In The Name Of God

HW07

Advanced Neuroscience

MohammadAmin Alamalhoda
97102099

■ Part1 - Simulation of evidence accumulation

□ Q01 and Q02

$$dX = Bdt + \sigma dW$$
$$\int_0^t \frac{dX}{dt} = \int_0^t Bdt + \int_0^t \sigma \frac{dW}{dt}$$
$$X(t) - X(0) = Bt + \int_0^t \sigma \frac{dW}{dt}$$
$$X(t) = Bt + \sigma \int_0^t \frac{dW}{dt}$$

So, Expected value of X is:

$$E[X] = E[Bt] + E[\int_0^t \frac{dW}{dt}]$$
$$E[X] = Bt + \sigma \int_0^t E[dW]$$
$$E[X] = Bt + \sigma \times 0$$
$$E[X] = Bt$$

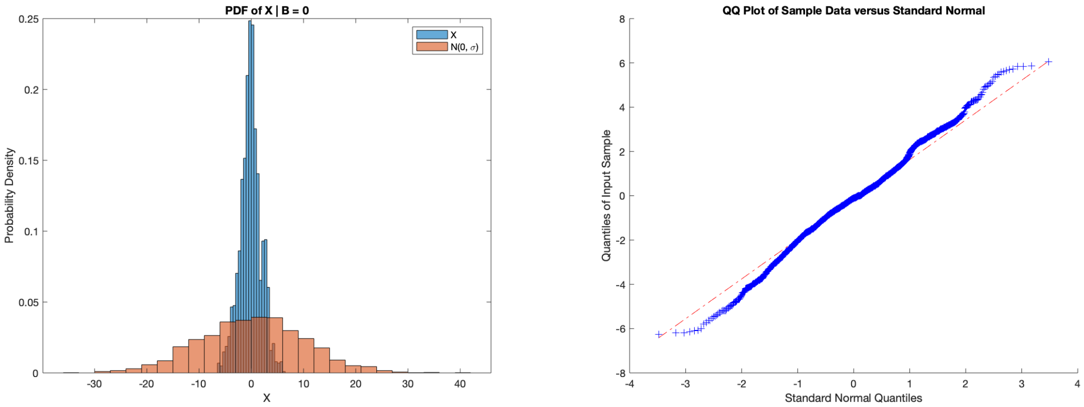
and Variance of the X is:

$$Var(X) = Var(Bt + \int_0^t \frac{dW}{dt})$$
$$Var(X) = Var(\sigma \int_0^t \frac{dW}{dt})$$
$$Var(X) = \sigma \int_0^t Var(\frac{d}{W})$$
$$Var(X) = \sigma t$$

Therefore:

$$X(t) \hookrightarrow \mathcal{N}(Bt, \sigma t)$$

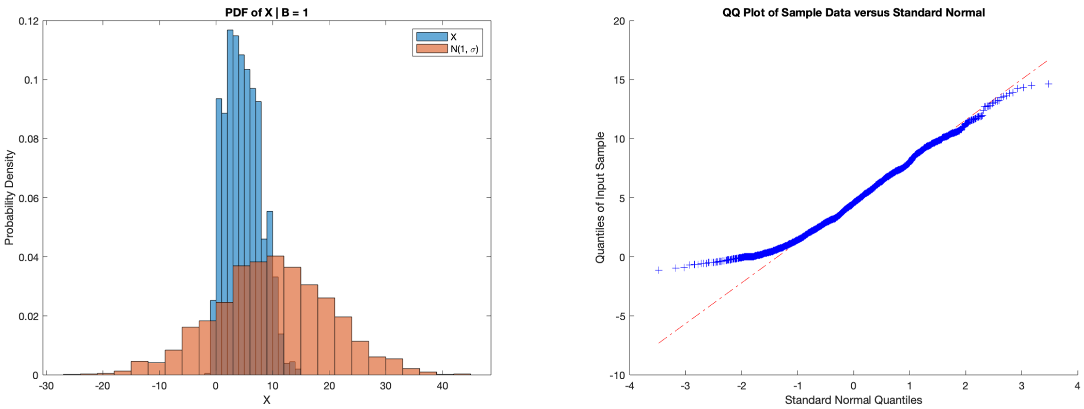
Bias = 0



(a) PDF of X(t) (b) QQPlot

Figure 1: The PDF of X(t) and QQ-Plot for checking the normality of the data when $bias = 0$, $\sigma = 1$, $time\ limit = 10$, and $dt = 0.01$

