2D Gabor functions and filters for image processing and computer vision

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Most of the images in this presentation were generated with the on-line simulation programs available at:

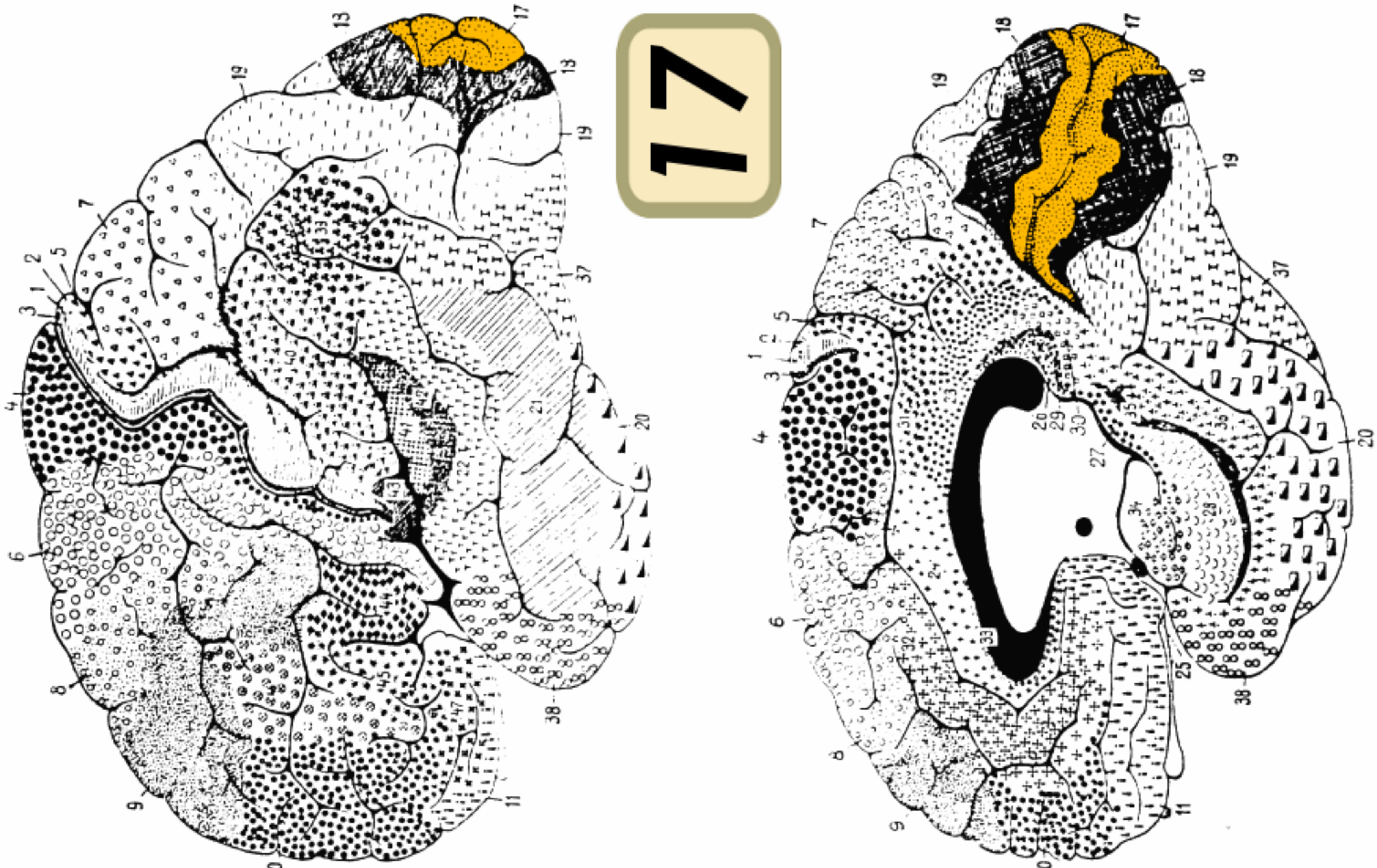
http://matlabserver.cs.rug.nl

Neurophysiologic background

university of groningen

Primary visual cortex (striate cortex or V1)

Brodmann area 17



Wikipedia.org

References to origin

Simple and complex cells:
respond to bars of given orientation

D.H. Hubel and T.N. Wiesel: Receptive fields, binocular interaction and functional architecture in the cat's visual cortex, Journal of Physiology (London), vol. 160, pp. 106--154, 1962.

D.H. Hubel and T.N. Wiesel: Sequence regularity and geometry of orientation columns in the monkey striate cortex, Journal of Computational Neurology, vol. 158, pp. 267--293, 1974.

D.H. Hubel: Exploration of the primary visual cortex, 1955-78, Nature, vol. 299, pp. 515--524, 1982.

Simple cells
and
Gabor filters

(or a Platonic view of reality)

Simple cells

Hubel and Wiesel named one type of cell "simple" because they shared the following properties:

• Their receptive fields have distinct excitatory and inhibitory regions.

• These regions follow the summation property.

• These regions have mutual antagonism - excitatory and inhibitory regions balance themselves out in diffuse lighting.

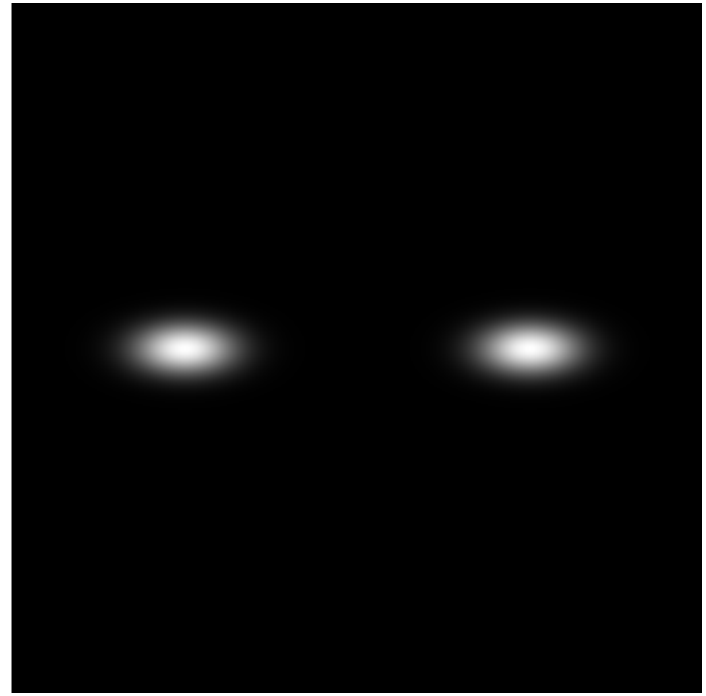
• It is possible to predict responses to stimuli given the map of excitatory and inhibitory regions.

In engineering terms:
a simple cell can be characterized by an impulse response.

Receptive field profiles of simple cells

Space domain

Frequency domain



How are they determined?

- recording responses to bars
- recording responses to gratings
- reverse correlation (spike-triggered average)

Why do simple cells respond to bars and gratings of given orientation?

References to origins – modeling

1D:

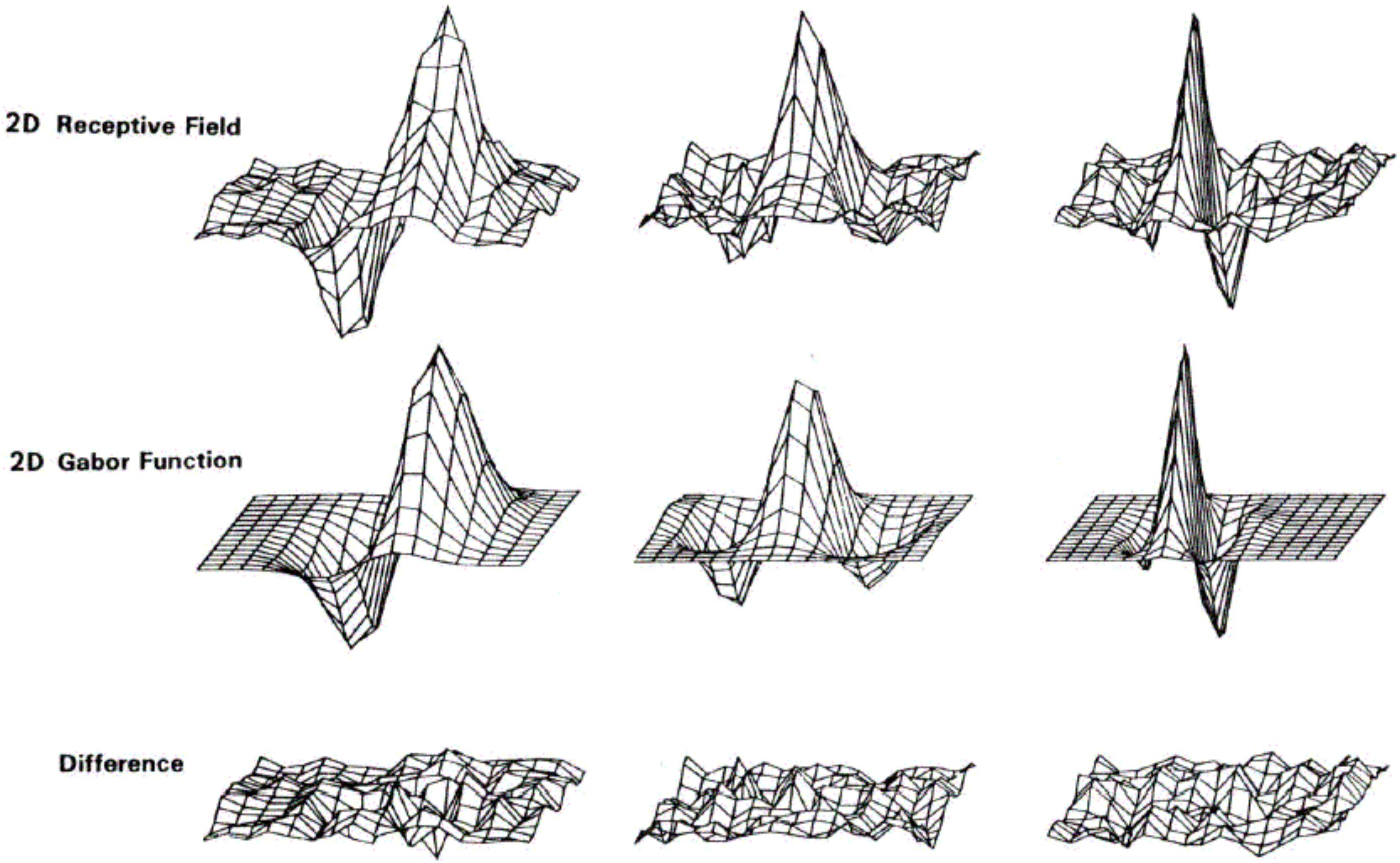
S. Marcelja: Mathematical description of the responses of simple cortical cells. Journal of the Optical Society of America 70, 1980, pp. 1297-1300.

2D:

J.G. Daugman: Uncertainty relations for resolution in space, spatial frequency, and orientation optimized by two-dimensional visual cortical filters, Journal of the Optical Society of America A, 1985, vol. 2, pp. 1160-1169.

J.P. Jones and A. Palmer: An evaluation of the two-dimensional Gabor filter model of simple receptive fields in cat striate cortex, Journal of Neurophysiology, vol. 58, no. 6, pp. 1233--1258, 1987

References to origins – modeling



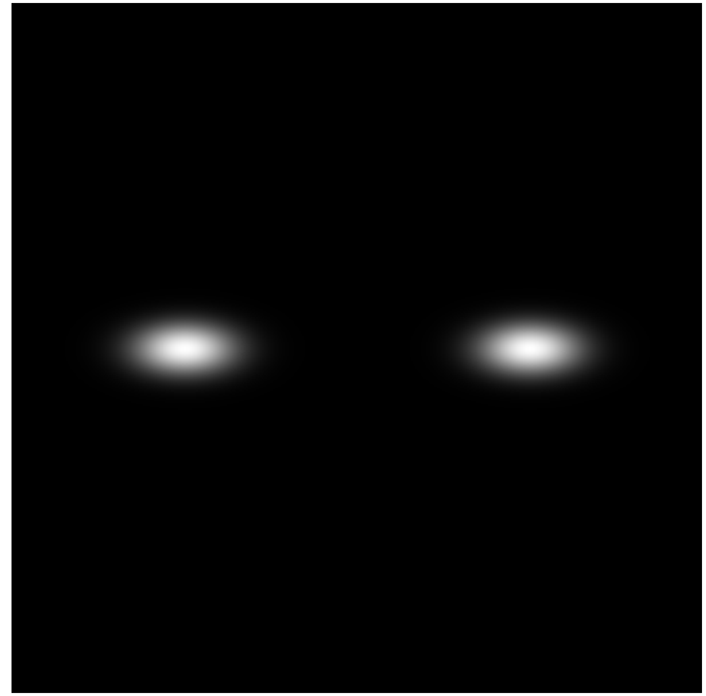
J.P. Jones and A. Palmer: An evaluation of the two-dimensional Gabor filter model of simple receptive fields in cat striate cortex, Journal of Neurophysiology, vol. 58, no. 6, pp. 1233--1258, 1987

2D Gabor functions

2D Gabor function

Space domain

Frequency domain



$$g_{\lambda,\theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda}+\varphi\right) \qquad (1)$$

$$\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}$$

Parameterization according to:

N. Petkov: Biologically motivated computationally intensive approaches to image pattern recognition, *Future Generation Computer Systems*, **11** (4-5), 1995, 451-465.

N. Petkov and P. Kruizinga: Computational models of visual neurons specialised in the detection of periodic and aperiodic oriented visual stimuli: bar and grating cells, *Biological Cybernetics*, **76** (2), 1997, 83-96.

P. Kruizinga and N. Petkov: Non-linear operator for oriented texture, *IEEE Trans. on Image Processing*, **8** (10), 1999, 1395-1407.

S.E. Grigorescu, N. Petkov and P. Kruizinga: Comparison of texture features based on Gabor filters, *IEEE Trans. on Image Processing*, **11** (10), 2002, 1160-1167.

N. Petkov and M. A. Westenberg: Suppression of contour perception by band-limited noise and its relation to non-classical receptive field inhibition, *Biological Cybernetics*, **88**, 2003, 236-246.

C. Grigorescu, N. Petkov and M. A. Westenberg: Contour detection based on nonclassical receptive field inhibition, *IEEE Trans. on Image Processing*, **12** (7), 2003, 729-739.

http://www.cs.rug.nl/~petkov/publications/journals

Preferred spatial frequency (1/*λ*) and size (*σ*)

$$g_{\lambda,\Theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda}+\varphi\right) \quad (1)$$

$$\begin{aligned} x' &= x\cos\Theta + y\sin\Theta \\ y' &= -x\sin\Theta + y\cos\Theta \end{aligned}$$

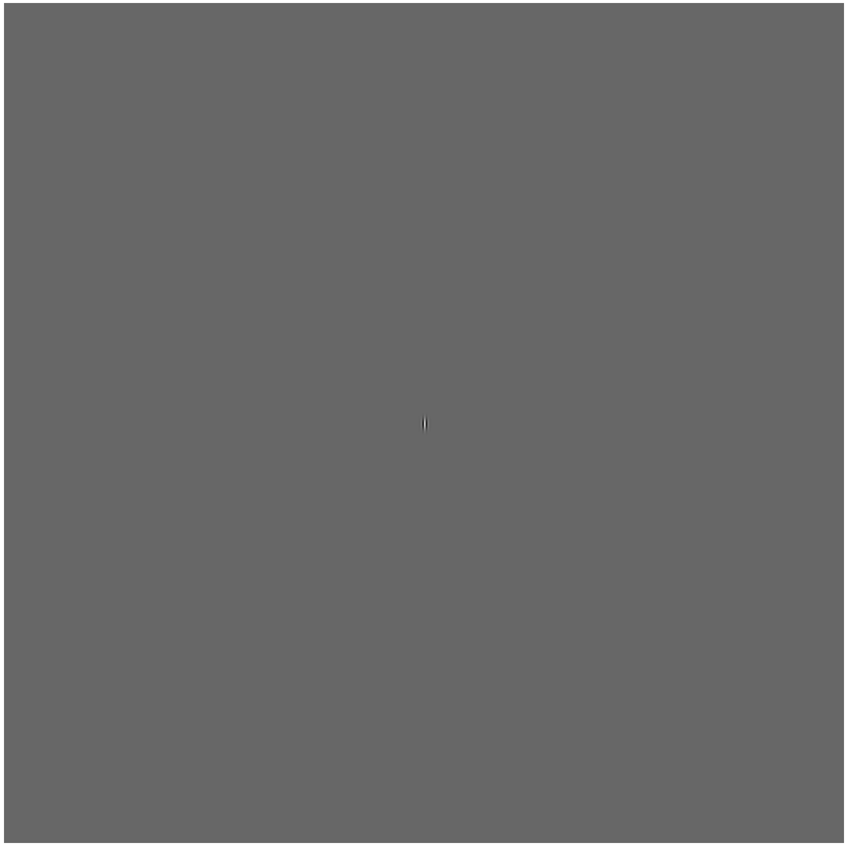
Preferred spatial frequency (1/*λ*) and size (*σ*) are not completely independent:

$$\boldsymbol{\sigma = a\lambda}$$

with *a* between 0.3 and 0.6 for most cells.
In the following, we use mostly *σ* = 0.56*λ*.

