Mathematical Statistics I
Assignment 3 - Questions

Due by April 24, 2024

1. Let N be a random variable taking values $1, 2, \ldots$ with known probabilities $p_1, p_2, \ldots$, where $\Sigma p_i = 1$. Having observed $N = n$, perform n Bernoulli trials with success probability θ, getting X successes.
   (a) Prove that the pair (X, N) is minimal sufficient and N is ancillary for θ.
   (b) Prove that the estimator X/N is unbiased for θ and has variance $\theta(1-\theta)\mathrm{E}(1/N)$.

2. Show that the minimal sufficient statistic for the uniform $(\theta, \theta+1)$, is not complete.

3. For each of the following pdfs let $X_1, \ldots, X_n$ be iid observations. Find a complete sufficient statistic, or show that one does not exist.
   (a) $f(x;\theta) = \frac{2x}{\theta^2}, \quad 0 < x < \theta, \quad \theta > 0$
   (b) $f(x;\theta) = e^{-(x-\theta)} \exp\left(-e^{-(x-\theta)}\right), \quad -\infty < x < \infty, \quad -\infty < \theta < \infty$
   (c) $f(x;\theta) = \binom{2}{x}\theta^x(1-\theta)^{2-x}, \quad x = 0, 1, 2, \quad 0 \leq \theta \leq 1$

4. Let X be one observation from the pdf

$$f(x;\theta) = \left(\frac{\theta}{2}\right)^{|x|}(1-\theta)^{1-|x|}, \quad x = -1, 0, 1, \quad 0 \leq \theta \leq 1.$$

   (a) Is X a complete sufficient statistic?
   (b) Is $|X|$ a complete sufficient statistic?
   (c) Does $f(x;\theta)$ belong to the exponential class?

5. Consider the following family of distributions:

$$\mathcal{P} = \left\{P_\lambda(X = x) : P_\lambda(X = x) = \lambda^x e^{-\lambda}/x!; x = 0, 1, 2, \ldots; \lambda = 0 \text{ or } 1\right\}.$$

   This is a Poisson family with λ restricted to be 0 or 1 . Show that the family $\mathcal{P}$ is not complete, demonstrating that completeness can be dependent on the range of the parameter.