

BELJING NORMAL UNIVERSITY
SCHOOL OF MATHEMATICS

Template

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目录

1	hpp	6
1.1	heading	6
1.2	debug.h	7
1.3	mod int	7
2	Shell Scripts	9
2.1	checker.sh	9
3	data structure	10
3.1	stack	10
3.2	queue	10
3.3	DSU	10
3.4	ST	10
3.5	cartesian tree	11
3.6	segment tree czr	11
3.7	segment tree	13
3.8	hjt segment tree	20
3.9	LiChao Tree	22
3.10	treap	23
3.11	splay	28
3.12	Link Cut Tree	31
3.13	ODT	32
3.14	tree in tree	32
3.15	heap	38
4	string	39
4.1	hash	39
4.2	Cantor Expansion	39
4.3	kmp	40
4.4	z function	40
4.5	Manacher	40
4.6	AC automaton	41
4.7	PAM	41
4.8	Suffix Array	42

4.9	trie	42
5	math - number theory	45
5.1	Eculid	45
5.2	inverse	46
5.3	sieve	47
5.4	杜教筛	48
5.5	extended Euler theorem	48
5.6	block	48
5.7	CRT & exCRT	49
5.8	BSGS & exBSGS	50
5.9	Miller Rabin	50
5.10	Pollard Rho	51
5.11	quadratic residu	52
5.12	Lucas	52
5.13	Wilson	54
5.14	LTE	55
5.15	Mobius inversion	56
6	math - polynomial	57
6.1	FTT	57
6.2	FWT and FMT	58
6.3	class polynomial	60
6.4	wsy poly	64
7	math - numerical analysis	71
7.1	Simpson	71
8	math - game theory	72
8.1	nim game	72
8.2	anti - nim game	72
9	math - linear algebra	73
9.1	matrix	73
9.2	matrix tree	74
9.3	linear basis	75
9.4	berlekamp massey	76
9.5	linear programming	77

10	math - complex number	78
11	graph	79
11.1	topsort	79
11.2	shortest path	79
11.3	minimum spanning tree	83
11.4	SCC	83
11.4.1	缩点	84
11.5	DCC	84
11.6	2 SAT	87
11.7	minimum ring	88
11.8	tree - center of gravity	88
11.9	tree - DSU on tree	88
11.10	tree - AHU	89
11.11	tree - LCA	90
11.12	tree - LLD	90
11.13	tree - HLD	91
11.14	tree - virtual tree	92
11.15	tree - pseudo tree	93
11.16	tree - prufer sequence	94
11.17	tree - divide and conquer on tree	94
11.18	network flow - maximal flow	96
11.19	network flow - minimum cost flow	99
11.20	network flow - minimal cut	101
11.21	matching - matching on bipartite graph	102
11.22	matching - matching on general graph	104
12	geometry	105
12.1	two demention	105
12.2	convex	106
13	sweep line	107
14	offline algorithm	108
14.1	discretization	108
14.2	Mo algorithm	108
14.3	CDQ	109

14.4 线段树分治	111
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1 hpp

1.1 heading

```

1  #include <bits/stdc++.h>
2
3  // using namespace std;
4
5  #define typet typename T
6  #define typeu typename U
7  #define types typename... Ts
8  #define tempt template <typet>
9  #define tempu template <typeu>
10 #define temps template <types>
11 #define tandu template <typet, typeu>
12
13 using LL = long long;
14 using i128 = __int128;
15 using PII = std::pair<int, int>;
16 /*
17 using UI = unsigned int;
18 using ULL = unsigned long long;
19 using ULL = unsigned long long;
20 using PIL = std::pair<int, LL>;
21 using PLI = std::pair<LL, int>;
22 using PLL = std::pair<LL, LL>;
23 */
24 using vi = std::vector<int>;
25 using vvi = std::vector<vi>;
26 using vl = std::vector<LL>;
27 using vvl = std::vector<vl>;
28 using vpi = std::vector<PII>;
29
30 #define ff first
31 #define ss second
32 #define all(v) v.begin(), v.end()
33 #define rall(v) v.rbegin(), v.rend()
34
35 #ifdef LOCAL
36 #include "../debug.h"
37 #else
38 #define debug(...) \
39     do { \
40     } while (false)
41 #endif
42
43 constexpr int mod = 998244353;
44 constexpr int inv2 = (mod + 1) / 2;
45 constexpr int inf = 0x3f3f3f3f;
46 constexpr LL INF = 1e18;
47 constexpr double pi = 3.141592653589793;
48 constexpr double eps = 1e-6;
49
50 constexpr int lowbit(int x) { return x & -x; }
51 /*
52 constexpr int add(int x, int y) { return x + y < mod ? x + y : x - mod + y; }
53 constexpr int sub(int x, int y) { return x < y ? mod + x - y : x - y; }
54 constexpr int mul(LL x, int y) { return x * y % mod; }
55 constexpr void Add(int& x, int y) { x = add(x, y); }
56 constexpr void Sub(int& x, int y) { x = sub(x, y); }
57 constexpr void Mul(int& x, int y) { x = mul(x, y); }
58 constexpr int pow(int x, int y, int z = 1) {
59     for (; y; y /= 2) {
60         if (y & 1) Mul(z, x);
61         Mul(x, x);
62     }
63     return z;
64 }
65 temps constexpr int add(Ts... x) {
66     int y = 0;
67     (... , Add(y, x));
68     return y;
69 }
70 temps constexpr int mul(Ts... x) {
71     int y = 1;
72     (... , Mul(y, x));
73     return y;
74 }
75 */
76
77 tandu bool Max(T& x, const U& y) { return x < y ? x = y, true : false; }
78 tandu bool Min(T& x, const U& y) { return x > y ? x = y, true : false; }
79

```

```

80 void solut() {
81
82 }
83
84 int main() {
85     std::ios::sync_with_stdio(false);
86     std::cin.tie(0);
87     std::cout.tie(0);
88
89     int t = 1;
90     std::cin >> t;
91     while (t--) {
92         solut();
93     }
94     return 0;
95 }

```

1.2 debug.h

```

1  template <typename T, typename U>
2  std::ostream& operator<<(std::ostream& os, const std::pair<T, U>& p) {
3      return os << '<' << p.first << ',' << p.second << '>';
4  }
5
6  template <
7      typename T, typename = decltype(std::begin(std::declval<T>())),
8      typename = std::enable_if_t<!std::is_same_v<T, std::string>>>
9  std::ostream& operator<<(std::ostream& os, const T& c) {
10     auto it = std::begin(c);
11     if (it == std::end(c)) return os << "{}";
12     for (os << '{' << *it; ++it != std::end(c); os << ',' << *it);
13     return os << '}';
14 }
15
16 #define debug(arg...) \
17     do { \
18         std::cerr << "[" #arg "]" :"; \
19         dbg(arg); \
20     } while (false)
21
22 template <typename... Ts>
23 void dbg(Ts... args) {
24     (... , (std::cerr << ' ' << args));
25     std::cerr << std::endl;
26 }

```

2 Shell Scripts

2.1 checker.sh

Linux version.

```
1 #!/bin/bash
2
3 cd "$1"
4
5 g++ -o main -O2 -std=c++17 -DLOCAL main.cpp -ftrapv -fsanitize=address,undefined
6
7 for input in *.in; do
8     output=${input%.*}.out
9     answer=${input%.*}.ans
10
11     ./main < $input > $output
12
13     echo "case ${input%.*}: "
14     echo "My: "
15     cat $output
16     echo "Answer: "
17     cat $answer
18
19 done
```

Windows version.

```
1 @echo off
2
3 cd %1
4
5 del .\main.exe
6
7 g++ -o main.exe main.cpp -DLOCAL -std=c++17 -ftrapv
8
9 for %%i in (*.in) do (
10     main.exe < %%i > %%~ni.out
11     echo case %%~ni:
12     echo My:
13     type %%~ni.out
14     echo Answer:
15     type %%~ni.ans
16 )
17
18 cd ../shell
```


3 mod int

```

1  template <int P>
2  struct Mint {
3      int v = 0;
4
5      // reflection
6      template <typet = int>
7      constexpr operator T() const {
8          return v;
9      }
10
11     // constructor //
12     constexpr Mint() = default;
13     template <typet>
14     constexpr Mint(T x) : v(x % P) {}
15     constexpr int val() const { return v; }
16     constexpr int mod() { return P; }
17
18     // io //
19     friend std::istream& operator>>(std::istream& is, Mint& x) {
20         LL y;
21         is >> y;
22         x = y;
23         return is;
24     }
25     friend std::ostream& operator<<(std::ostream& os, Mint x) { return os << x.v; }
26
27     // comparision //
28     friend constexpr bool operator==(const Mint& lhs, const Mint& rhs) { return lhs.v == rhs.v; }
29     friend constexpr bool operator!=(const Mint& lhs, const Mint& rhs) { return lhs.v != rhs.v; }
30     friend constexpr bool operator<(const Mint& lhs, const Mint& rhs) { return lhs.v < rhs.v; }
31     friend constexpr bool operator<=(const Mint& lhs, const Mint& rhs) { return lhs.v <= rhs.v; }
32     friend constexpr bool operator>(const Mint& lhs, const Mint& rhs) { return lhs.v > rhs.v; }
33     friend constexpr bool operator>=(const Mint& lhs, const Mint& rhs) { return lhs.v >= rhs.v; }
34
35     // arithmetic //
36     template <typet>
37     friend constexpr Mint power(Mint a, T n) {
38         Mint ans = 1;
39         while (n) {
40             if (n & 1) ans *= a;
41             a *= a;
42             n >>= 1;
43         }
44         return ans;
45     }
46     friend constexpr Mint inv(const Mint& rhs) { return power(rhs, P - 2); }
47     friend constexpr Mint operator+(const Mint& lhs, const Mint& rhs) {
48         return lhs.val() + rhs.val() < P ? lhs.val() + rhs.val() : lhs.val() - P + rhs.val();
49     }
50     friend constexpr Mint operator-(const Mint& lhs, const Mint& rhs) {
51         return lhs.val() < rhs.val() ? lhs.val() + P - rhs.val() : lhs.val() - rhs.val();
52     }
53     friend constexpr Mint operator*(const Mint& lhs, const Mint& rhs) {
54         return static_cast<LL>(lhs.val()) * rhs.val() % P;
55     }
56     friend constexpr Mint operator/(const Mint& lhs, const Mint& rhs) { return lhs * inv(rhs); }
57     Mint operator+() const { return *this; }
58     Mint operator-() const { return Mint() - *this; }
59     constexpr Mint& operator++() {
60         v++;
61         if (v == P) v = 0;
62         return *this;
63     }
64     constexpr Mint& operator--() {
65         if (v == 0) v = P;
66         v--;
67         return *this;
68     }
69     constexpr Mint& operator++(int) {
70         Mint ans = *this;
71         ++*this;
72         return ans;
73     }
74     constexpr Mint& operator--(int) {
75         Mint ans = *this;
76         --*this;
77         return ans;
78     }
79     constexpr Mint& operator+=(const Mint& rhs) {
80         v = v + rhs;
81         return *this;
82     }
83     constexpr Mint& operator-=(const Mint& rhs) {

```

```

84     v = v - rhs;
85     return *this;
86 }
87 constexpr Mint& operator*=(const Mint& rhs) {
88     v = v * rhs;
89     return *this;
90 }
91 constexpr Mint& operator/=(const Mint& rhs) {
92     v = v / rhs;
93     return *this;
94 }
95 };
96 using Z = Mint<998244353>;

```

4 data structure

4.1 stack

```

1 vi stk;
2 for (int i = 1; i <= n; i++){
3     while (!stk.empty() and stk.back() > a[i]) {
4         stk.pop_back();
5     }
6     stk.push_back(a[i]);
7 }

```

4.2 queue

```

1 std::deque<int> q;
2 for (int i = 1; i <= n; i++) {
3     while (!q.empty() and a[q.back()] >= a[i]) q.pop_back();
4     if (!q.empty() and i - q.front() >= k) q.pop_front();
5     q.push_back(i);
6 }

```

4.3 DSU

```

1 vi fa(n + 1);
2 std::iota(all(fa), 0);
3 std::function<void(int)> find = [&] (int x) -> int{
4     return x == fa[x] ? x : fa[x] = find(fa[x]);
5 };
6 auto merge = [&] (int x, int y) -> void{
7     x = find(x), y = find(y);
8     if (x == y) return;
9     /* operations */
10    fa[y] = x;
11 };

```

4.4 ST

用于解决可重复问题的数据结构。

可重复问题是指对运算 opt , 满足 $x \text{ opt } x = x$ 。

一维

```

1 vvi f(n + 1, vi(30));
2 vi Log2(n + 1);
3 auto ST_init = [&]() -> void {
4     for (int i = 1; i <= n; i++) {

```

```

5     f[i][0] = a[i];
6     if (i > 1) Log2[i] = Log2[i / 2] + 1;
7 };
8 int t = Log2[n];
9 for (int j = 1; j <= t; j++) {
10    for (int i = 1; i <= n - (1 << j) + 1; i++) {
11        f[i][j] = std::max(f[i][j - 1], f[i + (1 << (j - 1))][j - 1]);
12    }
13 }
14 };
15
16 auto ST_query = [&](int l, int r) -> int {
17     int t = Log2[r - l + 1];
18     return std::max(f[l][t], f[r - (1 << t) + 1][t]);
19 };

```

二维

```

1  std::vector f(n + 1, std::vector<std::array<std::array<int, 30>, 30>>(m + 1));
2  vi Log2(n + 1);
3  auto ST_init = [&]() -> void {
4      for (int i = 2; i <= std::max(n, m); i++) {
5          Log2[i] = Log2[i / 2] + 1;
6      }
7      for (int i = 2; i <= n; i++) {
8          for (int j = 2; j <= m; j++) {
9              f[i][j][0][0] = a[i][j];
10         }
11     }
12     for (int ki = 0; ki <= Log2[n]; ki++) {
13         for (int kj = 0; kj <= Log2[m]; kj++) {
14             if (!ki && !kj) continue;
15             for (int i = 1; i <= n - (1 << ki) + 1; i++) {
16                 for (int j = 1; j <= m - (1 << kj) + 1; j++) {
17                     if (ki) {
18                         f[i][j][ki][kj] =
19                             std::max(f[i][j][ki - 1][kj], f[i + (1 << (ki - 1))][j][ki - 1][kj]);
20                     } else {
21                         f[i][j][ki][kj] =
22                             std::max(f[i][j][ki][kj - 1], f[i][j + (1 << (kj - 1))][ki][kj - 1]);
23                     }
24                 }
25             }
26         }
27     }
28 };
29 auto ST_query = [&](int x1, int y1, int x2, int y2) -> int {
30     int ki = Log2[x2 - x1 + 1], kj = Log2[y2 - y1 + 1];
31     int t1 = f[x1][y1][ki][kj];
32     int t2 = f[x2 - (1 << ki) + 1][y1][ki][kj];
33     int t3 = f[x1][y2 - (1 << kj) + 1][ki][kj];
34     int t4 = f[x2 - (1 << ki) + 1][y2 - (1 << kj) + 1][ki][kj];
35     return std::max({t1, t2, t3, t4});
36 };

```

4.5 cartesian tree

一种特殊的平衡树，用元素的值作为平衡点节点的 *val*，元素的下标作为 *key*。

```

1  // cartesian tree //
2  vi ls(n + 1), rs(n + 1), stk(n + 1);
3  int top = 1;
4  for (int i = 1; i <= n; i++) {
5      int k = top;
6      while (k && a[stk[k]] > a[i]) k--;
7      if (k) rs[stk[k]] = i;
8      if (k < top) ls[i] = stk[k + 1];
9      stk[++k] = i;
10     top = k;
11 }

```

4.6 segment tree czr

区间加, 区间赋值, 区间求和, 区间最大值

```

1  const int N = 100010;
2  struct node {
3      int l, r;
4      ll sum, maxn, add, set;
5      bool addflag, setflag;
6  } tr[N << 2];
7
8  void push_up(int u) {
9      tr[u].sum = tr[u << 1].sum + tr[u << 1 | 1].sum;
10     tr[u].maxn = max(tr[u << 1].maxn, tr[u << 1 | 1].maxn);
11 }
12 // 0 4 0 0 0
13 // 2 6 2 2 2
14 // 2 6 2 4 4
15
16 void push_down(int u) {
17     auto& root = tr[u], &left = tr[u << 1], &right = tr[u << 1 | 1];
18     if (root.setflag) {
19         assert(!root.addflag);
20         left.add = 0, left.set = root.set, left.addflag = false, left.setflag = true;
21         right.add = 0, right.set = root.set, right.addflag = false, right.setflag = true;
22         left.sum = root.set * (left.r - left.l + 1);
23         right.sum = root.set * (right.r - right.l + 1);
24         left.maxn = root.set;
25         right.maxn = root.set;
26         root.set = 0, root.setflag = false;
27     }
28     if (root.addflag) {
29         assert(!root.setflag);
30         if (left.setflag) left.set += root.add;
31         else left.add += root.add, left.addflag = true;
32         if (right.setflag) right.set += root.add;
33         else right.add += root.add, right.addflag = true;
34
35         left.sum += root.add * (left.r - left.l + 1);
36         right.sum += root.add * (right.r - right.l + 1);
37         left.maxn += root.add;
38         right.maxn += root.add;
39         root.add = 0, root.addflag = false;
40     }
41     assert(root.add == 0);
42 }
43
44 void build(int u, int l, int r, vector<ll>& a) {
45     if (l == r) {
46         tr[u].l = tr[u].r = l, tr[u].sum = tr[u].maxn = a[l];
47         tr[u].add = 0, tr[u].set = 0;
48         tr[u].addflag = tr[u].setflag = false;
49     } else {
50         tr[u].l = l, tr[u].r = r, tr[u].add = 0, tr[u].set = 0;
51         tr[u].addflag = tr[u].setflag = false;
52         int mid = l + r >> 1;
53         build(u << 1, l, mid, a);
54         build(u << 1 | 1, mid + 1, r, a);
55         push_up(u);
56     }
57 }
58
59 // 区间加
60 void modify(int u, int l, int r, ll d) {
61     if (l > r) return;
62     if (tr[u].l >= l && tr[u].r <= r) {
63         if (tr[u].setflag) tr[u].set += d;
64         else tr[u].add += d, tr[u].addflag = true;
65         tr[u].sum += d * (tr[u].r - tr[u].l + 1);
66         tr[u].maxn += d;
67     } else {
68         push_down(u);
69         int mid = tr[u].l + tr[u].r >> 1;
70         if (l <= mid) modify(u << 1, l, r, d);
71         if (r > mid) modify(u << 1 | 1, l, r, d);
72         push_up(u);
73     }
74 }
75
76 // 区间赋值
77 void update(int u, int l, int r, ll x) {
78     if (l > r) return;
79     if (tr[u].l >= l && tr[u].r <= r) {
80         tr[u].set = x, tr[u].setflag = true;
81         tr[u].add = 0, tr[u].addflag = false;

```

```

82     tr[u].sum = x * (tr[u].r - tr[u].l + 1);
83     tr[u].maxn = x;
84 } else {
85     push_down(u);
86     int mid = tr[u].l + tr[u].r >> 1;
87     if (l <= mid) update(u << 1, l, r, x);
88     if (r > mid) update(u << 1 | 1, l, r, x);
89     push_up(u);
90 }
91 }
92
93 ll query_sum(int u, int l, int r) {
94     if (l > r) return 0;
95     if (tr[u].l >= l && tr[u].r <= r) return tr[u].sum;
96     else {
97         ll res = 0;
98         push_down(u);
99         int mid = tr[u].l + tr[u].r >> 1;
100        if (l <= mid) res += query_sum(u << 1, l, r);
101        if (r > mid) res += query_sum(u << 1 | 1, l, r);
102        return res;
103    }
104 }
105
106 ll query_maxn(int u, int l, int r) {
107     if (l > r) return -1e18;
108     if (tr[u].l >= l && tr[u].r <= r) return tr[u].maxn;
109     else {
110         ll res = -1e18;
111         push_down(u);
112         int mid = tr[u].l + tr[u].r >> 1;
113         if (l <= mid) res = max(res, query_maxn(u << 1, l, r));
114         if (r > mid) res = max(res, query_maxn(u << 1 | 1, l, r));
115         return res;
116     }
117 }
118
119 // 找到最小 i 使得 sum(l, i) >= k
120 ll find_presum_idx(int u, int l, int r, int x) {
121     if (tr[u].l == tr[u].r) return tr[u].l;
122     else {
123         push_down(u);
124         int mid = tr[u].l + tr[u].r >> 1;
125         if (r <= mid) {
126             return find_presum_idx(u << 1, l, r, x);
127         } else if (l > mid) {
128             return find_presum_idx(u << 1 | 1, l, r, x);
129         } else {
130             ll lsum = query_sum(u << 1, l, r);
131             if (lsum >= x) return find_presum_idx(u << 1, l, mid, x);
132             else return find_presum_idx(u << 1 | 1, mid + 1, r, x - lsum);
133         }
134     }
135 }

```

4.7 segment tree

维护半群

```

1 struct Info {
2     /* 重载 operator+ */
3 };
4
5 struct Tag {
6     /* 重载 operator== */
7 };
8
9 void infoApply(Info& a, int l, int r, const Tag& tag) {}
10
11 void tagApply(Tag& a, int l, int r, const Tag& tag) {}
12
13 template <class Info, class Tag>
14 class segTree {
15     #define ls i << 1
16     #define rs i << 1 | 1
17     #define mid ((l + r) >> 1)
18     #define lson ls, l, mid
19     #define rson rs, mid + 1, r
20
21     int n;
22     std::vector<Info> info;

```

```

23     std::vector<Tag> tag;
24
25 public:
26     segTree(const std::vector<Info>& init) : n(init.size() - 1) {
27         assert(n > 0);
28         info.resize(4 << std::lg(n));
29         tag.resize(4 << std::lg(n));
30         auto build = [&](auto dfs, int i, int l, int r) {
31             if (l == r) {
32                 info[i] = init[l];
33                 return;
34             }
35             dfs(dfs, lson);
36             dfs(dfs, rson);
37             push_up(i);
38         };
39         build(build, 1, 1, n);
40     }
41
42
43 private:
44     void push_up(int i) { info[i] = info[ls] + info[rs]; }
45
46
47     template <class... T>
48     void apply(int i, int l, int r, const T&... val) {
49         ::infoApply(info[i], l, r, val...);
50         ::tagApply(tag[i], l, r, val...);
51     }
52
53     void push_down(int i, int l, int r) {
54         if (tag[i] == Tag{}) return;
55         apply(lson, tag[i]);
56         apply(rson, tag[i]);
57         tag[i] = {};
58     }
59
60 public:
61     template <class... T>
62     void rangeApply(int ql, int qr, const T&... val) {
63         auto dfs = [&](auto dfs, int i, int l, int r) {
64             if (qr < l or r < ql) return;
65             if (ql <= l and r <= qr) {
66                 apply(i, l, r, val...);
67                 return;
68             }
69             push_down(i, l, r);
70             dfs(dfs, lson);
71             dfs(dfs, rson);
72             push_up(i);
73         };
74         dfs(dfs, 1, 1, n);
75     }
76
77     Info rangeAsk(int ql, int qr) {
78         Info res{};
79         auto dfs = [&](auto dfs, int i, int l, int r) {
80             if (qr < l or r < ql) return;
81             if (ql <= l and r <= qr) {
82                 res = res + info[i];
83                 return;
84             }
85             push_down(i, l, r);
86             dfs(dfs, lson);
87             dfs(dfs, rson);
88         };
89         dfs(dfs, 1, 1, n);
90         return res;
91     }
92
93 #undef rson
94 #undef lson
95 #undef mid
96 #undef rs
97 #undef ls
98 };

```

区间修改 (带 add 和 mul 的 lazy tag)

n 个数, m 次操作, 操作分为

1. $1 \ x \ y \ k$: 将区间 $[x, y]$ 中的数每个乘以 k .
2. $2 \ x \ y \ k$: 将区间 $[x, y]$ 中的数每个加上 k .
3. $3 \ x \ y$:

输出区间 $[x, y]$ 中数的和. (对 p 取模)

```

1 // Problem: P3373 【模板】线段树 2
2
3 struct Info {
4     LL sum = 0;
5
6     Info(LL _sum = 0) : sum(_sum) {}
7
8     Info operator+(const Info& b) const { return Info(add(sum + b.sum)); }
9 };
10
11 struct Tag {
12     LL add = 0, mul = 1;
13
14     Tag(LL _add = 0, LL _mul = 1) : add(_add), mul(_mul) {}
15
16     bool operator==(const Tag& b) const { return add == b.add and mul == b.mul; }
17 };
18
19 void infoApply(Info& a, int l, int r, const Tag& tag) {
20     a.sum = add(mul(a.sum, tag.mul), mul((r - l + 1), tag.add));
21 }
22
23 void tagApply(Tag& a, int l, int r, const Tag& tag) {
24     a.add = add(mul(a.add, tag.mul), tag.add);
25     a.mul = mul(a.mul, tag.mul);
26 }
27
28 template <class Info, class Tag>
29 class segTree {
30 #define ls i << 1
31 #define rs i << 1 | 1
32 #define mid ((l + r) >> 1)
33 #define lson ls, l, mid
34 #define rson rs, mid + 1, r
35
36     int n;
37     std::vector<Info> info;
38     std::vector<Tag> tag;
39
40 public:
41     segTree(const std::vector<Info>& init) : n(init.size() - 1) {
42         assert(n > 0);
43         info.resize(4 << std::lg(n));
44         tag.resize(4 << std::lg(n));
45         auto build = [&](auto dfs, int i, int l, int r) {
46             if (l == r) {
47                 info[i] = init[l];
48                 return;
49             }
50             dfs(dfs, lson);
51             dfs(dfs, rson);
52             push_up(i);
53         };
54         build(build, 1, 1, n);
55     }
56
57 private:
58     void push_up(int i) { info[i] = info[ls] + info[rs]; }
59
60
61     template <class... T>
62     void apply(int i, int l, int r, const T&... val) {
63         ::infoApply(info[i], l, r, val...);
64         ::tagApply(tag[i], l, r, val...);
65     }
66
67     void push_down(int i, int l, int r) {
68         if (tag[i] == Tag{}) return;
69         apply(lson, tag[i]);
70         apply(rson, tag[i]);
71         tag[i] = {};
72     }
73
74 public:
75     template <class... T>
76     void rangeMerge(int ql, int qr, const T&... val) {
77         auto dfs = [&](auto dfs, int i, int l, int r) {
78             if (qr < l or r < ql) return;
79             if (ql <= l and r <= qr) {
80                 apply(i, l, r, val...);
81                 return;
82             }
83             push_down(i, l, r);
84             dfs(dfs, lson);
85             dfs(dfs, rson);

```

```

86         dfs(dfs, rson);
87         push_up(i);
88     };
89     dfs(dfs, 1, 1, n);
90 }
91
92 Info rangeQuery(int ql, int qr) {
93     Info res{};
94     auto dfs = [&](auto dfs, int i, int l, int r) {
95         if (qr < l or r < ql) return;
96         if (ql <= l and r <= qr) {
97             res = res + info[i];
98             return;
99         }
100         push_down(i, l, r);
101         dfs(dfs, lson);
102         dfs(dfs, rson);
103     };
104     dfs(dfs, 1, 1, n);
105     return res;
106 }
107
108 #undef rson
109 #undef lson
110 #undef mid
111 #undef rs
112 #undef ls
113 };
114
115 int main() {
116     std::ios::sync_with_stdio(false);
117     std::cin.tie(0);
118     std::cout.tie(0);
119
120     int n, m, p;
121     std::cin >> n >> m >> p;
122     std::vector<Info> a(n + 1);
123     for (int i = 1; i <= n; i++) std::cin >> a[i].sum;
124     static segTree<Info, Tag> tr(a);
125
126     while (m--) {
127         int op, k, l, r;
128         std::cin >> op >> l >> r;
129         if (op == 1) {
130             std::cin >> k;
131             tr.rangeMerge(l, r, Tag(0, k));
132         } else if (op == 2) {
133             std::cin >> k;
134             tr.rangeMerge(l, r, Tag(k, 1));
135         } else {
136             std::cout << tr.rangeQuery(l, r).sum << '\n';
137         }
138     }
139
140     return 0;
141 }

```

动态开点权值线段树

如果实现 push up 记得先开点再 push.

```

1 // Problem: P3369 【模板】普通平衡树
2
3 struct node {
4     int id, l, r;
5     int ls, rs;
6     int sum;
7
8     node(int _id, int _l, int _r) : id(_id), l(_l), r(_r) {
9         ls = rs = 0;
10        sum = 0;
11    }
12 };
13
14
15 // Segment tree //
16 int idx = 1;
17 std::vector<node> tree = {node{0, 0, 0}};
18
19 auto new_node = [&](int l, int r) -> int {
20     tree.push_back(node(idx, l, r));
21     return idx++;
22 };

```



```

23
24 auto push_up = [&](int u) -> void {
25     tree[u].sum = 0;
26     if (tree[u].ls) tree[u].sum += tree[tree[u].ls].sum;
27     if (tree[u].rs) tree[u].sum += tree[tree[u].rs].sum;
28 };
29
30 auto build = [&]() { new_node(-10000000, 10000000); };
31
32 std::function<void(int, int, int, int)> insert = [&](int u, int l, int r, int x) {
33     if (l == r) {
34         tree[u].sum++;
35         return;
36     }
37     int mid = (l + r - 1) / 2;
38     if (x <= mid) {
39         if (!tree[u].ls) tree[u].ls = new_node(l, mid);
40         insert(tree[u].ls, l, mid, x);
41     } else {
42         if (!tree[u].rs) tree[u].rs = new_node(mid + 1, r);
43         insert(tree[u].rs, mid + 1, r, x);
44     }
45     push_up(u);
46 };
47
48 std::function<void(int, int, int, int)> remove = [&](int u, int l, int r, int x) {
49     if (l == r) {
50         if (tree[u].sum) tree[u].sum--;
51         return;
52     }
53     int mid = (l + r - 1) / 2;
54     if (x <= mid) {
55         if (!tree[u].ls) return;
56         remove(tree[u].ls, l, mid, x);
57     } else {
58         if (!tree[u].rs) return;
59         remove(tree[u].rs, mid + 1, r, x);
60     }
61     push_up(u);
62 };
63
64 std::function<int(int, int, int, int)> get_rank_by_key = [&](int u, int l, int r, int x) -> int {
65     if (l == r) {
66         return 1;
67     }
68     int mid = (l + r - 1) / 2;
69     int ans = 0;
70     if (x <= mid) {
71         if (!tree[u].ls) return 1;
72         ans = get_rank_by_key(tree[u].ls, l, mid, x);
73     } else {
74         if (!tree[u].rs) return tree[tree[u].ls].sum + 1;
75         if (!tree[u].ls) {
76             ans = get_rank_by_key(tree[u].rs, mid + 1, r, x);
77         } else {
78             ans = get_rank_by_key(tree[u].rs, mid + 1, r, x) + tree[tree[u].ls].sum;
79         }
80     }
81     return ans;
82 };
83
84 std::function<int(int, int, int, int)> get_key_by_rank = [&](int u, int l, int r, int x) -> int {
85     if (l == r) {
86         return l;
87     }
88     int mid = (l + r - 1) / 2;
89     if (tree[u].ls) {
90         if (x <= tree[tree[u].ls].sum) {
91             return get_key_by_rank(tree[u].ls, l, mid, x);
92         } else {
93             return get_key_by_rank(tree[u].rs, mid + 1, r, x - tree[tree[u].ls].sum);
94         }
95     } else {
96         return get_key_by_rank(tree[u].rs, mid + 1, r, x);
97     }
98 };
99
100 std::function<int(int)> get_prev = [&](int x) -> int {
101     int rank = get_rank_by_key(1, -10000000, 10000000, x) - 1;
102     debug(rank);
103     return get_key_by_rank(1, -10000000, 10000000, rank);
104 };
105
106 std::function<int(int)> get_next = [&](int x) -> int {
107     debug(x + 1);
108     int rank = get_rank_by_key(1, -10000000, 10000000, x + 1);
109     debug(rank);

```

```

110 |     return get_key_by_rank(1, -100000000, 100000000, rank);
111 | };

```

线段树上二分

```

1  int lc[M], rc[M], w[M], tot;
2  ll a[N];
3  void ins(int& o, ll l, ll r, ll x) {
4      if (!o) o = ++tot;
5      ++w[o];
6      if (l == r) return ;
7      ll md = l + r >> 1;
8      if (x <= md) ins(lc[o], l, md, x);
9      else ins(rc[o], md + 1, r, x);
10 };
11 int ask(int o, ll L, ll R, ll l, ll r) {
12     if (!o) return 0;
13     // assert(o);
14     if (l <= L && R <= r) return w[o];
15     ll md = L + R >> 1; int z = 0;
16     if (r >= L) z += ask(lc[o], L, md, l, r);
17     if (l < R) z += ask(rc[o], md + 1, R, l, r);
18     return z;
19 };
20 auto bs(int o, ll l, ll r, int k) {
21     // assert(k <= w[o]);
22     if (l == r) return std::make_pair(l, w[o] - k);
23     ll md = l + r >> 1;
24     if (w[rc[o]] >= k) return bs(rc[o], md + 1, r, k);
25     else return bs(lc[o], l, md, k - w[rc[o]]);
26 };

