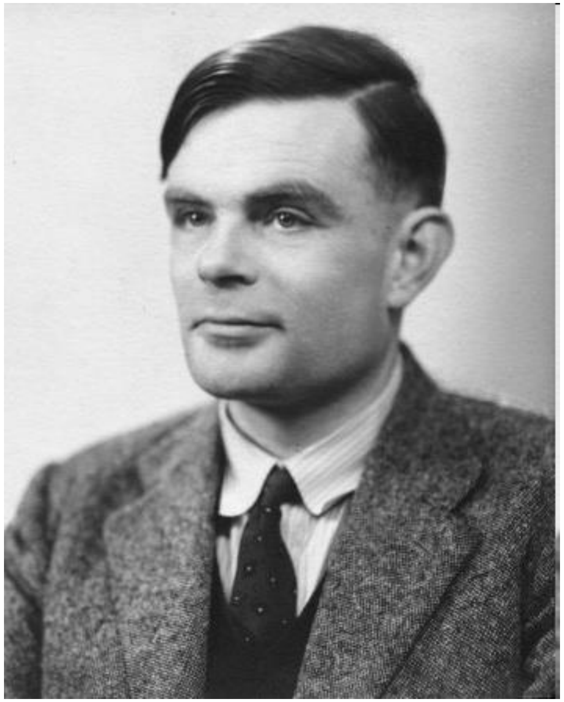
Theory of Computer Science

Alan Turing (1912-1954)

(A pioneer of automata theory)

- Father of Modern Computer Science
- English mathematician
- Studied abstract machines called ***Turing machines*** even before computers existed
- Heard of the Turing test?



Introduction

- One of the most fundamental courses on Computer Science
- Help you understand how people have thought about computer Science as a Science, past 50 years
- About what kind of things you can really compute mechanically
- How Fast? How much Space does it take?

Design a Machine

To accept all strings that end in 0

- 1010000
- 01000101

Design a Machine

To accept all valid Java Code

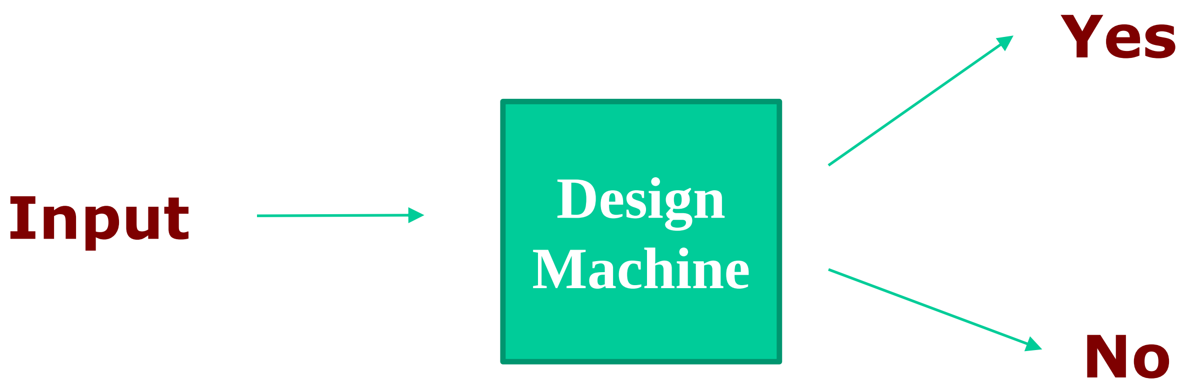
- Is it Possible?