Bias = 1



(a) PDF of X(t) (b) QQPlot

Figure 2: The PDF of X(t) and QQ-Plot for checking the normality of the data when $bias = 1$, $\sigma = 1$, $time\ limit = 10$, and $dt = 0.01$

As calculated in the last part and as can be seen in Figures 1 and 2, the distribution of the $X(t)$ is Normal. Quantile Quantile-Plots better show that the distribution of the $X(t)$ is Normal.

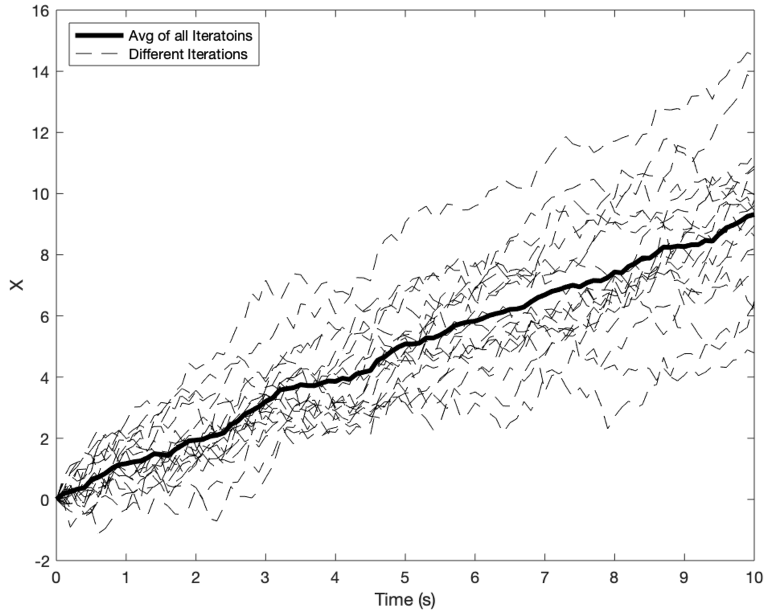


Figure 3: X(t) during different runs - $bias = 1$, $\sigma = 1$, $time\ limit = 10$, and $dt = 0.01$

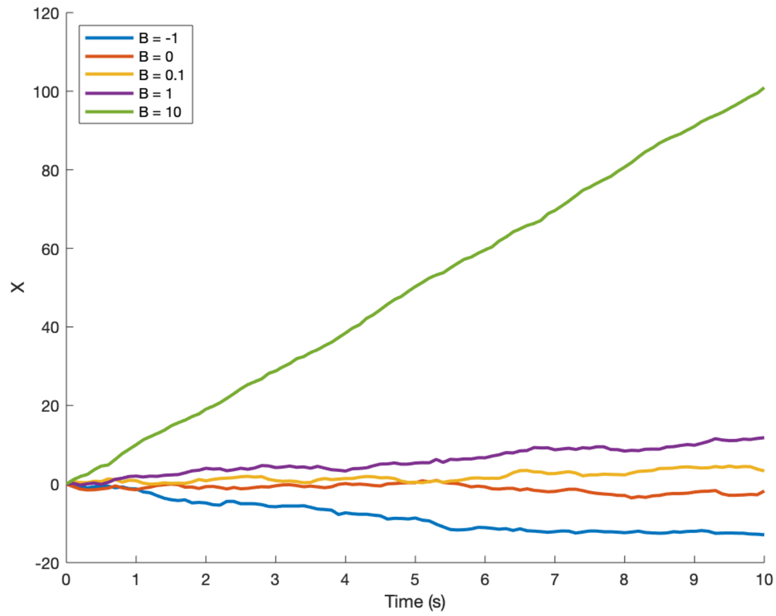


Figure 4: X(t) with different biases - $\sigma = 1$, $time\ limit = 10$, and $dt = 0.01$

Figures 3 and 4 verify the calculated Expected-Value and Variance of the $X(t)$. As can be seen in these Figures, the Expected-Value and Variance of the $X(t)$ increase with time.

