Variants of FCM for brain MRI segmentation

CS 736 Project

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Variants of FCM

In this project, we have implemented Fuzzy-C-Means and its 4 variants in reference to the research paper "*Fuzzy C-mean based brain MRI segmentation algorithms*" by *M. A. Balafar*.

Data used is the same as in Assignment 2 - *assignmentSegmentBrain.mat*

Variants implemented

1. FCM_S
2. FCM_S1
3. FCM_EN
4. FGFCM
5. FRFCM

Algorithms

FCM

Given

- Data = { y_j }, j = 1,...,N
- Number of clusters = K (known / fixed)

Memberships

- u_{jk} = membership (non-negative) of j-th point in k-th cluster
- For each datum, over all classes k, memberships u_{jk} sum to 1

Objective function to be minimized

- Penalize distance of datum j from mean of class k
- Weight penalty based on membership u_{jk}

$$\sum_{j=1}^{N} \sum_{k=1}^{K} u_{jk}^{q} (y_j - c_k)^2$$

q > 1: free parameter controling fuzziness of clusters/memberships

Constraints

$$\forall j, \sum_k u_{jk} = 1$$

- Positivity constraint on memberships gets satisfied automatically

FCM_S

- Modification to the standard FCM by incorporating neighborhood information in the objective function

$$J_q = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ij}^q d(x_i, \theta_j) + \frac{\alpha}{N_R} \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ij}^q \sum_{r \in N_i} d(x_r, \theta_j) \qquad \|x_r - v_i\|^2$$

where ***d*** is distance between data ***xi*** and centre of the cluster ***j***, ***θj*** and ***u*** is the fuzzy membership of data ***xi*** to cluster with centre ***θj*** . ***q*** specifies the degree of fuzziness in the clustering and ***α*** is the weight of neighborhood information. ***NR*** is the number of neighbors in a window around ***xi***, ***Ni*** is the set of neighbors and ***xr*** represents the neighbor of ***xi***

- Optimization done following the same method as described in class using method of Lagrange multipliers and performing alternate minimization.
- Shortcoming of FCM_S is that the neighbor term is computed in iteration which is time consuming

FCM_S1

- Proposed by Chen and Zhang
- Modification to the FCM_S algorithm's objective function to overcome FCM-S shortcoming

$$J_q = \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ij}^q d(x_i, \theta_j) + \alpha \sum_{i=1}^{n} \sum_{j=1}^{m} u_{ij}^q d(\bar{x}_i, \theta_j) \qquad \|\overline{x_k} - v_i\|^2$$

where $\bar{x}_i$ is the average of neighbors of pixel xi

- Optimization done following the same method as described in class using method of Lagrange multipliers and performing alternate minimization

FCM_EN

- Proposed by Szilágyi
- Modification to input data in order to speed up clustering process
- Linearly-weighted sum image of the original image and its average image used as the input image for clustering

$$S_i = \frac{1}{1+\beta}(X_i + \beta \bar{X}_i)$$

$$J_m = \sum_{i=1}^{c} \sum_{k=1}^{N} u_{ik}^{m} \|x_k - v_i\|^2$$

where ***β*** is the weight of neighborhood information

- Optimization done following the same method as described in class using method of Lagrange multipliers and performing alternate minimization

FGFCM

- Proposed by Cai et al.
- Improvement to FCM_EN in order to overcome shortcomings in adapting the common crucial parameter β.
- Input image used for clustering is generated as

$$\xi_i = \frac{\sum_{j \in N_i} S_{ij} x_j}{\sum_{j \in N_i} S_{ij}} \qquad S_{ir} = \begin{Bmatrix} S_{s\_ir} * S_{g\_ij}, & i \neq r \\ 0, & i = r \end{Bmatrix}$$

$$S_{s\_ir} = \exp\left(\frac{-\max(|x_i - x_r|, |y_i - y_r|)}{\beta_s} \right) \quad S_{g\_ir} = \exp\left(\frac{-||I_i - I_r||^2}{\beta_g * \sigma_i^2} \right) \quad \sigma_i = \sqrt{\frac{\sum_{r \in N_i} ||I_i - I_r||^2}{|N_i|}}$$

where ***rth*** pixel is a neighbor of pixel ***xi***; ***(xr, yr)*** is ***rth*** neighbor pixel, ***βs*** is the weight of spatial information; ***Ii*** is grey value of the pixel ***xi*** and ***Ir*** is that of ***rth*** neighbor

- Optimization done following the same method as described in class using method of Lagrange multipliers and performing alternate minimization

FRFCM based on morphological reconstruction

- Objective function is

$$J_m = \sum_{l=1}^{q} \sum_{k=1}^{c} \gamma_l u_{kl}^m \|\xi_l - v_k\|^2$$

- This is performed on gray level histogram to save computational time, hence gamma_l.
- u_kl is fuzzy membership of gray value l

$$\xi = R^C(f)$$

- Difference between this and others is that image is reconstructed by morphological reconstruction

FRFCM based on morphological reconstruction

- MR is able to preserve object contour and remove noise without knowing the noise type in advance.

$$R_f^{\delta}(g) = \delta_f^{(i)}(g),$$

$$R_f^{\varepsilon}(g) = \varepsilon_f^{(i)}(g),$$

$$\delta_f^{(1)}(g) = \delta(g) \wedge f$$

$$\delta_g^{(i)}(f) = \delta(\delta^{(i-1)}(g)) \wedge f$$

$$\varepsilon_f^{(1)}(g) = \varepsilon(g) \vee f, \; \varepsilon_g^{(i)}(f) = \varepsilon(\varepsilon^{(i-1)}(g)) \vee f$$

-

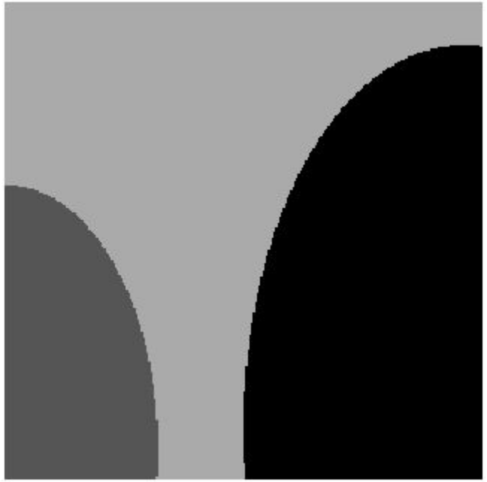
$$R^C(f) = R^{\varepsilon}_{R_f^{\delta}(\varepsilon(f))}\left(\delta\left(R_f^{\delta}\left(\varepsilon(f)\right)\right)\right)$$

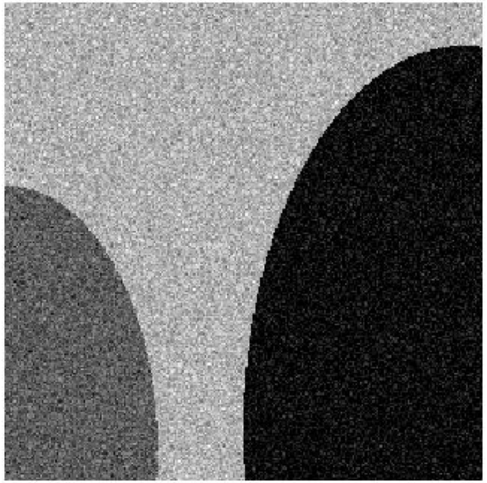
FRFCM based on morphological reconstruction

- f is original image, g is marker image, delta is dilation operation and epsilon is erosion operation.
- After optimizing, we get graylevel memberships and means. Using means and image data we can get original memberships.
- A median filter is used in membership matrix to computational time.

