RCC INSTITUTE OF INFORMATION TECHNOLOGY

A Unit of RCC Institute of Technology, an autonomous society of Department of Higher Education, Govt. of West Bengal.

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Department: computer science and Engineering

Course Name	:	Mathematics-II-A
Course Code	:	RCC-BSC-M-201
Topics covered	:	Measures of central tendencies, conditional density, probability distribution function, Marginal density.
CO covered	:	CO2, CO3 and CO4
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Score Obtained	:	
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Q No 17 Find the mean, median, mode of the following distribution table.

Time travel to work	Frequency
1 - 10	8
11 - 20	14
21 - 30	12
31 - 40	9
41 - 50	7

Ans: Step 1: Mean

Take the mid-points of the intervals:

- 1 - 10 : mid point = 5.5
- 11 - 20 : mid point = 15.5
- 21 - 30 : mid point = 25.5
- 31 - 40 : mid-point = 35.5
- 41 - 50 : mid-point = 45.5

Now, apply:

$$\text{Mean} = \frac{\sum f \cdot x}{\sum f}$$

$$= \frac{(8 \cdot 5.5 + 14 \cdot 15.5 + 12 \cdot 25.5 + 9 \cdot 35.5 + 7 \cdot 45.5)}{50}$$

$$= \frac{(44 + 217 + 306 + 319.5 + 318.5)}{50}$$

$$= \frac{1205}{50} = 24.1$$

$\therefore$ Mean = 24.1 minutes.

step 2: Median

Total frequency (N) = 50

Median class = the class containing the 25th value

cumulative frequencies:

- 1 - 10: 8
- 11 - 20: 22
- 21 - 30: 34 → Median class.

using the median formula:

$$\boxed{\text{Median} = L + \frac{\frac{N}{2} - F}{f} \cdot h}$$

where:

- L = lower boundary = 20.5
- N = 50
- F = cumulative frequency before median class = 22
- f = frequency of median class = 12
- h = class width = 10

$$\therefore \text{Median} = 20.5 + \frac{25-22}{12} \cdot 10$$

$$= 20.5 + \frac{3}{12} \cdot 10$$

$$= 20.5 + 2.5$$

$$= 23.$$

$$\boxed{\text{Median} = 23 \text{ minutes}}$$

step 3: Mode

Mode formula:

$$\boxed{Mode = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \cdot h}$$

Modal class = 11-20 (highest frequency = 14)

- $L = 10.5$
- $f_1 = 14$
- $f_0 = 8$
- $f_2 = 12$
- $h = 10$

$$Mode = 10.5 + \frac{14-8}{2(14)-8-12} \cdot 10$$
$$= 10.5 + \frac{6}{8} \cdot 10$$
$$= 10.5 + 7.5$$
$$= 18$$

$\therefore$ $\boxed{\text{Mode} = 18 \text{ minutes.}}$

QNo→2 Find the value of K and $P(X+Y<2)$ for the bivariate distribution.

$$f(x,y) = \begin{cases} K(3x+y) & \text{for } 1 \le x \le 3, 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Ans:- Given,

$$f(x,y) = \begin{cases} K(3x+y), & 1 \le x \le 3, 0 \le y \le 2 \\ 0, & \end{cases}$$

i) using total probability = 1

$$\int_1^3 \int_0^2 K(3x+y)\,dy\,dx = 1$$

Inner Integral:

$$\int_0^2 (3x+y)\,dy = \left[3xy + y^2/2\right]_0^2$$
$$= 3x(2) + \frac{4}{2} = 6x + 2.$$

Now Integrate :

$$\int_1^3 k(6x+2)\,dx = k\left[3x^2+2x\right]_1^3 = k[(27+6)-(3+2)]$$

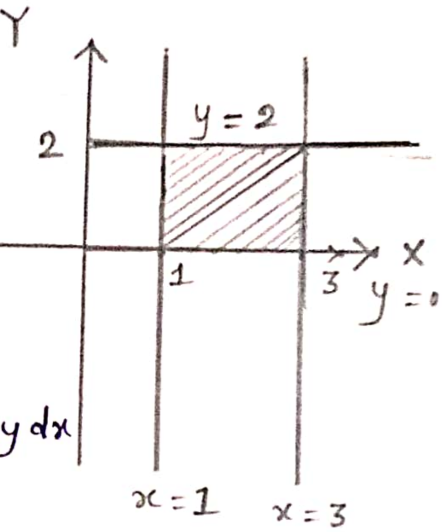
$$= k(33-5)$$

$$= 28k$$

$28k = 1$

$\boxed{\Rightarrow k = 1/28}$

ii) Find $p(X+Y<2)$



Region: $1 \leq x \leq 3,\ 0 \leq y \leq 2$

But $X+Y<2 \Rightarrow Y < 2-x$

Bounds: since $x \in [1,2) \Rightarrow y \in [0, 2-x]$

$$p(X+Y<2) = \int_1^2 \int_0^{2-x} \frac{1}{28}(3x+y)\,dy\,dx$$

Inner Integral:

$$\int_0^{2-x}(3x+y)\,dy = 3x(2-x) + \frac{(2-x)^2}{2} = 6x - 3x^2 + \frac{(4-4x+x^2)}{2} = 6x-3x^2+2-2x+\frac{x^2}{2}$$

$$= 4x - 3x^2 + 2 + \frac{x^2}{2}$$

Now integrate:

$$\int_1^2 \left(4x+2-\frac{5x^2}{2}\right)dx = \left[2x^2+2x-\frac{5x^3}{6}\right]_1^2$$

calculate:

At $x=2$: $2(4)+4-\frac{5(8)}{6} = 8+4-\frac{40}{6} = (12-6.67) = 5.33$

At $x=1$: $2+2-\frac{5}{6} = (4-0.83) = 3.17$

So result $= 5.33 - 3.17 = \boxed{2.16}$

$\therefore P = k(2.16) = \frac{1}{28} \times 2.16 \approx 0.077$

$\therefore \boxed{P \approx 0.077}$

Q No. 3) The joint PDF of the random variable X and Y is given as follows:

$$f(x,y) = \begin{cases} kx^2(8-y), & \text{when } x<y<2x,\ 0\le x\le 2 \\ 0, & \text{elsewhere.} \end{cases}$$

Find i) the value of k ii) the marginal probability density functions of X and Y. iii) the conditional density functions $f(x/Y)$ and $f(Y/x)$.

Ans:- Given;

$$f(x,y) = \begin{cases} kx^2(8-y), & x<y<2x,\ 0<x<2 \\ 0 & \text{elsewhere} \end{cases}$$

i) Finding k:

$$\int_0^2 \int_x^{2x} kx^2(8-y)\,dy\,dx = 1$$

Inner Integral:

$$\int_x^{2x} x^2(8-y)\,dy = x^2\left[8y-\frac{y^2}{2}\right]_x^{2x} = x^2\left[16x-2x^2-\left(8x-\frac{x^2}{2}\right)\right]$$

$$= x^2\left[8x-\frac{3x^2}{2}\right]$$

$$= 8x^3-\frac{3x^4}{2}$$

Now outer:

$$k\int_0^2\left(8x^3-\frac{3x^4}{2}\right)dx$$

$$= k\left[2x^4-\frac{3x^5}{10}\right]_0^2 = k\left[\left(32-\frac{96}{10}\right)\right] = k(32-9.6)$$

$$= k(22.4)$$

$$\Rightarrow \boxed{k = \frac{1}{22.4} = \frac{5}{112}}$$

ii) Marginal probability density functions of X and Y

1) Marginal Density $f_X(x)$:

we integrate $f(x,y)$ over y, from $y = x$ to $y = 2x$

$$f_X(x) = \int_{y=x}^{2x} f(x,y)\,dy = \int_{x}^{2x} \frac{5}{112}x^2(8-y)\,dy$$

Take constants out:

$$f_X(x) = \frac{5x^2}{112}\int_{x}^{2x}(8-y)\,dy$$

Now solve the integral

$$\int_{x}^{2x}(8-y)\,dy = \left[8y - \frac{y^2}{2}\right]_{x}^{2x} = \left[16x - \frac{(2x)^2}{2}\right] - \left[8x - \frac{x^2}{2}\right]$$

$$= (16x - 2x^2) - (8x - \tfrac{1}{2}x^2)$$

$$= 8x - \frac{3}{2}x^2.$$

Now plug into $f_X(x)$:

$$f_X(x) = \frac{5x^2}{112}\left(8x - \frac{3}{2}x^2\right) = \frac{5}{112}\left(8x^3 - \frac{3}{2}x^4\right)$$

So; the marginal density of f_X is:

$$\boxed{f_X(x) = \frac{5}{112}\left(8x^3 - \frac{3}{2}x^4\right), \quad \text{for } 0 \le x \le 2}$$

