Quantum information scrambling in three-qubit random mixed states

Amey Rodge
M.Sc. Physics
(21PHMP26)

Project guide
Prof. V. Subrahmanyam
Sop, UoH.

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Outline of the talk

- Introduction to information scrambling
- Mutual information and Tripartite Mutual Information (TMI)
- Density plot of TMI
- Generating random mixed states from random numbers
- Variation of TMI for random mixed states generated from distribution functions
- Summary

Scrambling of Quantum Information

- Classically - data scrambling →
  - Deleting data or data obfuscating
  - To preserve confidentiality of data
  - Irreversible process

- Information Scrambling →
  - Spread or dispersal of quantum information
  - Describes propagation and effective loss of information in quantum many body systems

How to measure ? how to quantify?

➢ Mutual information shared between two systems A and B

$$\boxed{I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB})}$$

- Von-Neumann Entropy : $S(\rho) = -Tr(\rho \log_2 \rho) = -\sum_k \lambda_k \log_2 \lambda_k \longrightarrow \rho_A = tr_B(\rho_{AB}) \longrightarrow S(\rho_A) = -Tr(\rho_A \log_2 \rho_A)$

$$0 \leq I(A:B) \leq 2$$

Pure bipartite separable states ← 0 ; 2 → Pure bipartite entangled states

Tripartite mutual Information (TMI)

$$I(A:B:C) = I(A:B) + I(A:C) - I(A:BC)$$

In terms of Von – Neumann entropy

$$I(A:B:C) = S(\rho_A) + S(\rho_B) + S(\rho_C) - S(\rho_{AB}) - S(\rho_{BC}) - S(\rho_{CA}) + S(\rho_{ABC})$$

Two cases

positive

$I(A:B:C)$

A with B and A with C share some redundant information

negative

$I(A:B:C)$

Information shared between A and BC as a system together is more

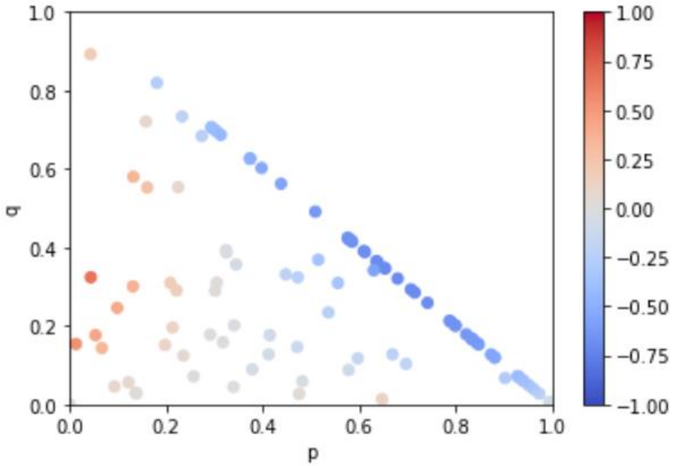
Signature of loss of information

INFORMATION SCRAMBLING

Illustration of Scrambling and TMI

- For state - $\rho = \frac{p}{3}\left|100+010+001\right\rangle\left\langle 100+010+001\right| + q\left|111\right\rangle\left\langle 111\right| + (1-p-q)\left|000\right\rangle\left\langle 000\right|$

Where, $p, q \geq 0 \text{ and } p+q \leq 1$



Here it is seen that TMI is negative for $p \geq 0.2$

Generating random mixed states

Random Numbers → Inverse transform sampling

$$\rho_{real} = \begin{bmatrix} p & A & B & C & \cdot & \cdot & \cdot & \cdot \\ A^* & q & H & \cdot & \cdot & \cdot & \cdot & \cdot \\ B^* & H^* & r & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & s & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & t & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & u & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & v & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1-p-q-r-s-t-u-v \end{bmatrix}$$

8x8 Hermitian matrix with real parameters [7+28 = 35 (independent parameters)]

Random Numbers → Inverse transform sampling

$$\rho_{complex} = \begin{bmatrix} p & A+iB & C+iD & E+iF & \cdot & \cdot & \cdot & \cdot \\ A-iB & q & O+iP & \cdot & \cdot & \cdot & \cdot & \cdot \\ C-iD & O-iP & r & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & s & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & t & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & u & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & v & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1-p-q-r-s-t-u-v \end{bmatrix}$$

