MŰEGYETEM 1782

Designing of a glued, laminated timber girder

2nd homework of REINFORCED CONCRETE BUILDINGS - BMEEOHSA-A2

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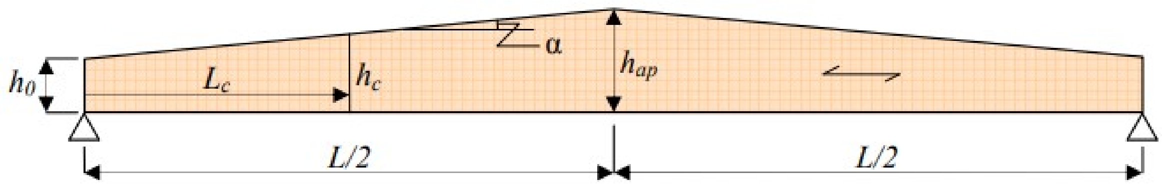
1. Given data:

1.1 Static structure:



$l_{eff} := 26.8\ m$ the effective length of the beam(in the figure L)

$b := 190\ mm$ the width of the beam

$h_{ap} := 1.4\ m$ the depth of the apex zone

$s := 1.5\ m$ the distance between braci ng positions along the tapered face

$\alpha := 2\ °$ angle of the apex zone

1.2 Material properties: Timber: GL32h

$f_{m.k} := 32\ \frac{N}{mm^2}$ Characteristic bending strength

$f_{t.90.k} := 0.5\ \frac{N}{mm^2}$ Characteristic tensile strength perpendicular to the grain

$f_{v.k} := 3.8\ \frac{N}{mm^2}$ Characteristic shear strength

$E_{0.05} := 11.1\ \frac{kN}{mm^2}$ Fifth percentile value of modulus of elasticity

$k_{mod} := 0.8$ Modification factor for duration of load and moisture content

$\rho_k := 380\ \frac{kg}{m^3}$ characteristic density

$\rho_{mean} := 460\ \frac{kg}{m^3}$ mean density

$\gamma_M := 1.25$ Partial factor for material properties

$f_{c.90.k} := 6.0\ \frac{N}{mm^2}$ Characteristic comp. strength perpendicular to the grain

$E_{0.g.mean} := 14.2\ \frac{kN}{mm^2}$ mean value of modulus of elasticity

1.3 Loads:

$g := 9.756\ \frac{m}{s^2}$ $\qquad g_{self} := (b \cdot h_{ap}) \cdot 2 \cdot g \cdot \rho_{mean} = 2.387\ \frac{kN}{m}$

$g_k := 1.1\ \frac{kN}{m}$ $\qquad \gamma_G := 1.35$ $\qquad \gamma_Q := 1.5$ $\qquad q_k := 2.1\ \frac{kN}{m}$

$p := g_k \cdot \gamma_G + q_k \cdot \gamma_Q + g_{self} \cdot \gamma_G = 7.858\ \frac{kN}{m}$

1.4 Ultimate limit state :

Design strength: $\qquad f_{m.d} := k_{mod} \cdot \frac{f_{m.k}}{\gamma_M} = 20.48\ MPa$

$f_{v.d} := k_{mod} \cdot \frac{f_{v.k}}{\gamma_M} = 2.432\ MPa$ $\qquad f_{t.90.d} := k_{mod} \cdot \frac{f_{t.90.k}}{\gamma_M} = 0.32\ MPa$

$f_{c.90.d} := k_{mod} \cdot \frac{f_{c.90.k}}{\gamma_M} = (3.84 \cdot 10^6)\ Pa$

$h_0 := -\tan(\alpha) \cdot 0.5\ l_{eff} + h_{ap} = 932.062\ mm$ $\qquad$ side dimension of the double tampered beam

Critical section with respect to bending, for a univerformly distributed load is at distance.