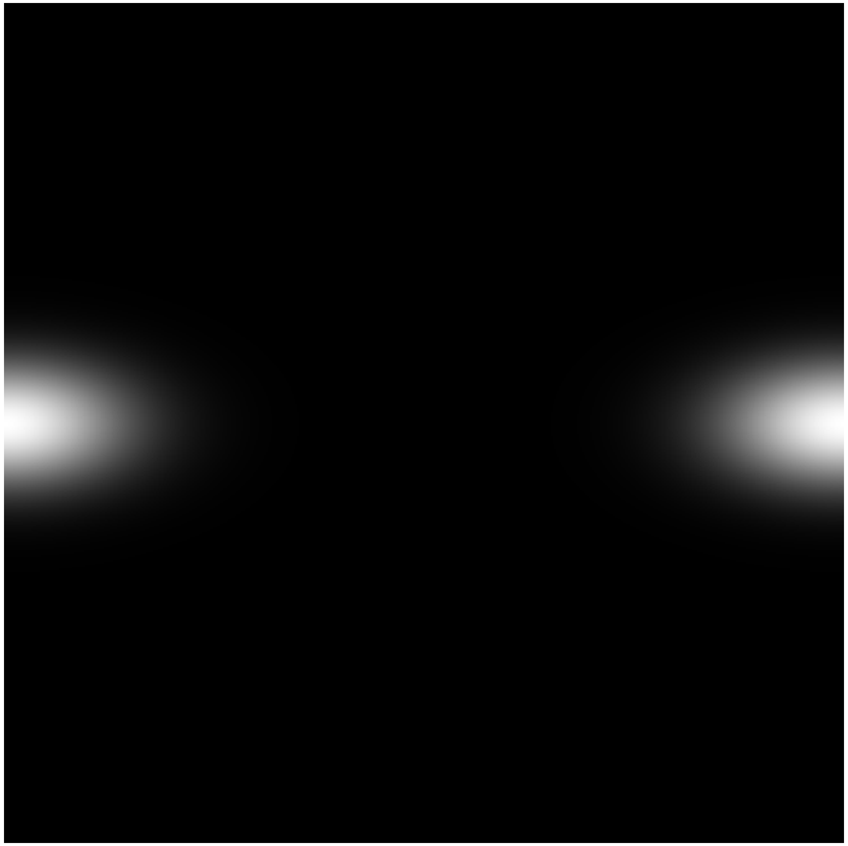
Preferred spatial frequency and size

Space domain



Wavelength = 2/512

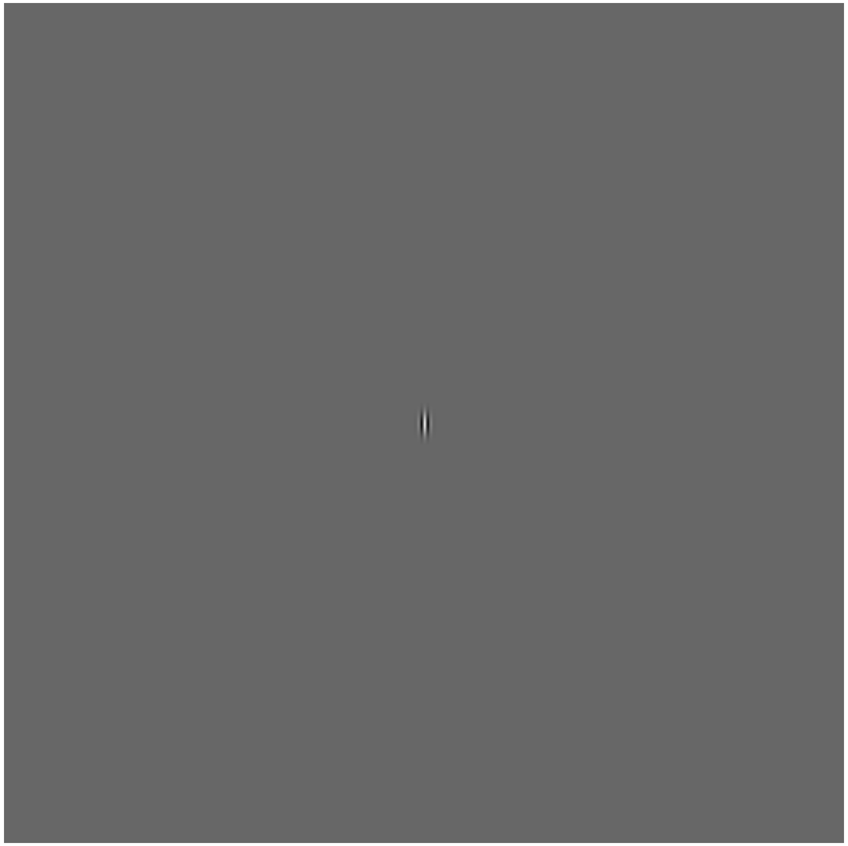
Frequency domain



Frequency = 512/2

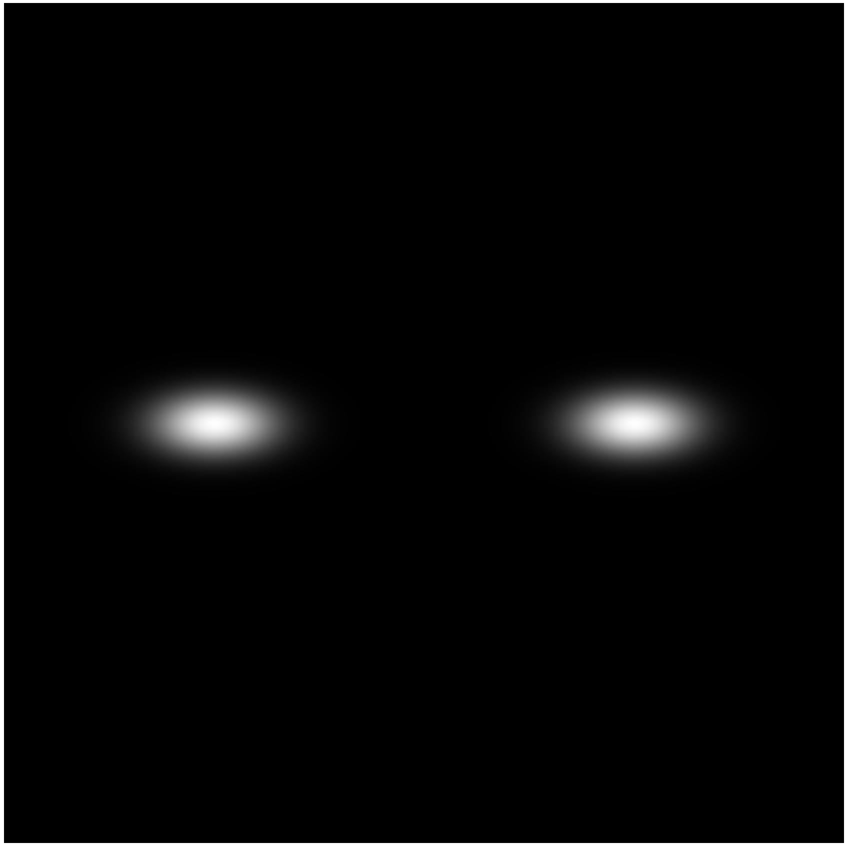
Preferred spatial frequency and size

Space domain



Wavelength = 4/512

Frequency domain



Frequency = 512/4

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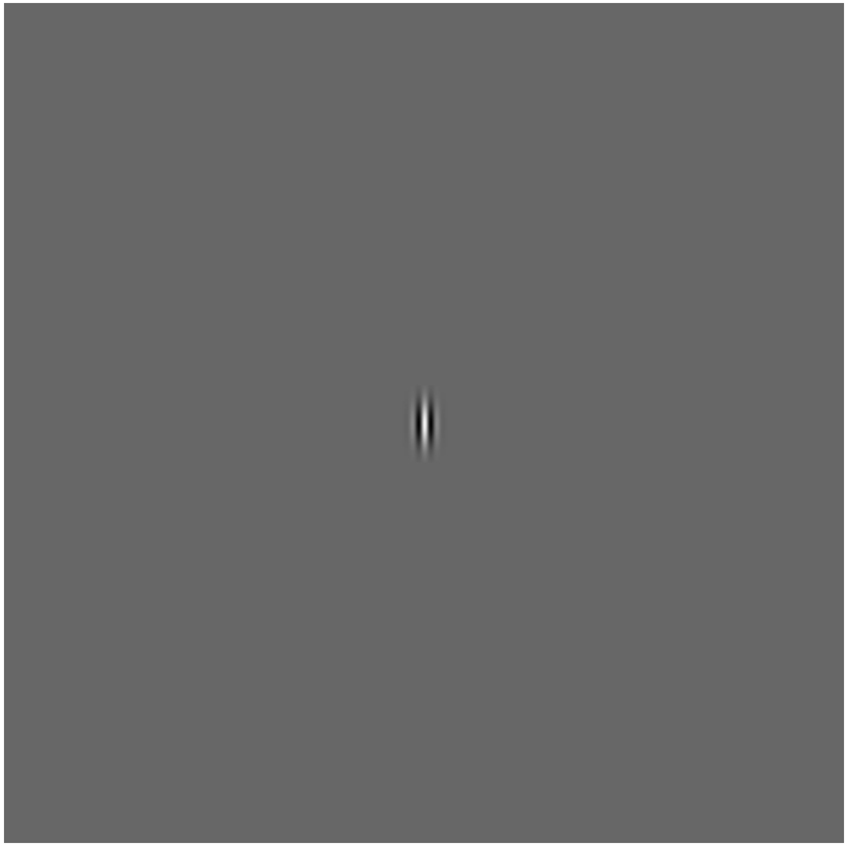
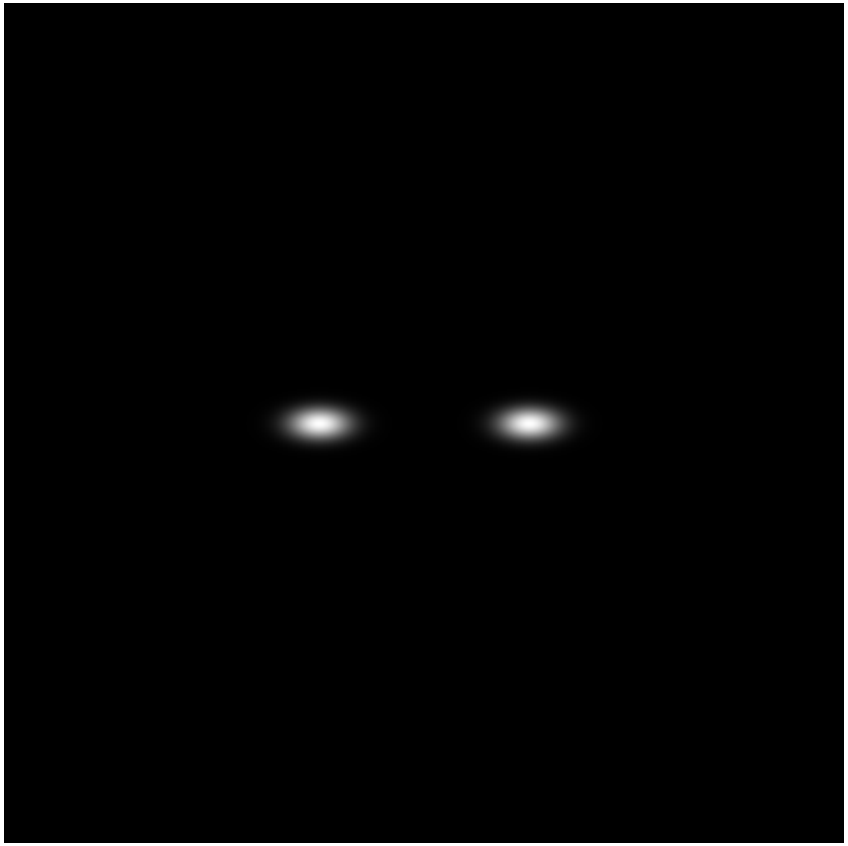
Preferred spatial frequency and size

Space domain



Wavelength = 8/512

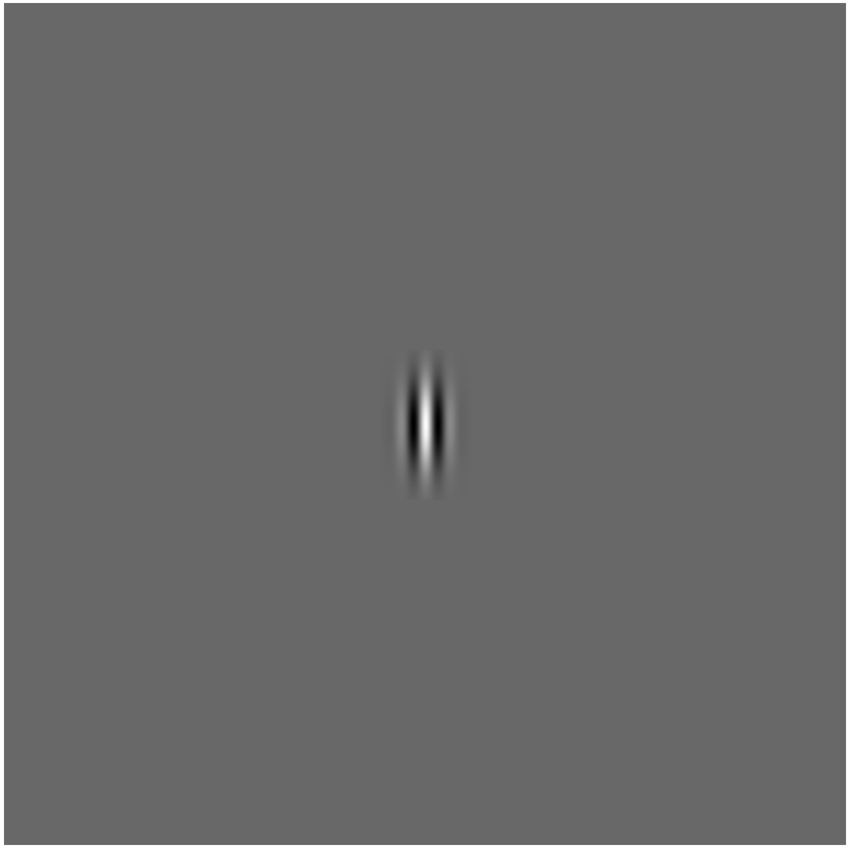
Frequency domain



Frequency = 512/8

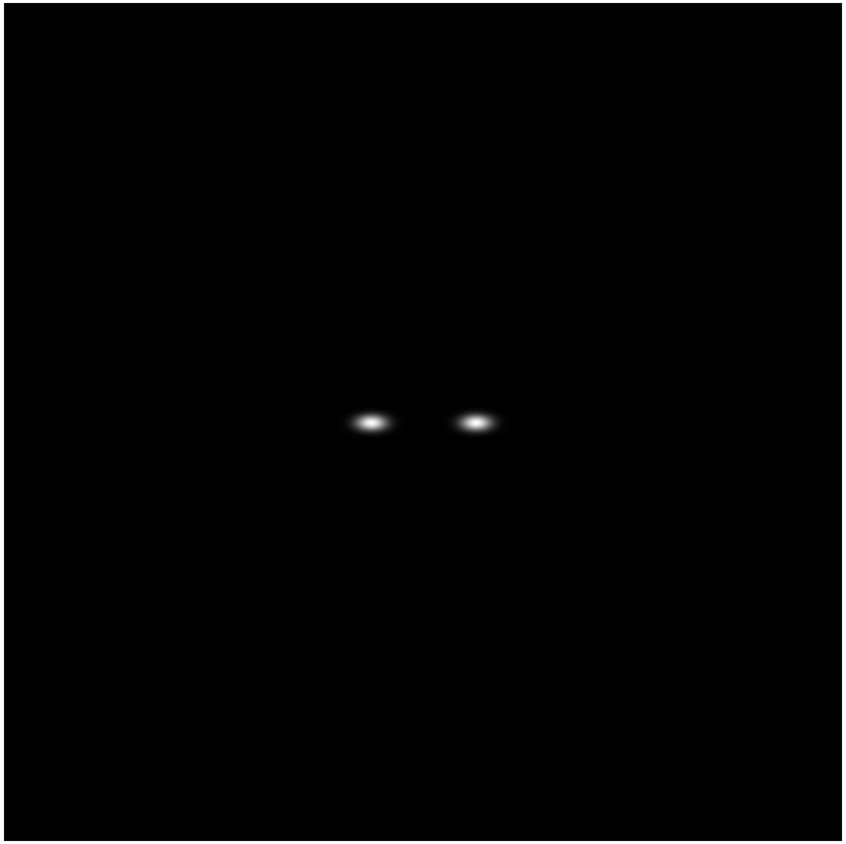
Preferred spatial frequency and size

Space domain



Wavelength = 16/512

Frequency domain



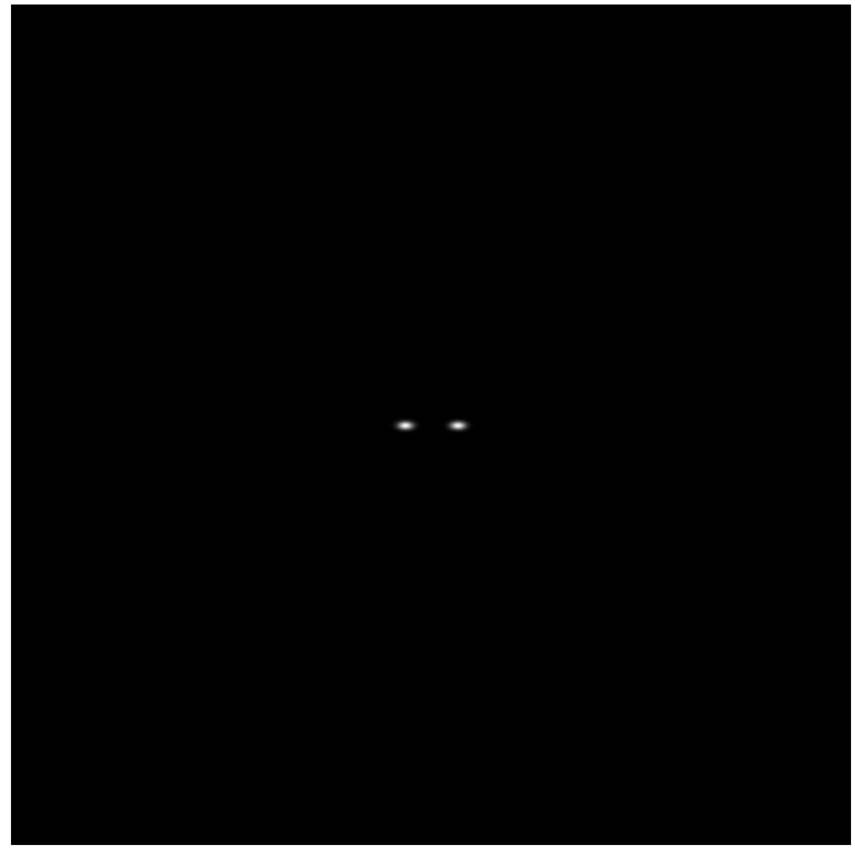
Frequency = 512/16

Preferred spatial frequency and size

Space domain

Wavelength = 32/512

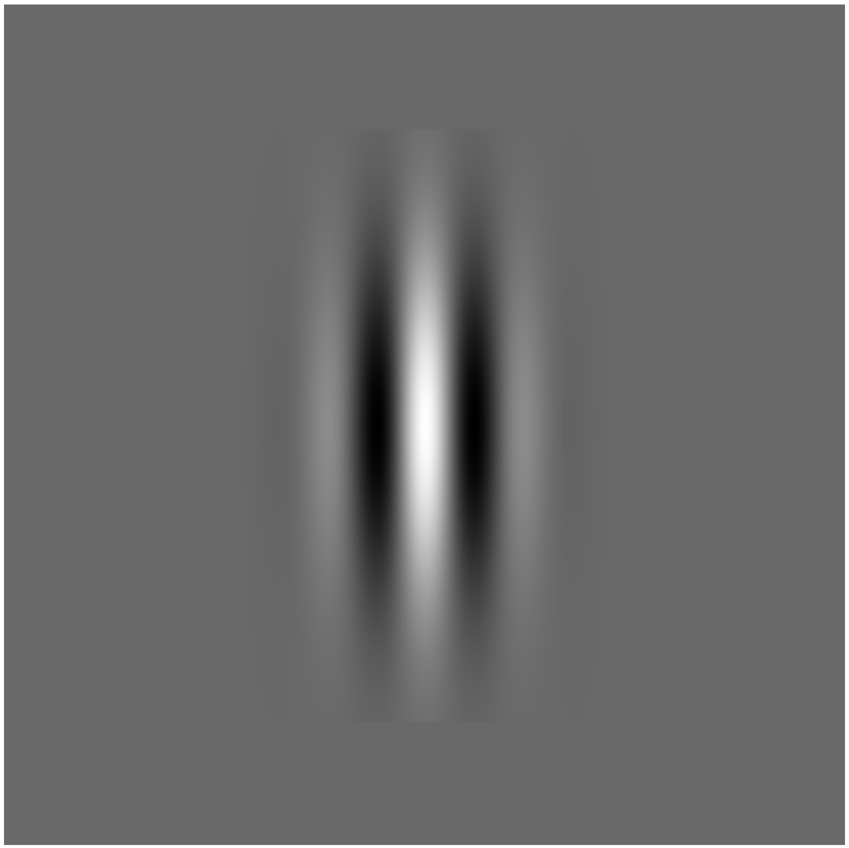
Frequency domain



Frequency = 512/32

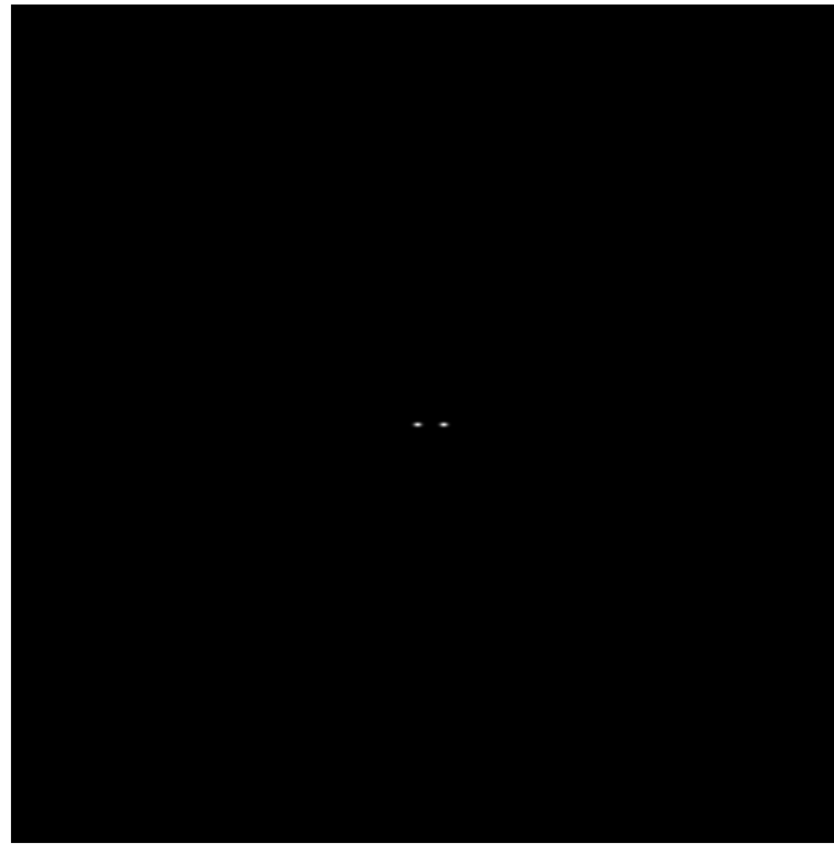
Preferred spatial frequency and size

Space domain



Wavelength = 64/512

Frequency domain



Frequency = 512/64

Orientation (θ)