8x8 Hermitian matrix with complex parameters [7+ 56 = 63 (independent parameters)]

➤ Steps

- CDF (Cumulative distribution function)
- Inverse CDF

$$F_x = P(X \leq x) \longrightarrow F^{-1}(X)$$

- **Gaussian Distribution**

$$F(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp - \frac{(x-\mu)^2}{2\sigma^2}$$

$$F_X(x) = \frac{1}{2}(1 + erf(\frac{x-\mu}{\sqrt{2}\sigma}) = u(say)$$

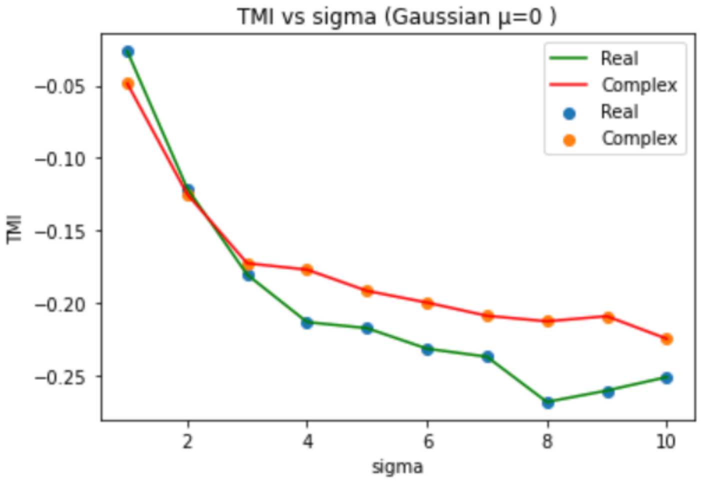
$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp -t^2 dt.$$

$$\boxed{F_X^{-1}(x) = \mu + \sqrt{2}\sigma erf^{-1}(2u-1)}$$

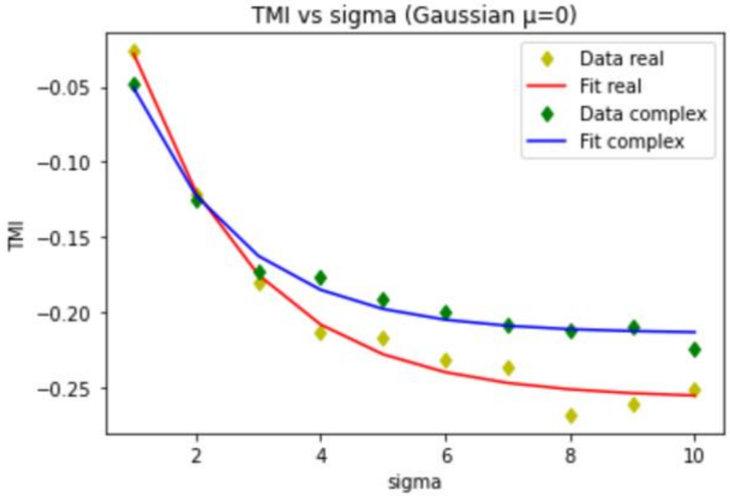
Desired sampling formula

Variation of TMI for three-qubit random mixed states

1. For random mixed states generated using gaussian distribution



TMI vs σ

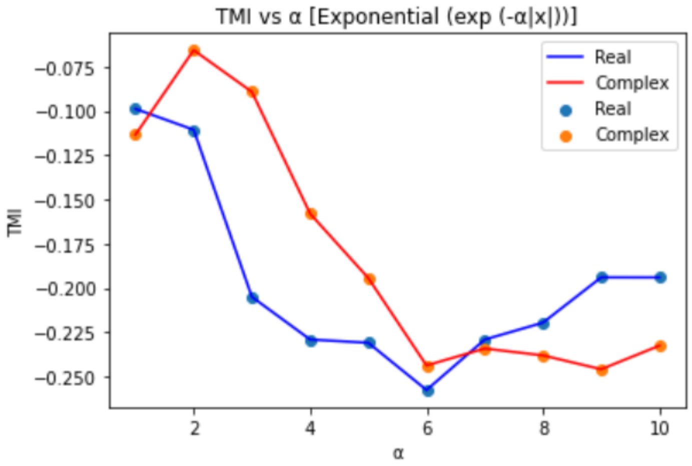


TMI vs σ σ fitted to exponential decay function

Variation of TMI with σ (keeping μ constant).

