Mathematical Statistics I
Assignment 6 - Questions

Due by June 5, 2024

1. Let $X_1, \ldots, X_{n+1}$ be iid Bernoulli(p), and define the function $h(p)$ by

$$h(p) = P\left(\sum_{i=1}^{n} X_i > X_{n+1}\right)$$

the probability that the first n observations exceed the $(n+1)$st.
(a) Show that

$$T\left(X_1, \ldots, X_{n+1}\right) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} X_i > X_{n+1} \\ 0 & \text{otherwise} \end{cases}$$

is an unbiased estimator of $h(p)$.
(b) Find the best unbiased estimator of $h(p)$.

2. Let a random sample of size n be taken from a distribution of the discrete type with pmf $f(x;\theta) = 1/\theta, x = 1, 2, \ldots, \theta$, zero elsewhere, where θ is an unknown positive integer.
(a) Show that the largest observation, say Y, of the sample is a complete sufficient statistic for θ.
(b) Prove that

$$\left[Y^{n+1} - (Y-1)^{n+1}\right] / \left[Y^{n} - (Y-1)^{n}\right]$$

is the unique MVUE of θ.

3. Let $X_1, \ldots, X_n$ be a random sample from exponential distribution with parameter θ. Prove $\sum_i X_i$ is complete and considering that, find $E(nX_{(1)} | \sum_i X_i)$.

4. Let $\left(X_1, \ldots, X_n\right), n > 2$, be a random sample from the uniform distribution on the interval $(\theta_1 - \theta_2, \theta_1 + \theta_2)$, where $\theta_1 \in \mathcal{R}$ and $\theta_2 > 0$. Find the UMVUE's of $\theta_j, j = 1, 2$.

5. For each of the following pdfs, let $X_1, \ldots, X_n$ be a sample from that distribution. In each case, find the best unbiased estimator of θ^r.
(a) $f(x;\theta) = e^{-(x-\theta)}, \quad x > \theta$
(b) $f(x;\theta) = \frac{e^{-x}}{e^{-\theta} - e^{-b}}, \quad \theta < x < b, \quad b$ known