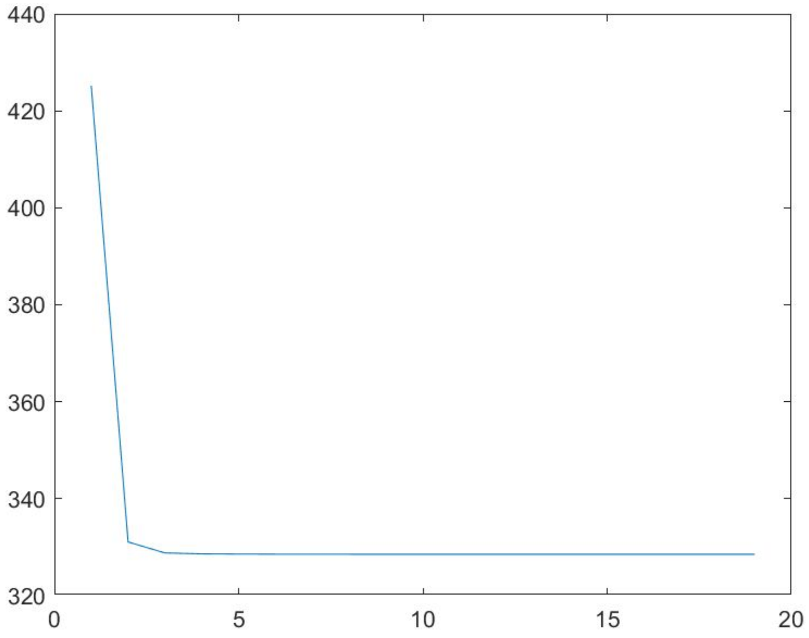
Results | Test Image 1

Original Image and Noisy Image



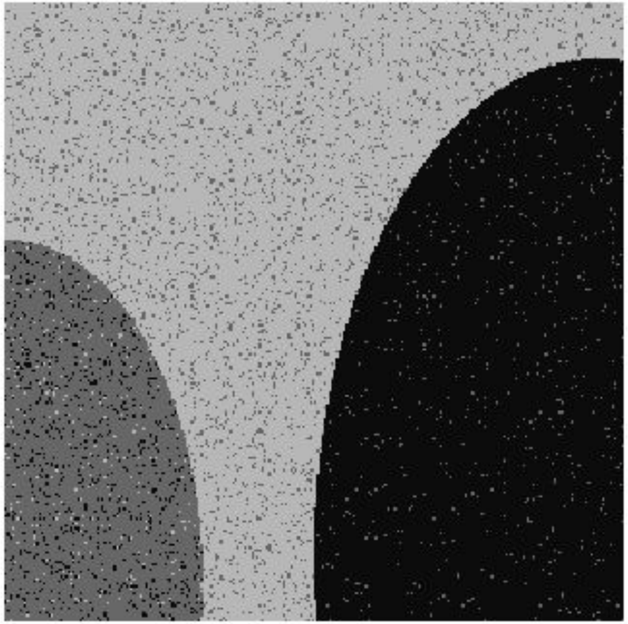


FCM | Objective Function vs Iterations

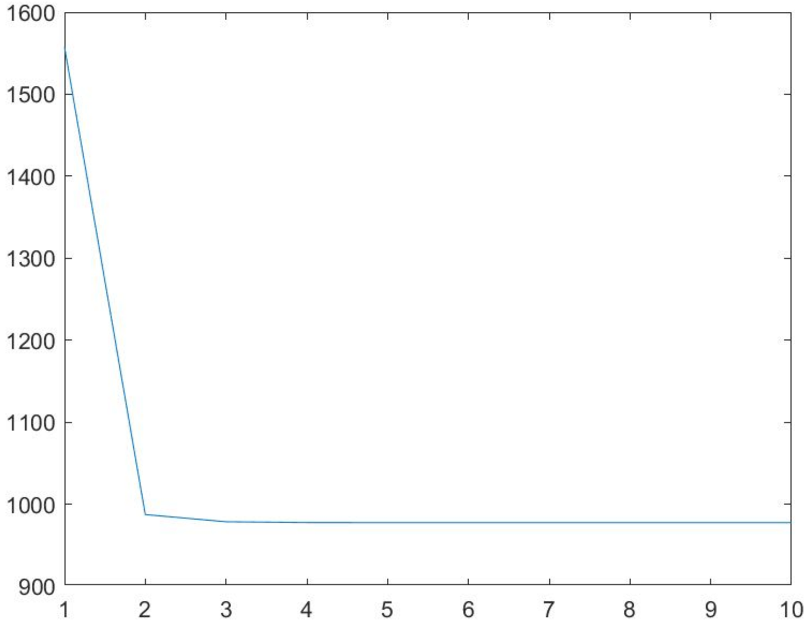


FCM | Segmentation

- MSE = 579.4981

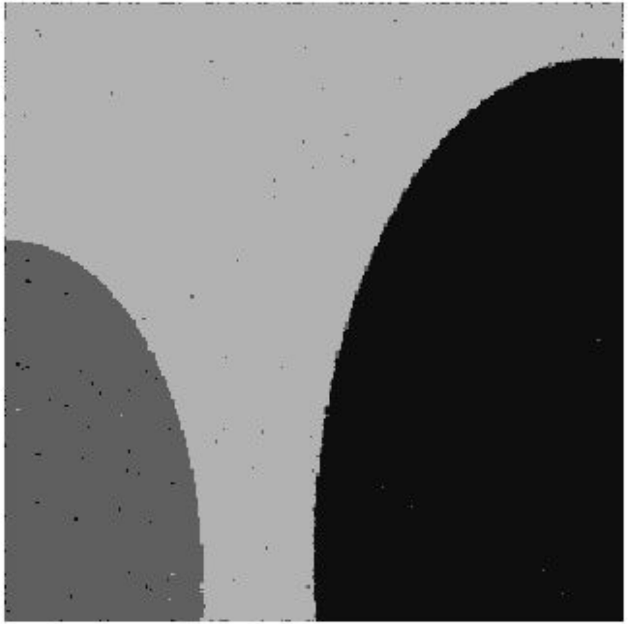


FCM_S | Objective Function vs Iterations

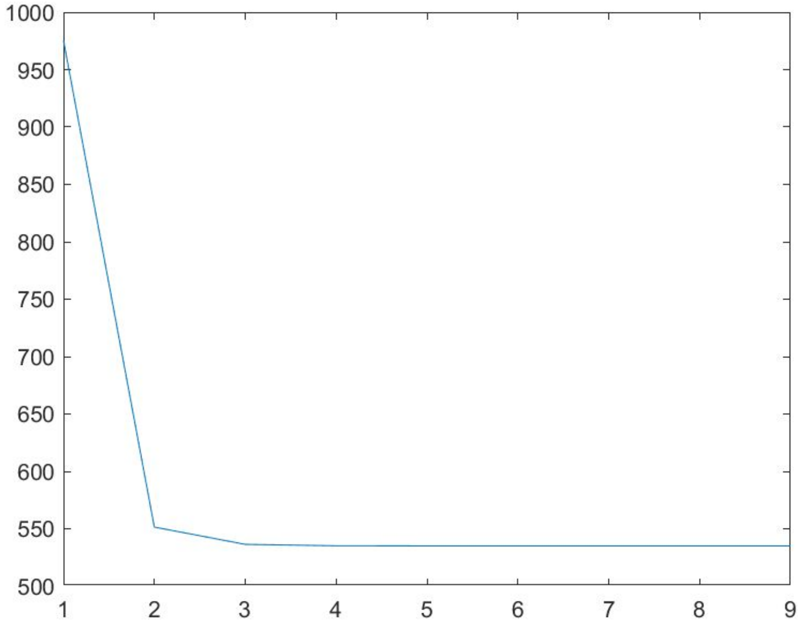


FCM_S | Segmentation

- MSE = 159.6948

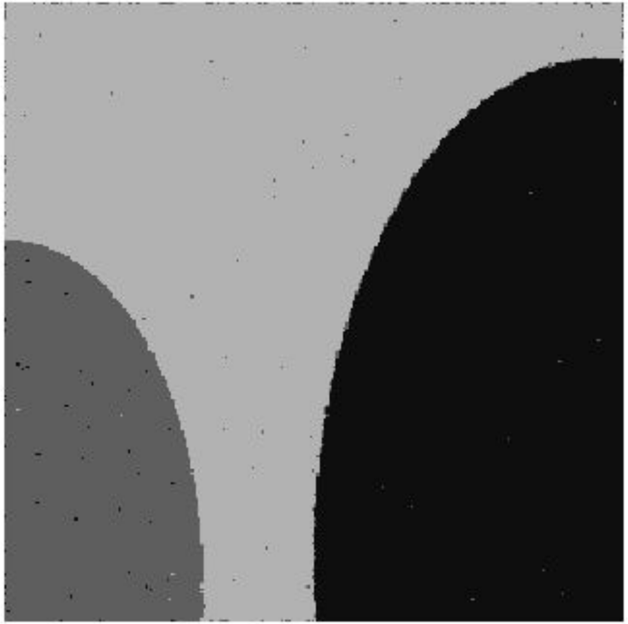


FCM_S1 | Objective Function vs Iterations

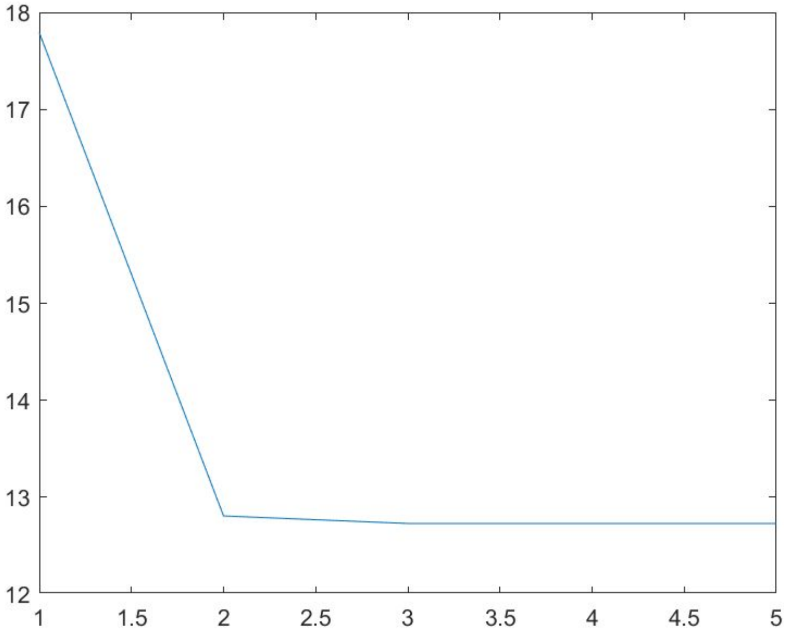


