In The Name Of God

HW07

Advanced Neuroscience

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■ Part1 - Simulation of evidence accumulation

□ Q01 and Q02

$$dX = Bdt + \sigma dW$$
$$\int_0^t \frac{dX}{dt} = \int_0^t Bdt + \int_0^t \sigma \frac{dW}{dt}$$
$$X(t) - X(0) = Bt + \int_0^t \sigma \frac{dW}{dt}$$
$$X(t) = Bt + \sigma \int_0^t \frac{dW}{dt}$$

So, Expected value of X is:

$$E[X] = E[Bt] + E[\int_0^t \frac{dW}{dt}]$$
$$E[X] = Bt + \sigma \int_0^t E[dW]$$
$$E[X] = Bt + \sigma \times 0$$
$$E[X] = Bt$$

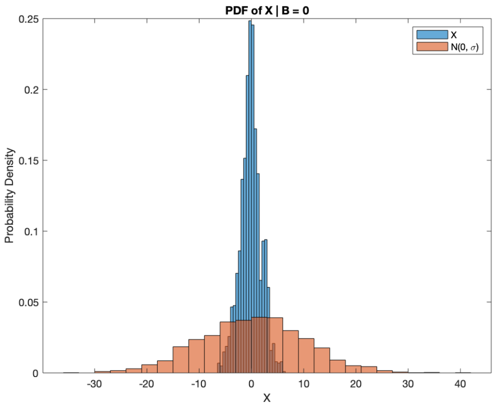
and Variance of the X is:

$$Var(X) = Var(Bt + \int_0^t \frac{dW}{dt})$$
$$Var(X) = Var(\sigma \int_0^t \frac{dW}{dt})$$
$$Var(X) = \sigma \int_0^t Var(\frac{d}{W})$$
$$Var(X) = \sigma t$$

Therefore:

$$X(t) \hookrightarrow \mathcal{N}(Bt, \sigma t)$$

Bias = 0

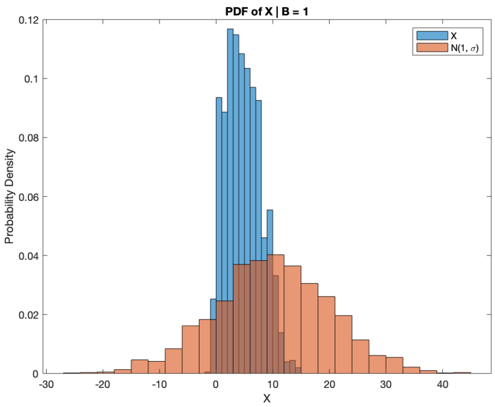


(a) PDF of X(t)

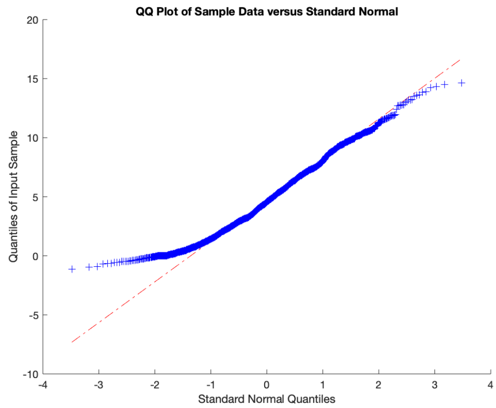
(b) QQPlot

Figure 1: The PDF of X(t) and QQ-Plot for checking the normality of the data when bias = 0

Bias = 1



(a) PDF of X(t)



(b) QQPlot

Figure 2: The PDF of X(t) and QQ-Plot for checking the normality of the data when bias = 1

As calculated in the last part and as can be seen in Figures 1 and 8, the distribution of the $X(t)$ is Normal. QQ-Plots better show that the distribution of the $X(t)$ is Normal.

