Modernizing the undergraduate statistics curriculum: what are the theoretical underpinnings?

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nhorton@amherst.edu, all slides from session at
https://github.com/Amherst-Statistics/Modernizing-Undergrad-Stat-Curric

Plan

- Motivation
- What we've been teaching
- What to teach?
- Some questions
- Some suggestions
- Closing thoughts

Motivation

"The ASA's 2014 guidelines for undergraduate programs in statistics stressed the importance of the integration of theory and practice to ensure that graduates have the capability to effectively extract meaning from data."

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- focus on **undergraduate**

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- goal: **integration of theory and practice**

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- goal: **extract meaning from data**

Learning outcomes (ASA guidelines)

- Bachelors graduates in statistics must be able to understand issues of design, confounding, and bias.
- They need to be know how to apply their knowledge of theoretical foundations to the sound analysis of data.
- They should be able to design studies, use graphical and other means to explore data, build and assess statistical models, employ a variety of formal inference procedures (including resampling methods), and draw appropriate scope of conclusions from the analysis.
- They need knowledge and experience applying a variety of statistical methods, assessing their appropriateness, and communicating results.
-

Where are we now?

"We believe that aspects of the traditional probability/inference sequence, with its emphasis on large sample size approximations and lists of distributions, does not fully capture current statistical practice" (ASA guidelines 2014)

JSM 2003 discussion

"The math stat course has not changed in 40 years, whereas statistics has changed enormously, so how could the course not be obsolete?" (David Moore)

JSM 2003 discussion

"Goethe: 'Theory is all grey, but the tree of life is green.' The main slam on the math stat course is that it is caught in a time warp. The table of contents was stunning because it is so much like the course we teach now. The worse thing is that it tends to bore the teachers, and if teachers are bored, students dont get the liveliness of the subject" (Brad Efron)

JSM 2003 discussion

"We should de-emphasize t-test, jackknife, named nonparametric methods, asymptotics (Ive never seen an infinite sample), Cramer-Rao lower bound. It should not be too much like a catalog. I would add modeling, computing (likelihood graphics), problem solving, decision calculations (how to think about the best thing to do), risk, odds, expectation, sample and experimental design (at least touch on that), foundational issues such as understanding p-values (direction of conditioning), worrying about whether they are any good" (Carl Morris)

Question #1: Early or late in the curriculum

- What is the optimal placement of the theoretical statistics course in an undergraduate statistics major?
- long stretch of linear prereqs between students and theoretical underpinnings

Question #2: Intro or non-intro course prereq

- historically only stats course a math major might take
- what does GAISE K-12 report imply regarding time to imbue statistical reasoning skills?

Question #3: Role of computing

- historically taught as chalk and talk
- incorporate technology

JSM 2003 discussion

"If we imagine a universe where computing preceded mathematics in the development of statistics, then introductory courses would not be the same; they would start with the easy stuff (nonparametrics) and work up to parametric stuff (the hard stuff)" (Brad Efron).

Question #4: Empirical vs. analytic problem-solving

- empirical simulations to check answers
- empirical simulation to gain insights

Example (Horton, TAS, 2013)

"Assume that we observe n iid observations from a normal distribution. (i) Use the IQR of the list to estimate σ. (ii) Use simulation to assess the variability of this estimator for samples of n $=$ 100 and n $=$ 400. (iii) How does the variability of this estimator compare to $\hat{\sigma}$ (usual estimator)?"

Question #5: Future doctoral students in statistics?

- small fraction of students in theoretical statistics
- University of Chicago advice: "The prerequisites for the masters program are calculus through Jacobians and multivariate integrals, linear/matrix algebra, and a year of elementary probability and statistics."
- "Applicants to the doctoral program should have that background plus additional courses in advanced mathematics, such as real or complex analysis, and/or in other disciplines such as computer science, economics, and the natural sciences."
- No math stat required!

Where are we now?

"A modern statistical theory course might, for example, include work on computer-intensive methods and non-parametric modeling. Such a course should provide students with an overview of statistics and statistical thinking that builds on their introductory statistics courses.
It may be useful to incorporate computing, data-related, and communication components in this class. If included early on in a student's program, it will help to provide a solid foundation for future courses and experiential opportunities" (ASA guidelines, 2014)

Cobb, Chilean Journal of Statistics (2010)

"When we teach statistics, what is it that we want our students to learn? Surely the most common answer must be that we want our students to learn to analyze data, and certainly I share that goal. But for some students, particularly those with a strong interest and ability in mathematics, I suggest a complementary goal, one that in my opinion has not received enough explicit attention: We want these mathematically inclined students to learn to solve methodological problems."