FCM_S1 | Segmentation

- MSE = 151.2929

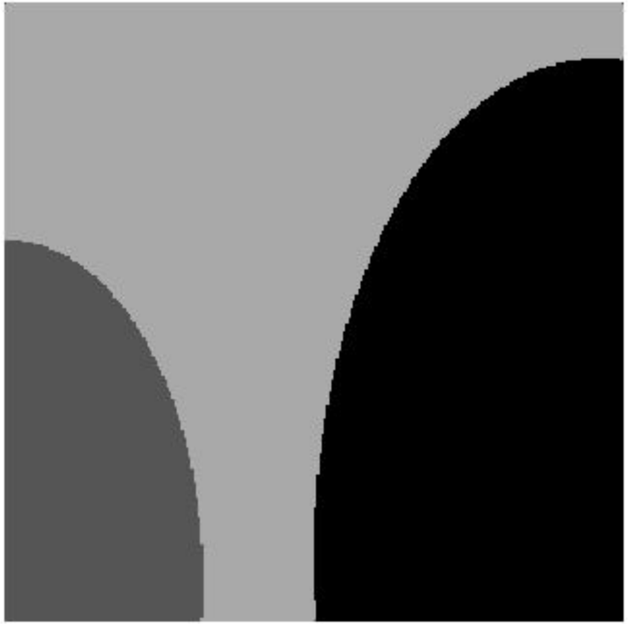


FCM_EN | Objective Function vs Iterations

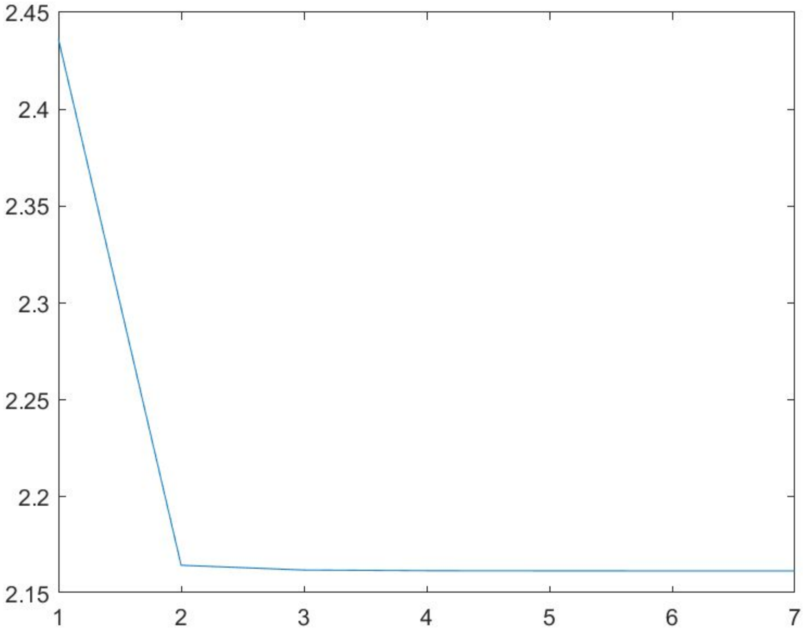


FCM_EN | Segmentation

- MSE = 0.5691

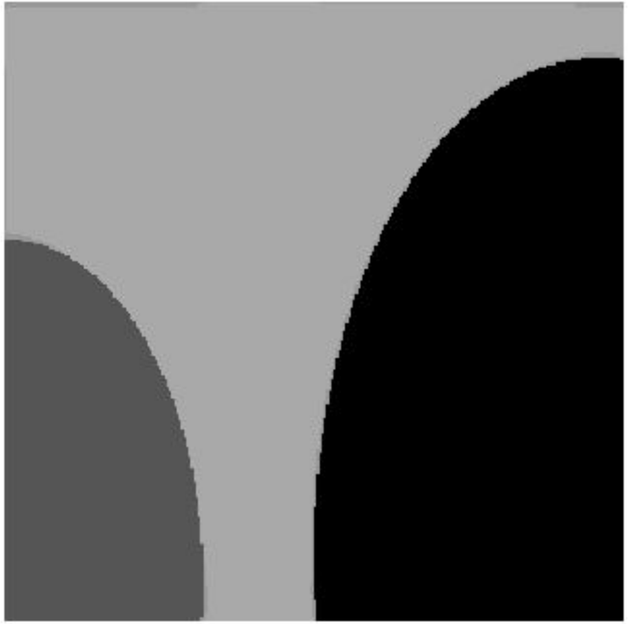


FGFCM | Objective Function vs Iterations



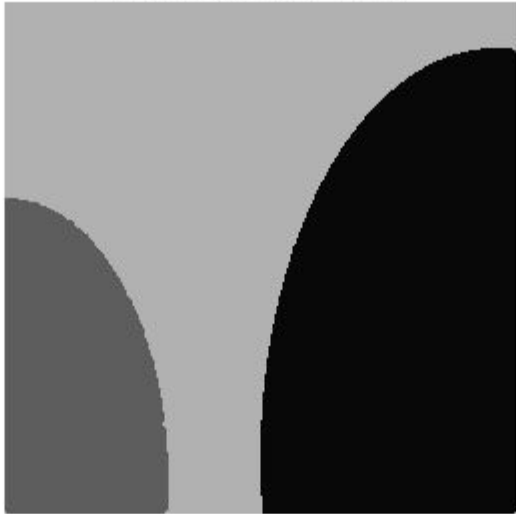
FGFCM | Segmentation

- MSE = 3.3623



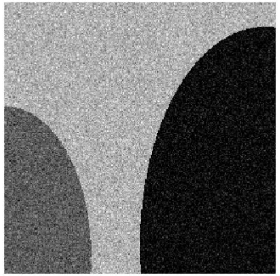
FRFCM | Segmentation

- MSE = 11

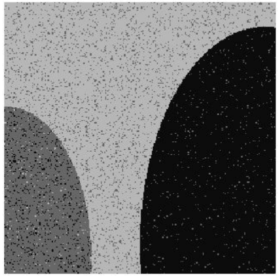


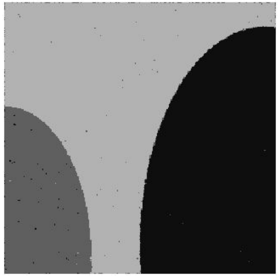
Segmentation

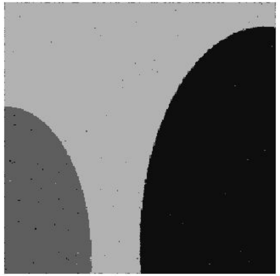
