Name of Student:	Roll No.	Date:

Experiment No. 4

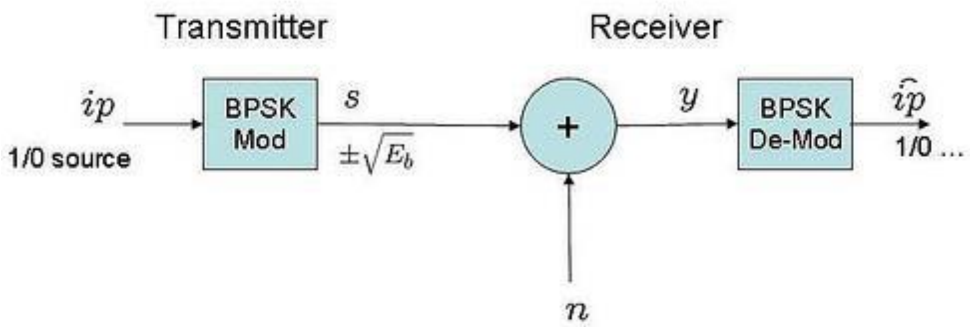
Aim: Simulate BER Performance over Rayleigh fading wireless channel with BPSK Transmission

Problem Statement:
Simulate BER Performance over Rayleigh Fading wireless channel with BPSK Transmission for SNR 0 to 60 db..

Objectives:
- What is a Rayleigh Fading
- Study of BPSK transmission

Software Requirements:
- Windows 7/10/11
- Matlab

THEORY: In this experiment, we will derive the theoretical equation for bit error rate (BER) with Binary Phase Shift Keying (BPSK) modulation scheme in Additive White Gaussian Noise (AWGN) channel. The BER results obtained using Matlab/Octave simulation scripts show good agreement with the derived theoretical results.
With Binary Phase Shift Keying (BPSK), the binary digits 1 and 0 maybe represented by the analog levels. The system model is as shown in the Figure below.



Channel Model
The transmitted waveform gets corrupted by noise , typically referred to as **Additive White**

Gaussian Noise (AWGN).

Additive: As the noise gets 'added' (and not multiplied) to the received signal
White: The spectrum of the noise if flat for all frequencies.

Gaussian: The values of the noise follows the Gaussian probability distribution function, with and .

function, $p(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$ with $\mu=0$ and $\sigma^2=\frac{N_0}{2}$.

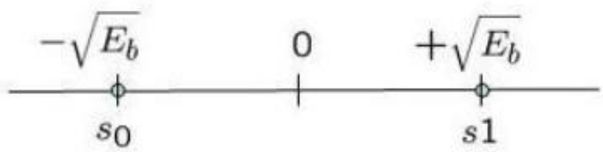
Computing the probability of error

$y=s_1+n$ when bit 1 is transmitted and

$y=s_0+n$ when bit 0 is transmitted.

$$p(y|s_0)=\frac{1}{\sqrt{\pi N_0}}e^{\frac{-(y+\sqrt{E_b})^2}{N_0}}$$

$$p(y|s_1)=\frac{1}{\sqrt{\pi N_0}}e^{\frac{-(y-\sqrt{E_b})^2}{N_0}}.$$



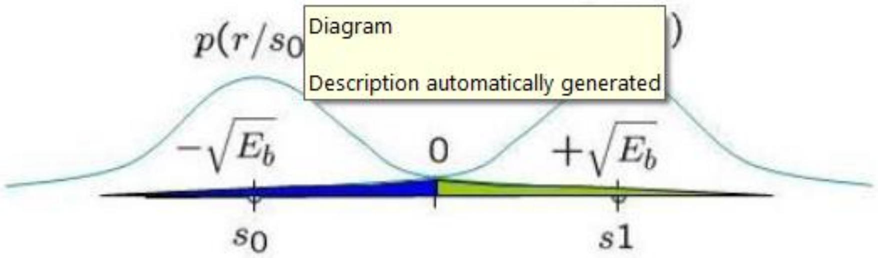


Figure: Conditional probability density function with BPSK modulation

Assuming that s_1 and s_0 are equally probable i.e. $p(s_1)=p(s_0)=1/2$, the **threshold 0** forms the optimal decision boundary.

- if the received signal is y is greater than 0, then the receiver assumes s_1 was transmitted.
- if the received signal is y is less than or equal to 0, then the receiver assumes s_0 was transmitted.

i.e.

$y>0 \Rightarrow s_1$ and

$y\le 0 \Rightarrow s_0$.

Probability of error given s_1 was transmitted.

With this threshold, the probability of error given s_1 is transmitted is (the area in blue region):

$$p(e|s_1)=\frac{1}{\sqrt{\pi N_0}}\int_{-\infty}^{0}e^{\frac{-(y-\sqrt{E_b})^2}{N_0}}dy=\frac{1}{\sqrt{\pi}}\int_{\sqrt{\frac{E_b}{N_0}}}^{\infty}e^{-z^2}dz=\frac{1}{2}erfc\left(\sqrt{\frac{E_b}{N_0}}\right),$$

Probability of error given s_0 was transmitted

Similarly the probability of error given s_0 is transmitted is (the area in green region):

$$p(e|s_0) = \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} e^{\frac{-(y+\sqrt{E_b})^2}{N_0}} dy = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right).$$

Total probability of bit error

$$P_b = p(s_1)p(e|s_1) + p(s_0)p(e|s_0).$$

Given that we assumed that s_1 and s_0 are equally probable i.e. $p(s_1) = p(s_0) = 1/2$, the **bit error probability** is,

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right).$$

Simulation model

Matlab/Octave source code for computing the bit error rate with BPSK modulation from theory and simulation. The code performs the following:

(a) Generation of random BPSK modulated symbols +1's and -1's

(b) Passing them through Additive White Gaussian Noise channel

(c) Demodulation of the received symbol based on the location in the constellation

(d) Counting the number of errors

(e) Repeating the same for multiple Eb/No value

Matlab Code:

```
clear
N = 10^6 % number of bits or symbols
rand('state',100); % initializing the rand() function
randn('state',200); % initializing the randn() function
% Transmitter
ip = rand(1,N)>0.5; % generating 0,1 with equal probability
s = 2*ip-1; % BPSK modulation 0 -> -1; 1 -> 1
n = 1/sqrt(2)*[randn(1,N) + j*randn(1,N)]; % white gaussian noise,
0dB variance
Eb_N0_dB = [-3:10]; % multiple Eb/N0 values
for ii = 1:length(Eb_N0_dB)
% Noise addition
y = s + 10^(-Eb_N0_dB(ii)/20)*n; % additive white gaussian noise
% receiver - hard decision decoding
ipHat = real(y)>0;
% counting the errors
nErr(ii) = size(find([ip- ipHat]),2);
end
simBer = nErr/N; % simulated ber
theoryBer = 0.5*erfc(sqrt(10.^(Eb_N0_dB/10))); % theoretical ber
% plot
```

```
close all
figure
semilogy(Eb_N0_dB,theoryBer,'b.-');
hold on
semilogy(Eb_N0_dB,simBer,'mx-');
axis([-3 10 10^-5 0.5])
grid on
legend('theory', 'simulation');
xlabel('Eb/No, dB');
ylabel('Bit Error Rate');
title('Bit error probability curve for BPSK modulation');
Result:
```

Conclusion

Checked By:

Name of Subject Teacher	Sign with Date