$L_c := l_{eff} \cdot \left(\frac{h_0}{2\ h_{ap}}\right) = 8.921\ m$ $\qquad$ from the support where,

$h_c := h_0 + (h_{ap} - h_0) \cdot 2\ \frac{L_c}{l_{eff}} = 1.244\ m$

Nominal bending stress at critical section:

$$\sigma_{m.\alpha.d} := \frac{0.5 \cdot p \cdot L_c \cdot (l_{eff} - L_c)}{b \cdot \frac{{h_c}^2}{6}} = 12.796\ MPa$$

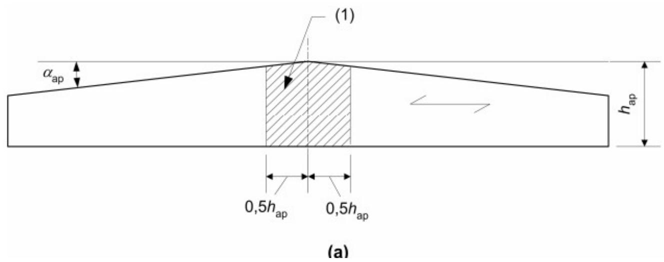
Verification of the failure condition:

$\sigma_{m.\alpha.d} < k_{m.\alpha} \cdot f_{m.d}$

where the stress modification factor due to compression at the tapered egde is defined by

$$k_{m.\alpha} := \frac{1}{\sqrt{1 + \left(\frac{f_{m.d}}{1.5\ f_{v.d}} \cdot \tan(\alpha)\right)^2 + \left(\frac{f_{m.d}}{f_{c.90.d}} \cdot \tan(\alpha)^2\right)^2}} = 0.981$$

$k_{m.\alpha} \cdot f_{m.d} = 20.097\ MPa$ $\qquad$ > $\qquad \sigma_{m.\alpha.d} = 12.796\ MPa$ $\qquad$ *sufficient!*



Apex zone

The apex bending stress should be calculated as follows:

$$\sigma_{m,d} = k_\ell \frac{6 M_{ap,d}}{b h_{ap}^2}$$

with:

$$k_\ell = k_1 + k_2 \left(\frac{h_{ap}}{r}\right) + k_3 \left(\frac{h_{ap}}{r}\right)^2 + k_4 \left(\frac{h_{ap}}{r}\right)^3$$

$$k_1 = 1 + 1,4 \tan \alpha_{ap} + 5,4 \tan^2 \alpha_{ap}$$

$$k_2 = 0,35 - 8 \tan \alpha_{ap}$$

$$k_3 = 0,6 + 8,3 \tan \alpha_{ap} - 7,8 \tan^2 \alpha_{ap}$$

$$k_4 = 6 \tan^2 \alpha_{ap}$$

$$r = r_{in} + 0,5\, h_{ap}$$

where:
M(ap.d) is the design moment at the apex
h(ap) is the depth of the beam at the apex
b is the width of the beam
r(in) is the inner radius (in the curved beam)
α(ap) is the angle of the taper in the middle of the apex zone

using above equation

$$k_l := 1 + 1.4\ \tan(\alpha) + 5.4 \cdot \tan(\alpha)^2 = 1.055$$

$$M_{ap.d} := p \cdot \frac{{l_{eff}}^2}{8} = 705.501\ kN \cdot m$$

$$\sigma_{m.d} := k_l \cdot 6\ \frac{M_{ap.d}}{b \cdot {h_{ap}}^2} = 11.997\ MPa$$

$k_r := 1$ reduction factor of the bending stresss in case of the double tampered beams

$k_r \cdot f_{m.d} = 20.48\ MPa$ > $\sigma_{m.d} = 11.997\ MPa$ the bending stress at apex is well below the limit.

Largest tensile stress perpendicular to grain is defined by:

$$k_p := 0.2 \cdot \tan(\alpha) = 0.007$$

$$\sigma_{t.90.d} := k_p \cdot 6 \cdot \frac{M_{ap.d}}{b \cdot h_{ap}^{\ 2}} = 0.079\ \mathbf{MPa}$$

The design requirement is: $\tau_d := 0\ \mathbf{MPa}$

$k_{dis} := 1.4$ for double tampered and curved beams

k_{vol} is a volume factor, which for solid timber is equal to 1, and for glued laminated timber and (LVL)

$$k_{vol} = \left(\frac{V_0}{V}\right)^{0.2}$$

$$V_0 := 0.01\ \mathbf{m}^3$$

$$V := b \cdot h_{ap}^{\ 2} \cdot (1 - 0.25\ \tan(\alpha)) = 0.369\ \mathbf{m}^3$$

calculated using the below table.