☐ Q03

$$Error = 1 - P(choice = 1 | X(t) \hookrightarrow \mathcal{N}(Bt,\, \sigma t))$$
$$Error = 1 - P(|X(t)| = 1 | X(t) \hookrightarrow \mathcal{N}(Bt,\, \sigma t))$$

Because the mean of this normal distribution is positive, to calculate the above probability I used the formula below:

$$Error = 1 - \frac{min(8sigma, \mu + 4sigma)}{8sigma}$$

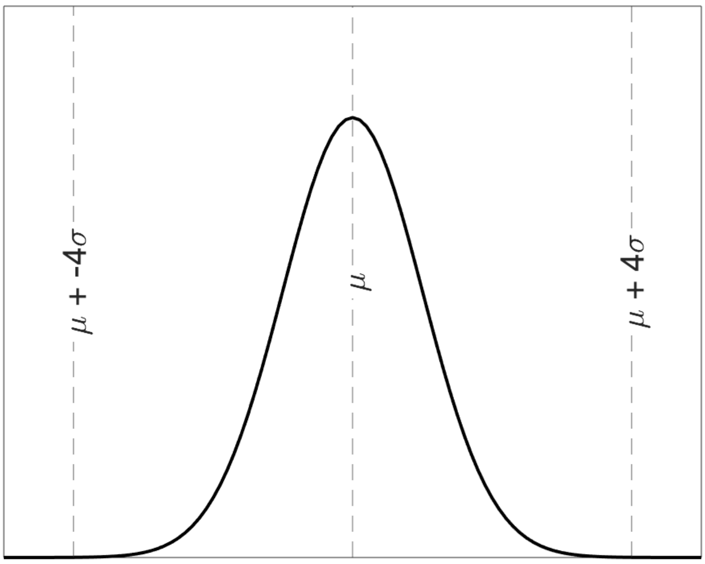
Where:

$$\mu = Bt$$
$$sigma = \sigma t$$

So:

$$Error(t) = 1 - \frac{min(8\sigma t, Bt + 4\sigma t)}{8\sigma t}$$

The theory of $\frac{min(8\sigma t, Bt+4\sigma t)}{8\sigma t}$ is that in a normal distribution the probability of values less than $\mu - 4\sigma$ or bigger than $\mu + 4\sigma$ is zero (Figure below).



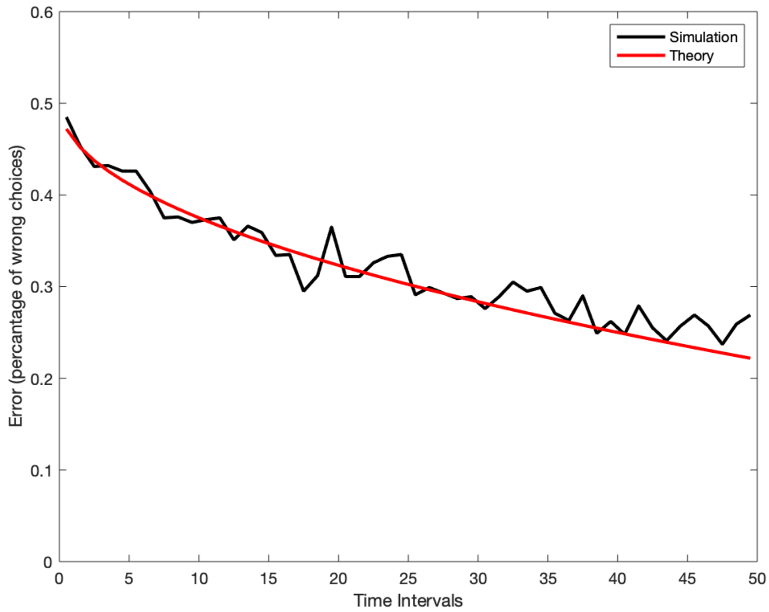


Figure 5: Stimulation and Theory of Choice Error Rate vs Time Limit- $bias = 0.1$, $\sigma = 1$, and $dt = 0.1$

As I expect and as calculated on the last page, Error increases when Time-Limit increases. This is shown in Figure 5.