```

(权值) 线段树合并分裂

首先村落里的一共有 n 座房屋, 并形成一棵树状结构. 然后救济粮分 m 次发放, 每次选择两个房屋 (x, y) , 然后对于 x 到 y 的路径上每座房子里发放一袋 z 类型的救济粮. 查询所有的救济粮发放完毕后, 每座房子里存放的最多的是哪种救济粮.

```

1  // Problem: P4556 [Vani有约会]雨天的尾巴 / 【模板】线段树合并
2
3  struct node {
4      int l, r, id;
5      int ls, rs;
6      int cnt, ans;
7
8      node(int _id, int _l, int _r) : id(_id), l(_l), r(_r) {
9          ls = rs = 0;
10         cnt = ans = 0;
11     }
12 };
13
14 int main() {
15     std::ios::sync_with_stdio(false);
16     std::cin.tie(0);
17     std::cout.tie(0);
18
19     int n, m;
20     std::cin >> n >> m;
21     vvi e(n + 1);
22     vi ans(n + 1);
23     for (int i = 1; i < n; i++) {
24         int u, v;
25         std::cin >> u >> v;
26         e[u].push_back(v);
27         e[v].push_back(u);
28     }
29
30     /* Segment tree */
31     int idx = 1;
32     vi rt(n + 1);
33     std::vector<node> tree = {node{0, 0, 0}};
34
35     auto new_node = [&](int l, int r) -> int {
36         tree.push_back(node(idx, l, r));
37         return idx++;
38     };

```

```

39
40 auto push_up = [&](int u) -> void {
41     if (!tree[u].ls) {
42         tree[u].cnt = tree[tree[u].rs].cnt;
43         tree[u].ans = tree[tree[u].rs].ans;
44     } else if (!tree[u].rs) {
45         tree[u].cnt = tree[tree[u].ls].cnt;
46         tree[u].ans = tree[tree[u].ls].ans;
47     } else {
48         if (tree[tree[u].rs].cnt > tree[tree[u].ls].cnt) {
49             tree[u].cnt = tree[tree[u].rs].cnt;
50             tree[u].ans = tree[tree[u].rs].ans;
51         } else {
52             tree[u].cnt = tree[tree[u].ls].cnt;
53             tree[u].ans = tree[tree[u].ls].ans;
54         }
55     }
56 };
57
58 std::function<void(int, int, int, int, int)> modify = [&](int u, int l, int r, int x, int k) {
59     if (l == r) {
60         tree[u].cnt += k;
61         tree[u].ans = 1;
62         return;
63     }
64     int mid = (l + r) >> 1;
65     if (x <= mid) {
66         if (!tree[u].ls) tree[u].ls = new_node(l, mid);
67         modify(tree[u].ls, l, mid, x, k);
68     } else {
69         if (!tree[u].rs) tree[u].rs = new_node(mid + 1, r);
70         modify(tree[u].rs, mid + 1, r, x, k);
71     }
72     push_up(u);
73 };
74
75 std::function<int(int, int, int, int)> merge = [&](int u, int v, int l, int r) -> int {
76     /* v 的信息传递给 u */
77     if (!u) return v;
78     if (!v) return u;
79     if (l == r) {
80         tree[u].cnt += tree[v].cnt;
81         return u;
82     }
83     int mid = (l + r) >> 1;
84     tree[u].ls = merge(tree[u].ls, tree[v].ls, l, mid);
85     tree[u].rs = merge(tree[u].rs, tree[v].rs, mid + 1, r);
86     push_up(u);
87     return u;
88 };
89
90 /* LCA */
91
92 for (int i = 1; i <= n; i++) {
93     rt[i] = idx;
94     new_node(1, 100000);
95 }
96
97 for (int i = 1; i <= m; i++) {
98     int u, v, w;
99     std::cin >> u >> v >> w;
100     int lca = LCA(u, v);
101     modify(rt[u], 1, 100000, w, 1);
102     modify(rt[v], 1, 100000, w, 1);
103     modify(rt[lca], 1, 100000, w, -1);
104     if (father[lca][0]) {
105         modify(rt[father[lca][0]], 1, 100000, w, -1);
106     }
107 }
108
109 /* dfs */
110 std::function<void(int, int)> Dfs = [&](int u, int fa) {
111     for (auto v : e[u]) {
112         if (v == fa) continue;
113         Dfs(v, u);
114         merge(rt[u], rt[v], 1, 100000);
115     }
116     ans[u] = tree[rt[u]].ans;
117     if (tree[rt[u]].cnt == 0) ans[u] = 0;
118 };
119
120 Dfs(1, 0);
121
122 for (int i = 1; i <= n; i++) {
123     std::cout << ans[i] << '\n';
124 }
125

```

```

126 |     return 0;
127 | }

```

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace Acc{
4      using i64=int64_t;
5      enum{N=200009,M=10000000};
6      i64 v[M];
7      int lc[M],rc[M],tot,a[N],r[N];
8      auto up=[](int o){
9          v[o]=v[lc[o]]+v[rc[o]];
10     };
11     void bd(int&o,int l,int r){
12         if(o==++tot,l==r)return cin>>v[o],void();
13         int md=l+r>>1;
14         bd(lc[o],l,md),bd(rc[o],md+1,r),up(o);
15     }
16     void spl(int&o,int&x,int l,int r,int L,int R){
17         if(l<=L&&R<=r)return o=x,x=0,void();
18         int md=L+R>>1;
19         o=++tot;
20         if(l<=md)spl(lc[o],lc[x],l,r,L,md);
21         if(r>md)spl(rc[o],rc[x],l,r,md+1,R);
22         up(o),up(x);
23     }
24     void mg(int&o,int x,int l,int r){
25         if(!o||!x)return o=x,void();
26         if(l==r)return v[o]+=v[x],void();
27         int md=l+r>>1;
28         mg(lc[o],lc[x],l,md);
29         mg(rc[o],rc[x],md+1,r);
30         up(o);
31     }
32     void ins(int&o,int l,int r,int x,int k){
33         if(!o)o=++tot;
34         if(v[o]!=x,l==r)return;
35         int md=l+r>>1;
36         x<=md?ins(lc[o],l,md,x,k):ins(rc[o],md+1,r,x,k);
37     }
38     i64 qry(int o,int l,int r,int L,int R){
39         if(!o)return 0;
40         if(l<=L&&R<=r)return v[o];
41         int md=L+R>>1;i64 z=0;
42         if(l<=md)z+=qry(lc[o],l,r,L,md);
43         if(r>md)z+=qry(rc[o],l,r,md+1,R);
44         return z;
45     }
46     int kth(int o,int l,int r,int k){
47         if(l==r)return l;
48         if(k>v[o])return -1;
49         int md=l+r>>1;
50         if(k<=v[lc[o]])return kth(lc[o],l,md,k);
51         else return kth(rc[o],md+1,r,k-v[lc[o]]);
52     }
53     auto work=[](){
54         int n,m,i,x,y,o=1;
55         for(cin>>n>>m,bd(r[1],1,n);m--;)switch(cin>>i,i){
56             case 0:cin>>i>>x>>y,spl(r[++o],r[i],x,y,1,n);break;
57             case 1:cin>>x>>y,mg(r[x],r[y],1,n);break;
58             case 2:cin>>i>>x>>y,ins(r[i],1,n,y,x);break;
59             case 3:cin>>i>>x>>y,cout<<qry(r[i],x,y,1,n)<<"\n";break;
60             case 4:cin>>i>>x,cout<<kth(r[i],1,n,x)<<"\n";break;
61         }
62     };
63 }
64 int main(){
65     ios::sync_with_stdio(0);
66     cin.tie(0),Acc::work();
67 }

```

4.8 hjt segment tree

第 1 个例题

n 个数, m 次操作, 操作分别为

1. v_i 1 loc_i value _{i} : 将第 v_i 个版本的 $a[loc_i]$ 修改为 $value_i$,

2. v_i 2 loc_i : 拷贝第 v_i 个版本, 并查询该版本的 $a[loc_i]$.

```

1 // 洛谷 P3919 【模板】可持久化线段树 1 (可持久化数组)
2
3 struct node {
4     int l, r, key;
5 };
6
7 int main() {
8     std::ios::sync_with_stdio(false);
9     std::cin.tie(0);
10    std::cout.tie(0);
11
12    int n, m;
13    std::cin >> n >> m;
14    vi a(n + 1);
15    for (int i = 1; i <= n; i++) {
16        std::cin >> a[i];
17    }
18
19    /* hjt segment tree */
20    int idx = 0;
21    vi root(m + 1);
22    std::vector<node> tr(n * 25);
23
24    std::function<int(int, int)> build = [&](int l, int r) -> int {
25        int p = ++idx;
26        if (l == r) {
27            tr[p].key = a[l];
28            return p;
29        }
30        int mid = (l + r) >> 1;
31        tr[p].l = build(l, mid);
32        tr[p].r = build(mid + 1, r);
33        return p;
34    };
35
36    std::function<int(int, int, int, int, int)> modify = [&](int p, int l, int r, int k,
37                                                            int x) -> int {
38        int q = ++idx;
39        tr[q].l = tr[p].l, tr[q].r = tr[p].r;
40        if (tr[q].l == tr[q].r) {
41            tr[q].key = x;
42            return q;
43        }
44        int mid = (l + r) >> 1;
45        if (k <= mid) {
46            tr[q].l = modify(tr[q].l, l, mid, k, x);
47        } else {
48            tr[q].r = modify(tr[q].r, mid + 1, r, k, x);
49        }
50        return q;
51    };
52
53    std::function<int(int, int, int, int)> query = [&](int p, int l, int r, int k) -> int {
54        if (tr[p].l == tr[p].r) {
55            return tr[p].key;
56        }
57        int mid = (l + r) >> 1;
58        if (k <= mid) {
59            return query(tr[p].l, l, mid, k);
60        } else {
61            return query(tr[p].r, mid + 1, r, k);
62        }
63    };
64
65    root[0] = build(1, n);
66
67    for (int i = 1; i <= m; i++) {
68        int op, ver, k, x;
69        std::cin >> ver >> op;
70        if (op == 1) {
71            std::cin >> k >> x;
72            root[i] = modify(root[ver], 1, n, k, x);
73        } else {
74            std::cin >> k;
75            root[i] = root[ver];
76            std::cout << query(root[ver], 1, n, k) << '\n';
77        }
78    }
79
80    return 0;
81 }

```

第 2 个例题

长度为 n 的序列 a , m 次查询, 每次查询 $[l, r]$ 中的第 k 小值.

```

1 // 洛谷P3834 【模板】可持久化线段树 2
2
3 struct node {
4     int l, r, cnt;
5 };
6
7 int main() {
8     std::ios::sync_with_stdio(false);
9     std::cin.tie(0);
10    std::cout.tie(0);
11
12    int n, m;
13    std::cin >> n >> m;
14    vi a(n + 1), v;
15    for (int i = 1; i <= n; i++) {
16        std::cin >> a[i];
17        v.push_back(a[i]);
18    }
19    std::sort(all(v));
20    v.erase(unique(all(v)), v.end());
21    auto find = [&](int x) -> int { return std::lower_bound(all(v), x) - v.begin() + 1; };
22
23    /* hjt segment tree */
24    std::vector<node> tr(n * 25);
25    vi root(n + 1);
26    int idx = 0;
27
28    std::function<int(int, int)> build = [&](int l, int r) -> int {
29        int p = ++idx;
30        if (l == r) return p;
31        int mid = (l + r) >> 1;
32        tr[p].l = build(l, mid), tr[p].r = build(mid + 1, r);
33        return p;
34    };
35
36    std::function<int(int, int, int, int)> modify = [&](int p, int l, int r, int x) -> int {
37        int q = ++idx;
38        tr[q] = tr[p];
39        if (tr[q].l == tr[q].r) {
40            tr[q].cnt++;
41            return q;
42        }
43        int mid = (l + r) >> 1;
44        if (x <= mid) {
45            tr[q].l = modify(tr[q].l, l, mid, x);
46        } else {
47            tr[q].r = modify(tr[q].r, mid + 1, r, x);
48        }
49        tr[q].cnt = tr[tr[q].l].cnt + tr[tr[q].r].cnt;
50        return q;
51    };
52
53    std::function<int(int, int, int, int, int)> query = [&](int p, int q, int l, int r,
54                                                         int x) -> int {
55        if (l == r) return l;
56        int cnt = tr[tr[p].l].cnt - tr[tr[q].l].cnt;
57        int mid = (l + r) >> 1;
58        if (x <= cnt) {
59            return query(tr[p].l, tr[q].l, l, mid, x);
60        } else {
61            return query(tr[p].r, tr[q].r, mid + 1, r, x - cnt);
62        }
63    };
64
65    root[0] = build(1, v.size());
66
67    for (int i = 1; i <= n; i++) {
68        root[i] = modify(root[i - 1], 1, v.size(), find(a[i]));
69    }
70    for (int i = 1; i <= m; i++) {
71        int l, r, k;
72        std::cin >> l >> r >> k;
73        std::cout << v[query(root[r], root[l - 1], 1, v.size(), k) - 1] << '\n';
74    }
75
76    return 0;
77 }
78

```

4.9 LiChao Tree

线段版本

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace Acc{
4  #define lc (o<<1)
5  #define rc (o<<1|1)
6  const int N = 4e5+10;
7  int v[N],n,l,r,z;
8  double k[N],b[N];
9  inline void r1(int&x){x=(x+z-1)%39989+1;}
10 inline void r2(int&x){x=(x+z-1)%1000000000+1;}
11 double f(int o,int x){
12     return k[o]*x+b[o];
13 }
14 int beat(int x,int a,int b){
15     double u=f(a,x),v=f(b,x);
16     return fabs(u-v)<=1e-8?a<b:u>v;
17 }
18 void add(int o,int L,int R,int x){
19     int md=L+R>>1;
20     if(l<=L&&R<=r){
21         if(!v[o])return (void)(v[o]=x);
22         if(beat(L,v[o],x) && beat(R,v[o],x))return;
23         if(beat(L,x,v[o]) && beat(R,x,v[o]))return (void)(v[o]=x);
24         if(beat(md,x,v[o]))swap(x,v[o]);
25         if(beat(L,x,v[o]))add(lc,L,md,x);
26         else add(rc,md+1,R,x);
27         return;
28     }
29     if(r>md)add(rc,md+1,R,x);
30     if(l<=md)add(lc,L,md,x);
31 }
32 int ask(int o,int L,int R){
33     if(L==R)return v[o];
34     int md=L+R>>1,h=l<=md?ask(lc,L,md):ask(rc,md+1,R);
35     return beat(l,h,v[o])?h:v[o];
36 }
37 void work(){
38     cin>>n;
39     for(int op,y1,y2,c=0;n--;){
40         cin>>op;
41         if(op){
42             cin>>l>>y1>>r>>y2,++c,r1(l),r2(y1),r1(r),r2(y2);
43             if(l==r)k[c]=0,b[c]=max(y1,y2);
44             else {
45                 if(l>r)swap(l,r),swap(y1,y2);
46                 k[c]=(y2-y1+0.)/(r-l),b[c]=y1-k[c]*l;
47             }
48             add(1,1,4e4+10,c);
49         }else cin>>l,r1(l),cout<<(z=ask(1,1,4e4+10))<<'\\n';
50     }
51 }
52 }
53 int main(){
54     return Acc::work(),0;
55 }

```

4.10 treap

旋转 treap

n 次操作, 操作分为如下 6 种:

1. 插入数 x ,
2. 删除数 x (若有多个相同的数, 只删除一个),
3. 查询数 x 的排名 (排名定义为小于 x 的数的个数 + 1),
4. 查询排名为 x 的数,
5. 求 x 的前驱 (前驱定义为小于 x 的最大数),

6. 求 x 的后继 (后继定义为大于 x 的最小数).

```

1 // Problem: P3369 【模板】普通平衡树
2
3 int n, root, idx;
4
5 struct node {
6     int l, r;
7     int key, val;
8     int cnt, size;
9 } treap[N];
10
11 void push_up(int p) {
12     treap[p].size = treap[treap[p].l].size + treap[treap[p].r].size + treap[p].cnt;
13 }
14
15 int get_node(int key) {
16     treap[++idx].key = key;
17     treap[idx].val = rand();
18     treap[idx].cnt = treap[idx].size = 1;
19     return idx;
20 }
21
22 void zig(int &p) {
23     // 右旋 //
24     int q = treap[p].l;
25     treap[p].l = treap[q].r, treap[q].r = p, p = q;
26     push_up(treap[p].r), push_up(p);
27 }
28
29 void zag(int &p) {
30     // 左旋 //
31     int q = treap[p].r;
32     treap[p].r = treap[q].l, treap[q].l = p, p = q;
33     push_up(treap[p].l), push_up(p);
34 }
35
36 void build() {
37     get_node(-inf), get_node(inf);
38     root = 1, treap[1].r = 2;
39     push_up(root);
40     if (treap[1].val < treap[2].val) zag(root);
41 }
42
43 void insert(int &p, int key) {
44     if (!p) {
45         p = get_node(key);
46     } else if (treap[p].key == key) {
47         treap[p].cnt++;
48     } else if (treap[p].key > key) {
49         insert(treap[p].l, key);
50         if (treap[treap[p].l].val > treap[p].val) zig(p);
51     } else {
52         insert(treap[p].r, key);
53         if (treap[treap[p].r].val > treap[p].val) zag(p);
54     }
55     push_up(p);
56 }
57
58 void remove(int &p, int key) {
59     if (!p) return;
60     if (treap[p].key == key) {
61         if (treap[p].cnt > 1) {
62             treap[p].cnt--;
63         } else if (treap[p].l || treap[p].r) {
64             if (!treap[p].r || treap[treap[p].l].val > treap[treap[p].r].val) {
65                 zig(p);
66                 remove(treap[p].r, key);
67             } else {
68                 zag(p);
69                 remove(treap[p].l, key);
70             }
71         } else {
72             p = 0;
73         }
74     } else if (treap[p].key > key) remove(treap[p].l, key);
75     } else {
76         remove(treap[p].r, key);
77     }
78     push_up(p);
79 }
80
81 int get_rank_by_key(int p, int key) {
82     // 通过数值找排名 //
83     if (!p) return 0;
84

```



```

85     if (treap[p].key == key) return treap[treap[p].l].size;
86     if (treap[p].key > key) return get_rank_by_key(treap[p].l, key);
87     return treap[treap[p].l].size + treap[p].cnt + get_rank_by_key(treap[p].r, key);
88 }
89
90 int get_key_by_rank(int p, int rank) {
91     // 通过排名找数值 //
92     if (!p) return inf;
93     if (treap[treap[p].l].size >= rank) return get_key_by_rank(treap[p].l, rank);
94     if (treap[treap[p].l].size + treap[p].cnt >= rank) return treap[p].key;
95     return get_key_by_rank(treap[p].r, rank - treap[treap[p].l].size - treap[p].cnt);
96 }
97
98 int get_prev(int p, int key) {
99     // 找前驱 //
100    if (!p) return -inf;
101    if (treap[p].key >= key) return get_prev(treap[p].l, key);
102    return max(treap[p].key, get_prev(treap[p].r, key));
103 }
104
105 int get_next(int p, int key) {
106     // 找后继 //
107     if (!p) return inf;
108     if (treap[p].key <= key) return get_next(treap[p].r, key);
109     return min(treap[p].key, get_next(treap[p].l, key));
110 }
111
112 int main() {
113     ios::sync_with_stdio(false);
114     cin.tie(0);
115     cout.tie(0);
116
117     cin >> n;
118     build();
119     rep(i, 1, n) {
120         int op, x;
121         cin >> op >> x;
122         if (op == 1) {
123             insert(root, x);
124         } else if (op == 2) {
125             remove(root, x);
126         } else if (op == 3) {
127             cout << get_rank_by_key(root, x) << '\n';
128         } else if (op == 4) {
129             cout << get_key_by_rank(root, x + 1) << '\n';
130         } else if (op == 5) {
131             cout << get_prev(root, x) << '\n';
132         } else {
133             cout << get_next(root, x) << '\n';
134         }
135     }
136     return 0;
137 }

```

无旋 treap

与旋转 Treap 同一个题目

```

1 struct node {
2     node *ch[2];
3     int key, val;
4     int cnt, size;
5
6     node(int _key) : key(_key), cnt(1), size(1) {
7         ch[0] = ch[1] = nullptr;
8         val = rand();
9     }
10
11     // node(node *_node) {
12     //     key = _node->key, val = _node->val, cnt = _node->cnt, size = _node->size;
13     // }
14
15     inline void push_up() {
16         size = cnt;
17         if (ch[0] != nullptr) size += ch[0]->size;
18         if (ch[1] != nullptr) size += ch[1]->size;
19     }
20 };
21
22 struct treap {
23     #define _2 second.first
24     #define _3 second.second

```

```

25 node *root;
26
27
28 pair<node *, node *> split(node *p, int key) {
29     if (p == nullptr) return {nullptr, nullptr};
30     if (p->key <= key) {
31         auto temp = split(p->ch[1], key);
32         p->ch[1] = temp.first;
33         p->push_up();
34         return {p, temp.second};
35     } else {
36         auto temp = split(p->ch[0], key);
37         p->ch[0] = temp.second;
38         p->push_up();
39         return {temp.first, p};
40     }
41 }
42
43 pair<node *, pair<node *, node *> > split_by_rank(node *p, int rank) {
44     if (p == nullptr) return {nullptr, {nullptr, nullptr}};
45     int ls_size = p->ch[0] == nullptr ? 0 : p->ch[0]->size;
46     if (rank <= ls_size) {
47         auto temp = split_by_rank(p->ch[0], rank);
48         p->ch[0] = temp._3;
49         p->push_up();
50         return {temp.first, {temp._2, p}};
51     } else if (rank <= ls_size + p->cnt) {
52         node *lt = p->ch[0];
53         node *rt = p->ch[1];
54         p->ch[0] = p->ch[1] = nullptr;
55         return {lt, {p, rt}};
56     } else {
57         auto temp = split_by_rank(p->ch[1], rank - ls_size - p->cnt);
58         p->ch[1] = temp.first;
59         p->push_up();
60         return {p, {temp._2, temp._3}};
61     }
62 }
63
64 node *merge(node *u, node *v) {
65     if (u == nullptr && v == nullptr) return nullptr;
66     if (u != nullptr && v == nullptr) return u;
67     if (v != nullptr && u == nullptr) return v;
68     if (u->val < v->val) {
69         u->ch[1] = merge(u->ch[1], v);
70         u->push_up();
71         return u;
72     } else {
73         v->ch[0] = merge(u, v->ch[0]);
74         v->push_up();
75         return v;
76     }
77 }
78
79 void insert(int key) {
80     auto temp = split(root, key);
81     auto l_tr = split(temp.first, key - 1);
82     node *new_node;
83     if (l_tr.second == nullptr) {
84         new_node = new node(key);
85     } else {
86         l_tr.second->cnt++;
87         l_tr.second->push_up();
88     }
89     node *l_tr_combined = merge(l_tr.first, l_tr.second == nullptr ? new_node : l_tr.second);
90     root = merge(l_tr_combined, temp.second);
91 }
92
93 void remove(int key) {
94     auto temp = split(root, key);
95     auto l_tr = split(temp.first, key - 1);
96     if (l_tr.second->cnt > 1) {
97         l_tr.second->cnt--;
98         l_tr.second->push_up();
99         l_tr.first = merge(l_tr.first, l_tr.second);
100     } else {
101         if (temp.first == l_tr.second) temp.first = nullptr;
102         delete l_tr.second;
103         l_tr.second = nullptr;
104     }
105     root = merge(l_tr.first, temp.second);
106 }
107
108 int get_rank_by_key(node *p, int key) {
109     auto temp = split(p, key - 1);
110     int ret = (temp.first == nullptr ? 0 : temp.first->size) + 1;
111     root = merge(temp.first, temp.second);

```

```

112     return ret;
113 }
114
115 int get_key_by_rank(node *p, int rank) {
116     auto temp = split_by_rank(p, rank);
117     int ret = temp._2->key;
118     root = merge(temp.first, merge(temp._2, temp._3));
119     return ret;
120 }
121
122 int get_prev(int key) {
123     auto temp = split(root, key - 1);
124     int ret = get_key_by_rank(temp.first, temp.first->size);
125     root = merge(temp.first, temp.second);
126     return ret;
127 }
128
129 int get_nex(int key) {
130     auto temp = split(root, key);
131     int ret = get_key_by_rank(temp.second, 1);
132     root = merge(temp.first, temp.second);
133     return ret;
134 }
135 };
136
137 treap tr;
138
139 int main() {
140     ios::sync_with_stdio(false);
141     cin.tie(0);
142     cout.tie(0);
143
144     srand(time(0));
145
146     int n;
147     cin >> n;
148     while (n--) {
149         int op, x;
150         cin >> op >> x;
151         if (op == 1) {
152             tr.insert(x);
153         } else if (op == 2) {
154             tr.remove(x);
155         } else if (op == 3) {
156             cout << tr.get_rank_by_key(tr.root, x) << '\n';
157         } else if (op == 4) {
158             cout << tr.get_key_by_rank(tr.root, x) << '\n';
159         } else if (op == 5) {
160             cout << tr.get_prev(x) << '\n';
161         } else {
162             cout << tr.get_nex(x) << '\n';
163         }
164     }
165     return 0;
166 }

```

用 01 trie 实现的一种方式

同样的题目, 注意使用 01 trie 只能存在非负数.

速度能快不少, 但只能单点操作, 而且有点费空间.

```

1 // 洛谷 P3369 【模板】普通平衡树
2
3 struct Treap {
4     int id = 1, maxlog = 25;
5     int ch[N * 25][2], siz[N * 25];
6
7     int newnode() {
8         id++;
9         ch[id][0] = ch[id][1] = siz[id] = 0;
10        return id;
11    }
12
13    void merge(int key, int cnt) {
14        int u = 1;
15        for (int i = maxlog - 1; i >= 0; i--) {
16            int v = (key >> i) & 1;
17            if (!ch[u][v]) ch[u][v] = newnode();
18            u = ch[u][v];
19            siz[u] += cnt;
20        }
21    }
22 }

```

```

21     }
22
23     int get_key_by_rank(int rank) {
24         int u = 1, key = 0;
25         for (int i = maxlog - 1; i >= 0; i--) {
26             if (siz[ch[u][0]] >= rank) {
27                 u = ch[u][0];
28             } else {
29                 key |= (1 << i);
30                 rank -= siz[ch[u][0]];
31                 u = ch[u][1];
32             }
33         }
34         return key;
35     }
36
37     int get_rank_by_key(int rank) {
38         int key = 0;
39         int u = 1;
40         for (int i = maxlog - 1; i >= 0; i--) {
41             if ((rank >> i) & 1) {
42                 key += siz[ch[u][0]];
43                 u = ch[u][1];
44             } else {
45                 u = ch[u][0];
46             }
47             if (!u) break;
48         }
49         return key;
50     }
51
52     int get_prev(int x) { return get_key_by_rank(get_rank_by_key(x)); }
53     int get_next(int x) { return get_key_by_rank(get_rank_by_key(x + 1) + 1); }
54 } treap;
55
56 const int num = 1e7;
57 int n, op, x;
58
59 int main() {
60     std::ios::sync_with_stdio(false);
61     std::cin.tie(0);
62     std::cout.tie(0);
63
64     std::cin >> n;
65     for (int i = 1; i <= n; i++) {
66         std::cin >> op >> x;
67         if (op == 1) {
68             treap.merge(x + num, 1);
69         } else if (op == 2) {
70             treap.merge(x + num, -1);
71         } else if (op == 3) {
72             std::cout << treap.get_rank_by_key(x + num) + 1 << '\n';
73         } else if (op == 4) {
74             std::cout << treap.get_key_by_rank(x) - num << '\n';
75         } else if (op == 5) {
76             std::cout << treap.get_prev(x + num) - num << '\n';
77         } else if (op == 6) {
78             std::cout << treap.get_next(x + num) - num << '\n';
79         }
80     }
81     return 0;
82 }

```

4.11 splay

文艺平衡树

初始为 1 到 n 的序列, m 次操作, 每次将序列下标为 $[l \sim r]$ 的区间翻转.

```

1 // 洛谷 P3391 【模板】文艺平衡树
2
3 struct node {
4     int ch[2], fa, key;
5     int siz, flag;
6
7     void init(int _fa, int _key) { fa = _fa, key = _key, siz = 1; }
8 };
9
10 struct splay {
11     node tr[N];
12     int n, root, idx;

```

```

13
14 bool get(int u) { return u == tr[tr[u].fa].ch[1]; }
15
16 void pushup(int u) { tr[u].siz = tr[tr[u].ch[0]].siz + tr[tr[u].ch[1]].siz + 1; }
17
18 void pushdown(int u) {
19     if (tr[u].flag) {
20         std::swap(tr[u].ch[0], tr[u].ch[1]);
21         tr[tr[u].ch[0]].flag ^= 1, tr[tr[u].ch[1]].flag ^= 1;
22         tr[u].flag = 0;
23     }
24 }
25
26 void rotate(int x) {
27     int y = tr[x].fa, z = tr[y].fa;
28     int op = get(x);
29     tr[y].ch[op] = tr[x].ch[op ^ 1];
30     if (tr[x].ch[op ^ 1]) tr[tr[x].ch[op ^ 1]].fa = y;
31     tr[x].ch[op ^ 1] = y;
32     tr[y].fa = x, tr[x].fa = z;
33     if (z) tr[z].ch[y == tr[z].ch[1]] = x;
34     pushup(y), pushup(x);
35 }
36
37 void opt(int u, int k) {
38     for (int f = tr[u].fa; f = tr[u].fa, f != k; rotate(u)) {
39         if (tr[f].fa != k) rotate(get(u) == get(f) ? f : u);
40     }
41     if (k == 0) root = u;
42 }
43
44 void output(int u) {
45     pushdown(u);
46     if (tr[u].ch[0]) output(tr[u].ch[0]);
47     if (tr[u].key >= 1 && tr[u].key <= n) {
48         std::cout << tr[u].key << ' ';
49     }
50     if (tr[u].ch[1]) output(tr[u].ch[1]);
51 }
52
53 void insert(int key) {
54     idx++;
55     tr[idx].ch[0] = root;
56     tr[idx].init(0, key);
57     tr[root].fa = idx;
58     root = idx;
59     pushup(idx);
60 }
61
62 int kth(int k) {
63     int u = root;
64     while (1) {
65         pushdown(u);
66         if (tr[u].ch[0] && k <= tr[tr[u].ch[0]].siz) {
67             u = tr[u].ch[0];
68         } else {
69             k -= tr[tr[u].ch[0]].siz + 1;
70             if (k <= 0) {
71                 opt(u, 0);
72                 return u;
73             } else {
74                 u = tr[u].ch[1];
75             }
76         }
77     }
78 }
79
80 } splay;
81
82 int n, m, l, r;
83
84 int main() {
85     std::ios::sync_with_stdio(false);
86     std::cin.tie(0);
87     std::cout.tie(0);
88
89     std::cin >> n >> m;
90     splay.n = n;
91     splay.insert(-inf);
92     rep(i, 1, n) splay.insert(i);
93     splay.insert(inf);
94     rep(i, 1, m) {
95         std::cin >> l >> r;
96         l = splay.kth(l), r = splay.kth(r + 2);
97         splay.opt(l, 0), splay.opt(r, 1);
98         splay.tr[splay.tr[r].ch[0]].flag ^= 1;
99     }

```

```

100     splay.output(splay.root);
101
102     return 0;
103 }

```

普通平衡树

n 次操作, 操作分为如下 6 种:

1. 插入数 x 2. 删除数 x (若有多个相同的数, 只删除一个) 3. 查询数 x 的排名 (排名定义为小于 x 的数的个数 + 1) 4. 查询排名为 x 的数 5. 求 x 的前驱 (前驱定义为小于 x 的最大数) 6. 求 x 的后继 (后继定义为大于 x 的最小数)

```

1 // 洛谷 P3369 【模板】普通平衡树
2
3 struct node {
4     int ch[2], fa, key, siz, cnt;
5
6     void init(int _fa, int _key) { fa = _fa, key = _key, siz = cnt = 1; }
7
8     void clear() { ch[0] = ch[1] = fa = key = siz = cnt = 0; }
9 };
10
11 struct splay {
12     node tr[N];
13     int n, root, idx;
14
15     bool get(int u) { return u == tr[tr[u].fa].ch[1]; }
16
17     void pushup(int u) { tr[u].siz = tr[tr[u].ch[0]].siz + tr[tr[u].ch[1]].siz + tr[u].cnt; }
18
19     void rotate(int x) {
20         int y = tr[x].fa, z = tr[y].fa;
21         int op = get(x);
22         tr[y].ch[op] = tr[x].ch[op ^ 1];
23         if (tr[x].ch[op ^ 1]) tr[tr[x].ch[op ^ 1]].fa = y;
24         tr[x].ch[op ^ 1] = y;
25         tr[y].fa = x, tr[x].fa = z;
26         if (z) tr[z].ch[y == tr[z].ch[1]] = x;
27         pushup(y), pushup(x);
28     }
29
30     void opt(int u, int k) {
31         for (int f = tr[u].fa; f = tr[u].fa, f != k; rotate(u)) {
32             if (tr[f].fa != k) {
33                 rotate(get(u) == get(f) ? f : u);
34             }
35         }
36         if (k == 0) root = u;
37     }
38
39     void insert(int key) {
40         if (!root) {
41             idx++;
42             tr[idx].init(0, key);
43             root = idx;
44             return;
45         }
46         int u = root, f = 0;
47         while (1) {
48             if (tr[u].key == key) {
49                 tr[u].cnt++;
50                 pushup(u), pushup(f);
51                 opt(u, 0);
52                 break;
53             }
54             f = u, u = tr[u].ch[tr[u].key < key];
55             if (!u) {
56                 idx++;
57                 tr[idx].init(f, key);
58                 tr[f].ch[tr[f].key < key] = idx;
59                 pushup(idx), pushup(f);
60                 opt(idx, 0);
61                 break;
62             }
63         }
64     }
65
66     // 返回节点编号 //
67     int kth(int rank) {