JSM 2003 discussion

"There was a pure mathematician at Texas–R.L. Moore. He would grab some smart freshmen and sit them down day one and give them research problems to work on right away (topological). Within the first minutes of lecture, he would ask a question and would get mad if students did not know the answer. We could do that too" (Carl Morris).

(highly) Modified Moore-method theoretical statistics

- Moore introduced a method for graduate mathematics instruction that consisted primarily of individual student work on challenging proofs.
- Cohen described an adaptation with less explicit competition suitable for undergraduate students at a liberal arts college.
- This article details an adaptation of this modified Moore method to teach mathematical statistics, and describes ways that such an approach helps engage students and foster the teaching of statistics.
- Groups of students worked a set of three difficult problems (some theoretical, some applied) every two weeks.

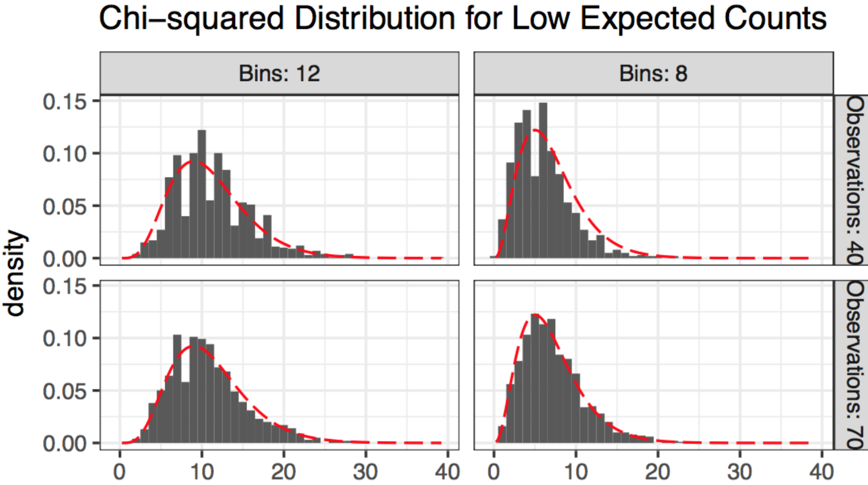
(highly) Modified Moore-method theoretical statistics

- Class time was devoted to coaching sessions with the instructor, group meeting time, and class presentations.
- R was used to estimate solutions empirically, where analytic results were intractable, as well as to provide an environment to undertake simulation studies with the aim of deepening understanding and complementing analytic solutions.
- Each group presented comprehensive solutions to complement oral presentations.
- Development of parallel techniques for empirical and analytic problem solving was an explicit goal of the course, which also attempted to communicate ways that statistics can be used to tackle interesting problems.
- The group problem-solving component and use of technology allowed students to attempt much more challenging questions than they could otherwise solve.

Nolan and Speed (Statlabs, 2000)

Perform a simulation study on the sensitivity of the χ^2 test for the uniform distribution to expected cell counts below 5. Simulate the distribution of the test statistics for 40, 50, 60, and 70 observations from a uniform distribution using 8, 10, and 12 equal-length bins.

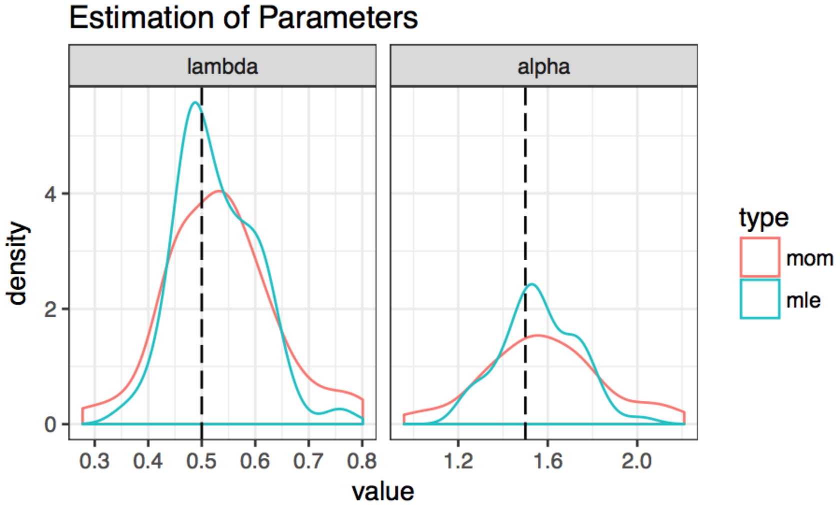
Nolan and Speed (Statlabs, 2000)



