Metropolis-Hastings

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Programming for Scientists

March 24, 2009

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Random Hill Climbing

1. $x_0 \leftarrow$ random
2. For $t \in \{1 \ldots N\}$
   a. $c \leftarrow x_{t-1} + \mathcal{N}(0, \sigma)$
   b. If $f(c) < f(x_{t-1}), x_t \leftarrow c$
   c. Else, $x_t \leftarrow x_{t-1}$

Random Numbers

Many algorithms need random numbers.

Pseudo-Random Numbers

PRN are deterministically generated numbers that look like random.

Pseudo Random Number Example

$$x_{n+1} = 1103515245 x_n + 12345 \mod 2^{32}$$

Pseudo Random Sequences

Each number is generated based on the previous.

Seeding the Sequence

```
import random
random.seed(0)
print random.random()
print random.random()
print random.random()
print

random.seed(0)
print random.random()
print random.random()
print random.random()
```

prints

```
0.844421851525
0.75795440294
0.420571580831

0.844421851525
0.75795440294
0.420571580831
```

Seeding the Sequence (II)

```
import numpy.random
numpy.random.seed(0)
print numpy.random.random()
print numpy.random.random()
print numpy.random.random()
print

numpy.random.seed(0)
print numpy.random.random()
print numpy.random.random()
print numpy.random.random()
```

prints

```
0.548813503927
0.715189366372
0.602763376072

0.548813503927
0.715189366372
0.602763376072
```

Sampling From a Distribution

Given a random number uniformly distributed on $[0, 1]$, how do we sample from $p(x)$?