Design a Machine

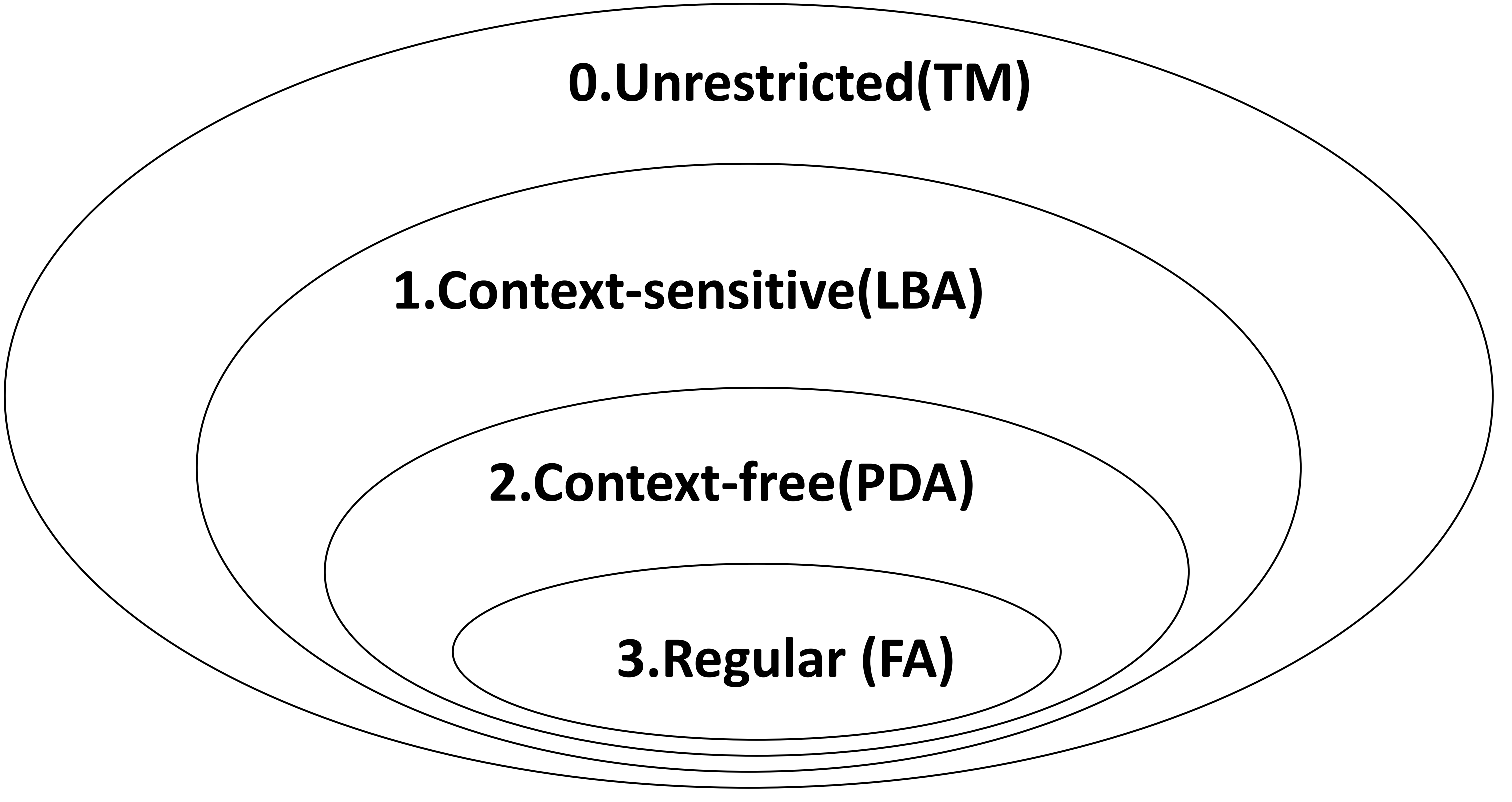
To accept all valid Java Codes that never goes into infinite

- Is it Possible?

What will we do in this Subject?



The Chomsky Hierarchy



SETS

SET

A **'Set'** is any well defined collection of objects called the elements or members of the set.

Following are some examples of sets:

- The collection of all planets in our universe.
- The collection of all states in India.
- The collection of vowels in English alphabets, i.e.
  A = {a, e, i, o, u}.
- The collection of first five natural numbers.
  A = {1,2,3,4,5}

$x \in S$ means "x is an element of set S."
$x \notin S$ means "x is not an element of set S."

Note:

- {1, 2, 3} is the set containing “1” and “2” and “3.”

- {1, 1, 2, 3, 3} = {1, 2, 3} since repetition is irrelevant.
  {6, 11} = {11, 6} = {11, 6, 6, 11}

- {1, 2, 3} = {3, 2, 1} since sets are unordered.

- {1, 2, 3, ..., 100},where the ellipsis ("...") indicates that the list continues in the obvious way.
  Ellipsis may also be used where sets have infinitely many members. Thus, the set of positive even numbers can be written as {2, 4, 6, 8, ... }.

- {0,1, 2, 3, …} is a way we denote an infinite set (in this case, the natural numbers).

Some Important Sets:

- N : *For the set of natural numbers*
- Z or I : *For the set of integers*
- Z^+ or I^+ : *For the set of all positive integers*
- Q : *For the set of all rational numbers*
- Q^+ : *For the set of all positive rational numbers*
- R : *For the set of all real numbers*
- R^+ : *For the set of all positive real numbers*
- C : *For the set of all complex numbers*

Representation of a Set

A set is often represented in the following two ways:

(I) Roster form (Tabular form)

-set is described by listing elements separated by commas, within braces { }.

Eg: the set of even natural numbers {2, 4, 6, 8, ...}.

(II)Set Builder form(Rule form)

- set is described by a characterizing property P(x) of its element x.

{x : P(x) holds} or {x / P(x) holds}

The symbol '|' or ':' is read as 'such that'.