```

```

68     int u = root;
69     while (1) {
70         if (tr[u].ch[0] && rank <= tr[tr[u].ch[0]].siz) {
71             u = tr[u].ch[0];
72         } else {
73             rank -= tr[tr[u].ch[0]].siz + tr[u].cnt;
74             if (rank <= 0) {
75                 opt(u, 0);
76                 return u;
77             } else {
78                 u = tr[u].ch[1];
79             }
80         }
81     }
82 }
83
84 // 返回排名 //
85 int nlt(int key) {
86     int rank = 0, u = root;
87     while (1) {
88         if (tr[u].key > key) {
89             u = tr[u].ch[0];
90         } else {
91             rank += tr[tr[u].ch[0]].siz;
92             if (tr[u].key == key) {
93                 opt(u, 0);
94                 return rank + 1;
95             }
96             rank += tr[u].cnt;
97             if (tr[u].ch[1]) {
98                 u = tr[u].ch[1];
99             } else {
100                 return rank + 1;
101             }
102         }
103     }
104 }
105
106 int get_prev(int key) { return kth(nlt(key) - 1); }
107
108 int get_next(int key) { return kth(nlt(key) + 1); }
109
110 void remove(int key) {
111     nlt(key);
112     if (tr[root].cnt > 1) {
113         tr[root].cnt--;
114         pushup(root);
115         return;
116     }
117     int u = root, l = get_prev(key);
118     tr[tr[u].ch[1]].fa = l;
119     tr[l].ch[1] = tr[u].ch[1];
120     tr[u].clear();
121     pushup(root);
122 }
123
124 void output(int u) {
125     if (tr[u].ch[0]) output(tr[u].ch[0]);
126     std::cout << tr[u].key << ' ';
127     if (tr[u].ch[1]) output(tr[u].ch[1]);
128 }
129
130 } splay;
131
132 int n, op, x;
133
134 int main() {
135     std::ios::sync_with_stdio(false);
136     std::cin.tie(0);
137     std::cout.tie(0);
138
139     splay.insert(-inf), splay.insert(inf);
140
141     std::cin >> n;
142     for (int i = 1; i <= n; i++) {
143         std::cin >> op >> x;
144         if (op == 1) {
145             splay.insert(x);
146         } else if (op == 2) {
147             splay.remove(x);
148         } else if (op == 3) {
149             std::cout << splay.nlt(x) - 1 << endl;
150         } else if (op == 4) {
151             std::cout << splay.tr[splay.kth(x + 1)].key << endl;
152         } else if (op == 5) {
153             std::cout << splay.tr[splay.get_prev(x)].key << endl;
154         } else if (op == 6) {

```

```

155     std::cout << splay.tr[splay.get_next(x)].key << endl;
156 }
157 }
158
159 return 0;
160 }

```

4.12 Link Cut Tree

```

1 struct LCT{
2     int v[N],r[N],f[N],s[N][2],st[N],tp;
3     void pu(int x){v[x]=a[x]^v[s[x][0]]^v[s[x][1]];}
4     void flp(int x){r[x]^=1,std::swap(s[x][0],s[x][1]);}
5     void pd(int x){if(r[x])flp(s[x][0]),flp(s[x][1]),r[x]=0;}
6     bool isrt(int x){return s[f[x]][0]!=x&&s[f[x]][1]!=x;}
7     void rtt(int x){
8         int y=f[x],z=f[y],k=(s[y][1]==x);if(!isrt(y)) s[z][y==s[z][1]]=x;
9         f[x]=z,f[y]=x,f[s[x][k^1]]=y,s[y][k]=s[x][k^1],s[x][k^1]=y,pu(y),pu(x);}
10    void spl(int x){
11        st[tp++]=x;for(int i=x;!isrt(i);i=f[i])st[tp++]=f[i];
12        while(tp)pd(st[--tp]);
13        while(!isrt(x)){
14            if(!isrt(f[x]))rtt((s[f[x]][0]==x)^(s[f[f[x]]][0]==f[x])?x:f[x]);
15            rtt(x);
16        }pu(x);
17    }
18    void acc(int x){for(int y=0;x;y=x,x=f[x]) spl(x),s[x][1]=y,pu(x);}
19    void mkrt(int x){acc(x),spl(x),flp(x);}
20    int fdrt(int x){acc(x),spl(x);while(s[x][0])x=s[x][0];spl(x);return x;}
21    void cut(int x,int y){mkrt(x);if(x==fdrt(y)&&f[y]==x&&!s[y][0])s[x][1]=f[y]=0,pu(x);}
22    void lk(int x,int y){mkrt(x);if(x!=fdrt(y))f[x]=y;}
23 }t;

```

4.13 ODT

```

1 struct T{
2     int l,r,v;
3     T(int a,int b=-1,int c=-1):l(a),r(b),v(c){}
4     bool operator<(const T&_){return l<_.l;}
5 };
6 set<T>s;
7 auto spl(int p){
8     auto it=s.lower_bound(p);
9     if(it!=end(s) && it->l==p)return it;
10    --it;
11    int l=it->l,r=it->r,v=it->v;
12    s.erase(it),s.insert(T(l,p-1,v));
13    return s.insert(T(p,r,v)).first;
14 }
15 void asgn(int l,int r,int v){
16     auto ed=spl(r+1),bg=spl(l);
17     s.erase(bg,ed);
18     auto i=s.insert(T(l,r,v)).first,j=prev(i);
19     if(i!=begin(s)&&j->v==v)l=j->l,s.erase(j);
20     if((j=next(i))!=end(s)&&j->v==v)r=j->r,s.erase(j);
21     s.erase(i),s.insert(T(l,r,v));
22 }

```

4.14 tree in tree

线段树套线段树

n 个三维数对 (a_i, b_i, c_i) , 设 $f(i)$ 表示 $a_j \leq a_i$ 且 $b_j \leq b_i$ 且 $c_j \leq c_i$ 且 $i \neq j$ 的个数. 输出 $f(i)$ ($0 \leq i \leq n-1$) 的值.

```

1 // 洛谷 P3810 【模板】三维偏序（陌上花开）
2
3 struct node1 {
4     int l, r, root;
5 } tr1[N << 2];

```



```

6
7 struct node2 {
8     int ch[2], cnt;
9 } tr2[N << 7];
10
11 struct node {
12     int x, y, z, cnt;
13
14     bool operator==(const node& a) { return (x == a.x && y == a.y && z == a.z); }
15 } data[N];
16
17 bool cmp(node a, node b) {
18     if (a.x != b.x) return a.x < b.x;
19     if (a.y != b.y) return a.y < b.y;
20     return a.z < b.z;
21 }
22
23 int root_tot, n, m, ans[N], anss[N];
24
25 void build(int u, int l, int r) {
26     tr1[u].l = l, tr1[u].r = r;
27     if (l != r) {
28         int mid = (l + r) >> 1;
29         build(u << 1, l, mid);
30         build(u << 1 | 1, mid + 1, r);
31     }
32 }
33
34 void modify_2(int& u, int l, int r, int pos) {
35     if (u == 0) u = ++root_tot;
36     tr2[u].cnt++;
37     if (l == r) return;
38     int mid = (l + r) >> 1;
39     if (pos <= mid) {
40         modify_2(tr2[u].ch[0], l, mid, pos);
41     } else {
42         modify_2(tr2[u].ch[1], mid + 1, r, pos);
43     }
44 }
45
46 int query_2(int& u, int l, int r, int x, int y) {
47     if (u == 0) return 0;
48     if (x <= l && r <= y) return tr2[u].cnt;
49     int mid = (l + r) >> 1, ans = 0;
50     if (x <= mid) ans += query_2(tr2[u].ch[0], l, mid, x, y);
51     if (mid < y) ans += query_2(tr2[u].ch[1], mid + 1, r, x, y);
52     return ans;
53 }
54
55 void modify_1(int u, int l, int r, int t) {
56     modify_2(tr1[u].root, 1, m, data[t].z);
57     if (l == r) return;
58     int mid = (l + r) >> 1;
59     if (data[t].y <= mid) {
60         modify_1(u << 1, l, mid, t);
61     } else {
62         modify_1(u << 1 | 1, mid + 1, r, t);
63     }
64 }
65
66 int query_1(int u, int l, int r, int t) {
67     if (1 <= l && r <= data[t].y) return query_2(tr1[u].root, 1, m, 1, data[t].z);
68     int mid = (l + r) >> 1, ans = 0;
69     if (1 <= mid) ans += query_1(u << 1, l, mid, t);
70     if (mid < data[t].y) ans += query_1(u << 1 | 1, mid + 1, r, t);
71     return ans;
72 }
73
74 int main() {
75     std::ios::sync_with_stdio(false);
76     std::cin.tie(0);
77     std::cout.tie(0);
78
79     std::cin >> n >> m;
80     rep(i, 1, n) {
81         int x, y, z;
82         std::cin >> x >> y >> z;
83         data[i] = {x, y, z};
84     }
85     std::sort(data + 1, data + n + 1, cmp);
86     build(1, 1, m);
87     rep(i, 1, n) {
88         modify_1(1, 1, m, i);
89         ans[i] = query_1(1, 1, m, i);
90     }
91     per(i, n - 1, 1) {

```

```

93 |         if (data[i] == data[i + 1]) ans[i] = ans[i + 1];
94 |     }
95 |     rep(i, 1, n) anss[ans[i]]++;
96 |     rep(i, 1, n) std::cout << anss[i] << endl;
97 |
98 |     return 0;
99 | }

```

线段树套平衡树

长度为 n 的序列和 m 此操作, 包含 5 种操作:

1. $l\ r\ k$: 询问区间 $[l \sim r]$ 中数 k 的排名.
2. $l\ r\ k$: 询问区间 $[l \sim r]$ 中排名为 k 的数.
3. $pos\ k$: 将序列中 pos 位置的数修改为 k .
4. $l\ r\ k$: 询问区间 $[l \sim r]$ 中数 k 的前驱.
5. $l\ r\ k$: 询问区间 $[l \sim r]$ 中数 k 的后继.

treap 实现

```

1 | // 洛谷 P3380 【模板】二逼平衡树（树套树）
2 |
3 | int n, m, op, l, r, pos, key, root_tot;
4 | int a[N];
5 |
6 | struct node2 {
7 |     node2 *ch[2];
8 |     int key, val;
9 |     int cnt, size;
10 |
11 |     node2(int _key) : key(_key), cnt(1), size(1) {
12 |         ch[0] = ch[1] = nullptr;
13 |         val = rand();
14 |     }
15 |
16 |     // node2(node2 *_node2) {
17 |     //     key = _node2->key, val = _node2->val, cnt = _node2->cnt, size = _node2->size;
18 |     // }
19 |
20 |     inline void push_up() {
21 |         size = cnt;
22 |         if (ch[0] != nullptr) size += ch[0]->size;
23 |         if (ch[1] != nullptr) size += ch[1]->size;
24 |     }
25 | };
26 |
27 | struct treap {
28 |     ...
29 | };
30 |
31 | treap tr2[N << 4];
32 |
33 | struct node1 {
34 |     int l, r, root;
35 | } tr1[N << 4];
36 |
37 | void build(int u, int l, int r) {
38 |     tr1[u] = {l, r, u};
39 |     root_tot = std::max(root_tot, u);
40 |     if (l == r) return;
41 |     int mid = (l + r) >> 1;
42 |     build(u << 1, l, mid), build(u << 1 | 1, mid + 1, r);
43 | }
44 |
45 | void modify(int u, int pos, int key) {
46 |     tr2[u].insert(key);
47 |     if (tr1[u].l == tr1[u].r) return;
48 |     int mid = (tr1[u].l + tr1[u].r) >> 1;
49 |     if (pos <= mid){
50 |         modify(u << 1, pos, key);
51 |     }
52 |     else{
53 |         modify(u << 1 | 1, pos, key);

```

```

54     }
55 }
56
57 int get_rank_by_key_in_interval(int u, int l, int r, int key) {
58     if (l <= tr1[u].l && tr1[u].r <= r) return tr2[u].get_rank_by_key(tr2[u].root, key) - 2;
59     int mid = (tr1[u].l + tr1[u].r) >> 1, ans = 0;
60     if (l <= mid) ans += get_rank_by_key_in_interval(u << 1, l, r, key);
61     if (mid < r) ans += get_rank_by_key_in_interval(u << 1 | 1, l, r, key);
62     return ans;
63 }
64
65 int get_key_by_rank_in_interval(int u, int l, int r, int rank) {
66     int L = 0, R = 1e8;
67     while (L < R) {
68         int mid = (L + R + 1) / 2;
69         if (get_rank_by_key_in_interval(1, l, r, mid) < rank){
70             L = mid;
71         }
72         else{
73             R = mid - 1;
74         }
75     }
76     return L;
77 }
78
79 void change(int u, int pos, int pre_key, int key) {
80     tr2[u].remove(pre_key);
81     tr2[u].insert(key);
82     if (tr1[u].l == tr1[u].r) return;
83     int mid = (tr1[u].l + tr1[u].r) >> 1;
84     if (pos <= mid){
85         change(u << 1, pos, pre_key, key);
86     }
87     else{
88         change(u << 1 | 1, pos, pre_key, key);
89     }
90 }
91
92 int get_prev_in_interval(int u, int l, int r, int key) {
93     if (l <= tr1[u].l && tr1[u].r <= r) return tr2[u].get_prev(key);
94     int mid = (tr1[u].l + tr1[u].r) >> 1, ans = -inf;
95     if (l <= mid) ans = std::max(ans, get_prev_in_interval(u << 1, l, r, key));
96     if (mid < r) ans = std::max(ans, get_prev_in_interval(u << 1 | 1, l, r, key));
97     return ans;
98 }
99
100 int get_nex_in_interval(int u, int l, int r, int key) {
101     if (l <= tr1[u].l && tr1[u].r <= r) return tr2[u].get_nex(key);
102     int mid = (tr1[u].l + tr1[u].r) >> 1, ans = inf;
103     if (l <= mid) ans = std::min(ans, get_nex_in_interval(u << 1, l, r, key));
104     if (mid < r) ans = std::min(ans, get_nex_in_interval(u << 1 | 1, l, r, key));
105     return ans;
106 }
107
108 int main() {
109     std::ios::sync_with_stdio(false);
110     std::cin.tie(0);
111     std::cout.tie(0);
112
113     srand(time(0));
114
115     std::cin >> n >> m;
116     build(1, 1, n);
117     rep(i, 1, n) {
118         std::cin >> a[i];
119         modify(1, i, a[i]);
120     }
121     rep(i, 1, root_tot) { tr2[i].insert(inf), tr2[i].insert(-inf); }
122     rep(i, 1, m) {
123         std::cin >> op;
124         if (op == 1) {
125             std::cin >> l >> r >> key;
126             std::cout << get_rank_by_key_in_interval(1, l, r, key) + 1 << endl;
127         } else if (op == 2) {
128             std::cin >> l >> r >> key;
129             std::cout << get_key_by_rank_in_interval(1, l, r, key) << endl;
130         } else if (op == 3) {
131             std::cin >> pos >> key;
132             change(1, pos, a[pos], key);
133             a[pos] = key;
134         } else if (op == 4) {
135             std::cin >> l >> r >> key;
136             std::cout << get_prev_in_interval(1, l, r, key) << endl;
137         } else if (op == 5) {
138             std::cin >> l >> r >> key;
139             std::cout << get_nex_in_interval(1, l, r, key) << endl;
140         }

```

```

141     }
142
143     return 0;
144 }

```

然而洛谷上的会 T 两个点, Loj 和 ACwing 上的能过.

Splay 实现

```

1 // 洛谷 P3380 【模板】二逼平衡树（树套树）
2
3 int n, m, op, l, r, pos, key, root_tot;
4 int a[N];
5
6 struct node{
7     int ch[2], fa, key, siz, cnt;
8
9     void init(int _fa, int _key){
10         fa = _fa, key = _key, siz = cnt = 1;
11     }
12
13     void clear(){
14         ch[0] = ch[1] = fa = key = siz = cnt = 0;
15     }
16 }tr[N * 30];
17
18 struct splay{
19
20     int idx;
21
22     bool get(int u){
23         return u == tr[tr[u].fa].ch[1];
24     }
25
26     void pushup(int u){
27         tr[u].siz = tr[tr[u].ch[0]].siz + tr[tr[u].ch[1]].siz + tr[u].cnt;
28     }
29
30     void rotate(int x){
31         int y = tr[x].fa, z = tr[y].fa;
32         int op = get(x);
33         tr[y].ch[op] = tr[x].ch[op ^ 1];
34         if(tr[x].ch[op ^ 1]) tr[tr[x].ch[op ^ 1]].fa = y;
35         tr[x].ch[op ^ 1] = y;
36         tr[y].fa = x, tr[x].fa = z;
37         if(z) tr[z].ch[y == tr[z].ch[1]] = x;
38         pushup(y), pushup(x);
39     }
40
41     void opt(int& root, int u, int k){
42         for(int f = tr[u].fa; f = tr[u].fa, f != k; rotate(u)){
43             if(tr[f].fa != k) rotate(get(u) == get(f) ? f : u);
44         }
45         if(k == 0) root = u;
46     }
47
48     void insert(int& root, int key){
49         if(tr[root].siz == 0){
50             idx++;
51             tr[idx].init(0, key);
52             root = idx;
53             return;
54         }
55         int u = root, f = 0;
56         while(1){
57             if(tr[u].key == key){
58                 tr[u].cnt++;
59                 pushup(u), pushup(f);
60                 opt(root, u, 0);
61                 break;
62             }
63             f = u, u = tr[u].ch[tr[u].key < key];
64             if(!u){
65                 idx++;
66                 tr[idx].init(f, key);
67                 tr[f].ch[tr[f].key < key] = idx;
68                 pushup(idx), pushup(f);
69                 opt(root, idx, 0);
70                 break;
71             }
72         }
73     }
74
75     int kth(int& root, int rank){
76         int u = root;

```

```

77     while(1){
78         if(tr[u].ch[0] && rank <= tr[tr[u].ch[0]].siz) u = tr[u].ch[0];
79         else{
80             rank -= tr[tr[u].ch[0]].siz + tr[u].cnt;
81             if(rank <= 0){
82                 opt(root, u, 0);
83                 return u;
84             }
85             else u = tr[u].ch[1];
86         }
87     }
88 }
89
90 int nlt(int& root, int key){
91     int rank = 0, u = root;
92     while(1){
93         if(tr[u].key > key) u = tr[u].ch[0];
94         else{
95             rank += tr[tr[u].ch[0]].siz;
96             if(tr[u].key == key){
97                 opt(root, u, 0);
98                 return rank + 1;
99             }
100             rank += tr[u].cnt;
101             if(tr[u].ch[1]) u = tr[u].ch[1];
102             else return rank + 1;
103         }
104     }
105 }
106
107 int get_prev(int& root, int key){
108     return kth(root, nlt(root, key) - 1);
109 }
110
111 int get_next(int& root, int key){
112     return kth(root, nlt(root, key) + 1);
113 }
114
115 void remove(int& root, int key){
116     nlt(root, key);
117     if(tr[root].cnt > 1){
118         tr[root].cnt--;
119         pushup(root);
120         return;
121     }
122     int u = root, l = get_prev(root, key);
123     tr[tr[u].ch[1]].fa = l;
124     tr[l].ch[1] = tr[u].ch[1];
125     tr[u].clear();
126     pushup(root);
127 }
128
129 void output(int u){
130     if(tr[u].ch[0]) output(tr[u].ch[0]);
131     std::cout << tr[u].key << ' ';
132     if(tr[u].ch[1]) output(tr[u].ch[1]);
133 }
134
135 }splay;
136
137 struct node1{
138     int l, r, root;
139 }tr1[N * 4];
140
141 void build(int u, int l, int r){
142     tr1[u] = {l, r, u};
143     root_tot = splay.idx = std::max(root_tot, u);
144     if(l == r) return;
145     int mid = (l + r) >> 1;
146     build(u << 1, l, mid), build(u << 1 | 1, mid + 1, r);
147 }
148
149 void modify(int u, int pos, int key){
150     splay.insert(tr1[u].root, key);
151     if(tr1[u].l == tr1[u].r) return;
152     int mid = (tr1[u].l + tr1[u].r) >> 1;
153     if(pos <= mid) modify(u << 1, pos, key);
154     else modify(u << 1 | 1, pos, key);
155 }
156
157 int get_rank_by_key_in_interval(int u, int l, int r, int key){
158     if(l <= tr1[u].l && tr1[u].r <= r)
159         return splay.nlt(tr1[u].root, key) - 2;
160     int mid = (tr1[u].l + tr1[u].r) >> 1, ans = 0;
161     if(l <= mid) ans += get_rank_by_key_in_interval(u << 1, l, r, key);
162     if(mid < r) ans += get_rank_by_key_in_interval(u << 1 | 1, l, r, key);
163     return ans;

```

```

164 }
165
166 int get_key_by_rank_in_interval(int u, int l, int r, int rank){
167     int L = 0, R = 1e8;
168     while(L < R){
169         int mid = (L + R + 1) / 2;
170         if(get_rank_by_key_in_interval(1, l, r, mid) < rank) L = mid;
171         else R = mid - 1;
172     }
173     return L;
174 }
175
176 void change(int u, int pos, int pre_key, int key){
177     splay.remove(tr1[u].root, pre_key);
178     splay.insert(tr1[u].root, key);
179     if(tr1[u].l == tr1[u].r) return;
180     int mid = (tr1[u].l + tr1[u].r) >> 1;
181     if(pos <= mid) change(u << 1, pos, pre_key, key);
182     else change(u << 1 | 1, pos, pre_key, key);
183 }
184
185 int get_prev_in_interval(int u, int l, int r, int key){
186     if(l <= tr1[u].l && tr1[u].r <= r)
187         return tr[splay.get_prev(tr1[u].root, key)].key;
188     int mid = (tr1[u].l + tr1[u].r) >> 1, ans = -inf;
189     if(l <= mid) ans = std::max(ans, get_prev_in_interval(u << 1, l, r, key));
190     if(mid < r) ans = std::max(ans, get_prev_in_interval(u << 1 | 1, l, r, key));
191     return ans;
192 }
193
194
195 int get_next_in_interval(int u, int l, int r, int key){
196     if(l <= tr1[u].l && tr1[u].r <= r)
197         return tr[splay.get_next(tr1[u].root, key)].key;
198     int mid = (tr1[u].l + tr1[u].r) >> 1, ans = inf;
199     if(l <= mid) ans = std::min(ans, get_next_in_interval(u << 1, l, r, key));
200     if(mid < r) ans = std::min(ans, get_next_in_interval(u << 1 | 1, l, r, key));
201     return ans;
202 }
203
204 int main(){
205     std::ios::sync_with_stdio(false);
206     std::cin.tie(0);
207     std::cout.tie(0);
208
209     srand(time(0));
210
211     std::cin >> n >> m;
212     build(1, 1, n);
213     rep(i, 1, n){
214         std::cin >> a[i];
215         modify(1, i, a[i]);
216     }
217     rep(i, 1, root_tot){
218         splay.insert(tr1[i].root, inf), splay.insert(tr1[i].root, -inf);
219     }
220     rep(i, 1, m){
221         std::cin >> op;
222         if(op == 1){
223             std::cin >> l >> r >> key;
224             std::cout << get_rank_by_key_in_interval(1, l, r, key) + 1 << endl;
225         }
226         else if(op == 2){
227             std::cin >> l >> r >> key;
228             std::cout << get_key_by_rank_in_interval(1, l, r, key) << endl;
229         }
230         else if(op == 3){
231             std::cin >> pos >> key;
232             change(1, pos, a[pos], key);
233             a[pos] = key;
234         }
235         else if(op == 4){
236             std::cin >> l >> r >> key;
237             std::cout << get_prev_in_interval(1, l, r, key) << endl;
238         }
239         else if(op == 5){
240             std::cin >> l >> r >> key;
241             std::cout << get_next_in_interval(1, l, r, key) << endl;
242         }
243     }
244 }
245
246 return 0;
247 }

```

然而洛谷, ACwing 能过, Loj T 一堆。

4.15 heap

左偏树

```

1 struct leftist{
2     int d[N],lc[N],rc[N],rt[N],val[N],vis[N];
3     int find(int x){return rt[x]=rt[x]==x?x:find(rt[x]);}
4     int merge(int x,int y){
5         if(!x || !y)return x|y;
6         if(val[x]>val[y] || (val[x]==val[y] && x>y))swap(x,y);
7         rc[x]=merge(rc[x],y);
8         if(d[lc[x]]<d[rc[x]])swap(lc[x],rc[x]);
9         d[x]=d[rc[x]]+1;
10        return x;
11    }
12    void onion(int x,int y){
13        if(vis[x]||vis[y])return;
14        x=find(x),y=find(y);
15        if(x!=y)rt[x]=rt[y]=merge(x,y);
16    }
17    int ask(int x){
18        if(vis[x])return -1;
19        x=find(x);
20        vis[x]=1;
21        rt[lc[x]]=rt[rc[x]]=rt[x]=merge(lc[x],rc[x]);
22        lc[x]=rc[x]=d[x]=0;
23        return val[x];
24    }
25    void init(int* a){
26        d[0]=-1;
27        for(int i=1;i<=n;i++)val[i]=a[i];
28        for(int i=1;i<=n;i++)rt[i]=i;
29    }
30 }t;

```

5 string

5.1 hash

```

1 const int B1 = 3937;
2 const int B2 = 13331;
3 const int P = 1e9 + 7;
4
5 auto preHash = [&](const std::string& s, int B) {
6     int n = s.size();
7     std::vector<int> h(n + 1), p(n + 1);
8     p[0] = 1;
9     for (int i = 1; i <= n; ++i) {
10         p[i] = 111 * B * p[i - 1] % P;
11         h[i] = (111 * B * h[i - 1] + s[i - 1]) % P;
12     }
13     return std::make_pair(h, p);
14 };
15 auto f = [&](const auto& o, int l, int r){
16     auto& [h, p] = o;
17     return (h[r - 1] - 111 * p[r - 1 + 1] * h[l] % P + P) % P;
18     /*
19     若采用 1 下标
20     return (h[r] - 111 * p[r - 1 + 1] * h[l - 1] % P + P) % P;
21     */
22 };
23
24 auto _s = std::make_pair(preHash(s, B1), preHash(s, B2));
25 auto _t = std::make_pair(preHash(t, B1), preHash(t, B2));
26 auto check = [&](const auto& _s, int l1, int r1, const auto& _t, int l2, int r2) {
27     return getHashV(_s.first, l1, r1) == getHashV(_t.first, l2, r2)
28         and getHashV(_s.second, l1, r1) == getHashV(_t.second, l2, r2);
29 };

```

5.2 Cantor Expansion

一个排列 a_1, a_2, \dots, a_n 的排名为

$$\sum_{i=1}^n \left((n-i)! \sum_{j=i}^n [a_j < a_i] \right)$$

```

1  std::cin >> n, fac[0] = 1;
2  for (int i = 1; i <= n; ++i) {
3      std::cin >> a[i];
4      fac[i] = 1ll * fac[i - 1] * i % P;
5  }
6  auto ins = [&](int x) {
7      for (; x <= n; x += x & -x) ++t[x];
8  };
9  auto ask = [&](int x) {
10     int z = 0;
11     for (; x; x ^= x & -x) z += t[x];
12     return z;
13 };
14 int z = 0;
15 for (int i = n; i; --i) {
16     z = (z + 1ll * fac[n - i] * ask(a[i])) % P;
17     ins(a[i]);
18 }
19 std::cout << ++z << '\n';

```

5.3 kmp

```

1  auto get_next = [&](const std::string& s) -> vi {
2      int n = s.length();
3      vi next(n);
4      for (int i = 1; i < n; i++) {
5          int j = next[i - 1];
6          while (j > 0 and s[i] != s[j]) j = next[j - 1];
7          if (s[i] == s[j]) j++;
8          next[i] = j;
9      }
10     return next;
11 };

```

```

1  char s[N];
2  int f[N][20], d[N];
3  void work(){
4      int n,i,j=0,q,x,y;
5      for(cin>>s+1,n=strlen(s+1),i=2;i<=n;f[i][0]=j,d[i]=d[j]+1,++i){
6          while(j&&s[i]!=s[j+1])j=f[j][0];
7          if(s[i]==s[j+1])++j;
8      }
9      for(j=1;j<20;++j)for(i=1;i<=n;++i)f[i][j]=f[f[i][j-1]][j-1];
10     for(cin>>q;q--;cout<<f[x][0]<<'\n'){
11         if(cin>>x>>y,d[x]<d[y])swap(x,y);
12         for(i=19;~i;--i)if(d[f[x][i]]>=d[y])x=f[x][i];
13         for(i=19;~i;--i)if(f[x][i]!=f[y][i])x=f[x][i],y=f[y][i];
14     }
15 }

```

5.4 z function

```

1  auto z_function = [&](const std::string& s) -> vi {
2      int n = s.size();
3      vi z(n);
4      for (int i = 1, l = 0, r = 0; i < n; i++) {
5          if (i <= r and z[i - l] < r - i + 1) {
6              z[i] = z[i - l];
7          } else {
8              z[i] = std::max(0, r - i + 1);
9              while (z[i] + i < n and s[z[i]] == s[z[i] + i]) z[i]++;
10             }
11             if (z[i] + i - 1 > r) {
12                 l = i;
13                 r = z[i] + i - 1;
14             }
15         }
16     }

```



```

15     }
16     return z;
17 };

```

5.5 Manacher

$p[i] - 1$ 为无 # 的回文直径, $p[i] / 2$ 为无 # 的回文半径.

```

1 auto Manacher = [&](const std::string& t) {
2     std::string s = "#";
3     for (char c : t) s += c, s += '#';
4     int i, o = 0, r = 0, n = s.size();
5     std::vector<int> p(n, 1), q(n);
6     for (i = 0; i < n; ++i) {
7         if (i <= r) p[i] = std::min(r - i + 1, p[2 * o - i]);
8         for (; p[i] <= i && s[i + p[i]] == s[i - p[i]]; ++p[i]);
9         if (i + p[i] - 1 > r) r = i + p[i] - 1, o = i;
10    }
11    return p;
12 };

```

5.6 AC automaton

$fail[i]$ 表示结点 i 的最长真 border, $t[i][c]$ 不存在时跳 $fail$.

```

1 int p[N], t[N][26], f[N], q[N];
2 string s;
3 void work() {
4     int n, l = 1, r = 0, i, j, x, y, sz = 0;
5     for (cin >> n, i = 1; i <= n; p[i] = x, ++i)
6         for (cin >> s, x = j = 0; j < s.size(); x = t[x][y], ++j)
7             if (!t[x][y = s[j] - 97]) t[x][y] = ++sz;
8     for (i = 0; i < 26; ++i) if (x = t[0][i]) q[++r] = x;
9     while (l <= r) for (x = q[l++], i = 0; i < 26; ++i)
10        if (int&v = t[x][i]) f[q[++r] = v] = t[f[x]][i]; else v = t[f[x]][i];
11 }

```

5.7 PAM

```

1 template<const int M = 26>
2 struct PAM {
3     struct T {
4         int len, d, fa, ch[M];
5         T() : len(), d(), fa(), ch() {}
6     };
7     int las;
8     string s;
9     vector<T> t;
10    vector<int> bl;
11    size_t count() const {
12        return t.size() - 2;
13    }
14    const T& operator[] (const size_t& p) const {
15        return t[p];
16    }
17    const T& ask(const size_t& p) const {
18        return t[bl[p]];
19    }
20    int gf(int o, int p) {
21        while (p - t[o].len - 1 < 0 || s[p - t[o].len - 1] != s[p]) o = t[o].fa;
22        return o;
23    }
24    void append(int c) {
25        int p = s.size(), o;
26        s += c, o = gf(las, p);
27        if (t[o].ch[c] == 0) {
28            t.emplace_back();
29            t.back().len = t[o].len + 2;
30            t.back().fa = t[gf(t[o].fa, p)].ch[c];
31            t.back().d = t[t.back().fa].d + 1;
32            t[o].ch[c] = t.size() - 1;
33        }

```

```

34     bl.emplace_back(las = t[o].ch[c]);
35 }
36 PAM() : las(), s(), t(2) {
37     t[0].fa = t[1].fa = 1, t[1].len = -1;
38 }
39 PAM(const string& str, int h) : las(), s(), t(2) {
40     t[0].fa = t[1].fa = 1, t[1].len = -1;
41     for (char c : str) append(c - h);
42 }
43 };