□ Q04

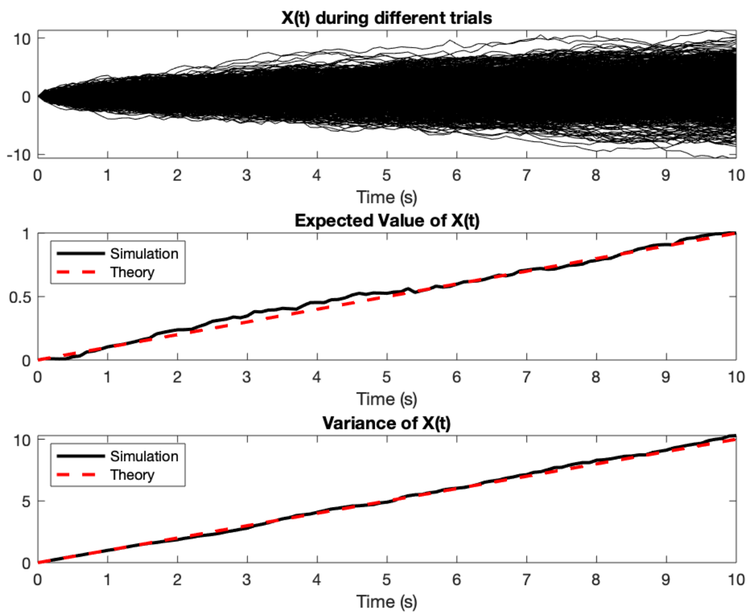
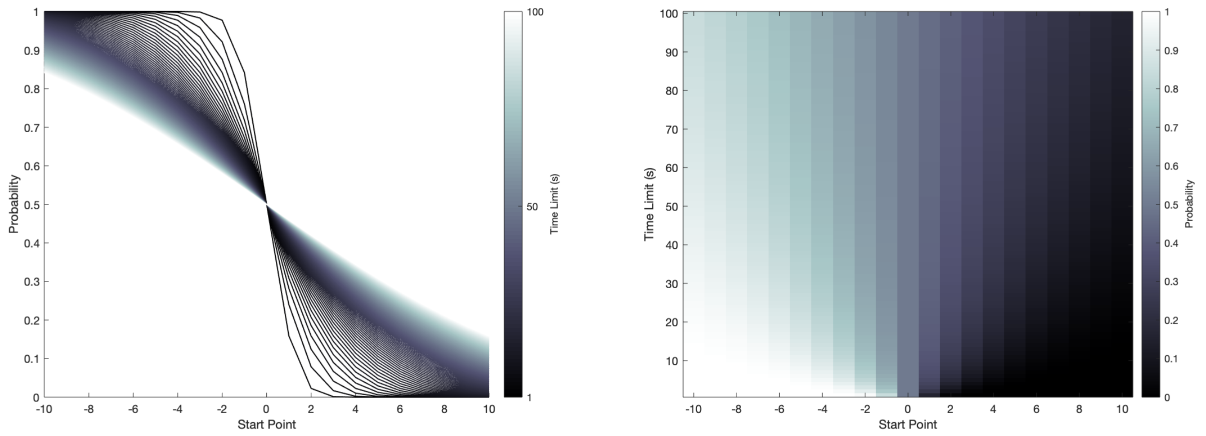


Figure 6: X(t), Expected Value, and Variance of the X(t) vs Time - $bias = 0.1$, $\sigma = 1$, and $dt = 0.1$

As can be seen in Figure 6, the Expected-Value and Variance of the $X(t)$ increase with time which is Consistent with our theoretical calculations in Q01.