Eg: 1) Set of vowels of English

Roster form :- A= {a,e,i,o,u}

Set Builder Form:- A= {x: x is an English vowel}

2) Set of first ten natural numbers

Roster form :- A= {1,2,3,4,5,6,7,8,9,10}

Set Builder Form:- A= $\{x: x \in N \text{ and } 1 \leq x \leq 10\}$

3) $\{x: x$ is an integer and $4 < x \leq 8\} = \{5,6,7,8\}$

4) {x: x is a month with exactly 30 days}={April,June,September,November}

Types of Sets

Empty set:

A set containing no element is called an empty set. It is also known as null set or void set. It is denoted by {},Φ

Eg:

(a) $A = \left\{ x \in R \,/\, x^2 = -10 \right\} = \phi$

(b) $B = \text{set of immortal man} = \phi$

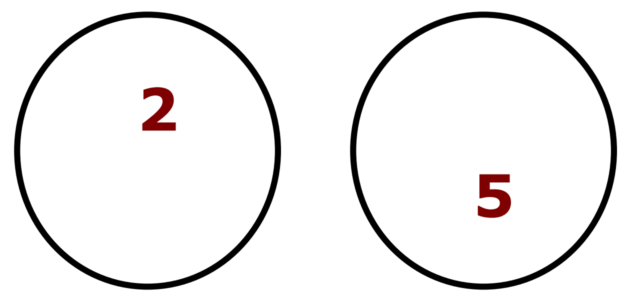
Singleton set:

A set having exactly one element is called singleton set.

For example,

{2}, {0}, {5} are singleton sets.

A={x:x is a solution of the equation 2x-5=0}



Finite set:

A set consisting of a finite number of elements is called a finite set.

Eg:

- {1, 2, 4, 6} is a finite set because it has four elements.
- The set of months is a finite set because it has 12 elements.

Infinite set:

A set which is not a finite set, i.e., a set consisting of infinite number of elements is called an infinite set.

Eg:

(i) The set of all straight line in a given plane.
(ii) The set of all natural numbers.
(iii) The set of real numbers between ‘1’ and ‘2’.

Cardinal number of a set

The number of elements in a finite set is called cardinal number or order of the set. It is referred as the cardinality of the set A and is denoted by n(A),|A| or #(A). For example, if

$$A = \{1, 2, 3, 4, 5\} \Rightarrow n(A) = 5 \text{ or } o(A) = 5.$$

If $S = \{3,3,3,3,3\}$, $|S| = 1$

If $S = \varnothing$ then $|S| = 0$.

Set of sets:

A set S having all its elements as sets is called set of sets.

Eg:

S = { {1, 2}, {2, 4}, {3, 5, 9} }

But S = { {1, 2}, 4, {3, 5, 9} } is not a set of sets

as element 4 ϵ S is not a set.

Equivalent and Equal Sets

Equivalent sets:

Two finite sets A and B are equivalent if their cardinal number is same, i.e. n(A) = n(B).

Equal sets:

Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

For example:

A = {4, 5, 6} and
B = {a, b, c} are equivalent ie. A Ξ B

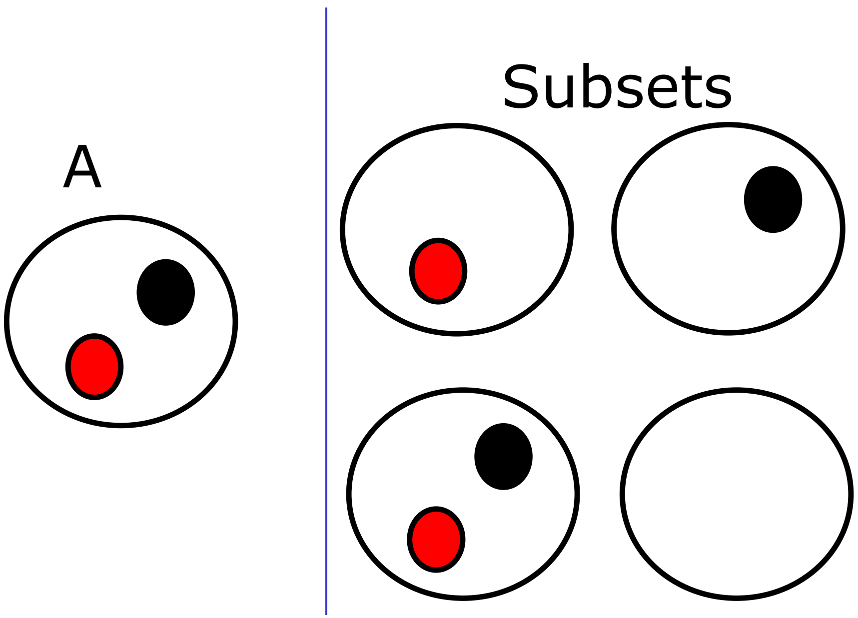
but

A = {4, 5, 6} and
C = {6, 5, 4} are equal, ie. A = C.

Subsets:

A set A is said to be a subset of a set B, written as $A \subseteq B$, if each element of A is also an element of B.

$A \subseteq B$, if $x \in A \Rightarrow x \in B$



Eg:

Let A = {2, 4, 6, 8},

B = {2, 4, 6, 8, 10, 12},

then $A \subseteq B$.