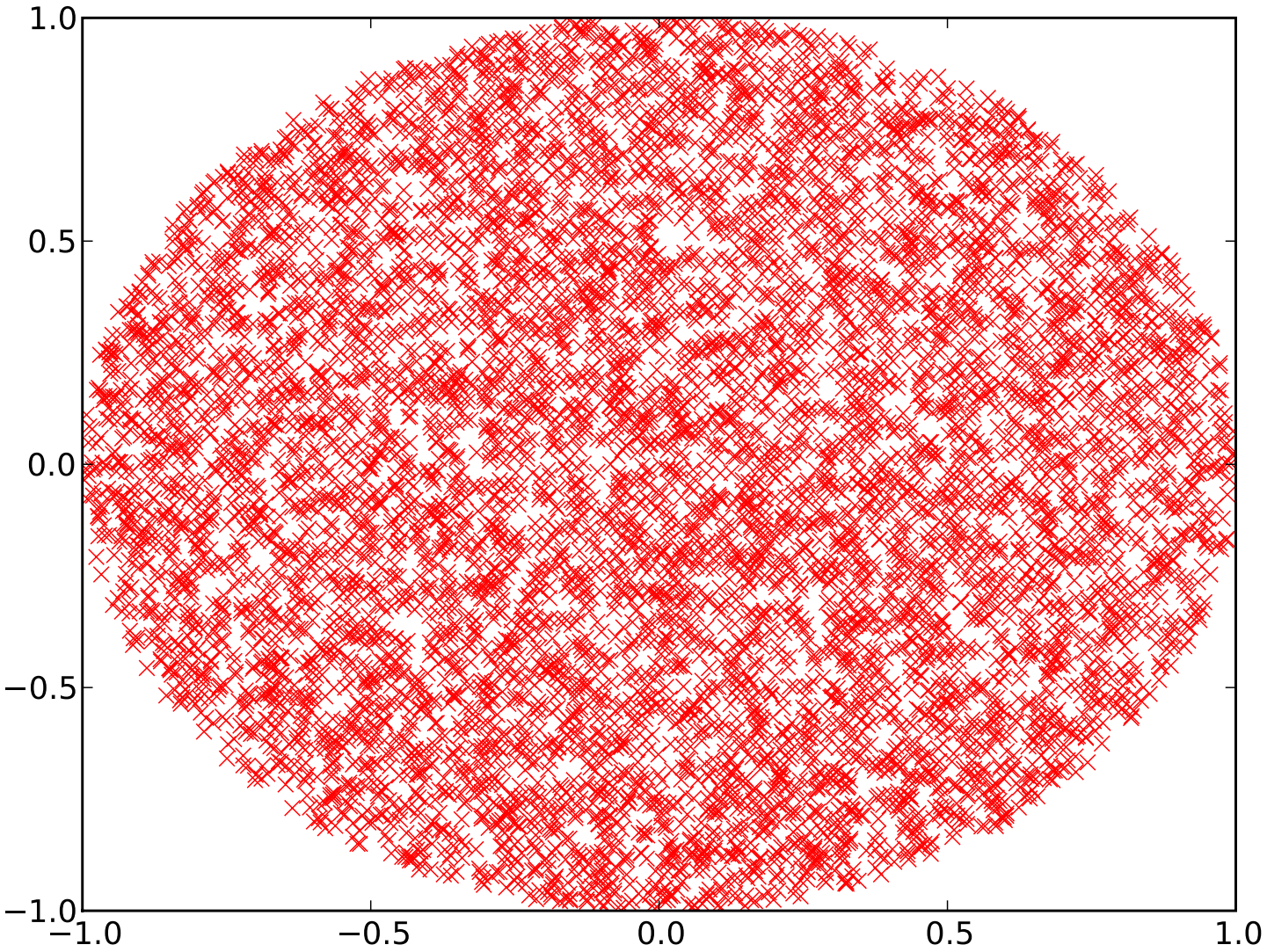
In Practice

Check out what's already written (scipy.stats has plenty).