Original



Noisy



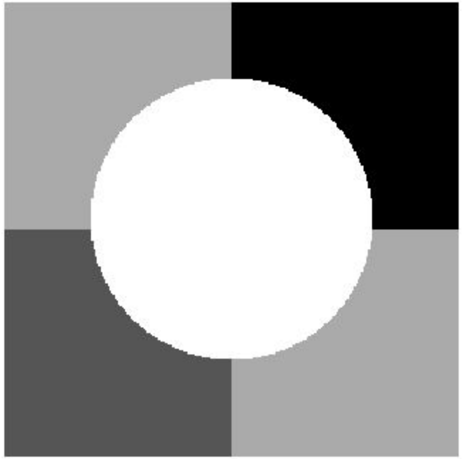


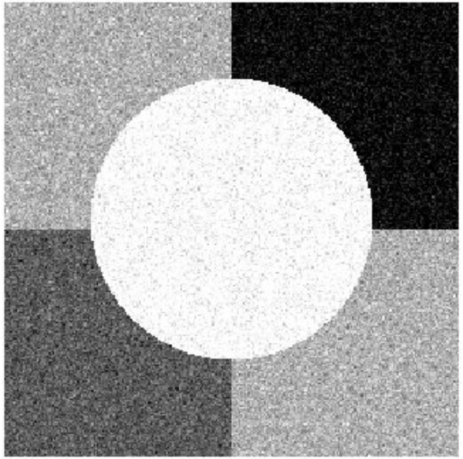


	FCM	FCM_S	FCM_S1	FCM_EN	FRFCM	FGFCM
MSE	579	159	151	0.56	3.3	11

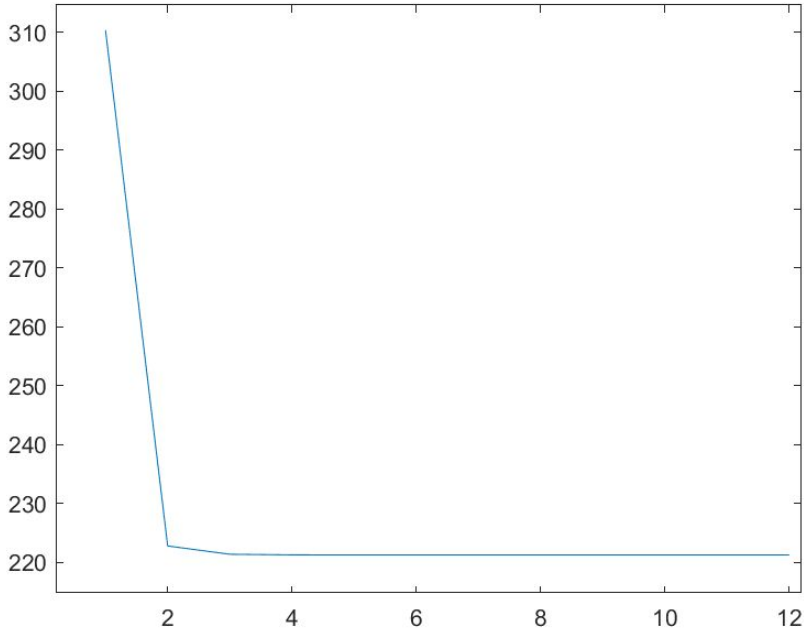
Results | Test Image 2

Original Image and Noisy Image



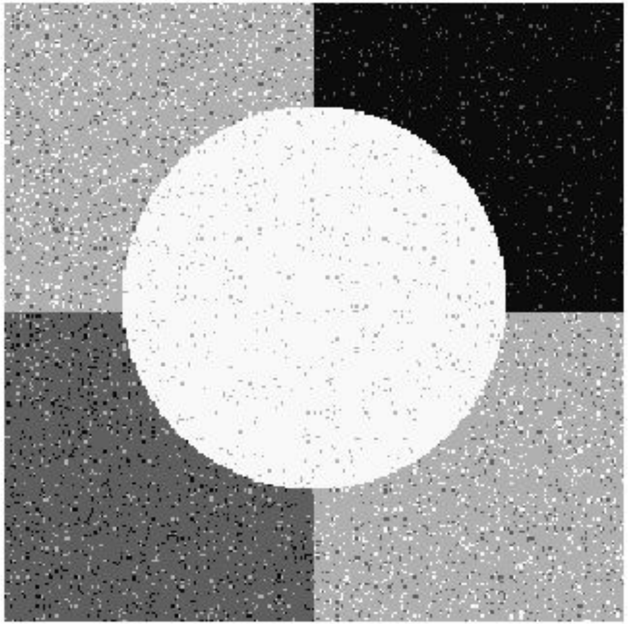


FCM | Objective Function vs Iterations

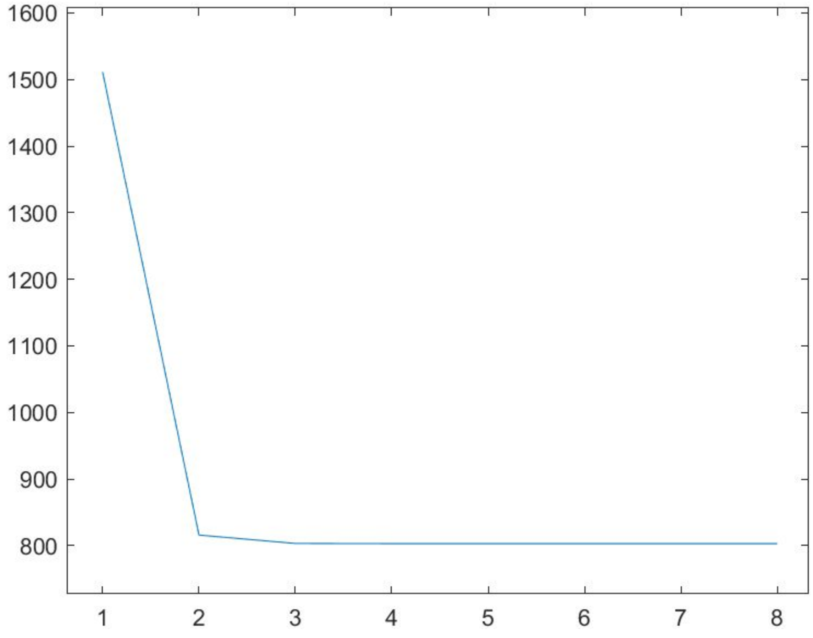


FCM | Segmentation

- MSE = 582.6100

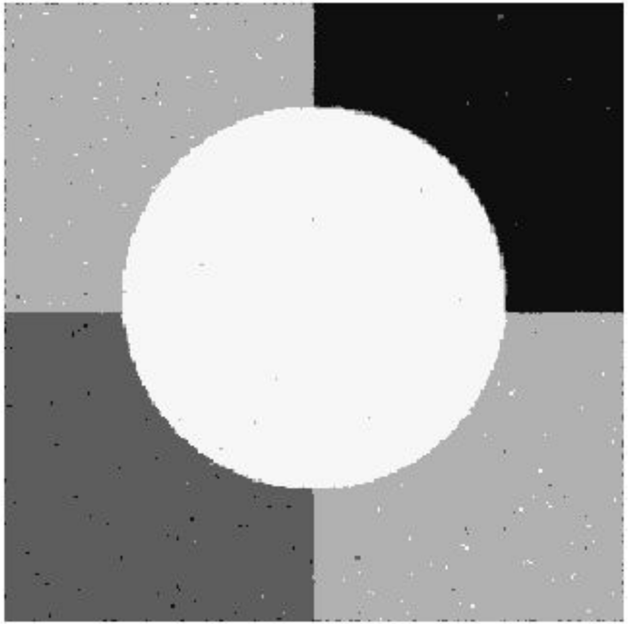


FCM_S | Objective Function vs Iterations

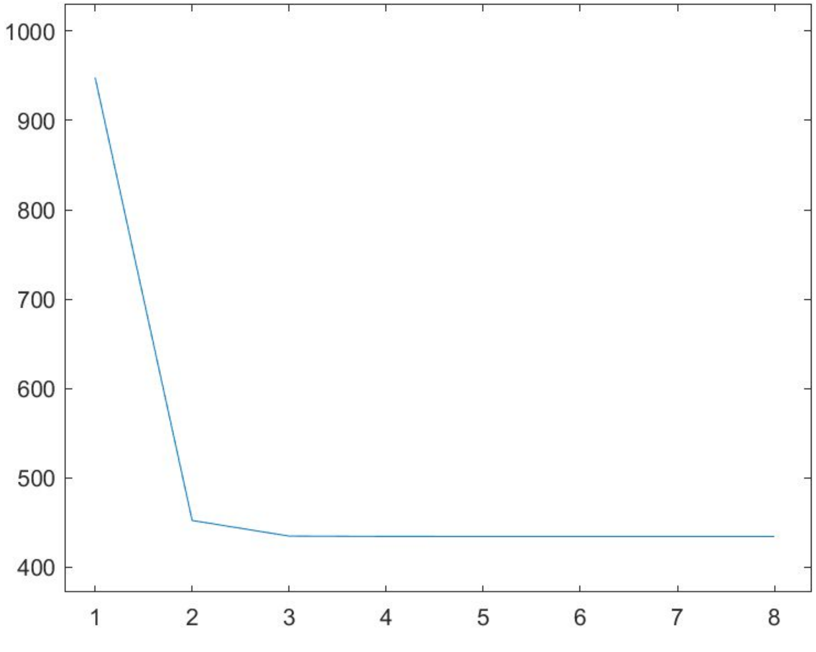