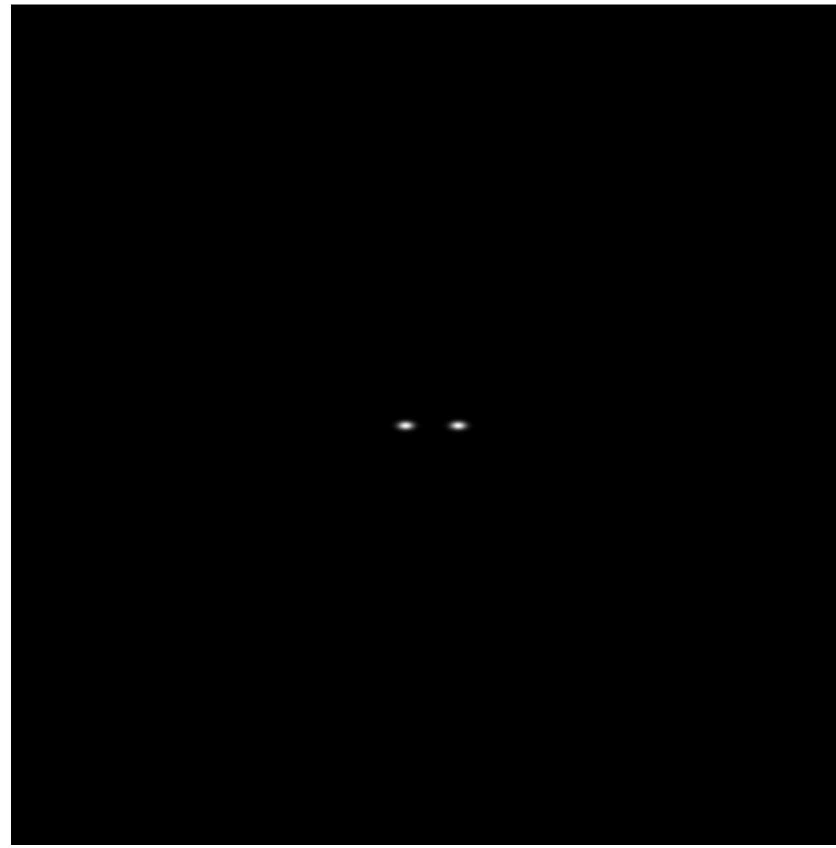
$$g_{\lambda,\Theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda} + \varphi\right) \quad (1)$$

$$\begin{aligned} x' &= \quad x\cos\Theta + y\sin\Theta \\ y' &= -x\sin\Theta + y\cos\Theta \end{aligned}$$

Orientation

Space domain

Frequency domain

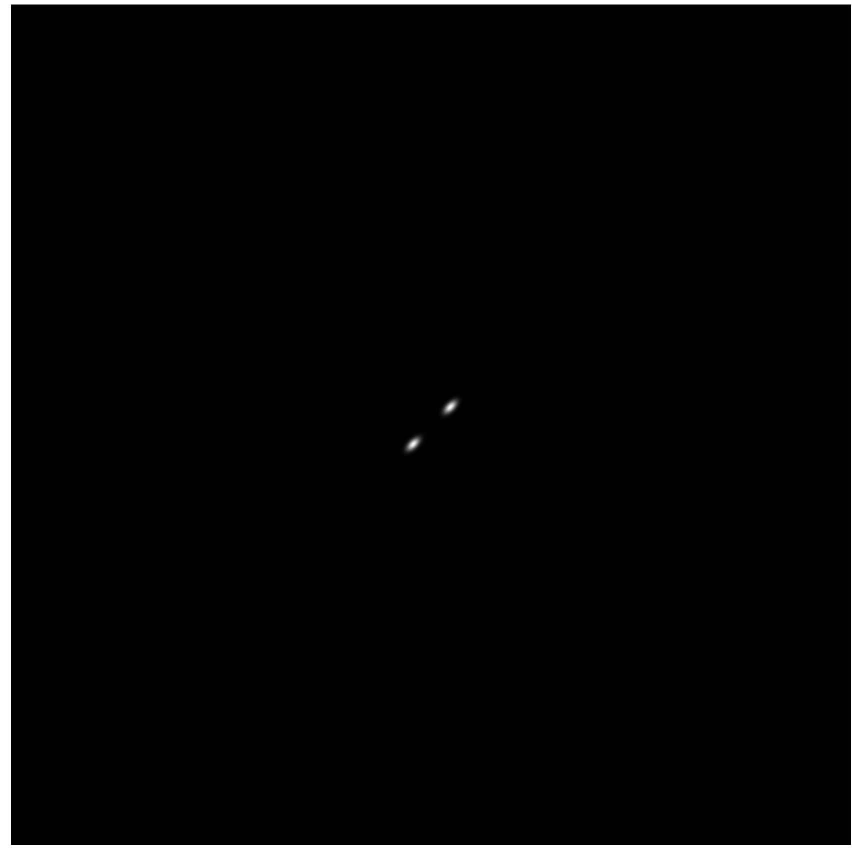


Orientation = 0

Orientation

Space domain

Frequency domain

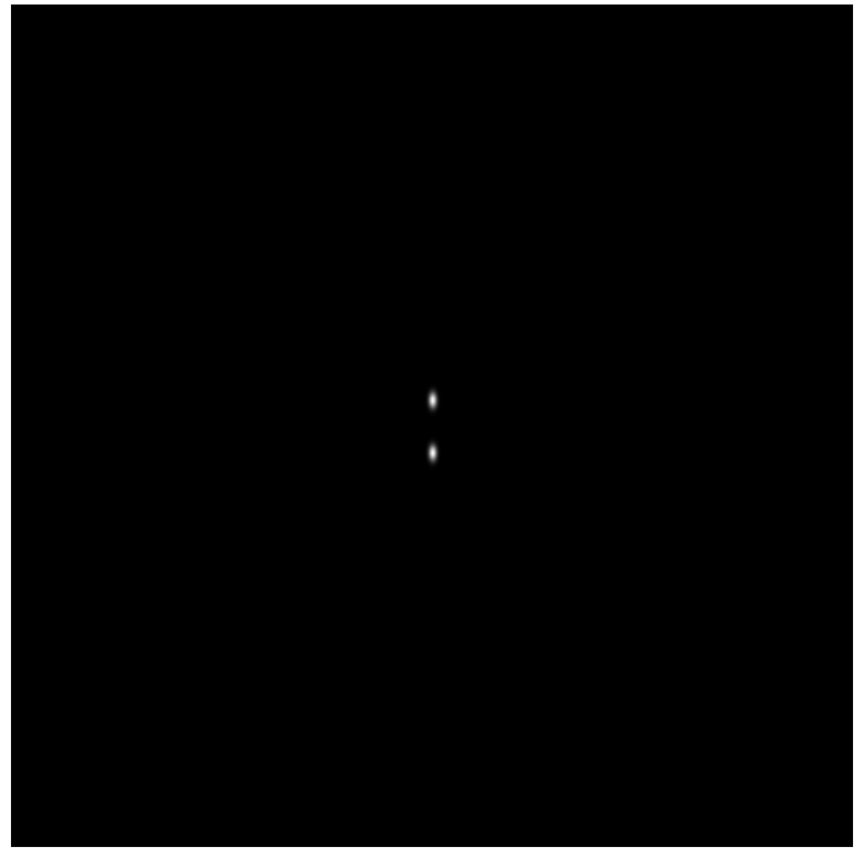


Orientation = 45

Orientation

Space domain

Frequency domain



Orientation = 90

Symmetry (phase offset φ)

$$g_{\lambda,\Theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda} + \varphi\right) \qquad (1)$$

$$\begin{aligned} x' &= x\cos\Theta + y\sin\Theta \\ y' &= -x\sin\Theta + y\cos\Theta \end{aligned}$$

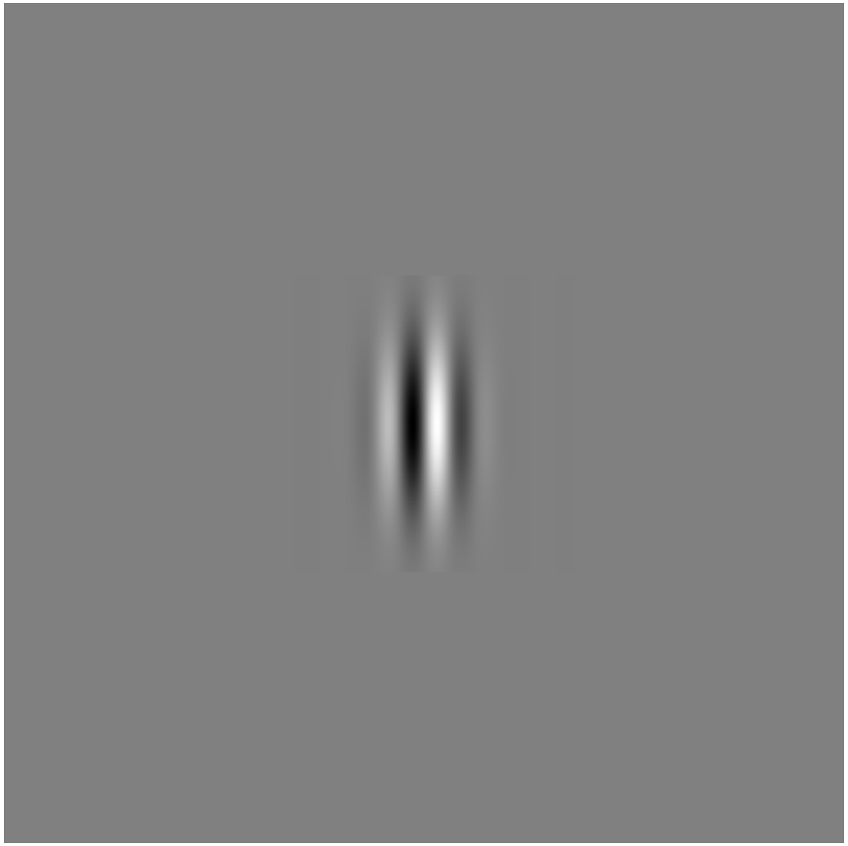
Symmetry (phase offset)

Space domain

Phase offset = 0

(symmetric function)

Space domain



Phase offset = -90

(anti-symmetric function)

Spatial aspect ratio (γ)

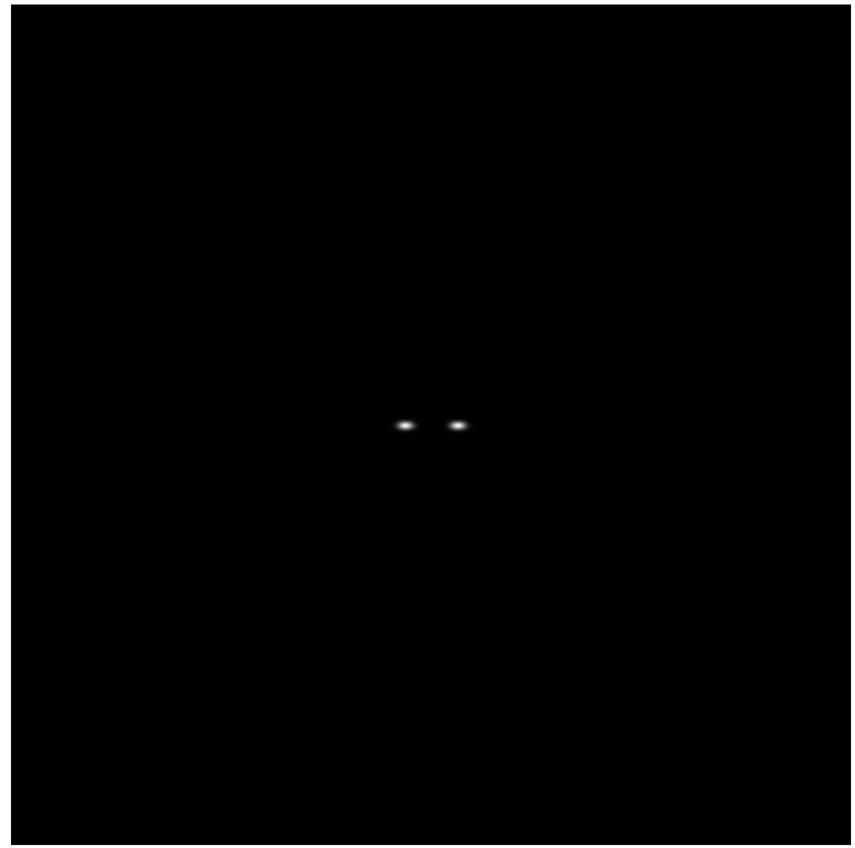
$$g_{\lambda,\Theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda} + \varphi\right) \quad (1)$$

$$\begin{aligned} x' &= \quad x\cos\Theta + y\sin\Theta \\ y' &= -x\sin\Theta + y\cos\Theta \end{aligned}$$

Spatial aspect ratio

Space domain

Frequency domain

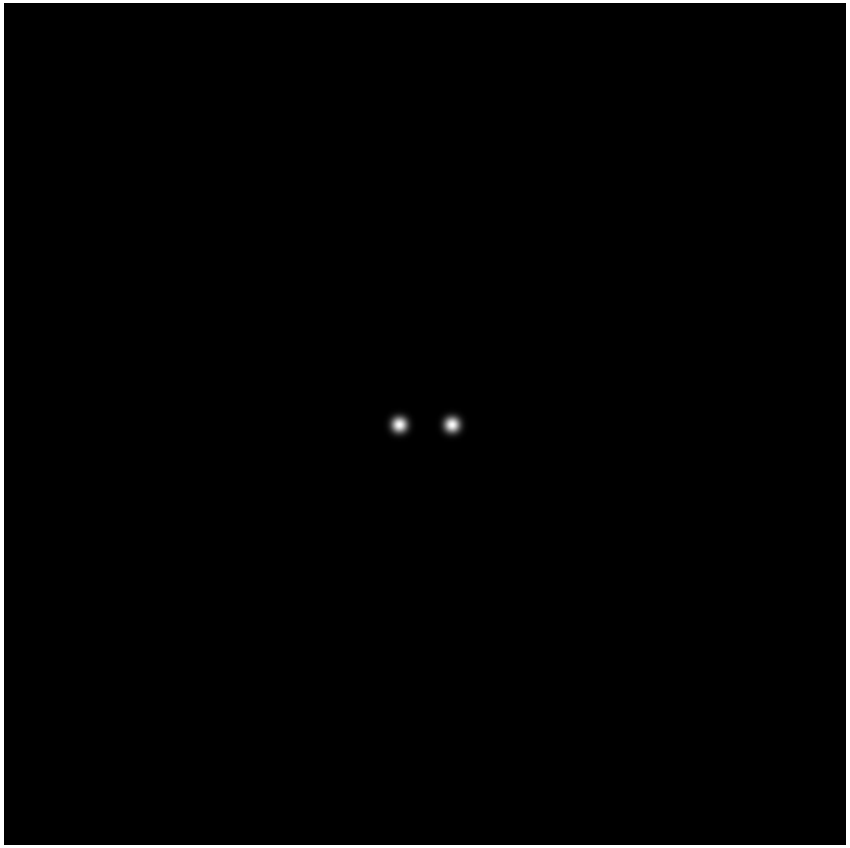


Aspect ratio = 0.5

Spatial aspect ratio

Space domain

Frequency domain



Aspect ratio = 1

Spatial aspect ratio

Space domain

Frequency domain

Aspect ratio = 2

(does not occur)

Bandwidth – related to the ratio *σ/λ*

Half-response spatial frequency bandwidth *b* (in octaves)

$$b = \log_2 \frac{\frac{\sigma}{\lambda}\pi + \sqrt{\frac{\ln 2}{2}}}{\frac{\sigma}{\lambda}\pi - \sqrt{\frac{\ln 2}{2}}}, \quad \frac{\sigma}{\lambda} = \frac{1}{\pi}\sqrt{\frac{\ln 2}{2}} \cdot \frac{2^b + 1}{2^b - 1} \qquad (2)$$

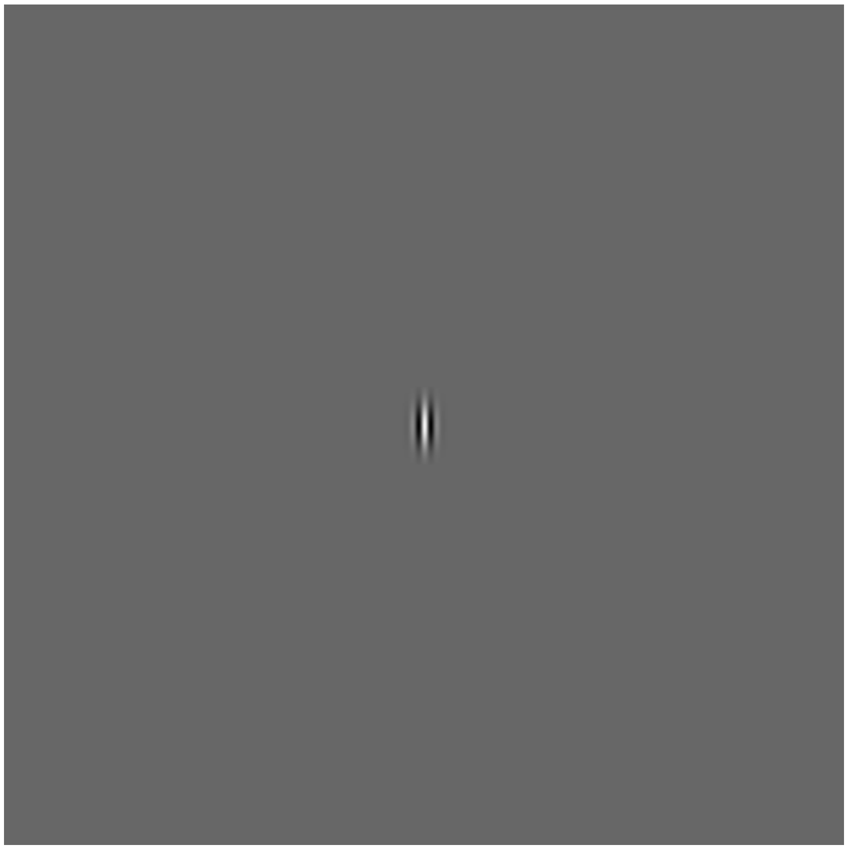
$$g_{\lambda,\Theta,\varphi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2 + \gamma^2 y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda} + \varphi\right) \qquad (1)$$