```

5.8 Suffix Array

```

1 auto SA = [](std::string s) {
2     int n = s.size(), m = 128, i, j, l;
3     std::vector<int> ct(m), sa(n), rk(n), h(n), a(n);
4     for (i = 0; i < n; ++i) ++ct[rk[i] = s[i]];
5     for (i = 1; i < m; ++i) ct[i] += ct[i - 1];
6     for (i = n - 1; ~i; --i) sa[--ct[rk[i]]] = i;
7     for (l = 1; l < n; l *= 2) {
8         for (j = 0, i = n - 1; i >= n - l; --i) a[j++] = i;
9         for (i = 0; i < n; ++i) if (sa[i] >= l) a[j++] = sa[i] - l;
10        ct = std::vector<int>(m);
11        for (i = 0; i < n; ++i) ++ct[rk[a[i]]];
12        for (i = 1; i < m; ++i) ct[i] += ct[i - 1];
13        for (i = n - 1; ~i; --i) sa[--ct[rk[a[i]]]] = a[i];
14        std::swap(rk, a), rk[sa[0]] = 0;
15        for (i = 1; i < n; ++i) {
16            rk[sa[i]] = rk[sa[i - 1]] + (a[sa[i]] != a[sa[i - 1]] || a[(sa[i] + l) % n] != a[(sa[i - 1] + l) % n]);
17        }
18        if ((m = rk[sa[n - 1]] + 1) == n) break;
19    }
20    for (i = j = 0; i + 1 < n; h[rk[i++]] = j) {
21        for (j ? --j : 0; s[i + j] == s[sa[rk[i] - 1] + j]; ++j);
22    }
23    sa.erase(sa.begin());
24    rk.erase(rk.begin());
25    h.erase(h.begin());
26    return make_tuple(sa, rk, h);
27 };
28 //h[i] : LCP(rk[i], rk[i - 1])

```

5.9 trie

普通字典树 (单词匹配)

```

1 int cnt;
2 std::vector<std::array<int, 26>> trie(n + 1);
3 vi exist(n + 1);
4
5 auto insert = [&](const std::string& s) -> void {
6     int p = 0;
7     for (const auto ch : s) {
8         int c = ch - 'a';
9         if (!trie[p][c]) trie[p][c] = ++cnt;
10        p = trie[p][c];
11    }
12    exist[p] = true;
13 };
14
15 auto find = [&](const string& s) -> bool {
16     int p = 0;
17     for (const auto ch : s) {
18         int c = ch - 'a';
19         if (!trie[p][c]) return false;
20        p = trie[p][c];
21    }
22    return exist[p];
23 };

```

01 字典树 (求最大异或值)

给定 n 个数, 取两个数进行异或运算, 求最大异或值.

```

1 // trie //
2 int cnt = 0;
3 std::vector<std::array<int, 2>> trie(N);
4
5 auto insert = [&](int x) -> void {
6     int p = 0;
7     for (int i = 30; i >= 0; i--) {
8         int c = (x >> i) & 1;
9         if (!trie[p][c]) trie[p][c] = ++cnt;
10        p = trie[p][c];
11    }
12 };
13
14 auto find = [&](int x) -> int {
15     int sum = 0, p = 0;
16     for (int i = 30; i >= 0; i--) {
17         int c = (x >> i) & 1;
18         if (trie[p][c ^ 1]) {
19             p = trie[p][c ^ 1];
20             sum += (1 << i);
21         } else {
22             p = trie[p][c];
23         }
24     }
25     return sum;
26 };

```

字典树合并

来自浙大城市学院 2023 校赛 E 题。

给定一棵根为 1 的树, 每个点的点权为 w_i . 一共 q 次询问, 每次给出一对 u, v , 询问以 v 为根的子树上的点与 u 的权值最大异或值.

```

1 int main() {
2     std::ios::sync_with_stdio(false);
3     std::cin.tie(0);
4     std::cout.tie(0);
5
6     int n, m;
7     std::cin >> n;
8     vi w(n + 1);
9     for (int i = 1; i <= n; i++) {
10        std::cin >> w[i];
11    }
12
13    vvi e(n + 1);
14    for (int i = 1; i < n; i++) {
15        int u, v;
16        std::cin >> u >> v;
17        e[u].push_back(v);
18        e[v].push_back(u);
19    }
20
21    /* 离线询问 */
22    std::cin >> m;
23    std::vector<vpi> q(n + 1);
24    vi ans(m + 1);
25    for (int i = 1; i <= m; i++) {
26        int u, v;
27        std::cin >> u >> v;
28        q[v].emplace_back(u, i);
29    }
30
31    /* 01 trie */
32    std::vector<std::array<int, 2>> tr(1);
33
34    auto new_node = [&]() -> int {
35        tr.emplace_back();
36        return tr.size() - 1;
37    };
38
39    vi id(n + 1);
40
41    auto insert = [&](int root, int x) {

```

```

42     int p = root;
43     for (int i = 29; i >= 0; i--) {
44         int c = x >> i & 1;
45         if (!tr[p][c]) tr[p][c] = new_node();
46         p = tr[p][c];
47     }
48 };
49
50 auto query = [&](int root, int x) -> int {
51     int ans = 0, p = root;
52     for (int i = 29; i >= 0; i--) {
53         int c = x >> i & 1;
54         if (tr[p][c ^ 1]) {
55             p = tr[p][c ^ 1];
56             ans += (1 << i);
57         } else {
58             p = tr[p][c];
59         }
60     }
61     return ans;
62 };
63
64 std::function<int(int, int)> merge = [&](int a, int b) -> int {
65     // b 的信息挪到 a 上 //
66     if (!a) return b;
67     if (!b) return a;
68     tr[a][0] = merge(tr[a][0], tr[b][0]);
69     tr[a][1] = merge(tr[a][1], tr[b][1]);
70     return a;
71 };
72
73 std::function<void(int, int)> dfs = [&](int u, int fa) {
74     id[u] = new_node();
75     insert(id[u], w[u]);
76     for (auto v : e[u]) {
77         if (v == fa) continue;
78         dfs(v, u);
79         id[u] = merge(id[u], id[v]);
80     }
81     for (auto [v, i] : q[u]) {
82         ans[i] = query(id[u], w[v]);
83     }
84 };
85 dfs(1, 0);
86
87 for (int i = 1; i <= m; i++) std::cout << ans[i] << endl;
88
89 return 0;
90 }

```

6 math - number theory

6.1 Eculid

欧几里得算法

```
1 std::gcd(a, b)
```

扩展欧几里得算法

```
1 auto exgcd = [&](LL a, LL b, LL& x, LL& y) {
2     LL x1 = 1, x2 = 0, x3 = 0, x4 = 1;
3     while (b != 0) {
4         LL c = a / b;
5         std::tie(x1, x2, x3, x4, a, b) =
6             std::make_tuple(x3, x4, x1 - x3 * c, x2 - x4 * c, b, a - b * c);
7     }
8     x = x1, y = x2;
9 };
```

```
1 auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
2     if (!b) {
3         x = 1, y = 0;
4         return;
5     }
6     self(self, b, a % b, y, x);
7     y -= a / b * x;
8 };
```

```
1 auto exgcd = [&](auto&& self, LL a, LL b, LL& x, LL& y) {
2     if (!b) {
3         x = 1, y = 0;
4         return a;
5     }
6     LL d = self(self, b, a % b, y, x);
7     y -= a / b * x;
8     return d;
9 };
```

类欧几里得算法

一般形式: 求 $f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor$

$f(a, b, c, n)$ 可以单独求.

$$f(a, b, c, n) = nm - f(c, c - b - 1, a, m - 1)$$

```
1 LL f(LL a, LL b, LL c, LL n) {
2     if (a == 0) return ((b / c) * (n + 1));
3     if (a >= c || b >= c)
4         return f(a % c, b % c, c, n) + (a / c) * n * (n + 1) / 2 + (b / c) * (n + 1);
5     LL m = (a * n + b) / c;
6     LL v = f(c, c - b - 1, a, m - 1);
7     return n * m - v;
8 }
```

更进一步, 求: $g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai+b}{c} \rfloor$ 以及 $h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai+b}{c} \rfloor^2$

直接记吧.

$$g(a, b, c, n) = \lfloor \frac{mn(n+1) - f(c, c-b-1, a, m-1) - h(c, c-b-1, a, m-1)}{2} \rfloor$$

$$h(a, b, c, n) = nm(m+1) - 2f(c, c-b-1, a, m-1) - 2g(c, c-b-1, a, m-1) - f(a, b, c, n)$$

```

1  const int inv2 = 499122177;
2  const int inv6 = 166374059;
3
4  LL f(LL a, LL b, LL c, LL n);
5  LL g(LL a, LL b, LL c, LL n);
6  LL h(LL a, LL b, LL c, LL n);
7
8  struct data {
9      LL f, g, h;
10 };
11
12 data calc(LL a, LL b, LL c, LL n) {
13     LL ac = a / c, bc = b / c, m = (a * n + b) / c, n1 = n + 1, n21 = n * 2 + 1;
14     data d;
15     if (a == 0) {
16         d.f = bc * n1 % mod;
17         d.g = bc * n % mod * n1 % mod * inv2 % mod;
18         d.h = bc * bc % mod * n1 % mod;
19         return d;
20     }
21     if (a >= c || b >= c) {
22         d.f = n * n1 % mod * inv2 % mod * ac % mod + bc * n1 % mod;
23         d.g =
24             ac * n % mod * n1 % mod * n21 % mod * inv6 % mod + bc * n % mod * n1 % mod * inv2 % mod;
25         d.h = ac * ac % mod * n % mod * n1 % mod * n21 % mod * inv6 % mod +
26             bc * bc % mod * n1 % mod + ac * bc % mod * n % mod * n1 % mod;
27         d.f %= mod, d.g %= mod, d.h %= mod;
28         data e = calc(a % c, b % c, c, n);
29         d.h += e.h + 2 * bc % mod * e.f % mod + 2 * ac % mod * e.g % mod;
30         d.g += e.g, d.f += e.f;
31         d.f %= mod, d.g %= mod, d.h %= mod;
32         return d;
33     }
34     data e = calc(c, c - b - 1, a, m - 1);
35     d.f = n * m % mod - e.f, d.f = (d.f % mod + mod) % mod;
36     d.g = m * n % mod * n1 % mod - e.h - e.f, d.g = (d.g * inv2 % mod + mod) % mod;
37     d.h = n * m % mod * (m + 1) % mod - 2 * e.g - 2 * e.f - d.f;
38     d.h = (d.h % mod + mod) % mod;
39     return d;
40 }

```

6.2 inverse

线性递推

$$a^{-1} \equiv -\lfloor \frac{p}{a} \rfloor \times (p \% a)^{-1}$$

```

1  vi inv(n + 1);
2  auto sieve_inv = [&](int n) {
3      inv[1] = 1;
4      for (int i = 2; i <= n; i++) {
5          inv[i] = 1ll * (p - p / i) * inv[p % i] % p;
6      }
7  };

```

求 n 个数的逆元

```

1  auto get_inv = [&](const vi& a) {
2      int n = a.size();
3      vi b(n), f(n), ivf(n);
4      f[0] = a[0];
5      for (int i = 1; i < n; i++) {
6          f[i] = 1ll * f[i - 1] * a[i] % p;
7      }
8      ivf.back() = quick_power(f.back(), p - 2, p);
9      for (int i = n - 1; i; i--) {
10         ivf[i - 1] = 1ll * ivf[i] * a[i] % p;
11     }
12     b[0] = ivf[0];
13     for (int i = 1; i < n; i++) {
14         b[i] = 1ll * ivf[i] * f[i - 1] % p;
15     }
16     return b;
17 };

```

6.3 sieve

素数

```

1 vi prime, is_prime(n + 1, 1);
2 auto Euler_sieve = [&](int n){
3     for (int i = 2; i <= n; i++) {
4         if (is_prime[i]) prime.push_back(i);
5         for (auto p : prime) {
6             if (i * p > n) break;
7             is_prime[i * p] = 0;
8             if (i % p == 0) break;
9         }
10    }
11 };

```

欧拉函数

```

1 vi phi(n + 1), prime;
2 vi is_prime(n + 1, 1);
3 auto get_phi = [&](int n) {
4     int cnt = 0;
5     phi[1] = 1;
6     for (int i = 2; i <= n; i++) {
7         if (is_prime[i]) {
8             prime.push_back(i);
9             phi[i] = i - 1;
10        }
11        for (auto p : prime) {
12            if (i * p > n) break;
13            is_prime[i * p] = 0;
14            if (i % p) {
15                phi[i * p] = phi[i] * phi[p];
16            } else {
17                phi[i * p] = phi[i] * p;
18                break;
19            }
20        }
21    }
22 };

```

约数和

$$d(n) = \sum_{k|n} k$$

```

1 vi g(n + 1), d(n + 1), prime;
2 vi is_prime(n + 1, 1);
3 auto get_d = [&](int n) {
4     int tot = 0;
5     g[1] = d[1] = 1;
6     for (int i = 2; i <= n; i++) {
7         if (is_prime[i]) {
8             prime.push_back(i);
9             d[i] = g[i] = i + 1;
10        }
11        for (auto p : prime) {
12            if (i * p > n) break;
13            is_prime[i * p] = 0;
14            if (i % p == 0) {
15                g[i * p] = g[i] * p + 1;
16                d[i * p] = d[i] / g[i] * g[i * p];
17                break;
18            } else {
19                d[i * p] = d[i] * d[p];
20                g[i * p] = 1 + p;
21            }
22        }
23    }
24 };

```

莫比乌斯函数

```

1 vi mu(n + 1), prime;
2 vi is_prime(n + 1, 1);
3 auto get_mu = [&](int n) {
4     mu[1] = 1;
5     for (int i = 2; i <= n; i++) {
6         if (is_prime[i]) {
7             prime.push_back(i);
8             mu[i] = -1;
9         }
10        for (auto p : prime) {
11            if (i * p > n) break;
12            is_prime[i * p] = 0;
13            if (i % p == 0) {
14                mu[i * p] = 0;
15                break;
16            }
17            mu[i * p] = -mu[i];
18        }
19    }
20 };

```

6.4 杜教筛

$$g(1)S(n) = \sum_{i=1}^n (f * g)(i) - \sum_{i=2}^n g(i)S\left(\left\lfloor \frac{n}{i} \right\rfloor\right).$$

```

1 ll calc_sum_mu(int n) {
2     if (n < N) return S_mu[n];
3     if (mp_mu[n] != 0) return mp_mu[n];
4     ll ans = 0;
5     for (ll l = 2, r; l <= n; l = r + 1) {
6         r = n / (n / l);
7         ans = ans + (r - l + 1) * calc_sum_mu(n / l);
8     }
9     return mp_mu[n] = 1 - ans;
10 }

```

6.5 extended Euler theorem

当 $\gcd(a, p) \neq 1$ 时, 有

$$a^b \equiv \begin{cases} a^b & b \leq \varphi(p) \\ a^{b \bmod \varphi(p) + \varphi(p)} & b > \varphi(p) \end{cases} \pmod{p}$$

6.6 block

分块的逻辑

下取整 $\lfloor \frac{n}{g} \rfloor = k$ 的分块 ($g \leq n$)

```

1 for (int l = 1, r, k; l <= n; l = r + 1) {
2     k = n / l;
3     r = n / (n / l);
4     debug(l, r, k);
5 }

```

$k = \lfloor \frac{n}{g} \rfloor$ 从大到小遍历 $\lfloor \frac{n}{g} \rfloor$ 的所有取值, $[l, r]$ 对应的是 g 取值的区间.

```
1 // n = 11
```



```

2 [l, r, k] : 1 1 11
3 [l, r, k] : 2 2 5
4 [l, r, k] : 3 3 3
5 [l, r, k] : 4 5 2
6 [l, r, k] : 6 11 1

```

上取整 $\lceil \frac{n}{g} \rceil = k$ 的分块 ($g < n$)

```

1 for(int l = 1, r, k; l < n; l = r + 1){
2     k = (n + l - 1) / l;
3     r = (n + k - 2) / (k - 1) - 1;
4     debug(l, r, k);
5 }

```

$k = \lceil \frac{n}{g} \rceil$ 从大到小遍历 $\lceil \frac{n}{g} \rceil$ 的所有取值, $[l, r]$ 对应的是 g 取值的区间。

```

1 // n = 11
2 [l, r, k] : 1 1 11
3 [l, r, k] : 2 2 6
4 [l, r, k] : 3 3 4
5 [l, r, k] : 4 5 3
6 [l, r, k] : 6 10 2

```

一般形式

$$\sum_{i=1}^n f(i) \lfloor \frac{n}{i} \rfloor$$

设 $s(i)$ 为 $f(i)$ 的前缀和。

```

1 for (int l = 1, r; l <= n; l = r + 1) {
2     r = n / (n / l);
3     ans += (s[r] - s[l - 1]) * (n / l);
4 }

```

$$\sum_{i=1}^n f(i) \lfloor \frac{a}{i} \rfloor \lfloor \frac{b}{i} \rfloor$$

```

1 for (int l = 1, r, r1, r2; l <= n; l = r + 1) {
2     if (a / l) {
3         r1 = a / (a / l);
4     } else {
5         r1 = n;
6     }
7     if (b / l) {
8         r2 = b / (b / l);
9     } else {
10        r2 = n;
11    }
12    r = min(min(r1, r2), n);
13    ans += (s[r] - s[l - 1]) * (a / l) * (b / l);
14 }

```

6.7 CRT & exCRT

求解

$$\begin{cases} N \equiv a_1 \pmod{m_1} \\ N \equiv a_2 \pmod{m_2} \\ \dots \\ N \equiv a_n \pmod{m_n} \end{cases}$$

$$\text{有 } N \equiv \sum_{i=1}^k a_i \times \text{inv}\left(\frac{M}{m_i}, m_i\right) \times \left(\frac{M}{m_i}\right) \pmod{M}$$

```

1 auto crt = [&](int n, const vi& a, const vi& m) -> LL{
2     LL ans = 0, M = 1;
3     for(int i = 1; i <= n; i++) M *= m[i];
4     for(int i = 1; i <= n; i++){
5         ans = (ans + a[i] * inv(M / m[i], m[i]) * (M / m[i])) % M;

```

```

6 |     }
7 |     return (ans % M + M) % M;
8 | };

```

扩展中国剩余定理

```

1 | auto excrt = [&](int n, const vi& a, const vi& m) -> LL{
2 |     LL A = a[1], M = m[1];
3 |     for (int i = 2; i <= n; i++) {
4 |         LL x, y, d = std::gcd(M, m[i]);
5 |         exgcd(M, m[i], x, y);
6 |         LL mod = M / d * m[i];
7 |         x = x * (a[i] - A) / d % (m[i] / d);
8 |         A = ((M * x + A) % mod + mod) % mod;
9 |         M = mod;
10 |    }
11 |    return A;
12 | };

```

6.8 BSGS & exBSGS

求解满足 $a^x \equiv b \pmod p$ 的 x

```

1 | /* return value = -1e18 means no solution */
2 | auto BSGS = [&](LL a, LL b, LL p) {
3 |     if (1 % p == b % p) return 0ll;
4 |     LL k = std::sqrt(p) + 1;
5 |     std::unordered_map<LL, LL> hash;
6 |     for (LL i = 0, j = b % p; i < k; i++) {
7 |         hash[j] = i;
8 |         j = j * a % p;
9 |     }
10 |    LL ak = 1;
11 |    for (int i = 1; i <= k; i++) ak = ak * a % p;
12 |    for (int i = 1, j = ak; i <= k; i++) {
13 |        if (hash.count(j)) return 1ll * i * k - hash[j];
14 |        j = 1ll * j * ak % p;
15 |    }
16 |    return -INF;
17 | };

```

$(a, p) \neq 1$ 的情形

```

1 | /* return value < 0 means no solution */
2 | auto exBSGS = [&](auto&& self, LL a, LL b, LL p) {
3 |     b = (b % p + p) % p;
4 |     if (1ll % p == b % p) return 0ll;
5 |     LL x, y, d = std::gcd(a, p);
6 |     exgcd(exgcd, a, p, x, y);
7 |     if (d > 1) {
8 |         if (b % d != 0) return -INF;
9 |         exgcd(exgcd, a / d, p / d, x, y);
10 |        return self(self, a, b / d * x % (p / d), p / d) + 1;
11 |    }
12 |    return BSGS(a, b, p);
13 | };

```

6.9 Miller Rabin

原理基于：对奇素数 p , $a^2 \equiv 1 \pmod p$ 的解为 $x \equiv 1 \pmod p$ 或 $x \equiv p - 1 \pmod p$, 以及费马小定理.

随机一个底数 x , 将 $a^{p-1} \pmod p$ 的指数 $p-1$ 分解为 $a \times 2^b$, 计算出 x^a , 之后进行最多 b 次平方操作, 若发现非平凡平方根时即可判断出其不是素数, 否则通过此轮测试.

$test_time$ 为测试次数, 建议设为不小于 8 的整数以保证正确率, 但也不宜过大, 否则会影响效率.

```

1 | auto miller_rabin = [&](LL n) -> bool {
2 |     if (n <= 3) return n == 2 || n == 3;
3 |     LL a = n - 1, b = 0;
4 |     while (!(a & 1)) a >>= 1, b++;
5 |     for (int i = 1, j; i <= 10; i++) { /* test time = 10 */
6 |         LL x = rand() % (n - 2) + 2, v = quick_power(x, a, n);

```

```

7         if (v == 1 || v == n - 1) continue;
8         for (j = 0; j < b; j++) {
9             if (v == n - 1) break;
10            v = (i28) v * v % n;
11        }
12        if (j >= b) return false;
13    }
14    return true;
15 };

```

事实上底数没必要随机 10 次, 检验如下数即可. 快速幂记得要 i128.

1. int 范围: 2, 7, 61.

2. LL 范围: 2, 325, 9375, 28178, 450775, 9780504, 1795265022.

```

1  vl vv = {2, 3, 5, 7, 11, 13, 17, 23, 29};
2  auto miller_rabin = [&](LL n) -> bool {
3      auto test = [&](LL n, int a) {
4          if (n == a) return true;
5          if (n % 2 == 0) return false;
6          LL d = (n - 1) >> __builtin_ctzll(n - 1);
7          LL r = quick_power(a, d, n);
8          while (d < n - 1 and r != 1 and r != n - 1) {
9              d <<= 1;
10             r = (i128) r * r % n;
11         }
12         return r == n - 1 or d & 1;
13     };
14     if (n == 2 or n == 3) return true;
15     for (auto p : vv) {
16         if (test(n, p) == 0) return false;
17     }
18     return true;
19 }

```

6.10 Pollard Rho

能在 $O(n^{\frac{1}{4}})$ 的时间复杂度随机出一个 n 的非平凡因数.

```

1  auto pollard_rho = [&](LL x) -> LL{
2      LL s = 0, t = 0, val = 1;
3      LL c = rand() % (x - 1) + 1;
4      for(int goal = 1;; goal <<= 1, s = t, val = 1){
5          for(int step = 1; step <= goal; step++){
6              t = ((i128) t * t + c) % x;
7              val = (i128) val * abs(t - s) % x;
8              if(step % 127 == 0){
9                  LL d = std::gcd(val, x);
10                 if(d > 1) return d;
11             }
12         }
13         LL d = std::gcd(val, x);
14         if(d > 1) return d;
15     }
16 };

```

利用 Miller Rabin 和 Pollard Rho 进行素因数分解

```

1  auto factorize = [&](LL a) -> vl{
2      vl ans, stk;
3      for (auto p : prime) {
4          if (p > 1000) break;
5          while (a % p == 0) {
6              ans.push_back(p);
7              a /= p;
8          }
9          if (a == 1) return ans;
10     }
11     /* 先筛小素数, 再跑 Pollard-Rho */
12     stk.push_back(a);
13     while (!stk.empty()) {
14         LL b = stk.back();
15         stk.pop_back();
16         if (miller_rabin(b)) {

```

```

17         ans.push_back(b);
18         continue;
19     }
20     LL c = b;
21     while (c >= b) c = pollard_rho(b);
22     stk.push_back(c);
23     stk.push_back(b / c);
24 }
25 return ans;
26 };

```

6.11 quadratic residu

Cipolla 算法

```

1 auto cipolla = [&](int x) {
2     std::srand(time(0));
3     auto check = [&](int x) -> bool { return pow(x, (mod - 1) / 2) == 1; };
4     if (!x) return 0;
5     if (!check(x)) return -1;
6     int a, b;
7     while (1) {
8         a = rand() % mod;
9         b = sub(mul(a, a), x);
10        if (!check(b)) break;
11    }
12    PII t = {a, 1};
13    PII ans = {1, 0};
14    auto mulp = [&](PII x, PII y) -> PII {
15        auto [x1, x2] = x;
16        auto [y1, y2] = y;
17        int c = add(mul(x1, y1), mul(x2, y2, b));
18        int d = add(mul(x1, y2), mul(x2, y1));
19        return {c, d};
20    };
21    for (int i = (mod + 1) / 2; i; i >>= 1) {
22        if (i & 1) ans = mulp(ans, t);
23        t = mulp(t, t);
24    }
25    return std::min(ans.ff, mod - ans.ff);
26 }

```

6.12 Lucas

卢卡斯定理

用于求大组合数, 并且模数是一个不大的素数.

$$\binom{n}{m} \bmod p = \binom{\lfloor n/p \rfloor}{\lfloor m/p \rfloor} \cdot \binom{n \bmod p}{m \bmod p} \bmod p$$

$\binom{n \bmod p}{m \bmod p}$ 可以直接计算, $\binom{\lfloor n/p \rfloor}{\lfloor m/p \rfloor}$ 可以继续使用卢卡斯计算.

递归至 $m = 0$ 的时候, 返回 1.

p 不太大, 一般在 10^5 左右.

```

1 auto C = [&](LL n, LL m, LL p) -> LL {
2     if (n < m) return 0;
3     if (m == 0) return 1;
4     return fac[n] * inv_fac[m] % p * inv_fac[n - m] % p;
5 };
6
7 auto lucas = [&](auto&& self, LL n, LL m, LL p) -> LL {
8     if (n < m) return 0;
9     if (m == 0) return 1;
10    return C(n % p, m % p, p) * self(self, n / p, m / p, p) % p;
11 }

```

素数在组合数中的次数

Legengre 给出一种 $n!$ 中素数 p 的幂次的计算方式为:

$$\sum_{1 \leq j} \left\lfloor \frac{n}{p^j} \right\rfloor.$$

另一种计算方式利用 p 进制下各位数字和:

$$v_p(n!) = \frac{n - S_p(n)}{p - 1}.$$

则有

$$v_p(C_m^n) = \frac{S_p(n) + S_p(m - n) - S_p(m)}{p - 1}.$$

扩展卢卡斯定理

计算

$$\binom{n}{m} \bmod p,$$

p 可能为合数.

第一部分: CRT.

原问题变成求

$$\left\{ \begin{array}{l} \binom{n}{m} \equiv a_1 \bmod p_1^{\alpha_1} \\ \binom{n}{m} \equiv a_2 \bmod p_2^{\alpha_2} \\ \dots \\ \binom{n}{m} \equiv a_k \bmod p_k^{\alpha_k} \end{array} \right.$$

在求出 a_i 之后就可以利用 CRT 求出答案.

第二部分: 移除分子分母中的素数

问题转换成求解

$$\binom{n}{m} \bmod q^k.$$

等价于

$$\frac{\frac{n!}{q^x}}{\frac{m!}{q^y} \frac{(n-m)!}{q^z}} q^{x-y-z} \bmod q^k,$$

其中 x 表示 $n!$ 中 q 的次数, y, z 同理.

第三部分: 威尔逊定理的推论

问题转换为求

$$\frac{n!}{q^x} \bmod q^k.$$

可以利用威尔逊定理的推论.

```
1 auto exLucas = [&](LL n, LL m, LL p) {
2     auto inv = [&](LL a, LL p) {
3         LL x, y;
```

```

4 |     exgcd(a, p, x, y);
5 |     return (x % p + p) % p;
6 | };
7 |
8 | auto func = [&](auto&& self, LL n, LL pi, LL pk) {
9 |     if (!n) return 1ll;
10 |     LL ans = 1;
11 |     for (LL i = 2; i <= pk; i++) {
12 |         if (i % pi) ans = ans * i % p;
13 |     }
14 |     ans = quick_power(ans, n / pk, pk);
15 |     for (LL i = 2; i <= n % pk; i++) {
16 |         if (i % pi) ans = ans * i % pk;
17 |     }
18 |     ans = ans * self(self, n / pi, pi, pk) % pk;
19 |     return ans;
20 | };
21 |
22 | auto multiLucas = [&](LL n, LL m, LL pi, LL pk) {
23 |     LL cnt = 0;
24 |     for (LL i = n; i; i /= pi) cnt += i / pi;
25 |     for (LL i = m; i; i /= pi) cnt -= i / pi;
26 |     for (LL i = n - m; i; i /= pi) cnt -= i / pi;
27 |     LL ans = quick_power(pi, cnt, pk) * func(func, n, pi, pk) % pk;
28 |     ans = ans * inv(func(func, m, pi, pk), pk) % pk;
29 |     ans = ans * inv(func(func, n - m, pi, pk), pk) % pk;
30 |     return ans;
31 | };
32 |
33 | auto crt = [&](const vl& a, const vl& m, int k) {
34 |     LL ans = 0;
35 |
36 |     for (int i = 0; i < k; i++) {
37 |         ans = (ans + a[i] * inv(p / m[i], m[i]) * (p / m[i])) % p;
38 |     }
39 |     return (ans % p + p) % p;
40 | };
41 |
42 | vl a, prime;
43 | LL pp = p;
44 | for (int i = 2; i * i <= pp; i++) {
45 |     if (pp % i) continue;
46 |     prime.push_back(i);
47 |     while (pp % i == 0) {
48 |         prime.back() *= i;
49 |         pp /= i;
50 |     }
51 |     a.push_back(multiLucas(n, m, i, prime.back()));
52 | }
53 | if (pp > 1) {
54 |     prime.push_back(pp);
55 |     a.push_back(multiLucas(n, m, pp, pp));
56 | }
57 | return crt(a, prime, a.size());
58 | };

```

6.13 Wilson

简单结论

对于素数 p 有

$$(p-1)! \equiv -1 \pmod{p}.$$

推论

令 $(n!)_p$ 表示不大于 n 且不被 p 整除的正整数的乘积.

特殊情形: n 为素数 p 时即为上述结论.

一般结论: 对素数 p 和正整数 q 有

$$((p^q)!)_p \equiv \pm 1 \pmod{p^q}.$$

详细定义:

$$((p^q)!)_p = \begin{cases} 1 & \text{if } p = 2 \text{ and } q \geq 3, \\ -1 & \text{other wise.} \end{cases}$$

更进一步的推论

6.14 LTE

将素数 p 在整数 n 中的个数记为 $v_p(n)$.

$$(n, p) = 1$$

对所有素数 p 和满足 $(n, p) = 1$ 的整数 n , 有

1. 若 $p \mid x - y$, 则有

$$v_p(x^n - y^n) = v_p(x - y).$$

2. 若 $p \mid x - y$, 则对奇数 n 有

$$v_p(x^n + y^n) = v_p(x + y).$$

p 是奇素数

对所有奇素数 p 有

1. 若 $p \mid x - y$, 则有

$$v_p(x^n - y^n) = v_p(x - y) + v_p(n).$$

2. 若 $p \mid x - y$, 则对奇数 n 有

$$v_p(x^n + y^n) = v_p(x + y) + v_p(n).$$

$$p = 2$$

对 $p = 2$ 且 $p \mid x - y$ 有

1. 对奇数 n 有

$$v_2(x^n - y^n) = v_2(x - y).$$

2. 对偶数 n 有

$$v_2(x^n - y^n) = v_2(x - y) + v_2(x + y) + v_2(n) - 1.$$

除此之外, 对上述 x, y, n , 若 $4 \mid x - y$, 有

1. $v_2(x + y) = 1$.

2. $v_2(x^n - y^n) = v_2(x - y) + v_2(n)$.

6.15 Mobius inversion

莫比乌斯函数

$$\mu(n) = \begin{cases} 1 & n = 1, \\ 0 & n \text{ 含有平方因子}, \\ (-1)^k & k \text{ 为 } n \text{ 的本质不同素因子个数}. \end{cases}$$

性质

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & n = 1 \\ 0 & n \neq 1 \end{cases}.$$

$$\varphi(n) = \sum_{d|n} d \cdot \mu\left(\frac{n}{d}\right).$$

反演结论

$$[gcd(i, j) = 1] = \sum_{d|gcd(i, j)} \mu(d).$$

$O(n \log n)$ 求莫比乌斯函数

```

1 mu[1] = 1;
2 for (int i = 1; i <= n; i++){
3     for (int j = i + i; j <= n; j += i){
4         mu[j] -= mu[i];
5     }
6 }
```

莫比乌斯变换

设 $f(n), F(n)$.

1. $F(n) = \sum_{d|n} f(d)$, 则 $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$.
2. $F(n) = \sum_{n|d} f(d)$, 则 $f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) F(d)$.

