MŰEGYETEM 1782

Calculation of a prestressed concrete beam according to the Eurocode 2

Reinforced concrete structure BMEEOHSA-A2
Homework-1

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Initial data:

Length of the beam: $l_n := 20\ m$

Length of the supported part: $v := 21\ cm$

Distance of the longitudinal axises: $a := 7\ m$

Dead load: $g_k := 1\ \dfrac{kN}{m^2}$

Live load: $q_k := 1.4\ \dfrac{kN}{m^2}$

Concrete: C50/60

Steel: S500B

Prestressed tendons: FP-100/1770-R2

1. Data

1.1 Materials

Concrete C50/60

Characteristic value of compression stress: $f_{ck} := 50 \frac{N}{mm^2}$ $\gamma_c := 1.5$

Adverse loading effect factor: $\alpha := 1$

Design value of compression stress: $f_{cd} := \alpha \cdot \frac{f_{ck}}{\gamma_c} = 33.333 \frac{N}{mm^2}$

A prospective value of tensile stress: $f_{ctm} := 0.3 \cdot \left(f_{ck} \cdot \frac{N^{\frac{1}{2}}}{mm}\right)^{\frac{2}{3}} = 4.072 \frac{N}{mm^2}$

The prospective value of young's modulus: $E_{cm} := 37 \frac{kN}{mm^2}$

The design value of young's modulus: $E_{cd} := \frac{E_{cm}}{\gamma_c} = 24.667 \frac{kN}{mm^2}$

The maximum compression of concrete: $\varepsilon_{cu} := 0.35\%$

Reinforcement: S500B

The value of the young's modulus: $E_s := 200 \frac{kN}{mm^2}$

Characteristic value of yield stress: $f_{yk} := 500 \frac{N}{mm^2}$ $\gamma_s := 1.15$

Design value of yield stress: $f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \frac{N}{mm^2}$

The maximum elastic elongation: $\varepsilon_{sy} := \frac{f_{yd}}{E_s}$ $\varepsilon_{sy} := 0.22\%$

Characteristic value of maximum elongation: $\varepsilon_{su} := 5\%$ "B" ductility category

Prestressed tendon: FP 100/1770-R2

$A_t := 100\ mm^2$

$\sigma_{u.t} := 1770 \frac{N}{mm^2}$

R2- relaxation category

the young's modulus of the tendon: $E_p := 195 \frac{kN}{mm^2}$

the stress according to the agreed 0.1% yield stress: $f_{p0.1k} := 1500 \ \frac{N}{mm^2}$

the design value of the ultimate stress: $f_{pd} := \frac{f_{p0.1k}}{\gamma_s} = 1.304 \ \frac{kN}{mm^2}$

the area of the outer diameter of the tendon: $\phi_p := 12.9 \ mm$

the area of the cross section of one tendon: $A_{p100} := 100 \ mm^2$

the maximum elongation: $\varepsilon_{py} := \frac{f_{pd}}{E_p} = 0.007$ $\varepsilon_{py} := 0.7\%$

the characteristic value of maximum elongation: $\varepsilon_{pu} := 4\%$

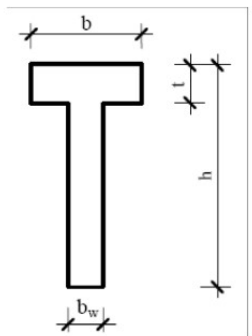
1.2 Cross section

$h := 1200 \ mm$

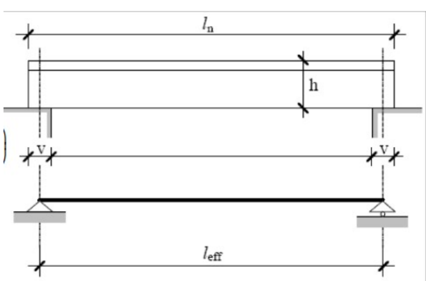
$b := 500 \ mm$

$t := 200 \ mm$

$b_w := 200 \ mm$



1.3 Static structure:



the effective length of the beam:

$$l_{eff} := l_n - 2 \ min\left(\frac{h}{2}, \frac{v}{2}\right) = 19.79 \ m$$

$$A_i := b \cdot t + b_w \cdot (h - t) = 0.3 \ m^2$$

1.4 Loads

the length between the beams: $a := 7 \ m$

the permanent load: $g_a := 1 \ \frac{kN}{m^2}$

the imposed/snow load: $q_{mk} := 1.4 \ \frac{kN}{m^2}$

calculation of loads for the prestressed girder:

the weight of the concrete: $q_{rc} := 25\ \dfrac{kN}{m^3}$

the value of the dead load: $g_{1.k} := \left(b \cdot t + b_w \cdot (h - t)\right) \cdot q_{rc} = 7.5\ \dfrac{kN}{m}$

the value of the permanent load: $g_{2.k} := a \cdot g_a \quad g_{2.k} = 7\ \dfrac{kN}{m}$

the safety factor the permanent load: $\gamma_G := 1.35$

the value of the imposed load: $q_k := a \cdot q_{mk} = 9.8\ \dfrac{kN}{m}$

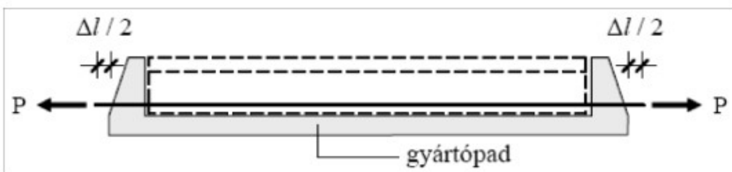
the safety factor of the imposed load: $\gamma_Q := 1.5$

$\psi_1 := 0.7 \quad \psi_2 := 0.6$

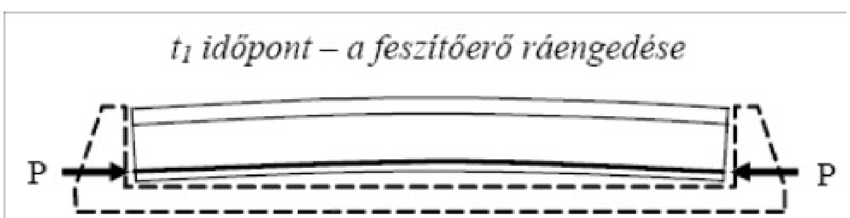
2. The loads and stresses in the different examining periods:

2.1 Examining periods

Fabrication bench

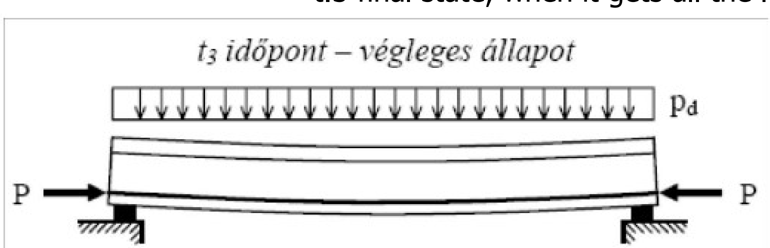


t.1 the moment when the beam gets the prestressed load



t.2 in the middle state (when the keep the beam outside of the factory, and when they are transporting to the site.

t.3 final state, when it gets all the loads.