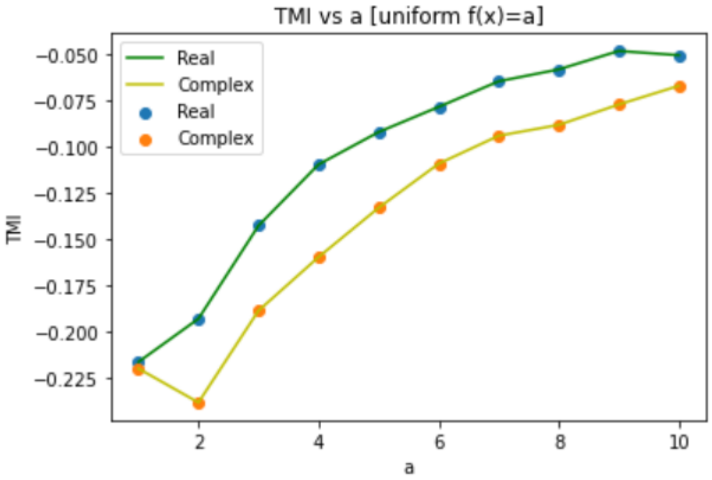
Variation of TMI for three-qubit random mixed states

2. For exponential distribution



Variation of TMI with α

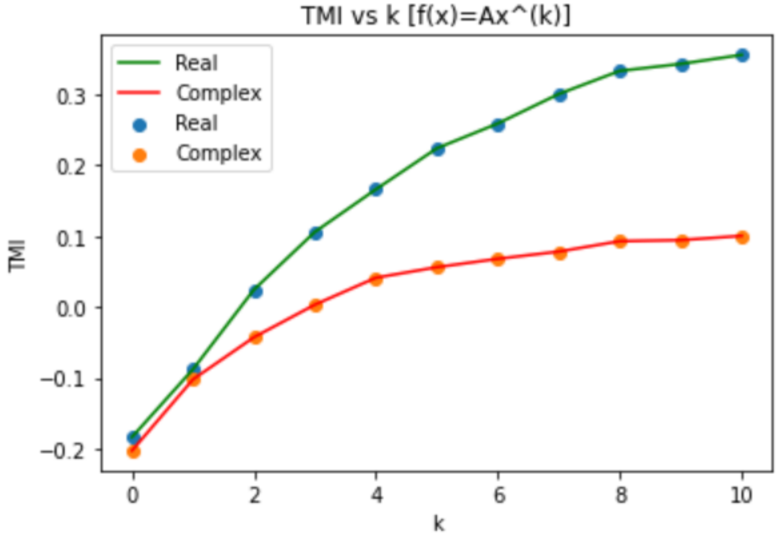
3. For uniform distribution



Variation of TMI with a

Variation of TMI for three-qubit random mixed states

4. For distribution $F(x) = Ax^{|k|}$



Variation of TMI with k

TMI starts becoming positive for values of k ≈ 3

Summary

- Random numbers were generated from different distributions using inverse transform sampling method and hence random three-qubit mixed states are generated
- The variation of average tripartite mutual information for real as well as complex parameters of random mixed state has been studied by varying corresponding parameter of various distribution functions.

[https://github.com/Ameyrodge/information_scrambling]

THANK YOU

Mixed states

- These are defined by density operator → $\rho = \sum_i P_i \left|\psi_i\right\rangle \left\langle\psi_i\right|$

$$\boxed{(1)\ \rho^{\dagger} = \rho,\ (2)\ 0 \leq \lambda \leq 1,\ (3)\ Tr(\rho) = 1,\ (4)\ \rho^2 \neq \rho,\ (5)\ Tr(\rho^2) \leq 1}$$

