Determine the closest player to a point

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December 2018

We want to compute, for a moving player on a domain how much time it takes to reach the point $(0, 0)$.

At the initial time the velocity of the player at position $\boldsymbol{x}(t = 0) = (x_0, y_0)$ is $\boldsymbol{u}(x_0, y_0, t = 0) = (u_0, v_0)$. We will consider that the player will use a constant force (per unit mass) in a given direction, of strength $|\boldsymbol{F}|^2 = F_x^2 + F_y^2$. This is an assumption which allows to find a simple analytical solution. In particular, it allows to consider the two directions separately.

Newton's law writes

$$d_t^2 x = F_x \tag{1}$$

so that we have

$$x(t) = x_0 + u_0 t + \frac{1}{2} F_x t^2. \tag{2}$$

We evaluate this expression at $x = 0$, and want to determine at which time this point is reached. Let us first determine the force per mass F_x,

$$F_x = -2 \frac{x_0 + u_0 t}{t^2}. \tag{3}$$

and similarly

$$F_y = -2 \frac{y_0 + v_0 t}{t^2}. \tag{4}$$

Since $|\boldsymbol{F}|^2 = F_x^2 + F_y^2$, we have,

$$F^2 = \frac{4}{t^4} \left((x_0 + u_0 t)^2 + (y_0 + v_0 t)^2 \right). \tag{5}$$

Yielding the fourth order polynomial for t,

$$t^4 - \frac{4}{F^2} \left((x_0 + u_0 t)^2 + (y_0 + v_0 t)^2 \right) = 0. \tag{6}$$

This equation has formally 4 solutions. However, only one of these is the physical time for a player to reach the origin. The constraints to choose the correct solution are that the time needs to be positive and real.

In practice, in a script, one can evaluate all four analytical solutions of this equation and choose the smallest positive real solution. The adjustable parameter is the value of F. It should be of order of magnitude $1 < F < 10\ ms^{-2}$.