In The Name Of God

HW07

Advanced Neuroscience

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■ Part1 - Simulation of evidence accumulation

□ Q01 and Q02

$$dX = Bdt + \sigma dW$$
$$\int_0^t \frac{dX}{dt} = \int_0^t Bdt + \int_0^t \sigma \frac{dW}{dt}$$
$$X(t) - X(0) = Bt + \int_0^t \sigma \frac{dW}{dt}$$
$$X(t) = Bt + \sigma \int_0^t \frac{dW}{dt}$$

So, Expected value of X is:

$$E[X] = E[Bt] + E[\int_0^t \frac{dW}{dt}]$$
$$E[X] = Bt + \sigma \int_0^t E[dW]$$
$$E[X] = Bt + \sigma \times 0$$
$$E[X] = Bt$$

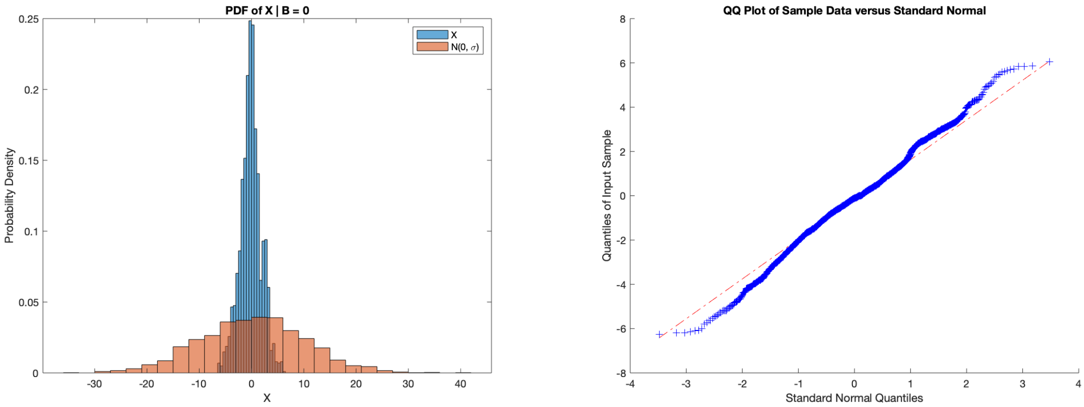
and Variance of the X is:

$$Var(X) = Var(Bt + \int_0^t \frac{dW}{dt})$$
$$Var(X) = Var(\sigma \int_0^t \frac{dW}{dt})$$
$$Var(X) = \sigma \int_0^t Var(\frac{d}{W})$$
$$Var(X) = \sigma t$$

Therefore:

$$X(t) \hookrightarrow \mathcal{N}(Bt, \sigma t)$$

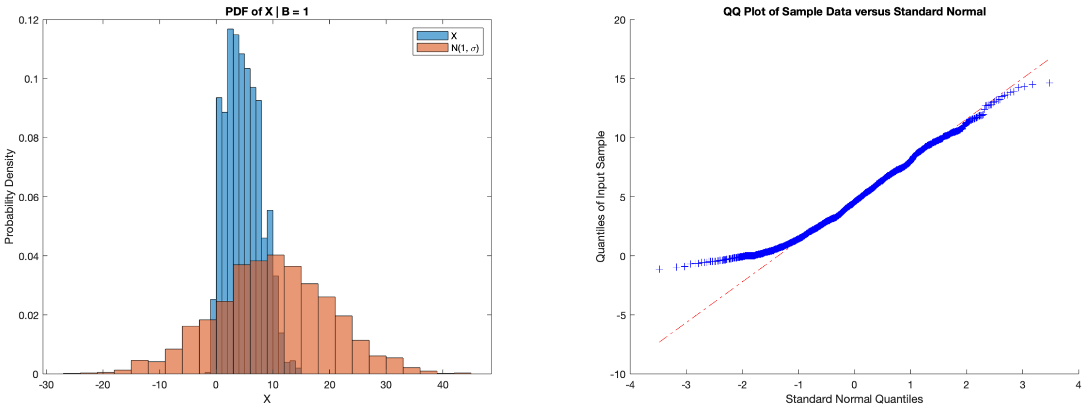
Bias = 0



(a) PDF of X(t) (b) QQPlot

Figure 1: The PDF of X(t) and QQ-Plot for checking the normality of the data when bias = 0

Bias = 1



(a) PDF of X(t) (b) QQPlot

Figure 2: The PDF of X(t) and QQ-Plot for checking the normality of the data when bias = 1

As calculated in the last part and as can be seen in Figures 1 and 14, the distribution of the $X(t)$ is Normal. QQ-Plots better show that the distribution of the $X(t)$ is Normal.