7 math - polynomial

7.1 FTT

FFT 与拆系数 FFT

```

1  const int sz = 1 << 23;
2  int rev[sz];
3  int rev_n;
4  void set_rev(int n) {
5      if (n == rev_n) return;
6      for (int i = 0; i < n; i++) rev[i] = (rev[i / 2] | (i & 1) * n) / 2;
7      rev_n = n;
8  }
9  template void butterfly(T* a, int n) {
10     set_rev(n);
11     for (int i = 0; i < n; i++) {
12         if (i < rev[i]) std::swap(a[i], a[rev[i]]);
13     }
14 }
15
16 namespace Comp {
17
18     long double pi = 3.141592653589793238;
19
20     template struct complex {
21         T x, y;
22         complex(T x = 0, T y = 0) : x(x), y(y) {}
23         complex operator+(const complex& b) const { return complex(x + b.x, y + b.y); }
24
25         complex operator-(const complex& b) const { return complex(x - b.x, y - b.y); }
26
27         complex operator*(const complex& b) const {
28             return complex<T>(x * b.x - y * b.y, x * b.y + y * b.x);
29         }
30         complex operator~() const { return complex(x, -y); }
31         static complex unit(long double rad) { return complex(std::cos(rad), std::sin(rad)); }
32     };
33
34 } // namespace Comp
35
36 struct fft_t {
37     typedef Comp::complex<double> complex;
38     complex wn[sz];
39
40     fft_t() {
41         for (int i = 0; i < sz / 2; i++) {
42             wn[sz / 2 + i] = complex::unit(2 * Comp::pi * i / sz);
43         }
44         for (int i = sz / 2 - 1; i; i--) wn[i] = wn[i * 2];
45     }
46
47     void operator()(complex* a, int n, int type) {
48         if (type == -1) std::reverse(a + 1, a + n);
49         butterfly(a, n);
50         for (int i = 1; i < n; i *= 2) {
51             const complex* w = wn + i;
52             for (complex* b = a, t; b != a + n; b += i + 1) {
53                 t = b[i];
54                 b[i] = *b - t;
55                 *b = *b + t;
56                 for (int j = 1; j < i; j++) {
57                     t = (++b)[i] * w[j];
58                     b[i] = *b - t;
59                     *b = *b + t;
60                 }
61             }
62         }
63         if (type == 1) return;
64         for (int i = 0; i < n * 2; i++) ((double*) a)[i] /= n;
65     }
66 } FFT;
67
68 typedef decltype(FFT)::complex complex;
69
70 vi fft(const vi& f, const vi& g) {
71     static complex ff[sz];
72     int n = f.size(), m = g.size();
73     vi h(n + m - 1);
74     if (std::min(n, m) <= 50) {
75         for (int i = 0; i < n; i++) {

```

```

76         for (int j = 0; j < m; ++j) {
77             h[i + j] += f[i] * g[j];
78         }
79     }
80     return h;
81 }
82 int c = 1;
83 while (c + 1 < n + m) c *= 2;
84 std::memset(ff, 0, sizeof(decltype(*(ff))) * (c));
85 for (int i = 0; i < n; i++) ff[i].x = f[i];
86 for (int i = 0; i < m; i++) ff[i].y = g[i];
87 FFT(ff, c, 1);
88 for (int i = 0; i < c; i++) ff[i] = ff[i] * ff[i];
89 FFT(ff, c, -1);
90 for (int i = 0; i + 1 < n + m; i++) h[i] = std::llround(ff[i].y / 2);
91 return h;
92 }
93
94 vi mtt(const vi& f, const vi& g) {
95     static complex ff[3][sz], gg[2][sz];
96     static int s[3] = {1, 31623, 31623 * 31623};
97     int n = f.size(), m = g.size();
98     vi h(n + m - 1);
99     if (std::min(n, m) <= 50) {
100         for (int i = 0; i < n; ++i) {
101             for (int j = 0; j < m; ++j) {
102                 Add(h[i + j], mul(f[i], g[j]));
103             }
104         }
105         return h;
106     }
107     int c = 1;
108     while (c + 1 < n + m) c *= 2;
109     for (int i = 0; i < 2; ++i) {
110         std::memset(ff[i], 0, sizeof(decltype(*(ff[i]))) * (c));
111         std::memset(gg[i], 0, sizeof(decltype(*(ff[i]))) * (c));
112         for (int j = 0; j < n; ++j) ff[i][j].x = f[j] / s[i] % s[1];
113         for (int j = 0; j < m; ++j) gg[i][j].x = g[j] / s[i] % s[1];
114         FFT(ff[i], c, 1);
115         FFT(gg[i], c, 1);
116     }
117     for (int i = 0; i < c; ++i) {
118         ff[2][i] = ff[1][i] * gg[1][i];
119         ff[1][i] = ff[1][i] * gg[0][i];
120         gg[1][i] = ff[0][i] * gg[1][i];
121         ff[0][i] = ff[0][i] * gg[0][i];
122     }
123     for (int i = 0; i < 3; ++i) {
124         FFT(ff[i], c, -1);
125         for (int j = 0; j + 1 < n + m; ++j) {
126             Add(h[j], mul(std::llround(ff[i][j].x) % mod, s[i]));
127         }
128     }
129     FFT(gg[1], c, -1);
130     for (int i = 0; i + 1 < n + m; ++i) {
131         Add(h[i], mul(std::llround(gg[1][i].x) % mod, s[1]));
132     }
133     return h;
134 }

```

7.2 FWT and FMT

```

1 void FMTor(int f[]) {
2     for (int i = 0; i < n; ++i)
3         for (int j = 0; j < m; ++j)
4             if (j >> i & 1) f[j] = (f[j] + f[j ^ 1 << i]) % P;
5 }
6 void FMToriv(int f[]) {
7     for (int i = 0; i < n; ++i)
8         for (int j = 0; j < m; ++j)
9             if (j >> i & 1) f[j] = (f[j] - f[j ^ 1 << i] + P) % P;
10 }
11 void FMTand(int f[]) {
12     for (int i = 0; i < n; ++i)
13         for (int j = 0; j < m; ++j)
14             if (~j >> i & 1) f[j] = (f[j] + f[j ^ 1 << i]) % P;
15 }
16 void FMTandiv(int f[]) {
17     for (int i = 0; i < n; ++i)
18         for (int j = 0; j < m; ++j)
19             if (~j >> i & 1) f[j] = (f[j] - f[j ^ 1 << i] + P) % P;
20 }

```

```

21 void FWT(int f[]) {
22     for (int len = 1; len < m; len *= 2) {
23         for (int i = 0; i < m; i += len * 2) {
24             for (int j = i; j < i + len; ++j) {
25                 int x = f[j], y = f[j + len];
26                 f[j] = (x + y) % P;
27                 f[j + len] = (x - y + P) % P;
28             }
29         }
30     }
31 }
32 void FWTiv(int f[]) {
33     for (int len = 1; len < m; len *= 2) {
34         for (int i = 0; i < m; i += len * 2) {
35             for (int j = i; j < i + len; ++j) {
36                 int x = f[j], y = f[j + len];
37                 f[j] = 111 * (x + y) * iv2 % P;
38                 f[j + len] = 111 * (x - y + P) * iv2 % P;
39             }
40         }
41     }
42 }

```

and

$$C_i = \sum_{i=j&k} A_j B_k$$

分治过程

$$\text{FWT}[A] = \text{merge}(\text{FWT}[A_0] + \text{FWT}[A_1], \text{FWT}[A_1]),$$

$$\text{UFWT}[A'] = \text{merge}(\text{UFWT}[A'_0] - \text{UFWT}[A'_1], \text{UFWT}[A'_1]).$$

```

1  /* mod 998244353 */
2  auto FWT_and = [&](vi v, int type) -> vi {
3      int n = v.size();
4      for (int mid = 1; mid < n; mid <= 1) {
5          for (int block = mid < 1, j = 0; j < n; j += block) {
6              for (int i = j; i < j + mid; i++) {
7                  LL x = v[i], y = v[i + mid];
8                  if (type == 1) {
9                      v[i] = add(x, y);
10                 } else {
11                     v[i] = sub(x, y);
12                 }
13             }
14         }
15     }
16     return v;
17 };

```

or

$$C_i = \sum_{i=j|k} A_j B_k$$

分治过程

$$\text{FWT}[A] = \text{merge}(\text{FWT}[A_0], \text{FWT}[A_0] + \text{FWT}[A_1]),$$

$$\text{UFWT}[A'] = \text{merge}(\text{UFWT}[A'_0], -\text{UFWT}[A'_0] + \text{UFWT}[A'_1]).$$

```

1  /* mod 998244353 */
2  auto FWT_or = [&](vi v, int type) -> vi {
3      int n = v.size();
4      for (int mid = 1; mid < n; mid <= 1) {
5          for (int block = mid < 1, j = 0; j < n; j += block) {
6              for (int i = j; i < j + mid; i++) {
7                  LL x = v[i], y = v[i + mid];
8                  if (type == 1) {
9                      v[i + mid] = add(x, y);

```

```

10         } else {
11             v[i + mid] = sub(y, x);
12         }
13     }
14 }
15 }
16 return v;
17 };

```

xor

$$C_i = \sum_{i=j \oplus k} A_j B_k$$

分治过程

$$\text{FWT}[A] = \text{merge}(\text{FWT}[A_0] + \text{FWT}[A_1], \text{FWT}[A_0] - \text{FWT}[A_1]),$$

$$\text{UFWT}[A'] = \text{merge}\left(\frac{\text{UFWT}[A'_0] + \text{UFWT}[A'_1]}{2}, \frac{\text{UFWT}[A'_0] - \text{UFWT}[A'_1]}{2}\right).$$

```

1  /* mod 998244353 */
2  auto FWT_xor = [&](vi v, int type) -> vi {
3      int n = v.size();
4      for (int mid = 1; mid < n; mid <= 1) {
5          for (int block = mid << 1, j = 0; j < n; j += block) {
6              for (int i = j; i < j + mid; i++) {
7                  LL x = v[i], y = v[i + mid];
8                  v[i] = add(x, y);
9                  v[i + mid] = sub(x, y);
10                 if (type == -1) {
11                     Mul(v[i], inv2);
12                     Mul(v[i + mid], inv2);
13                 }
14             }
15         }
16     }
17     return v;
18 };

```

统一地，

```

1  a = FWT(a, 1), b = FWT(b, 1);
2  for (int i = 0; i < (1 << n); i++) {
3      c[i] = mul(a[i], b[i]);
4  }
5  c = FWT(c, -1);

```

7.3 class polynomial

```

1  class polynomial : public vi {
2      public:
3          polynomial() = default;
4          polynomial(const vi& v) : vi(v) {}
5          polynomial(vi&& v) : vi(std::move(v)) {}
6
7          int degree() { return size() - 1; }
8
9          void clearzero() {
10             while (size() && !back()) pop_back();
11         }
12 };
13
14
15 polynomial& operator+=(polynomial& a, const polynomial& b) {
16     a.resize(std::max(a.size(), b.size()), 0);
17     for (int i = 0; i < b.size(); i++) {
18         Add(a[i], b[i]);
19     }
20     a.clearzero();
21     return a;
22 }
23

```

```

24 polynomial operator+(const polynomial& a, const polynomial& b) {
25     polynomial ans = a;
26     return ans += b;
27 }
28
29 polynomial& operator-=(polynomial& a, const polynomial& b) {
30     a.resize(std::max(a.size(), b.size()), 0);
31     for (int i = 0; i < b.size(); i++) {
32         Sub(a[i], b[i]);
33     }
34     a.clearzero();
35     return a;
36 }
37
38 polynomial operator-(const polynomial& a, const polynomial& b) {
39     polynomial ans = a;
40     return ans -= b;
41 }
42
43 class ntt_t {
44 public:
45     static const int maxbit = 22;
46     static const int sz = 1 << maxbit;
47     static const int mod = 998244353;
48     static const int g = 3;
49
50     std::array<int, sz + 10> w;
51     std::array<int, maxbit + 10> len_inv;
52
53     ntt_t() {
54         int wn = pow(g, (mod - 1) >> maxbit);
55         w[0] = 1;
56         for (int i = 1; i <= sz; i++) {
57             w[i] = mul(w[i - 1], wn);
58         }
59         len_inv[maxbit] = pow(sz, mod - 2);
60         for (int i = maxbit - 1; ~i; i--) {
61             len_inv[i] = add(len_inv[i + 1], len_inv[i + 1]);
62         }
63     }
64
65     void operator()(vi& v, int& n, int type) {
66         int bit = 0;
67         while ((1 << bit) < n) bit++;
68         int tot = (1 << bit);
69         v.resize(tot, 0);
70         vi rev(tot);
71         n = tot;
72         for (int i = 0; i < tot; i++) {
73             rev[i] = rev[i >> 1] >> 1;
74             if (i & 1) {
75                 rev[i] |= tot >> 1;
76             }
77         }
78         for (int i = 0; i < tot; i++) {
79             if (i < rev[i]) {
80                 std::swap(v[i], v[rev[i]]);
81             }
82         }
83         for (int midd = 0; (1 << midd) < tot; midd++) {
84             int mid = 1 << midd;
85             int len = mid << 1;
86             for (int i = 0; i < tot; i += len) {
87                 for (int j = 0; j < mid; j++) {
88                     int w0 = v[i + j];
89                     int w1 = mul(
90                         w[type == 1 ? (j << maxbit - midd - 1) : (len - j << maxbit - midd - 1)],
91                         v[i + j + mid]);
92                     v[i + j] = add(w0, w1);
93                     v[i + j + mid] = sub(w0, w1);
94                 }
95             }
96         }
97         if (type == -1) {
98             for (int i = 0; i < tot; i++) {
99                 v[i] = mul(v[i], len_inv[bit]);
100             }
101         }
102     }
103 } NTT;

```

```

1 polynomial& operator*=(polynomial& a, const polynomial& b) {
2     if (!a.size() || !b.size()) {
3         a.resize(0);
4         return a;
5     }
6     polynomial tmp = b;
7     int deg = a.size() + b.size() - 1;
8     int temp = deg;
9
10    // 项数较小直接硬算
11
12    if ((LL) a.size() * (LL) b.size() <= (LL) deg * 50LL) {
13        tmp.resize(0);
14        tmp.resize(deg, 0);
15        for (int i = 0; i < a.size(); i++) {
16            for (int j = 0; j < b.size(); j++) {
17                tmp[i + j] = add(tmp[i + j], mul(a[i], b[j]));
18            }
19        }
20        a = tmp;
21        return a;
22    }
23
24    // 项数较多跑 NTT
25
26    NTT(a, deg, 1);
27    NTT(tmp, deg, 1);
28    for (int i = 0; i < deg; i++) {
29        Mul(a[i], tmp[i]);
30    }
31    NTT(a, deg, -1);
32    a.resize(temp);
33    return a;
34 }
35
36 polynomial operator*(const polynomial& a, const polynomial& b) {
37     polynomial ans = a;
38     return ans *= b;
39 }

```

逆

```

1 polynomial inverse(const polynomial& a) {
2     polynomial ans({pow(a[0], mod - 2)});
3     polynomial temp;
4     polynomial tempa;
5     int deg = a.size();
6     for (int i = 0; (1 << i) < deg; i++) {
7         tempa.resize(0);
8         tempa.resize(1 << i << 1, 0);
9         for (int j = 0; j != tempa.size() and j != deg; j++) {
10             tempa[j] = a[j];
11         }
12         temp = ans * (polynomial({2}) - tempa * ans);
13         if (temp.size() > (1 << i << 1)) {
14             temp.resize(1 << i << 1, 0);
15         }
16         temp.clearzero();
17         std::swap(temp, ans);
18     }
19     ans.resize(deg);
20     return ans;
21 }

```

对数

```

1 polynomial differential(const polynomial& a) {
2     if (!a.size()) {
3         return a;
4     }
5     polynomial ans(vi(a.size() - 1));
6     for (int i = 1; i < a.size(); i++) {
7         ans[i - 1] = mul(a[i], i);
8     }
9     return ans;
10 }
11
12 polynomial integral(const polynomial& a) {

```

```

13     polynomial ans(vi(a.size() + 1));
14     for (int i = 0; i < a.size(); i++) {
15         ans[i + 1] = mul(a[i], pow(i + 1, mod - 2));
16     }
17     return ans;
18 }
19
20 polynomial ln(const polynomial& a) {
21     int deg = a.size();
22     polynomial da = differential(a);
23     polynomial inva = inverse(a);
24     polynomial ans = integral(da * inva);
25     ans.resize(deg);
26     return ans;
27 }

```

指数

```

1 polynomial exp(const polynomial& a) {
2     polynomial ans({1});
3     polynomial temp;
4     polynomial tempa;
5     polynomial tempaa;
6     int deg = a.size();
7     for (int i = 0; (1 << i) < deg; i++) {
8         tempa.resize(0);
9         tempa.resize(1 << i << 1, 0);
10        for (int j = 0; j != tempa.size() and j != deg; j++) {
11            tempa[j] = a[j];
12        }
13        tempaa = ans;
14        tempaa.resize(1 << i << 1);
15        temp = ans * (tempa + polynomial({1}) - ln(tempaa));
16        if (temp.size() > (1 << i << 1)) {
17            temp.resize(1 << i << 1, 0);
18        }
19        temp.clearzero();
20        std::swap(temp, ans);
21    }
22    ans.resize(deg);
23    return ans;
24 }

```

根号

```

1 polynomial sqrt(polynomial& a) {
2     polynomial ans({cipolla(a[0])});
3     if (ans[0] == -1) return ans;
4     polynomial temp;
5     polynomial tempa;
6     polynomial tempaa;
7     int deg = a.size();
8     for (int i = 0; (1 << i) < deg; i++) {
9         tempa.resize(0);
10        tempa.resize(1 << i << 1, 0);
11        for (int j = 0; j != tempa.size() and j != deg; j++) {
12            tempa[j] = a[j];
13        }
14        tempaa = ans;
15        tempaa.resize(1 << i << 1);
16        temp = (tempa * inverse(tempaa) + ans) * inv2;
17        if (temp.size() > (1 << i << 1)) {
18            temp.resize(1 << i << 1, 0);
19        }
20        temp.clearzero();
21        std::swap(temp, ans);
22    }
23    ans.resize(deg);
24    return ans;
25 }
26
27 // 特判 //
28
29 int cnt = 0;
30 for (int i = 0; i < a.size(); i++) {
31     if (a[i] == 0) {
32         cnt++;
33     } else {

```

```

34     break;
35 }
36 }
37 if (cnt) {
38     if (cnt == n) {
39         for (int i = 0; i < n; i++) {
40             std::cout << "0 ";
41         }
42         std::cout << endl;
43         return 0;
44     }
45     if (cnt & 1) {
46         std::cout << "-1" << endl;
47         return 0;
48     }
49     polynomial b(vi(a.size() - cnt));
50     for (int i = cnt; i < a.size(); i++) {
51         b[i - cnt] = a[i];
52     }
53     a = b;
54 }
55 a.resize(n - cnt / 2);
56 a = sqrt(a);
57 if (a[0] == -1) {
58     std::cout << "-1" << endl;
59     return 0;
60 }
61 reverse(all(a));
62 a.resize(n);
63 reverse(all(a));

```

7.4 wsy poly

```

1  #include <bits/stdc++.h>
2
3  using ul = std::uint32_t;
4  using li = std::int32_t;
5  using ll = std::int64_t;
6  using ull = std::uint64_t;
7  using llf = long double;
8  using lf = double;
9  using vul = std::vector<ul>;
10 using vvul = std::vector<vul>;
11 using pulb = std::pair<ul, bool>;
12 using vpulb = std::vector<pulb>;
13 using vvpulb = std::vector<vpulb>;
14 using vb = std::vector<bool>;
15
16 const ul base = 998244353;
17
18 std::mt19937 rnd;
19
20 ul plus(ul a, ul b) { return a + b < base ? a + b : a + b - base; }
21
22 ul minus(ul a, ul b) { return a < b ? a + base - b : a - b; }
23
24 ul mul(ul a, ul b) { return ull(a) * ull(b) % base; }
25
26 void exgcd(li a, li b, li& x, li& y) {
27     if (b) {
28         exgcd(b, a % b, y, x);
29         y -= x * (a / b);
30     } else {
31         x = 1;
32         y = 0;
33     }
34 }
35
36 ul inverse(ul a) {
37     li x, y;
38     exgcd(a, base, x, y);
39     return x < 0 ? x + li(base) : x;
40 }
41
42 ul pow(ul a, ul b) {
43     ul ret = 1;
44     ul temp = a;
45     while (b) {
46         if (b & 1) {
47             ret = mul(ret, temp);
48         }
49         temp = mul(temp, temp);
50         b >>= 1;

```



```

51     }
52     return ret;
53 }
54
55
56 ul sqrt(ul x) {
57     ul a;
58     ul w2;
59     while (true) {
60         a = rnd() % base;
61         w2 = minus(mul(a, a), x);
62         if (pow(w2, base - 1 >> 1) == base - 1) {
63             break;
64         }
65     }
66     ul b = base + 1 >> 1;
67     ul rs = 1, rt = 0;
68     ul as = a, at = 1;
69     ul qs, qt;
70     while (b) {
71         if (b & 1) {
72             qs = plus(mul(rs, as), mul(mul(rt, at), w2));
73             qt = plus(mul(rs, at), mul(rt, as));
74             rs = qs;
75             rt = qt;
76         }
77         b >>= 1;
78         qs = plus(mul(as, as), mul(mul(at, at), w2));
79         qt = plus(mul(as, at), mul(as, at));
80         as = qs;
81         at = qt;
82     }
83     return rs + rs < base ? rs : base - rs;
84 }
85
86 ul log(ul x, ul y, bool initied = false) {
87     static std::map<ul, ul> bs;
88     const ul d = std::round(std::sqrt(1l(base - 1)));
89     if (!initied) {
90         bs.clear();
91         for (ul i = 0, j = 1; i != d; ++i, j = mul(j, x)) {
92             bs[j] = i;
93         }
94     }
95     ul temp = inverse(pow(x, d));
96     for (ul i = 0, j = 1;; i += d, j = mul(j, temp)) {
97         auto it = bs.find(mul(y, j));
98         if (it != bs.end()) {
99             return it->second + i;
100         }
101     }
102 }
103
104 ul powroot(ul x, ul y, bool initied = false) {
105     const ul g = 3;
106     ul lgx = log(g, x, initied);
107     li s, t;
108     exgcd(y, base - 1, s, t);
109     if (s < 0) {
110         s += base - 1;
111     }
112     return pow(g, ull(s) * ull(lgx) % (base - 1));
113 }
114
115 class polynomial : public vul {
116 public:
117     void clearzero() {
118         while (size() && !back()) {
119             pop_back();
120         }
121     }
122     polynomial() = default;
123     polynomial(const vul& a) : vul(a) {}
124     polynomial(vul&& a) : vul(std::move(a)) {}
125     ul degree() const { return size() - 1; }
126     ul operator()(ul x) const {
127         ul ret = 0;
128         for (ul i = size() - 1; ~i; --i) {
129             ret = mul(ret, x);
130             ret = plus(ret, vul::operator[](i));
131         }
132         return ret;
133     }
134 };
135
136 polynomial& operator+=(polynomial& a, const polynomial& b) {
137     a.resize(std::max(a.size(), b.size()), 0);

```

```

138     for (ul i = 0; i != b.size(); ++i) {
139         a[i] = plus(a[i], b[i]);
140     }
141     a.clearzero();
142     return a;
143 }
144
145 polynomial operator+(const polynomial& a, const polynomial& b) {
146     polynomial ret = a;
147     return ret += b;
148 }
149
150 polynomial& operator+=(polynomial& a, const polynomial& b) {
151     a.resize(std::max(a.size(), b.size()), 0);
152     for (ul i = 0; i != b.size(); ++i) {
153         a[i] = minus(a[i], b[i]);
154     }
155     a.clearzero();
156     return a;
157 }
158
159 polynomial operator-(const polynomial& a, const polynomial& b) {
160     polynomial ret = a;
161     return ret -= b;
162 }
163
164 class ntt_t {
165 public:
166     static const ul lgsz = 20;
167     static const ul sz = 1 << lgsz;
168     static const ul g = 3;
169     ul w[sz + 1];
170     ul leninv[lgsz + 1];
171     ntt_t() {
172         ul wn = pow(g, (base - 1) >> lgsz);
173         w[0] = 1;
174         for (ul i = 1; i <= sz; ++i) {
175             w[i] = mul(w[i - 1], wn);
176         }
177         leninv[lgsz] = inverse(sz);
178         for (ul i = lgsz - 1; ~i; --i) {
179             leninv[i] = plus(leninv[i + 1], leninv[i + 1]);
180         }
181     }
182     void operator()(vul& v, ul& n, bool inv) {
183         ul lgn = 0;
184         while ((1 << lgn) < n) {
185             ++lgn;
186         }
187         n = 1 << lgn;
188         v.resize(n, 0);
189         for (ul i = 0, j = 0; i != n; ++i) {
190             if (i < j) {
191                 std::swap(v[i], v[j]);
192             }
193             ul k = n >> 1;
194             while (k & j) {
195                 j &= ~k;
196                 k >>= 1;
197             }
198             j |= k;
199         }
200         for (ul lgmid = 0; (1 << lgmid) != n; ++lgmid) {
201             ul mid = 1 << lgmid;
202             ul len = mid << 1;
203             for (ul i = 0; i != n; i += len) {
204                 for (ul j = 0; j != mid; ++j) {
205                     ul t0 = v[i + j];
206                     ul t1 =
207                         mul(w[inv ? (len - j << lgsz - lgmid - 1) : (j << lgsz - lgmid - 1)],
208                             v[i + j + mid]);
209                     v[i + j] = plus(t0, t1);
210                     v[i + j + mid] = minus(t0, t1);
211                 }
212             }
213         }
214         if (inv) {
215             for (ul i = 0; i != n; ++i) {
216                 v[i] = mul(v[i], leninv[lgn]);
217             }
218         }
219     }
220 } ntt;
221
222 polynomial& operator*=(polynomial& a, const polynomial& b) {
223     if (!b.size() || !a.size()) {
224         a.resize(0);

```

```

225     return a;
226 }
227 polynomial temp = b;
228 ul npmp1 = a.size() + b.size() - 1;
229 if (ull(a.size()) * ull(b.size()) <= ull(npmp1) * ull(50)) {
230     temp.resize(0);
231     temp.resize(npmp1, 0);
232     for (ul i = 0; i != a.size(); ++i) {
233         for (ul j = 0; j != b.size(); ++j) {
234             temp[i + j] = plus(temp[i + j], mul(a[i], b[j]));
235         }
236     }
237     a = temp;
238     a.clearzero();
239     return a;
240 }
241 ntt(a, npmp1, false);
242 ntt(temp, npmp1, false);
243 for (ul i = 0; i != npmp1; ++i) {
244     a[i] = mul(a[i], temp[i]);
245 }
246 ntt(a, npmp1, true);
247 a.clearzero();
248 return a;
249 }
250
251 polynomial operator*(const polynomial& a, const polynomial& b) {
252     polynomial ret = a;
253     return ret *= b;
254 }
255
256 polynomial& operator*=(polynomial& a, ul b) {
257     if (!b) {
258         a.resize(0);
259         return a;
260     }
261     for (ul i = 0; i != a.size(); ++i) {
262         a[i] = mul(a[i], b);
263     }
264     return a;
265 }
266
267 polynomial operator*(const polynomial& a, ul b) {
268     polynomial ret = a;
269     return ret *= b;
270 }
271
272 polynomial inverse(const polynomial& a, ul lgdeg) {
273     polynomial ret({inverse(a[0])});
274     polynomial temp;
275     polynomial tempa;
276     for (ul i = 0; i != lgdeg; ++i) {
277         tempa.resize(0);
278         tempa.resize(1 << i << 1, 0);
279         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
280             tempa[j] = a[j];
281         }
282         temp = ret * (polynomial({2}) - tempa * ret);
283         if (temp.size() > (1 << i << 1)) {
284             temp.resize(1 << i << 1, 0);
285         }
286         temp.clearzero();
287         std::swap(temp, ret);
288     }
289     return ret;
290 }
291
292 void quotientremain(const polynomial& a, polynomial b, polynomial& q, polynomial& r) {
293     if (a.size() < b.size()) {
294         q = polynomial();
295         r = std::move(a);
296         return;
297     }
298     std::reverse(b.begin(), b.end());
299     auto ta = a;
300     std::reverse(ta.begin(), ta.end());
301     ul n = a.size() - 1;
302     ul m = b.size() - 1;
303     ta.resize(n - m + 1);
304     ul lgnmmp1 = 0;
305     while ((1 << lgnmmp1) < n - m + 1) {
306         ++lgnmmp1;
307     }
308     q = ta * inverse(b, lgnmmp1);
309     q.resize(n - m + 1);
310     std::reverse(b.begin(), b.end());
311     std::reverse(q.begin(), q.end());

```

```

312     r = a - b * q;
313 }
314
315 polynomial mod(const polynomial& a, const polynomial& b) {
316     polynomial q, r;
317     quotientremain(a, b, q, r);
318     return r;
319 }
320
321 polynomial quotient(const polynomial& a, const polynomial& b) {
322     polynomial q, r;
323     quotientremain(a, b, q, r);
324     return q;
325 }
326
327 polynomial sqrt(const polynomial& a, ul lgdeg) {
328     polynomial ret({sqrt(a[0])});
329     polynomial temp;
330     polynomial tempa;
331     for (ul i = 0; i != lgdeg; ++i) {
332         tempa.resize(0);
333         tempa.resize(1 << i << 1, 0);
334         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
335             tempa[j] = a[j];
336         }
337         temp = (tempa * inverse(ret, i + 1) + ret) * (base + 1 >> 1);
338         if (temp.size() > (1 << i << 1)) {
339             temp.resize(1 << i << 1, 0);
340         }
341         temp.clearzero();
342         std::swap(temp, ret);
343     }
344     return ret;
345 }
346
347 polynomial differential(const polynomial& a) {
348     if (!a.size()) {
349         return a;
350     }
351     polynomial ret(vul(a.size() - 1, 0));
352     for (ul i = 1; i != a.size(); ++i) {
353         ret[i - 1] = mul(a[i], i);
354     }
355     return ret;
356 }
357
358 polynomial integral(const polynomial& a) {
359     polynomial ret(vul(a.size() + 1, 0));
360     for (ul i = 0; i != a.size(); ++i) {
361         ret[i + 1] = mul(a[i], inverse(i + 1));
362     }
363     return ret;
364 }
365
366 polynomial ln(const polynomial& a, ul lgdeg) {
367     polynomial da = differential(a);
368     polynomial inva = inverse(a, lgdeg);
369     polynomial ret = integral(da * inva);
370     if (ret.size() > (1 << lgdeg)) {
371         ret.resize(1 << lgdeg);
372         ret.clearzero();
373     }
374     return ret;
375 }
376
377 polynomial exp(const polynomial& a, ul lgdeg) {
378     polynomial ret({1});
379     polynomial temp;
380     polynomial tempa;
381     for (ul i = 0; i != lgdeg; ++i) {
382         tempa.resize(0);
383         tempa.resize(1 << i << 1, 0);
384         for (ul j = 0; j != tempa.size() && j != a.size(); ++j) {
385             tempa[j] = a[j];
386         }
387         temp = ret * (polynomial({1}) - ln(ret, i + 1) + tempa);
388         if (temp.size() > (1 << i << 1)) {
389             temp.resize(1 << i << 1, 0);
390         }
391         temp.clearzero();
392         std::swap(temp, ret);
393     }
394     return ret;
395 }
396
397 polynomial pow(const polynomial& a, ul k, ul lgdeg) { return exp(ln(a, lgdeg) * k, lgdeg); }
398

```

```

399 polynomial alpi[1 << 16][17];
400
401 polynomial getalpi(const ul x[], ul l, ul lgrml) {
402     if (lgrml == 0) {
403         return alpi[l][lgrml] = vul({minus(0, x[l]), 1});
404     }
405     return alpi[l][lgrml] = getalpi(x, l, lgrml - 1) * getalpi(x, l + (1 << lgrml - 1), lgrml - 1);
406 }
407
408 void multians(const polynomial& f, const ul x[], ul y[], ul l, ul lgrml) {
409     if (f.size() <= 700) {
410         for (ul i = l; i != l + (1 << lgrml); ++i) {
411             y[i] = f(x[i]);
412         }
413         return;
414     }
415     if (lgrml == 0) {
416         y[l] = f(x[l]);
417         return;
418     }
419     multians(mod(f, alpi[l][lgrml - 1]), x, y, l, lgrml - 1);
420     multians(mod(f, alpi[l + (1 << lgrml - 1)][lgrml - 1]), x, y, l + (1 << lgrml - 1), lgrml - 1);
421 }
422
423 ul sqrt(ul x) {
424     ul a;
425     ul w2;
426     while (true) {
427         a = rnd() % base;
428         w2 = minus(mul(a, a), x);
429         if (pow(w2, base - 1 >> 1) == base - 1) {
430             break;
431         }
432     }
433     ul b = base + 1 >> 1;
434     ul rs = 1, rt = 0;
435     ul as = a, at = 1;
436     ul qs, qt;
437     while (b) {
438         if (b & 1) {
439             qs = plus(mul(rs, as), mul(mul(rt, at), w2));
440             qt = plus(mul(rs, at), mul(rt, as));
441             rs = qs;
442             rt = qt;
443         }
444         b >>= 1;
445         qs = plus(mul(as, as), mul(mul(at, at), w2));
446         qt = plus(mul(as, at), mul(as, at));
447         as = qs;
448         at = qt;
449     }
450     return rs + rs < base ? rs : base - rs;
451 }
452
453 ul log(ul x, ul y, bool initd = false) {
454     static std::map<ul, ul> bs;
455     const ul d = std::round(std::sqrt(1f(base - 1)));
456     if (!initd) {
457         bs.clear();
458         for (ul i = 0, j = 1; i != d; ++i, j = mul(j, x)) {
459             bs[j] = i;
460         }
461     }
462     ul temp = inverse(pow(x, d));
463     for (ul i = 0, j = 1;; i += d, j = mul(j, temp)) {
464         auto it = bs.find(mul(y, j));
465         if (it != bs.end()) {
466             return it->second + i;
467         }
468     }
469 }
470
471 ul powroot(ul x, ul y, bool initd = false) {
472     const ul g = 3;
473     ul lgx = log(g, x, initd);
474     li s, t;
475     exgcd(y, base - 1, s, t);
476     if (s < 0) {
477         s += base - 1;
478     }
479     return pow(g, ull(s) * ull(lgx) % (base - 1));
480 }
481
482 ul n;
483
484 int main() {
485     std::scanf("%u", &n);

```

```

486 |     polynomial f;
487 |     for (ul i = 0; i <= n; ++i) {
488 |         ul t;
489 |         std::scanf("%u", &t);
490 |         f.push_back(t % base);
491 |     }
492 |     polynomial g = exp(ln(f * inverse(f[0]), 17) * inverse(3), 17) * powroot(f[0], 3);
493 |     while (g.size() <= n) {
494 |         g.push_back(0);
495 |     }
496 |     for (ul i = 0; i <= n; ++i) {
497 |         if (i) {
498 |             std::putchar(' ');
499 |         }
500 |         std::printf("%u", g[i]);
501 |     }
502 |     std::putchar('\n');
503 |     return 0;
504 | }

```

Lagrange interpolation

一般的插值

给出一个多项式 $f(x)$ 上的 n 个点 (x_i, y_i) , 求 $f(k)$.