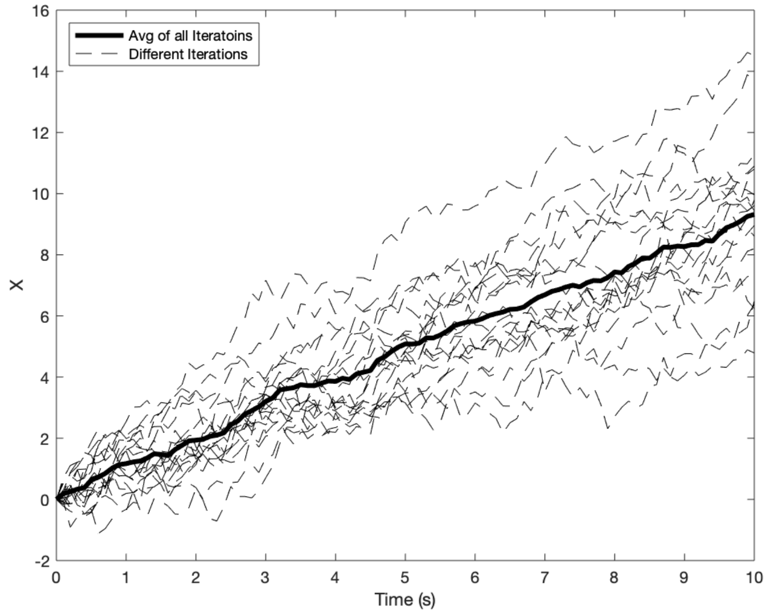


Figure 3: X(t) during different runs

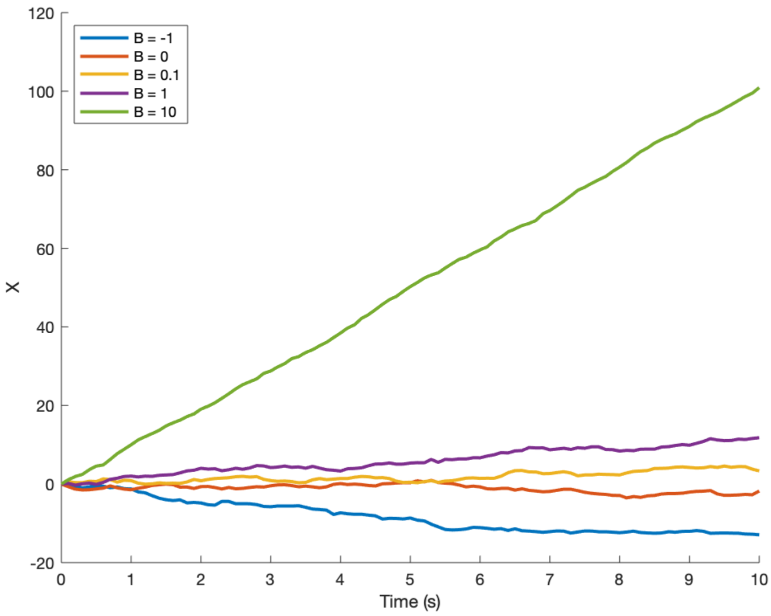


Figure 4: X(t) with different biases

Figures 3 and 3 verify the calculated Expected-Value and Variance of the $X(t)$. As can be see in these Figures, the Expected-Value and Variance of the $X(t)$ increases with time.

□ Q03

$$Error = 1 - P(choice = 1|X(t) \hookrightarrow \mathcal{N}(Bt,\, \sigma t))$$
$$Error = 1 - P(|X(t)| = 1|X(t) \hookrightarrow \mathcal{N}(Bt,\, \sigma t))$$

Because the mean of this normal distribution is positive, to calculate the above probability I used the formula below :

$$Error = 1 - \frac{min(8sigma, \mu + 4sigma)}{8sigma}$$

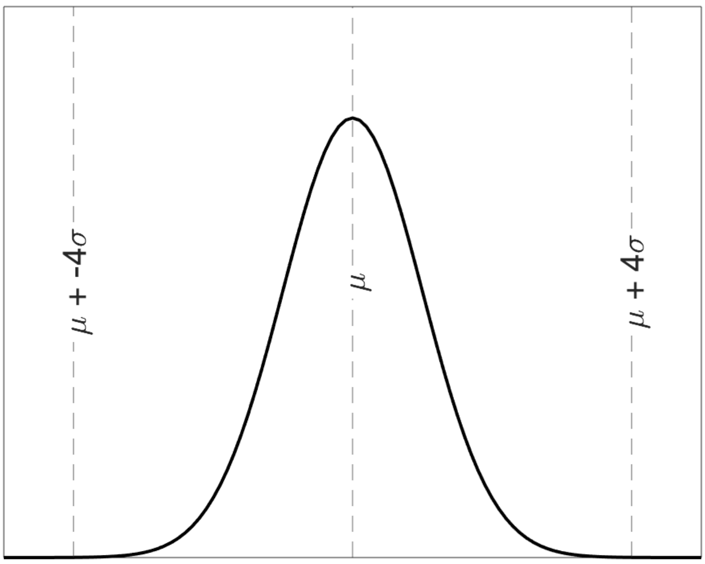
Where:

$$\mu = Bt$$
$$sigma = \sigma t$$

So:

$$Error(t) = 1 - \frac{min(8\sigma t, Bt + 4\sigma t)}{8\sigma t}$$

The theory of $\frac{min(8\sigma t, Bt+4\sigma t)}{8\sigma t}$ is that in a normal distribution is visible in the following Figure.



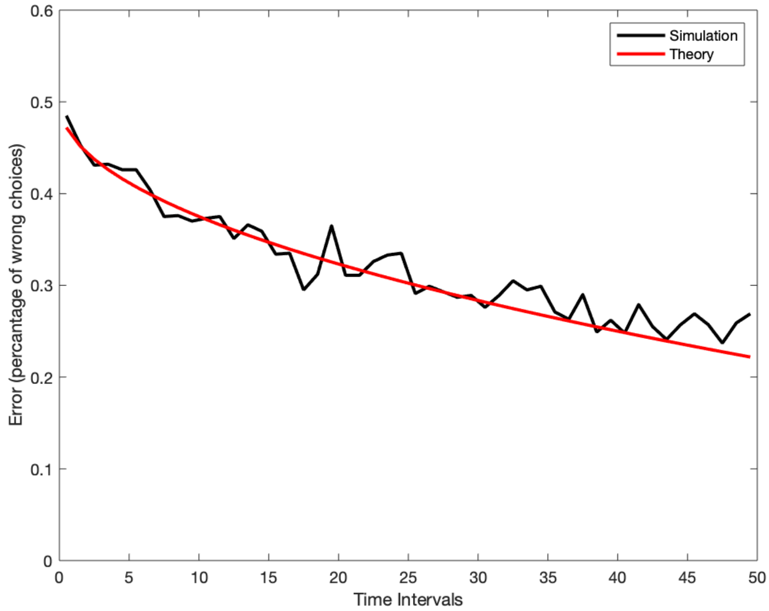


Figure 5: Stimulation and Theory of Choice Error Rate vs Time Limit

As I expect and as calculated in the last page, Error increases when Time-Limit increases. This is show in the Figure 5.

☐ Q04

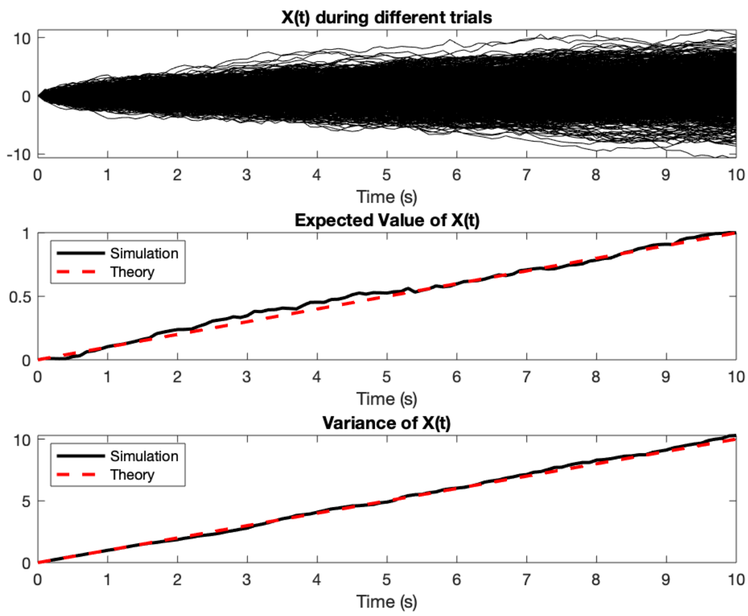
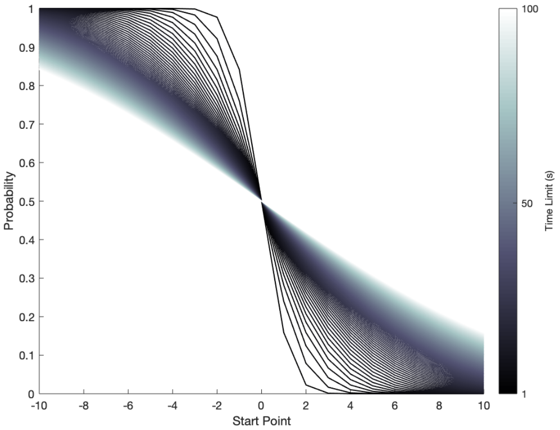