FCM_S | Segmentation

- MSE = 139.7833

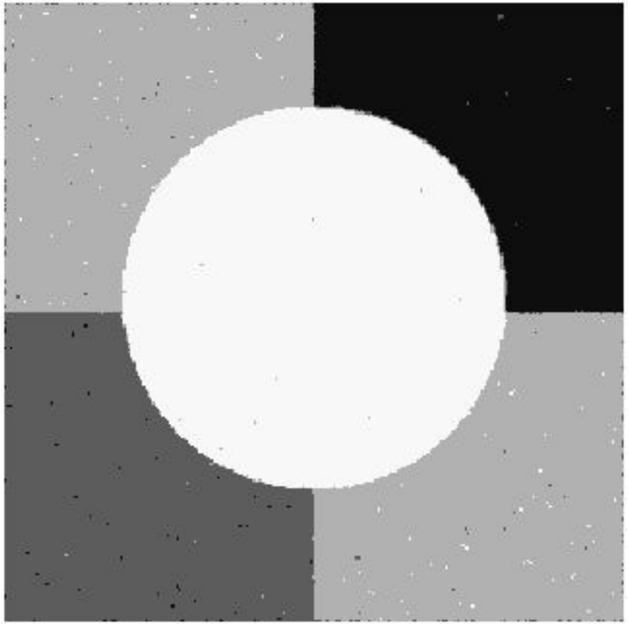


FCM_S1 | Objective Function vs Iterations

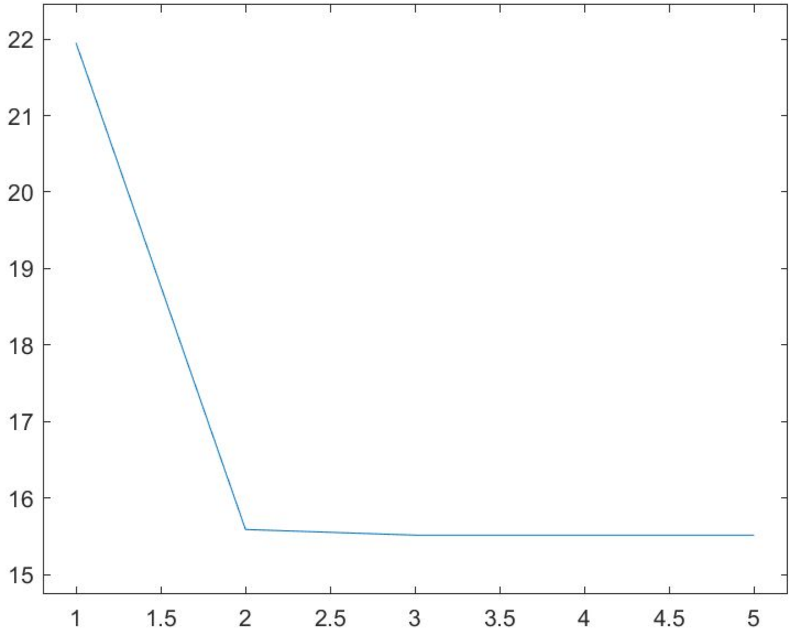


FCM_S1 | Segmentation

- MSE = 133.4468

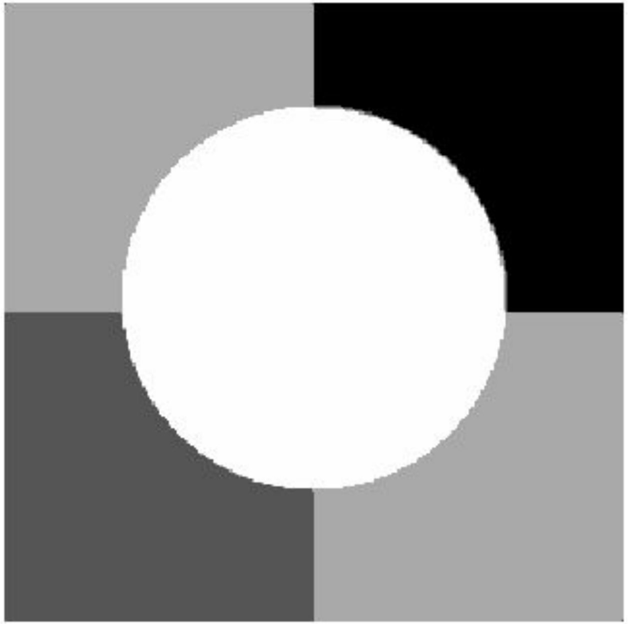


FCM_EN | Objective Function vs Iterations

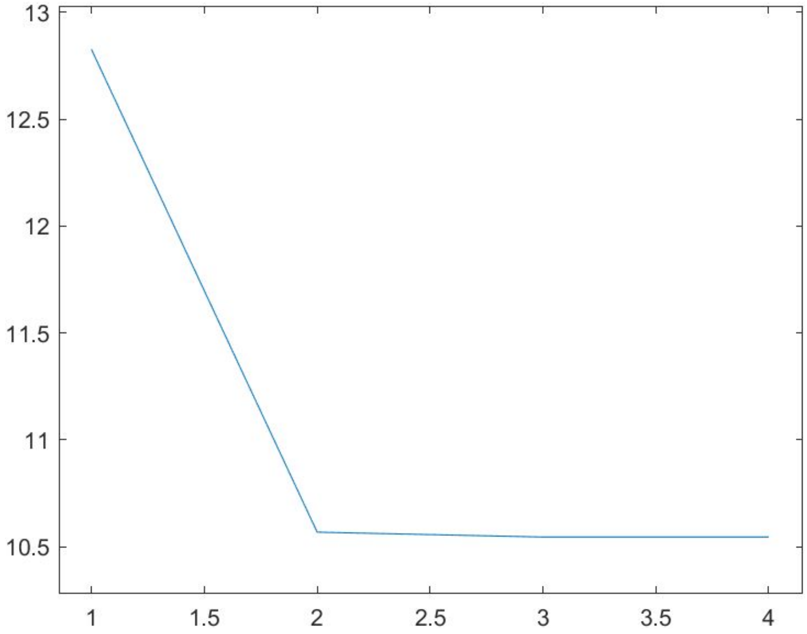


FCM_EN | Segmentation

- MSE = 23.3744

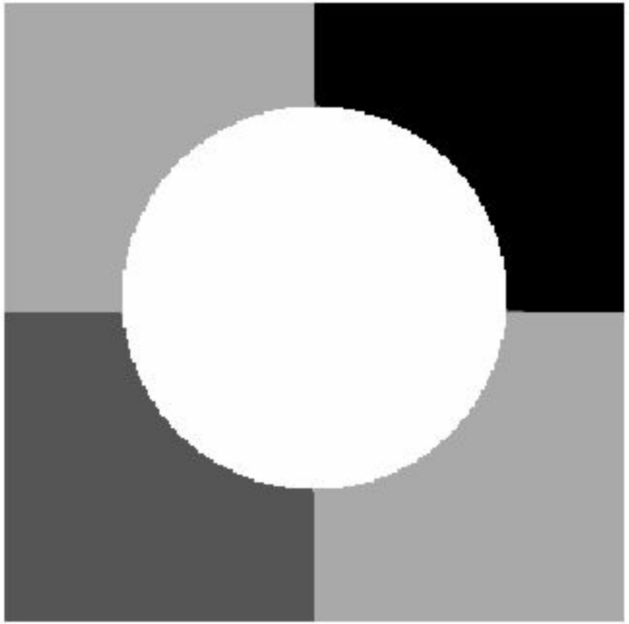


FGFCM | Objective Function vs Iterations



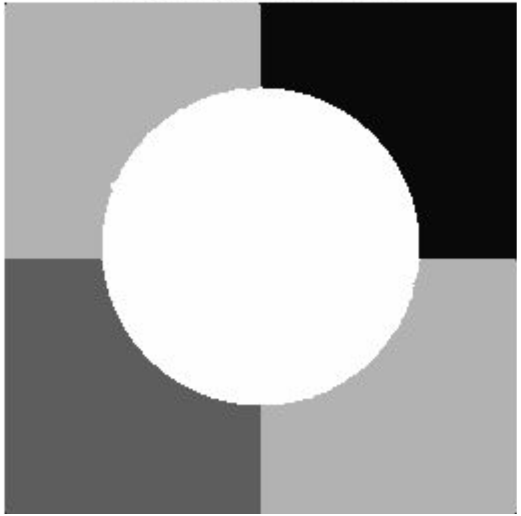
FGFCM | Segmentation

- MSE = 1.8561