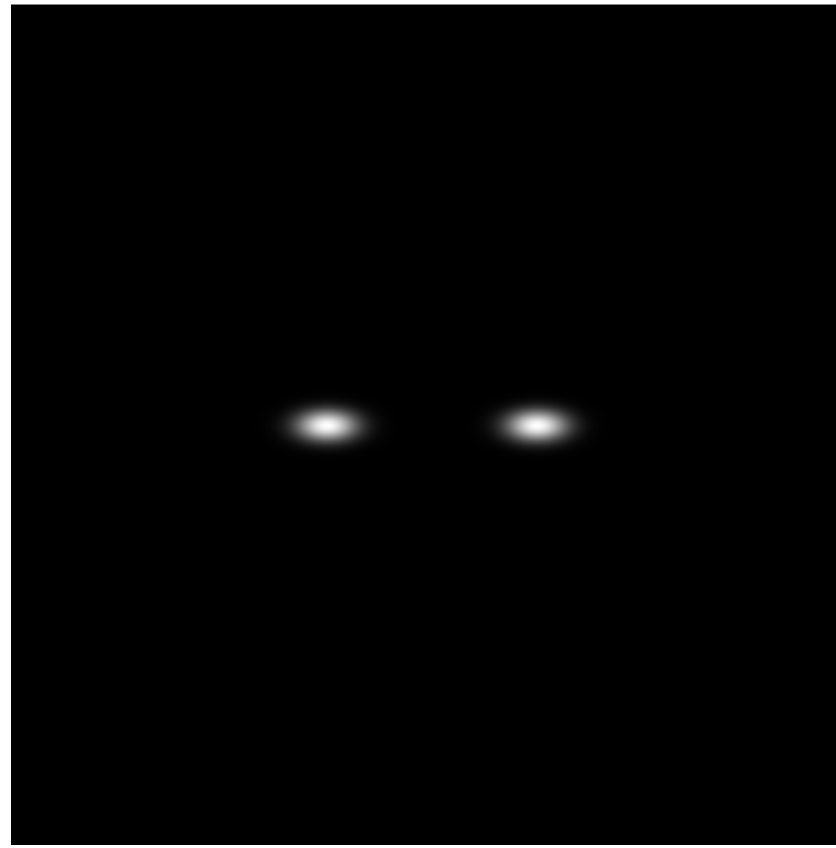
$$\begin{aligned} x' &= x\cos\Theta + y\sin\Theta \\ y' &= -x\sin\Theta + y\cos\Theta \end{aligned}$$

Bandwidth

Space domain

Frequency domain



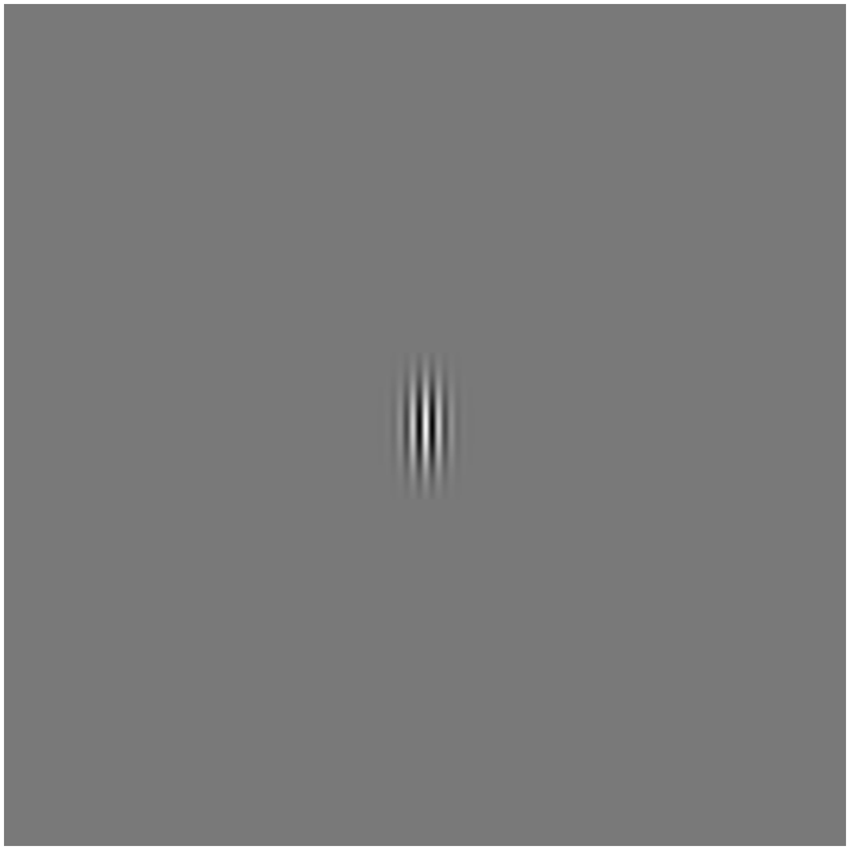


Bandwidth = 1 ($\sigma = 0.56\lambda$)

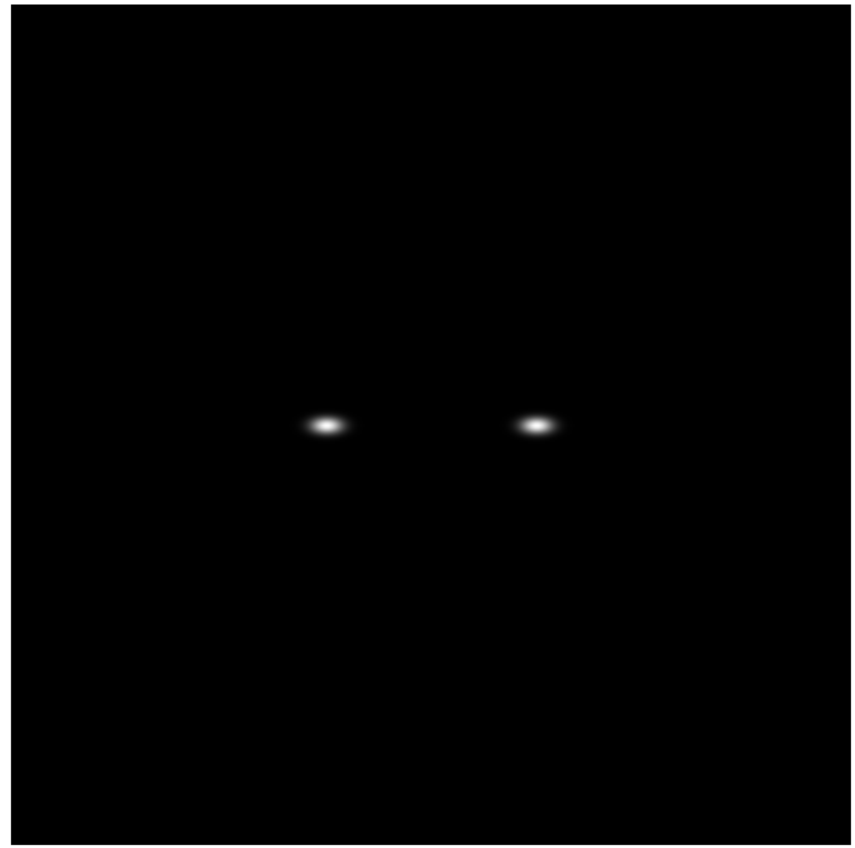
Wavelength = 8/512

Bandwidth

Space domain



Frequency domain

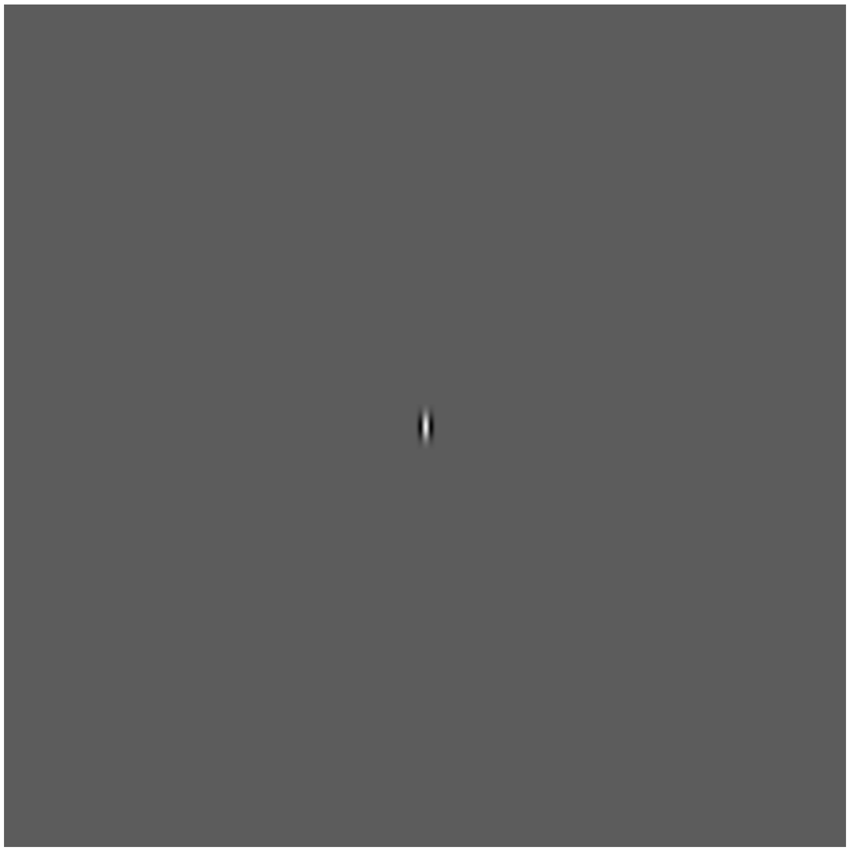


Bandwidth = 0.5

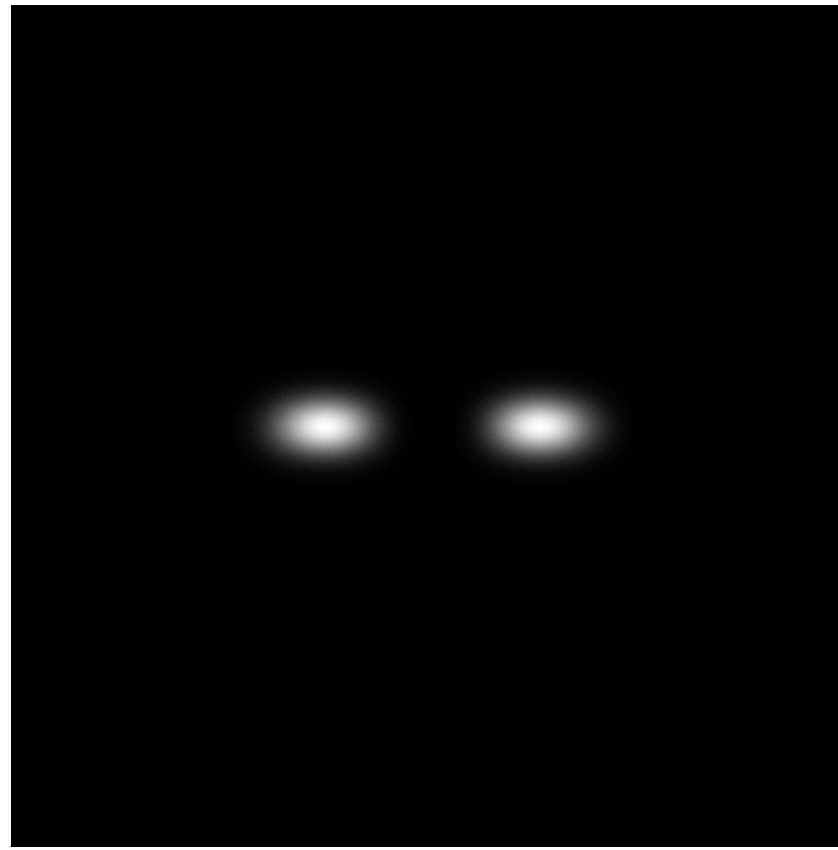
Wavelength = 8/512

Bandwidth

Space domain



Frequency domain



Bandwidth = 2

Wavelength = 8/512

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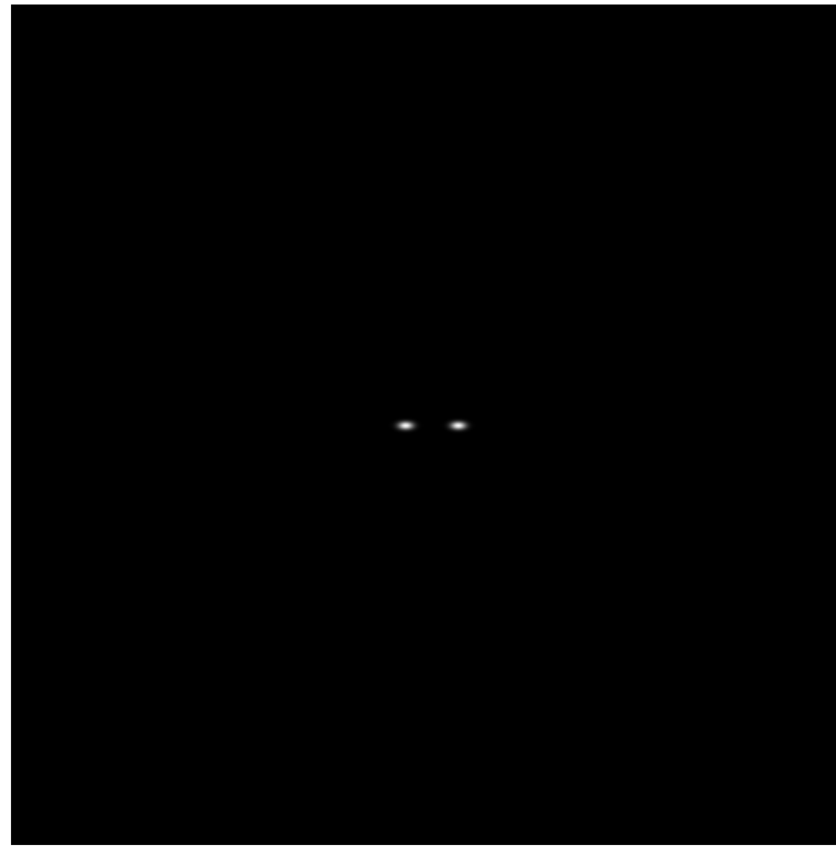
Bandwidth

Space domain

Frequency domain



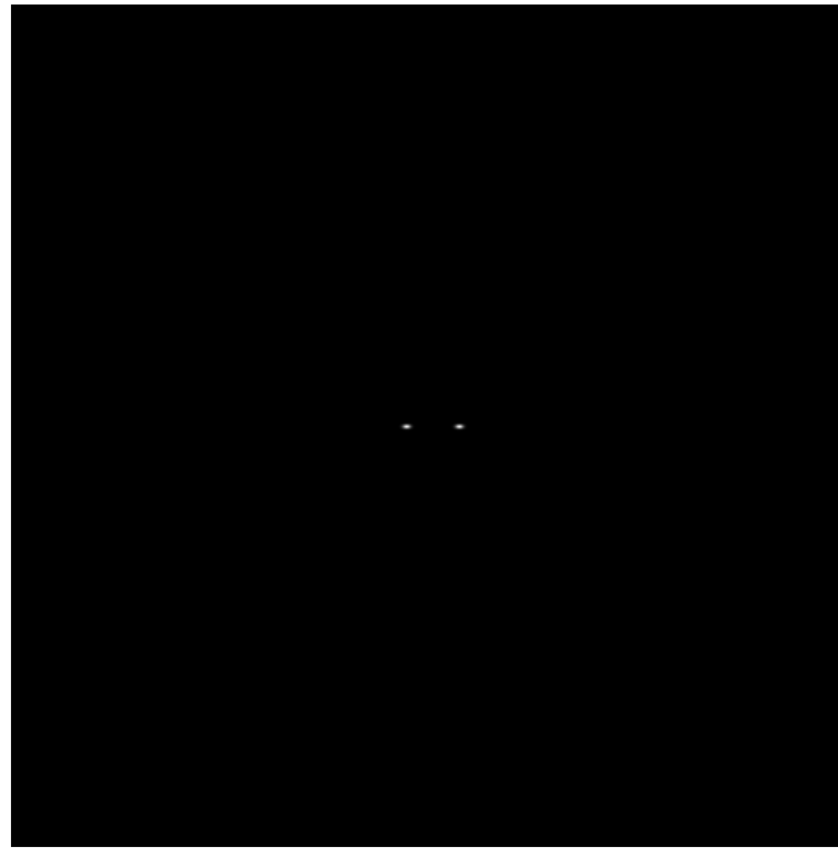
Bandwidth = 1 ($\sigma = 0.56\lambda$)