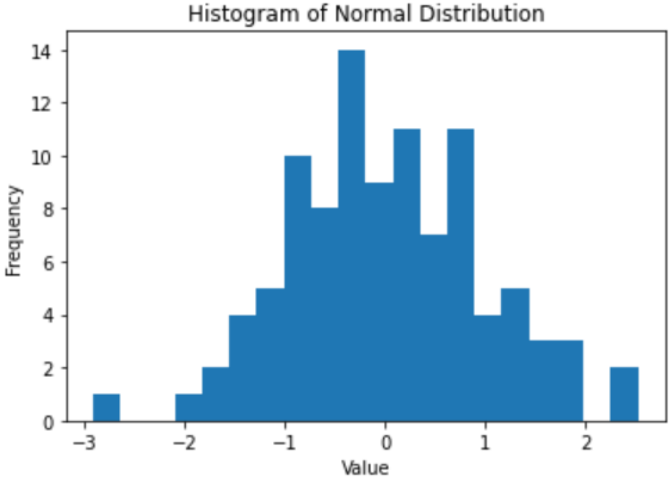
To construct density matrix (mixed state) from H

1. Find eigenvalues λ_i and eigenvectors ψ_i of H.
2. Take absolute values of $\lambda_i's$ and normalize $\lambda_i's$ so that they sum to 1
3. now construct density matrix ρ (mixed state)

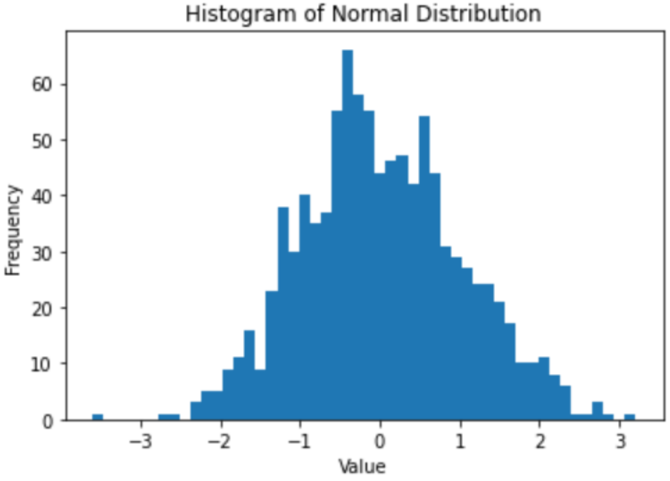
$$\rho = \sum_i \lambda_i \left|\psi_i\right\rangle \left\langle\psi_i\right|$$

Histogram of random numbers generated using gaussian distribution

```
[-1.366 -0.926  0.307 -0.861  1.317 -1.579  0.663  1.306  1.479 -0.171
 -1.836  0.19   0.189  0.701 -0.798 -0.33   0.015 -0.426 -1.093 -0.443
 -0.246 -0.598  0.514  1.191 -1.065 -0.993  0.439  0.22   2.297 -0.078
 -0.95  -1.216 -1.278 -0.33   0.498  0.584 -0.314 -0.233 -0.259 -0.601
  0.855  1.362  2.529 -0.212 -0.654  0.862 -0.015 -0.45  -1.474 -0.542
 -1.31  -0.487  0.852 -0.874 -0.113 -0.674  0.084  1.837  0.785  0.053
 -1.242  1.652  0.905  1.617 -0.852 -0.811  1.069  0.852 -0.301 -0.785
 -1.545 -0.451 -0.731  0.639  0.326 -0.224  0.786  0.369  0.348  1.954
  0.054  0.537  1.053  1.215 -0.064 -0.536  0.759 -0.879 -2.915  0.288
  0.41  -0.054 -1.75  -0.349  0.324  0.149  0.807  0.318  1.85   1.118]
```



Sample size 100 (μ =0, σ = 1)



Sample size 1000 (μ =0, σ = 1)

$$
\begin{array}{cccccccc}
\rho_{23} & & & & \rho_{13} & & \rho_{12} & \\
\end{array}
$$