Scipy.stats

- Normal
- Exponential
- Poisson
- ...

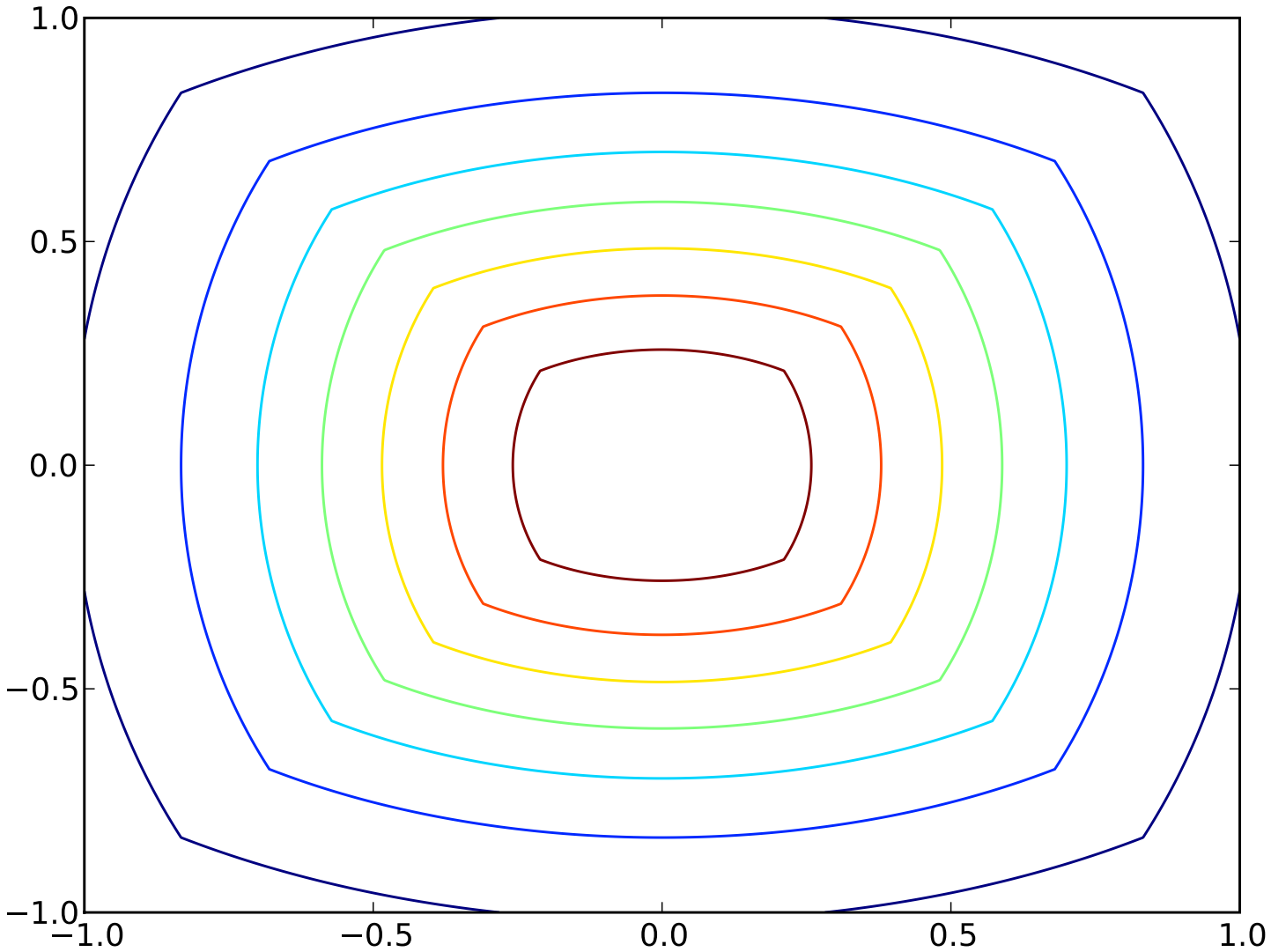
How do I Sample a sphere?