插值的结果是

$$f(x) = \sum_{i=1}^n y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j},$$

直接带入 k 并且取模即可, 时间复杂度 $O(n^2)$.

```

1 | auto lagrange = (const vi& x, const vi& y, int n, int k) {
2 |     for (int i = 1; i <= n; i++) {
3 |         LL s1 = y[i] % mod, s2 = 1ll;
4 |         for (int j = 1; j <= n; j++) {
5 |             if (i != j) {
6 |                 s1 = s1 * (k - x[j]) % mod;
7 |                 s2 = s2 * (x[i] - x[j]) % mod;
8 |             }
9 |         }
10 |         Add(ans, mul(s1, quick_power(s2, mod - 2, mod)));
11 |     }
12 |     return ans;
13 | };

```

坐标连续的插值

给出的点是 (i, y_i) .

$$\begin{aligned}
 f(x) &= \sum_{i=1}^n y_i \prod_{j \neq i} \frac{x - x_j}{x_i - x_j} \\
 &= \sum_{i=1}^n y_i \prod_{j \neq i} \frac{x - j}{i - j} \\
 &= \sum_{i=1}^n y_i \cdot \frac{\prod_{j=1}^n (x - j)}{(x - i)(-1)^{n+1-i}(i - 1)!(n + 1 - i)!} \\
 &= \left(\prod_{j=1}^n (x - j) \right) \left(\sum_{i=1}^n \frac{(-1)^{n+1-i} y_i}{(x - i)(i - 1)!(n + 1 - i)!} \right),
 \end{aligned}$$

时间复杂度为 $O(n)$.

8 math - numerical analysis

8.1 Simpson

```

1  #include <bits/stdc++.h>
2  using namespace std;
3  const double eps = 1e-8;
4  double a, b, c, d, l, r;
5  double f(double x) {
6      return pow(x, (a / x) - x);
7  }
8  double simpson(double l, double r) {
9      double mid = (l + r) / 2.0;
10     return (f(l) + 4.0 * f(mid) + f(r)) * (r - l) / 6.0;
11 }
12 double asr(double l, double r, double eps, double ans) {
13     double mid = (l + r) / 2.0, l_ = simpson(l, mid), r_ = simpson(mid, r);
14     if (fabs(l_ + r_ - ans) <= 15.0 * eps) return l_ + r_ + (l_ + r_ - ans) / 15.0;
15     return asr(l, mid, eps / 2.0, l_) + asr(mid, r, eps / 2.0, r_);
16 }
17 int main() {
18     ios::sync_with_stdio(false), cin.tie(nullptr), cout.tie(nullptr);
19     cin >> a;
20     if (a < 0) return cout << "orz", 0;
21     cout << fixed << setprecision(5) << asr(eps, 15.0, eps, simpson(l, r));
22     return 0;
23 }

```

9 math - game theory

9.1 nim game

若 nim 和为 0, 则先手必败.

暴力打表.

```

1 vi SG(21, -1); /* 记忆化 */
2 std::function<int(int, int)> sg = [&](int x) -> int {
3     if (/* 为最终态 */) return SG[x] = 0;
4     if (SG[x] != -1) return SG[x];
5     vi st;
6     for (/* 枚举所有可到达的状态 y */) {
7         st.push_back(sg(y));
8     }
9     std::sort(all(st));
10    for (int i = 0; i < st.size(); i++) {
11        if (st[i] != i) return SG[x] = i;
12    }
13    return SG[x] = st.size();
14 };

```

9.2 anti - nim game

若

1. 所有堆的石子均为一个, 且 nim 和不为 0,
2. 至少有一堆石子超过一个, 且 nim 和为 0,

则先手必败.

10 math - linear algebra

10.1 matrix

Gauss Elimination

```

1  using db = double;
2  const int N = 109;
3  const db eps = 1e-6;
4  db a[N][N];
5  auto work = []() {
6      int n;
7      cin >> n;
8      for (int i = 1; i <= n; ++i) {
9          for (int j = 1; j <= n + 1; ++j) {
10             cin >> a[i][j];
11         }
12     }
13     for (int i = 1; i <= n; ++i) {
14         if (fabs(a[i][i]) < eps) {
15             for (int j = i + 1; j <= n; ++j) {
16                 if (fabs(a[j][i]) > eps) {
17                     for (int k = 1; k <= n + 1; k++) {
18                         swap(a[i][k], a[j][k]);
19                     }
20                     break;
21                 }
22             }
23         }
24         if (fabs(a[i][i]) < eps) {
25             cout << "No Solution\n";
26             return ;
27         }
28         for (int j = 1; j <= n; ++j) if (i != j) {
29             db d = -a[j][i] / a[i][i];
30             for (int k = 1; k <= n + 1; ++k) {
31                 a[j][k] += a[i][k] * d;
32             }
33         }
34     }
35     cout << fixed << setprecision(2);
36     for (int i = 1; i <= n; ++i) {
37         cout << a[i][n + 1] / a[i][i] << '\n';
38     }
39 };

```

determinant mod non-prime

```

1  int a[609][609];
2  int main() {
3      int n, P, z = 1;
4      std::cin >> n >> P;
5      for (int i = 1; i <= n; ++i) {
6          for (int j = 1; j <= n; ++j) std::cin >> a[i][j];
7      }
8      for (int i = 1; i <= n; ++i) {
9          for (int j = i + 1; j <= n; ++j) {
10             while (a[j][i]) {
11                 z = -z;
12                 int d = a[i][i] / a[j][i];
13                 for (int k = i; k <= n; ++k) {
14                     int x = (a[i][k] - 111 * d * a[j][k]) % P;
15                     a[i][k] = a[j][k], a[j][k] = x;
16                 }
17             }
18         }
19         z = 111 * z * a[i][i] % P;
20     }
21     std::cout << (z + P) % P;
22 }

```

determinant mod 998244353

```

1 auto det = [&](int n, vvi e) -> int {
2     int ans = 1;
3     for (int i = 1; i <= n; i++) {
4         if (a[i][i] == 0) {
5             for (int j = i + 1; j <= n; j++) {
6                 if (a[j][i] != 0) {
7                     for (int k = i; k <= n; k++) {
8                         std::swap(a[i][k], a[j][k]);
9                     }
10                    ans = sub(mod, ans);
11                    break;
12                }
13            }
14        }
15        if (a[i][i] == 0) return 0;
16        Mul(ans, a[i][i]);
17        int x = pow(a[i][i], mod - 2);
18        for (int k = i; k <= n; k++) {
19            Mul(a[i][k], x);
20        }
21        for (int j = i + 1; j <= n; j++) {
22            int x = a[j][i];
23            for (int k = i; k <= n; k++) {
24                Sub(a[j][k], mul(a[i][k], x));
25            }
26        }
27    }
28    return ans;
29 };

```

matrix multiplication

$A_{n \times m}$ 与 $B_{m \times k}$ 相乘并模 998244353.

```

1 auto matrix_mul = [&](int n, int m, int k, const vvi& a, const vvi& b) -> vvi {
2     vvi c(n + 1, vi(k + 1));
3     for (int i = 1; i <= n; i++) {
4         for (int l = 1; l <= m; l++) {
5             int x = a[i][l];
6             for (int j = 1; j <= k; j++) {
7                 Add(c[i][j], mul(x, b[l][j]));
8             }
9         }
10    }
11    return c;
12 };

```

10.2 matrix tree

```

1 const int N=33,M=152599,P=998244353;
2 int qpow(int a,int b=P-2){
3     int r=1;for(;b>=1;a=1ll*a*a%P)if(b&1)r=1ll*r*a%P;return r;
4 }
5 struct T{int x,y,z;T(int a=0,int b=0,int c=0):x(a),y(b),z(c){}}e[N*N];
6 struct F{
7     int a,b;
8     F():a(),b(){}
9     F(int x,int y):a(x),b(y){}
10    F operator+(const F&_)const{return F((a+_a)%P,(b+_b)%P);}
11    F operator+=(const F&_){return *this=*this+_;}
12    F operator-(const F&_)const{return F((a-_a+P)%P,(b-_b+P)%P);}
13    F operator-=(const F&_){return *this=*this-_;}
14    F operator*(const F&_)const{return F((1ll*a*_a+1ll*b*_b)%P,1ll*b*_b)%P);}
15    F operator*=(const F&_){return *this=*this*_;}
16    int operator&()const{return b?2:(a?1:0);}
17    bool operator!()const{return !a&&!b;}
18    F operator~()const{
19        int d=qpow(b);
20        return F((P-1ll*a*d%P*d%P)%P,d);
21    }
22 };
23 int fa[N],phi[M],n,m;
24 int gf(int x){return x==fa[x]?x:fa[x]=gf(fa[x]);}
25 int cal(int p){
26     F a[N][N],d,iv,z=F(0,1);
27     int i,j,k,l,x=0;iota(fa,fa+n+1,0);
28     for(i=1;i<=m;++i)if(e[i].z%p==0){

```

```

29         if((j=gf(e[i].x))!=(k=gf(e[i].y)))fa[j]=k;
30         j=e[i].x,k=e[i].y,l=e[i].z,++x;
31         a[j][k]-=F(1,1),a[k][j]-=F(1,1);
32         a[j][j]+=F(1,1),a[k][k]+=F(1,1);
33     }
34     for(j=0,i=1;i<=n;++i)if(fa[i]==i)++j;
35     if(j>1 || x<n-1)return 0;
36     for(i=1;i<=n;++i){
37         for(k=i,j=i+1;j<=n;++j)if(&a[j][i]>&a[k][i])k=j;
38         if(k!=i)swap(a[i],a[k]),z*=F(0,P-1);
39         if(!a[i][i])return 0;
40         for(z*=a[i][i],iv=~a[i][i],j=i;j<=n;++j)a[j][i]*=iv;
41         for(j=i+1;j<=n;++j)for(d=a[j][i],k=i;k<=n;++k)a[j][k]-=a[i][k]*d;
42     }
43     return z.a;
44 }
45 void work(){
46     int h=0,i,j,x,y,z;
47     for(cin>>n>>m,i=1;i<=m;++i)cin>>x>>y>>z,e[i]=T(x,y,z),h=max(h,z);
48     iota(phi+1,phi+h+1,1);
49     for(i=1;i<=h;++i)for(j=i<=1;j<=h;j+=i)phi[j]=(phi[j]-phi[i]+P)%P;
50     for(z=0,i=1;i<=h;++i)z=(z+1ll*phi[i]*cal(i)%P)%P;
51     cout<<z<<"\n";
52 }

```

10.3 linear basis

```

1  template<typename T, const int M = sizeof(T) * 8>
2  struct Liner_Basis {
3      T a[M];
4      size_t sz;
5      Liner_Basis() : a(), sz() {}
6      size_t size() const {
7          return sz;
8      }
9      void clear() {
10         memset(a, 0, sizeof a);
11     }
12     bool ins(T x) {
13         for (size_t i = M - 1; ~i && x; --i)
14             if (x >> i & 1) {
15                 if (a[i]) x ^= a[i];
16                 else return a[i] = x, true;
17             }
18         return false;
19     }
20     Liner_Basis& operator+=(const Liner_Basis&_) {
21         for (T x : _.a) if (x) this->ins(x);
22         return *this;
23     }
24     Liner_Basis operator+(const Liner_Basis&_) {
25         Liner_Basis z = *this;
26         return z += _;
27     }
28     T qry(T x = 0) {
29         for (size_t i = M - 1; ~i; --i)
30             if ((x ^ a[i]) > x) x ^= a[i];
31         return x;
32     }
33 };
34 template<typename T>
35 using LB = Liner_Basis<T>;

```

```

1  struct Liner_Basis {
2      using u64 = unsigned long long;
3      static const size_t M = 60;
4      u64 a[M + 1];
5      size_t sz;
6      size_t size() {
7          return sz;
8      }
9      Liner_Basis& operator+=(u64 x) {
10         for (size_t i = M; ~i && x; --i)
11             if (x >> i & 1)
12                 if (a[i]) x ^= a[i];
13                 else return a[i] = x, ++sz, *this;
14         return *this;
15     }
16     Liner_Basis& operator+=(const Liner_Basis&_) {
17         for (u64 x : _.a) if (x) *this += x;
18         return *this;
19     }

```

```

19 }
20 Liner_Basis operator+(u64 x) {
21     Liner_Basis z = *this;
22     return z += x;
23 }
24 Liner_Basis operator+(const Liner_Basis&_) {
25     Liner_Basis z = *this;
26     return z += _;
27 }
28 u64 qry(u64 x = 0) {
29     for (size_t i = M; ~i; --i)
30         if ((x ^ a[i]) > x) x ^= a[i];
31     return x;
32 }
33 u64 rank(u64 x) {
34     u64 h = 1, z = 0;
35     for (size_t i = 0; i <= M; ++i)
36         if (a[i]) {
37             if (x >> i & 1) z += h;
38             h <<= 1;
39         }
40     return z;
41 }
42 u64 kth(u64 x) {
43     u64 z = 0;
44     for (size_t i = M; ~i; --i)
45         if (x >> i & 1) z ^= a[i];
46     return z;
47 }
48 }v;
49 using LB = Liner_Basis;

```

10.4 berlekamp massey

```

1  const int p = 998244353;
2  auto power = [](int a, int b = p - 2) {
3      int z = 1;
4      while (b) {
5          if (b & 1) z = 1ll * z * a % p;
6          a = 1ll * a * a % p, b >>= 1;
7      }
8      return z;
9  };
10 vector<int> berlekamp_massey(const vector<int> &a) {
11     vector<int> v, last; // v is the answer, 0-based, p is the module
12     int k = -1, delta = 0;
13
14     for (int i = 0; i < (int)a.size(); i++) {
15         int tmp = 0;
16         for (int j = 0; j < (int)v.size(); j++)
17             tmp = (tmp + (long long)a[i - j - 1] * v[j]) % p;
18
19         if (a[i] == tmp) continue;
20
21         if (k < 0) {
22             k = i;
23             delta = (a[i] - tmp + p) % p;
24             v = vector<int>(i + 1);
25
26             continue;
27         }
28
29         vector<int> u = v;
30         int val = (long long)(a[i] - tmp + p) * power(delta, p - 2) % p;
31
32         if (v.size() < last.size() + i - k) v.resize(last.size() + i - k);
33
34         (v[i - k - 1] += val) %= p;
35
36         for (int j = 0; j < (int)last.size(); j++) {
37             v[i - k + j] = (v[i - k + j] - (long long)val * last[j]) % p;
38             if (v[i - k + j] < 0) v[i - k + j] += p;
39         }
40
41         if ((int)u.size() - i < (int)last.size() - k) {
42             last = u;
43             k = i;
44             delta = a[i] - tmp;
45             if (delta < 0) delta += p;
46         }
47     }
48
49     for (auto &x : v) x = (p - x) % p;

```

```
50 | v.insert(v.begin(), 1);  
51 |  
52 | return v;  
53 | // forall i, sum_{j=0}^m a_{i-j} v_j = 0  
54 | }
```

10.5 linear programming

11 math - complex number

```

1  tandu struct Comp {
2      T a, b;
3
4      Comp(T _a = 0, T _b = 0) { a = _a, b = _b; }
5
6      Comp operator+(const Comp& x) const { return Comp(a + x.a, b + x.b); }
7
8      Comp operator-(const Comp& x) const { return Comp(a - x.a, b - x.b); }
9
10     Comp operator*(const Comp& x) const { return Comp(a * x.a - b * x.b, a * x.b + b * x.a); }
11
12     bool operator==(const Comp& x) const { return a == x.a and b == x.b; }
13
14     T real() { return a; }
15
16     T imag() { return b; }
17
18     U norm() { return (U) a * a + (U) b * b; }
19
20     Comp conj() { return Comp(a, -b); }
21
22     Comp operator/(const Comp& x) const {
23         Comp y = x;
24         Comp c = Comp(a, b) * y.conj();
25         T d = y.norm();
26         return Comp(c.a / d, c.b / d);
27     }
28 };
29
30 typedef Comp<LL, LL> complex;
31
32 complex gcd(complex a, complex b) {
33     LL d = b.norm();
34     if (d == 0) return a;
35     std::vector<complex> v(4);
36     complex c = a * b.conj();
37     auto fdiv = [&](LL a, LL b) -> LL { return a / b - ((a ^ b) < 0 && (a % b)); };
38     v[0] = complex(fdiv(c.real(), d), fdiv(c.imag(), d));
39     v[1] = v[0] + complex(1, 0);
40     v[2] = v[0] + complex(0, 1);
41     v[3] = v[0] + complex(1, 1);
42     for (auto& x : v) {
43         x = a - x * b;
44     }
45     std::sort(all(v), [&](complex a, complex b) { return a.norm() < b.norm(); });
46     return gcd(b, v[0]);
47 };

```

12 graph

12.1 topsort

```

1  vi top;
2  auto top_sort = [&]() -> bool {
3      vi d(n + 1);
4      std::queue<int> q;
5      for (int i = 1; i <= n; i++) {
6          d[i] = e[i].size();
7          if (!d[i]) q.push(i);
8      }
9      while (!q.empty()) {
10         int u = q.front();
11         q.pop();
12         top.push_back(u);
13         for (auto v : e[u]) {
14             d[v]--;
15             if (!d[v]) q.push(v);
16         }
17     }
18     if (top.size() != n) return false;
19     return true;
20 };

```

12.2 shortest path

Floyd

```

1  auto floyd = [&]() -> vvi {
2      vvi dist(n + 1, vi(n + 1, inf));
3      for (int i = 1; i <= n; i++) {
4          for (int j = 1; j <= n; j++) {
5              Min(dist[i][j], e[i][j]);
6          }
7          dist[i][i] = 0;
8      }
9      for (int k = 1; k <= n; k++) {
10         for (int i = 1; i <= n; i++) {
11             for (int j = 1; j <= n; j++) {
12                 Min(dist[i][j], dist[i][k] + dist[k][j]);
13             }
14         }
15     }
16     return dist;
17 };

```

Dijkstra

```

1  auto dijkstra = [&](int s) -> vl {
2      vl dist(n + 1, INF);
3      vi vis(n + 1, 0);
4      dist[s] = 0;
5      std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
6      q.emplace(0LL, s);
7      while (!q.empty()) {
8          auto [dis, u] = q.top();
9          q.pop();
10         if (vis[u]) continue;
11         vis[u] = 1;
12         for (const auto& [v, w] : e[u]) {
13             if (dist[v] > dis + w) {
14                 dist[v] = dis + w;
15                 q.emplace(dist[v], v);
16             }
17         }
18     }
19     return dist;
20 };

```

Bellman - Fold

```

1  int n, m, s;
2  int dist[N];
3  struct node{
4      int from, to, w;
5  }edge[M];
6  void bellman_fold(int s){
7      memset(dist, 0x3f, sizeof(dist));
8      dist[s] = 0;
9      for(int i = 1; i <= n; i++){
10         bool flag = true;
11         for(int j = 1; j <= m; j++){
12             int a = edge[j].from, b = edge[j].to, w = edge[j].w;
13             if(dist[a] == 0x3f3f3f3f) continue;
14             if(dist[b] > dist[a] + w){
15                 dist[b] = dist[a] + w;
16                 flag = false;
17             }
18         }
19         if(flag) break;
20     }
21 }

```

SPFA

```

1  int n, m, s;
2  vl dist(n + 1, INF);
3  std::vector<bool> vis(n + 1);
4  std::vector<PLI > e(n + 1);
5
6  void spfa(int s){
7      rep(i, 1, n) dist[i] = INF;
8      dist[s] = 0;
9      std::queue<int> q;
10     q.push(s);
11     vis[s] = true;
12     while(q.size()){
13         auto u = q.front();
14         q.pop();
15         vis[u] = false;
16         for(auto j : e[u]){
17             int v = j.ff; LL w = j.ss;
18             if(dist[v] > dist[u] + w){
19                 dist[v] = dist[u] + w;
20                 if(!vis[v]){
21                     q.push(v);
22                     vis[v] = true;
23                 }
24             }
25         }
26     }
27 }

```

Johnson

```

1  auto johnson = [&]() -> vl {
2      /* 负环 */
3      vl dist1(n + 1);
4      vl vis(n + 1), cnt(n + 1);
5      auto spfa = [&]() -> bool {
6          std::queue<int> q;
7          for (int u = 1; u <= n; u++) {
8              q.push(u);
9              vis[u] = false;
10         }
11         while (!q.empty()) {
12             auto u = q.front();
13             q.pop();
14             vis[u] = false;
15             for (auto [v, w] : e[u]) {
16                 if (dist1[v] > dist1[u] + w) {
17                     dist1[v] = dist1[u] + w;
18                     Max(cnt[v], cnt[u] + 1);
19                     if (cnt[v] >= n) return true;
20                     if (!vis[v]) {
21                         q.push(v);

```



```

22         vis[v] = true;
23     }
24 }
25 }
26 }
27 return false;
28 };
29
30 /* dijkstra */
31 vl dist2(n + 1);
32 auto dijkstra = [&](int s) {
33     for (int u = 1; u <= n; u++) {
34         dist2[u] = 1e9;
35         vis[u] = false;
36     }
37     dist2[s] = 0;
38     std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
39     q.emplace(0, s);
40     while (!q.empty()) {
41         auto [d, u] = q.top();
42         q.pop();
43         if (vis[u]) continue;
44         vis[u] = true;
45         for (const auto& [v, w] : e[u]) {
46             if (dist2[v] > d + w) {
47                 dist2[v] = d + w;
48                 q.emplace(dist2[v], v);
49             }
50         }
51     }
52 };
53
54 if (spfa()) return vvl{};
55 for (int u = 1; u <= n; u++) {
56     for (auto& [v, w] : e[u]) {
57         w += dist1[u] - dist1[v];
58     }
59 }
60 vvl dist(n + 1, vl(n + 1));
61 for (int u; u <= n; u++) {
62     dijkstra(u);
63     for (int v = 1; v <= n; v++) {
64         if (dist2[v] == 1e9) {
65             dist[u][v] = INF;
66         } else {
67             dist[u][v] = dist2[v] + dist1[v] - dist1[u];
68         }
69     }
70 }
71 return dist;
72 };

```

最短路计数 - Dijkstra

```

1 auto dijkstra = [&](int s) -> std::pair<vl, vi> {
2     vl dist(n + 1, INF);
3     vi cnt(n + 1), vis(n + 1);
4     dist[s] = 0;
5     cnt[s] = 1;
6     std::priority_queue<PLI, std::vector<PLI>, std::greater<PLI>> q;
7     q.emplace(0LL, s);
8     while (!q.empty()) {
9         auto [dis, u] = q.top();
10        q.pop();
11        if (vis[u]) continue;
12        vis[u] = 1;
13        for (const auto& [v, w] : e[u]) {
14            if (dist[v] > dis + w) {
15                dist[v] = dis + w;
16                cnt[v] = cnt[u];
17                q.push({dist[v], v});
18            } else if (dist[v] == dis + w) {
19                // cnt[v] += cnt[u];
20                cnt[v] += cnt[u];
21                cnt[v] %= 100003;
22            }
23        }
24    }
25    return {dist, cnt};
26 };

```

最短路计数 - Floyd

```

1 auto floyd() = [&] -> std::pair<vvi, vvi> {
2     vvi dist(n + 1, vi(n + 1, inf));
3     vvi cnt(n + 1, vi(n + 1));
4     for (int i = 1; i <= n; i++) {
5         for (int j = 1; j <= n; j++) {
6             Min(dist[i][j], e[i][j]);
7         }
8         dist[i][i] = 0;
9     }
10    for (int k = 1; k <= n; k++) {
11        for (int i = 1; i <= n; i++) {
12            for (int j = 1; j <= n; j++) {
13                if (dist[i][j] == dist[i][k] + dist[k][j]) {
14                    cnt[i][j] += cnt[i][k] * cnt[k][j];
15                } else if (dist[i][j] > dist[i][k] + dist[k][j]) {
16                    cnt[i][j] = cnt[i][k] * cnt[k][j];
17                    dist[i][j] = dist[i][k] + dist[k][j];
18                }
19            }
20        }
21    }
22    return {dist, cnt};
23 };

```

负环

判断的是最短路长度.

```

1 auto spfa = [&]() -> bool {
2     std::queue<int> q;
3     vi vis(n + 1), cnt(n + 1);
4     for (int i = 1; i <= n; i++) {
5         q.push(i);
6         vis[i] = true;
7     }
8     while (!q.empty()) {
9         auto u = q.front();
10        q.pop();
11        vis[u] = false;
12        for (const auto& [v, w] : e[u]) {
13            if (dist[v] > dist[u] + w) {
14                dist[v] = dist[u] + w;
15                cnt[v] = cnt[u] + 1;
16                if (cnt[v] >= n) return true;
17                if (!vis[v]) {
18                    q.push(v);
19                    vis[v] = true;
20                }
21            }
22        }
23    }
24    return false;
25 }

```

分层最短路

有一个 n 个点 m 条边的无向图, 你可以选择 k 条道路以零代价通行, 求 s 到 t 的最小花费。

```

1 int main() {
2     std::ios::sync_with_stdio(false);
3     std::cin.tie(0);
4     std::cout.tie(0);
5
6     int n, m, k, s, t;
7     std::cin >> n >> m >> k;
8     std::cin >> s >> t;
9     std::vector<std::vector<PIL>> e(n * (k + 1) + 1);
10    for (int i = 1; i <= m; i++) {
11        int a, b, c;
12        std::cin >> a >> b >> c;
13        e[a].emplace_back(b, c);
14        e[b].emplace_back(a, c);
15        for (int j = 1; j <= k; j++) {
16            e[a + (j - 1) * n].emplace_back(b + j * n, 0);

```

```

17         e[b + (j - 1) * n].emplace_back(a + j * n, 0);
18         e[a + j * n].emplace_back(b + j * n, c);
19         e[b + j * n].emplace_back(a + j * n, c);
20     }
21 }
22
23 auto dijkstra = [&](int s) -> vl {};
24
25 vl dist = dijkstra(s);
26 LL ans = INF;
27 for (int i = t; i <= n * (k + 1); i += n) {
28     Min(ans, dist[i]);
29 }
30
31 std::cout << ans << endl;
32
33 return 0;
34 }

```

12.3 minimum spanning tree

Kruskal

```

1 std::vector<std::tuple<int, int, int>> edge;
2 auto kruskal = [&]() -> int {
3     std::sort(all(edge), [&](std::tuple<int, int, int> a, std::tuple<int, int, int> b) {
4         auto [x1, y1, w1] = a;
5         auto [x2, y2, w2] = b;
6         return w1 < w2;
7     });
8     int res = 0, cnt = 0;
9     for (int i = 0; i < m; i++) {
10         auto [a, b, w] = edge[i];
11         a = find(a), b = find(b);
12         if (a != b) {
13             fa[a] = b;
14             res += w;
15             /* res = std::max(res, w); */
16             cnt++;
17         }
18     }
19     if (cnt < n - 1) return -1;
20     return res;
21 }

```

12.4 SCC

Tarjan

```

1 vi dfn(n + 1), low(n + 1), stk(n + 1), belong(n + 1);
2 int timestamp = 0, top = 0, scc_cnt = 0;
3 std::vector<bool> in_stk(n + 1);
4 auto tarjan = [&](auto&& self, int u) -> void {
5     dfn[u] = low[u] = ++timestamp;
6     stk[++top] = u;
7     in_stk[u] = true;
8     for (const auto& v : e[u]) {
9         if (!dfn[v]) {
10             self(self, v);
11             Min(low[u], low[v]);
12         } else if (in_stk[v]) {
13             Min(low[u], dfn[v]);
14         }
15     }
16     if (dfn[u] == low[u]) {
17         scc_cnt++;
18         int v;
19         do {
20             v = stk[top--];
21             in_stk[v] = false;
22             belong[v] = scc_cnt;
23         } while (v != u);
24     }
25 };

```

12.4.1 缩点

12.5 DCC

点双连通分量

求点双连通分量.

```

1 vi dfn(n + 1), low(n + 1), is_bcc(n + 1), stk;
2 int timestamp = 0, bcc_cnt = 0, root = 0;
3 vvi bcc(2 * n + 1);
4 std::function<void(int, int)> tarjan = [&](int u, int fa) {
5     dfn[u] = low[u] = ++timestamp;
6     int child = 0;
7     stk.push_back(u);
8     if (u == root and e[u].empty()) {
9         bcc_cnt++;
10        bcc[bcc_cnt].push_back(u);
11        return;
12    }
13    for (auto v : e[u]) {
14        if (!dfn[v]) {
15            tarjan(v, u);
16            low[u] = std::min(low[u], low[v]);
17            if (low[v] >= dfn[u]) {
18                child++;
19                if (u != root or child > 1) {
20                    is_bcc[u] = 1;
21                }
22                bcc_cnt++;
23                int z;
24                do {
25                    z = stk.back();
26                    stk.pop_back();
27                    bcc[bcc_cnt].push_back(z);
28                } while (z != v);
29                bcc[bcc_cnt].push_back(u);
30            }
31        } else if (v != fa) {
32            low[u] = std::min(low[u], dfn[v]);
33        }
34    }
35 };
36 for (int i = 1; i <= n; i++) {
37     if (!dfn[i]) {
38         root = i;
39         tarjan(i, i);
40     }
41 }

```

求割点.

```

1 vi dfn(n + 1), low(n + 1), is_bcc(n + 1);
2 int timestamp = 0, bcc = 0, root = 0;
3 std::function<void(int, int)> tarjan = [&](int u, int fa) {
4     dfn[u] = low[u] = ++timestamp;
5     int child = 0;
6     for (auto v : e[u]) {
7         if (!dfn[v]) {
8             tarjan(v, u);
9             low[u] = std::min(low[u], low[v]);
10            if (low[v] >= dfn[u]) {
11                child++;
12                if ((u != root or child > 1) and !is_bcc[u]) {
13                    bcc++;
14                    is_bcc[u] = 1;
15                }
16            }
17        } else if (v != fa) {
18            low[u] = std::min(low[u], dfn[v]);
19        }
20    }
21 };
22 for (int i = 1; i <= n; i++) {
23     if (!dfn[i]) {
24         root = i;
25         tarjan(i, i);
26     }
27 }

```

边双连通分量

求边双连通分量.

```

1  std::vector<vpi> e(n + 1);
2  for (int i = 1; i <= m; i++) {
3      int u, v;
4      std::cin >> u >> v;
5      e[u].emplace_back(v, i);
6      e[v].emplace_back(u, i);
7  }
8  vi dfn(n + 1), low(n + 1), is_ecc(n + 1), fa(n + 1), stk;
9  int timestamp = 0, ecc_cnt = 0;
10 vvi ecc(2 * n + 1);
11 std::function<void(int, int)> tarjan = [&](int u, int id) {
12     low[u] = dfn[u] = ++timestamp;
13     stk.push_back(u);
14     for (auto [v, idx] : e[u]) {
15         if (!dfn[v]) {
16             tarjan(v, idx);
17             low[u] = std::min(low[u], low[v]);
18         } else if (idx != id) {
19             low[u] = std::min(low[u], dfn[v]);
20         }
21     }
22     if (dfn[u] == low[u]) {
23         ecc_cnt++;
24         int v;
25         do {
26             v = stk.back();
27             stk.pop_back();
28             ecc[ecc_cnt].push_back(v);
29         } while (v != u);
30     }
31 };
32 for (int i = 1; i <= n; i++) {
33     if (!dfn[i]) {
34         tarjan(i, 0);
35     }
36 }

```

求桥. (可能有诈)

```

1  vi dfn(n + 1), low(n + 1), is_ecc(n + 1), fa(n + 1);
2  int timestamp = 0, ecc = 0;
3  std::function<void(int, int)> tarjan = [&](int u, int faa) {
4      fa[u] = faa;
5      low[u] = dfn[u] = ++timestamp;
6      for (auto v : e[u]) {
7          if (!dfn[v]) {
8              tarjan(v, u);
9              low[u] = std::min(low[u], low[v]);
10             if (low[v] > dfn[u]) {
11                 is_ecc[v] = 1;
12                 ecc++;
13             }
14         } else if (dfn[v] < dfn[u] && v != faa) {
15             low[u] = std::min(low[u], dfn[v]);
16         }
17     }
18 };
19 for (int i = 1; i <= n; i++) {
20     if (!dfn[i]) {
21         tarjan(i, i);
22     }
23 }

```

```

1  // 割点
2  std::vector<int> G[N];
3  int dfn[N], low[N], is_cut[N], tm, rt;
4  void tar(int u) {
5      int c = 0;
6      dfn[u] = low[u] = ++tm;
7      for (int v : G[u]) {
8          if (!dfn[v]) {
9              ++c, tar(v);
10             low[u] = std::min(low[u], low[v]);
11             if (low[v] == dfn[u]) is_cut[u] = 1;
12         } else low[u] = std::min(low[u], dfn[v]);
13     }
14     if (u == rt) is_cut[u] = c > 1;
15 }