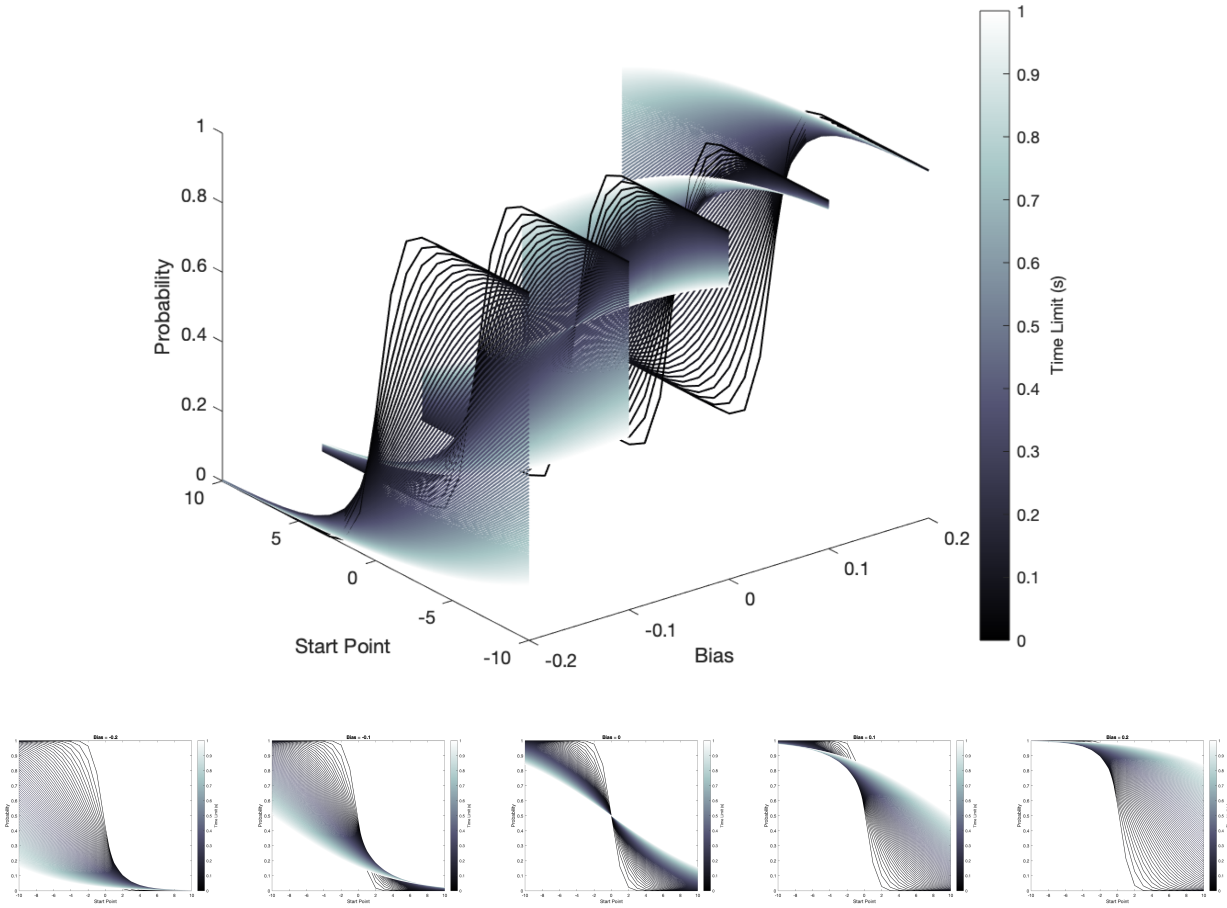
☐ Q05



(a) Probability of Correct Choice vs Start Point during different Time-Limits

(b) Probability of Correct Choice vs Start Point and Time-Limit

Figure 7: Probability of Right Choice vs Start Point vs Time-Limit When $bias = 0$, $\sigma = 1$, and $dt = 0.1$



(a) Bias = -0.2

(b) Bias = -0.1

(c) Bias =0

(d) Bias = 0.1

(e) Bias = 0.2

Figure 8: Probability of Right Choice vs Start Point vs Time-Limit vs Bias - $\sigma = 1$ and $dt = 0.1$

As I expect, the probability of choosing the right choice will decay with time, so if you choose the same choice in times 1 and 10, you are more confident when choosing at time 1. This is shown in Figures 7 and 8.