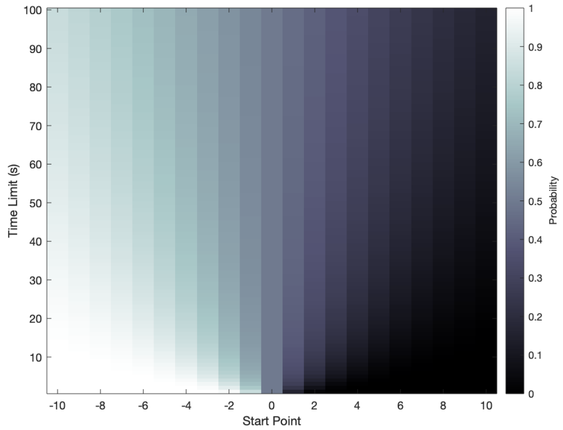
Figure 6: X(t), Expected Value, and Variance of the X(t) vs Time

As can be seen in Figure 6, the Expected-Value and Variance of the $X(t)$ increases with time which Consistent with our theoretical calculations in Q01.

☐ Q05

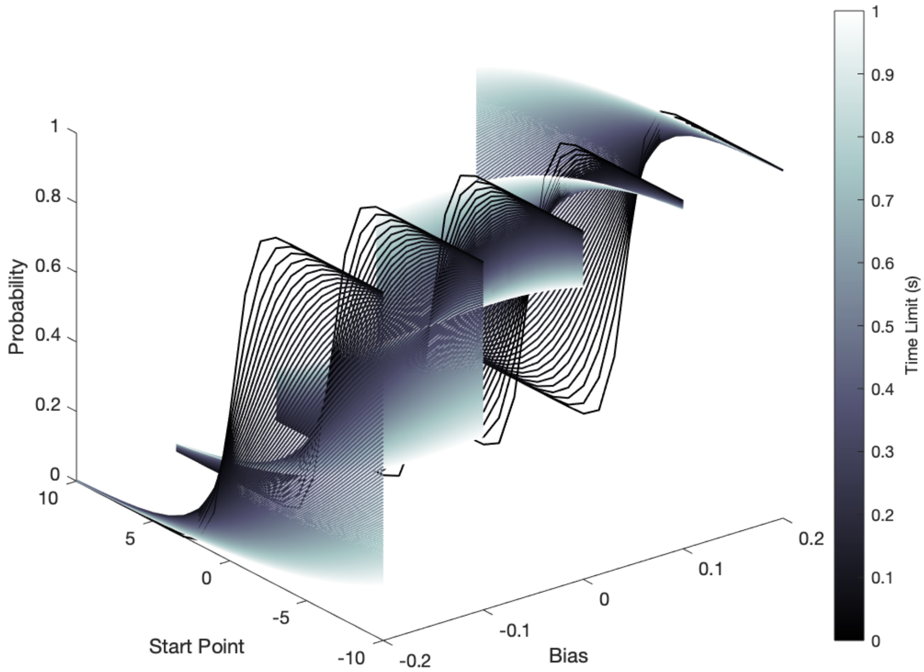


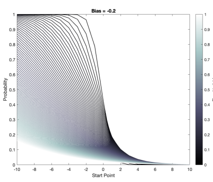
(a) Probability of Correct Choice vs Start Point during different Time-Limits



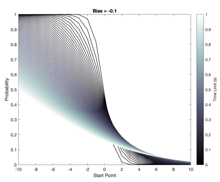
(b) Probability of Correct Choice vs Start Point and Time-Limit

