Mathematical Statistics I
Recitation Session 7

Theorem. *Let $\mathcal{U}$ be the set of all unbiased estimators of 0 with finite variances and T be an unbiased estimator of ϑ with $E\left(T^2\right) < \infty$. A necessary and sufficient condition for $T(X)$ to be a UMVUE of ϑ is that $E[T(X)U(X)] = 0$ for any $U \in \mathcal{U}$ and any $P \in \mathcal{P}$.*

Theorem. *Uniformly minimum-variance unbiased estimator (UMVUE) is unique, whenever it exists.*

Exercise. Let $X_1, \ldots, X_n$ be i.i.d from uniform distribution on the interval $(0, \theta)$ with parameter space $\Theta = [1, \infty)$. Find UMVUE of θ. Note that the maximum order statistic, $X_{(n)}$, is sufficient for θ but not complete because $\theta \geq 1$.

Exercise. Let $(X_1, \ldots, X_n)$ be a random sample from the exponential distribution with density $\theta^{-1} e^{-(x-a)/\theta} I_{(a,\infty)}(x)$, where $a \leq 0$ and θ is known. Obtain a UMVUE of a. Note that the minimum order statistic, $X_{(1)}$, is sufficient for a but not complete because $a \leq 0$.

Exercise. Let X be an observation from uniform $(\theta, \theta + 1), \theta \in \mathbb{R}$. Show that there is no UMVUE of $g(\theta)$ for any non-constant differentiable function g.