2.2 Loads

at the time when it gets the prestressing (t1): $g_{1.k} = 7.5\ \frac{kN}{m}$

At the final state (t3):

- for the ultimate limit state: $p_d := \gamma_G \cdot (g_{1.k} + g_{2.k}) + \gamma_Q \cdot q_k = 34.275\ \frac{kN}{m}$

-for the cracking state of the beam (bequently load combination):

$$p_{ser.b} := g_{1.k} + g_{2.k} + \psi_1 \cdot q_k = 21.36\ \frac{kN}{m}$$

-for the counting the deflection(quasi- permanent load combination):

$$p_{ser.c} := g_{1.k} + g_{2.k} + \psi_2 \cdot q_k = 20.38\ \frac{kN}{m}$$

2.3 The Calculated stresses

Bending moment in the middle of the span:
when it gets the prestressing force(t1):

$$M_g := g_{1.k} \cdot \frac{{l_{eff}}^2}{8} = 367.166\ kN \cdot m$$

In the final state (t3):

- for calculating the ultimate limit state : $M_{ed} := p_d \cdot \frac{{l_{eff}}^2}{8} = \left(1.678 \cdot 10^3\right)\ kN \cdot m$
- for the cracking state of the beam (bequently load combination):

$$M_{ser.b} := p_{ser.b} \cdot \frac{{l_{eff}}^2}{8} = \left(1.046 \cdot 10^3\right)\ kN \cdot m$$

- for the counting the deflection (quazi- permanent load combination):

$$M_{ser.c} := p_{ser.c} \cdot \frac{{l_{eff}}^2}{8} = 997.713\ kN \cdot m$$

The shear stress:

- in the final state (t3): $V_{ed} := p_d \cdot \frac{l_{eff}}{2} = 339.151\ kN$

3. Determination of the amount of the reinforcement and tendons

the effective length: $d := 0.9\ h = \left(1.08 \cdot 10^3\right)\ mm$ $x_c := 105.29\ mm$
moment at the neutral axis of the tensile steel bars:
$M_{ed} = \left(1.678 \cdot 10^6\right)\ kN \cdot mm$

$$r_0 := \mathbf{root}\left(\left(b \cdot x_c \cdot \alpha \cdot f_{cd} \cdot \left(d - \frac{x_c}{2}\right) - M_{ed}\right), x_c\right) = 97.632\ mm$$

Checking the assumptions: $x_c = 105.29\ mm$ $t = 200\ mm$ it was ok!

$\xi_c := \frac{x_c}{d} = 0.097$ $\qquad$ $\zeta_{c0} := \frac{560}{700 + f_{yd} \cdot \frac{mm^2}{N}} = 0.493$

$\xi_c < \xi_{c0}$ the assumption was ok!

The required amount of steel bars from M.Ed:

$A_{s.szuks} := b \cdot x_c \cdot \frac{f_{cd}}{f_{yd}} = \left(4.036 \cdot 10^3\right) mm^2$ $\quad A_{s.szuks}$ means the area of both kind of bars (the reinforcement and the tendons)

The ratio between the tendons and the reinforcement:

$\chi := 0.7$

Determination of the reinforcement:

$A_{st.szuks} := (1 - \chi) \cdot A_{s.szuks} = \left(1.211 \cdot 10^3\right) mm^2$

Used reinforcement: 5*20 $\phi_{st} := 20\ mm$ $\quad A_{st} := 5 \cdot \phi_{st}^{\ 2} \cdot \frac{\pi}{4} = \left(1.571 \cdot 10^3\right) mm^2$

Determination of the prestressed tendons:

$A_{p.szuks} := \chi \cdot A_{s.szuks} \cdot \frac{f_{yd}}{f_{pd}} = 941.761\ mm^2$ $\qquad A_{p100} := 100\ mm^2$

Required number of tendons: $n_{szuks} := \frac{A_{p.szuks}}{A_{p100}} = 9.418$

Applied number of tendons: $n_{alk} := 12$

Preliminary prestressing strength:

$\sigma_{p0} := 1200\ \frac{N}{mm^2}\ <\ 0.85 \cdot f_{p0.1k} = \left(1.275 \cdot 10^3\right) \frac{N}{mm^2}$

4. The checking of the load bearing capacity of the beam

4.1 Cross section

calculating the required concrete cover:

- Environmental class: XC3 (room with limited humidity)
- Structural class: S4 (planned lifetime 50 years)
- Concrete cover due to cohesion requirement $\quad c_{min.b} := \phi_{st}$
- Concrete cover due to durability: $\quad c_{min.d} := 20\ mm$
- The minimum concrete cover: $\quad c_{min} := \max\left(c_{min.b}, c_{min.d}, 10\ mm\right) = 20\ mm$
- Addiction due to positioning non accuracy: $\Delta c_{dev} := 5\ mm$
- Concrete cover the calculations: $c := c_{min} + \Delta c_{dev} = 25\ mm$
- The minimum amount of space between reinforcements and tendons:
- the biggest diameter of the aggregate: $d_g := 14\ mm$
- the minimum space between the rebars: $\Delta s := \max\left(\phi_{st}, d_g + 4\ mm, 20\ mm\right) = 20\ mm$
- the min.horizontal space between tendonds:

$\Delta px := \max\left(d_g + 5\ mm, 2 \cdot \phi_p, 20\ mm\right) = 25.8\ mm$

- the min.vertical space between tendons: $\Delta py_{min} := \max\left(d_g, 2 \cdot \phi_p\right) = 25.8\ mm$

$\Delta py := 30\ mm$

the required number of tendons was 12 which can be put in 3 rows.
the required number of reinforcement is $5\phi 20$

Applied amount of reinforcement and tendons, and effective length:

- Reinforcement:

$\phi_{dt} := 20\ mm \quad \phi_w := 8\ mm$

$$a_{st} := c + \phi_w + \frac{\phi_{st}}{2} = 43\ mm$$

$$d_{st} := h - a_{st} = \left(1.157 \cdot 10^3\right)\ mm$$

$$A_{st} = \left(1.571 \cdot 10^3\right)\ mm^2$$

- Tendons:

$$d_{p0} := h - c - \phi_w - \phi_{st} - 70\ mm - \frac{\phi_p}{2} = \left(1.071 \cdot 10^3\right)\ mm$$

$$d_{p1} := d_{p0} - 2 \cdot \frac{\phi_p}{2} - \Delta py = \left(1.028 \cdot 10^3\right)\ mm$$

$$d_{p2} := d_{p1} - 2 \cdot \frac{\phi_p}{2} - \Delta py = 984.75\ mm$$

$A_{p0} := 4 \cdot A_{p100} = 400\ mm^2 \qquad A_{p2} := 4 \cdot A_{p100}$

$A_{p1} := 4 \cdot A_{p100} = 400\ mm^2$

$$A_p := A_{p0} + A_{p1} + A_{p2} = \left(1.2 \cdot 10^3\right)\ mm^2$$

$$d_p := \frac{\left(d_{p0} + d_{p1} + d_{p2}\right)}{3} = \left(1.028 \cdot 10^3\right)\ mm$$

4.2 The moment of getting the pressed force

The grade of concrete at the beginning: C40/50

$$f_{ck0} := 40\ \frac{N}{mm^2}$$

$$f_{ctd0} := 1.5\ \frac{N}{mm^2} \qquad f_{ctk0} := 1.5 \cdot f_{ctd0}$$

the design of the young's modulus:

$$E_{cd0} := \left(\frac{22}{\gamma_c} \cdot \left(\frac{f_{ck} + 8 \cdot \left(\frac{N}{mm^2}\right)}{10}\right)^{0.3} \cdot \left(\left(\frac{N}{mm^2}\right)\right)^{0.7} \cdot 1000\right) = 24.852\ \frac{kN}{mm^2}$$

$$\alpha_{es0} := \frac{E_s}{E_{cd0}} = 8.048 \qquad \alpha_{ep0} := \frac{E_p}{E_{cd0}} = 7.846$$

Ideal cross section behaviour in elastic, non- cracked state. (if x2>t):

Area of the cross section:

$A_{i10} := b \cdot t + b_w \cdot (h-t) + (\alpha_{es0} - 1) \cdot A_{st} + (\alpha_{ep0} - 1) \cdot A_p = (3.193 \cdot 10^5)\ mm^2$

Static moment:

$$s_{xi10} := b \cdot \frac{t^2}{2} + b_w \cdot (h-t) \cdot \frac{(t+h)}{2} + (\alpha_{es0} - 1) \cdot A_{st} \cdot d_{st} + (\alpha_{ep0} - 1) \cdot A_p \cdot d_p = (1.713 \cdot 10^8)\ \text{ɱ...}$$

Neutral axis: $x_{i10} := \frac{s_{xi10}}{A_{i10}} = 536.357\ mm$ > $t = 200\ mm$ The assumption OK!

$$I_{xi10} := \frac{b \cdot t^3}{12} + b \cdot t \cdot \left(x_{i10} - \frac{t}{2}\right)^2 + \frac{b_w \cdot (h-t)^3}{12} + b_w \cdot (h-t) \cdot \left(\frac{h}{2} + \frac{t}{2} - x_{i10}\right)^2 = (4.764 \cdot 10 \ldots$$
$$+ (\alpha_{es0} - 1) \cdot A_{st} \cdot (d_{st} - x_{i10})^2 + (\alpha_{ep0} - 1) \cdot A_p \cdot (d_p - x_{i10})^2$$

$I_{xi10} = (4.764 \cdot 10^{10})\ mm^4$

Checking at the midspan:

The prestressing force at the beginning, and the bending moment from it:

$N_{p0} := A_p \cdot \sigma_{p0} = (1.44 \cdot 10^3)\ kN$

$M_{p0} := A_p \cdot \sigma_{p0} \cdot (d_p - x_{i10}) = 707.462\ kN \cdot m$

the stress in the lower extreme fibre:

$$\sigma_{c.a} := \frac{-N_{p0}}{A_{i10}} + \frac{M_g - M_{p0}}{I_{xi10}} \cdot (h - x_{i10}) = -9.25\ \frac{N}{mm^2}$$

$\sigma_{c.a} = -9.25\ \frac{N}{mm^2}$ < $0.6 \cdot f_{ck0} = 24\ \frac{N}{mm^2}$ ok!

the stress in the upper extreme fibre

$\sigma_{c.f} := \frac{-N_{p0}}{A_{i10}} - \frac{M_g - M_{p0}}{I_{xi10}} \cdot x_{i10} = -0.679\ \frac{N}{mm^2}$ < $f_{ctd0} = 1.5\ \frac{N}{mm^2}$ ok!

Checking at the end of the beam:

The anchored length of the tendons:

$\eta_{p.1} := 3.2$ (in case of 7 vein tendons)

$\eta_1 := 0.7$ (general case of anchor)

Gripping strength: $f_{bpt} := \eta_{p.1} \cdot \eta_1 \cdot f_{ctd0} = 3.36 \ \frac{N}{mm^2}$

$\alpha_1 := 1.25$

$\alpha_2 := 0.19$

The anchored length: $I_{pt} := \alpha_1 \cdot \alpha_2 \cdot \phi_p \cdot \frac{\sigma_{p0}}{f_{bpt}} = 1.094 \ m$

The design value of the anchored length: $I_{ptd} := 0.8 \cdot I_{pt} = 0.875 \ m$

The force from the dead load:

$$R_g := g_{1.k} \cdot \frac{l_{eff}}{2} = 74.213 \ kN$$

Bending moment from the considered support point:

$$M_{gv} := R_g \cdot \left(I_{ptd} - \frac{v}{2}\right) - g_{1.k} \cdot \frac{\left(I_{ptd} - \frac{v}{2}\right)^2}{2} = 54.945 \ kN \cdot m$$

the stress in the lower extreme fibre:

$$\sigma_{c.a2} := \frac{-N_{p0}}{A_{i10}} + \frac{M_{gv} - M_{p0}}{I_{xi10}} \cdot \left(h - x_{i10}\right) = -13.599 \ \frac{N}{mm^2}$$

$\sigma_{c.a2} = -13.599 \ \frac{N}{mm^2} \ < \ 0.6 \cdot f_{ck0} = 24 \ \frac{N}{mm^2}$ ok!

the stress in the upper extreme fibre:

$$\sigma_{c.f2} := \frac{-N_{p0}}{A_{i10}} - \frac{M_{gv} - M_{p0}}{I_{xi10}} \left(x_{i10}\right) = 2.836 \ \frac{N}{mm^2}$$

$\sigma_{c.f2} = 2.836 \ \frac{N}{mm^2} \ > \ f_{ctd0} = 1.5 \ \frac{N}{mm^2}$ tendons have enough materials to remain same under the stress values. ok!