Wavelength = 32/512

Bandwidth

Space domain

Frequency domain



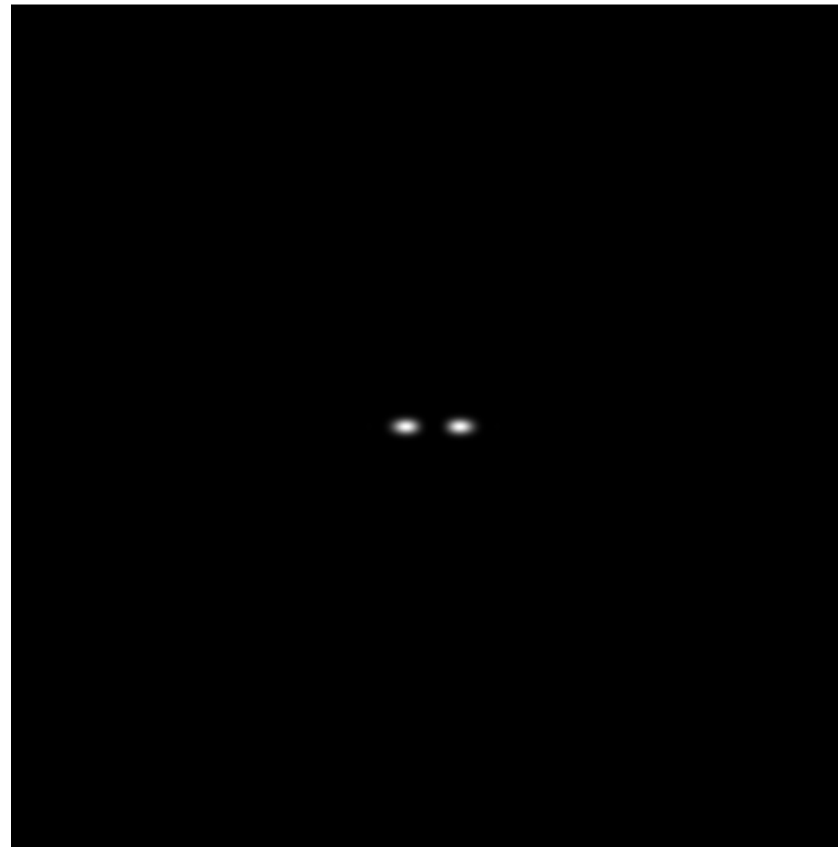
Bandwidth = 0.5

Wavelength = 32/512

Bandwidth

Space domain

Frequency domain



Bandwidth = 2

Wavelength = 32/512

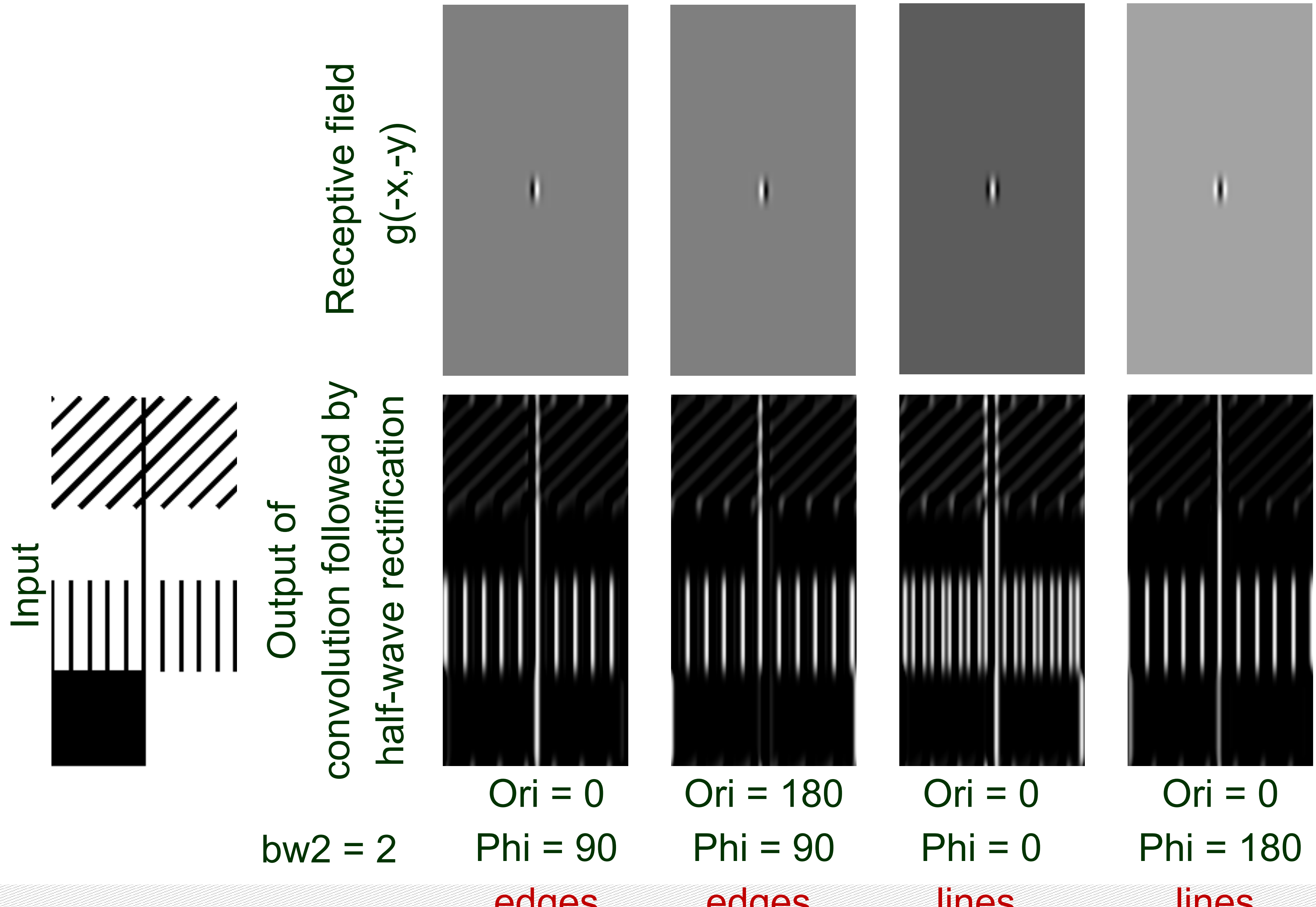
Semi-linear 2D Gabor filter

$$R = | g * I |^{+}$$

i.e., the response R is obtained by convolution (*) of the input I with a Gabor function g, followed by half-wave rectification ($|.|^{+}$)

Semi-linear Gabor filter

What is it useful for?

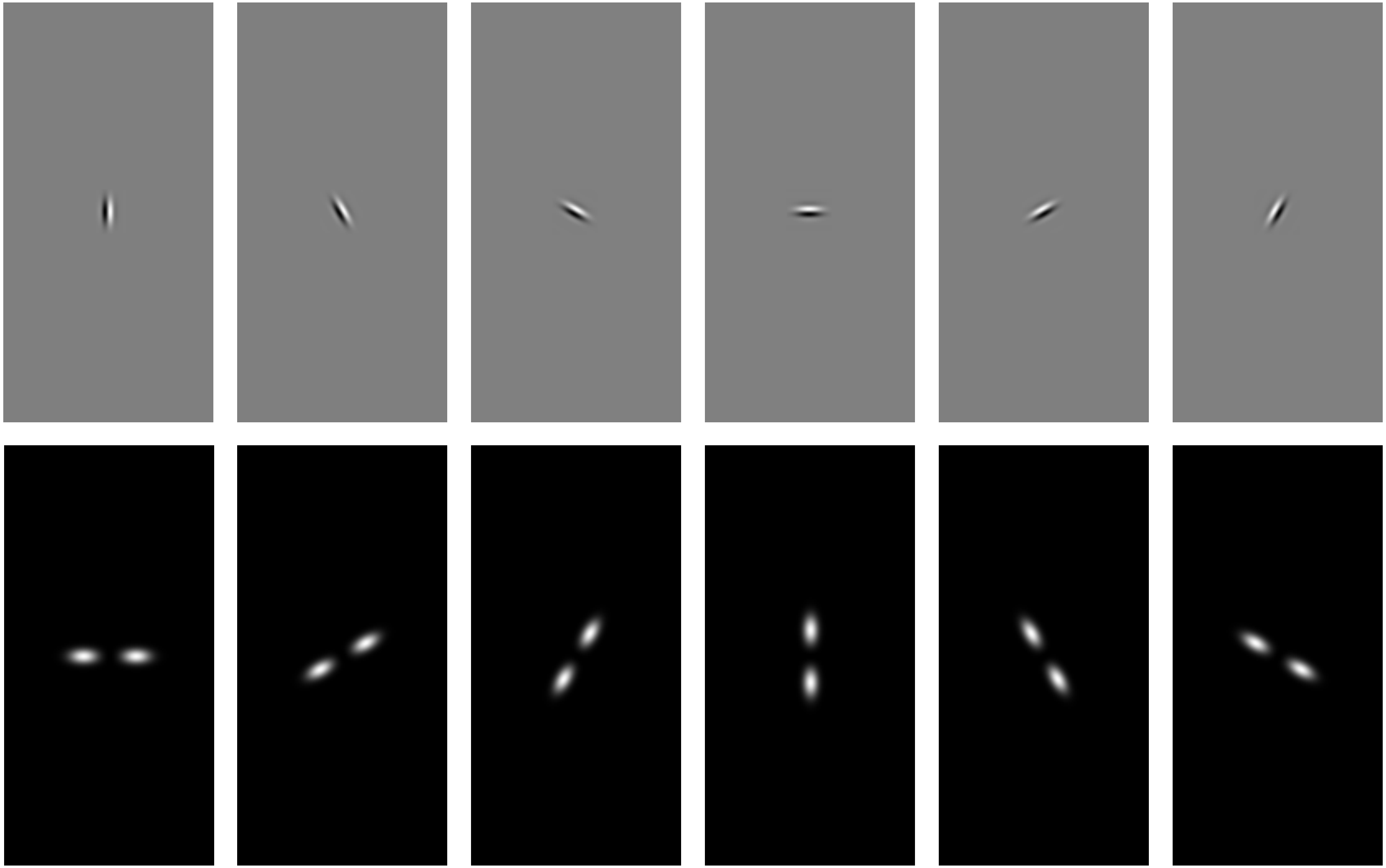


Bank of semi-linear Gabor filters

Which (an how many) orientations to use?

Receptive field g(-x,-y)

frequency domain



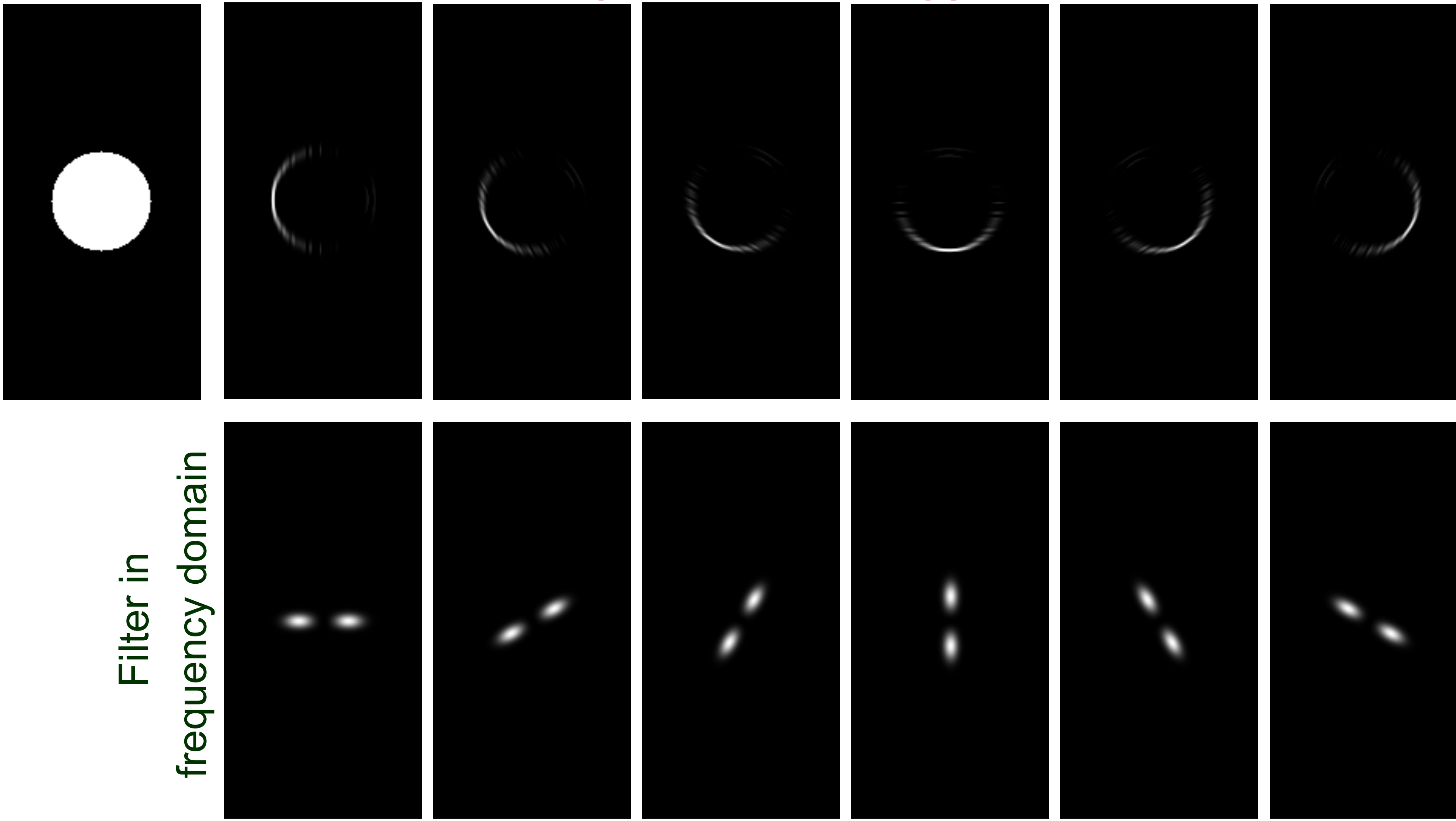
Ori = 0 30 60 90 120 150

For filters with s.a.r=0.5 and bw=2, good coverage of angles with 6 orientations

Bank of semi-linear Gabor filters

Which (and how many) orientations to use?

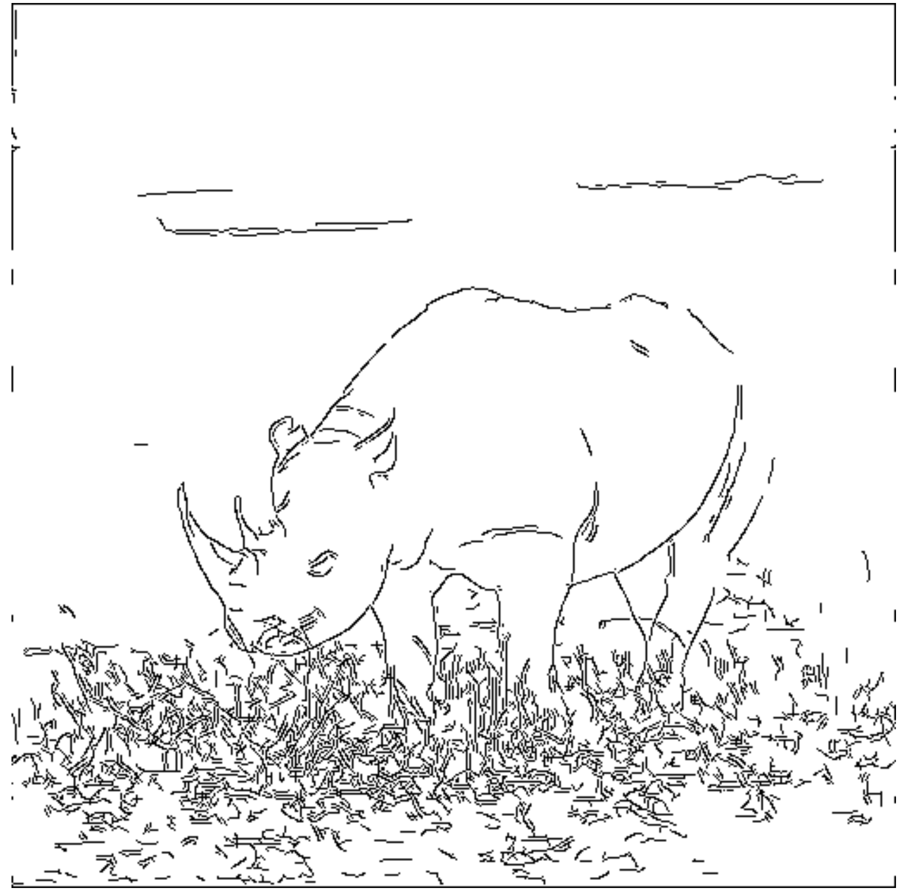
Input Output



Ori = 0 30 60 90 120 150

For filters with sar=0.5 and bw=2, good coverage of angles with 12 orientations

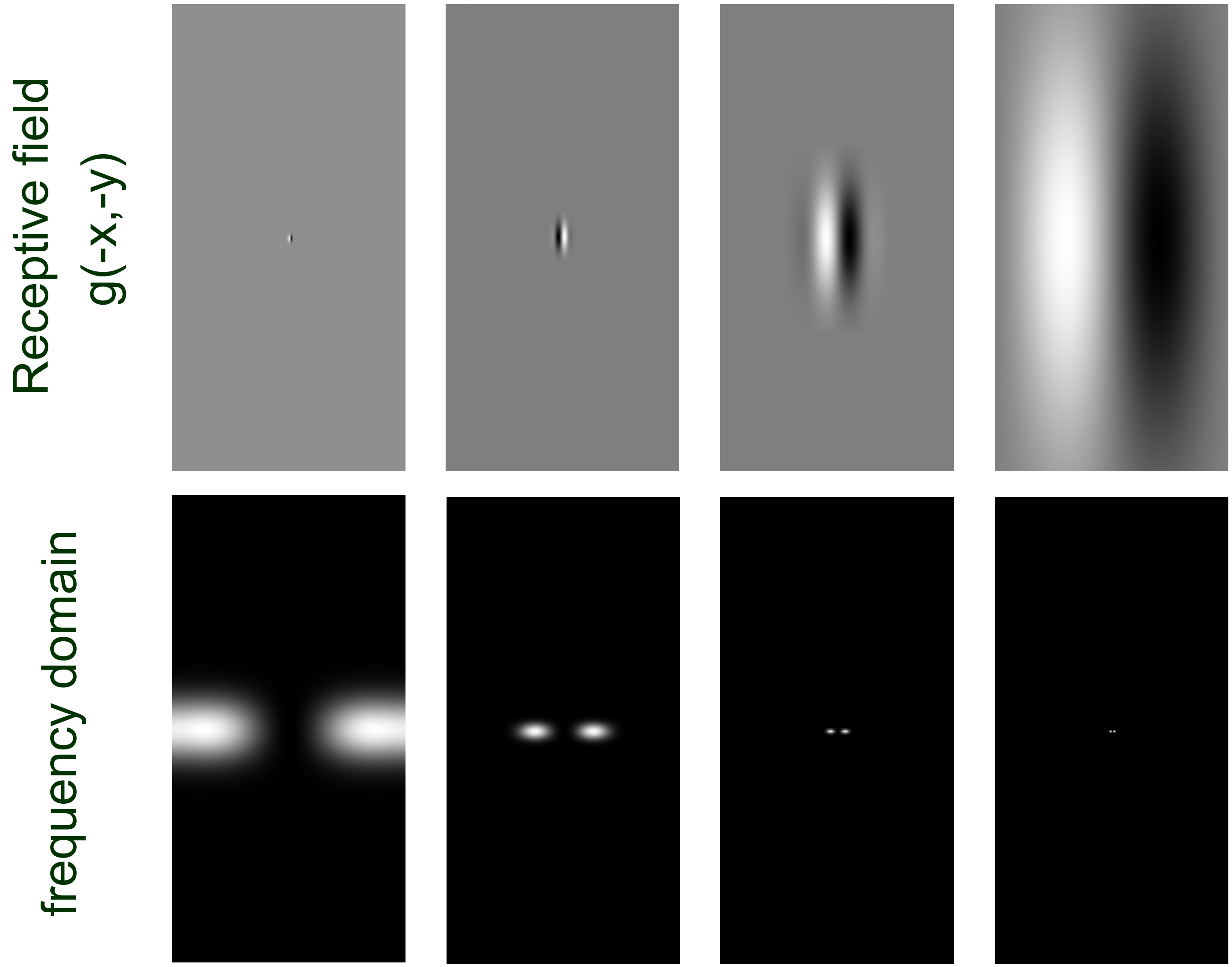
Bank of semi-linear Gabor filters



Result of superposition of the outputs of 12 semi-linear anti-symmetric (phi=90) Gabor filters with wavelength = 4, bandwidth = 2, spatial aspect ratio = 0.5 (after thinning and thresholding lt = 0.1, ht = 0.15).