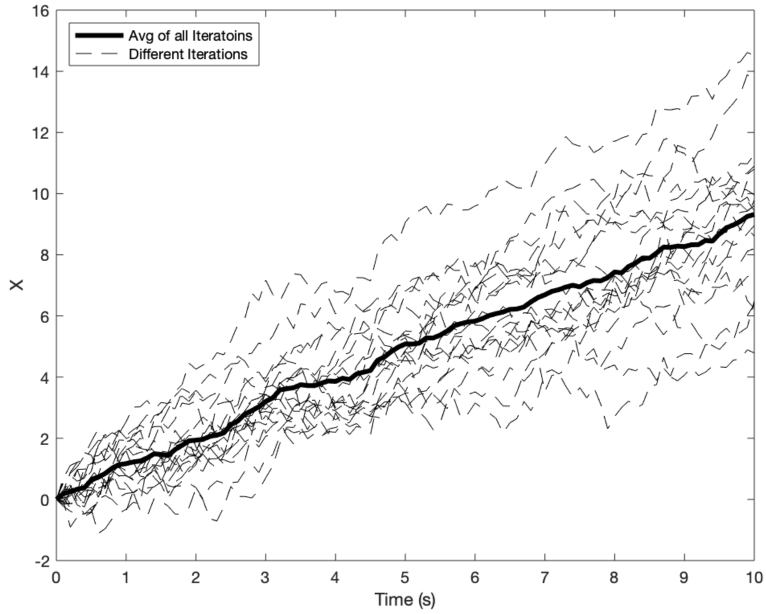


Figure 3: X(t) during different runs

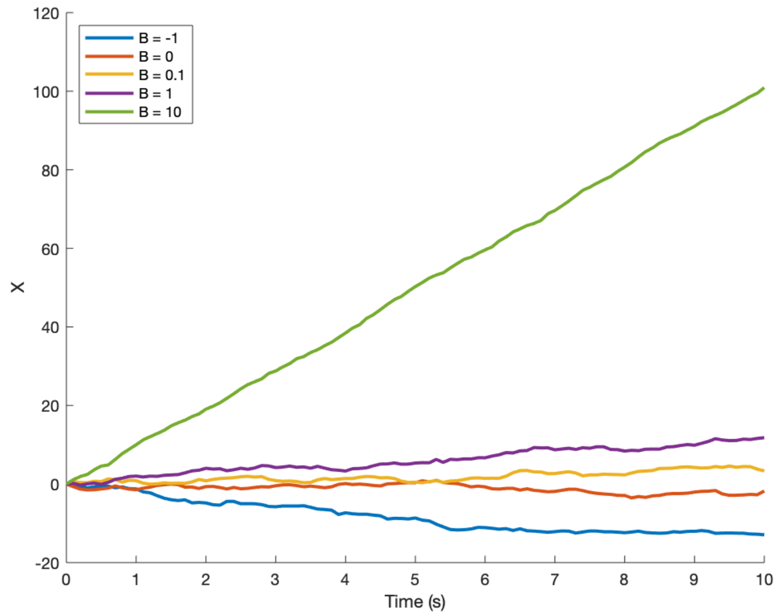


Figure 4: X(t) with different biases

Figures 3 and 3 verify the calculated Expected-Value and Variance of the $X(t)$. As can be see in these Figures, the Expected-Value and Variance of the $X(t)$ increases with time.

☐ Q03

$$Error = 1 - P(choice = 1|X(t) \hookrightarrow \mathcal{N}(Bt,\, \sigma t))$$
$$Error = 1 - P(|X(t)| = 1|X(t) \hookrightarrow \mathcal{N}(Bt,\, \sigma t))$$