Table 6.6 The stressed volume of the apex zone

Figure reference	Beam type	Stressed volume† (V)	Maximum allowable value of the stressed volume*
Figure 6.7a	Curved beam	$\frac{\beta\pi}{180} b(h_{ap}^2 + 2h_{ap}r_{in})$	$\frac{2}{3}V_c$
Figure 6.7b	Double tapered beam	$bh_{ap}^2(1 - 0.25\tan(\alpha_{ap}))$	$\frac{2}{3}V_{dt}$
Figure 6.7c	Pitched cambered beam	$b\left(\sin(\alpha_{ap})\cos(\alpha_{ap})(r_{in} + h_{ap})^2 - r_{in}^2\frac{\alpha_{ap}\pi}{180}\right)$	$\frac{2}{3}V_{pc}$

$$k_{vol} := \left(\frac{V_0}{V}\right)^{0.2} = 0.486$$

$$\frac{\tau_d}{f_{v.d}} + \frac{\sigma_{t.90.d}}{k_{dis} \cdot k_{vol} \cdot f_{t.90.d}} = 0.365 \qquad < \qquad 1 \qquad sufficient!$$

The shear stresses should, according to the current version of the code, not exceed the shear strength . However, a modification to the code, reducing the width of the section by a "cracking" factor, will most likely be made in the near future. The value for glulam is 0.67 .

Maximum shear stress, at the support

$$A := b \cdot h_0$$

$$\tau_d := \frac{3}{2} \cdot \frac{V}{A} = 3.127\ \mathbf{m}$$

1.5 Serviceability limit state:

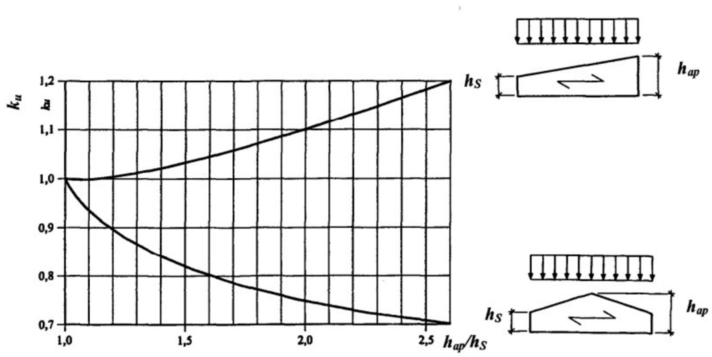


Fig. 10.2 Coefficient k_u

$\dfrac{h_{ap}}{h_0}=1.502$ by interpolation:

$k_u:=0.8104$

Instantaneous deflection:

$p_G:=\gamma_G\cdot\left(g_k+g_{self}\right)=4.708\ \dfrac{kN}{m}$ $\qquad p_Q:=\gamma_Q\cdot q_k=3.15\ \dfrac{kN}{m}$

$$w_{ins.g}:=k_u\cdot 5\cdot p_G\cdot\frac{{l_{eff}}^4}{384\cdot E_{0.g.mean}\cdot b\cdot\dfrac{{h_{ap}}^3}{12}}=0.042\ m$$

$$w_{ins.q}:=k_u\cdot 5\cdot p_Q\cdot\frac{{l_{eff}}^4}{384\cdot E_{0.g.mean}\cdot b\cdot\dfrac{{h_{ap}}^3}{12}}=0.028\ m$$

Glued Laminated timber	EN 14080	0,60	0,80	2,00

$k_{def}:=0.8$ because the of table shown above. kdef is independent on the material type, standard, service class. in this case, service class is 2.

$\psi_{2.1}:=0.2$

$$w_{net.fin}:=w_{ins.g}\cdot\left(1+k_{def}\right)+w_{ins.q}\cdot\left(1+\psi_{2.1}\cdot k_{def}\right)=0.107\ m$$

$$\frac{l_{eff}}{250} = 0.107\ m$$

Table 7.2 Examples of limiting values for deflections of beams

	w_{inst}	$w_{net,fin}$	w_{fin}
Beam on two supports	$\ell/300$ to $\ell/500$	$\ell/250$ to $\ell/350$	$\ell/150$ to $\ell/300$
Cantilevering beams	$\ell/150$ to $\ell/250$	$\ell/125$ to $\ell/175$	$\ell/75$ to $\ell/150$

if using above table for the recommended range of limiting values of the deflections for beams- the value is exactly same as the limitation number.