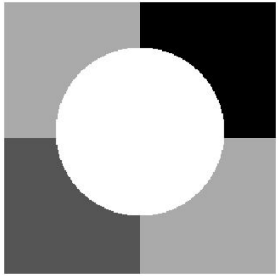


FRFCM | Segmentation

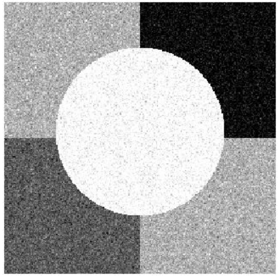
- MSE = 4



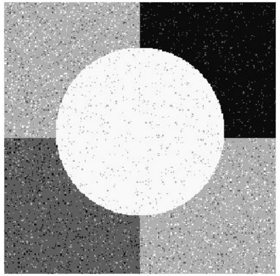
Segmentation

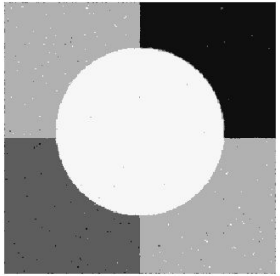


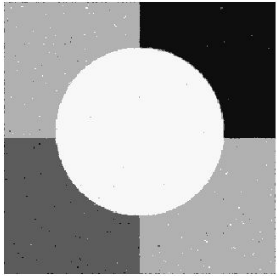
Original

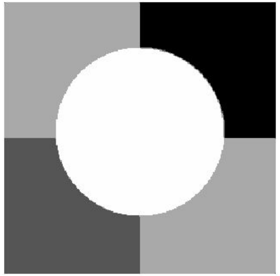


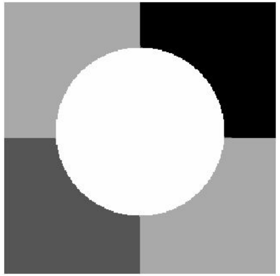
Noisy

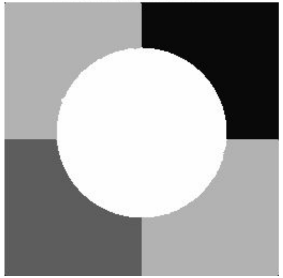










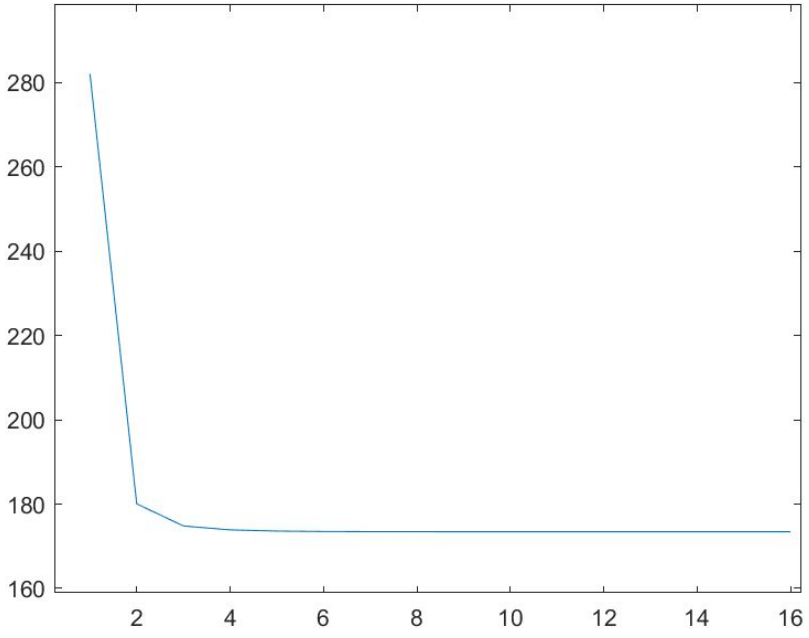


	FCM	FCM_S	FCM_S1	FCM_EN	FRFCM	FGFCM
MSE	582	139	133	23.3	1.85	4

Results | Brain Image

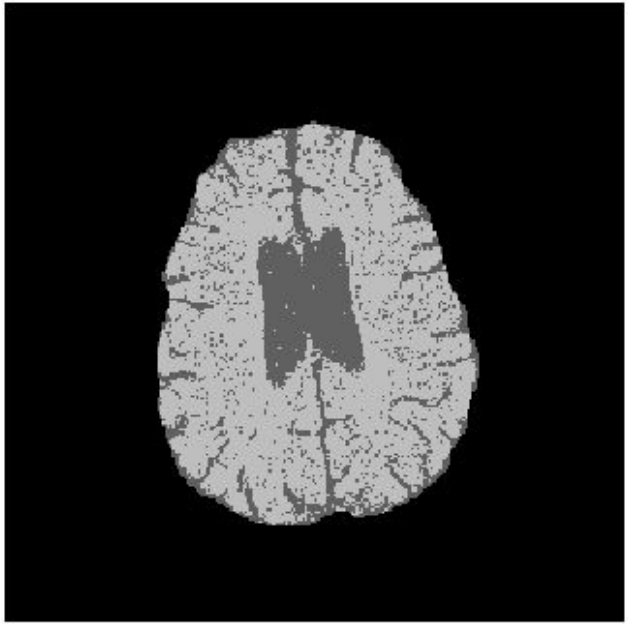