□ Q06 and Q07

I aim to calculate the distribution of the reaction time (when $x(t)$ reaches α as a threshold), so:

$$T_\alpha = inf\{t > 0 | X_t = \alpha\}$$

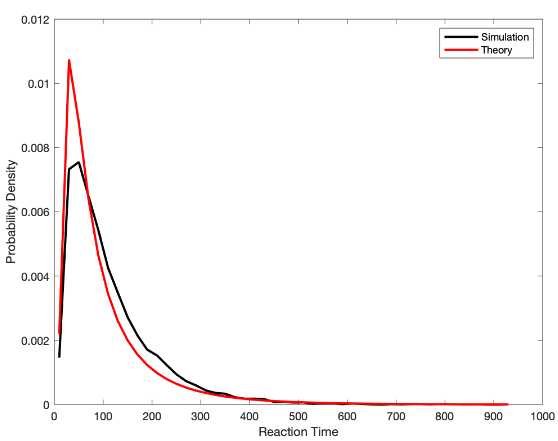
Where:

$$dX = Bdt + \sigma dW$$
$$X(t) \hookrightarrow \mathcal{N}(Bt, \sigma t)$$

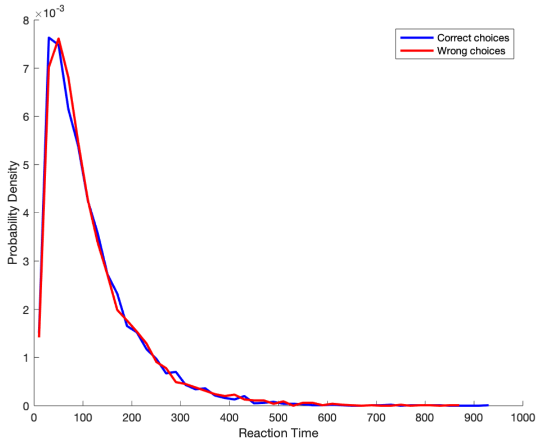
So:

$$T_\alpha = IG(\frac{\alpha}{B}, (\frac{\alpha}{\sigma})^2)$$

Where IG is inverse gaussian distribution.



(a) All CHoices



(b) Correct and Wrong Choices

Figure 9: PDF of Reaction Times - $bias = 0.1$, $\sigma = 1$, $dt = 0.01$, and $X_0 = 0$

As can be seen in Figure 9, the simulated result is consistent with my theoretical calculations.

☐ Q08

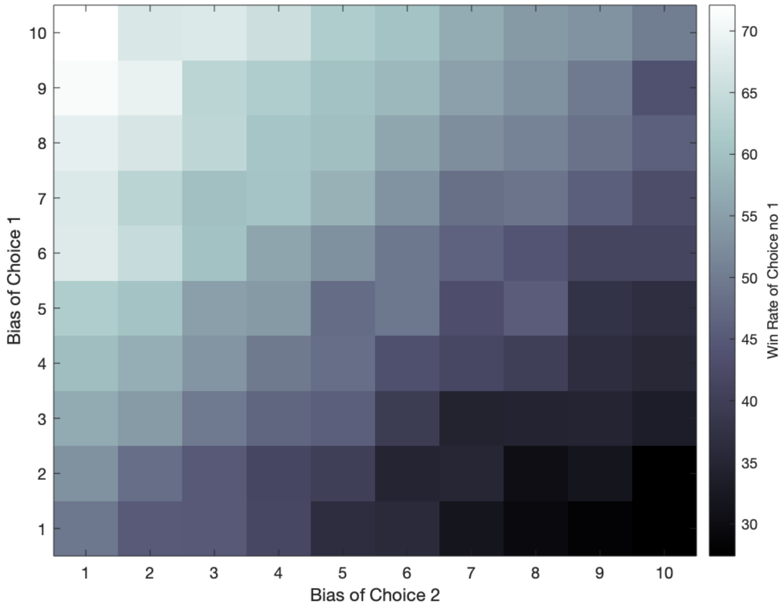


Figure 10: Win Rate of Choice 1 vs Bias of Choice 1 and Bias of Choice 2 - $\sigma = 1$, $dt = 0.01$, and $threshold = 10$

As can be seen in Figure 10, the win rate of choice 1 is bigger when the bias of choice 1 is bigger than the bias of choice 2 and it makes sense, because if the bias of a choice is way bigger than the bias of the other choice, it will reach the threshold faster and will win the race.