Because the mean of this normal distribution is positive, to calculate the above probability I used the formula below :

$$Error = 1 - \frac{min(8sigma, \mu + 4sigma)}{8sigma}$$

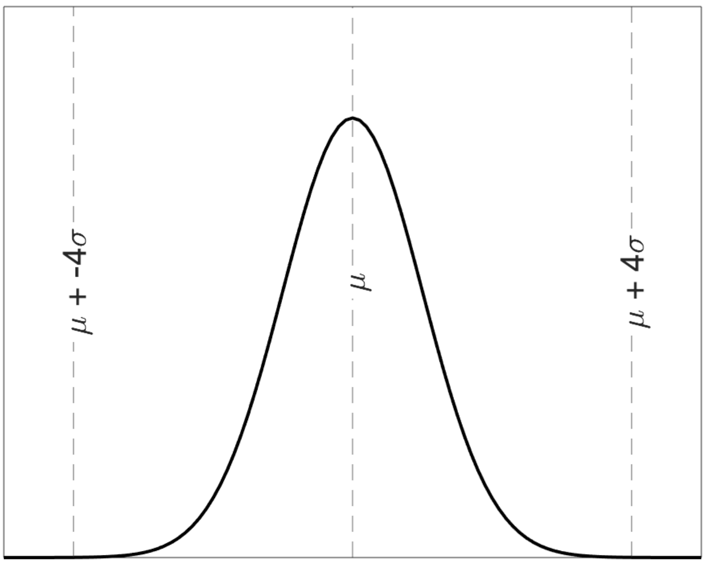
Where:

$$\mu = Bt$$
$$sigma = \sigma t$$

So:

$$Error(t) = 1 - \frac{min(8\sigma t, Bt + 4\sigma t)}{8\sigma t}$$

The theory of $\frac{min(8\sigma t, Bt+4\sigma t)}{8\sigma t}$ is that in a normal distribution is visible in the following Figure.



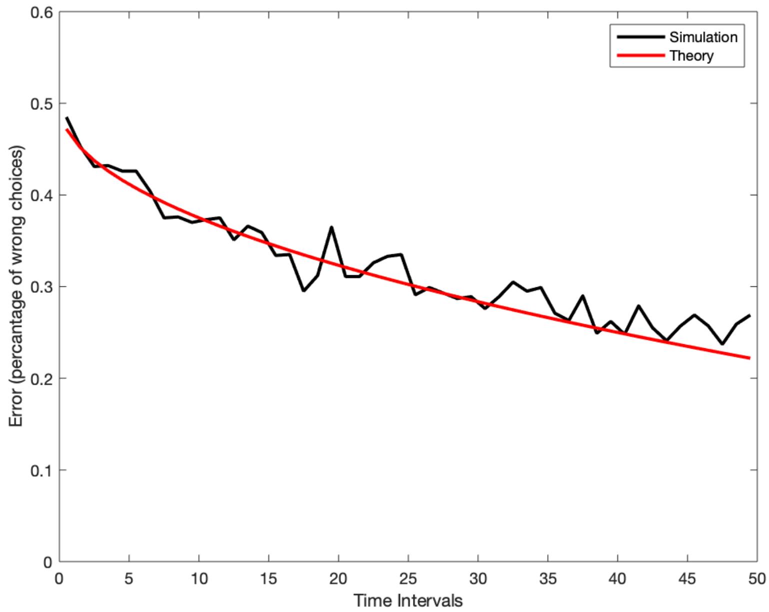


Figure 5: Stimulation and Theory of Choice Error Rate vs Time Limit

As I expect and as calculated in the last page, Error increases when Time-Limit increases. This is show in the Figure 5.

