Stochastic Processes
Assignment 2 - Questions

Due on March 17, 2023

1. Two people, A and B, play the game of matching pennies: at each time n, each person has a penny and must secretly turn the penny to heads or tails. The players then reveal their choices simultaneously. If the pennies match (both heads or both tails), Player A wins the penny. If the pennies do not match (one heads and one tails), Player B wins the penny. Suppose the players have between them a total of 5 pennies. If at any time one player has all of the pennies, to keep the game going, he gives one back to the other player and the game will continue.

a) Show that this game can be formulated as a Markov chain.

b) If Player A starts with 2 pennies and Player B with 3, what is the probability that A will lose his pennies first?

2. Suppose we have n red balls and n blue balls and we put them into 2 boxes that each box has n balls. Every time step you take one ball (chosen randomly) from each box, swap the balls and place them back in the boxes. Let X_m be the number of red balls in the first box after m time steps. Find the transition function of the Markov chain.

3. Consider the Markov chain with state space {0,1,2,3} and transition matrix:

$$P = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ \frac{3}{4} & 0 & \frac{1}{4} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a) Draw the accessibility diagram and determine the recurrent ,transient, absorbing states.

b) Find the expected time until absorption occurs.

c) The process is started in state 1; find the probability that it is in state 3 after two steps.

4. Suppose the weather is one of two types: sunny or rainy. If it is rainy, there's a 25% chance it will rain the next day. If it is sunny, there is a 10% chance it will rain the next day. The past Sunday and Tuesday were sunny, but you were away and don't know what the weather was on Monday. What's the probability that it was sunny on Monday ?

5. Consider the Markov chain with state space {0,1,2} and transition matrix:

$$P = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

If A={1,2} and X_0=0 :

a) Find the distribution of T_A.(Remind: $T_A = min\{n \geq 1, X_n \in A\}$ is first hitting time)

b) Compute $\mathbb{E}[T_A]$.

6. Charlie has been arrested and has 2 dollars; he can be free if he has 7 dollars. A police agrees to make a series of bets with him. If Charlie bets X dollars, he wins X dollars with probability 0.3 and loses X dollars with probability 0.7. Find the probability that he wins 7 dollars before losing all of his money if :

a) He bets 1 dollar each time.

b) He bets, each time ,as much as possible but not more than necessary to bring his fortune up to 7 dollars.

c) Which strategy(a or b) gives Charlie the better chance to be free?