Mathematical Statistics I
Assignment 1 - Questions

Due by February 28, 2024

1. If X is $N\left(\mu, \sigma^2\right)$, show that $E(|X-\mu|) = \sigma\sqrt{2/\pi}$.

2. Let X_1 and X_2 have chi-square distributions with r_1 and r_2 degrees of freedom, respectively, where $r_1 < r_2$. Let $Y = X_2 - X_1$ and assume Y and X_1 be independent random variables. Show that Y has a chi-square distribution with $r_2 - r_1$ degrees of freedom.

3. Let $X_1, \ldots, X_n$ be i.i.d. random variables having the exponential distribution $E(a, \theta), a \in \mathbb{R}$, and $\theta > 0$. Show that the smallest order statistic, $X_{(1)}$, has the exponential distribution $E(a, \theta/n)$ and that $Y = 2\sum_{i=1}^{n}\left(X_i - X_{(1)}\right)/\theta$ has the chi-square distribution χ^2_{2n-2}. (Note that Y and $X_{(1)}$ are independent, you can assume this without proof, providing its proof will be considered as a bonus point.)

4. Show that if the distribution of a positive random variable X is in a scale family, then the distribution of $\log X$ is in a location family.

5. Let X be a random variable having the gamma distribution $\Gamma(\alpha, \gamma)$ with a known α and an unknown $\gamma > 0$ and let $Y = \sigma \log X$.
   (a) Show that if $\sigma > 0$ is unknown, then the distribution of Y is in a location-scale family.
   (b) Show that if $\sigma > 0$ is known, then the distribution of Y is in an exponential family.