نام و نام خانوادگی: امین غلامی
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تمرین سری سوم
روز و ساعت کلاس: یکشنبه‌ها و سه‌شنبه‌ها ۱۲ – ۱۰:۳۰

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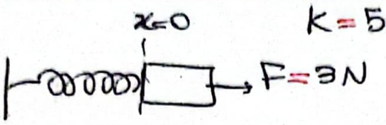
۱ معلومات مسئله عبارت‌اند از:



$k = 50 N/m$

حل: الف)

$$W_T = \Delta K \xrightarrow{v_1=v_2=0} W_T = 0 \Rightarrow W_s + W_F = 0$$

$$\Rightarrow -\frac{1}{2}kx^2 + Fx = 0 \Rightarrow x\left(-\frac{1}{2}kx + F\right) = 0 \Rightarrow x = \frac{2F}{k} = 0.12m$$

$$\Rightarrow \boxed{x = 12cm}$$

ب)

$$W_F = \vec{F}\cdot\vec{x} = |F||x|\cos\varphi = 3\times0,12\times1$$

$$\boxed{W_F = 0,36 J}$$

پ)

$$W_s = -\frac{1}{2}kx^2 = -\frac{1}{2}(50)(0,12)^2 = \boxed{-0,36J}$$

$$W_s = -W_F$$

ت)

$$\Delta K = W_T \Rightarrow K = W_s + W_F = -\frac{1}{2}kx^2 + Fx$$

$$\xrightarrow[\text{مشتق برحسب } x]{K_{max}} K' = -kx + F = 0 \Rightarrow x = \frac{F}{k} = \frac{3}{50} = 0,06m = \boxed{6cm}$$

ث)

$$K = -\frac{1}{2}kx^2 + Fx$$

$$K = -\frac{1}{2}(50)(0,06)^2 + (3)(0,06) = \boxed{0,09 J}$$

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۲ معلومات مسئله عبارت‌اند از:

$t = T$ ، $p' = p + dp$

$v_0 = 0$

حل:

$$W_T = \Delta K \xrightarrow{p=\frac{W}{t}} pt = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 \Rightarrow v = \sqrt{\frac{2pt}{m}}$$

$$v = \frac{dx}{dt} = \sqrt{\frac{2pt}{m}} \xrightarrow[\text{نامعین می‌گیریم}]{\text{از دو طرف انتگرال}} x\Big|_0^x = \int_0^t \sqrt{\frac{2pt}{m}}\,dt \Rightarrow x = \sqrt{\frac{2p}{m}}\int_0^t \sqrt{t}\,dt$$

$$\Rightarrow x = \sqrt{\frac{2p}{m}}\left(\frac{2}{3}\sqrt{t^3}\right) = \sqrt{\frac{8pt^3}{9m}} \quad \text{عدد ثابت}$$

$$x^2 = \frac{8pt^3}{9m} \xrightarrow{\text{مشتق از طرفین}} 0 = \frac{8}{9m}\left(t^3dp + 3t^2dt\cdot p\right)$$

$$\Rightarrow t\,dp + 3dt\cdot p = 0 \Rightarrow 3dt = -t\frac{dp}{p} \Rightarrow dt = -\frac{t}{3}\cdot\frac{dp}{p}$$

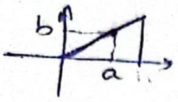
$$\boxed{dt = -\frac{T}{3}\frac{dp}{p}}$$

۳ معلومات مسئله عبارت اند از : $\vec{F}=2\beta xy\vec{i}+\beta x^2\vec{j}$ $m=2kg$, $v_0=2\frac{m}{s}$

حل :

$$W=\int_{r_i}^{r_f}\vec{F}\,d\vec{r}=\int_{r_i}^{r_f}(2\beta xy\,\vec{i}+\beta x^2\vec{j})\cdot(dx\,\vec{i}+dy\,\vec{j})$$

$y=\frac{b}{a}x$



$dy=\frac{b}{a}dx$

$\vec{r_i}=0\vec{i}+0\vec{j}$

$\vec{r_f}=a\vec{i}+b\vec{j}$

$$W=\int_{r_i}^{r_f}\frac{2b}{a}\beta x^2\,dx\,\vec{i}+\left(\frac{b}{a}\right)\beta x^2 dx\,\vec{j}$$

$$W=\frac{b}{a}\beta\int_{r_i}^{r_f}2x^2dx\,\vec{i}+x^2dx\,\vec{j}$$

$$W=\frac{b}{a}\beta\int_0^a 2x^2dx\,\vec{i}+x^2dx\,\vec{j}=\frac{b}{a}\beta\left(\frac{2}{3}x^3+\frac{x^3}{3}\right)\Big|_0^a$$

$$W=\frac{b}{a}\beta(a^3)=b\beta a^2$$

$$W_T=\Delta K\Rightarrow b\beta a^2=\frac{1}{2}m(v^2-v_0^2)\Rightarrow b\beta a^2=v^2-4\Rightarrow\boxed{v=\sqrt{b\beta a^2+4}}$$