4.3 Calculating the prestressing losses

According to the beams slow deformation the prestressing force will decrease in the tendons. At the final state we examine the beam for this decreased stress. The losses are the following:

- shrinkage and creep of the concrete
- elastic deformation of the concrete
- relaxation of the steel tendons.
- from the steaming (if they used it at the prefabrication)

Shrinkage of concrete

We can determine the shrinkage of the concrete from the grade of concrete, from the relative humidity, the used beam sizes, and the planned lasting of the beam. At average building conditions the shrinkage of the concrete:~0,05%

Final value of the shrinkage of concrete: $\varepsilon_{cs} := 0.05\%$

Creeping of concrete

It depends from the used beam sizes, grade of concrete, type of the cement, relative humidity, and the planned lasting of the beam. At average conditions it is ~2,0.
Final value of the shrinkage of concrete: $f_i := 2$

From the relaxation of the tendons

If we don't find the exact value of the relaxation of the beams, we can use then the following calculations:

$f_{pk} := 1770 \frac{N}{mm^2}$ The starting stress of the prestressing:

$$\sigma_{p0} = \left(1.2 \cdot 10^3\right) \frac{N}{mm^2}$$

$$\mu := \frac{\sigma_{p0}}{f_{pk}} = 0.678$$

- The tendon's relaxation loss in 1000 hours (if the beams temperature is 20 °C) is 2,5%.
$\rho_{1000} := 0.025$

-The time which passed since the prestressing in hours.(we suppose that our building is meant for 50 years, so
the time which passed since: $t_p := 50 \cdot 365.25 \cdot 24 = 4.383 \cdot 10^5$

A: in case of tendons is 0.66 $A := 0.66$

B: incase the tendons is 9.1 $B := 9.1$

the loss the relaxations of the tendons:

$$\Delta\sigma_{pr} := A \cdot \frac{\sigma_{p0}}{1000} \cdot \rho_{1000} \cdot e^{B \cdot \mu} \cdot \left(\frac{t_p}{1000}\right)^{0.75 \cdot (1-\mu)} = 41.124 \ \frac{N}{mm^2}$$

From the shrinkage and the creeping of the concrete:
where: σcgp0 is the stress of the concrete at the quazi permanent state vicinity of the tendons.

$$\sigma_{cgp0} := \frac{-N_{p0}}{A_{i10}} + \frac{M_{ser.c} - M_{p0}}{I_{xi10}} \cdot \left(d_p - x_{i10}\right) = -1.517 \ \frac{N}{mm^2}$$

- the area of the concrete cross section:

$$A_c := b \cdot t + b_w \cdot (h - t) = \left(3 \cdot 10^5\right) \ mm^2$$

- the inertia of the concrete :

$$I_c := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(x_{i10} - \frac{t}{2}\right)^2 + b_w \cdot \frac{(h-t)^3}{12} + b_w \cdot (h-t) \cdot \left(\frac{h}{2} + \frac{t}{2} - x_{i10}\right)^2$$

$$I_c = \left(2.083 \cdot 10^{11}\right) \ mm^4$$

- the centroid of the concrete cross section (measured from the upper extreme fibre)

$$y_c := \frac{b \cdot \frac{t^2}{2} + b_w \cdot (h-t) \cdot \left(\frac{h+t}{2}\right)}{A_c} = 500 \ mm$$

- the distance between the tendons and the centroid of the concrete cross section:

$$z_{cp} := d_p - y_c = 527.65 \ mm$$

- the coefficient of the the tendon's youngs modulus and the final youngs modulus of the concrete:

$$E_p := 195 \cdot \frac{kN}{mm^2} \; E_s := 200 \cdot \frac{kN}{mm^2} \quad \alpha_p := \frac{E_p}{E_{cm}} = 5.27$$

substituting the values above we get the loss from the shrinkage and the relaxation:

$$\Delta\sigma_{p.t} := \frac{\varepsilon_{cs} \cdot E_p + 0.8 \cdot \Delta\sigma_{pr} + \alpha_p \cdot f_i \cdot \left|\sigma_{cgp0}\right|}{1 + \alpha_p \cdot \frac{A_p}{A_c} \cdot \left(1 + \frac{A_c \cdot {z_{cp}}^2}{I_c}\right) \cdot \left(1 + 0.8 \cdot f_i\right)} = 135.95 \; \frac{N}{mm^2}$$

From the steaming of the concrete
We suppose that to make the stiffening faster we use steaming.

Heat expansion coefficient: $\alpha_T := 10^{-5} \cdot \frac{1}{° \cdot C}$

the temperature difference: $\Delta_T := 40 \cdot ° \cdot C$

the loss from the steaming: $\Delta\sigma_{p.T} := \alpha_T \cdot \Delta_T \cdot E_p = 78 \; \frac{N}{mm^2}$

The effective value of the prestress:

$$\sigma_{p0} = \left(1.2 \cdot 10^3\right) \frac{N}{mm^2} \qquad \Delta\sigma_{p.T} = 78 \; \frac{N}{mm^2} \qquad \Delta\sigma_{p.t} = 135.95 \; \frac{N}{mm^2}$$

$$\sigma_{pm} := \sigma_{p0} - \Delta\sigma_{p.T} - \Delta\sigma_{p.t} = 986.05 \; \frac{N}{mm^2}$$

the effective prestress ratio:

$$\upsilon := \frac{\sigma_{pm}}{\sigma_{p0}} = 0.822$$

the effective prestress force:

$$A_p = \left(1.2 \cdot 10^3\right) mm^2 \quad N_{pm} := \sigma_{pm} \cdot A_p = \left(1.183 \cdot 10^3\right) kN$$

4.4. Checking of the ultimate load bearing capacity

We check it in the final state, and for the calculation we use the elastic-plastic behaviour of the tendons.
We suppose that xc<t and the reinforcement and the tendons have plastic behaviour.

$$x_c := \frac{A_{st} \cdot f_{yd} + A_p \cdot \sigma_{pm}}{b \cdot f_{cd}} = 111.973 \; mm$$

From the calculation above the height of the compressed zone can be calculated:

$x_c = 111.973 \; mm$ < $t = 200 \; mm$ Our assumption was right!