Figure 7: Probability of Right Choice vs Start Point vs Time-Limit When Bias = 0

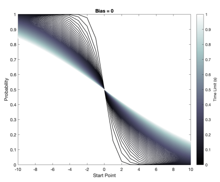




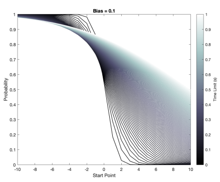
(a) Bias = -0.2



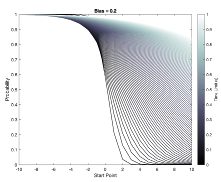
(b) Bias = -0.1



(c) Bias =0



(d) Bias = 0.1



(e) Bias = 0.2

Figure 8: Probability of Right Choice vs Start Point vs Time-Limit vs Bias

☐ Q06 and Q07

I aim to calculate distribution of the reaction time (when $x(t)$ reaches α as a threshold), so:

$$T_\alpha = inf\{t > 0 | X_t = \alpha\}$$

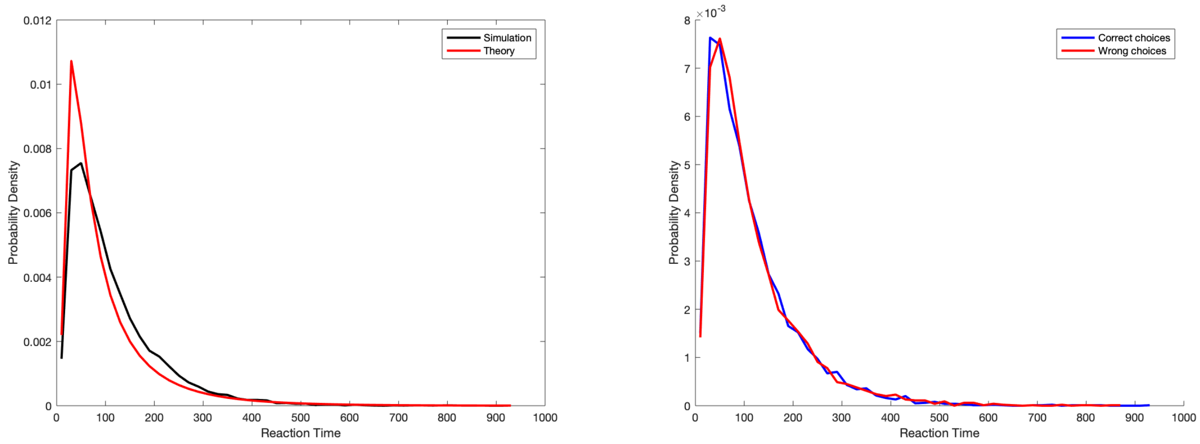
Where:

$$dX = Bdt + \sigma dW$$
$$X(t) \hookrightarrow \mathcal{N}(Bt, \sigma t)$$

So:

$$T_\alpha = IG(\frac{\alpha}{B}, (\frac{\alpha}{\sigma})^2)$$

Where IG is an inverse gaussian distribution.



(a) All CHoices

(b) Correct and Wrong Choices

Figure 9: PDF of Reaction Times

As can be seen in Figure 9, the simulated result is consistent with my theoretical calculations.

