Exercise 3 - Spatial Statistics

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Problem 1: Markov RF

Assume that we have observed seismic data over a domain $D \in \mathbb{R}^2$. We want to identify the underlying lithology distribution over D, the underlying lithology of a point is either sand or shale, $\{1, 0\}$ respectively.

The observations have been collected on a regular (75×75) grid L_d, with seismic data being $\{d(\mathbf{x}); \mathbf{x} \in L_d\}$. Where $d(\mathbf{x}) \in \mathbb{R}$.

We have observed the lithology distribution in a geologically comparable domain $D_c \in \mathbb{R}^2$. Assume that this was collected on a regular (66×66) grid L_{D_c}.

We assume that the underlying lithology distribution can be represented by a Mosaic RF $\{l(\mathbf{x}); \mathbf{x} \in L_D\}, l(\mathbf{x}) \in \{0, 1\}$.

Problem 1a)

We start by looking at L_d. Let the seismic data collection procedure follow the following likelihood model:

$$[d_i|\mathbf{l}] = \begin{cases} 0.02 + U_i \text{ if sand, } l_i = 0 \\ 0.08 + U_i \text{ if shale, } l_i = 1 \end{cases}$$

$i = 1, 2, \ldots, n$. With U_i being identically independently distributed $U_i \sim N(0, 0.06^2)$. This would make each observation point d_i conditionally independent on $\mathbf{l}$. That will say:

$$p(d_i|\mathbf{l}) = p(d_i|l_i) = \phi(d_i|\mu = 0.02 + 0.06 l_i, \sigma^2 = 0.06^2) \tag{1}$$

Where ϕ is the pdf of the normal distribution. As all observations are independent we thus have:

$$p(\mathbf{d}|\mathbf{l}) = \prod_{i=1}^{n} p(d_i|l_i) = \prod_{i=1}^{n} \phi(d_i|\mu = 0.02 + 0.06 l_i, \sigma^2 = 0.06^2) \tag{2}$$

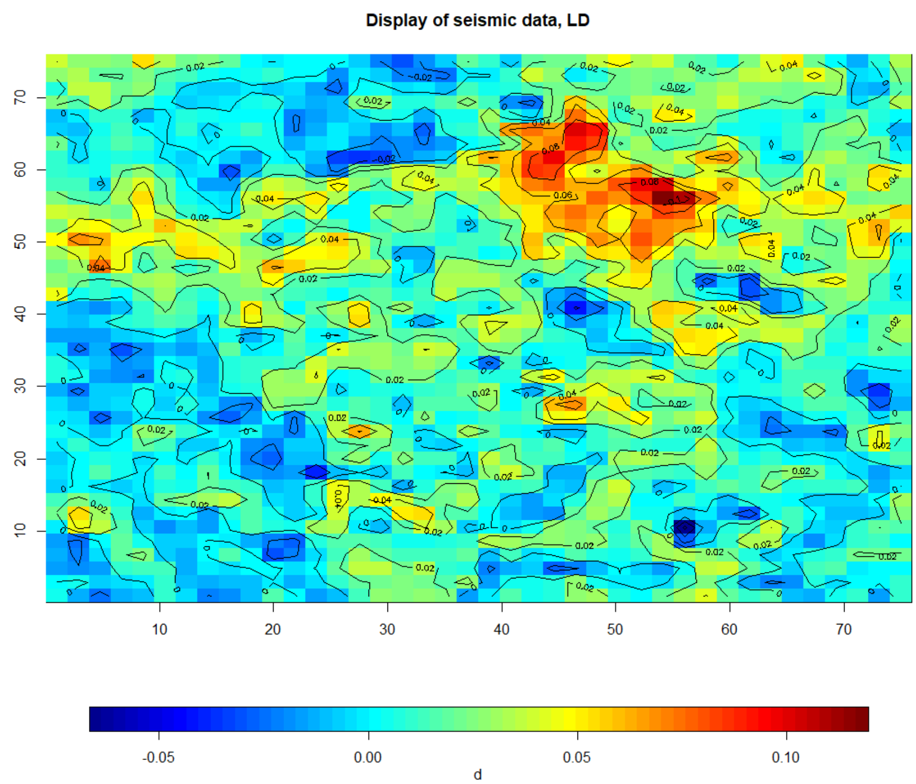

Figure 1: Display of seismic data L_D.

We display the observations from L_D as a map in Figure 1, there seems to be one large gathering where $d(\mathbf{x})$ takes on relatively large values, there also seems to be some smaller gatherings of large $d(\mathbf{x})$ in areas centered around the large one.

Problem 1b)