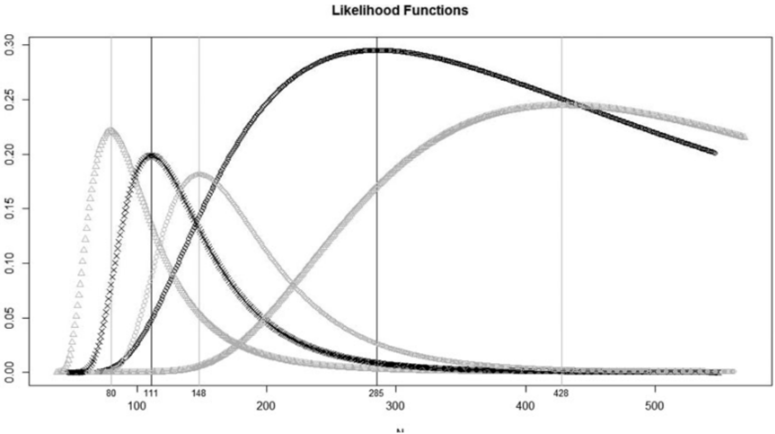
Rice Exercise 8.12

Suppose that you had to choose either the method of moments estimates or the maximum likelihood estimates in Example C of Section 8.4 and C of Section 8.5. Which would you choose and why?

Rice Exercise 8.12



Likelihood functions from a capture-recapture problem



Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report 2016

revised GAISE College Report (2016)

1. Teach statistical thinking.
    - Teach statistics as an investigative process of problem-solving and decision-making.
    - Give students experience with multivariable thinking.
2. Focus on conceptual understanding.
3. Integrate real data with a context and purpose.
4. Foster active learning.
5. Use technology to explore concepts and analyze data.
6. Use assessments to improve and evaluate student learning.

Feedback from modified Moore method (debrief with students)

"Although your students may want you to tell them 'the key points to absolutely know,' you believe strongly that they must work their way toward knowledge mastery in this course. To assist them in achieving this end, you have structured the course in ways that require them to work individually and collaboratively–with guidance from you–as they become more expert and reflective learners."

Feedback from modified Moore method (debrief with students)

"Many of your students are uneasy with this approach and un- sure of themselves: they want to know the right answers, the correct way to think, hence their request for more input from you. Their unease marks them as less sophisticated about real learning and/or timid about undertaking independent intellectual journeys. You might have an explicit discussion with your students about your pedagogy and your learning goals for them."

Summary of modified Moore method

- group work (not individual)
- cooperation (not competition)
- major use of R as environment to explore (Horton TAS 2004)
- github to help collaboration (Bryan, https://happygitwithr.com)

Back to the questions

1. Early in the curriculum (to allow reinforcement)
2. Intro course as prereq (to build on a prior foundation)
3. Key role of computing (and in probability...)
4. Teaching problem solving (empirical and analytic)
5. Different pathways for future doctoral students

Items 1-4 are necessary (but not sufficient) for Hilary to hire the graduates...

JSM 2003

Carl Morris: But the better you are at math, the better you are at developing appropriate mathematical models.
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shameless plug for Hardin and Horton (2017, Notices AMS)

Closing thoughts

- It's never been easier to extract meaning from data (improved tools)
- How do we ensure that statistics remains a vibrant choice for our students?