☐ Q04

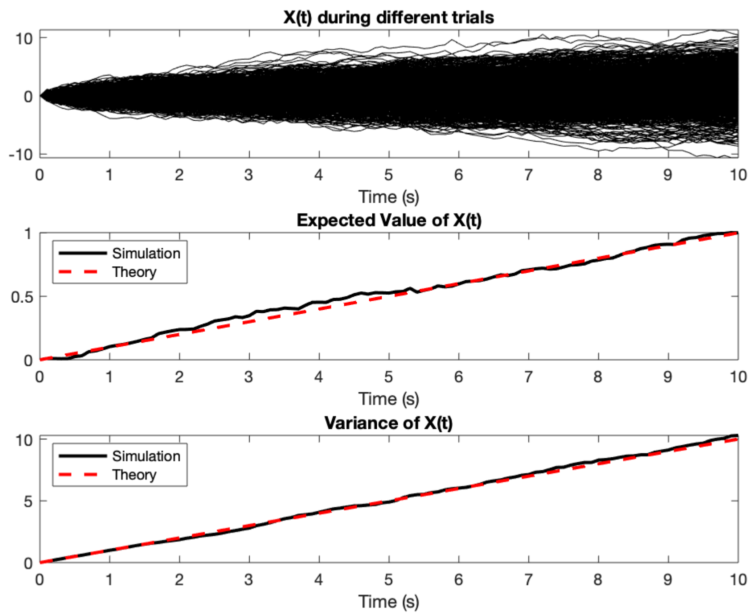
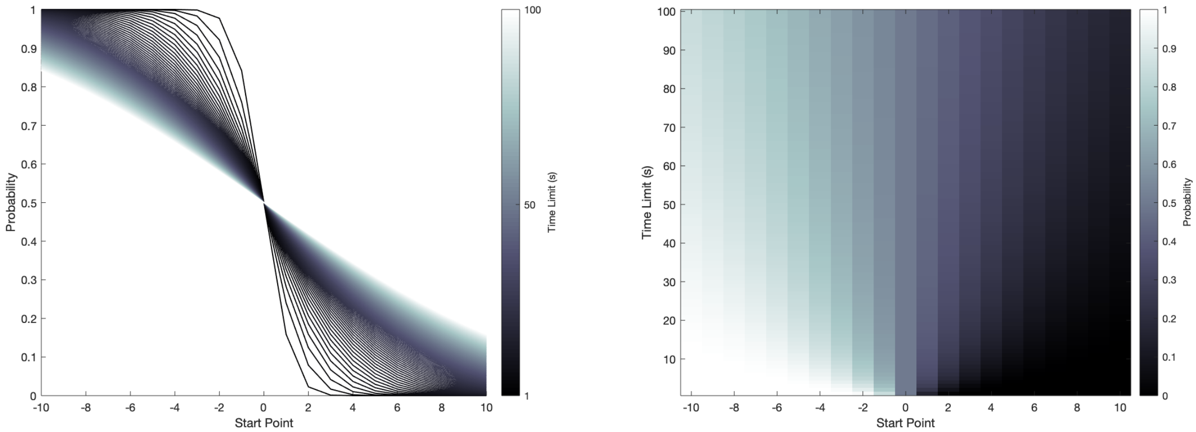