2) Marginal Density $f_Y(y)$:

Now integrate over x, but we must find limits from the domain:

- Given $x < y \le 2x \Rightarrow y/2 \le x < y$

So:
$$f_Y(Y) = \int_{x=y/2}^{y} f(x,y)\,dx = \int_{y/2}^{y} \frac{5}{112}x^2(8-y)\,dx$$

Factor out constants

$$f_Y(y) = \frac{5(8-y)}{112}\int_{y/2}^{y} x^2\,dx = \frac{5(8-y)}{112}\left[\frac{x^3}{3}\right]_{y/2}^{y}$$

Evaluate the definite integral:

$$y^3/3 - \frac{(y/2)^3}{3} = \frac{y^3}{3} - \frac{y^3}{24} = \frac{8y^3 - y^3}{24} = \frac{7y^3}{24}$$

Now plug in:

$$f_Y(Y) = \frac{5(8-y)}{112} \cdot \frac{7y^3}{24} = \frac{35(8-y)y^3}{2688}$$

So; marginal density of y is

$$\boxed{f_Y(y) = \frac{35(8-y)y^3}{2688}, \quad \text{for } 0 \le y \le 4.}$$

iii) 1) conditional density of Y given $X=x$;

$$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)}.$$

we already had,

- $f(x,y) = \frac{5}{112} x^2(8-y)$
- $f_X(x) = \frac{5}{112}\left(8x^3 - \frac{3}{2}x^4\right)$

So, $$f_{Y|X}(y|x) = \frac{x^2(8-y)}{\left(8x^3 - \frac{3}{2}x^4\right)} = \frac{x^2(8-y)}{x^3\left(8-\frac{3}{2}x\right)} = \boxed{\frac{8-y}{x\left(8x-\frac{3}{2}x\right)}}$$

Domain: $x \in (0,2]$, $y \in (x, 2x]$

$$= \boxed{\frac{x^2(8-y)}{8x^3 - \frac{3}{2}x^4}}$$

2) conditional density of X given $Y=y$;

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)}$$

Earlier we found,

- $f_Y(y) = \frac{35(8-y)y^3}{2688}$

So,

$$f_{X|Y}(x|y) = \frac{\frac{5}{122}x^2(8-y)}{\frac{35(8-y)y^3}{2688}} = \boxed{\frac{24x^2}{7y^3}}$$

Domain: $y \in [0,4]$, $x \in [0, y/2]$

QNo:-4. An unbiased coin is tossed three times. Let x = number of heads $Y = |$number of heads $-$ tails$|$. Find joint distribution of (X, Y). Find the marginal distribution of X and Y. Are X and Y independent?

Ans:- Let • X = number of heads

$Y = |$heads $-$ tails$|$

Total outcomes = 8

Listing outcomes:

outcomes	X	Y
HHH	3	3
HHT	2	1
HTH	2	1
THH	2	1
HTT	1	1
THT	1	1
TTH	1	1
TTT	0	3

Joint distribution of (X, Y):

X	Y	P (X, Y)
0	3	1/8
1	1	3/8
2	1	3/8
3	3	1/8

Marginal of X:

- $p(X=0) = 1/8$
- $p(X=1) = 3/8$
- $p(X=2) = 3/8$
- $p(X=3) = 1/8$.

To find $f_X(x)$ sum over all values of Y for each $X = x$

$$f_X(x) = \sum_y p(X=x, Y=y)$$

X	$f_X(x)$
0	$0 + 1/8 = 1/8$
1	$\frac{3}{8} + 0 = 3/8$
2	$3/8 + 0 = 3/8$
3	$0 + 1/8 = 1/8$.

Marginal of Y:

- $p(Y=1) = 6/8 = 3/4$
- $p(Y=3) = 2/8 = 1/4$

for Y

Y	$f_Y(y)$
1	$\frac{3}{8} + \frac{3}{8} = 3/4$
3	$1/8 + 1/8 = 1/4$

Now, we need to check if $p(X,Y) = p(X)p(Y)$ or not.

Let us try for $(1,1)$.

$p(X=1, Y=1) = \frac{3}{8}$ and $p(X=1)\,p(Y=1)$

$$= \frac{3}{8} \times \frac{3}{4} = \frac{9}{32} \neq \frac{3}{8}.$$

$\therefore$ $\boxed{X \text{ and } Y \text{ are not independent}}$.

———— x ————