Checking of the elongation of the reinforcement:

$$\varepsilon_s := \varepsilon_{cu} \cdot \frac{d_{st} - 1.25 \cdot x_c}{1.25 \cdot x_c} = 0.025$$

$\varepsilon_{sy} = 0.002$ < $\varepsilon_s = 0.025$ < $\varepsilon_{su} = 0.05$

Checking of the tendons:

$$\varepsilon_p := \varepsilon_{cu} \cdot \frac{d_p - 1.25 \cdot x_c}{1.25 \cdot x_c} + \frac{\sigma_{pm}}{E_p} = 0.027 \quad \varepsilon_{py} = 0.007 \quad \varepsilon_{pu} = 0.04$$

$$prestressing\_strand\_strain := \left\| \begin{array}{l} \text{if } \varepsilon_{py} < \varepsilon_p < \varepsilon_{pu} \\ \quad \left\| \text{"good"} \right. \\ \text{else} \\ \quad \left\| \text{"not good"} \right. \end{array} \right| = \text{"good"}$$

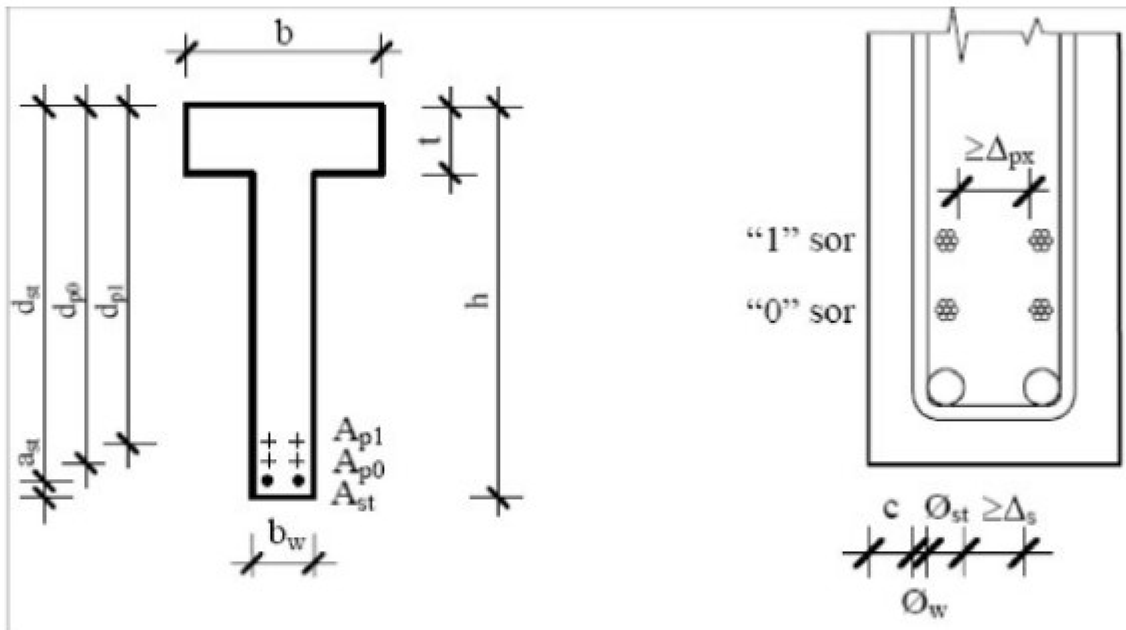
The calculation of the bending moment resistance:

$$M_{rd} := b \cdot x_c \cdot f_{cd} \cdot \frac{x_c}{2} + A_{st} \cdot f_{yd} \cdot \left(d_{st} - x_c\right) + A_p \cdot f_{pd} \cdot \left(d_p - x_c\right) = \left(2.251 \cdot 10^3\right) \; kN \cdot m$$

$M_{rd} = \left(2.251 \cdot 10^3\right) \; kN \cdot m$ > $M_{ed} = \left(1.678 \cdot 10^3\right) \; kN \cdot m$

Right. The assumption is okay.

Annex:



Applied amount of reinforcement and tendons.

3rd homework

4.5. Design of the shear reinforcement

The calculation of the shear reinforcement is calculated in (t3) final state.
The substitute of the effective height for the tensile reinforcement.

$$d_h := \frac{E_s \cdot A_{st} \cdot d_{st} + E_p \cdot A_p \cdot d_p}{E_s \cdot A_{st} + E_p \cdot A_p} = \left(1.102 \cdot 10^3\right)\ mm \qquad p_d = 34.275\ \frac{kN}{m}$$

the value of the shear force: $V_{ed} = 339.151\ kN$

reduction of the shear force: $V_{Ed.red} := V_{ed} - d_h \cdot p_d = 301.388\ kN$

Shear resistance of the section without shear reinforcement.

$$k := min\left(1 + \sqrt{\frac{200}{d_h \cdot \frac{1}{mm}}}, 2\right) = 1.426$$

Proportional area for the longitudinal reinforcement for which the ancoured reinforcement and the gripping of the tendons.

$$\rho_1 := min\left(\frac{A_{st} + A_p}{b_w \cdot d}, 0.02\right) = 0.013$$

the stress of the concrete from the prestress:

$$\sigma_{cp} := min\left(\frac{N_{pm}}{A_c}, 0.2 \cdot f_{cd}\right) = 3.944\ \frac{N}{mm^2}$$

$$v_{min} := \left(0.035 \cdot k^{1.5} \cdot f_{ck}^{\ 0.5} \cdot \frac{\sqrt{N}}{mm} + 0.15 \cdot \sigma_{cp}\right) \cdot b_w \cdot d_h = 223.241\ kN$$

the shear resistance:

$$V_{Rd.c} := \max\left(\left(\frac{0.18}{\gamma_c} \cdot k \cdot \left(100 \cdot \rho_1 \cdot f_{ck} \cdot \frac{N^2}{mm^4}\right)^{\frac{1}{3}} + 0.15 \cdot \sigma_{cp}\right) \cdot b_w \cdot d_h, v_{min}\right)$$

$V_{ed} = 339.151\ kN$ > $V_{Rd.c} = 281.314\ kN$ We don't need to use shear reinforcement.