Original Image and Noisy Image

FCM | Objective Function vs Iterations

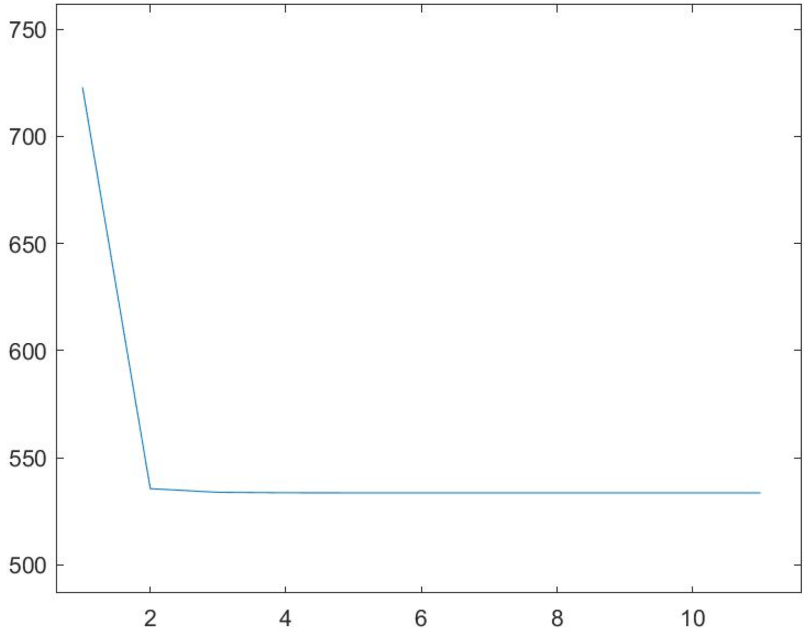


FCM | Segmentation

- MSE = 242.6889

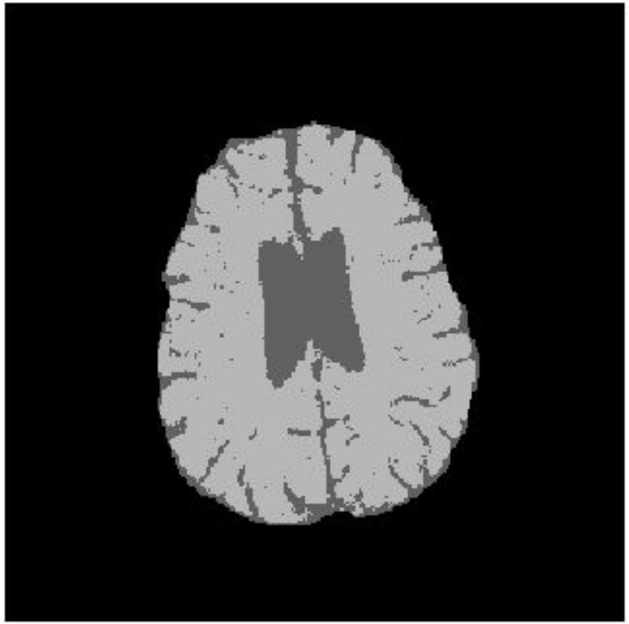


FCM_S | Objective Function vs Iterations

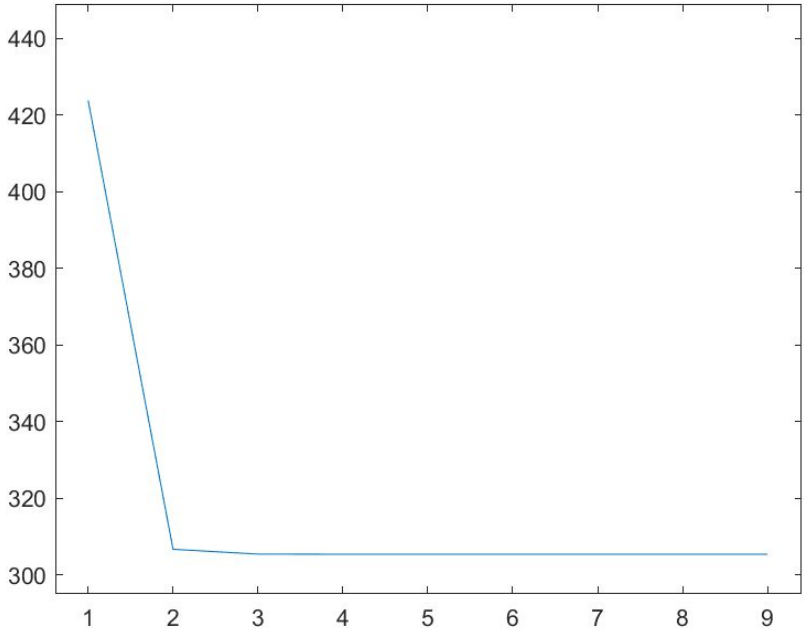


FCM_S | Segmentation

- MSE = 147.5855

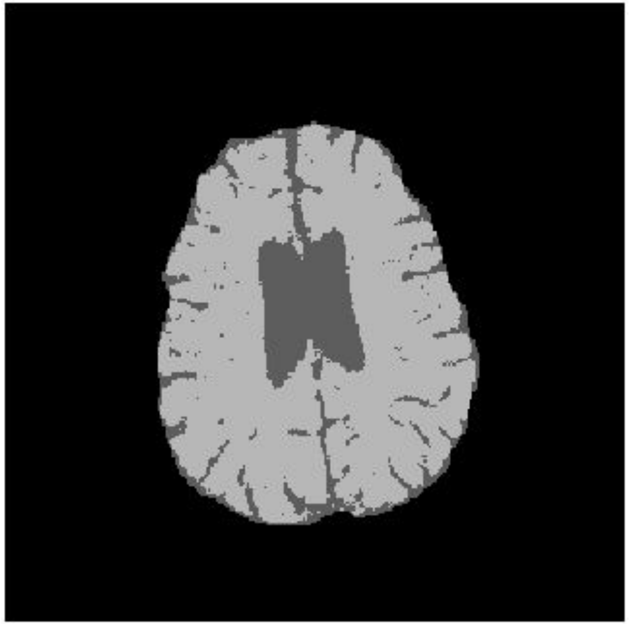


FCM_S1 | Objective Function vs Iterations

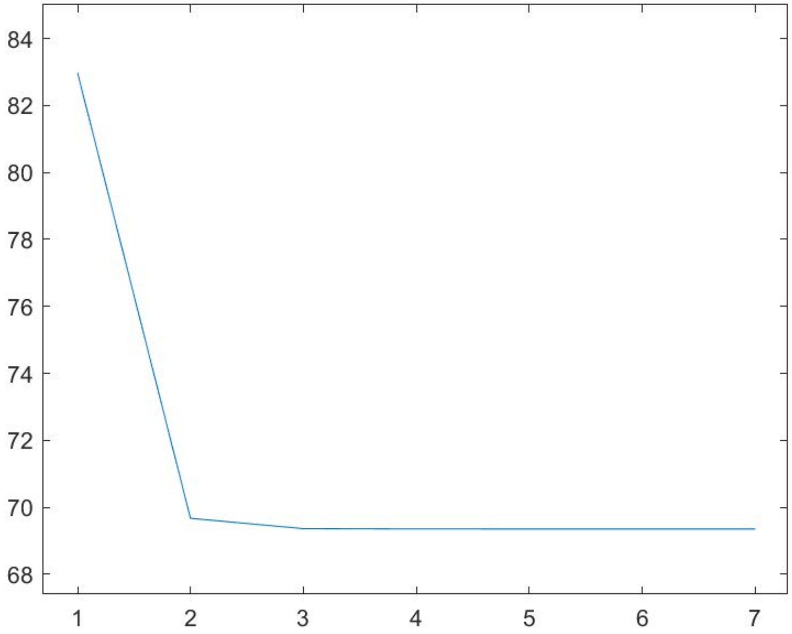


FCM_S1 | Segmentation

- MSE = 141.1271

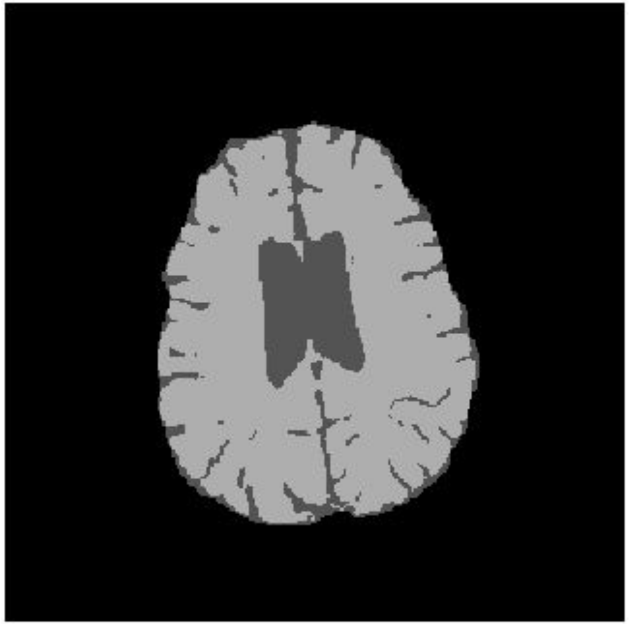


FCM_EN | Objective Function vs Iterations