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۴ معلومات مسئله عبارت اند از : $m=1800kg$, $d=3,7m$, $k=1,5\times10^5\frac{N}{m}$

$f_{fric}=4,4\times10^3N$

حل : الف)

$\oplus\downarrow$ جهت پایین مثبت

$$W_T=\Delta K\Rightarrow mgh-f_{fric}h=\frac{1}{2}mv^2-0$$

$$18\times10^3\times3,7-4,4\times10^3\times3,7=900v^2\Rightarrow v^2=\frac{50,32}{0,9}\Rightarrow\boxed{v\simeq7,4\frac{m}{s}}$$

ب) از زمان برخورد تا زمان فشردگی بیشینه : $W_T=\Delta K\Rightarrow W_g+W_{f_{fric}}+W_s=0-\frac{1}{2}mv_1^2$

$$\Rightarrow mgx-f_{fric}x-\frac{1}{2}kx^2=-\frac{1}{2}mv_1^2$$

$$\Rightarrow18\times10^3x-4,4\times10^3x-7,5\times10^4x^2=-900v_1^2=-50,32\times10^3$$

$$\xrightarrow{\div10^3}75x^2-13,6x-50,32=0\Rightarrow x\simeq-0,7\ \times\quad\boxed{x\simeq0,9m}$$

پ) $W_T=\Delta K\Rightarrow W_g+W_{f_{fric}}+W_s=0-0=0$

$$\Rightarrow-18\times10^3d-4,4\times10^3d+\frac{1}{2}\times15\times10^4((0,9)^2-0)\Rightarrow22,4d=60,75$$

$$\boxed{d\simeq2,7m}$$

ت) مکان تعادل فنر : $mg=kx\Rightarrow x=\frac{mg}{k}=\frac{18\times10^3}{15\times10^4}=1,2\times10^{-1}=0,12m$

از ابتدا تا توقف : $W_T=\Delta K=0\Rightarrow W_g+W_{f_{fric}}+W_s=0$

$$\Rightarrow18\times10^3(3,7+0,12)-4,4\times10^3\times\underset{\text{مسافت}}{d}+\frac{1}{2}\times15\times10^4(0-(0,12)^2)=0$$

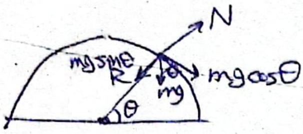
$$\Rightarrow68,76\times10^3-1,08\times10^3=4,4\times10^3d$$

$$\Rightarrow d=\frac{68,76-1,08}{4,4}\simeq\boxed{15,3m}$$

۵ معلومات مسئله عبارت اند از :

$v_0 = 0$

حل :



$F_c = \frac{mv_1^2}{R}$ در هنگام جدا شدن از سطح

$\sin\theta = \frac{h}{R} \Rightarrow mg\sin\theta - N = \frac{mv_1^2}{R} \Rightarrow \boxed{N = m\left(g\sin\theta - \frac{v_1^2}{R}\right)}$

$W_T = \Delta k \Rightarrow \frac{1}{2}mv_1^2 - 0 = mg(R-h) \Rightarrow \boxed{v_1^2 = 2g(R-h)}$ *

از زمان نشستن تا سر خوردن

$\Rightarrow N = m\left(g\sin\theta - \frac{2g(R-h)}{R}\right) = mg\left(\sin\theta - \frac{2(R-h)}{R}\right) = 0$

$\Rightarrow \sin\theta = \frac{2R-2h}{R} = 2 - \frac{2h}{R} \Rightarrow \frac{h}{R} = 2 - \frac{2h}{R} \Rightarrow \frac{3h}{R} = 2 \Rightarrow \boxed{h = \frac{2}{3}R}$

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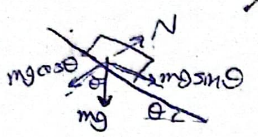
۶ معلومات مسئله عبارت اند از :

$\theta = 30°, L = 0/75m, h = 2m$

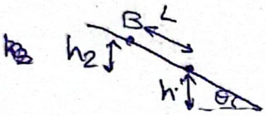
$\mu_k = 0/4 \ , \ v_A = 8\frac{m}{s}, v_B = ?$