The maximal shear value taken at the pure concrete semi-members of the section without collapse of these members

For normal concrete:

$$\beta_{ct} := 2.4 \quad \eta_1 := 1$$

$$V_c := \beta_{ct} \cdot \eta_1 \cdot 0.1 \left(f_{ck} \cdot \frac{N^2}{mm^4} \right)^{\frac{1}{3}} \cdot \left(1 + 1.2 \cdot \frac{\sigma_{cp}}{f_{cd}} \right) \cdot b_w \cdot 0.9 \cdot d_h = 200.247 \ kN$$

$$cot\theta := \frac{\left(1.2 + 1.4 \cdot \frac{\sigma_{cp}}{f_{cd}} \right)}{1 - \frac{V_c}{V_{Ed.red}}} = 4.069$$

restriction of cot $\quad 1.0 < cot\theta < 2.0 \qquad cot\theta := 2$

According to this used angle:

$$\alpha_{cw} := \begin{cases} 1 & \text{if } \sigma_{cp} = 0 \\ \left(1 + \frac{\sigma_{cp}}{f_{cd}} \right) & \text{if } 0 < \sigma_{cp} \le 0.25 \cdot f_{cd} \\ 1.25 & \text{if } 0.25 \cdot f_{cd} < \sigma_{cp} \le 0.5 \cdot f_{cd} \\ \left(2.5 \cdot \left(1 - \frac{\sigma_{cp}}{f_{cd}} \right) \right) & \text{if } 0.5 \cdot f_{cd} < \sigma_{cp} < f_{cd} \end{cases}$$

$$\alpha_{cw} = 1.118$$

Closed stirrups and the angle with the axis: $\quad \alpha_{sw} := 90 \cdot \frac{\pi}{180} = 1.571$

the efficiency coefficient:

$$\nu := 0.6 \cdot \left(1 - \frac{f_{ck}}{250} \cdot \frac{\frac{mm^2}{N}}{250} \right) = 0.6$$

the maximum resistance of shear resistance:

$$V_{Rd.max} := \alpha_{cw} \cdot b_w \cdot 0.9 \cdot d_h \cdot \nu \cdot f_{cd} \cdot \frac{\left(cot\theta + \cot\left(\alpha_{sw} \right) \right)}{1 + cot\theta^2}$$

$V_{Rd.max} = (1.773 \cdot 10^3)\ kN$ $\quad > \quad V_{ed} = 339.151\ kN$

the beam should be reinforced for shear

Calculating shear reinforcement

The used size of the stirrups: $\phi_w := 8\ mm$

The area of the cross section of the stirrups:

$$A_{sw} := 2 \cdot \phi_w^{\ 2} \cdot \frac{\pi}{4} = 100.531\ mm^2$$

the minimum distance between stirrups:

$$s_{min} := \frac{A_{sw}}{V_{Ed.red}} \cdot 0.9 \cdot d_h \cdot f_{yd} \cdot (cot\theta + \cot(\alpha_{sw})) \cdot \sin(\alpha_{sw}) = 287.617\ mm$$

Used stirrups distance: $s_{req1} := 250\ mm$

$$V_{Rd1} := \frac{A_{sw}}{s_{req1}} \cdot 0.9 \cdot d_h \cdot f_{yd} \cdot (cot\theta + \cot(\alpha_{sw})) \cdot \sin(\alpha_{sw})$$

$V_{Rd1} = 346.737\ kN$ $\quad > \quad V_{Ed.red} = 301.388\ kN$ $\quad Ok!$

Second stirrups distance: $s_{req2} := 300\ mm$

$$V_{Rd2} := \frac{A_{sw}}{s_{req2}} \cdot 0.9 \cdot d_h \cdot f_{yd} \cdot (cot\theta + \cot(\alpha_{sw})) \cdot \sin(\alpha_{sw}) = 288.948\ kN$$

Third stirrups distance: $s_{req3} := 350\ mm$

$$V_{Rd3} := \frac{A_{sw}}{s_{req3}} \cdot 0.9 \cdot d_h \cdot f_{yd} \cdot (cot\theta + \cot(\alpha_{sw})) \cdot \sin(\alpha_{sw}) = 247.669\ kN$$

-Checking the curtailments:

The maximum distance of the stirrups: $ok!$

$d_h = (1.102 \cdot 10^3)\ mm$ $\quad s_{max} := 0.75 \cdot d_h = 826.337\ mm$ $\quad > \quad s_{alk3}$

Checking proportional area of shear reinforcement:

$$\rho_w := \frac{A_{sw}}{s_{req3} \cdot b_w \cdot \sin(\alpha_{sw})} = 0.144\ 1\% \quad > \quad \rho_{w.min} := 0.08 \cdot \frac{\sqrt{f_{ck} \cdot \frac{N}{mm^2}}}{f_{yk}} = 0.113\ 1\%$$

$OK!$

5. Checking the deflection of the beam

In Serviceability limit state we have to check the deflection of the beam and the size of the cracks. But her the deflection will be presented.
First step is to examine if the beam cracks or not. The examination should be done with the bequently load combination.
the cross section in the non cracked elastic state, with the final grade of concrete.(considering x.ii>t).

$E_{cd}=24.667\ \dfrac{kN}{mm^2}$ $E_s=200\ \dfrac{kN}{mm^2}$ $\alpha_{es}:=\dfrac{E_s}{E_{cd}}=8.108$

$\alpha_{ep}:=\dfrac{E_p}{E_{cd}}=7.905$

The area of the cross section:

$$A_{iI}:=b\cdot t+b_w\cdot(h-t)+(\alpha_{es}-1)\cdot A_{st}+(\alpha_{ep}-1)\cdot A_p=0.319\ m^2$$

Static moment:

$$S_{xiI}:=b\cdot\frac{t^2}{2}+b_w\cdot(h-t)\cdot\frac{t+h}{2}+(\alpha_{es}-1)\cdot A_{st}\cdot d_{st}+(\alpha_{ep}-1)\cdot A_p\cdot d_p=\left(1.714\cdot 10^8\right)\ mm^3$$

Neutral axis:

$x_{iI}:=\dfrac{S_{xiI}}{A_{iI}}=536.65\ mm$ $\quad > \quad$ $t=200\ mm$

The assumption was right

$$I_{x.iI}:=\frac{b\cdot t^3}{12}+b\cdot t\cdot\left(x_{iI}-\frac{t}{2}\right)^2+\frac{b_w\cdot(h-t)^3}{12}+b_w\cdot(h-t)\cdot\left(\frac{h}{2}+\frac{t}{2}-x_{iI}\right)^2+(\alpha_{es}-1)\cdot A_{st}\cdot\left(d_{st}-x_{iI}\right)^2+(\alpha_{ep}-1)\cdot A_p\cdot\left(d_p-x_{iI}\right)^2=0.048\ m^4$$