```

```

16 int main() {
17     int n, m;
18     std::cin >> n >> m;
19     for (int x, y; m--;) {
20         std::cin >> x >> y;
21         G[x].emplace_back(y);
22         G[y].emplace_back(x);
23     }
24     for (rt = 1; rt <= n; ++rt) {
25         if (!dfn[rt]) tar(rt);
26     }
27     std::vector<int> cut_v;
28     for (int i = 1; i <= n; ++i) {
29         if (is_cut[i]) cut_v.emplace_back(i);
30     }
31     std::cout << cut_v.size() << '\n';
32     for (int x : cut_v) std::cout << x << ' ';
33 }
34
35
36 // 桥
37 std::vector<std::pair<int, int>> G[N], brg;
38 int dfn[N], low[N], rt, tm;
39 void tar(int u, int fa, int fr) {
40     dfn[u] = low[u] = ++tm;
41     for (auto[v, i] : G[u]) {
42         if (!dfn[v]) {
43             tar(v, u, i), low[u] = std::min(low[u], low[v]);
44         } else if (i != fr) {
45             low[u] = std::min(low[u], dfn[v]);
46         }
47     }
48     if (u != rt && dfn[u] == low[u]) {
49         brg.emplace_back(std::minmax(u, fa));
50     }
51 }
52 int main() {
53     int n, m;
54     std::cin >> n >> m;
55     for (int i = 1, x, y; i <= m; ++i) {
56         std::cin >> x >> y;
57         G[x].emplace_back(y, i);
58         G[y].emplace_back(x, i);
59     }
60     for (rt = 1; rt <= n; ++rt) {
61         if (!dfn[rt]) tar(rt, -1, -1);
62     }
63     std::sort(begin(brg), end(brg));
64     for (auto[u, v] : brg) {
65         std::cout << u << ' ' << v << '\n';
66     }
67 }
68
69
70 // 点双
71 std::vector<int> G[N];
72 std::vector<std::vector<int>> bcc;
73 int dfn[N], low[N], tm, st[N], tp, rt;
74 void tar(int u) {
75     dfn[u] = low[u] = ++tm, st[++tp] = u;
76     if (G[u].empty()) bcc.push_back({u});
77     for (int v : G[u]) {
78         if (!dfn[v]) {
79             tar(v), low[u] = std::min(low[u], low[v]);
80             if (low[v] == dfn[u]) {
81                 bcc.push_back({u});
82                 do {
83                     bcc.back().emplace_back(st[tp]);
84                 } while(st[tp--] != v);
85             }
86         } else low[u] = std::min(low[u], dfn[v]);
87     }
88 }
89 int main() {
90     std::ios::sync_with_stdio(0);
91     std::cin.tie(0);
92     int n, m;
93     std::cin >> n >> m;
94     for (int x, y; m--;) {
95         std::cin >> x >> y;
96         G[x].emplace_back(y);
97         G[y].emplace_back(x);
98     }
99     for (int i = 0; i < n; ++i) tar(i);
100     std::cout << bcc.size() << '\n';
101     for (auto v : bcc) {
102         std::cout << v.size() << ' ';

```

```

103     for (int x : v) std::cout << x << ' ';
104     std::cout << '\n';
105 }
106 }
107
108
109 // 边双
110 std::vector<std::pair<int, int>> G[N];
111 std::vector<std::vector<int>> becc;
112 int dfn[N], low[N], tm, st[N], tp;
113 void tar(int u, int fr) {
114     dfn[u] = low[u] = ++tm, st[++tp] = u;
115     for (auto [v, i] : G[u]) {
116         if (!dfn[v]) {
117             tar(v, i), low[u] = std::min(low[u], low[v]);
118         } else if (i != fr) {
119             low[u] = std::min(low[u], dfn[v]);
120         }
121     }
122     if (dfn[u] == low[u]) {
123         becc.emplace_back();
124         do {
125             becc.back().emplace_back(st[tp]);
126         } while (st[tp--] != u);
127     }
128 }
129 int main() {
130     int n, m;
131     std::cin >> n >> m;
132     for (int i = 1, x, y; i <= m; ++i) {
133         std::cin >> x >> y;
134         G[x].emplace_back(y, i);
135         G[y].emplace_back(x, i);
136     }
137     for (int i = 0; i < n; ++i) {
138         if (!dfn[i]) tar(i, -1);
139     }
140     std::cout << becc.size() << '\n';
141     for (auto& v : becc) {
142         std::cout << v.size() << ' ';
143         for (int x : v) std::cout << x << ' ';
144         std::cout << '\n';
145     }
146 }

```

12.6 2 SAT

给出 n 个集合, 每个集合有 2 个元素, 已知若干个数对 (a, b) , 表示 a 与 b 矛盾. 要从每个集合各选择一个元素, 判断能否一共选 n 个两两不矛盾的元素.

```

1 auto twoSat = [&](int n, const vpi& v) -> vi {
2     /* tarjan */
3     vvi e(2 * n);
4     vi dfn(2 * n), low(2 * n), stk(2 * n), belong(2 * n);
5     int timestamp = 0, top = 0, scc_cnt = 0;
6     std::vector<bool> in_stk(2 * n);
7
8     auto tarjan = [&](auto&& self, int u) -> void {
9         dfn[u] = low[u] = ++timestamp;
10        stk[++top] = u;
11        in_stk[u] = true;
12        for (const auto& v : e[u]) {
13            if (!dfn[v]) {
14                self(self, v);
15                Min(low[u], low[v]);
16            } else if (in_stk[v]) {
17                Min(low[u], dfn[v]);
18            }
19        }
20        if (dfn[u] == low[u]) {
21            scc_cnt++;
22            int v;
23            do {
24                v = stk[top--];
25                in_stk[v] = false;
26                belong[v] = scc_cnt;
27            } while (v != u);
28        }
29    };
30    /* end tarjan */
31 }

```

```

32     for (const auto& [a, b] : v) {
33         e[a].push_back(b ^ 1);
34         e[b].push_back(a ^ 1);
35     }
36     for (int i = 0; i < 2 * n; i++) {
37         if (!dfn[i]) tarjan(tarjan, i);
38     }
39     vi ans;
40     for (int i = 0; i < 2 * n; i += 2) {
41         if (belong[i] == belong[i + 1]) return vi{};
42         ans.push_back(belong[i] > belong[i + 1] ? i + 1 : i);
43     }
44     return ans;
45 };

```

上述将 i 与 $i + 1$ 作为一个集合里的元素, 编号为 0 至 $2n - 1$.

12.7 minimum ring

Floyd

```

1  auto min_circle = [&]() -> int {
2      vvi dist(n + 1, vi(n + 1, inf));
3      for (int i = 1; i <= n; i++) {
4          for (int j = 1; j <= n; j++) {
5              Min(dist[i][j], g[i][j]);
6          }
7          dist[i][i] = 0;
8      }
9      for (int k = 1; k <= n; k++) {
10         for (int i = 1; i < k; i++) {
11             for (int j = 1; j < i; j++) {
12                 Min(ans, dist[i][j] + g[i][k] + g[k][j]);
13             }
14         }
15         for (int i = 1; i <= n; i++) {
16             for (int j = 1; j <= n; j++) {
17                 Min(dist[i][j], dist[i][k] + dist[k][j]);
18             }
19         }
20     }
21     return ans;
22 };

```

tree - diameter

12.8 tree - center of gravity

```

1  int sum; /* 点权和 */
2  vi size(n + 1), weight(n + 1), depth(n + 1);
3  std::array<int, 2> centroid = {0, 0};
4  auto get_centroid = [&](auto&& self, int u, int fa) -> void {
5      size[u] = w[u];
6      weight[u] = 0;
7      for (auto v : e[u]) {
8          if (v == fa) continue;
9          self(self, v, u);
10         size[u] += size[v];
11         Max(weight[u], size[v]);
12     }
13     Max(weight[u], sum - size[u]);
14     if (weight[u] <= sum / 2) {
15         centroid[centroid[0] != 0] = u;
16     }
17 };

```

12.9 tree - DSU on tree

给出一棵 n 个节点以 1 为根的树, 每个节点染上一种颜色, 询问以 u 为节点的子树中有多少种颜色.


```

1 // Problem: U41492 树上数颜色
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, m, dfn = 0, cnttot = 0;
9     std::cin >> n;
10    vvi e(n + 1);
11    vi siz(n + 1), col(n + 1), son(n + 1), dfnl(n + 1), dfnr(n + 1), rank(n + 1);
12    vi ans(n + 1), cnt(n + 1);
13
14    for (int i = 1; i < n; i++) {
15        int u, v;
16        std::cin >> u >> v;
17        e[u].push_back(v);
18        e[v].push_back(u);
19    }
20    for (int i = 1; i <= n; i++) {
21        std::cin >> col[i];
22    }
23    auto add = [&](int u) -> void {
24        if (cnt[col[u]] == 0) cnttot++;
25        cnt[col[u]]++;
26    };
27    auto del = [&](int u) -> void {
28        cnt[col[u]]--;
29        if (cnt[col[u]] == 0) cnttot--;
30    };
31    auto dfs1 = [&](auto&& self, int u, int fa) -> void {
32        dfnl[u] = ++dfn;
33        rank[dfn] = u;
34        siz[u] = 1;
35        for (auto v : e[u]) {
36            if (v == fa) continue;
37            self(self, v, u);
38            siz[u] += siz[v];
39            if (!son[u] or siz[son[u]] < siz[v]) son[u] = v;
40        }
41        dfnr[u] = dfn;
42    };
43    auto dfs2 = [&](auto&& self, int u, int fa, bool op) -> void {
44        for (auto v : e[u]) {
45            if (v == fa or v == son[u]) continue;
46            self(self, v, u, false);
47        }
48        if (son[u]) self(self, son[u], u, true);
49        for (auto v : e[u]) {
50            if (v == fa or v == son[u]) continue;
51            rep(i, dfnl[v], dfnr[v]) { add(rank[i]); }
52        }
53        add(u);
54        ans[u] = cnttot;
55        if (op == false) {
56            rep(i, dfnl[u], dfnr[u]) { del(rank[i]); }
57        }
58    };
59    dfs1(dfs1, 1, 0);
60    dfs2(dfs2, 1, 0, false);
61    std::cin >> m;
62    for (int i = 1; i <= m; i++) {
63        int u;
64        std::cin >> u;
65        std::cout << ans[u] << endl;
66    }
67    return 0;
68 }

```

12.10 tree - AHU

```

1 std::map<vi, int> mapple;
2 std::function<int(vvi&, int, int)> tree_hash = [&](vvi& e, int u, int fa) -> int {
3     vi code;
4     if (u == 0) code.push_back(-1);
5     for (auto v : e[u]) {
6         if (v == fa) continue;
7         code.push_back(tree_hash(e, v, u));
8     }
9     std::sort(all(code));
10    int id = mapple.size();

```

```

11     auto it = mapple.find(code);
12     if (it == mapple.end()) {
13         mapple[code] = id;
14     } else {
15         id = it->ss;
16     }
17     return id;
18 };

```

12.11 tree - LCA

```

1  vvi e(n + 1), fa(n + 1, vi(50));
2  vi dep(n + 1);
3
4  auto dfs = [&](auto&& self, int u) -> void {
5      for (auto v : e[u]) {
6          if (v == fa[u][0]) continue;
7          dep[v] = dep[u] + 1;
8          fa[v][0] = u;
9          self(self, v);
10     }
11 };
12
13 auto init = [&]() -> void {
14     dep[root] = 1;
15     dfs(dfs, root);
16     for (int j = 1; j <= 30; j++) {
17         for (int i = 1; i <= n; i++) {
18             fa[i][j] = fa[fa[i][j - 1]][j - 1];
19         }
20     }
21 };
22 init();
23
24 auto LCA = [&](int a, int b) -> int {
25     if (dep[a] > dep[b]) std::swap(a, b);
26     int d = dep[b] - dep[a];
27     for (int i = 0; (1 << i) <= d; i++) {
28         if (d & (1 << i)) b = fa[b][i];
29     }
30     if (a == b) return a;
31     for (int i = 30; i >= 0 and a != b; i--) {
32         if (fa[a][i] == fa[b][i]) continue;
33         a = fa[a][i];
34         b = fa[b][i];
35     }
36     return fa[a][0];
37 };
38
39 auto dist = [&](int a, int b) -> int { return dep[a] + dep[b] - dep[LCA(a, b)] * 2; };

```

12.12 tree - LLD

```

1  const int N = 5e5 + 7;
2  int n, q, rt, g[N], d[N], f[N][21], son[N], dep[N], top[N], ans;
3  vi e[N], u[N], v[N];
4  ui s;
5  ll Ans;
6
7  inline ui get(ui x) {
8      return x ^= x << 13, x ^= x >> 17, x ^= x << 5, s = x;
9  }
10
11 void dfs(int x) {
12     dep[x] = d[x] = d[f[x][0]] + 1;
13     for (auto y : e[x]) {
14         f[y][0] = x;
15         for (int i = 0; f[y][i]; i++) f[y][i+1] = f[f[y][i]][i];
16         dfs(y);
17         if (dep[y] > dep[x]) dep[x] = dep[y], son[x] = y;
18     }
19 }
20
21 void dfs(int x, int p) {
22     top[x] = p;
23     if (x == p) {
24         for (int i = 0, o = x; i <= dep[x] - d[x]; i++)
25             u[x].pb(o), o = f[o][0];

```

```

26     for (int i = 0, o = x; i <= dep[x] - d[x]; i++)
27         v[x].pb(o), o = son[o];
28 }
29 if (son[x]) dfs(son[x], p);
30 for (auto y : e[x]) if (y != son[x]) dfs(y, y);
31 }
32
33 inline int ask(int x, int k) {
34     if (!k) return x;
35     x = f[x][g[k]], k -= 1 << g[k], k -= d[x] - d[top[x]], x = top[x];
36     return k >= 0 ? u[x][k] : v[x][-k];
37 }
38
39 int main() {
40     rd(n), rd(q), rd(s), g[0] = -1;
41     for (int i = 1; i <= n; i++)
42         rd(f[i][0]), e[f[i][0]].pb(i), g[i] = g[i>>1] + 1;
43     rt = e[0][0], dfs(rt), dfs(rt, rt);
44     for (int i = 1, x, k; i <= q; i++) {
45         x = (get(s) ^ ans) % n + 1;
46         k = (get(s) ^ ans) % d[x];
47         Ans ^= 1ll * i * (ans = ask(x, k));
48     }
49     print(Ans);
50     return 0;
51 }

```

12.13 tree - HLD

对一棵有根树进行如下 4 种操作:

1. 1 $x\ y\ z$: 将节点 x 到节点 y 的最短路径上所有节点的值加上 z .
2. 2 $x\ y$: 查询节点 x 到节点 y 的最短路径上所有节点的值和.
3. 3 $x\ z$: 将以节点 x 为根的子树上所有节点的值加上 z .
4. 4 x : 查询以节点 x 为根的子树上所有节点的值和.

```

1  /* HLD */
2  int cnt = 0;
3  vi son(n + 1), fa(n + 1), siz(n + 1), depth(n + 1);
4  vi dfn(n + 1), rank(n + 1), top(n + 1), botton(n + 1);
5
6  auto dfs1 = [&](auto&& self, int u) -> void {
7      son[u] = -1, siz[u] = 1;
8      for (auto v : e[u]) {
9          if (depth[v] != 0) continue;
10         depth[v] = depth[u] + 1;
11         fa[v] = u;
12         self(self, v);
13         siz[u] += siz[v];
14         if (son[u] == -1 or siz[v] > siz[son[u]]) son[u] = v;
15     }
16 };
17
18 auto dfs2 = [&](auto&& self, int u, int t) -> void {
19     top[u] = t;
20     dfn[u] = ++cnt;
21     rank[cnt] = u;
22     botton[u] = dfn[u];
23     if (son[u] == -1) return;
24     self(self, son[u], t);
25     Max(botton[u], botton[son[u]]);
26     for (auto v : e[u]) {
27         if (v != son[u] and v != fa[u]) {
28             self(self, v, v);
29             Max(botton[u], botton[v]);
30         }
31     }
32 };
33
34 depth[root] = 1;
35 dfs1(dfs1, root);
36 dfs2(dfs2, root, root);
37
38 /*
39

```

```

40  /* 求 LCA */
41  auto LCA = [&](int a, int b) -> int {
42      while (top[a] != top[b]) {
43          if (depth[top[a]] < depth[top[b]]) std::swap(a, b);
44          a = fa[top[a]];
45      }
46      return (depth[a] > depth[b] ? b : a);
47  };
48
49  /* 维护 u 到 v 的路径 */
50  while (top[u] != top[v]) {
51      if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
52      opt(dfn[top[u]], dfn[u]);
53      u = fa[top[u]];
54  }
55  if (dfn[u] > dfn[v]) std::swap(u, v);
56  opt(dfn[u], dfn[v]);
57
58  /* 维护 u 为根的子树 */
59  opt(dfn[u], botton[u]);
60
61  /*
62  线段树的 build() 函数中
63  if(l == r) tree[u] = {l, l, w[rank[l]], 0};
64  */
65  */
66
67  build(1, 1, n);
68
69  for (int i = 1; i <= m; i++) {
70      int op, u, v;
71      LL k;
72      std::cin >> op;
73      if (op == 1) {
74          std::cin >> u >> v >> k;
75          while (top[u] != top[v]) {
76              if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
77              modify(1, dfn[top[u]], dfn[u], k);
78              u = fa[top[u]];
79          }
80          if (dfn[u] > dfn[v]) std::swap(u, v);
81          modify(1, dfn[u], dfn[v], k);
82      } else if (op == 2) {
83          std::cin >> u >> v;
84          LL ans = 0;
85          while (top[u] != top[v]) {
86              if (depth[top[u]] < depth[top[v]]) std::swap(u, v);
87              ans = (ans + query(1, dfn[top[u]], dfn[u])) % p;
88              u = fa[top[u]];
89          }
90          if (dfn[u] > dfn[v]) std::swap(u, v);
91          ans = (ans + query(1, dfn[u], dfn[v])) % p;
92          std::cout << ans << endl;
93      } else if (op == 3) {
94          std::cin >> u >> k;
95          modify(1, dfn[u], botton[u], k);
96      } else {
97          std::cin >> u;
98          std::cout << query(1, dfn[u], botton[u]) % p << endl;
99      }
100  }

```

12.14 tree - virtual tree

```

1  auto build_vtree = [&](vi ver) -> void {
2      std::sort(all(ver), [&](int x, int y) { return dfn[x] < dfn[y]; });
3      vi stk = {1};
4      for (auto v : ver) {
5          int u = stk.back();
6          int lca = LCA(v, u);
7          if (lca != u) {
8              while (dfn[lca] < dfn[stk.end()[-2]]) {
9                  g[stk.end()[-2]].push_back(stk.back());
10                 stk.pop_back();
11             }
12             u = stk.back();
13             if (dfn[lca] != dfn[stk.end()[-2]]) {
14                 g[lca].push_back(u);
15                 stk.pop_back();
16                 stk.push_back(lca);
17             } else {
18                 g[lca].push_back(u);

```

```

19         stk.pop_back();
20     }
21 }
22     stk.push_back(v);
23 }
24 while (stk.size() > 1) {
25     int u = stk.end()[-2];
26     int v = stk.back();
27     g[u].push_back(v);
28     stk.pop_back();
29 }
30 };

```

12.15 tree - pseudo tree

```

1  /* ring detection (directed) */
2  vi vis(n + 1), fa(n + 1), ring;
3  auto dfs = [&](auto&& self, int u) -> bool {
4      vis[u] = 1;
5      for (const auto& v : e[u]) {
6          if (!vis[v]) {
7              fa[v] = u;
8              if (self(self, v)) {
9                  return true;
10             }
11         } else if (vis[v] == 1) {
12             ring.push_back(v);
13             for (auto x = u; x != v; x = fa[x]) {
14                 ring.push_back(x);
15             }
16             reverse(all(ring));
17             return true;
18         }
19     }
20     vis[u] = 2;
21     return false;
22 };
23 for (int i = 1; i <= n; i++) {
24     if (!vis[i]) {
25         if (dfs(dfs, i)) {
26             // operations //
27         }
28     }
29 }
30
31 /* cycle detection (undirected) */
32 vi vis(n + 1), ring;
33 vpi fa(n + 1);
34 auto dfs = [&](auto&& self, int u, int from) -> bool {
35     vis[u] = 1;
36     for (const auto& [v, id] : e[u]) {
37         if (id == from) continue;
38         if (!vis[v]) {
39             fa[v] = {u, id};
40             if (self(self, v, id)) {
41                 return true;
42             }
43         } else if (vis[v] == 1) {
44             ring.push_back(v);
45             for (auto x = u; x != v; x = fa[x].ff) {
46                 ring.push_back(x);
47             }
48             return true;
49         }
50     }
51     vis[u] = 2;
52     return false;
53 };
54 for (int i = 1; i <= n; i++) {
55     if (!vis[i]) {
56         if (dfs(dfs, i, 0)) {
57             // operations //
58         }
59     }
60 }

```

12.16 tree - prufer sequence

prufer 序列：每次选择一个编号最小的叶节点并删掉它，然后在序列中记录下它连接到的那个节点。

```

1 for(int i=1;i<n;i++)cin>>fa[i],d[fa[i]]++;
2 for(int i=1,j=1;i<n-1;i++,j++){
3     while(d[j])j++;
4     p[i]=fa[j];
5     while(i<n-1&&!--d[p[i]]&&j>p[i])p[i+1]=fa[p[i]],i++;
6 }

```

```

1 for(int i=1;i<n-1;i++)cin>>p[i],d[p[i]]++;
2 p[n-1]=n;
3 for(int i=1,j=1;i<n;i++,j++){
4     while(d[j])j++;
5     fa[j]=p[i];
6     while(i<n&&!--d[p[i]]&&j>p[i])fa[p[i]]=p[i+1],i++;
7 }

```

12.17 tree - divide and conquer on tree

点分治

第一个题

一棵 $n \leq 10^4$ 个点的树，边权 $w \leq 10^4$ 。 $m \leq 100$ 次询问树上是否存在长度为 $k \leq 10^7$ 的路径。

```

1 // 洛谷 P3806 【模板】点分治1
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, m, k;
9     std::cin >> n >> m;
10
11     std::vector<vpi> e(n + 1);
12     std::map<int, PII> mp;
13
14     for (int i = 1; i < n; i++) {
15         int u, v, w;
16         std::cin >> u >> v >> w;
17         e[u].emplace_back(v, w);
18         e[v].emplace_back(u, w);
19     }
20     for (int i = 1; i <= m; i++) {
21         std::cin >> k;
22         mp[i] = {k, 0};
23     }
24
25     /* centroid decomposition */
26     int top1 = 0, top2 = 0, root;
27     vi len1(n + 1), len2(n + 1), vis(n + 1);
28     static std::array<int, 20000010> cnt;
29
30     std::function<int(int, int)> get_size = [&](int u, int fa) -> int {
31         if (vis[u]) return 0;
32         int ans = 1;
33         for (auto [v, w] : e[u]) {
34             if (v == fa) continue;
35             ans += get_size(v, u);
36         }
37         return ans;
38     };
39
40     std::function<int(int, int, int, int)> get_root = [&](int u, int fa, int tot,
41                                                         int& root) -> int {
42         if (vis[u]) return 0;
43         int sum = 1, maxx = 0;
44         for (auto [v, w] : e[u]) {
45             if (v == fa) continue;
46             int tmp = get_root(v, u, tot, root);
47             Max(maxx, tmp);
48             sum += tmp;
49         }
50         Max(maxx, tot - sum);

```

```

51     if (2 * maxx <= tot) root = u;
52     return sum;
53 };
54
55 std::function<void(int, int, int)> get_dist = [&](int u, int fa, int dist) -> void {
56     if (dist <= 10000000) len1[++top1] = dist;
57     for (auto [v, w] : e[u]) {
58         if (v == fa or vis[v]) continue;
59         get_dist(v, u, dist + w);
60     }
61 };
62
63 auto solve = [&](int u, int dist) -> void {
64     top2 = 0;
65     for (auto [v, w] : e[u]) {
66         if (vis[v]) continue;
67         top1 = 0;
68         get_dist(v, u, w);
69         for (int i = 1; i <= top1; i++) {
70             for (int tt = 1; tt <= m; tt++) {
71                 int k = mp[tt].ff;
72                 if (k >= len1[i]) mp[tt].ss |= cnt[k - len1[i]];
73             }
74         }
75         for (int i = 1; i <= top1; i++) {
76             len2[++top2] = len1[i];
77             cnt[len1[i]] = 1;
78         }
79     }
80     for (int i = 1; i <= top2; i++) cnt[len2[i]] = 0;
81 };
82
83 std::function<void(int)> divide = [&](int u) -> void {
84     vis[u] = cnt[0] = 1;
85     solve(u, 0);
86     for (auto [v, w] : e[u]) {
87         if (vis[v]) continue;
88         get_root(v, u, get_size(v, u), root);
89         divide(root);
90     }
91 };
92
93 get_root(1, 0, get_size(1, 0), root);
94 divide(root);
95
96 for (int i = 1; i <= m; i++) {
97     if (mp[i].ss == 0) {
98         std::cout << "NAY" << endl;
99     } else {
100         std::cout << "AYE" << endl;
101     }
102 }
103
104 return 0;
105 }

```

第二个题

一棵 $n \leq 4 \times 10^4$ 个点的树, 边权 $w \leq 10^3$. 询问树上长度不超过 $k \leq 2 \times 10^4$ 的路径的数量.

```

1 // 洛谷 P4178 Tree
2
3 int main() {
4     std::ios::sync_with_stdio(false);
5     std::cin.tie(0);
6     std::cout.tie(0);
7
8     int n, k;
9     std::cin >> n;
10    std::vector<vpi> e(n + 1);
11    for (int i = 1; i < n; i++) {
12        int u, v, w;
13        std::cin >> u >> v >> w;
14        e[u].emplace_back(v, w);
15        e[v].emplace_back(u, w);
16    }
17    std::cin >> k;
18
19    /* centroid decomposition */
20    int root;
21    vi len, vis(n + 1);
22
23    std::function<int(int, int)> get_size = [&](int u, int fa) -> int {
24        if (vis[u]) return 0;
25        int ans = 1;

```

```

26     for (auto [v, w] : e[u]) {
27         if (v == fa) continue;
28         ans += get_size(v, u);
29     }
30     return ans;
31 };
32
33 std::function<int(int, int, int, int)> get_root = [&](int u, int fa, int tot,
34                                                     int& root) -> int {
35     if (vis[u]) return 0;
36     int sum = 1, maxx = 0;
37     for (auto [v, w] : e[u]) {
38         if (v == fa) continue;
39         int tmp = get_root(v, u, tot, root);
40         maxx = std::max(maxx, tmp);
41         sum += tmp;
42     }
43     maxx = std::max(maxx, tot - sum);
44     if (2 * maxx <= tot) root = u;
45     return sum;
46 };
47
48 std::function<void(int, int, int)> get_dist = [&](int u, int fa, int dist) -> void {
49     len.push_back(dist);
50     for (auto [v, w] : e[u]) {
51         if (v == fa || vis[v]) continue;
52         get_dist(v, u, dist + w);
53     }
54 };
55
56 auto solve = [&](int u, int dist) -> int {
57     len.clear();
58     get_dist(u, 0, dist);
59     std::sort(all(len));
60     int ans = 0;
61     for (int l = 0, r = len.size() - 1; l < r;) {
62         if (len[l] + len[r] <= k) {
63             ans += r - l++;
64         } else {
65             r--;
66         }
67     }
68     return ans;
69 };
70
71 std::function<int(int)> divide = [&](int u) -> int {
72     vis[u] = true;
73     int ans = solve(u, 0);
74     for (auto [v, w] : e[u]) {
75         if (vis[v]) continue;
76         ans -= solve(v, w);
77         get_root(v, u, get_size(v, u), root);
78         ans += divide(root);
79     }
80     return ans;
81 };
82
83 get_root(1, 0, get_size(1, 0), root);
84 std::cout << divide(root) << endl;
85
86 return 0;
87 }

```

12.18 network flow - maximal flow

Dinic

理论

通过 BFS 将网络根据点到原点的距离 (每条边长度定义为 1) 分层, 然后通过 DFS 暴力地在有效的网络中寻找增广路, 不断循环上述步骤直至图中不存在增广路.

BFS 逻辑:

$u \rightarrow v$ 的条件满足下面两条:

1. v 未必走过;

2. $e: u \rightarrow v$ 上还有残余流量, 即当前 e 的流量未达到其上限.

DFS 逻辑:

维护两个值: u : 当前搜索到哪个点; now : 可以增加的流量. $u \rightarrow v$ 的条件:

1. 在上一次 BFS 时, v 在 u 下面一层, 即 $d_v = d_u + 1$.
2. 递归 $\text{dfs}(v, \text{now})$, 这时可增加的流量上限要与 $e: u \rightarrow v$ 中可增加的流量上限取最小值, 递归结果大于零才意味着可以增加流量.