Bank of semi-linear Gabor filters

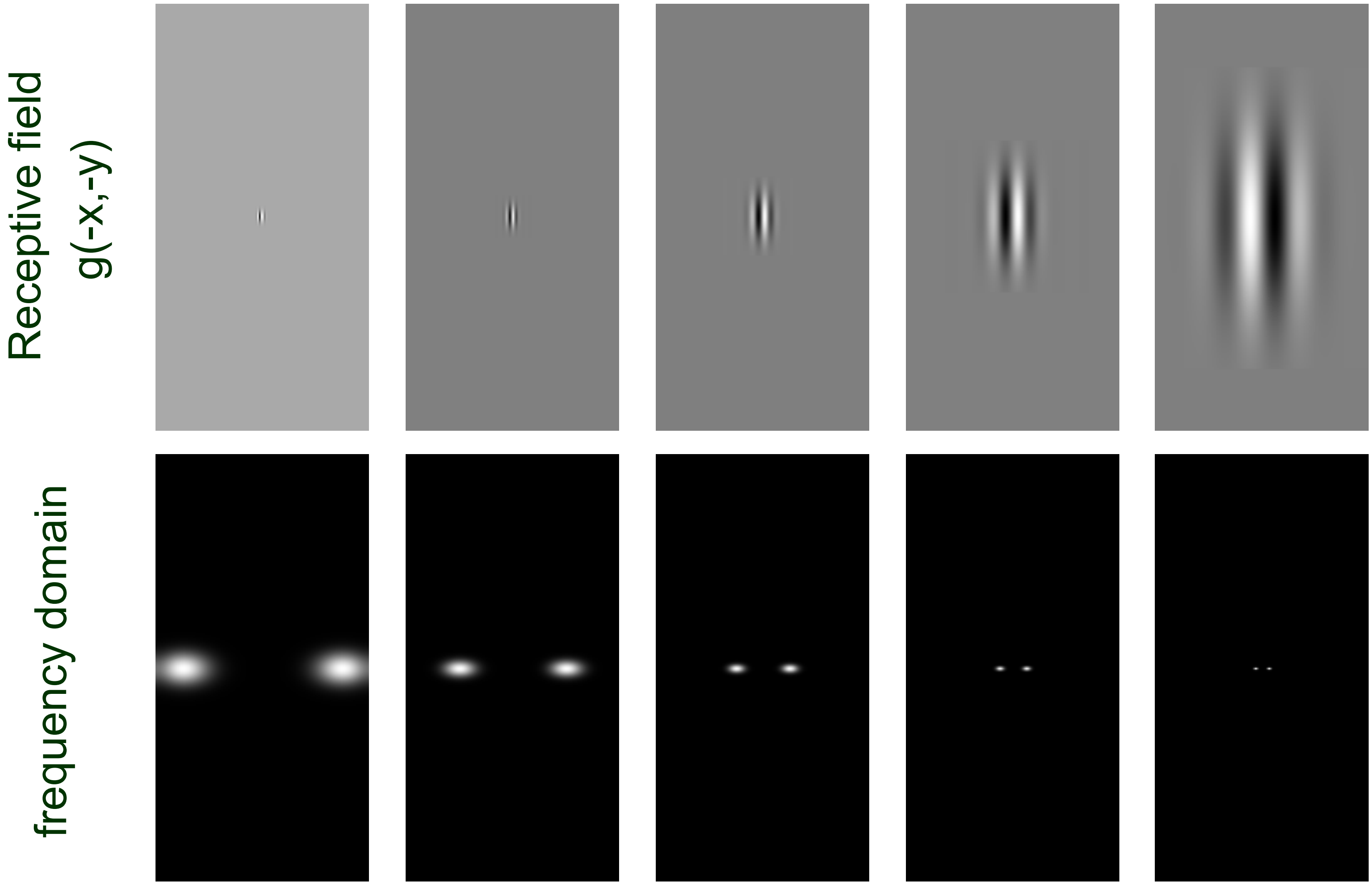
Which (and how many) frequencies to use?



Wavelength = 2 8 32 128 (s.a.r.=0.5)

For filters with bw=2, good coverage of frequencies with wavelength quadroppling

Bank of semi-linear Gabor filters



Wavelength = 2 4 8 16 32

For filters with bw=1, good coverage of frequencies with wavelength doubling

Complex cells
and
Gabor energy filters

References to origin - neurophysiology

Simple and complex cells:
respond to bars of given orientation

D.H. Hubel and T.N. Wiesel: Receptive fields, binocular interaction and functional architecture in the cat's visual cortex, Journal of Physiology (London), vol. 160, pp. 106--154, 1962.

D.H. Hubel and T.N. Wiesel: Sequence regularity and geometry of orientation columns in the monkey striate cortex, Journal of Computational Neurology, vol. 158, pp. 267--293, 1974.

D.H. Hubel: Exploration of the primary visual cortex, 1955-78, Nature, vol. 299, pp. 515--524, 1982.

Complex cells

Hubel and Wiesel named another type of cell “complex" because they contrasted simple cells in the following properties:

• Their receptive fields do not have distinct excitatory and inhibitory regions.

• Their response cannot be predicted by weighted summation.

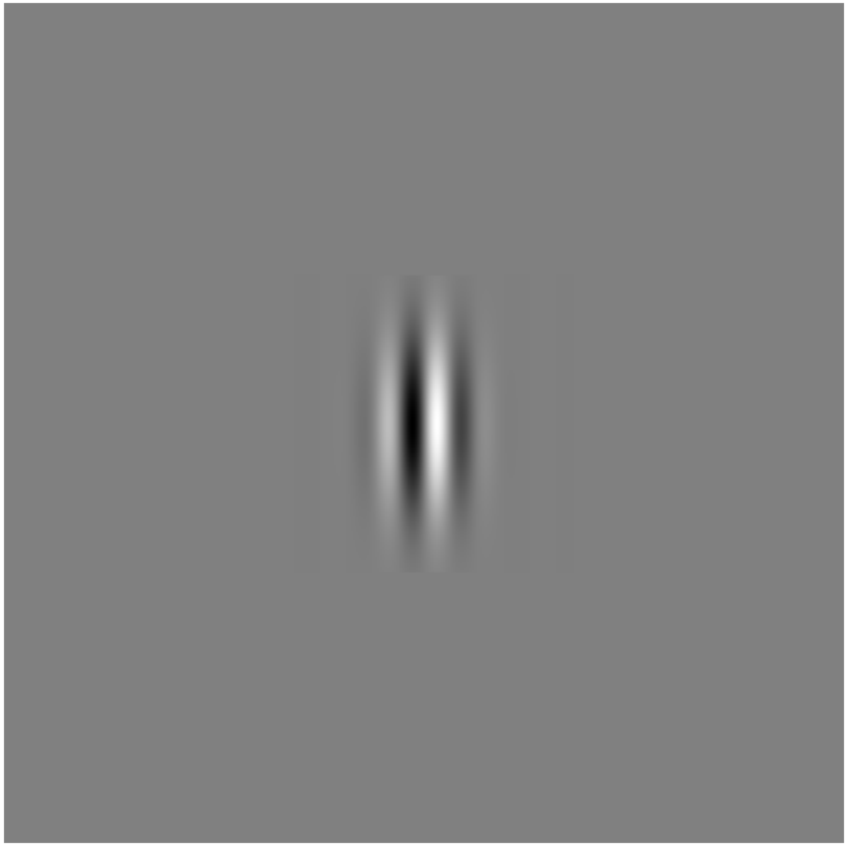
• Response is not modulated by the exact position of the optimal stimulus (bar or grating).

In engineering terms:
a complex cell cannot be characterized by an impulse response.

Gabor energy model of a complex cells

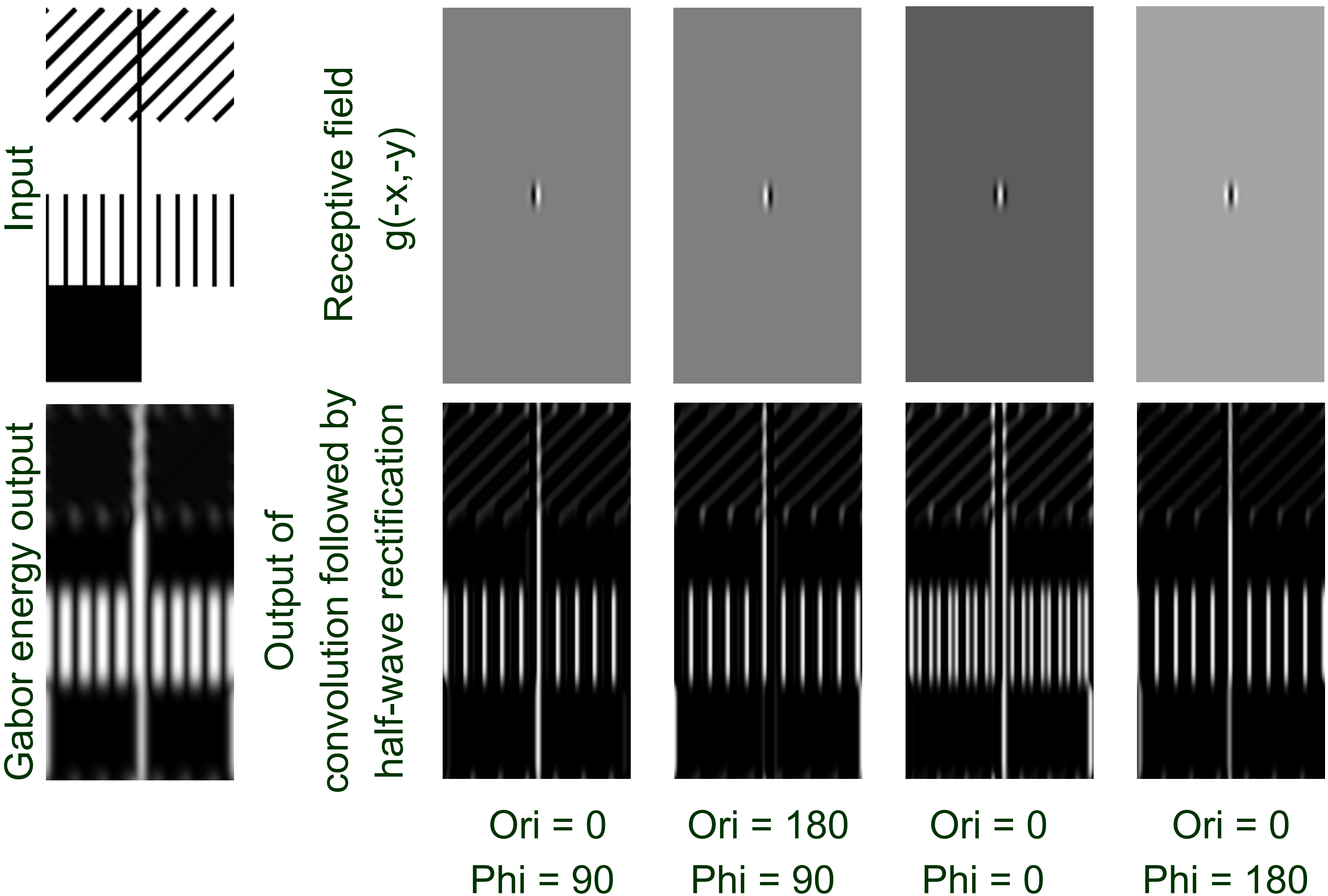
$$E_{\lambda,\sigma,\theta}(x,y) = \sqrt{R^2_{\lambda,\sigma,\theta,0}(x,y) + R^2_{\lambda,\sigma,\theta,-\frac{\pi}{2}}(x,y)}$$

Phase offset = 0
(symmetric function)



Phase offset = -90
(anti-symmetric function)

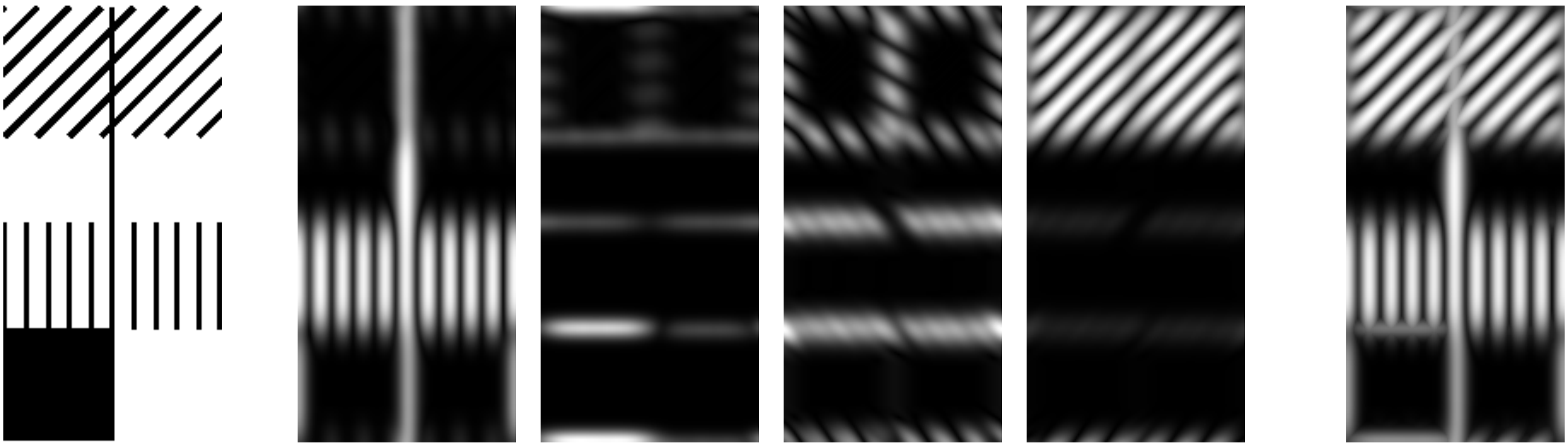
Gabor energy filter



Gabor energy filter

Input

Gabor energy output

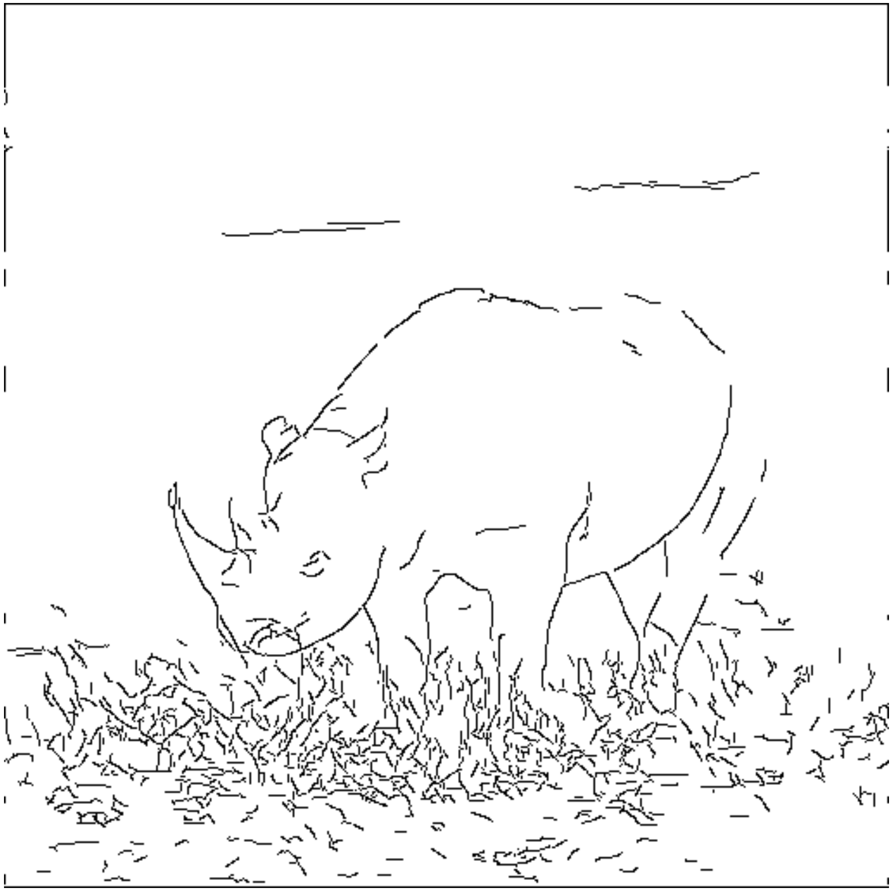


Orientation = 0 45 90 135 superposition

Result of superposition of the outputs of 4 Gabor energy filters (in [0,180)) with wavelength = 8, bandwidth = 1, spatial aspect ratio = 0.5

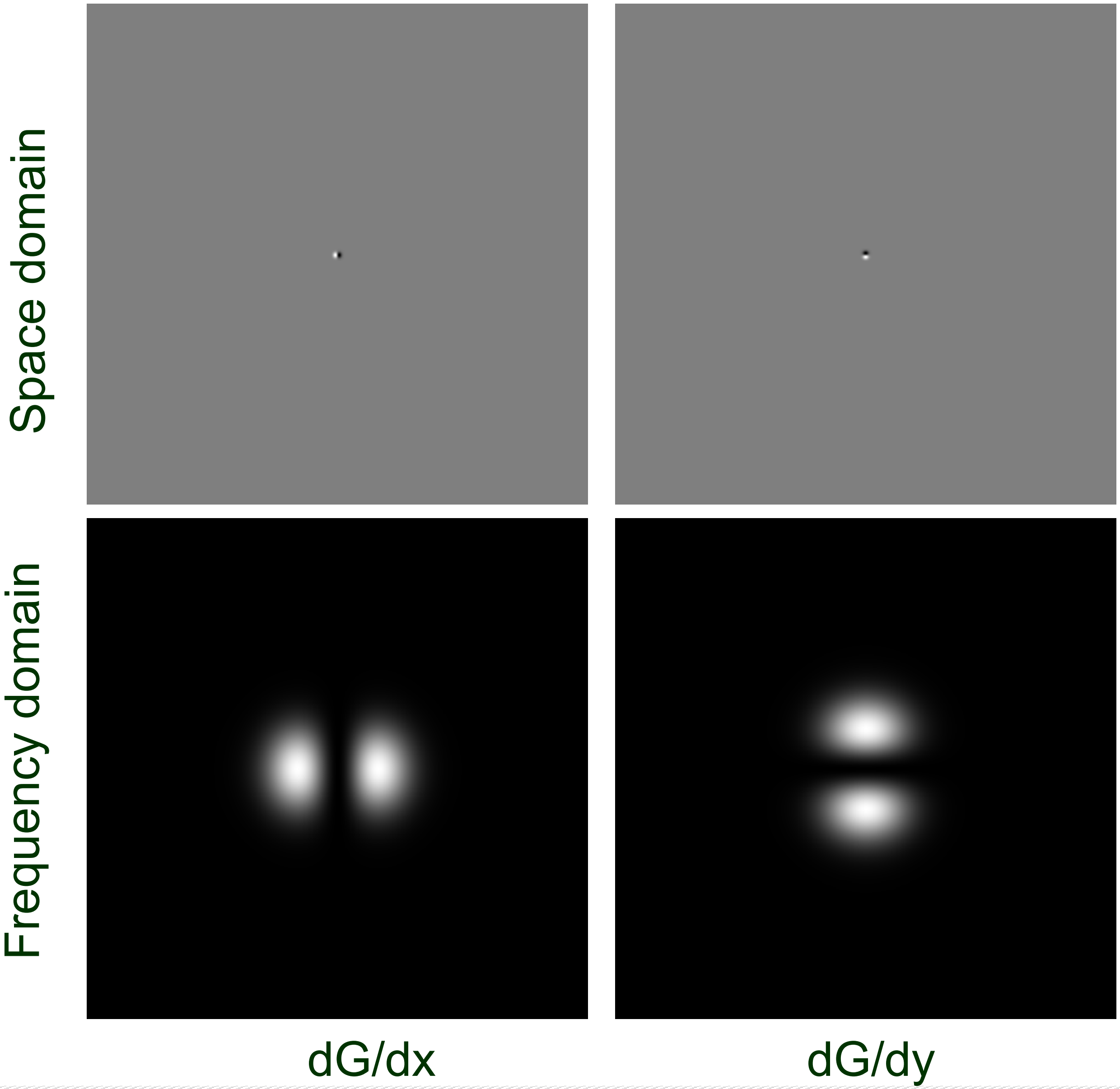
Bank of Gabor energy filters

How many orientations to use?

