

COMPUTER ENGINEERING DEPARTMENT

ASSIGNMENT NO-08

Sub: Theory of Computer Science

COURSE: T.E.

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Tutorial 8

1. Design PDA for

1] $L = \{a^{n-1} b^{2n+1} \mid n \geq 1\}$

2] $L = \{a^n b^m a^n \mid m, n \geq 1\}$

2.

Design a PDA to accept each of the following languages

i. $\{0^n 1^m 0^n \mid m, n \geq 1\}$

ii. $\{0^n 1^m 0^m 0^n \mid m, n \geq 1\}$

3. I] Design PDA to check even parenthesis over $\Sigma = \{0, 1\}$

II] Design DPDA for $L = \{x \mid N_a(x) > N_b(x)\}$

4. Generate Rule for the grammar

$$E \rightarrow E+E \mid E-E \mid (E) \mid id$$

5. Give the application of PDA

Q.1 Design PDA for

$$\text{① } L = \{ a^{n-1} b^{2n+1} \mid n \geq 1 \}$$

Ans:

$$G = \{ \text{bbb, abbabb, aabbabbb, ...} \}$$

Logic: For each a push two x into stack

For the first three b no operation (by pass)

For each b pop one x from stack

Machine is formally defined as

$$M = \{ Q, \Sigma, \delta, T, q_0, z_0, F \}$$

where,

$$Q = \{ q_0, q_1, q_F \}$$

$$\Sigma = \{ a, b \}$$

$z_0 \rightarrow$ stack symbol

$q_0 \rightarrow$ Initial state

$F \rightarrow q_F$

$$\text{Rules: } \delta(q_0, a, z_0) = (q_0, axz_0)$$

$$\delta(q_0, a, x) = (q_0, axxz_0)$$

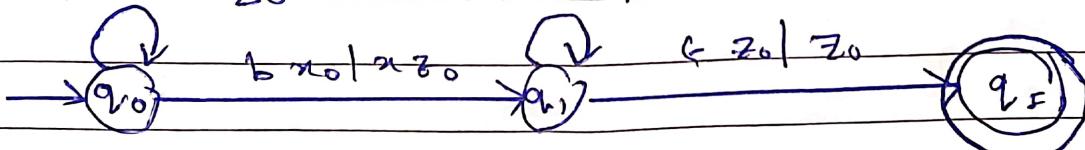
$$\delta(q_0, b, x) = (q_1, xz_0)$$

$$\delta(q_1, b, x) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_F, z_0)$$

$$\begin{array}{l} axz_0 | axxz_0 \\ a z_0 | axx z_0 \end{array}$$

$$bx | \epsilon$$



Simulation:

S($q_0, aaabbbaabbba, z_0$)

T($q_0, aabbbaabbba, \times \times z_0$)

T($q_0, abbbbbbba, \times \times \times z_0$)

T($q_1, bbbbbbba, \times \times \times \times z_0$)

T($q_1, bbbbbbba, \times \times \times \times \times z_0$)

T($q_1, bbbbbbba, \times \times \times \times \times \cancel{z_0}$)

T($q_1, bbbbbbba, \times \times \times \times \times z_0$)

T($q_1, bbbbbb, \times \times \times \times z_0$)

T($q_1, bbb, \times \times \times z_0$)

L($q_1, bb, \times \times z_0$)

T($q_1, b, \times z_0$)

L(q_1, ϵ, z_0)

T(q_F, z_0)

Accept

$$\textcircled{11} \quad L = \{ a^n b^m a^n \mid m, n \geq 1 \}$$

Ans:

Logic:

- For each a push $1x$ into the stack
- For b bypass all b 's.
- For each a pop $1x$ from the stack.

Machine definition:

$$\text{Let } M = (Q, \Sigma, \Gamma, q_0, z_0, F)$$

where,

$$Q = \{q_0, q_1, q_2, q_f\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{\star, z_0\}$$

Rules:

$$\delta(q_0, a, z_0) = (q_0, xz_0)$$

$$\delta(q_0, a, x) = (q_0, xxz_0)$$

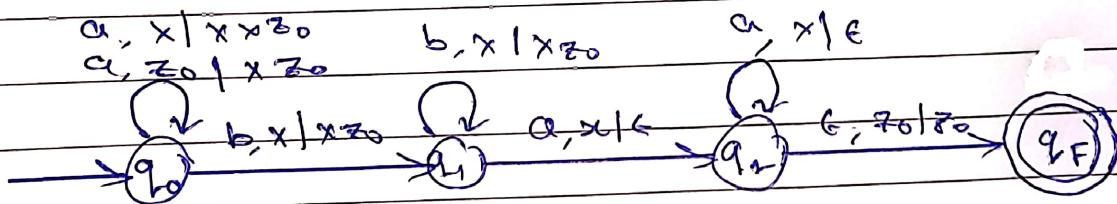
$$\delta(q_0, b, x) = (q_1, xz_0)$$

$$\delta(q_1, b, x) = (q_1, xz_0)$$

$$\delta(q_1, a, x) = (q_2, \epsilon)$$

$$\delta(q_2, a, x) = (q_2, \epsilon)$$

$$\delta(q_2, \epsilon, z_0) = (q_f, z_0)$$



Simulation: $\delta(q_0, aba, z_0)$

$$\vdash (q_0, ba, xz_0)$$

$$\vdash (q_1, a, xz_0)$$

$$\vdash (q_2, \epsilon, z_0)$$

$$\vdash (q_f, z_0)$$

Accepted

Q.2. Design a PDA to accept

$$\textcircled{1} \quad \{ 0^n 1^m 0^n \mid m, n \geq 1 \}$$

Ans:

Logic:

For each 0 push 1x into the stack

For 1 bypass all 1s.