Metropolis-Hastings

What if $P(x)$ is really complicated?

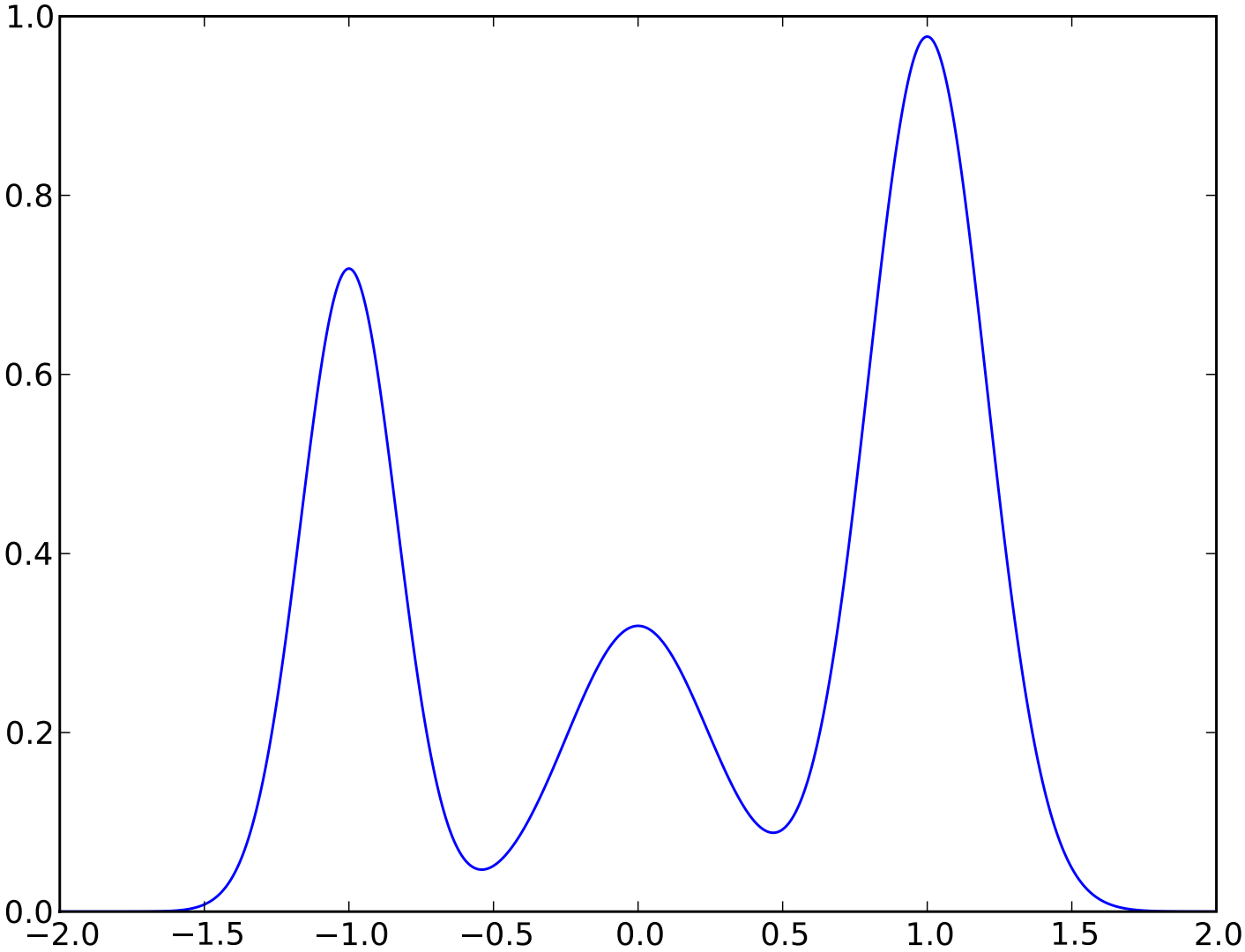
Example



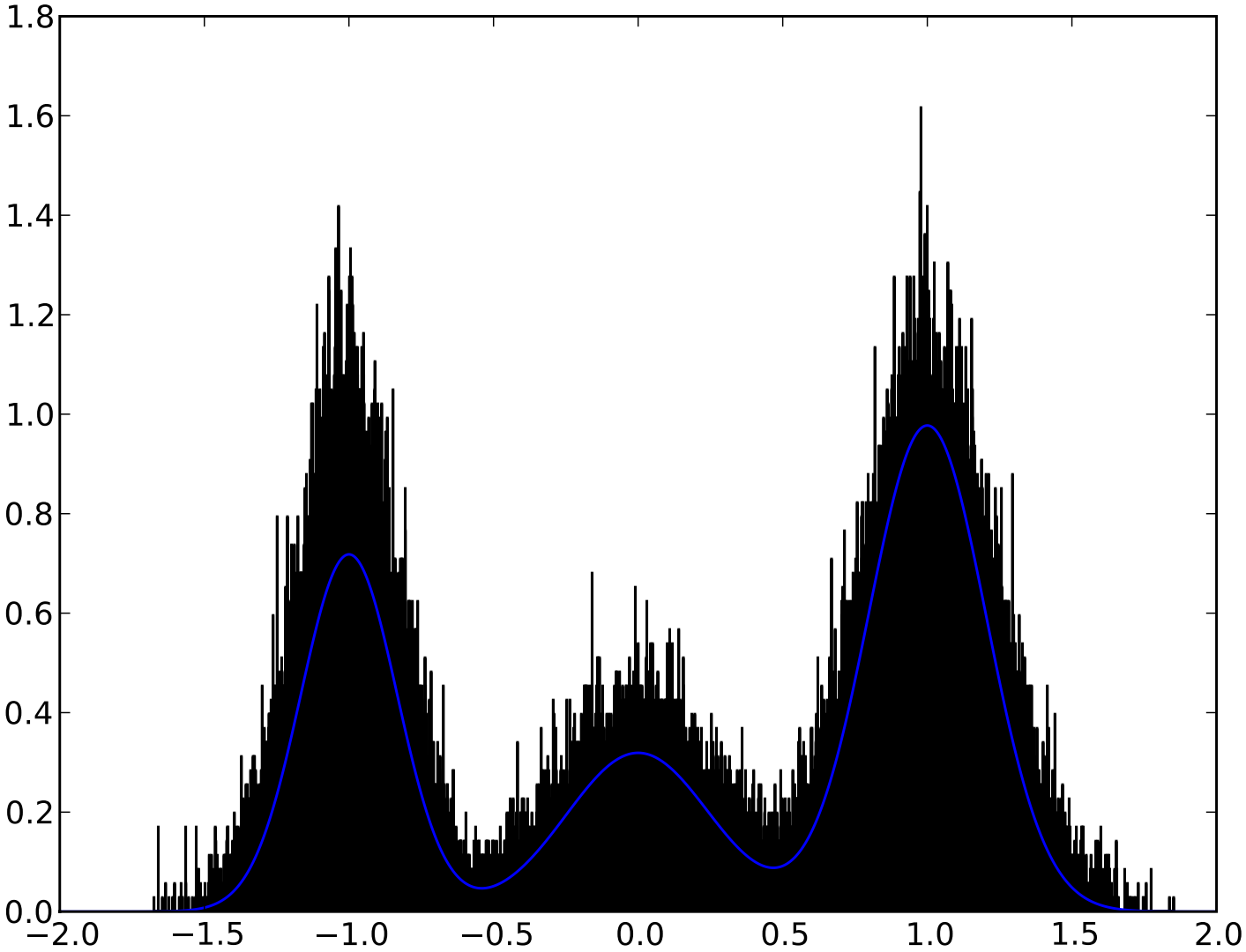
Metropolis-Hastings