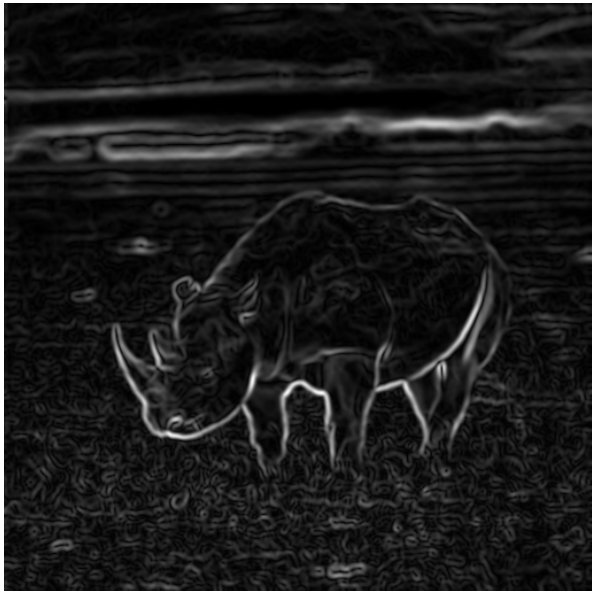


Result of superposition of the outputs of 6 Gabor energy filters (in [0,180)) with wavelength = 4, bandwidth = 2, spatial aspect ratio = 0.5 (after thinning and thresholding lt = 0.1, ht = 0.15).

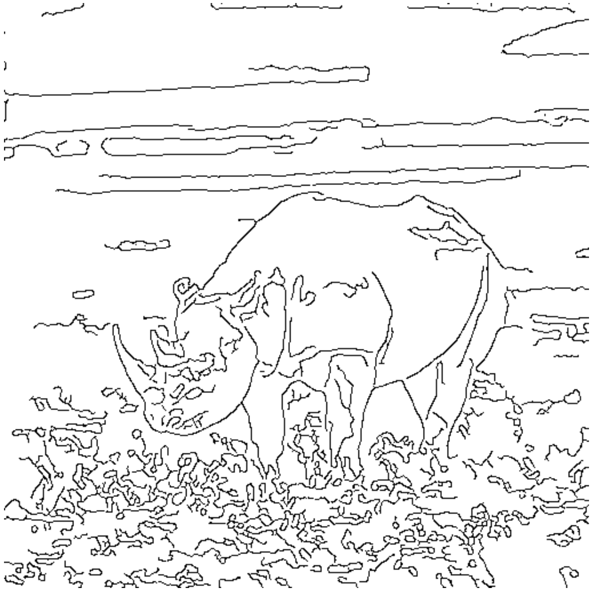
More efficient way to detect intensity changes by gradient computation



More efficient way to detect intensity changes

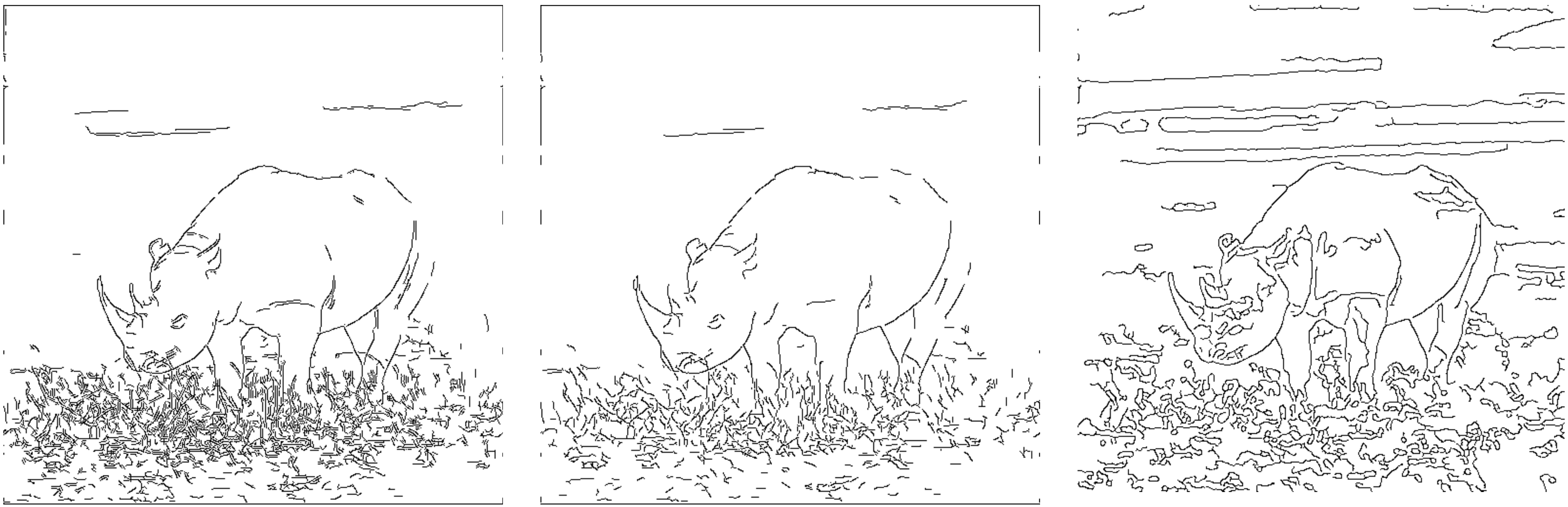


Gradient magnitude



Canny

Various ways to detect edges

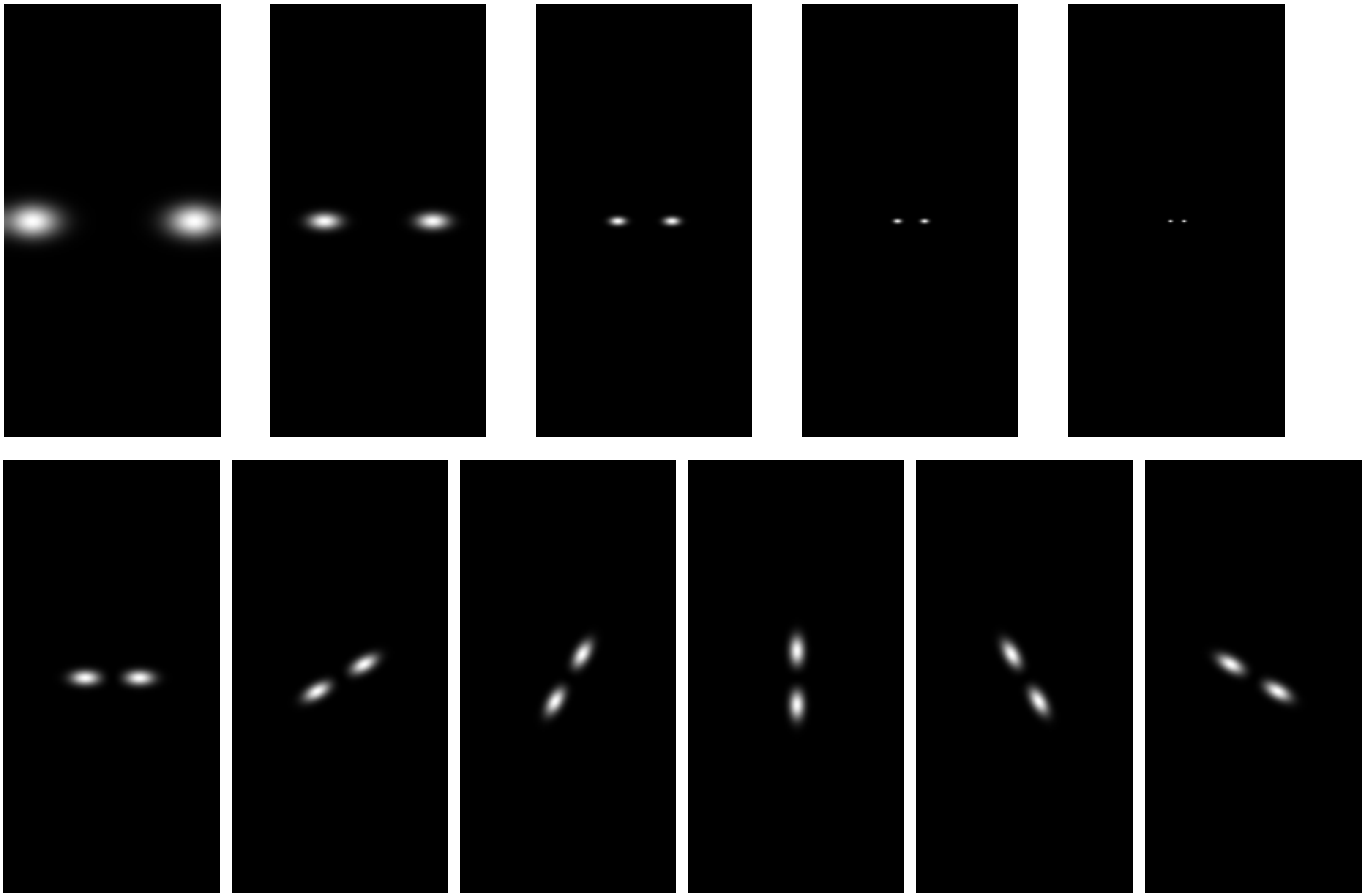


Gabor filter Gabor energy Canny

http://matlabserver.cs.rug.nl

Gabor filters for texture analysis

Filters in frequency domain



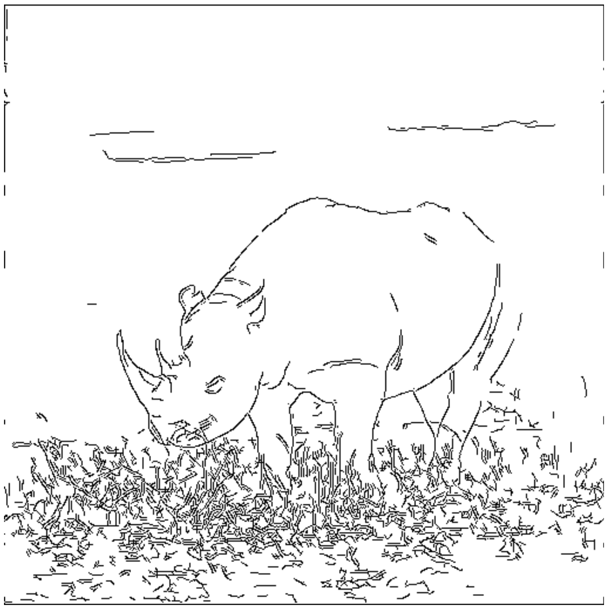
See e.g.

S.E. Grigorescu, N. Petkov and P. Kruizinga:
Comparison of texture features based on Gabor filters,
IEEE Trans. on Image Processing, **11** (10), 2002, 1160-1167.

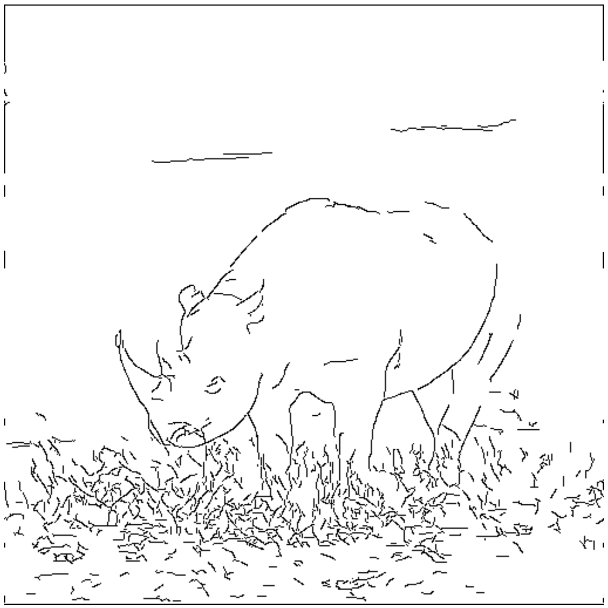
and references therein

http://matlabserver.cs.rug.nl

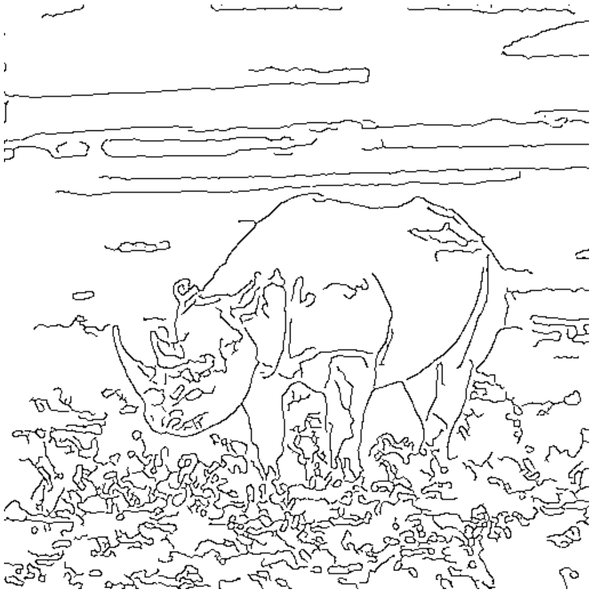
Problems with texture edges



Gabor filter



Gabor energy

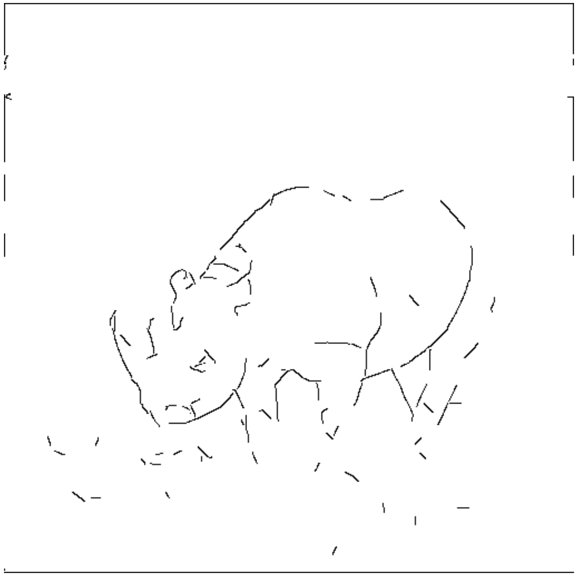


Canny

http://matlabserver.cs.rug.nl

Contour enhancement by suppression of texture

Canny



with surround suppression

[Petkov and Westenberg, Biol.Cyb. 2003]
[Grigorescu et al., IEEE-TIP 2003, IVC 2004]

Spatiotemporal (3D) Gabor filters

See

N. Petkov and E. Subramanian:
Motion detection, noise reduction, texture suppression and contour enhancement by spatiotemporal Gabor filters with surround inhibition,
Biological Cybernetics, **97** (5-6), 2007, 423-439.