Closing thoughts

> *Curriculum unavoidably involves decisions about scarce resources, so curricular innovation cannot escape being political, and of course "all politics is local" (ONeill and Hymel, 1995).*
>
> *Curriculum is political for economic reasons because, averaged over the long term, faculty FTEs and course offerings are at best a zero-sum game. Thus changing curriculum, like moving a graveyard, depends on local conditions: Whose cherished ancestry is uprooted by the change?*

(Cobb 'Mere renovation is too little, too late: we need to rethink our undergraduate curriculum from the ground up' arXiv 2015)

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Thoughts about textbooks (from Cobb ChJS)

Two important facts about textbooks for mathematical statistics:

1. The convex hull of content, organization, and approach has been expanding with Hubble-like acceleration. I take this rapid curricular expansion as a healthy symptom of our profession's growing discontent with the usual course.
2. Despite our rapidly expanding horizon of options, time remains an inelastic, zero-sum quantity, and so it is in the nature of the content of probability and mathematical statistics that any focus on abstract theory of inference takes time and attention away from the challenge of modeling data relationships.

Statistics is a subject with a vast field of application, involving problems which vary widely in their character and complexity. However, in tackling these, we use a relatively small core of central ideas and methods. In this book I have attempted to concentrate attention on these ideas, to place them in a general setting, and to illustrate them by relatively simple examples.

Wackerly, Mendenhall, and Scheaffer *Mathematical Statistics with Applications: 6th edition*, 2002

Talking with students taking or having completed a beginning course in mathematical statistics reveals a major flaw in many courses. Students can take the course and leave it without a clear understanding of the nature of statistics. Many see the theory as a collection of topics, weakly or strongly related, but fail to see that statistics is a theory of information with inference as its goal. Further, they may leave the course without an understanding of the important role played by statistics in scientific investigations.

Casella and Berger *Statistical Inference: 1st edition* 1991

The purpose of this book is to build theoretical statistics (as different from mathematical statistics) from the first principles of probability theory.

Nolan and Speed *StatLabs*, 2000

This book uses a model we have developed for teaching mathematical statistics through in-depth case studies. Traditional statistics texts have many small numerical examples in each chapter to illustrate a topic in statistical theory. Here, we instead make a case study the centerpiece of each chapter. The case studies raise interesting scientific questions, and figuring out how to answer a question is the starting point for developing statistical theory. ... We feel that this approach integrates theoretical and applied statistics in a way not commonly encountered in an undergraduate text

Casella and Berger *Statistical Inference: 2nd edition* 2002

We see the trend moving away from elegant proofs of special cases to algorithmic solutions of more complex and practical cases. ... Discussion of asymptotic methods has been greatly expanded. There is more emphasis on computing and simulation. ... Coverage of more applicable techniques (e.g., EM algorithm, bootstrapping) has been expanded or added.

Rice *Mathematical Statistics and Data Analysis: 3rd edition* 2007

Computation is an integral part of contemporary statistics. It is essential for data analysis and can be an aid to clarifying basic concepts.

Chihara and Hesterberg *Mathematical Statistics with Resampling and R*, 2011

MSRR is a one term undergraduate statistics textbook aimed at sophomores or juniors who have taken a course in probability but may not have had any previous exposure to statistics. What sets this book apart from other mathematical statistics texts is the use of modern resampling techniques–permutation tests and bootstrapping. We begin with permutation tests and bootstrap methods before introducing classical inference methods. Resampling helps students understand the meaning of sampling distributions, sampling variability, P-values, hypothesis tests, and confidence intervals.

Pruim *Foundations and Applications of Statistics: an Introduction using R*, 2011

This book is suitable for what is often a two-semester sequence in "mathematical statistics," but it is different in some important ways from many of the books written for such a course. I was trained as a mathematician first, and the book is clearly mathematical at some points, but the emphasis is on the statistics. Mathematics and computation are brought in where they are useful tools. The result is a book that stretches my students in different directions at different times—sometimes statistically, sometimes mathematically, sometimes computationally.

This book is intended as an upper level undergraduate or introductory graduate textbook in statistical thinking with a likelihood emphasis for students with a good knowledge of calculus and the ability to think abstractly. By "statistical thinking" is meant a focus on ideas that statisticians care about as opposed to technical details of how to put those ideas into practice. By "likelihood emphasis" is meant that the likelihood function and likelihood principle are unifying ideas throughout the text. Another unusual aspect is the use of statistical software as a pedagogical tool. That is, instead of viewing the computer merely as a convenient and accurate calculating device, we use computer calculation and simulation as another way of explaining and helping readers understand the underlying concepts.