The bending moment from the bequently load combination.

$$M_{ser.b}=1045.69\ kN\cdot m$$

Bending moment from the potential prestressing force:

$$M_{pm}:=A_p\cdot\sigma_{pm}\cdot\left(d_p-x_{iI}\right)=580.98\ kN\cdot m$$

Cracking bending moment:

$$M_{cr}:=\frac{I_{x.iI}}{h-x_{iI}}\cdot\left(f_{ctm}+\frac{N_{pm}}{A_{iI}}+\frac{M_{pm}}{I_{x.iI}}\cdot\left(h-x_{iI}\right)\right)=1140.081\ kN\cdot m$$

$M_{cr}>M_{ser.b}$ So the cross section at the calculation of the deflection can be calculated as non cracked!

The deflection of the beam with the assumption of non-cracked elastic cross section

$\phi_t := 2$ $\quad E_{cm} = 37 \ \frac{kN}{mm^2}$ $\quad E_{c.eff} := \frac{E_{cm}}{1+\phi_t} = 12.333 \ \frac{kN}{mm^2}$

upper extreme fibre:

$$\sigma_{cf.I} := \frac{-N_{pm}}{A_{iI}} - \frac{M_{ser.b} - M_{pm}}{I_{x.iI}} \cdot x_{iI} = -8.933 \ \frac{N}{mm^2}$$

lower extreme fibre:

$$\sigma_{ca.I} := \frac{-N_{pm}}{A_{iI}} + \frac{M_{ser.b} - M_{pm}}{I_{x.iI}} \cdot \left(h - x_{iI}\right) = 2.759 \ \frac{N}{mm^2}$$

the σca.I fictitious tensile stress, without the consideration of going over the limited tensile stress.
The curveture of the beam at the midspan:

$$k_I := \frac{\sigma_{ca.I} + \left(-1 \cdot \sigma_{cf.I}\right)}{h \cdot E_{c.eff}} = \left(7.9 \cdot 10^{-4}\right) \ \frac{1}{m}$$

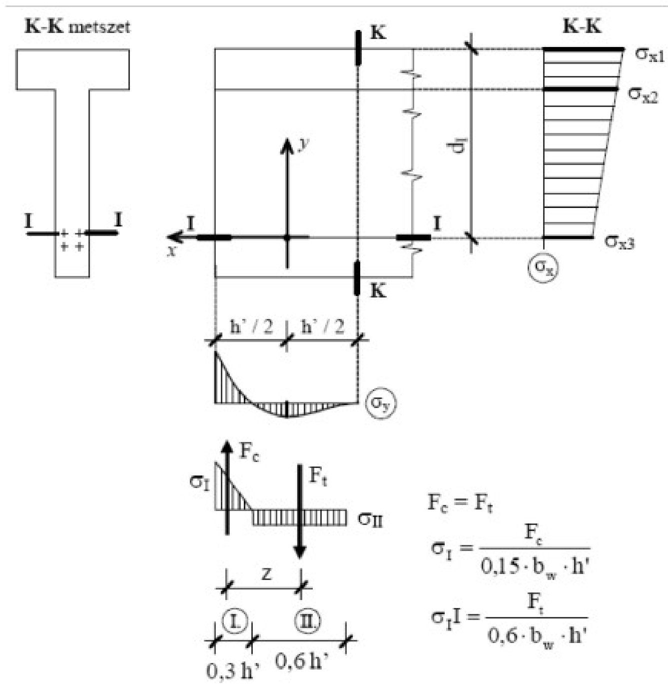
The value of deflection with the assumption of non cracked, elastic cross section.:

$$y_I := \frac{5}{48} \cdot k_I \cdot {l_{eff}}^2 = 32.227 \ mm$$

Checking the deflection: $\quad \frac{l_{eff}}{500} = 39.58 \ mm \quad > \quad y_I = 32.227 \ mm$

OK!

6. The examination of the end of the beam



The design value of the anchored length for the examination at the end of the beam

$I_{pt} = 1.094\ m$ $\qquad$ $l_{ptd2} := I_{pt} \cdot 1.2 = 1.313\ m$

The length of the examination area:

$$h' := \max\left(\sqrt{h^2 + \left(0.6 \cdot l_{ptd2}\right)^2}, l_{ptd2}\right) = 1.436\ m$$

The distance of the K - K section from the end of the beam :

$$\xi := h' - \frac{v}{2} = 1.331\ m$$

The bending moment in the K - K section, at final state:

$$M_{Ed\xi} := \frac{p_d \cdot l_{eff}}{2} \cdot \xi - p_d \cdot \frac{\xi^2}{2} = 420.903\ kN \cdot m$$

the effective height for the I - I horizontal section
$d_I := d_{p2} = 984.75\ mm$

The value of the horizontal stesses (with the consideration of non cracked elastic behaviour):

$$\sigma_{x1} := \frac{-N_{pm}}{A_{iI}} - \frac{M_{Ed\xi} - M_{pm}}{I_{x.iI}} \cdot x_{iI} = -0.002 \ \frac{kN}{mm^2}$$

$$\sigma_{x2} := \frac{-N_{pm}}{A_{iI}} - \frac{M_{Ed\xi} - M_{pm}}{I_{x.iI}} \cdot \left(x_{iI} - t\right) = -0.003 \ \frac{kN}{mm^2}$$

$$\sigma_{x3} := \frac{-N_{pm}}{A_{iI}} + \frac{M_{Ed\xi} - M_{pm}}{I_{x.iI}} \cdot \left(d_I - t\right) = -0.006 \ \frac{kN}{mm^2}$$

The bending moment of the horizontal loads for the I - I section:

$$M_x := \left|\sigma_{x1}\right| \cdot t \cdot b \cdot \left(d_I - \frac{t}{2}\right) + \frac{\left|\sigma_{x2} - \sigma_{x1}\right|}{2} \cdot t \cdot b \cdot \left(d_I - \frac{2}{3} \cdot t\right) + (0) + \left|\sigma_{x2}\right| \cdot \left(d_I - t\right) \cdot b_w \cdot \frac{d_I - t}{2} + \frac{\left|\sigma_{x3} - \sigma_{x2}\right|}{2} \cdot \left(d_I - t\right) \cdot b_w \cdot \frac{d_I - t}{3} = 432.726 \ kN \cdot m$$

