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Basic Concept of Complex Number

Complex Number: An expression of the form

$$a+ib \text{ or } a+bi$$

with a and b are real numbers then the expression is known as the complex number. It is denoted by a single letter, i.e.,

$$z=a+ib \text{ or } z=a+bi\,.$$

[Note: Here, i is known as imaginary number and $i=\sqrt{-1}$ or $i^2=-1$]

The number a is called the ***real part*** of z and is denoted by Re(z), and the number b is called the ***imaginary part*** of z and is denoted by Im(z). Thus,

$$\operatorname{Re}(x+iy)=x \qquad \operatorname{Im}(x+iy)=y$$

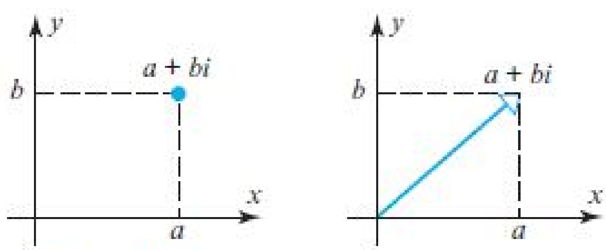
$$\operatorname{Re}(3+2i)=3 \qquad \operatorname{Im}(3+2i)=2$$

$$\operatorname{Re}(-3-2i)=-3 \qquad \operatorname{Im}(-3-2i)=-2$$

$$\operatorname{Re}(2i)=0 \qquad \operatorname{Im}(2i)=2$$

The Complex Plane:

A complex number $z = a + bi$ can be associated with the ordered pair (a, b) of real numbers and represented geometrically by a point or a vector in the xy-plane (Figure B.1). We call this the ***complex plane***.



▲ Figure B.1

Points on the x-axis have an imaginary part of zero and hence correspond to real numbers, whereas points on the y-axis have a real part of zero and correspond to pure imaginary numbers. Accordingly, we call the x-axis the ***real axis*** and the y-axis the ***imaginary axis*** (Figure B.2).

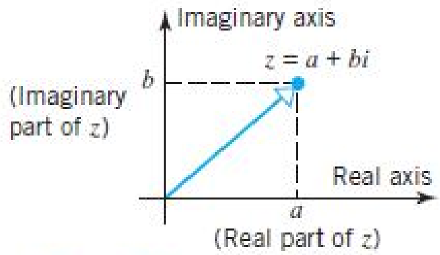


Figure B.2

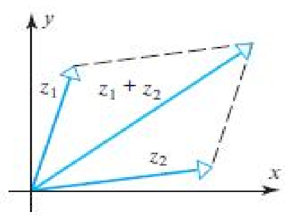
Addition, Subtraction, Multiplication and Division:

Complex numbers are added, subtracted, and multiplied in accordance with the standard rules of algebra but with $i^2 = -1$

Example: Let $z_1 = a + ib$ and $z_2 = c + di$

1. **Addition:**

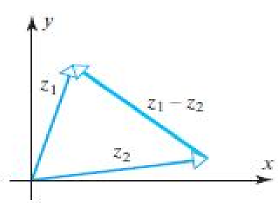
   Therefore $z_1 + z_2 = a + ib + c + di = (a + c) + i(b + d)$



The sum of two complex numbers

2. **Subtraction:**

   Therefore $z_1 - z_2 = (a + ib) - (c + di) = (a - c) + i(b - d)$



The difference of two complex numbers

3. **Multiplication:**

Therefore $z_1z_2 = (a+ib)(c+di) = a(c+di) + ib(c+di)$

$$\Rightarrow z_1z_2 = ac + adi + ibc + bdi^2 = ac + adi + ibc + bd(-1) = (ac - bd) + i(ad + bc)$$

4. **Division:**

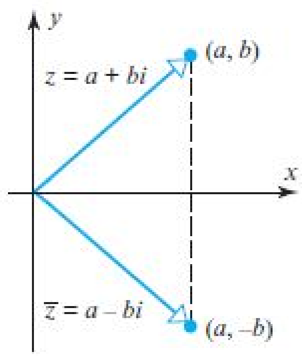
Therefore $\dfrac{z_1}{z_2} = \dfrac{a+ib}{c+di} = \dfrac{(a+ib)(c-di)}{(c+di)(c-di)} = \dfrac{ac + ibc - adi - bdi^2}{c^2 - (di)^2}$

$$\frac{z_1}{z_2} = \frac{(ac+bd) + i(bc - ad)}{c^2 + d^2} = \frac{ac+bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}$$

Complex conjugate: If $z = a + ib$ is a complex number, then the ***complex conjugate*** of *z*, or more simply, the ***conjugate*** of *z*, is denoted by $\bar{z}$ (read, "z bar") and is defined by

$$\bar{z} = \overline{a+ib} = a - ib$$

Numerically, $\bar{z}$ is obtained from *z* by reversing the sign of the imaginary part, and geometrically it is obtained by reflecting the vector for *z* about the real axis



Examples:

$z = 3 - 2i$	$\bar{z} = 3 + 2i$
$z = -3 + 2i$	$\bar{z} = -3 - 2i$
$z = i$	$\bar{z} = -i$
$z = 9$	$\bar{z} = 9$

THEOREM 1 The following results hold for any complex numbers *z*, z_1, and z_2.

a. $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

b. $\overline{z_1 . z_2} = \overline{z_1} . \overline{z_2}$

c. $\bar{\bar{z}} = z$

d. $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$

e. $\overline{z_1 / z_2} = \overline{z_1} / \overline{z_2}$

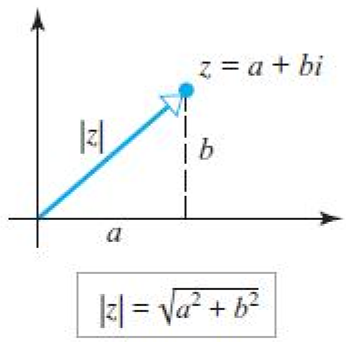
Modulus: The product of a complex number $z = a + ib$ and its conjugate $\bar{z} = a - ib$ is a non-negative real number. i.e,

$$z\bar{z} = (a+ib)(a-ib) = a^2 + b^2$$

And the length of the vector corresponding to z is recognize as,

$$\sqrt{z\bar{z}} = \sqrt{a^2 + b^2}$$

This length is called the modulus or absolute value of z and denoted it by $|z|$



Example: Find the modulus of the following complex number,

i. $z = 3 - 2i$

ii. $z = -3 + 2i$

iii. $z = i$

Solution:

i) $|z| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$

ii) $|z| = \sqrt{(-3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}$

iii) $|z| = \sqrt{1^2} = \sqrt{1} = 1$

THEOREM 2 The following results hold for any complex numbers z, z_1, and z_2.

a. $|\bar{z}| = |z|$

b. $|z_1 . z_2| = |z_1||z_2|$

c. $|z_1 + z_2| \le |z_1| + |z_2|$

d. $|z_1 / z_2| = |z_1| / |z_2|$

Assignment

1. Identify the Re(z) and Im(z) of the following complex number also draw the complex plane:

   i. $z = 1 - 2i$ ii. $z = 1 - i$ iii. $z = 5 + 3i$ iv. $z = -7i$

2. Express z_1/z_2 in the form $a + bi$. Where $z_1 = -7i$ and $z_2 = 5 + 3i$.

3. Prove the Theorem 1 and Theorem 2, for the following complex number,

   i) $z = 1 - i$ ii) $z = 5 + 3i$

4. Find the modulus of the following complex number,

   a. $z = 1 - 2i$ b. $z = 1 - i$ c. $z = 5 + 3i$ d. $z = -7i$