حل :



$\Sigma F_y = 0 \Rightarrow N = mg\cos\theta$

$f_k = \mu_k N = (0/4)(m)(10)(\cos30°) \simeq 3/4m$



$\frac{h_2}{L} = \sin\theta \Rightarrow h_2 = L\sin\theta = (0/75)(0/5) = 0/375m$

$\Delta E_{mech(A \to B)} = W_f \Rightarrow \Delta U_g + \Delta k = -f_k d$

$\Rightarrow mg(h+h_2) + \frac{1}{2}m(v_B^2 - v_A^2) = \underset{-3/4m}{-f_k \times L} \xrightarrow{\div m} 10(2+0/375) + \frac{1}{2}(v_B^2 - 64) = -2/55$

$\Rightarrow 23/75 + \frac{1}{2}v_B^2 - 32 = -2/55 \Rightarrow v_B^2 = 11/4 \Rightarrow \boxed{v_B \simeq 3/5\frac{m}{s}}$

سپس به نقطه B می رسد.

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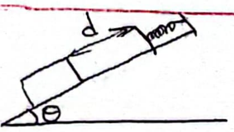
۷ معلومات مسئله عبارت اند از :

$\theta = 30° \quad k, m, d$



حل : الف)

$\Delta E_{mech} = 0 \Rightarrow \Delta U_g + \Delta U_s + \Delta E_k = 0$

$\Delta h = \Delta h_1 + \Delta h_2 = d\sin\theta + \frac{d}{5}\sin\theta = \frac{6d}{5}\sin\theta = \left(\frac{6d}{5}\right)\left(\frac{1}{2}\right) = \frac{3d}{5}$

$\Rightarrow W\Delta h + \frac{1}{2}kx^2 + \Delta E_k = 0 \Rightarrow \frac{3d}{5}W + \frac{1}{2}k\left(\frac{d}{5}\right)^2 + \Delta E_k = 0$

$\Rightarrow \boxed{|\Delta E_k| = \frac{3d}{5}W + \frac{1}{50}kd^2}$ ، $\Delta E_k < 0$

ب)

$\Delta E_{mech} = 0 \Rightarrow \Delta U_g + \Delta U_s + \Delta E_k = 0 \quad \Delta h = \left(d + \frac{3d}{2}\right)\sin\theta = \frac{5d}{4}$

$\Rightarrow W \cdot \frac{5d}{4} + \frac{1}{2}k\left(\frac{3d}{2}\right)^2 + (0 - K_1) = 0 \Rightarrow \boxed{K_1 = \frac{5d}{4}W + \frac{9}{8}kd^2}$

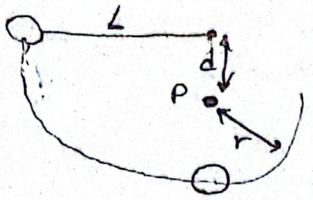
پ)

$|\Delta E_k| = (0/6)(0/4)(10) + \frac{1}{50}(100)(0/16) = 2/4 + 0/32 = \boxed{2/72J}$

کاهش

$K_1 = (1/25)(0/4)(10) + \frac{9}{8}(100)(0/16) = 5 + 18 = \boxed{23J}$

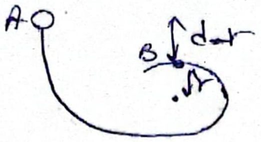
۸ معلومات مسئله عبارت اند از:



$L = 120\,cm = 1/2\,m$

$\min(d) = ?$

حل:



در نقطه B: $N + mg = \frac{mv^2}{r}$

$N = m(\frac{v^2}{r} - g)$

$\Longrightarrow v_{min} = \sqrt{rg}$

حداقل سرعت در نقطه بالای دایره

$\Delta E_{mech} = 0$

$\Delta U_g + \Delta k = 0 \Longrightarrow mg(r-d) + \frac{1}{2}m(v_B^2 - v_A^2) = 0$

$\Longrightarrow mg(r-d) + \frac{1}{2}mgr \Longrightarrow d - r = \frac{r}{2} \Longrightarrow d = \frac{3r}{2}$

$L = d + r = \frac{5}{2}r = 1/2 \Longrightarrow r = 0/48\,m \Longrightarrow d_{min} = \frac{3}{2}(0/48) = \boxed{0/72\,m}$

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۹ معلومات مسئله عبارت اند از:

$U = \frac{a}{r^2} - \frac{b}{r}$

$a, b > 0$

حل: الف)

$F = \frac{dU}{dr} = \frac{-2ar}{r^4} + \frac{b}{r^2} = \frac{-2a}{r^3} + \frac{b}{r^2}$