☐ Q08

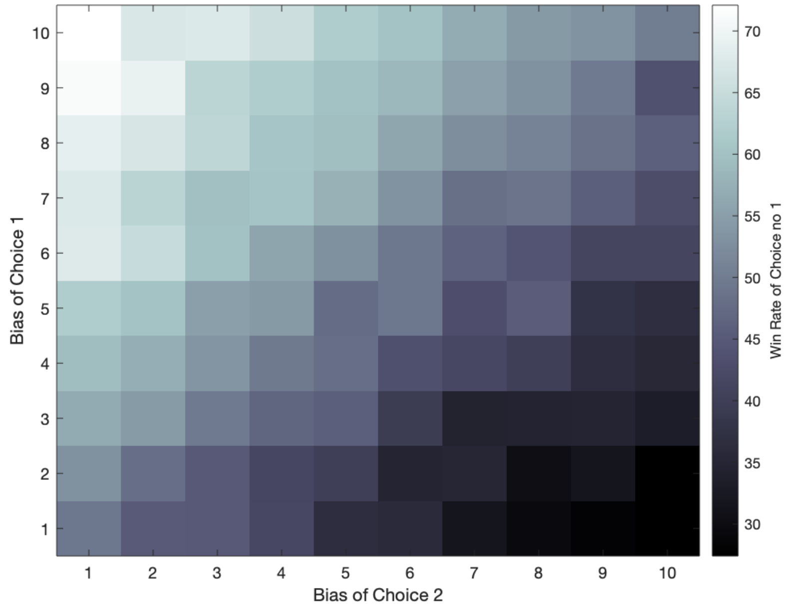


Figure 10: Win Rate of Choice 1 vs Bias of Choice 1 and Bias of Choice 2

As can be seen in Figure 10, the win rate of choice 1 is bigger when bias of choice 1 is bigger than bias of choice 2 and it makes sense, because if bias of a choice is way bigger than bias of the other choice, it will reach the threshold faster and will win the race.

☐ Q09

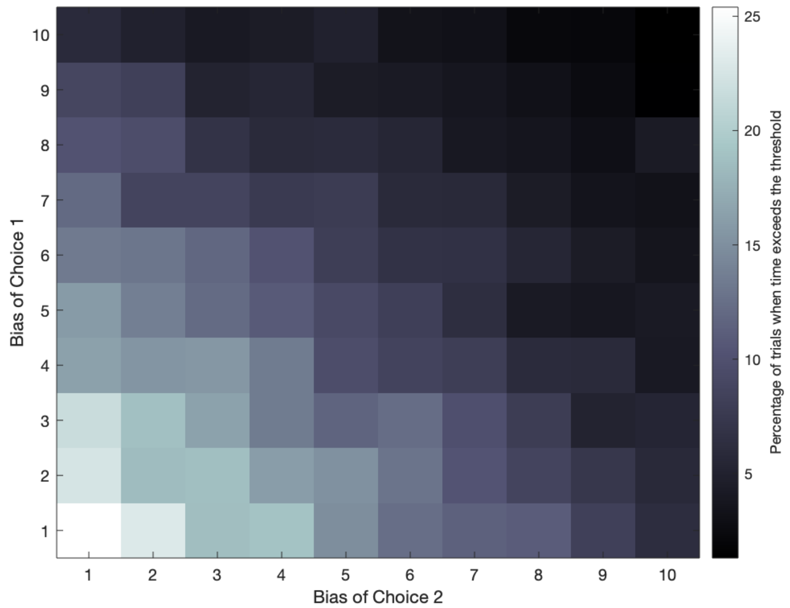


Figure 11: Percentage of Trials when Time Exceeds Time-Limit vs Bias of Choice 1 and Bias of Choice 2

When bias bias of the choices are small, it will take more time for each of them to reach the threshold, so for small biases more trials will exceed the time-limit and via versa for big biases.

■ Part2 - Simulation of the interaction between area MT and LIP

□ Q01

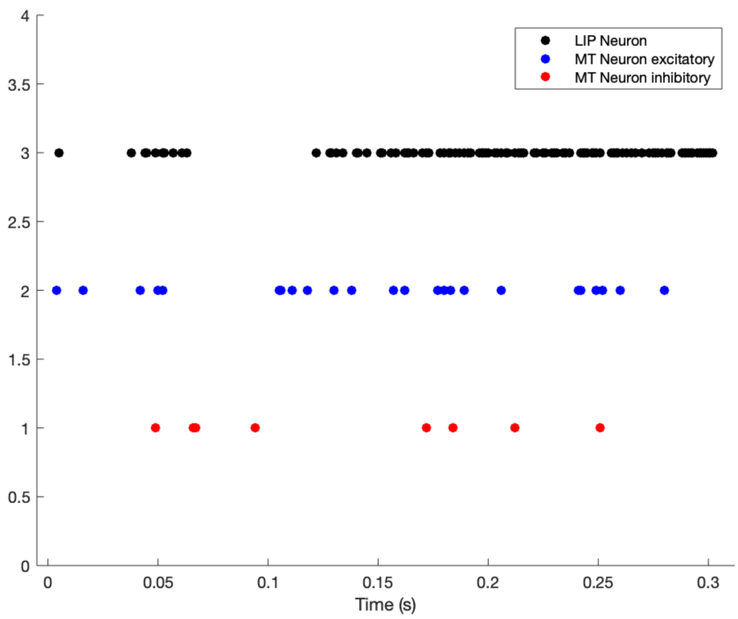


Figure 12: Simulation of two Area MT Neurons connected to one LIP Neuron

As can be seen in Figure 12, when the excitatory neuron has a high firing rate and the inhibitory neuron has low firing rate, the LIP neuron starts firing with high rate.