☐ Q09

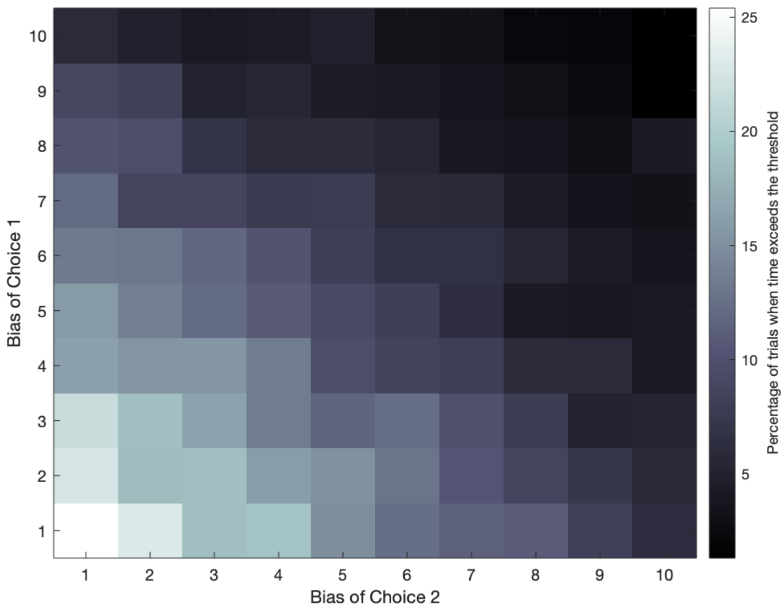


Figure 11: Percentage of Trials when Time Exceeds Time-Limit vs Bias of Choice 1 and Bias of Choice 2 - $\sigma = 1$, $dt = 0.01$, $time\ limit = 2$, and $threshold = 10$

When the bias of the choices is small, it will take more time for each of them to reach the threshold, so for small biases, more trials will exceed the time limit and via versa for big biases.

■ Part2 - Simulation of the interaction between area MT and LIP

□ Q01

$$MT\ P\ Values = \begin{bmatrix}0.1 & 0.05\end{bmatrix}$$
$$LIP\ Weigths = \begin{bmatrix}0.1 & -0.2\end{bmatrix}$$
$$LIP\ Threshold = 50$$

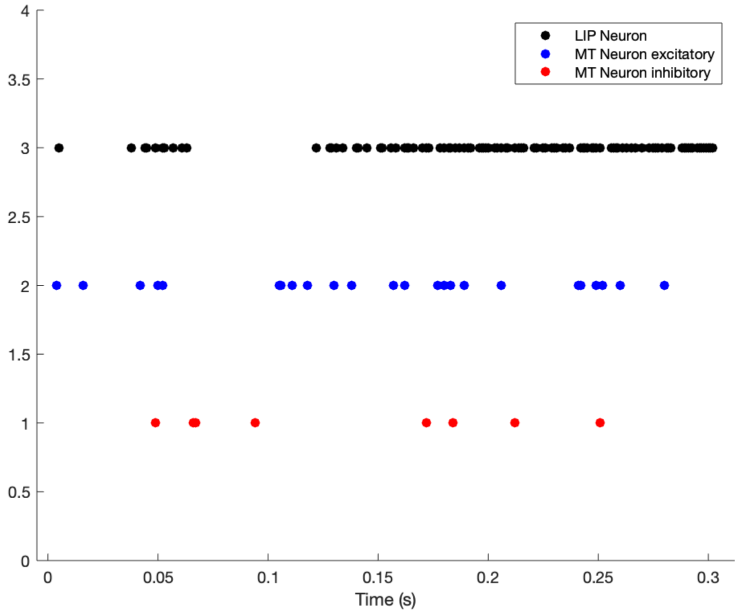


Figure 12: Simulation of two Area MT Neurons connected to one LIP Neuron

As can be seen in Figure 12, when the excitatory neuron has a high firing rate and the inhibitory neuron has a low firing rate, the LIP neuron starts firing with a high rate.