B $\supseteq$ A: B is a superset of A

B contains A

Proper subset: A set A is said to be a proper subset of a set B if every element of A is an element of B and B has at least one element which is not an element of A.

$A \subset B$ means "A is a proper subset of B."

$A \subseteq B$, and $A \neq B$.

A subset which is not proper is called improper subset.

For example:

Let A = {1, 2, 3}, B = {2, 3, 4, 1, 5}, then $A \subset B$

Thus, if A is a proper subset of B, then there exists an element $x \in B$ such that $x \notin A$.

For example, $\{1\} \subset \{1, 2, 3\}$ but $\{1, 4\} \not\subset \{1, 2, 3\}$.

$\{1,2,3\} \subseteq \{1,2,3,4,5\}$

$\{1,2,3\} \subset \{1,2,3,4,5\}$

Some Results on Subsets

(i) Every set is a subset of itself.

(ii) The empty set ϕ is a subset of every set.

(iii) The total number of subsets of a finite set containing n elements is 2^n.

Proof : We know that ${}^nC_r$ denotes the number of ways for choosing r things from n different things. Therefore each selection of r things gives a subset of the set A containing r elements.

$\therefore$ The number of subsets of A having no element ${}^nC_0$

The number of subsets of A having one element ${}^nC_1$

The number of subsets of A having two elements = ${}^nC_2$

- -

- -

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The number of subsets of A having n elements = ${}^nC_n$.

Hence, the total number of subsets of A

$$= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$$

[$\because$ We know that $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + \ldots + {}^nC_nx^n$.

Putting x = 1, we get $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$.]

Power Set

The set of all the subsets of a given set A is said to be the **power set** of A and is denoted by P(A).

i.e. $P(A)=\{S|S\subseteq A\}\Rightarrow S\in P(A)\Leftrightarrow S\subseteq A$

Also, $\phi\in P(A)$ and $A\in P(A)$ for all sets A.

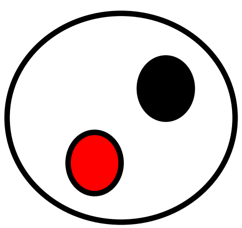
The power set of a finite set with *n* elements has 2^n elements.

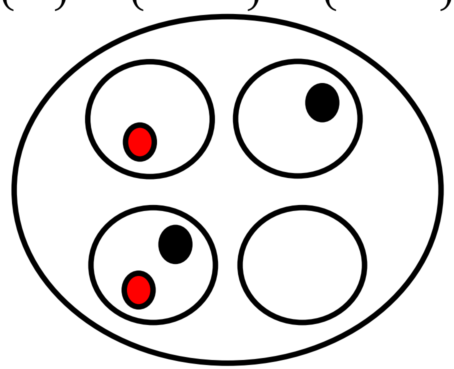
For example,

if A = {a, b, c}, then

$P(A)=\{\phi,\ \{a\},\ \{b\},\{c\},\ \{a,b\},\ \{a,c\},\{b,c\},\{a,b,c\}\}.$

A





P(A)

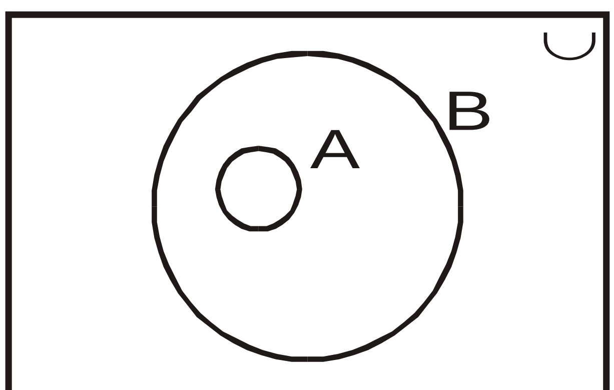
Universal Set

Any set which is super set of all the sets under consideration is called the **universal set** and is denoted by Ω or $\cup$.

For example:

(i) When we are using sets containing natural numbers then N is the universal set.

Euler-Venn Diagram- **a pictorial representation of sets**



$A \subset B$

Set Operations

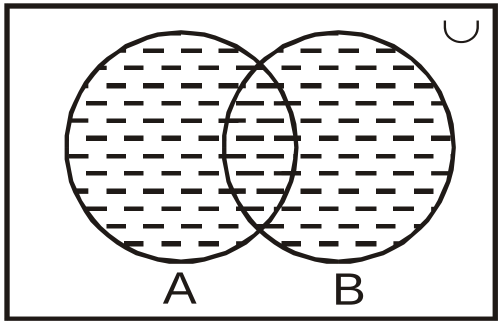
Union of sets:

The *union* of *A* and *B*, denoted by $A \cup B$, is the set of all things which are members of either *A* or *B*.