For each 0 pop 1x from the stack

Machine Definition:

$$\text{Let } M = \{ Q, \Sigma, \Gamma, \delta, q_0, z_0, F \}$$

where,

$$Q = \{ q_0, q_1, q_2, q_F \}$$

$$\Sigma = \{ 0, 1 \}$$

$$\Gamma = \{ x, z_0 \}$$

Rule:

$$\delta(q_0, 0, z_0) = (q_0, xz_0)$$

$$\delta(q_0, 0, x) = (q_0, xxz_0)$$

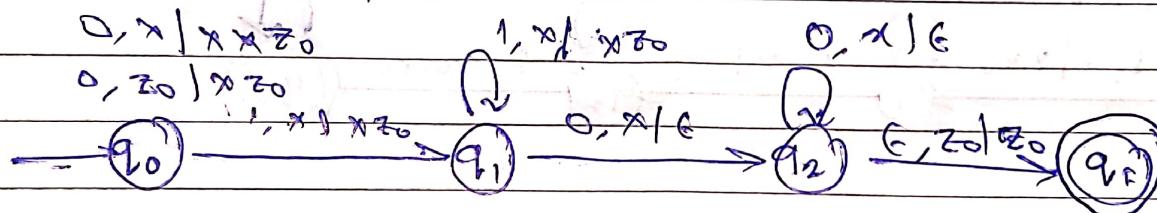
$$\delta(q_0, 1, x) = (q_1, xz_0)$$

$$\delta(q_1, 1, x) = (q_1, xz_0)$$

$$\delta(q_1, 0, x) = (q_2, \epsilon)$$

$$\delta(q_2, 0, x) = (q_2, \epsilon)$$

$$\delta(q_2, \epsilon, z_0) = (q_F, \epsilon)$$



Simulation: $\delta(q_0, 010, z_0)$

$$\vdash (q_0, 10, xz_0)$$

$$\vdash (q_1, 0, xz_0)$$

$$\vdash (q_1, \epsilon, z_0)$$

$$\vdash (q_F, z_0)$$

Accepted

11) $\{0^n 1^m 0^n \mid m, n \geq 1\}$

Ans:

Logic:

For each leading 0 push 1x into the stack

For each 1 push "1x" into the stack

x's on the stack are matched with trailing 0's in the input, pop them.

Machine Definition.

$$\text{Let } M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$$

where,

$$Q = \{q_0, q_1, q_2, q_f\}$$

$$\Sigma = \{0, 1\}$$

$$\Gamma = \{x, z_0\}$$

Rule:

$$\delta(q_0, 0, z_0) = (q_0, xz_0)$$

$$\delta(q_0, 0, x) = (q_0, xxz_0)$$

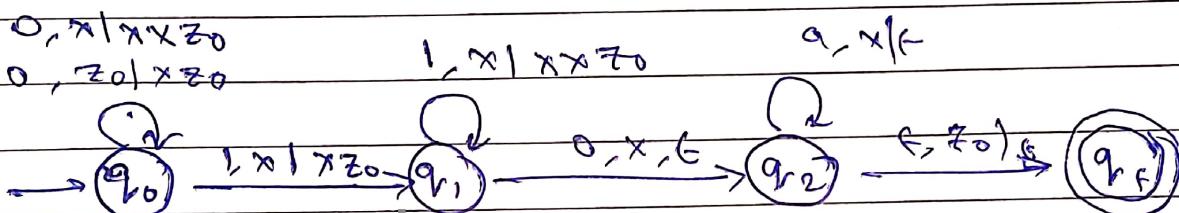
$$\delta(q_0, 1, x) = (q_1, xz_0)$$

$$\delta(q_1, 1, x) = (q_1, xxz_0)$$

$$\delta(q_1, 0, x) = (q_2, \epsilon)$$

$$\delta(q_2, 0, x) = (q_2, \epsilon)$$

$$\delta(q_2, \epsilon, z_0) = (q_f, \epsilon)$$



Simulation: $\delta(q_0, 0100, z_0)$

$$\vdash (q_0, 100, xz_0)$$

$$\vdash (q_1, 00, xxz_0)$$

$$\vdash (q_2, 0, xz_0)$$

$$\vdash (q_2, \epsilon, z_0)$$

$$\vdash (q_f, z_0)$$

Accepted

Q.3

① Design PDA to check even parentheses over $\Sigma = \{0, 1\}$

Ano:

Logic:

For input '(', push it into the stack

For every ')', pop from the stack

Machine definition

Let $m = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

where,

$$Q = \{q_0, q_f\}$$

$$\Sigma = \{(), [], []\}$$

$$\Gamma = \{z_0, (), []\}$$

Rule:

$$\delta(q_0, (, z_0) = (q_0 (z_0))$$

$$\delta(q_0, [, z_0) = (q_0 [z_0))$$

$$\delta(q_0, (, ()) = (q_0 (()))$$

$$\delta(q_0, [], []) = (q_0 [] []))$$

$$\delta(q_0, (, []) = (q_0 ([])$$

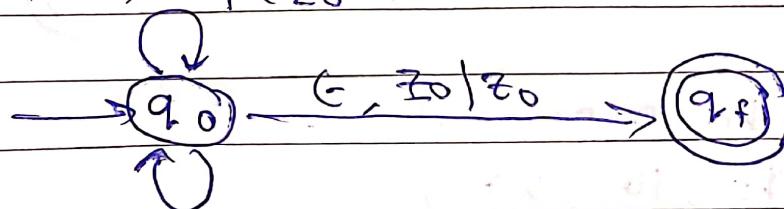
$$\delta(q_0, (), ()) = (q_0 (()))$$

$$\delta(q_0, [], ()) = (q_0 ([] ()))$$

$$\delta(q_0, (, ()) = (q_0 (()))$$

$$\delta(q_0, (, (()) = (q_0 (()))$$

$$\delta(q_0, (, (()) = (q_0 (()))$$



$C, [] \Sigma ($

$[], C, [] \Sigma ($

$), (\mid G$

$] - [\mid \epsilon$

$$Q_{\text{new}}(q^*) = (Q(q^*))_3$$
$$Q_{\text{new}}(q^*) = (Q(q^*))_6$$

Simulation:

$\delta(q_0, ([], z_0)) \leftarrow$
+ $(q_0, [], x(z_0))$
- $(q_0, [], z_0)$
+ $(q_0, (), z_0)$
+ $(q_0, (\ell), z_0)$
- (q_0, z_0)

Accepted

⑪ Design DPDA for $L = \{ x \mid N_a(x) > N_b(x) \}$

Answ.

Logic

For each 'a' push 1x into stack

For each 'b' pop 1x from stack

Pop x till stack is empty reading null string

Machine definition

$$\text{Let } M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$$

where,

$$Q = \{ q_0, q_1, q_2, q_f \}$$

$$\Sigma = \{ a, b \}$$

$$\Gamma = \{ x, z_0 \}$$

Rule:

$$\delta(q_0, a, z_0) = (q_1, xz_0)$$

$$\delta(q_0, a, x) = (q_0, xx)$$

$$\delta(q_0, b, x) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, x) = (q_2, \epsilon)$$

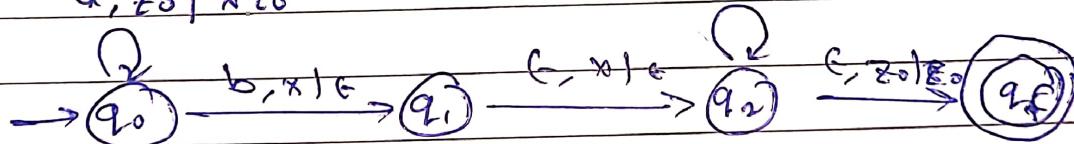
$$\delta(q_2, \epsilon, x) = (q_2, \epsilon)$$

$$\delta(q_2, \epsilon, z_0) = (q_f, z_0)$$

$a, x \mid xx$

$a, z_0 \mid xz_0$

$\epsilon, x \mid \epsilon$



Simulation: $\delta(q_0, aab, z_0)$

$\vdash (q_0, ab, xz_0)$

$\vdash (q_0, b, xxz_0)$

$\vdash (q_1, \epsilon, xz_0)$

$\vdash (q_2, \epsilon, z_0)$

$\vdash (q_f, z_0)$

Accepted

Q.4. Generate Rule for Grammar

$$E \rightarrow E + E \mid E - E \mid (E) \mid id$$

Ans:

Rules :

$$\delta(q_0, \epsilon, z_0) = \{(q_1, \Sigma, z_0)\}$$

$$\delta(q_1, +, E) = \{(q_1, E)\}$$

$$\delta(q_1, -, E) = \{(\epsilon q_1, E)\}$$

$$\delta(q_1, (, E) = \{(q_1, EA)\}$$

$$\delta(q_1, id, E) = \{(q_1, \epsilon)\}$$

$$\delta(q_1,), A) = \{(\epsilon q_1, \epsilon)\}$$

$$\delta(q_1, \epsilon, z_0) = \{(q_f, z_0)\}$$

Q.5. Give the application of PDA.

Ans:

Applications of PDA

- PDA is a machine for CFL.
- A string belonging to a CFL can be recognized by a PDA.
- PDA is extensively used for parsing.
- PDA is an abstract machine; it can also be used for giving proofs of lemma on CFL.