优化:

1. 一次可以处理多条增广路.
2. 每一条有向边事实上只会增加一次流量, 引入 cur 记录处理到了每个点的哪一条边以加快 DFS.

```

1 struct edge {
2     int from, to;
3     LL cap, flow;
4
5     edge(int u, int v, LL c, LL f) : from(u), to(v), cap(c), flow(f) {}
6 };
7
8 struct Dinic {
9     int n, m = 0, s, t;
10    std::vector<edge> e;
11    vi g[N];
12    int d[N], cur[N], vis[N];
13
14    void init(int n) {
15        for (int i = 0; i < n; i++) g[i].clear();
16        e.clear();
17        m = 0;
18    }
19
20    void add(int from, int to, LL cap) {
21        e.push_back(edge(from, to, cap, 0));
22        e.push_back(edge(to, from, 0, 0));
23        g[from].push_back(m++);
24        g[to].push_back(m++);
25    }
26
27    bool bfs() {
28        for (int i = 1; i <= n; i++) {
29            vis[i] = 0;
30        }
31        std::queue<int> q;
32        q.push(s), d[s] = 0, vis[s] = 1;
33        while (!q.empty()) {
34            int u = q.front();
35            q.pop();
36            for (int i = 0; i < g[u].size(); i++) {
37                edge& ee = e[g[u][i]];
38                if (!vis[ee.to] and ee.cap > ee.flow) {
39                    vis[ee.to] = 1;
40                    d[ee.to] = d[u] + 1;
41                    q.push(ee.to);
42                }
43            }
44        }
45        return vis[t];
46    }
47
48    LL dfs(int u, LL now) {
49        if (u == t || now == 0) return now;
50        LL flow = 0, f;
51        for (int& i = cur[u]; i < g[u].size(); i++) {
52            edge& ee = e[g[u][i]];
53            edge& er = e[g[u][i] ^ 1];
54            if (d[u] + 1 == d[ee.to] and (f = dfs(ee.to, std::min(now, ee.cap - ee.flow))) > 0) {
55                ee.flow += f, er.flow -= f;
56                flow += f, now -= f;
57                if (now == 0) break;
58            }
59        }
60        return flow;

```

```

61     }
62
63     LL dinic() {
64         LL ans = 0;
65         while (bfs()) {
66             for (int i = 1; i <= n; i++) cur[i] = 0;
67             ans += dfs(s, INF);
68         }
69         return ans;
70     }
71 } maxf;

```

HLPP

抄板子吧, 别管原理了, 留一个图吧.

```

1  struct HLPP {
2      int n, m = 0, s, t;
3      std::vector<edge> e;      /* 边 */
4      std::vector<node> nd;     /* 点 */
5      std::vector<int> g[N];    /* 点的连边编号 */
6      std::priority_queue<node> q;
7      std::queue<int> qq;
8      bool vis[N];
9      int cnt[N];
10
11     void init() {
12         e.clear();
13         nd.clear();
14         for (int i = 0; i <= n + 1; i++) {
15             nd.pushback(node(inf, i, 0));
16             g[i].clear();
17             vis[i] = false;
18         }
19     }
20
21     void add(int u, int v, LL w) {
22         e.pushback(edge(u, v, w));
23         e.pushback(edge(v, u, 0));
24         g[u].pushback(m++);
25         g[v].pushback(m++);
26     }
27
28     void bfs() {
29         nd[t].hight = 0;
30         qq.push(t);
31         while (!qq.empty()) {
32             int u = qq.front();
33             qq.pop();
34             vis[u] = false;
35             for (auto j : g[u]) {
36                 int v = e[j].to;
37                 if (e[j].cap == 0 && nd[v].hight > nd[u].hight + 1) {
38                     nd[v].hight = nd[u].hight + 1;
39                     if (vis[v] == false) {
40                         qq.push(v);
41                         vis[v] = true;
42                     }
43                 }
44             }
45         }
46         return;
47     }
48
49     void _push(int u) {
50         for (auto j : g[u]) {
51             edge &ee = e[j], &er = e[j ^ 1];
52             int v = ee.to;
53             node &nu = nd[u], &nv = nd[v];
54             if (ee.cap && nv.hight + 1 == nu.hight) {
55                 LL flow = std::min(ee.cap, nu.flow);
56                 ee.cap -= flow, er.cap += flow;
57                 nu.flow -= flow, nv.flow += flow;
58                 if (vis[v] == false && v != t && v != s) {
59                     q.push(nv);
60                     vis[v] = true;
61                 }
62                 if (nu.flow == 0) break;
63             }
64         }
65     }
66 }

```

```

67 void relabel(int u) {
68     nd[u].hight = inf;
69     for (auto j : g[u]) {
70         int v = e[j].to;
71         if (e[j].cap && nd[v].hight + 1 < nd[u].hight) {
72             nd[u].hight = nd[v].hight + 1;
73         }
74     }
75 }
76
77 LL hlpp() {
78     bfs();
79     if (nd[s].hight == inf) return 0;
80     nd[s].hight = n;
81     for (int i = 1; i <= n; i++) {
82         if (nd[i].hight < inf) cnt[nd[i].hight]++;
83     }
84     for (auto j : g[s]) {
85         int v = e[j].to;
86         int flow = e[j].cap;
87         if (flow) {
88             e[j].cap -= flow, e[j ^ 1].cap += flow;
89             nd[s].flow -= flow, nd[v].flow += flow;
90             if (vis[v] == false && v != s && v != t) {
91                 q.push(nd[v]);
92                 vis[v] = true;
93             }
94         }
95     }
96     while (!q.empty()) {
97         int u = q.top().id;
98         q.pop();
99         vis[u] = false;
100         _push(u);
101         if (nd[u].flow) {
102             cnt[nd[u].hight]--;
103             if (cnt[nd[u].hight] == 0) {
104                 for (int i = 1; i <= n; i++) {
105                     if (i != s && i != t && nd[i].hight > nd[u].hight && nd[i].hight < n + 1) {
106                         nd[i].hight = n + 1;
107                     }
108                 }
109             }
110             relabel(u);
111             cnt[nd[u].hight]++;
112             q.push(nd[u]);
113             vis[u] = true;
114         }
115     }
116     return nd[t].flow;
117 }
118 } maxf;

```

12.19 network flow - minimum cost flow

在网络中获得最大流的同时要求费用最小。

Dinic + SPFA

```

1 struct edge {
2     int from, to;
3     LL cap, cost;
4
5     edge(int u, int v, LL c, LL w) : from(u), to(v), cap(c), cost(w) {}
6 };
7
8 struct MCMF {
9     int n, m = 0, s, t;
10    std::vector<edge> e;
11    vi g[N];
12    int cur[N], vis[N];
13    LL dist[N], minc;
14
15    void init(int n) {
16        for (int i = 0; i < n; i++) g[i].clear();
17        e.clear();
18        minc = m = 0;
19    }
20 }

```

```

21 void add(int from, int to, LL cap, LL cost) {
22     e.push_back(edge(from, to, cap, cost));
23     e.push_back(edge(to, from, 0, -cost));
24     g[from].push_back(m++);
25     g[to].push_back(m++);
26 }
27
28 bool spfa() {
29     rep(i, 1, n) { dist[i] = INF, cur[i] = 0; }
30     std::queue<int> q;
31     q.push(s), dist[s] = 0, vis[s] = 1;
32     while (!q.empty()) {
33         int u = q.front();
34         q.pop();
35         vis[u] = 0;
36         for (int j = cur[u]; j < g[u].size(); j++) {
37             edge& ee = e[g[u][j]];
38             int v = ee.to;
39             if (ee.cap && dist[v] > dist[u] + ee.cost) {
40                 dist[v] = dist[u] + ee.cost;
41                 if (!vis[v]) {
42                     q.push(v);
43                     vis[v] = 1;
44                 }
45             }
46         }
47     }
48     return dist[t] != INF;
49 }
50
51 LL dfs(int u, LL now) {
52     if (u == t) return now;
53     vis[u] = 1;
54     LL ans = 0;
55     for (int& i = cur[u]; i < g[u].size() && ans < now; i++) {
56         edge &ee = e[g[u][i]], &er = e[g[u][i] ^ 1];
57         int v = ee.to;
58         if (!vis[v] && ee.cap && dist[v] == dist[u] + ee.cost) {
59             LL f = dfs(v, std::min(ee.cap, now - ans));
60             if (f) {
61                 minc += f * ee.cost, ans += f;
62                 ee.cap -= f;
63                 er.cap += f;
64             }
65         }
66     }
67     vis[u] = 0;
68     return ans;
69 }
70
71 PLL mcmf() {
72     LL maxf = 0;
73     while (spfa()) {
74         LL tmp;
75         while ((tmp = dfs(s, INF))) maxf += tmp;
76     }
77     return std::makepair(maxf, minc);
78 }
79 } minc_maxf;

```

Primal-Dual 原始对偶算法

```

1 struct edge {
2     int from, to;
3     LL cap, cost;
4
5     edge(int u, int v, LL c, LL w) : from(u), to(v), cap(c), cost(w) {}
6 };
7
8 struct node {
9     int v, e;
10
11     node(int _v = 0, int _e = 0) : v(_v), e(_e) {}
12 };
13
14 const int maxn = 5000 + 10;
15
16 struct MCMF {
17     int n, m = 0, s, t;
18     std::vector<edge> e;
19     vi g[maxn];
20     int dis[maxn], vis[maxn], h[maxn];
21     node p[maxn * 2];

```

```

22
23 void add(int from, int to, LL cap, LL cost) {
24     e.push_back(edge(from, to, cap, cost));
25     e.push_back(edge(to, from, 0, -cost));
26     g[from].push_back(m++);
27     g[to].push_back(m++);
28 }
29
30 bool dijkstra() {
31     std::priority_queue<PII, std::vector<PII>, std::greater<PII>> q;
32     for (int i = 1; i <= n; i++) {
33         dis[i] = inf;
34         vis[i] = 0;
35     }
36     dis[s] = 0;
37     q.push({0, s});
38     while (!q.empty()) {
39         int u = q.top().ss;
40         q.pop();
41         if (vis[u]) continue;
42         vis[u] = 1;
43         for (auto i : g[u]) {
44             edge ee = e[i];
45             int v = ee.to, nc = ee.cost + h[u] - h[v];
46             if (ee.cap and dis[v] > dis[u] + nc) {
47                 dis[v] = dis[u] + nc;
48                 p[v] = node(u, i);
49                 if (!vis[v]) q.push({dis[v], v});
50             }
51         }
52     }
53     return dis[t] != inf;
54 }
55
56 void spfa() {
57     std::queue<int> q;
58     for (int i = 1; i <= n; i++) h[i] = inf;
59     h[s] = 0, vis[s] = 1;
60     q.push(s);
61     while (!q.empty()) {
62         int u = q.front();
63         q.pop();
64         vis[u] = 0;
65         for (auto i : g[u]) {
66             edge ee = e[i];
67             int v = ee.to;
68             if (ee.cap and h[v] > h[u] + ee.cost) {
69                 h[v] = h[u] + ee.cost;
70                 if (!vis[v]) {
71                     vis[v] = 1;
72                     q.push(v);
73                 }
74             }
75         }
76     }
77 }
78
79 PLL mcmf() {
80     LL maxf = 0, minc = 0;
81     spfa();
82     while (dijkstra()) {
83         LL minf = INF;
84         for (int i = 1; i <= n; i++) h[i] += dis[i];
85         for (int i = t; i != s; i = p[i].v) minf = std::min(minf, e[p[i].e].cap);
86         for (int i = t; i != s; i = p[i].v) {
87             e[p[i].e].cap -= minf;
88             e[p[i].e ^ 1].cap += minf;
89         }
90         maxf += minf;
91         minc += minf * h[t];
92     }
93     return std::make_pair(maxf, minc);
94 }
95 } minc_maxf;

```

存在负环的网络

12.20 network flow - minimal cut

最小割解决的问题是将图中的点集 V 划分成 S 与 T , 使得 S 与 T 之间的连边的容量总和最小.

最大流最小割定理

网络中 s 到 t 的最大流流量的值等于所要求的最小割的值, 所以求最小割只需要跑 Dinic 即可.

获得 S 中的所有点

在 Dinic 的 bfs 函数中, 每次将所有点的 d 数组值改为无穷大, 最后跑完最大流之后 d 数组不为无穷大的就是和源点一起在 S 集中的点.

例子

最小割的本质是对图中点集进行 2-划分, 网络流只是求解答案的手段.

1. 在图中花费最小的代价断开一些边使得源点 s 无法流到汇点 t .
直接跑最大流就得到了答案.

2. 在图中删除最少的点使得源点 s 无法流到汇点 t .

对每个点进行拆点, 在 i 与 i' 之间建立容量为 1 的有向边.

12.21 matching - matching on bipartite graph

二分图最大匹配

Kuhn-Munkres

时间复杂度: $O(n^3)$.

```

1 auto KM = [&](int n1, int n2, vvi e) -> std::pair<vi, vi> {
2     vi vis(n2 + 1);
3     vi l(n1 + 1, -1), r(n2 + 1, -1);
4     std::function<bool(int)> dfs = [&](int u) -> bool {
5         for (auto v : e[u]) {
6             if (!vis[v]) {
7                 vis[v] = 1;
8                 if (r[v] == -1 or dfs(r[v])) {
9                     r[v] = u;
10                    return true;
11                }
12            }
13        }
14        return false;
15    };
16    for (int i = 1; i <= n1; i++) {
17        std::fill(all(vis), 0);
18        dfs(i);
19    }
20    for (int i = 1; i <= n2; i++) {
21        if (r[i] == -1) continue;
22        l[r[i]] = i;
23    }
24    return {l, r};
25 };
26 auto [mchl, mchr] = KM(n1, n2, e);
27 std::cout << mchl.size() - std::count(all(mchl), -1) << endl;

```

Hopcroft-Karp

据说时间复杂度是 $O(m\sqrt{n})$ 的, 但是快的飞起.

```

1 vpi e(m);
2 auto hopcroft_karp = [&](int n, int m, vpi& e) -> std::pair<vi, vi> {

```

```

3   vi g(e.size()), l(n + 1, -1), r(m + 1, -1), d(n + 2);
4   for (auto [u, v] : e) d[u]++;
5   std::partial_sum(all(d), d.begin());
6   for (auto [u, v] : e) g[--d[u]] = v;
7   for (vi a, p, q(n + 1);) {
8       a.assign(n + 1, -1);
9       p.assign(n + 1, -1);
10      int t = 1;
11      for (int i = 1; i <= n; i++) {
12          if (l[i] == -1) {
13              q[t++] = a[i] = p[i] = i;
14          }
15      }
16      bool match = false;
17      for (int i = 1; i < t; i++) {
18          int u = q[i];
19          if (l[a[u]] != -1) continue;
20          for (int j = d[u]; j < d[u + 1]; j++) {
21              int v = g[j];
22              if (r[v] == -1) {
23                  while (v != -1) {
24                      r[v] = u;
25                      std::swap(l[u], v);
26                      u = p[u];
27                  }
28                  match = true;
29                  break;
30              }
31              if (p[r[v]] == -1) {
32                  q[t++] = v = r[v];
33                  p[v] = u;
34                  a[v] = a[u];
35              }
36          }
37      }
38      if (!match) break;
39  }
40  return {l, r};
41 };

```

二分图最大权匹配

Kuhn-Munkres

注意是否为完美匹配, 非完美选 0, 完美选 $-INF$. (存疑)

```

1   auto KM = [&](int n, vvl e) -> std::tuple<LL, vi, vi> {
2       vl la(n + 1), lb(n + 1), pp(n + 1), vx(n + 1);
3       vi l(n + 1, -1), r(n + 1, -1);
4       vi va(n + 1), vb(n + 1);
5       LL delta;
6       auto bfs = [&](int x) -> void {
7           int a, y = 0, y1 = 0;
8           std::fill(all(pp), 0);
9           std::fill(all(vx), INF);
10          r[y] = x;
11          do {
12              a = r[y], delta = INF, vb[y] = 1;
13              for (int b = 1; b <= n; b++) {
14                  if (!vb[b]) {
15                      if (vx[b] > la[a] + lb[b] - e[a][b]) {
16                          vx[b] = la[a] + lb[b] - e[a][b];
17                          pp[b] = y;
18                      }
19                      if (vx[b] < delta) {
20                          delta = vx[b];
21                          y1 = b;
22                      }
23                  }
24              }
25              for (int b = 0; b <= n; b++) {
26                  if (vb[b]) {
27                      la[r[b]] -= delta;
28                      lb[b] += delta;
29                  } else
26                     vx[b] -= delta;
31              }
32              y = y1;
33          } while (r[y] != -1);
34          while (y) {
35              r[y] = r[pp[y]];

```

```
36 |         y = pp[y];
37 |     }
38 | };
39 | for (int i = 1; i <= n; i++) {
40 |     std::fill(all(vb), 0);
41 |     bfs(i);
42 | }
43 | LL ans = 0;
44 | for (int i = 1; i <= n; i++) {
45 |     if (r[i] == -1) continue;
46 |     l[r[i]] = i;
47 |     ans += e[r[i]][i];
48 | }
49 | return {ans, l, r};
50 | };
51 |
52 | auto [ans, mchl, mchr] = KM(n, e);
```

12.22 matching - matching on general graph

13 geometry

13.1 two demention

点与向量

```

1  tandu struct pnt {
2      T x, y;
3
4      pnt(T _x = 0, T _y = 0) { x = _x, y = _y; }
5
6      pnt operator+(const pnt& a) const { return pnt(x + a.x, y + a.y); }
7
8      pnt operator-(const pnt& a) const { return pnt(x - a.x, y - a.y); }
9
10     /*
11     bool operator<(const pnt& a) const {
12         if (std::is_same<T, double>::value) {
13             if (fabs(x - a.x) < eps) return y < a.y;
14         } else {
15             if (x == a.x) return y < a.y;
16         }
17         return x < a.x;
18     }
19     */
20
21     /* 注意数乘会不会爆 int */
22     pnt operator*(const T k) const { return pnt(k * x, k * y); }
23
24     U operator*(const pnt& a) const { return (U) x * a.x + (U) y * a.y; }
25
26     U operator^(const pnt& a) const { return (U) x * a.y - (U) y * a.x; }
27
28     U dist(const pnt a) { return ((U) a.x - x) * ((U) a.x - x) + ((U) a.y - y) * ((U) a.y - y); }
29
30     U len() { return dist(pnt(0, 0)); }
31
32     /* a, b, c 成逆时针 */
33     friend U area(pnt a, pnt b, pnt c) { return (b - a) ^ (c - a); }
34
35     /* 两向量夹角, 返回 cos 值 */
36     double get_angle(pnt a) {
37         return (double) (pnt(x, y) * a) / sqrt((double) pnt(x, y).len() * (double) a.len());
38     }
39 };

```

线段

```

1  struct line {
2      point a, b;
3
4      line(point _a = {}, point _b = {}) { a = _a, b = _b; }
5
6      /* 交点类型为 double */
7      friend point iPoint(line p, line q) {
8          point v1 = p.b - p.a;
9          point v2 = q.b - q.a;
10         point u = q.a - p.a;
11         return q.a + (q.b - q.a) * ((u ^ v1) * 1. / (v1 ^ v2));
12     }
13
14     /* 极角排序 */
15     bool operator<(const line& p) const {
16         double t1 = std::atan2((b - a).y, (b - a).x);
17         double t2 = std::atan2((p.b - p.a).y, (p.b - p.a).x);
18         if (fabs(t1 - t2) > eps) {
19             return t1 < t2;
20         }
21         return ((p.a - a) ^ (p.b - a)) > eps;
22     }
23 };

```

13.2 convex

2D

```

1 auto andrew = [&](std::vector<point>& v) -> std::vector<point> {
2     std::sort(all(v));
3     std::vector<point> stk;
4     for (int i = 0; i < n; i++) {
5         point x = v[i];
6         while (stk.size() > 1 and ((stk.end()[-1] - stk.end()[-2]) ^ (x - stk.end()[-2])) <= 0) {
7             stk.pop_back();
8         }
9         stk.push_back(x);
10    }
11    int tmp = stk.size();
12    for (int i = n - 2; i >= 0; i--) {
13        point x = v[i];
14        while (stk.size() > tmp and ((stk.end()[-1] - stk.end()[-2]) ^ (x - stk.end()[-2])) <= 0) {
15            stk.pop_back();
16        }
17        stk.push_back(x);
18    }
19    return stk;
20 };

```

旋转卡壳

```

1 #include<cstdio>
2 #include<algorithm>
3 #define db double
4 namespace Acc{
5     const int N = 5e4+10;
6     struct node{
7         int x,y;
8     }a[N],stk[N];
9     db cmp(node a,node b,node c){return 1.*(c.x-a.x)*(c.y-b.y)-1.*(c.x-b.x)*(c.y-a.y);}
10    int dis(node a,node b){return ((a.x-b.x)*(a.x-b.x)+(a.y-b.y)*(a.y-b.y));}
11    int n,tp,ans;
12    void work(){
13        scanf("%d",&n);
14        for(int i=1;i<=n;i++)scanf("%d%d",&a[i].x,&a[i].y);
15        std::sort(a+1,a+n+1,[&](node a,node b)->bool{return a.x<b.x || (a.x==b.x && a.y<b.y)});
16        stk[1]=a[1],tp=1;
17        for(int i=2;i<=n;i++){
18            while(tp>1 && cmp(stk[tp-1],stk[tp],a[i])<=0)tp--;
19            stk[++tp]=a[i];
20        }
21        int tmp=tp;
22        for(int i=n-1;i>=1;i--){
23            while(tp>tmp && cmp(stk[tp-1],stk[tp],a[i])<=0)tp--;
24            stk[++tp]=a[i];
25        }
26        for(int i=1,j=3;i<tp;i++){
27            while(cmp(stk[i],stk[i+1],stk[j])<cmp(stk[i],stk[i+1],stk[j+1]))j=j%(tp-1)+1;
28            ans=std::max(ans,std::max(dis(stk[i],stk[j]),dis(stk[i+1],stk[j])));
29        }
30        printf("%d",ans);
31    }
32 }
33 int main(){
34     return Acc::work(),0;
35 }

```

half plane

```

1 auto halfPlane = [&](std::vector<line>& ln) -> std::vector<point> {
2     std::sort(all(ln));
3     ln.erase(
4         unique(
5             all(ln),
6             [](line& p, line& q) {
7                 double t1 = std::atan2((p.b - p.a).y, (p.b - p.a).x);
8                 double t2 = std::atan2((q.b - q.a).y, (q.b - q.a).x);
9                 return fabs((t1 - t2)) < eps;

```

```

10     }},
11     ln.end());
12     auto check = [&](line p, line q, line r) -> bool {
13         point a = iPoint(p, q);
14         return ((r.b - r.a) ^ (a - r.a)) < -eps;
15     };
16     line q[ln.size() + 2];
17     int hh = 1, tt = 0;
18     q[++tt] = ln[0];
19     q[++tt] = ln[1];
20     for (int i = 2; i < (int) ln.size(); i++) {
21         while (hh < tt and check(q[tt - 1], q[tt], ln[i])) tt--;
22         while (hh < tt and check(q[hh + 1], q[hh], ln[i])) hh++;
23         q[++tt] = ln[i];
24     }
25     while (hh < tt and check(q[tt - 1], q[tt], q[hh])) tt--;
26     while (hh < tt and check(q[hh + 1], q[hh], q[tt])) hh++;
27     q[tt + 1] = q[hh];
28     std::vector<point> ans;
29     for (int i = hh; i <= tt; i++) {
30         ans.push_back(iPoint(q[i], q[i + 1]));
31     }
32     return ans;
33 };

```

14 sweep line

矩形面积并

```

1  #define int long long
2  const int N = 2e5+10;
3  int b[N<<1], n, len, ans;
4  struct node{
5      int y1, y2, x, k;
6  } a[N<<1];
7  struct Seg{
8      #define lc (o<<1)
9      #define rc (o<<1|1)
10     static const int N = 5e6+10;
11     int sum[N], cnt[N], tag[N];
12     void push_up(int o, int l, int r){
13         if(sum[o]) cnt[o] = b[r+1] - b[l];
14         else cnt[o] = cnt[lc] + cnt[rc];
15     }
16     void add(int o, int l, int r, int L, int R, int k){
17         if(r < L || l > R) return;
18         if(l == L && r == R) return (void)(sum[o] += k, push_up(o, l, r));
19         int mid = L + R >> 1;
20         if(r <= mid) add(lc, l, r, L, mid, k);
21         else if(l > mid) add(rc, l, r, mid+1, R, k);
22         else add(lc, l, mid, L, mid, k), add(rc, mid+1, r, mid+1, R, k);
23         push_up(o, L, R);
24     }
25 #undef lc
26 #undef rc
27 } t;
28 void work(){
29     cin >> n;
30     for(int i = 1; i <= n; i++){
31         cin >> x1 >> y1 >> x2 >> y2;
32         b[i*2-1] = y1, b[i*2] = y2, a[i*2-1] = {y1, y2, x1, 1}, a[i*2] = {y1, y2, x2, -1};
33     }
34     n <<= 1;
35     sort(b+1, b+n+1), len = unique(b+1, b+n+1) - b - 1;
36     for(int i = 1; i <= n; i++){
37         a[i].y1 = lower_bound(b+1, b+len+1, a[i].y1) - b, a[i].y2 = lower_bound(b+1, b+len+1, a[i].y2) - b;
38         sort(a+1, a+n+1, [](node a, node b) -> bool {return a.x < b.x;});
39         for(int i = 1; i <= n; i++){
40             t.add(1, a[i].y1, a[i].y2-1, 1, len-1, a[i].k);
41             ans += t.cnt[1] * (a[i+1].x - a[i].x);
42         }
43     }
44     cout << ans;
45 }
46 #undef int

```

15 offline algorithm

15.1 discretization

```

1 std::sort(all(a));
2 a.erase(unique(all(a)), a.end());
3 auto get_id = [&](const int& x) -> int { return lower_bound(all(a), x) - a.begin() + 1; };

```

15.2 Mo algorithm

普通莫队

```

1 int block = n / sqrt(2 * m / 3);
2 std::sort(all(q), [&](node a, node b) {
3     return a.l / block == b.l / block ? (a.r == b.r ? 0 : ((a.l / block) & 1) ^ (a.r < b.r))
4         : a.l < b.l;
5 });
6
7 auto move = [&](int x, int op) -> void {
8     if (op == 1) {
9         /* operations */
10    } else {
11        /* operations */
12    }
13 };
14
15 for (int k = 1, l = 1, r = 0; k <= m; k++) {
16     node Q = q[k];
17     while (l > Q.l) {
18         move(--l, 1);
19     }
20     while (r < Q.r) {
21         move(++r, 1);
22     }
23     while (l < Q.l) {
24         move(l++, -1);
25     }
26     while (r > Q.r) {
27         move(r--, -1);
28     }
29 }

```

回滚莫队

例题：P5906 给定一个序列，多次询问一个区间中相同数的最远间隔距离。

```

1 #include<bits/stdc++.h>
2 namespace Acc {
3     const int N = 200009;
4     int a[N], b[N], id[N], f[N], g[N], p[N], z[N];
5     std::pair<int, int> st[N];
6     struct T {
7         int l, r, o;
8     } q[N];
9     auto work = []() {
10         int n, m;
11         std::cin >> n;
12         for (int i = 1; i <= n; ++i) {
13             std::cin >> a[i], b[i] = a[i];
14         }
15         std::sort(b + 1, b + n + 1);
16         int ct = std::unique(b + 1, b + n + 1) - b - 1;
17         for (int i = 1; i <= n; ++i) {
18             a[i] = std::lower_bound(b + 1, b + ct + 1, a[i]) - b;
19         }
20         std::cin >> m;
21         for (int i = 1; i <= m; ++i) {
22             auto&[l, r, o] = q[i];
23             std::cin >> l >> r, o = i;
24         }
25         int B = ceil(n / sqrt(m));
26         for (int i = 1; i <= n; ++i) {

```

```

27     id[i] = (i - 1) / B + 1;
28 }
29 std::sort(q + 1, q + m + 1, [](T a, T b) {
30     return id[a.l] == id[b.l] ? a.r < b.r : a.l < b.l;
31 });
32 int ans = 0, L = 1, R = 0;
33 for (int i = 1; i <= m; ++i) {
34     auto[l, r, o] = q[i];
35     if (id[l] != id[q[i - 1].l]) {
36         ans = 0;
37         R = std::min(n, id[l] * B), L = R + 1;
38         memset(f + 1, 0, ct << 2);
39         memset(g + 1, 0, ct << 2);
40     }
41     if (id[l] == id[r]) {
42         for (int j = 1; j <= r; ++j) {
43             if (p[a[j]] == 0) p[a[j]] = j;
44             else ans = std::max(ans, j - p[a[j]]);
45         }
46         for (int j = 1; j <= r; ++j) p[a[j]] = 0;
47         z[o] = ans, ans = 0;
48     } else {
49         while (R < r) {
50             ++R, g[a[R]] = R;
51             if (f[a[R]] == 0) f[a[R]] = R;
52             else ans = std::max(ans, R - f[a[R]]);
53         }
54         int las = ans, t = L;
55         while (l < L) {
56             --L;
57             int x = f[a[L]], y = g[a[L]];
58             st[L] = std::make_pair(x, y);
59             f[a[L]] = L;
60             if (g[a[L]] == 0) g[a[L]] = L;
61             else ans = std::max(ans, g[a[L]] - L);
62         }
63         z[o] = ans;
64         for (int j = 1; j < t; ++j) {
65             auto[x, y] = st[j];
66             f[a[j]] = x, g[a[j]] = y;
67         }
68         ans = las, L = t;
69     }
70 }
71 for (int i = 1; i <= m; ++i) {
72     std::cout << z[i] << '\n';
73 }
74 };
75 }
76 int main() {
77     std::ios::sync_with_stdio(0);
78     std::cin.tie(0), Acc::work();
79 }

```

15.3 CDQ

n 个三维数对 (a_i, b_i, c_i) , 设 $f(i)$ 表示 $a_j \leq a_i, b_j \leq b_i, c_j \leq c_i (i \neq j)$ 的个数. 输出 $f(i)$ ($0 \leq i \leq n-1$) 的值.

```

1 // 洛谷 P3810 【模板】三维偏序（陌上花开）
2
3 struct data {
4     int a, b, c, cnt, ans;
5
6     data(int _a = 0, int _b = 0, int _c = 0, int _cnt = 0, int _ans = 0) {
7         a = _a, b = _b, c = _c, cnt = _cnt, ans = _ans;
8     }
9
10    bool operator!=(data x) {
11        if (a != x.a) return true;
12        if (b != x.b) return true;
13        if (c != x.c) return true;
14        return false;
15    }
16 };
17
18 int main() {
19     std::ios::sync_with_stdio(false);
20     std::cin.tie(0);
21     std::cout.tie(0);
22 }

```

```

23
24     int n, k;
25     std::cin >> n >> k;
26     static data v1[N], v2[N];
27     for (int i = 1; i <= n; i++) {
28         std::cin >> v1[i].a >> v1[i].b >> v1[i].c;
29     }
30
31     std::sort(v1 + 1, v1 + n + 1, [&](data x, data y) {
32         if (x.a != y.a) return x.a < y.a;
33         if (x.b != y.b) return x.b < y.b;
34         return x.c < y.c;
35     });
36
37     int t = 0, top = 0;
38     for (int i = 1; i <= n; i++) {
39         t++;
40         if (v1[i] != v1[i + 1]) {
41             v2[++top] = v1[i];
42             v2[top].cnt = t;
43             t = 0;
44         }
45     }
46
47     vi tr(N);
48
49     auto add = [&](int pos, int val) -> void {
50         while (pos <= k) {
51             tr[pos] += val;
52             pos += lowbit(pos);
53         }
54     };
55
56     auto query = [&](int pos) -> int {
57         int ans = 0;
58         while (pos > 0) {
59             ans += tr[pos];
60             pos -= lowbit(pos);
61         }
62         return ans;
63     };
64
65     std::function<void(int, int)> CDQ = [&](int l, int r) -> void {
66         if (l == r) return;
67         int mid = (l + r) >> 1;
68         CDQ(l, mid), CDQ(mid + 1, r);
69         std::sort(v2 + 1, v2 + mid + 1, [&](data x, data y) {
70             if (x.b != y.b) return x.b < y.b;
71             return x.c < y.c;
72         });
73         std::sort(v2 + mid + 1, v2 + r + 1, [&](data x, data y) {
74             if (x.b != y.b) return x.b < y.b;
75             return x.c < y.c;
76         });
77         int i = l, j = mid + 1;
78         while (j <= r) {
79             while (i <= mid && v2[i].b <= v2[j].b) {
80                 add(v2[i].c, v2[i].cnt);
81                 i++;
82             }
83             v2[j].ans += query(v2[j].c);
84             j++;
85         }
86         for (int ii = l; ii < i; ii++) {
87             add(v2[ii].c, -v2[ii].cnt);
88         }
89         return;
90     };
91
92     CDQ(1, top);
93     vi ans(n + 1);
94     for (int i = 1; i <= top; i++) {
95         ans[v2[i].ans + v2[i].cnt] += v2[i].cnt;
96     }
97     for (int i = 1; i <= n; i++) {
98         std::cout << ans[i] << endl;
99     }
100
101     return 0;
102 }

```

```

1 #include<bits/stdc++.h>
2 using namespace std;
3 namespace Acc {
4     const int N = 2e5 + 9;
5     struct T {

```

```

6      int x, y, z, o;
7      bool operator==(const T& b) const {
8          return x == b.x && y == b.y && z == b.z;
9      }
10     bool operator<(const T& b) const {
11         if (x != b.x) return x < b.x;
12         if (y != b.y) return y < b.y;
13         return z < b.z;
14     }
15     }a[N];
16     long long z[N];
17     int m, t[N], f[N];
18     auto ins = [](int x, int k) {
19         for (; x <= m; x += x & -x) t[x] += k;
20     };
21     auto ask = [](int x) {
22         int z = 0;
23         for (; x; x ^= x & -x) z += t[x];
24         return z;
25     };
26     void CDQ(int l, int r) {
27         if (l == r) return;
28         int md = l + r >> 1;
29         CDQ(l, md), CDQ(md + 1, r);
30         int i, j;
31         for (i = l, j = md + 1; j <= r; ++j) {
32             for (; i <= md && a[i].y <= a[j].y; ++i) ins(a[i].z, 1);
33             f[a[j].o] += ask(a[j].z);
34         }
35         for (--i; i >= l; --i) ins(a[i].z, -1);
36         inplace_merge(a + l, a + md + 1, a + r + 1, [](T a, T b) {
37             return a.y < b.y;
38         });
39     }
40     auto work = []() {
41         int n;
42         cin >> n >> m;
43         for (int i = 1; i <= n; ++i) {
44             auto&[x, y, z, o] = a[i];
45             cin >> x >> y >> z;
46         }
47         sort(a + 1, a + n + 1);
48         for (int i = 1; i <= n; ++i) a[i].o = i;
49         CDQ(1, n);
50         sort(a + 1, a + n + 1);
51         for (int i = n - 1; i; --i) {
52             if (a[i] == a[i + 1]) f[i] = f[i + 1];
53         }
54         for (int i = 1; i <= n; ++i) ++z[f[i]];
55         for (int i = 0; i < n; ++i) cout << z[i] << '\n';
56     };
57 }
58 int main() {
59     ios::sync_with_stdio(0);
60     cin.tie(0), Acc::work();
61 }

```

15.4 线段树分治

```

1  #include<bits/stdc++.h>
2  using namespace std;
3  namespace Acc {
4      const int N = 1e5;
5      pair<int, int> q[N * 2];
6      vector<int> v[N * 4];
7      int n, l, r, p;
8      int fa[N * 2], sz[N * 2];
9      pair<int, int> st[N * 2];
10     int tp;
11     void ins(int o, int L, int R) {
12         if (r < L || l > R) return;
13         if (l <= L && R <= r) {
14             v[o].emplace_back(p);
15             return;
16         }
17         int md = L + R >> 1;
18         ins(o << 1, L, md);
19         ins(o << 1 | 1, md + 1, R);
20     }
21     auto gf = [](int x) {
22         while (x != fa[x]) x = fa[x];
23         return x;
24     };

```

```

24 };
25 auto mg = [](int x, int y) {
26     x = gf(x), y = gf(y);
27     if (x != y) {
28         if (sz[x] < sz[y]) swap(x, y);
29         fa[y] = x, sz[x] += sz[y];
30         st[++tp] = {x, y};
31     }
32 };
33 void dfs(int o, int L, int R) {
34     int lastp = tp;
35     for (int i : v[o]) {
36         auto[x, y] = q[i];
37         mg(x, y + n), mg(x + n, y);
38         if (gf(x) == gf(x + n)) {
39             for (int i = L; i <= R; ++i) {
40                 cout << "No\n";
41             }
42             goto _;
43         }
44     }
45     if (L == R) {
46         cout << "Yes\n";
47     } else {
48         int md = L + R >> 1;
49         dfs(o << 1, L, md);
50         dfs(o << 1 | 1, md + 1, R);
51     }
52     _:
53     for (; tp > lastp; --tp) {
54         auto[x, y] = st[tp];
55         fa[y] = y, sz[x] -= sz[y];
56     }
57 }
58 auto work = []() {
59     int m, k;
60     cin >> n >> m >> k;
61     for (int i = 1; i <= m; ++i) {
62         int x, y;
63         cin >> x >> y >> l >> r;
64         if (++l <= r) {
65             q[p = i] = {x, y}, ins(1, 1, k);
66         }
67     }
68     iota(fa + 1, fa + n * 2 + 1, 1);
69     fill(sz + 1, sz + n * 2 + 1, 1);
70     dfs(1, 1, k);
71 };
72 }
73 int main() {
74     ios::sync_with_stdio(0);
75     cin.tie(0), Acc::work();
76 }

```