☐ Q02

$$LIP\ Weigths = \begin{bmatrix} 0.1 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}$$

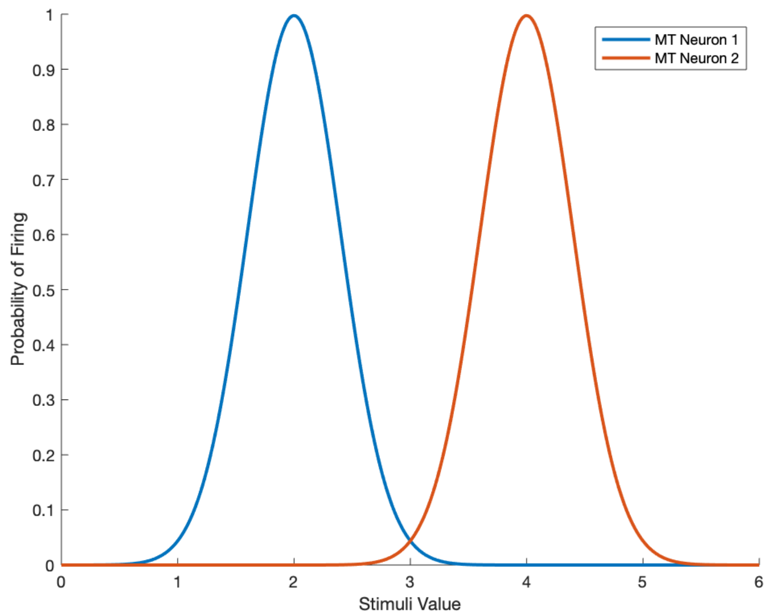
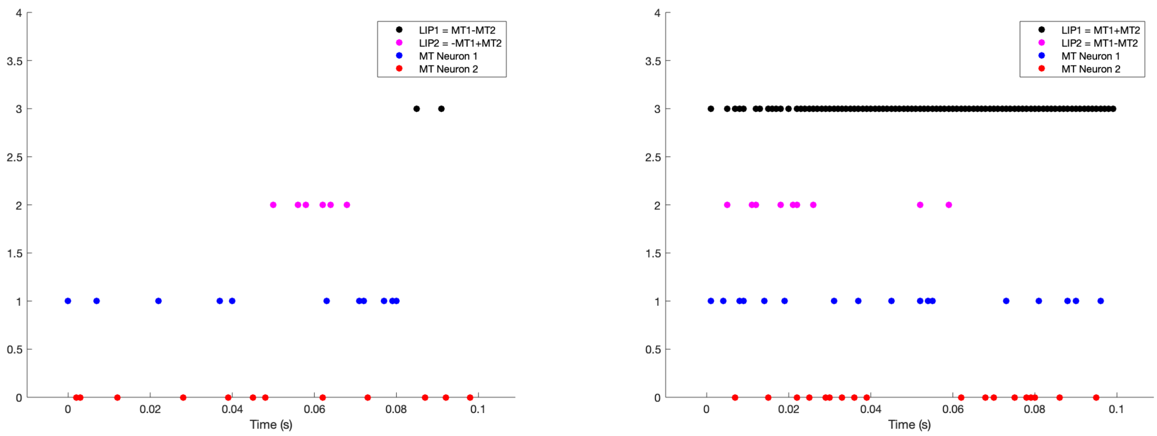


Figure 13: Tuning Curves of Area MT Neurons

Figure 13 shows tuning curve for two neuron from the area MT.



(a) LIP1 = MT1-MT2, LIP2 = -MT1+MT2

(b) LIP1 = MT1+MT2, LIP2 = MT1-MT2

Figure 14: Simulation of two Area MT Neurons connected to two LIP Neuron

As can be seen in Figure 14, the activity of the LIP neurons depends on their connection weights with the area MT neuron and when the inhibitory connections have large values the LIP neurons won't fire.