$\xrightarrow[F=0]{\text{در حالت تعادل}} \frac{2a}{r^3} = \frac{b}{r^2} \Longrightarrow br = 2a \Longrightarrow \boxed{r_0 = \frac{2a}{b}\,m}$

با توجه به اینکه در تعادل است پس پایدار است و نیروی خارجی وارد نمی‌شود

ب)

$F_{max} \Longrightarrow F' = 0 \Longrightarrow \frac{(+2a)(3r^2)}{r^6} - \frac{2br}{r^4} = 0$

$\Longrightarrow \frac{+6a}{r^4} - \frac{2b}{r^3} = 0 \Longrightarrow 2br = +6a \Longrightarrow r_* = 3\frac{a}{b}$

$\Longrightarrow F_{max} = F(\frac{3a}{b}) = \frac{-2a}{27\frac{a^3}{b^3}} + \frac{b}{9\frac{a^2}{b^2}} = \frac{-2}{27}\frac{b^3}{a^2} + \frac{1}{9}\frac{b^3}{a^2}$

$\Longrightarrow \boxed{F_{max} = \frac{1}{27}\frac{b^3}{a^2}\,N}$

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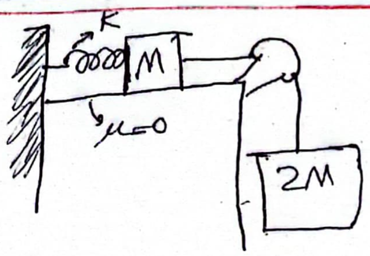
۱۰ معلومات مسئله عبارت اند از:



حل: الف)

$\Delta E_{mech_{\text{سامانه}}} = 0 \Longrightarrow \Delta K + \Delta U_s + \Delta U_g = 0$

$\Longrightarrow (K_2 - 0) + \frac{1}{2}k(d^2 - 0) - 2Mgd = 0 \Longrightarrow \boxed{K_2 = -\frac{1}{2}kd^2 + 2Mgd}$

ب)

$k_{tot} = k_M + k_{2M} = \frac{k_{2M}}{2} + k_{2M} = \frac{3}{2}k_{2M} \Longrightarrow k_{2M} = \frac{2}{3}k_{tot}$

$\Longrightarrow k_{2M} = \frac{2}{3}(2Mgd - \frac{1}{2}kd^2) = \boxed{\frac{4}{3}Mgd - \frac{1}{3}kd^2}$

ج)

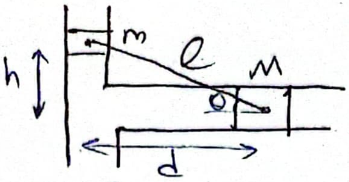
$v_{2M} = 0$

$K_{2M} = 0 \Longrightarrow \frac{2}{3}(2Mgd - \frac{1}{2}kd^2) = 0 \Longrightarrow 2Mgd = \frac{1}{2}kd^2$

$\Longrightarrow 4Mg = kd \Longrightarrow \boxed{d_{max} = \frac{4Mg}{k}}$

۱۱ معلومات مسئله عبارت اند از:

$v_0 = 0 \quad \mu = 0$



حل:

$$\Delta E_{mech\,system} = 0 \Rightarrow \Delta K + \Delta U = 0$$

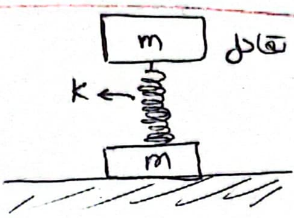
$$\Rightarrow \frac{1}{2}Mv_M^2 + \frac{1}{2}mv_m^2 - mgh = 0 \Rightarrow \frac{1}{2}mv_m^2 = mgh - \frac{1}{2}Mv_M^2$$

$$\Rightarrow v_m^2 = 2gh - \frac{M}{m}v_M^2$$

$$h=0 \Rightarrow [illegible] \; \underset{\sin\theta = 0}{l = d} \Rightarrow v_M = 0 \qquad \Rightarrow v_m^2 = 2gh$$

$$\boxed{v_m = \sqrt{2gh}}$$

۱۲ معلومات مسئله عبارت اند از:



حل:

$$W_T = \Delta K \qquad W_g + W_S = \Delta K \Rightarrow -mgh + \frac{1}{2}kh^2 = \frac{1}{2}mv^2$$

$$\Delta l_{min} \Rightarrow v = 0 \Rightarrow -mgh + \frac{1}{2}kh^2 = 0$$

$$\Rightarrow \frac{1}{2}kh = mg \Rightarrow h = \frac{2mg}{k} \quad ?$$