$$\therefore\ A \bigcup B = \{x : x \in A \ \text{ or } \ x \in B\}.$$

Clearly, if $x \in A \bigcup B \Leftrightarrow x \in A \ \text{ or } \ x \in B$

and $x \notin A \bigcup B \Leftrightarrow x \notin A \ \text{ and } \ x \notin B$



$A \cup B$

- **{1, 2} ∪ {red, white} ={1, 2, red, white}.**
- **If A = {Charlie, Lucy, Linus}, and B = {Lucy, Desi}, then**

  A ∪ B = {Charlie, Lucy, Linus, Desi}

Generalised definition

If $A_1, A_2, \ldots A_n$ is a finite family of sets, then their union is

$$A_1 \bigcup A_2 \bigcup A_3 \bigcup \ldots \bigcup A_n \ \text{ or } \ \bigcup_{i=1}^{n} A_n.$$

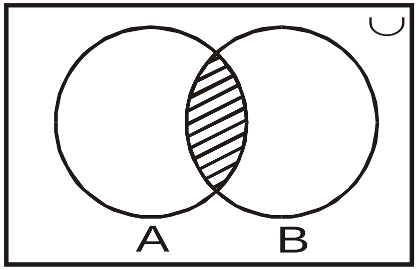
Intersection of sets:

The *intersection* of *A* and *B*, denoted by *A* ∩ *B*, is the set of all things which are members of both *A* and *B*.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

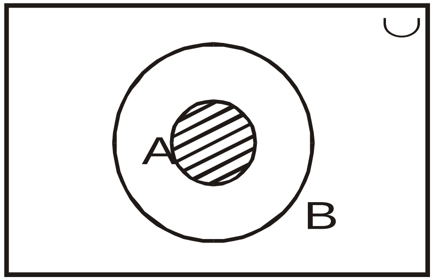
Clearly, if $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

and $x \notin A \cap B \Leftrightarrow x \notin A$ or $x \notin B$.



$A \cap B$

- If A = {Charlie, Lucy, Linus}, and
  B = {Lucy, Desi}, then
  A ∩ B = {Lucy}



$A \cap B$

- {1, 2} ∩ {red, white} = ∅.
- {1, 2, green} ∩ {red, white, green} = {green}.
- {1, 2} ∩ {1, 2} = {1, 2}.

Generalised definition

The intersection of sets $A_1, A_2, \ldots A_n$ is the set of all the elements which are common to all the sets $A_1, A_2, \ldots A_n$. It is denoted by

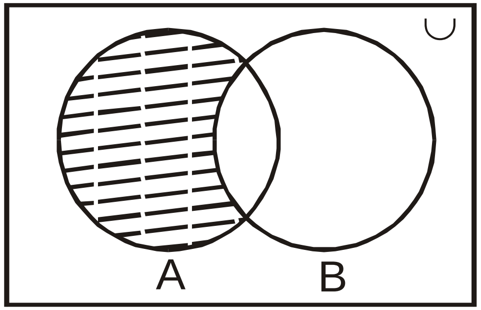
$$A_1 \cap A_2 \cap A_3 \ldots \cap A_n \text{ or } \bigcap_{i=1}^{n} A_i$$

Difference of sets

The difference of two sets A and B, denoted by A-B or A \ B, is a set which contains those elements of A which are not in B.

$$A - B = \{x : x \in A \ \text{ and } \ x \notin B\}$$

$$B - A = \{x : x \in B \ \text{ and } \ x \notin A\}$$



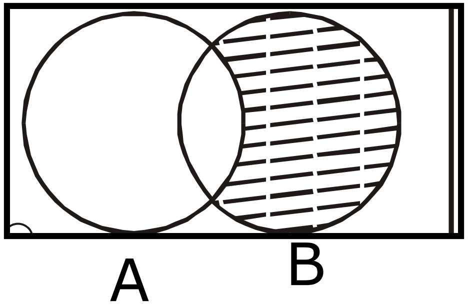
A-B

Eg:

If A={1,2,3,4} and B={3,4,5,6}

then A-B={1,2}

B-A={5,6}

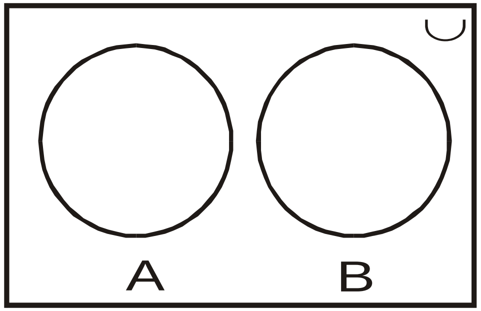


B-A

Disjoint sets:

Sets whose intersection is empty are called *disjoint* sets

$$A \bigcap B = \phi$$



$A \cap B$

Eg:

Let A={a,b,c,d,e} and B= {x,y,z}

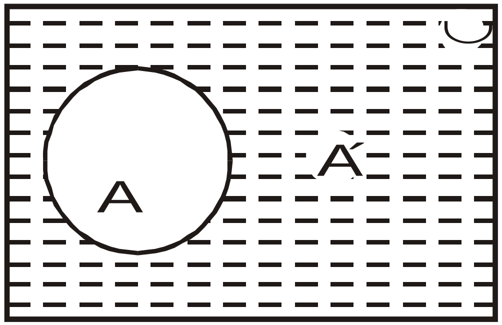
Then, clearly $A \bigcap B = \phi$

Complement of set (A′, A^c, $\overline{A}$)

$$\overline{A} = \{x : x \notin A\}$$

A′=U-A= $\{x : x \in U \text{ and } x \notin A\}$

$$\boxed{x \in A' \Leftrightarrow x \notin A}$$



A′

Eg: If U={1,2,3,4,5} and
A={12,3}

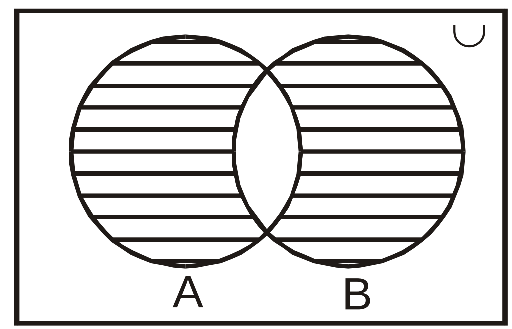
Then, A′=U-A = {4,5}

Symmetric Difference of sets

The ***symmetric difference of sets A and B***, denoted by $A \oplus B$ *or* $A \ \Delta\ B$, consists of those elements which belong to either A or B but not both. ie.

$$A \oplus B = \{\, x : (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$$

$$= (A - B) \cup (B - A)$$



$A \oplus B$

$A \ \Delta\ B = (A \cup B) - (A \cap B)$

$x \in A \ \Delta\ B \Rightarrow x \notin A \cap B$

Eg:

If A={1,2,3,4} and B={3,4,5,6}

then A-B={1,2}

B-A={5,6}

$A \oplus B = (A - B) \cup (B - A)$

$= \{1,2,5,6\}$

Cartesian Product

The *Cartesian product* of two sets A and B, denoted by $A \times B$ is the set of all ordered pairs (a, b) such that a is a member of A and b is a member of B.

$$A \times B = \{ (a, b) : a \in A \text{ and } b \in B\}$$

Let A and B be finite sets, then

| A × B | = | B × A | = | A | × | B |.

$$\boldsymbol{A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n): a_1 \in A_1, a_2 \in A_2, \ldots, a_n \in A_n\}}$$

Eg:

- **{**1, 2} × {red, white} = {(1, red), (1, white), (2, red), (2, white)}.
- {1, 2} × {a, b} = {(1, a), (1, b), (2, a), (2, b)}.

Binary and unary operation

A **unary operation** is an operation with only one operand, i.e. a single input. Specifically, it is a function f :A→ A where *A* is a set .

In this case *f* is called a unary operation on *A*.

Common notations are

prefix (e.g. +, −, not), postfix (e.g. n!), functional notation (e.g. sin x or sin (x)), and superscripts (e.g. complement A, transpose A^T).

A **binary operation** is a calculation involving two operands.

Let *A*, *B* and *C* be three sets .

Then a relation * from $A \times B \to C$ is a binary relation. a*b=c

Typical examples of binary operations are the addition (+) and multiplication (×) of numbers and matrices as well as composition of functions on a single set.

<u>In sets:</u>

unary operation: complement

binary operation : union, intersection, difference, symmetric difference.

Remark:

(i) $U' = \{x : x \in U \text{ and } x \notin U\} = \phi.$

(ii) $\phi' = \{x : x \in U \text{ and } x \notin \phi\} = U.$

(iii) $(A')' = \{x : x \in U \text{ and } x \notin A'\}$

$$= \{x : x \in U \text{ and } x \in A\} = A$$

(iv) $A \cup A' = \{x : x \in U \text{ and } x \in A\} \cup \{x : x \in U \text{ and } x \notin A\} = U$

(v) $A \cap A' = \{x : x \in U \text{ and } x \in A\} \cap \{x : x \in U \text{ and } x \notin A\} = \phi$

Procedure for Proving Equality of Sets

As we have discussed earlier that two sets A and B are said to be equal if every element of set A is an element of set B and every element of B is an element of A.

It is clear that $$A = B \Leftrightarrow A \subseteq B \text{ and } B \subseteq A.$$

i.e. $$A = B \Leftrightarrow [x \in A \Leftrightarrow x \in B].$$

Algebra of Sets

Idempotent laws

For any set A, we have

(i) $A \cup A = A$

(ii) $A \cap A = A$

Identity laws

For any set A, we have

(i) $A \cup \phi = A$

(ii) $A \cap U = A$

Involution Laws

$(A')' = A$

Complement Laws

(i) $A \cup A' = U$

(ii) $A \cap A' = \Phi$

(iii) $U' = \Phi$

(iv) $\Phi' = U$

Commutative laws

For any two sets A and B, we have

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

i.e. union and intersection are commutative.

