

MAT 102: Mathematics II

Fall 24 Problem List

Chapter (1.5 → 3.1)

Chapter 1.5: Eigen Values and Eigen Vectors:

1. Find all the *Eigen values* and *Eigen vectors* of the following matrices:

a.
$$\begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}$$

b.
$$\begin{pmatrix} 5 & 2 & 1 \\ 0 & 5 & 3 \\ 1 & 2 & 5 \end{pmatrix}$$

c.
$$\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

d.
$$\begin{pmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 2 & 3 & 4 \end{pmatrix}$$

Chapter 2.1: Complex Variables:

1. Find the *polar form* and *Euler form* of the following complex numbers:

a. $z = -1 - \sqrt{3} i$

b. $z = 1 - \sqrt{3} i$

c. $z = -\sqrt{2} - \sqrt{6} i$

d. $z = -\sqrt{3} i$

e. $z = 1$

f. $z = -i$

2. Evaluate z^3 , z^5 , z^{11} of the following complex numbers by using *De Moivre's formula*

a. $z = 1 - \sqrt{3} i$

b. $z = \frac{1}{i} - 2i^3 + \sqrt{3} i$

c. $z = \sqrt{2}i^2 - \sqrt{6} i^3$

d. $z = 5i^7 + 3i^9$

e. $z = 1$

f. $z = -\sqrt{\frac{1}{2}} + \sqrt{\frac{3}{2}}i^3$

3. Check each of the following function for ***analyticity*** by using the Cauchy-Riemann equation, and if so, then find the ***complex derivative***, $[f'(x)]$

a. $f(z) = e^z$

b. $f(z) = e^{z^2}$

c. $f(z) = \frac{\bar{z}}{|z|}$

d. $f(z) = 2z^3 - 4z + 1$

e. $f(z) = z^2 + 5iz + 3 - i$

f. $f(z) = ze^{-z}$

g. $f(z) = \sin(2z)$

h. $f(z) = |z|^2$

Chapter 2.2: Harmonic Functions:

1. Examine the following function u is ***harmonic*** or not, if it is harmonic then Construct the ***conjugate harmonic function*** v . Also Setup the ***complex variables***, $f(z) = u + iv$

a. $u(x, y) = 2xy + 3xy^2 - 2y^3$

b. $u(x, y) = \frac{1}{2}\ln(x^2 + y^2) + \frac{1}{2}\ln e^y$

c. $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$

d. $u(x, y) = xe^x \cos y - ye^x \sin y$

e. $u(x, y) = e^{-2xy} \sin(x^2 - y^2)$

f. $u(x, y) = \frac{1}{2}\ln \sqrt{x^2 + y^2}$

Chapter 3.1: Fourier Series and Transforms:

1. Construct the ***Fourier series*** of the following functions in the interval $-\pi \leq x \leq \pi$:

a. $F(x) = e^x$

b. $F(x) = x^2$

c. $F(x) = x^3$

d. $F(x) = x^2 + x$

e. $F(x) = \sin(nx)$

f. $F(x) = \cos(2x)$

2. Evaluate the ***Fourier transform(Sine/Cosine)*** of the following functions:

a. $F(x) = e^{-x}; 0 \leq x$

b. $F(x) = 3x; 0 < x < 6$

c. $F(x) = \sin(nx); 0 < x < \pi$

d. $F(x) = e^{mx}; 0 < x < \pi$