Figure 6: X(t), Expected Value, and Variance of the X(t) vs Time

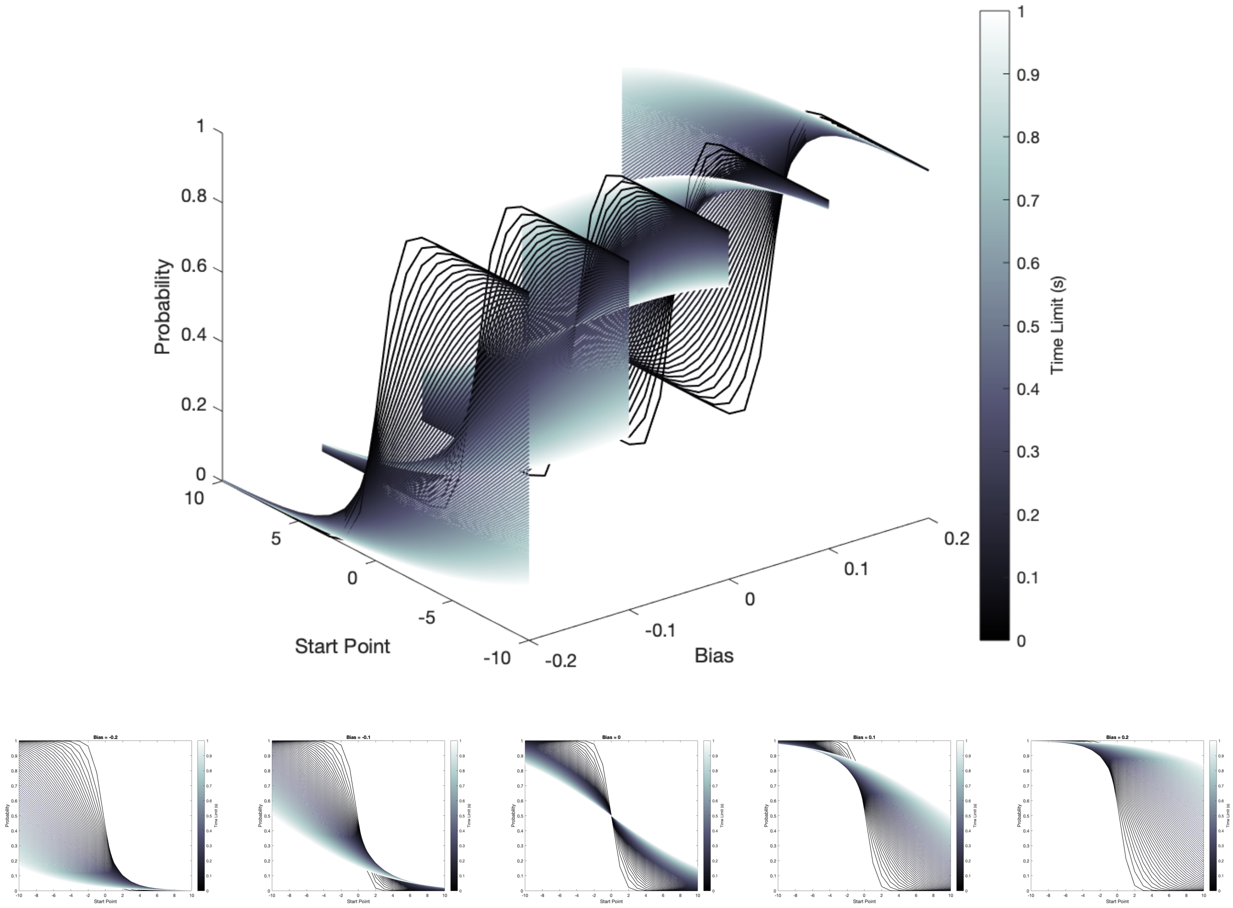
□ Q05



(a) Probability of Correct Choice vs Start Point during different Time-Limits

(b) Probability of Correct Choice vs Start Point and Time-Limit

Figure 7: Probability of Right Choice vs Start Point vs Time-Limit



(a) Bias = -0.2

(b) Bias = -0.1

(c) Bias =0

(d) Bias = 0.1

(e) Bias = 0.2

Figure 8: Probability of Right Choice vs Start Point vs Time-Limit vs Bias

□ Q06 and Q07

I aim to calculate distribution of the reaction time (when $x(t)$ reaches α as a threshold), so:

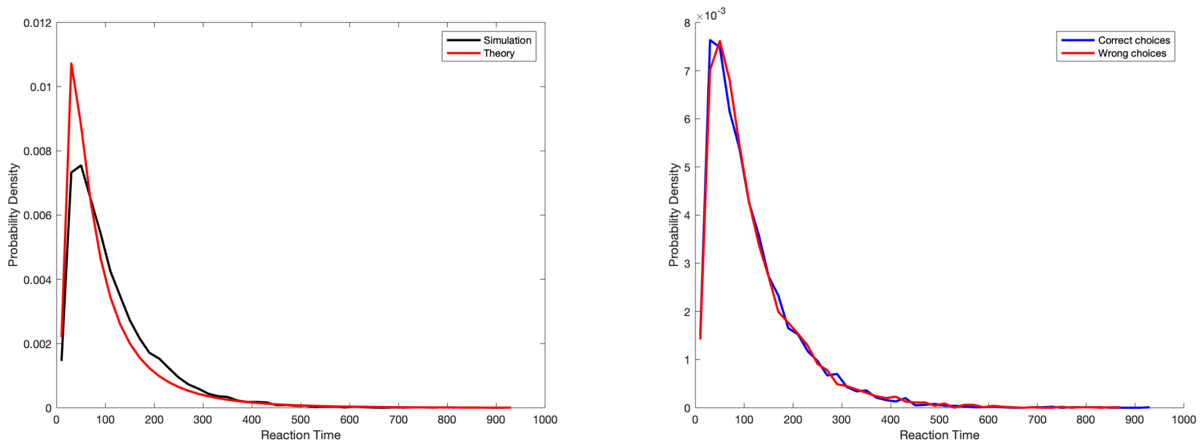
$$T_\alpha = inf\{t > 0 | X_t = \alpha\}$$