Proof: As we know that two sets X and Y are equal if

$X \subseteq Y$ and $Y \subseteq X$.

(i) Let x be any arbitrary element of $A \cup B$

$$\Rightarrow x \in A \cup B \Rightarrow x \in A \text{ or } x \in B$$
$$\Rightarrow x \in B \text{ or } x \in A$$
$$\Rightarrow x \in B \cup A \qquad \therefore A \cup B \subseteq B \cup A \quad \text{...(i)}$$

(ii) Similarly, let y be any arbitrary element of $B \cup A$

$$\Rightarrow y \in B \cup A \Rightarrow y \in B \text{ or } y \in A$$
$$\Rightarrow y \in A \text{ or } y \in B$$
$$\Rightarrow y \in A \cup B \qquad \therefore B \cup A \subseteq A \cup B \quad \text{...(ii)}$$

From (i) and (ii)

$$\boxed{A \cup B = B \cup A}$$

Associative laws

If A, B and C are any three sets, then

(i) $(A \cup B) \cup C = A \cup (B \cup C)$

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

i.e. union and intersection are associative.

Proof:

(i) Let x be any arbitrary element of $(A \cup B) \cup C$.

$$\begin{aligned}
\therefore\ x \in (A \cup B) \cup C \ &\Rightarrow\ x \in (A \cup B) \text{ or } x \in C \\
&\Rightarrow\ (x \in A \text{ or } x \in B) \text{ or } x \in C \\
&\Rightarrow\ x \in A \text{ or } (x \in B \text{ or } x \in C) \\
&\Rightarrow\ x \in A \text{ or } (x \in B \cup C) \\
&\Rightarrow\ x \in A \cup (B \cup C)
\end{aligned}$$

$$\therefore\ (A \cup B) \cup C \subseteq A \cup (B \cup C) \qquad \text{...(i)}$$

(ii) Similarly, let y be any arbitrary element of $A \bigcup (B \bigcup C)$

$$
\begin{aligned}
\therefore\ y \in A \bigcup (B \bigcup C) &\Rightarrow y \in A \text{ or } y \in (B \bigcup C)\\
&\Rightarrow y \in A \text{ or } (y \in B \text{ or } y \in C)\\
&\Rightarrow (y \in A \text{ or } y \in B) \text{ or } y \in C\\
&\Rightarrow (y \in A \bigcup B) \text{ or } y \in C\\
&\Rightarrow y \in (A \bigcup B) \bigcup C
\end{aligned}
$$

$$\therefore\ A \bigcup (B \bigcup C) \subseteq (A \bigcup B) \bigcup C \qquad \ldots\text{(ii)}$$

From (i) and (ii)

$$(A \bigcup B) \bigcup C = A \bigcup (B \bigcup C)$$

Proved.

Distributive laws

If A, B and C are any three sets, then

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ie, union and intersection are distributive over intersection and union respectively.

Proof:

(i) Let x be any arbitrary element of $A \cup (B \cap C)$

$\therefore\ x \in A \cup (B \cap C) \Rightarrow x \in A \text{ or } x \in (B \cap C)$

$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C)$

$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$

[$\because$ 'or' is distributive over 'and']

$\Rightarrow x \in (A \cup B) \text{ and } x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\therefore\ A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ ...(i)

(ii) Similarly, let y be any arbitrary element of $(A \cup B) \cap (A \cup C)$.

$$\therefore\ y \in (A \cup B) \cap (A \cup C) \Rightarrow y \in (A \cup B) \text{ and } y \in (A \cup C)$$

$$\Rightarrow (y \in A \text{ or } y \in B) \text{ and } (y \in A \text{ or } y \in C)$$

$$\Rightarrow y \in A \text{ or } (y \in B \text{ and } y \in C)$$

$$\Rightarrow y \in A \text{ or } (y \in B \cap C)$$

$$\Rightarrow y \in A \cup (B \cap C)$$

$$\therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \qquad ...(ii)$$

From (i) and (ii),

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Proved.

De Morgan's Law

If A and B are any two sets, then

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Proof:

(i) Let x be an arbitrary element of the set $(A \cup B)'$.

$$\therefore\ x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$$
$$\Rightarrow x \notin A \text{ and } x \notin B$$
$$\Rightarrow x \in A' \text{ and } x \in B'$$
$$\Rightarrow x \in A' \cap B'$$

$\therefore\ (A \cup B)' \subseteq A' \cap B'$ ...(i)

(ii)Now, let y be an arbitrary element of $A' \cap B'$

$$\therefore\ y \in A' \cap B' \Rightarrow y \in A' \text{ and } y \in B'$$
$$\Rightarrow y \notin A \text{ and } y \notin B$$
$$\Rightarrow y \notin A \cup B$$
$$\Rightarrow y \in (A \cup B)'$$

$\therefore\ A' \cap B' \subseteq (A \cup B)'$ ...(ii)

From (i) and (ii),

$$(A \cup B)' = A' \cap B'$$

Q: For any two sets A and B, prove that

$$A \bigcup B = A \bigcap B \Leftrightarrow A = B.$$

First, let A = B.

