarrow aBB$.

After removing these, we get: P: $S' \rightarrow bA \mid aB$,
$S \rightarrow bA \mid aB$,
$A \rightarrow bX \mid aS \mid a$,
$B \rightarrow aY \mid bS \mid b$,
$X \rightarrow AA$,
$Y \rightarrow BB$

(V) Now change the productions

$S' \rightarrow bA$, $S' \rightarrow aB$, $S \rightarrow bA$, $S \rightarrow aB$,
$A \rightarrow bX$, $A \rightarrow aS$, $B \rightarrow aY$, $B \rightarrow bS$.

Finally we get:

P: $S' \rightarrow VA \mid UB$,
$S \rightarrow VA \mid UB$,
$A \rightarrow VX \mid US \mid a$,
$B \rightarrow UY \mid VS \mid b$,
$X \rightarrow AA$,
$Y \rightarrow BB$,
$U \rightarrow a$,
$V \rightarrow b$.

Which is the required Chomsky Normal Form for the given CFG.