- $x_0 \leftarrow$ random
- For $t \in \{1 \cdots T\}$:
  1. $c \leftarrow$ sample from $Q(x'|x_t)$
  2. $a \leftarrow \frac{P(c)Q(x_t|c)}{P(x_t)Q(c|x_t)}$
  3. $r \leftarrow U(0,1)$
  4. If $r < a : x_{t+1} \leftarrow c$
  5. Else $x_{t+1} \leftarrow x_t$

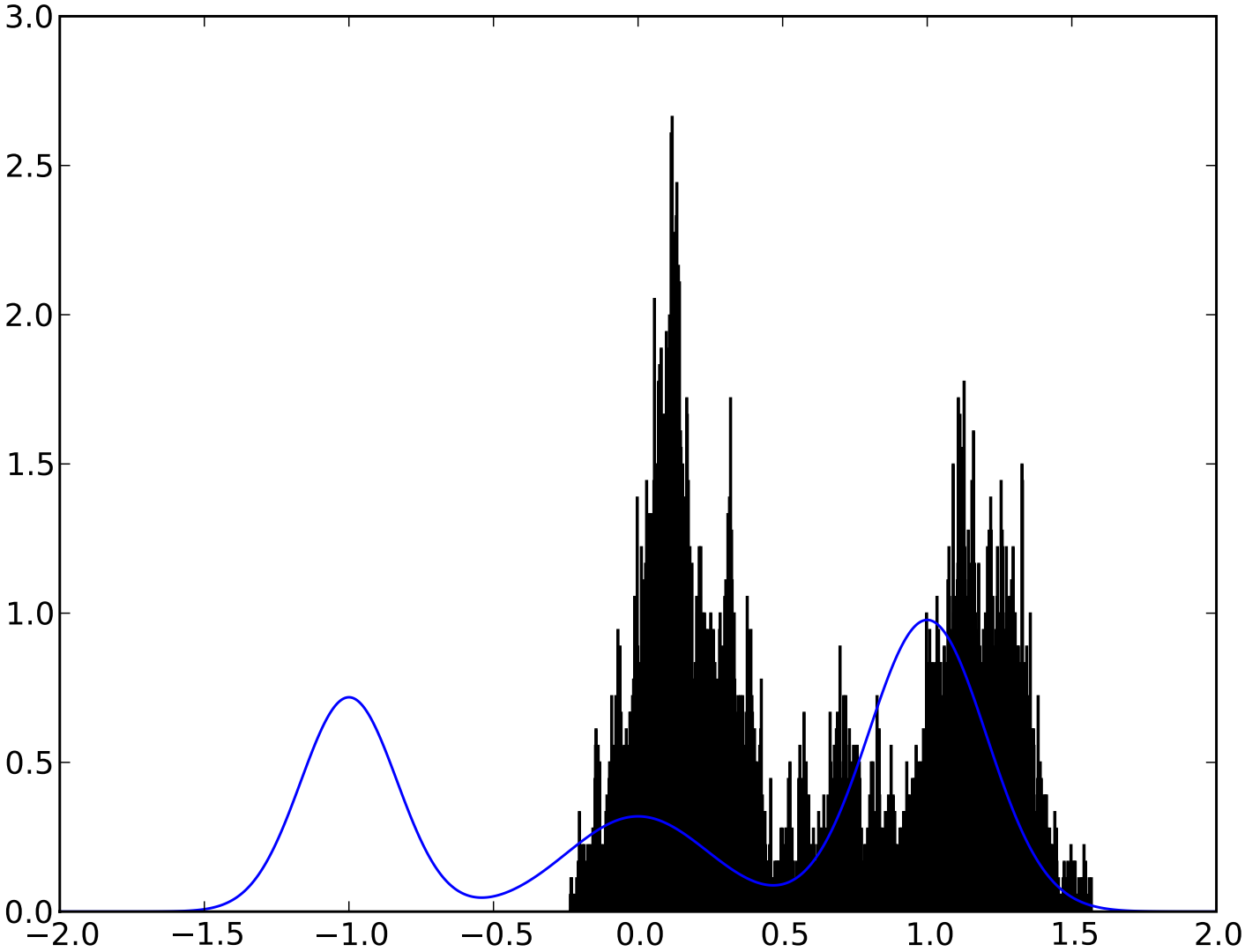
Metropolis-Hastings Example (Simple)