☐ Q02

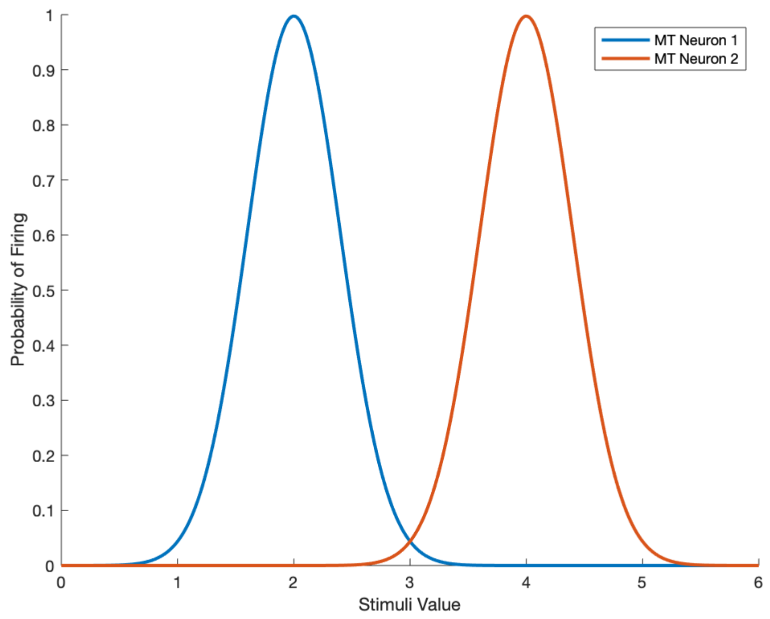
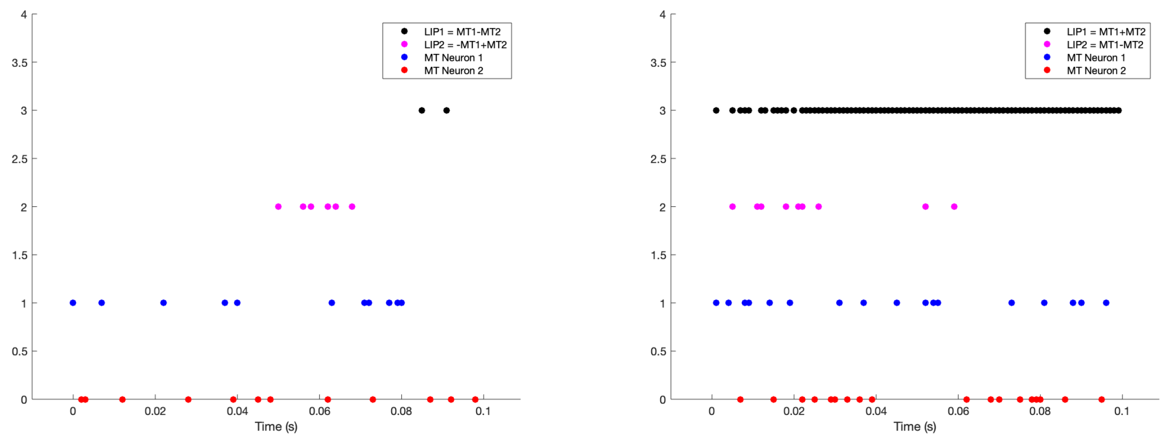


Figure 13: Tuning Curves of Area MT Neurons

Figure 13 shows tuning curve for two neuron from the area MT.



(a) LIP1 = MT1-MT2, LIP2 = -MT1+MT2 (b) LIP1 = MT1+MT2, LIP2 = MT1-MT2

Figure 14: Simulation of two Area MT Neurons connected to two LIP Neuron

As can be seen in Figure 14, the activity of the LIP neurons depend on their connection weights with area MT neuron, if the inhibitory connections have large values, the LIP neurons won't fire.