The level arm vertical Ft and Fc forces:

$$z := 0.5 \cdot h' = 717.75 \ mm$$

The vertical tensile force from the bending moment equilibrium K - K and I - I sections

$$F_t := \frac{M_x}{z} = 602.892 \ kN \qquad F_c := F_t$$

The required amount of reinforcement(closed stirrups):

$$A_{sw.req} := \frac{F_t}{f_{yd}} = 1386.651 \ mm^2$$

the required amount of stirrups:

$$n := \frac{A_{sw.req}}{A_{sw}} = 13.793$$

the used number of stirrups: $n_{app} := 14$

The stirrups should be distributed in the range:

$$0.6 \cdot h' = 861.3 \ mm$$

Required spacing of the stirrups :

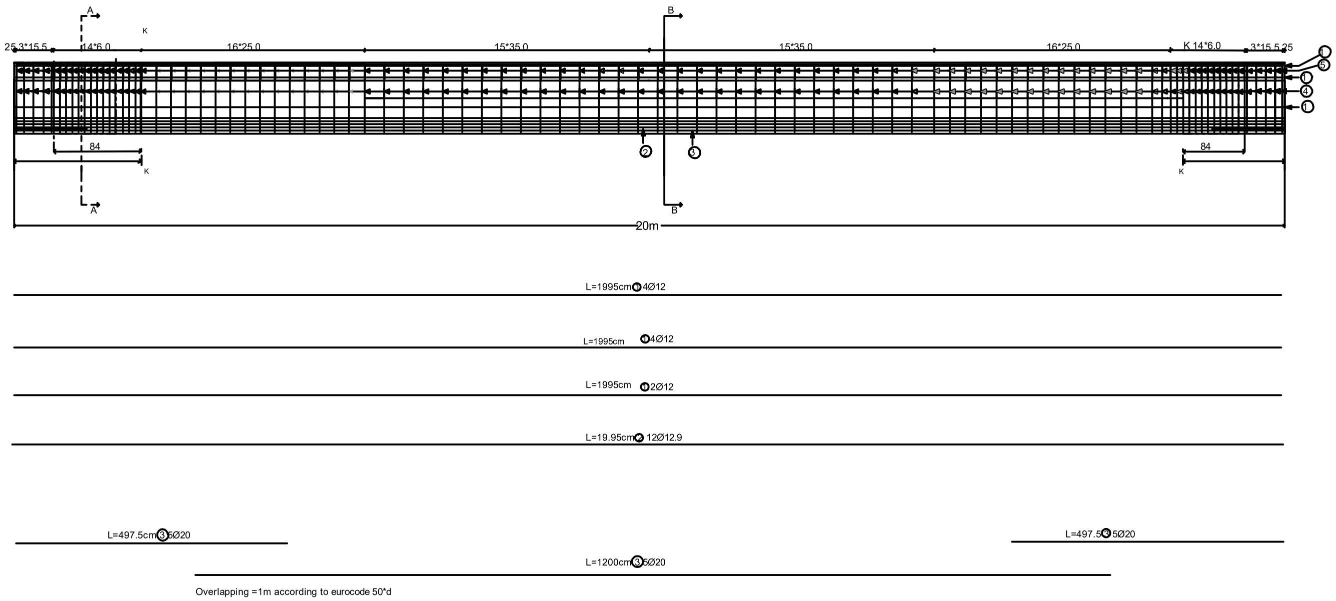
$$0.6 \cdot \frac{h'}{n_{app}} = 61.521 \ mm$$

Applied spacing of the stirrups :

$$S_{app} := 60 \ mm$$

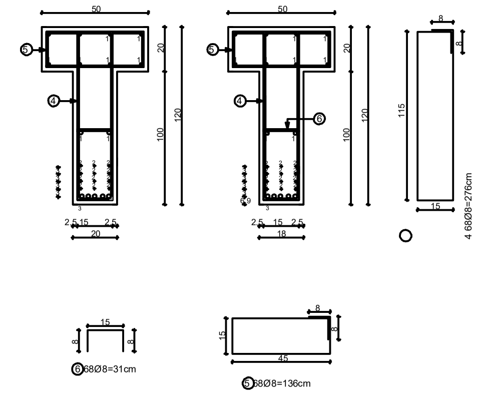
$$0.3 \cdot h' = 430.65 \ mm$$

Reinforcement Plan of Beam (S=1:25)



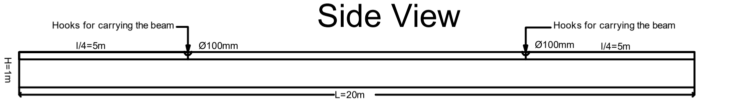
Section A-A (S=1:10)

Section B-B (S=1:10)

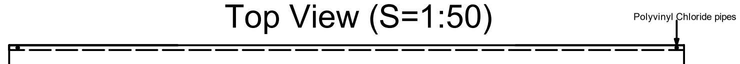


Form Work Plan (S=1:50)

Side View



Top View (S=1:50)



Notes:

Eurocode,Hungarian National annex 1992-1-1

Materials:
Concrete: 50/60-XC3-S3
Reinforcement: S500B
Prestressed Tendons: Fp-100/1770-R2
Concrete cover:25mm
Structural class:S3(planned life time is 50 years)

Prestressing loses=986.05N/mm2 σ

The loss of the relaxation of the tendons:σ_{pr}=41.124 N/mm2
The loss from the shrinkage and creeping of concrete : σ_{pt}=135.95N/mm2
The effective prestress ratio: ν=0.822

Live load:qk=1 kN/m
Dead load:gk=1.4 kN/m

Masses:
Concrete : 15685.9 kg
Tendons:198 kg
Reinforcement:616.63 kg
Girder:16500.53kg

Reinforcement schedule

bar mark	number of bars	Φ	length of one bar[m]	total length of each bar[m]			
				Φ8	Φ12	Φ12.9	Φ20
1	10	12	19.95		199.5		
2	10	12.9	20			200	
3	5	20	21.95				109.75
4	96	8	2.76	264.96			
5	96	8	1.36	130.56			
6	96	8	0.31	29.76			
Total length for each diameter [m]				425.28	199.5	200	109.75
mass of 1m [kg/m]				0.395	0.89	0.99	2.47
mass for each diameter [kg]				167.99	177.56	198	271.08
total mass [kg]				814.63			

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Reinforced Concrete Buildings		S=1:25 S=1:10
Consultant	Roszevák Zs	27-05-2022

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