Where:

$$dX = Bdt + \sigma dW$$
$$X(t) \hookrightarrow \mathcal{N}(Bt, \sigma t)$$

So:

$$T_\alpha = IG(\frac{\alpha}{B}, (\frac{\alpha}{\sigma})^2)$$



(a) All CHoices (b) Correct and Wrong Choices

Figure 9: PDF of Reaction Times

☐ Q08

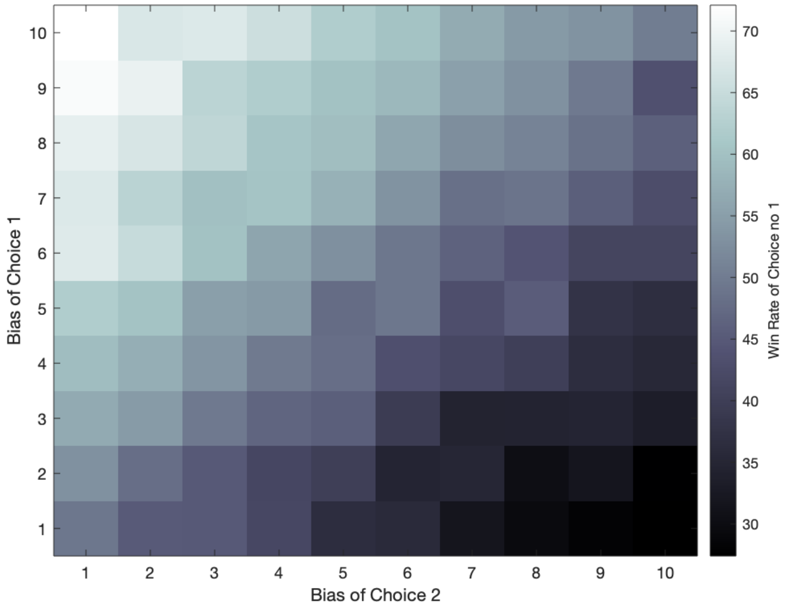


Figure 10: Win Rate of Choice 1 vs Bias of Choice 1 and Bias of Choice 2

☐ Q09

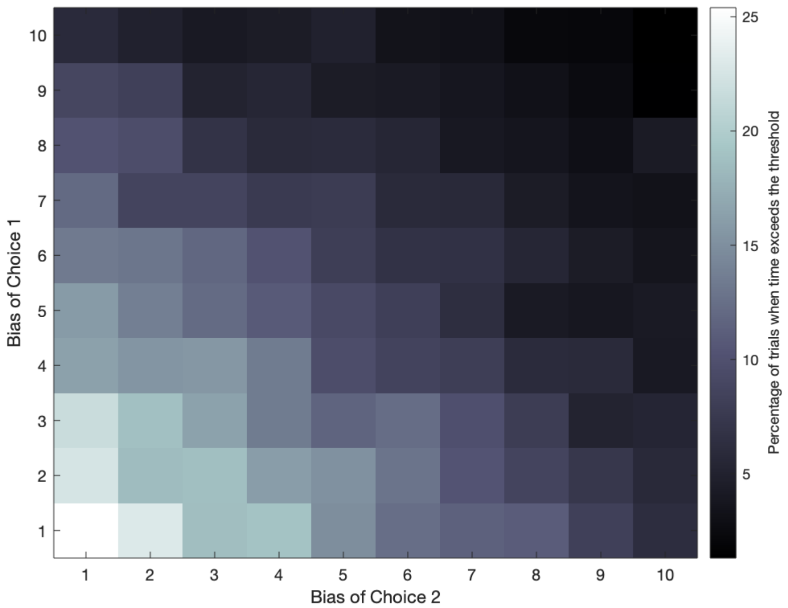


Figure 11: Percentage of Trials when Time Exceeds Time-Limit vs Bias of Choice 1 and Bias of Choice 2