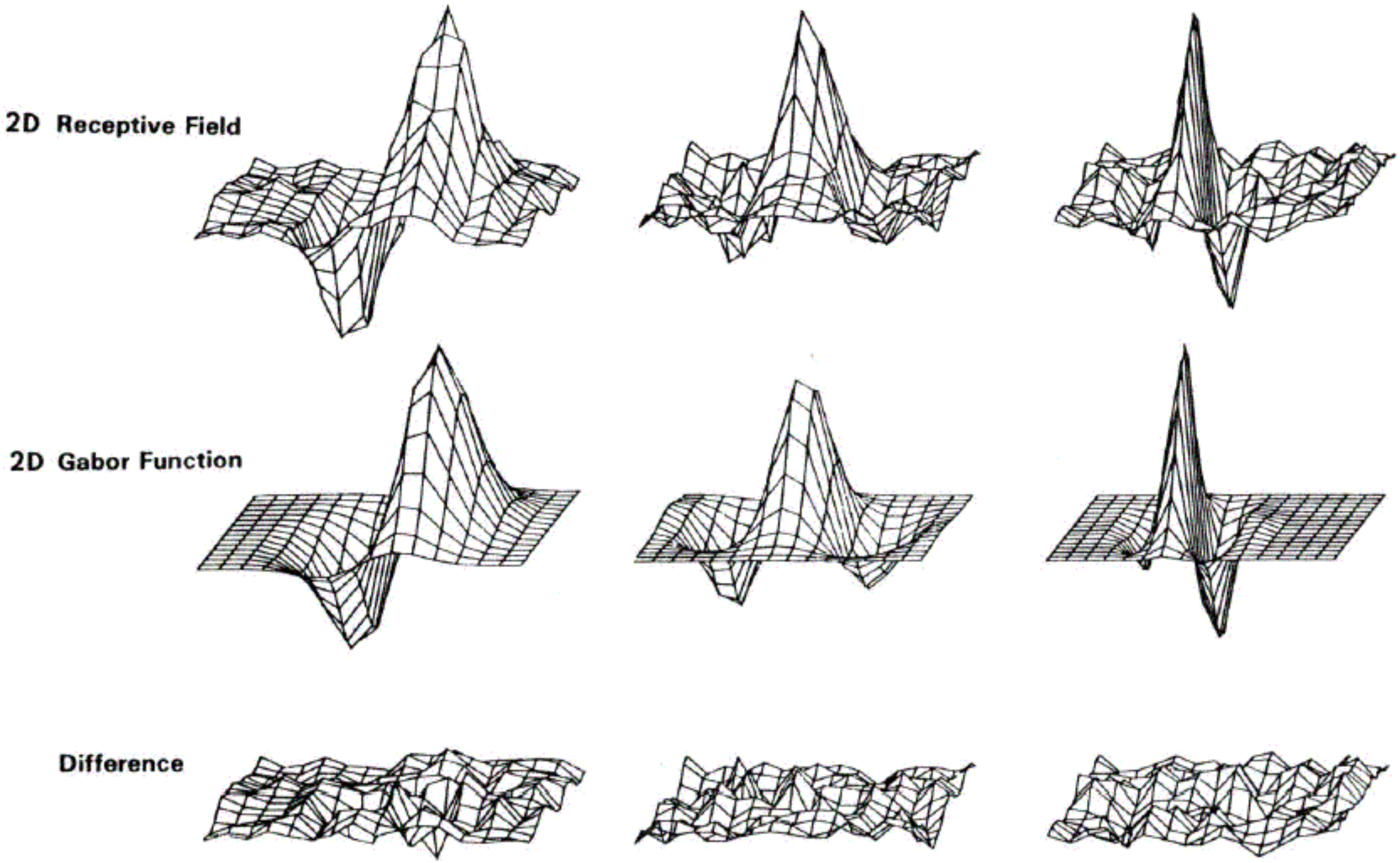
and references therein

http://www.cs.rug.nl/~petkov/publications/journals

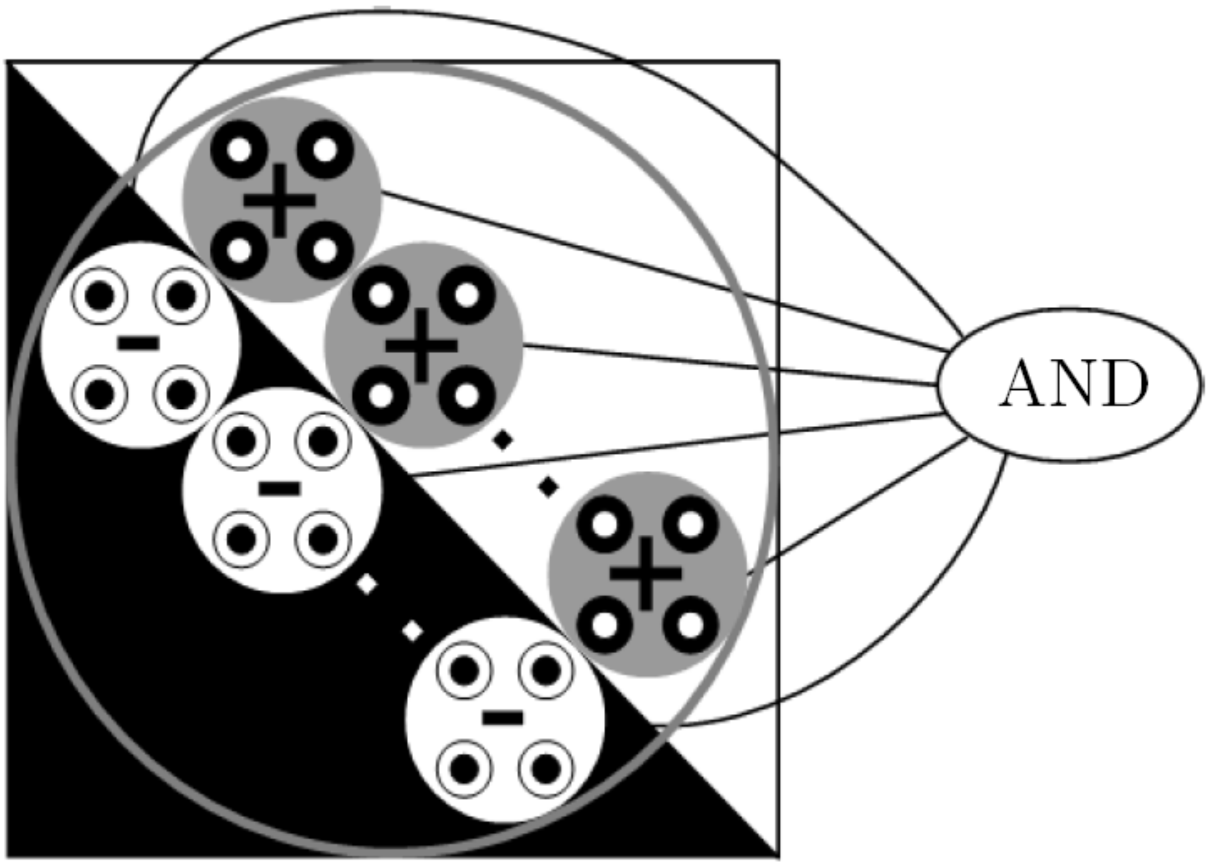
Complex cells
and
CORF filters

(or a non-Gaborian, less Platonic view of reality)

References to origins – modeling

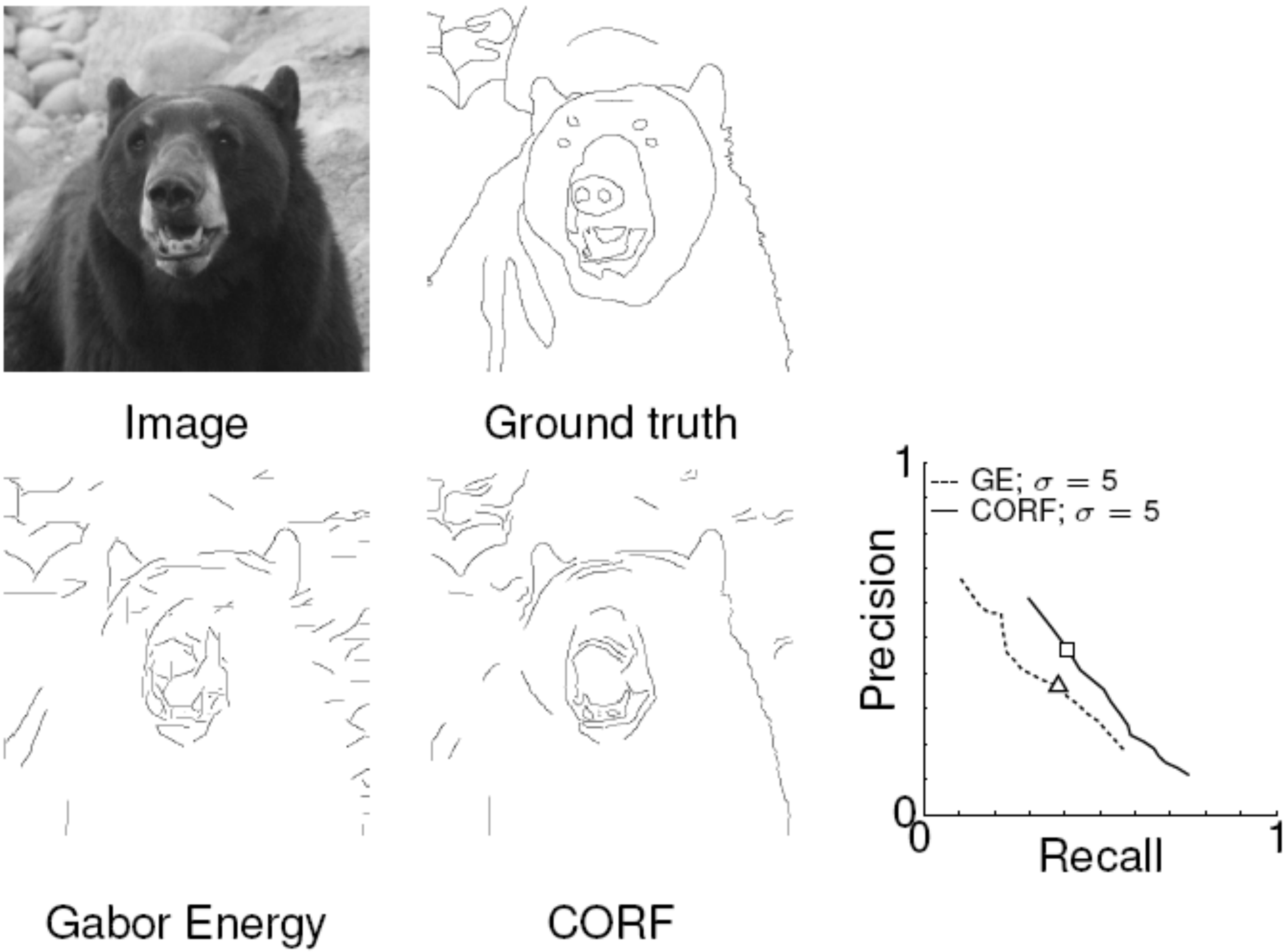


V1 complex cells modeled by
CORF: Combination Of LGN Receptive Fields



[Azzopardi and Petkov, 2011]

V1 complex cells modeled by CORF: Combination Of LGN Receptive Fields



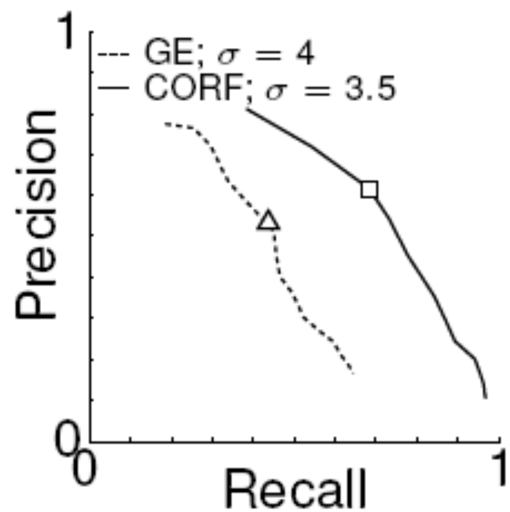
V1 complex cells modeled by CORF: Combination Of LGN Receptive Fields

Image

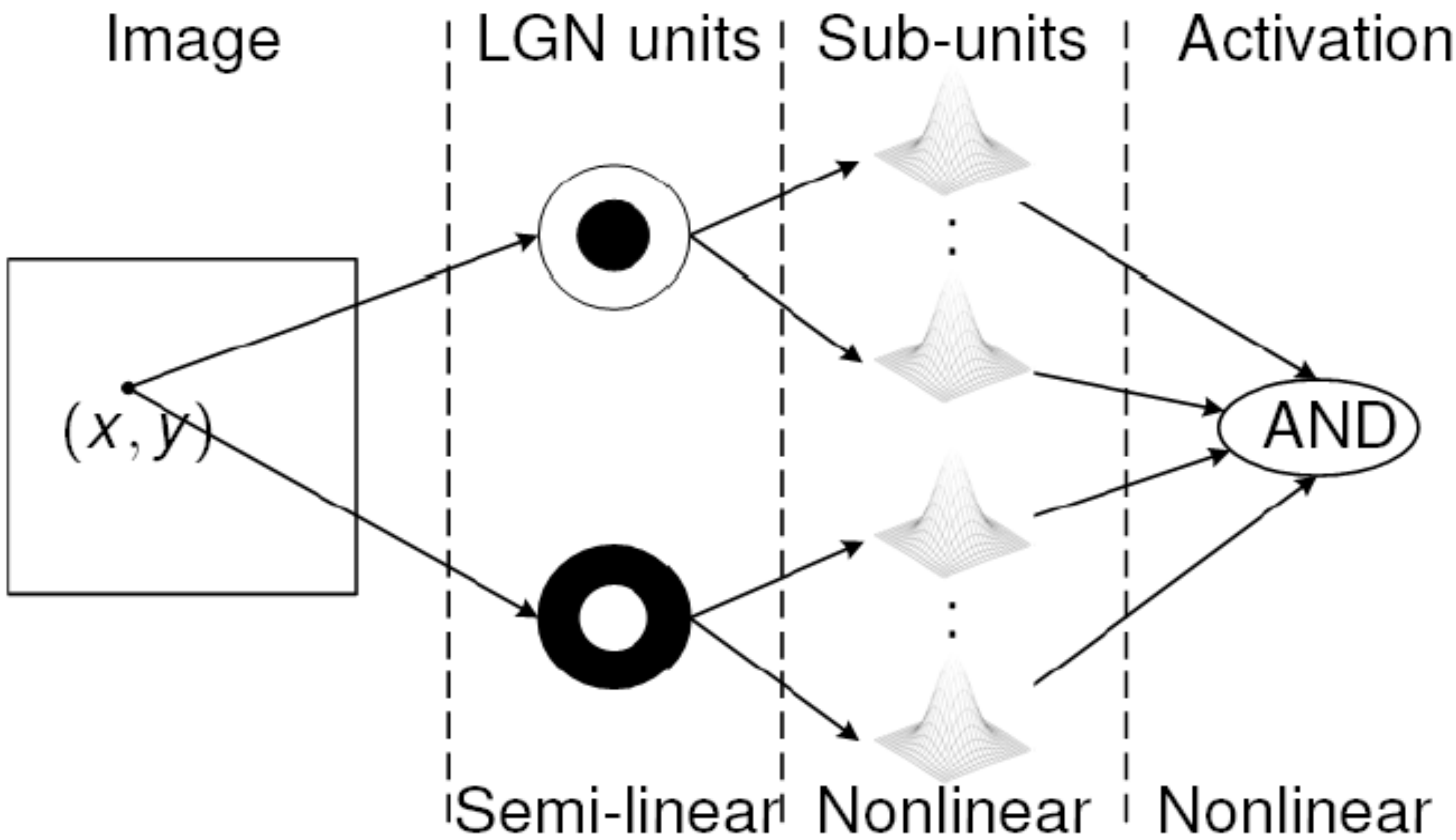
Ground truth

Gabor Energy

CORF

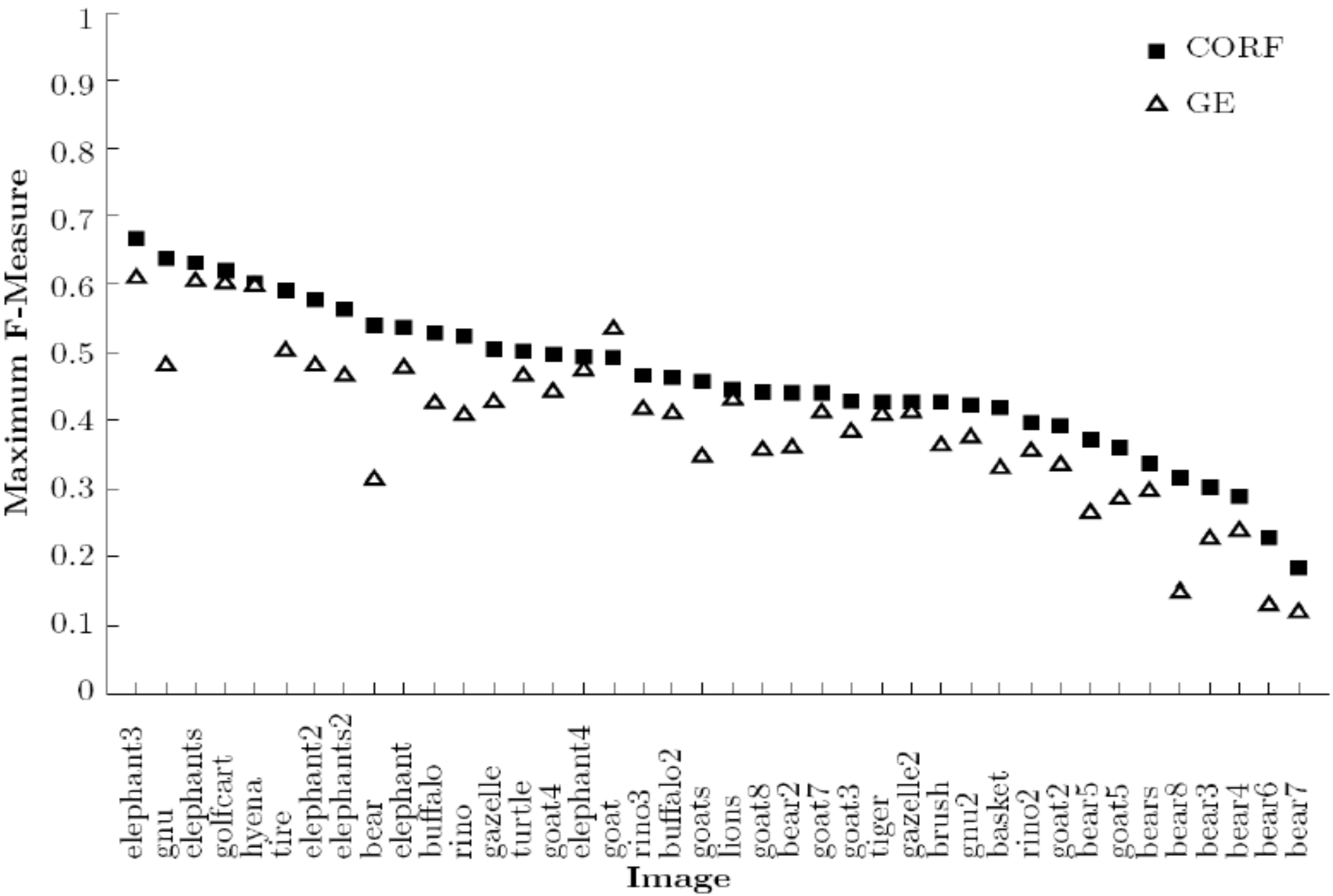


V1 complex cells modeled by CORF: Combination Of LGN Receptive Fields

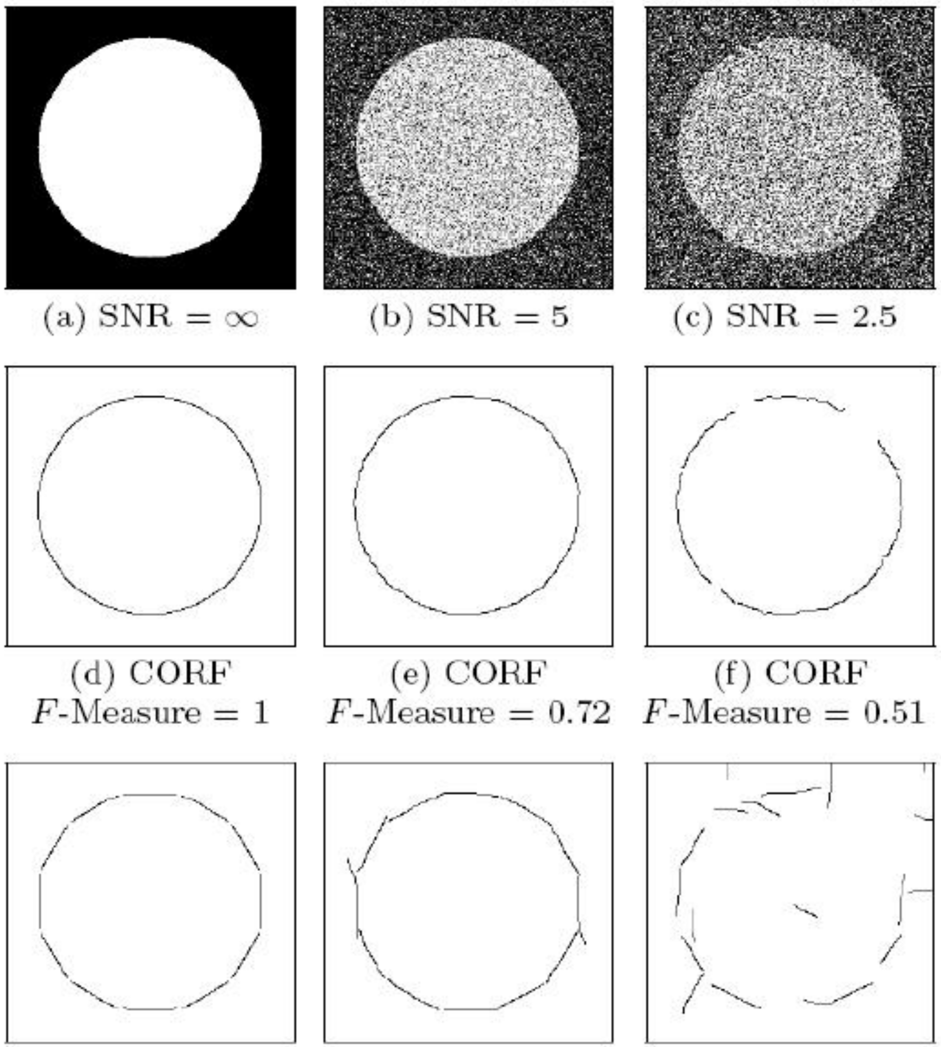


university of groningen

V1 complex cells modeled by CORF: Combination Of LGN Receptive Fields



V1 complex cells modeled by
CORF: Combination Of LGN Receptive Fields



(a) SNR = ∞ (b) SNR = 5 (c) SNR = 2.5

(d) CORF F-Measure = 1 (e) CORF F-Measure = 0.72 (f) CORF F-Measure = 0.51

(g) GE F-Measure = 0.48 (h) GE F-Measure = 0.43 (i) GE F-Measure = 0.17

CORF is more effective than GE

- Better contour integration
- More robust to noise
- Better edge localization

Conclusions or morals

Computational models (of V1/2) are approximations

Plato: use Gabor energy (for aesthetic reasons)

Popper: use CORF (for practical empiricism)