$$
\hat{\rho}_{123} = \begin{pmatrix}
\rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} & \rho_{15} & \rho_{16} & \rho_{17} & \rho_{18} \\
\rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} & \rho_{25} & \rho_{26} & \rho_{27} & \rho_{28} \\
\rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} & \rho_{35} & \rho_{36} & \rho_{37} & \rho_{38} \\
\rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} & \rho_{45} & \rho_{46} & \rho_{47} & \rho_{48} \\
\rho_{51} & \rho_{52} & \rho_{53} & \rho_{54} & \rho_{55} & \rho_{56} & \rho_{57} & \rho_{58} \\
\rho_{61} & \rho_{62} & \rho_{63} & \rho_{64} & \rho_{65} & \rho_{66} & \rho_{67} & \rho_{68} \\
\rho_{71} & \rho_{72} & \rho_{73} & \rho_{74} & \rho_{75} & \rho_{76} & \rho_{77} & \rho_{78} \\
\rho_{81} & \rho_{82} & \rho_{83} & \rho_{84} & \rho_{85} & \rho_{86} & \rho_{87} & \rho_{88}
\end{pmatrix}
$$

$$
\hat{\rho}_1 = Tr_2[\hat{\rho}] = \begin{pmatrix} \rho_{11} & \rho_{13} \\ \rho_{31} & \rho_{33} \end{pmatrix} + \begin{pmatrix} \rho_{22} & \rho_{24} \\ \rho_{42} & \rho_{44} \end{pmatrix} = \begin{pmatrix} \rho_{11} + \rho_{22} & \rho_{13} + \rho_{24} \\ \rho_{31} + \rho_{42} & \rho_{33} + \rho_{44} \end{pmatrix}
$$

$$
\hat{\rho}_2 = Tr_1[\hat{\rho}] = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} + \begin{pmatrix} \rho_{33} & \rho_{34} \\ \rho_{43} & \rho_{44} \end{pmatrix} = \begin{pmatrix} \rho_{11} + \rho_{33} & \rho_{12} + \rho_{34} \\ \rho_{21} + \rho_{43} & \rho_{22} + \rho_{44} \end{pmatrix}
$$

((Reduced density operator $\hat{\rho}_{23}$))

$$
\hat{\rho}_{23} = Tr_1[\hat{\rho}] = \begin{pmatrix}
\rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\
\rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\
\rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\
\rho_{41} & \rho_{42} & \rho_{43} & \rho_{44}
\end{pmatrix} + \begin{pmatrix}
\rho_{55} & \rho_{56} & \rho_{57} & \rho_{58} \\
\rho_{65} & \rho_{66} & \rho_{67} & \rho_{68} \\
\rho_{75} & \rho_{76} & \rho_{77} & \rho_{78} \\
\rho_{85} & \rho_{86} & \rho_{87} & \rho_{88}
\end{pmatrix}
$$

((Reduced density operator $\hat{\rho}_{13}$))

$$
\hat{\rho}_{13} = Tr_2[\hat{\rho}] = \begin{pmatrix}
\rho_{11} & \rho_{12} & \rho_{15} & \rho_{16} \\
\rho_{21} & \rho_{22} & \rho_{25} & \rho_{26} \\
\rho_{51} & \rho_{52} & \rho_{55} & \rho_{56} \\
\rho_{61} & \rho_{62} & \rho_{65} & \rho_{66}
\end{pmatrix} + \begin{pmatrix}
\rho_{33} & \rho_{34} & \rho_{37} & \rho_{38} \\
\rho_{43} & \rho_{44} & \rho_{47} & \rho_{48} \\
\rho_{73} & \rho_{74} & \rho_{77} & \rho_{78} \\
\rho_{83} & \rho_{84} & \rho_{87} & \rho_{88}
\end{pmatrix}
$$

((Reduced density operator $\hat{\rho}_{12}$))

$$
\hat{\rho}_{12} = Tr_3[\hat{\rho}] = \begin{pmatrix}
\rho_{11} & \rho_{13} & \rho_{15} & \rho_{17} \\
\rho_{31} & \rho_{33} & \rho_{35} & \rho_{37} \\
\rho_{51} & \rho_{53} & \rho_{55} & \rho_{57} \\
\rho_{71} & \rho_{73} & \rho_{75} & \rho_{77}
\end{pmatrix} + \begin{pmatrix}
\rho_{22} & \rho_{24} & \rho_{26} & \rho_{28} \\
\rho_{42} & \rho_{44} & \rho_{46} & \rho_{48} \\
\rho_{62} & \rho_{64} & \rho_{66} & \rho_{68} \\
\rho_{82} & \rho_{84} & \rho_{86} & \rho_{88}
\end{pmatrix}
$$