Then $A \bigcup B = A$ and $A \bigcap B = A$

$$\Rightarrow A \bigcup B = A \bigcap B$$

$$\therefore A = B \Rightarrow A \bigcup B = A \bigcap B \quad ...(i)$$

Conversely, let $A \bigcup B = A \bigcap B$.

$\therefore x \in A \Rightarrow x \in A \bigcup B$

$\Rightarrow x \in A \bigcap B$

$\Rightarrow x \in B$

$\Rightarrow x \in A$ and $x \in B \quad \therefore A \subseteq B \quad ...(ii)$

Now let $y \in B \Rightarrow y \in A \bigcup B$

$\Rightarrow y \in A \bigcap B$

$\Rightarrow y \in A$

$\Rightarrow y \in A$ and $y \in B \quad \therefore B \subseteq A \quad ...(iii)$

From (ii) and (iii), we get A = B

$$\text{Thus, } A \bigcup B = A \bigcap B \Rightarrow A = B \quad ...(iv)$$

From (i) and (iv), $A \bigcup B = A \bigcap B \Leftrightarrow A = B$

Q: If $a \in N$ such that $aN = \{ax : x \in N\}$ describe the set $3N \cap 7N$.

Sol: We have $aN = \{ax : x \in N\}$

$\therefore\ 3N = \{3x : x \in N\} = \{3, 6, 9, 12, 15, \ldots\}$

$7N = \{7x : x \in N\} = \{7, 14, 21, 28, \ldots\}$

Hence, $3N \cap 7N = \{21, 42, 63, \ldots\} = \{21x : x \in N\} = 21N$

Note that $aN \cap bN = cN$ where c = LCM of a, b.

Q:If A, B and C are any three sets, then prove that

$$A-(B\cap C)=(A-B)\cup(A-C).$$

Let x be any element of $A-(B\cap C)$

$$\therefore\ x\in A-(B\cap C)\Rightarrow x\in A \text{ and } x\notin(B\cap C)$$
$$\Rightarrow\ x\in A \text{ and } (x\notin B \text{ or } x\notin C)$$
$$\Rightarrow\ (x\in A \text{ and } x\notin B) \text{ or } (x\in A \text{ and } x\notin C)$$
$$\Rightarrow\ x\in(A-B) \text{ or } x\in(A-C)$$
$$\Rightarrow\ x\in(A-B)\cup(A-C)$$
$$\therefore\ A-(B\cap C)\subseteq(A-B)\cup(A-C) \qquad ...(i)$$

Again y be any element of $(A-B)\cup(A-C)$.

$$\therefore\ y\in(A-B)\cup(A-C)\Rightarrow y\in(A-B) \text{ or } y\in(A-C)$$
$$\Rightarrow\ (y\in A \text{ and } y\notin B) \text{ or } (y\in A \text{ and } y\notin C)$$
$$\Rightarrow\ y\in A \text{ and } (y\notin B \text{ or } y\notin C)$$
$$\Rightarrow\ y\in A \text{ and } (y\notin(B\cap C))$$
$$\Rightarrow\ y\in A-(B\cap C)$$
$$\therefore\ (A-B)\cup(A-C)\subseteq A-(B\cap C) \qquad ...(ii)$$

From (i) and (ii),

$$A-(B\cap C)=(A-B)\cup(A-C)$$ Proved.

Q: Let A, B and C be three sets such that $A \bigcup B = C$ and $A \bigcap B = \phi$. then prove that $A = C - B$

$\because$ We have $A \bigcup B = C$.

$$\therefore\ C - B = (A \bigcup B) - B$$

$$= (A \bigcup B) \bigcap B' \qquad [\because\ X - Y = X \bigcap Y']$$

$$= (A \bigcap B') \bigcup (B \bigcap B') \qquad \text{[By distributive law]}$$

$$= (A \bigcap B') \bigcup \phi$$

$$= A \bigcap B'$$

$$= A - B$$

$$= A \qquad [\because\ A \bigcap B = \phi]$$

Proved.

If A, B and C are the sets such that $A \subset B$, then prove that $C - B \subset C - A$.

Let x be any arbitrary element of C – B.

$$\therefore \quad x \in C - B \Rightarrow x \in C \text{ and } x \notin B$$

$$\Rightarrow x \in C \text{ and } x \notin A \qquad [\because A \subset B]$$

$$\Rightarrow x \in C - A$$

$$\therefore \quad C - B \subset C - A \qquad \text{Proved.}$$