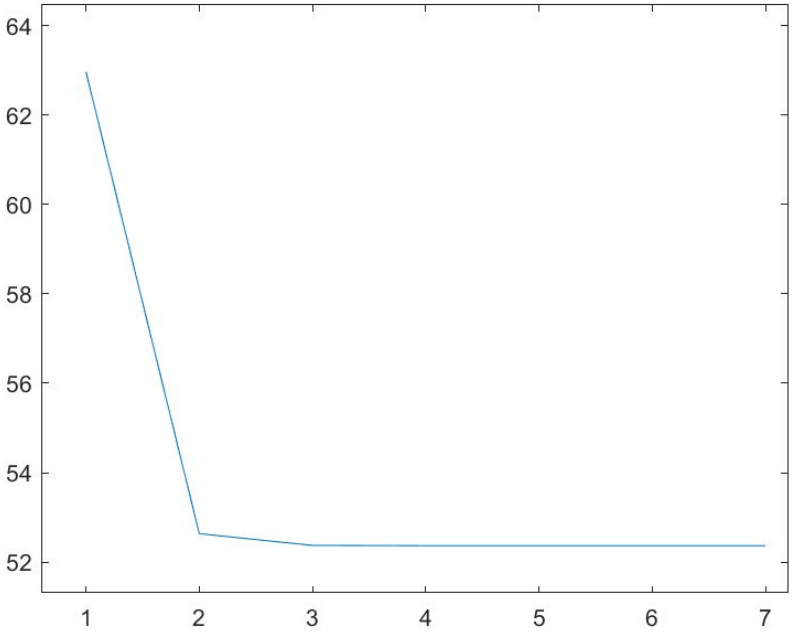


FCM_EN | Segmentation

- MSE = 109.7403



FGFCM | Objective Function vs Iterations



FGFCM | Segmentation

- MSE = 131.9464

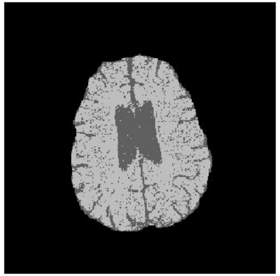
FRFCM | Segmentation

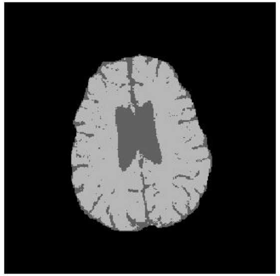
- MSE = 212

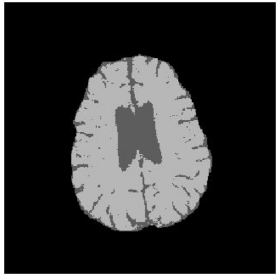
Segmentation

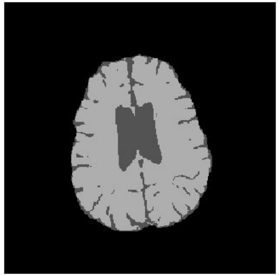
Original

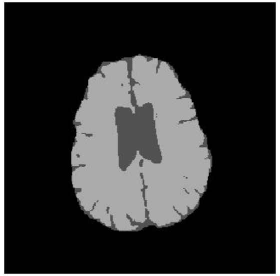
Noisy











	FCM	FCM_S	FCM_S1	FCM_EN	FGFCM	FRFCM
MSE	242	147	141	109	131	212

Thank You

Prof. Suyash P. Awate