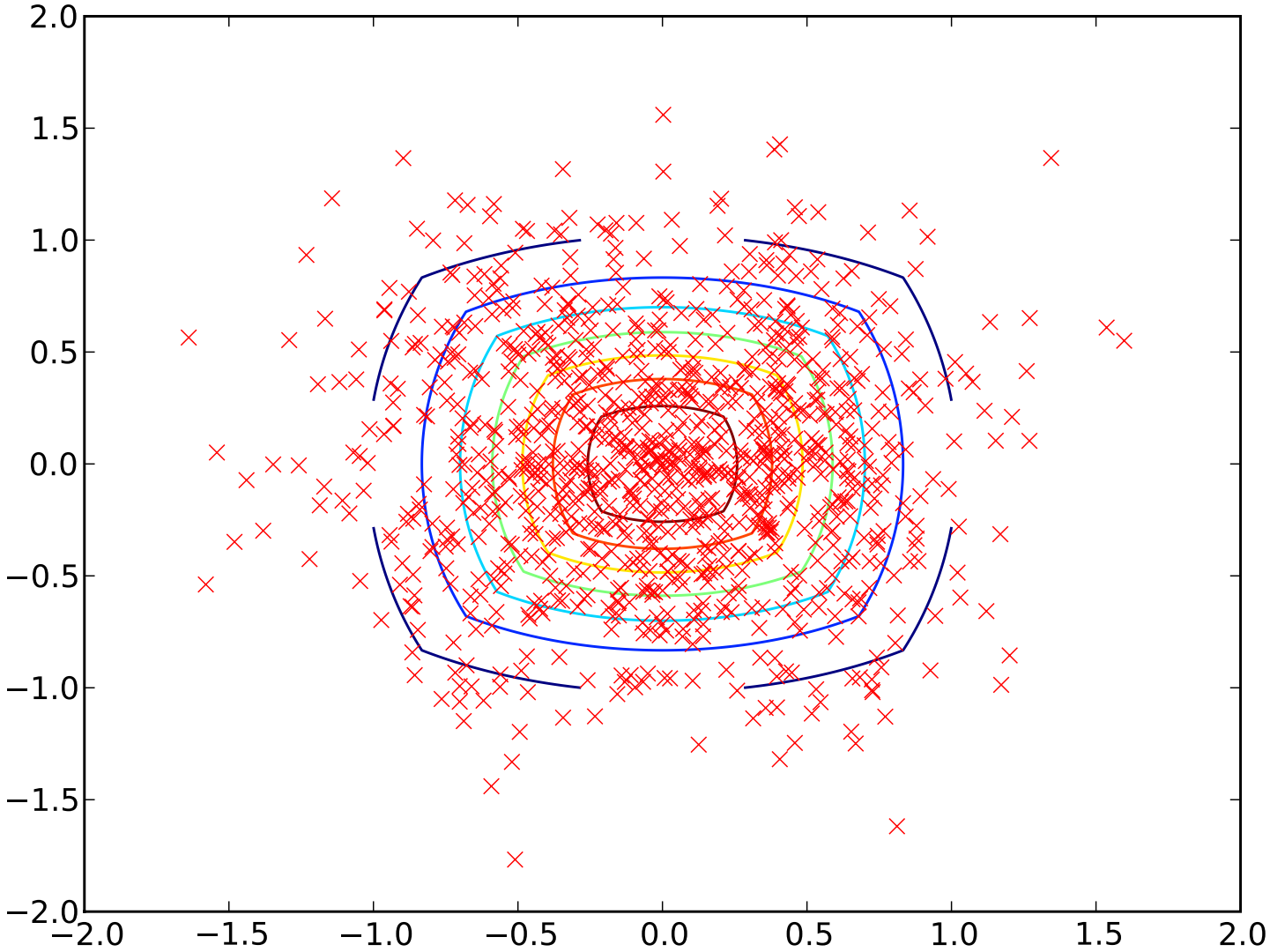
Metropolis-Hastings Example



Metropolis-Hastings Example



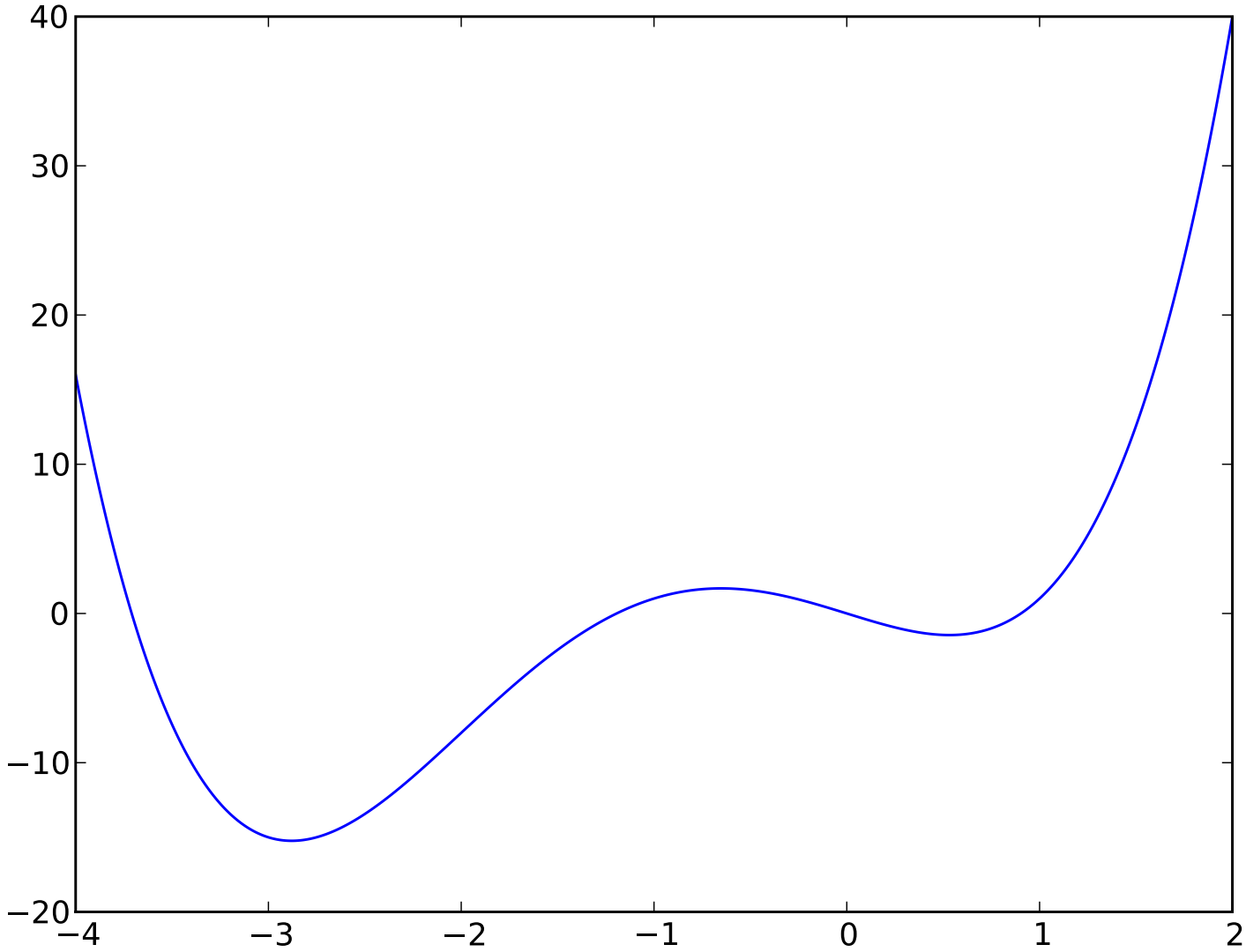
Metropolis-Hastings More Complex Example