■ Part2 - Simulation of the interaction between area MT and LIP

□ Q01

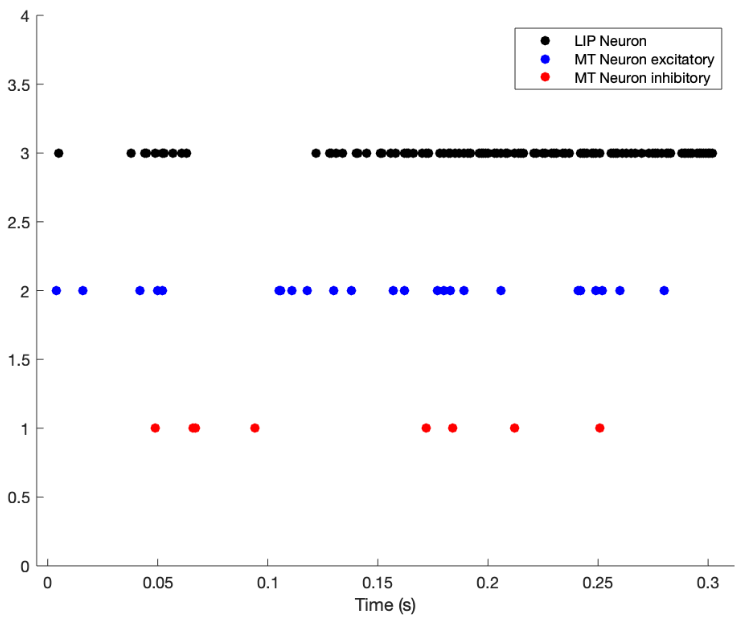


Figure 12: Simulation of two Area MT Neurons and one LIP Neuron

□ Q02

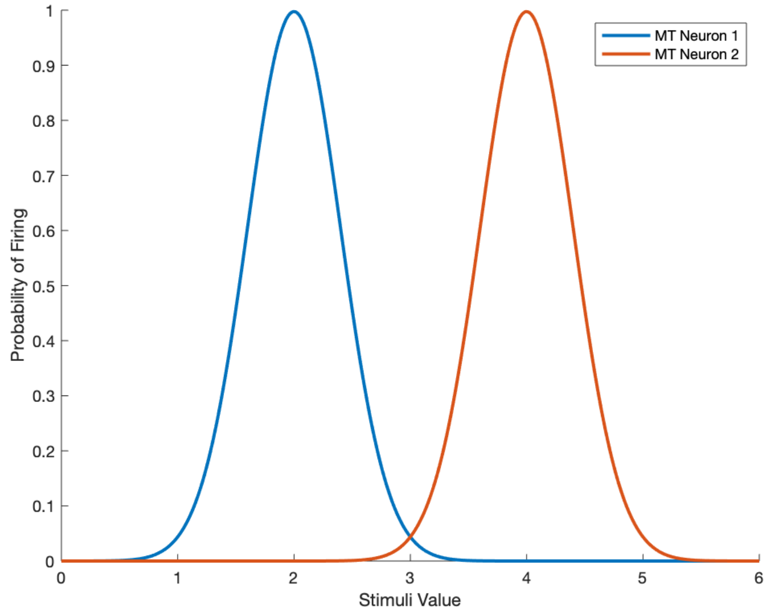
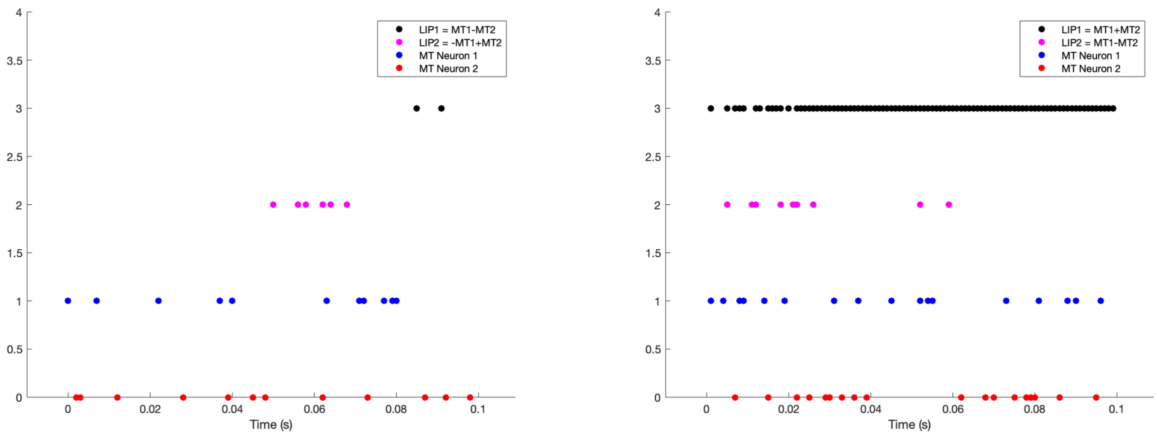


Figure 13: Tuning Curves of Area MT Neurons



(a) LIP1 = MT1-MT2, LIP2 = -MT1+MT2

(b) QQPlot

Figure 14: LIP1 = MT1+MT2, LIP2 = MT1-MT2