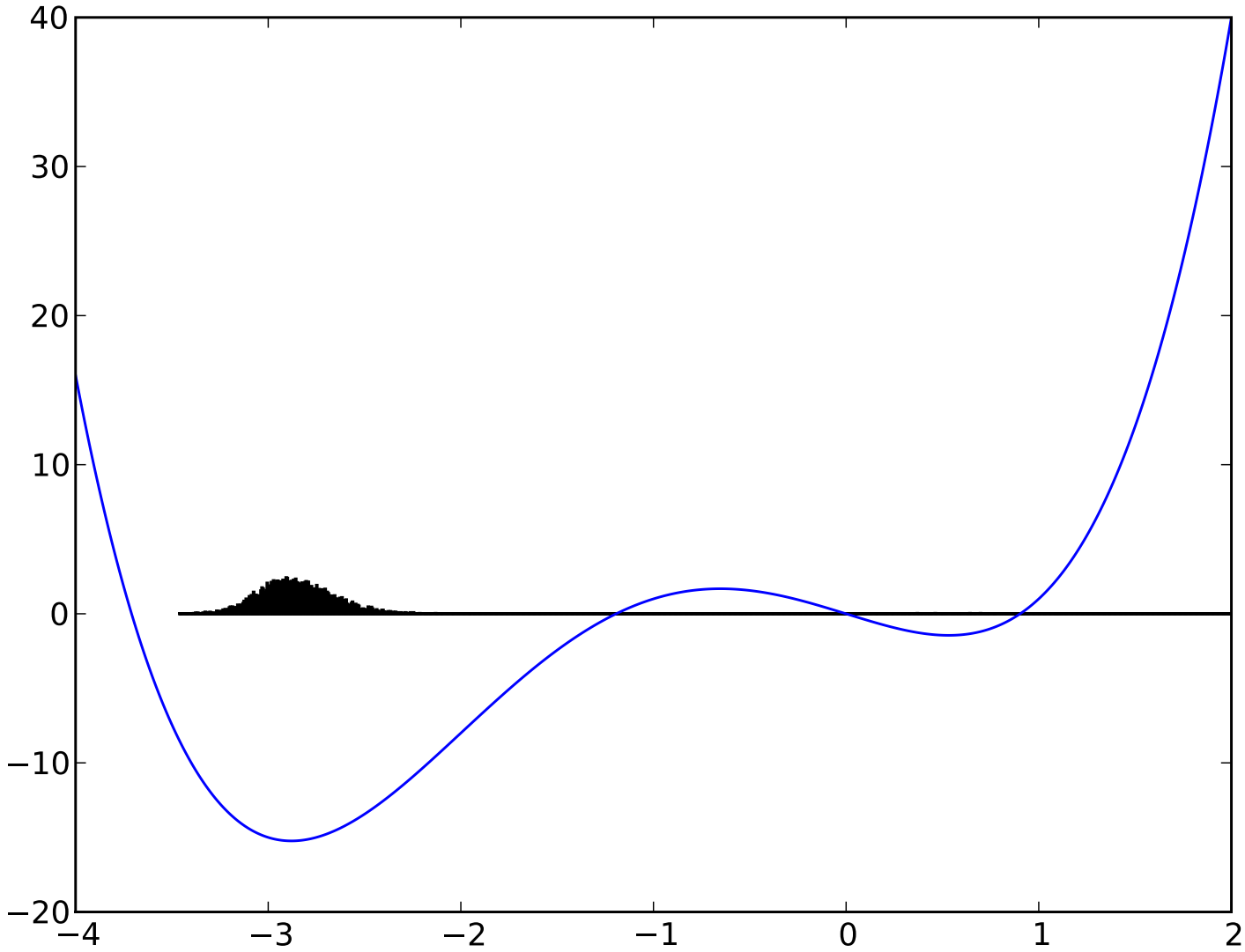
MCMC for Minimisation

$$P(x) = k \exp(-\beta f(x))$$

Minimisation



Minimisation



Simulated Annealing Trick

$$P(x)/P(x') = \exp\left(T(f(x) - f(x'))\right)$$

Simulated Annealing Trick

$$P(x)/P(x') = \exp\left(T(f(x) - f(x'))\right)$$

Start hot, reduce the temperature.

Simulated Annealing

Starts like Metropolis-Hastings, finished like Random Hill Climbing.